Mediation, Time and Causality: Lessons from Continuous-Time Modeling

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Overview

- Current commonly used **longitudinal models** can lead to **misleading conclusions** regarding causal relationships

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- Current commonly used **longitudinal models** can lead to **misleading conclusions** regarding causal relationships
- How time is treated determines what we interpret as **total, direct and indirect** effects

Motivating Context

- Experience Sampling Methodology (ESM) Data
  - Many **repeated measurements** - multiple times a day for many days

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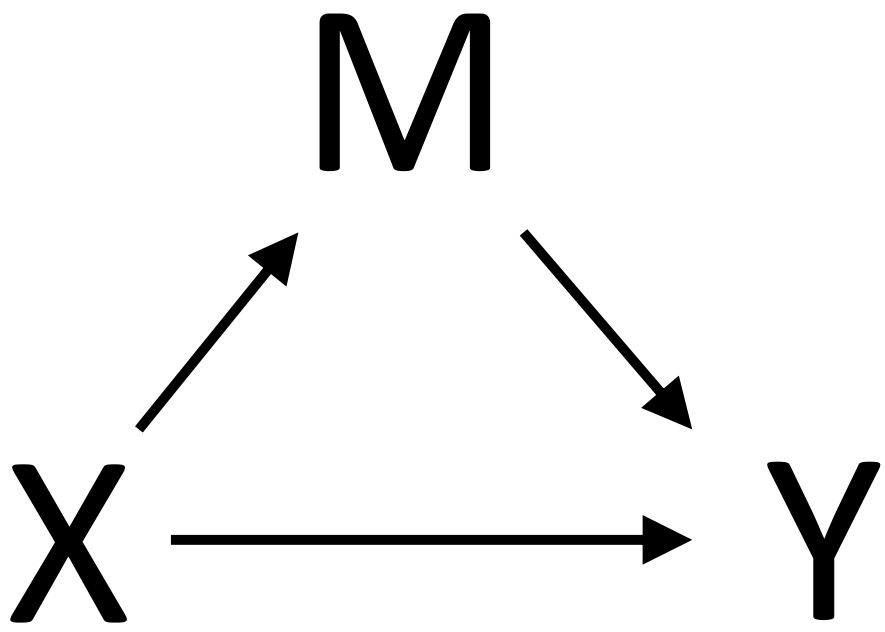
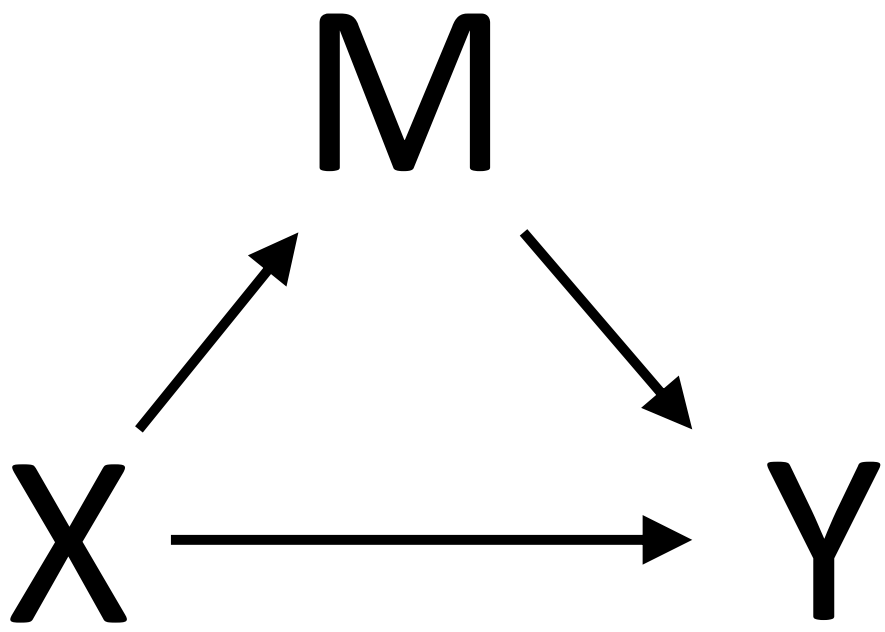
  - Many **repeated measurements** - multiple times a day for many days
- Dynamical Systems and Psychology
  - Psychopathologies as systems of **time-varying symptoms** (e.g Bringmann et al. 2013; van de Leemput et al. 2014)

Mediation



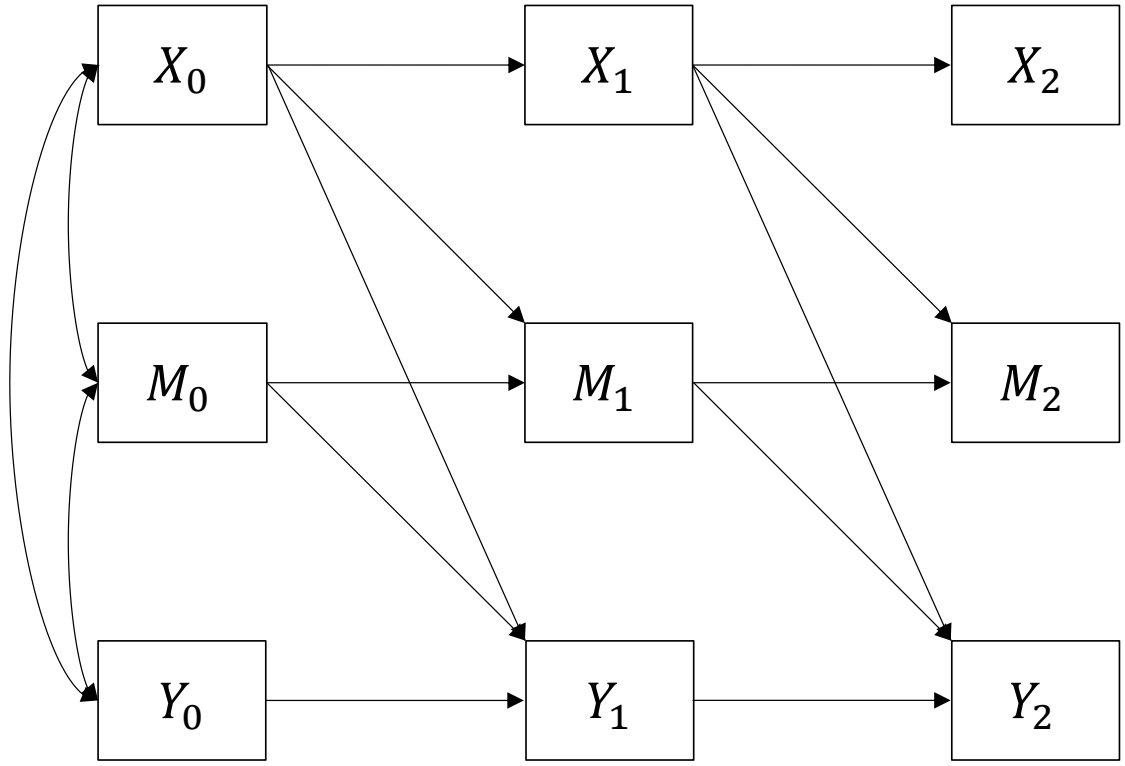
Mediation



Caveat: **Time-order** is needed!

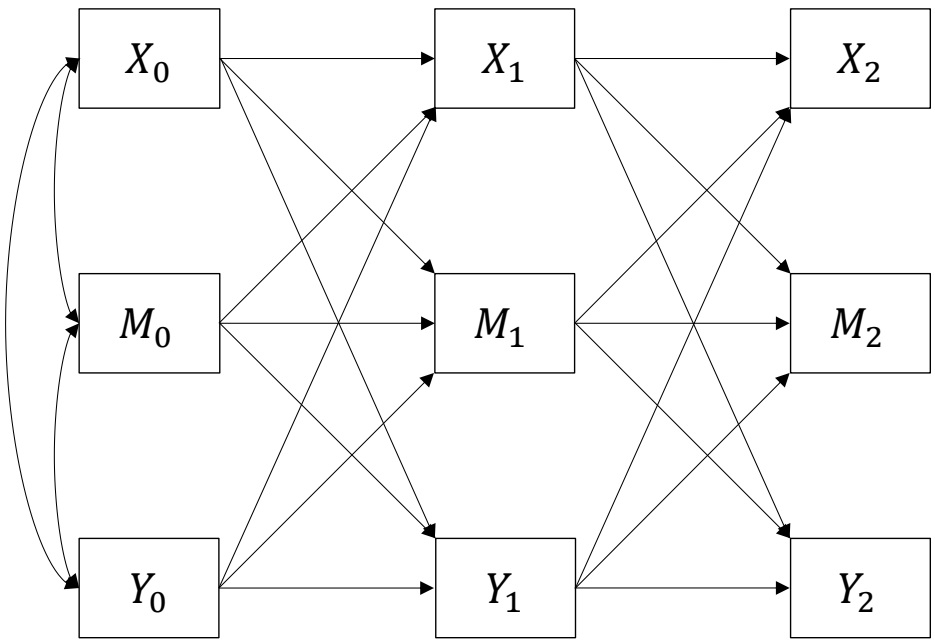
Longitudinal Mediation

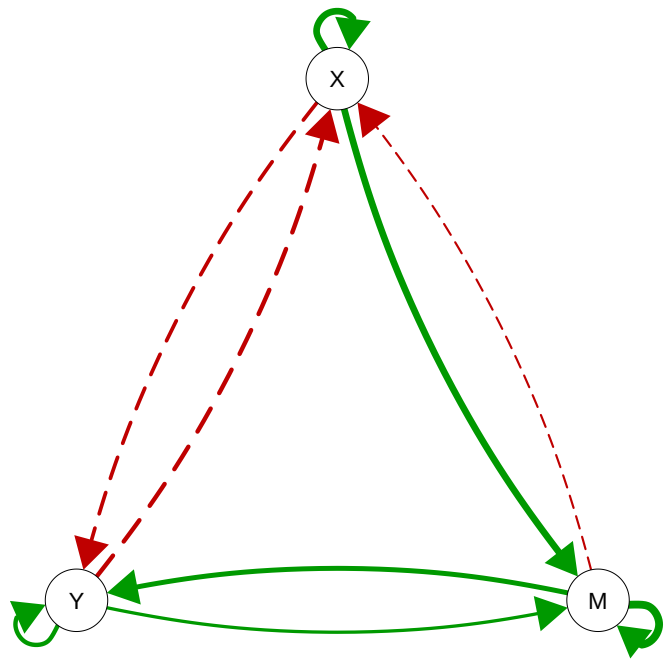
Cole & Maxwell (2003, 2007)



General VAR(1) Model

$$\boldsymbol{Z}_m = \boldsymbol{B}\boldsymbol{Z}_{m-1} + \boldsymbol{w}_m$$





VAR(1) Models and Time

The VAR(1) model is a **Discrete Time** model

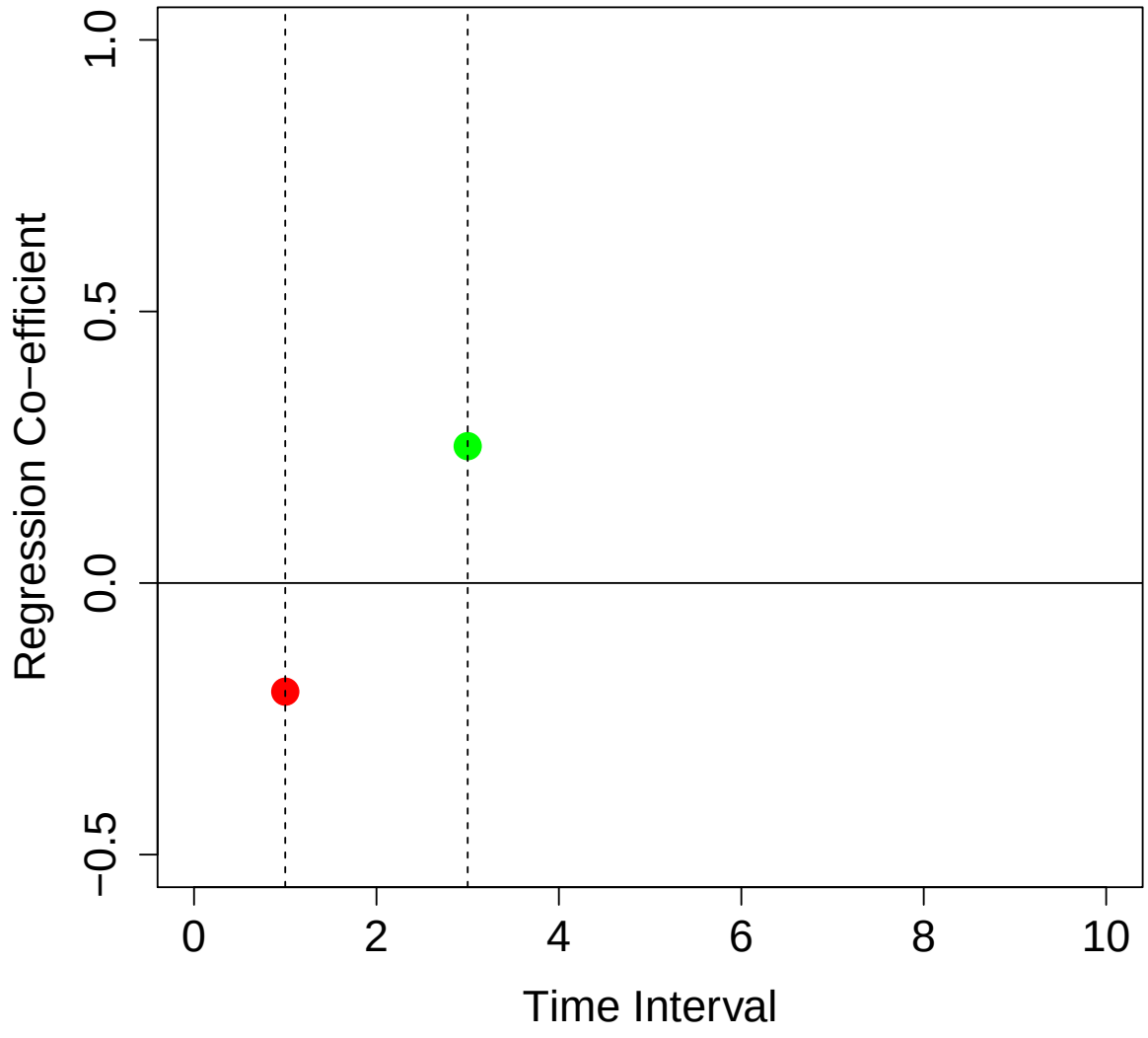
- Amount of time between measurements Δt_m ignored
- Assumes equal time-intervals between measurements

$$\boldsymbol{Z}_m = \boldsymbol{B}\boldsymbol{Z}_{m-1} + \boldsymbol{w}_m$$

$$\Leftrightarrow$$

$$\boldsymbol{Z}(t_m) = \boldsymbol{B}(\Delta t_m)\boldsymbol{Z}(t_m - \Delta t_m) + \boldsymbol{w}(\Delta t_m)$$

Time-Interval Dependency



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Problem occurs if processes continue to exist and influence one another **in-between measurement occasions**

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Alternative: Continuous Time Modeling

Continuous Time Model

Based on a First-Order Stochastic Differential Equation (e.g. Oud & Jansen, 2000; Oravecz et al., 2011)

$$\frac{d\boldsymbol{Z}(t)}{dt} = \boldsymbol{A}\boldsymbol{Z}(t) + G\frac{d\boldsymbol{W}(t)}{dt}$$

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Related to the VAR(1) model

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$$\boldsymbol{Z}(t_m) = \boldsymbol{B}(\Delta t_m)\boldsymbol{Z}(t_m - \Delta t_m) + \boldsymbol{w}(\Delta t_m)$$

Via the well-known equality

$$\boldsymbol{B}(\Delta t_m) = e^{\boldsymbol{A}\Delta t_m}$$

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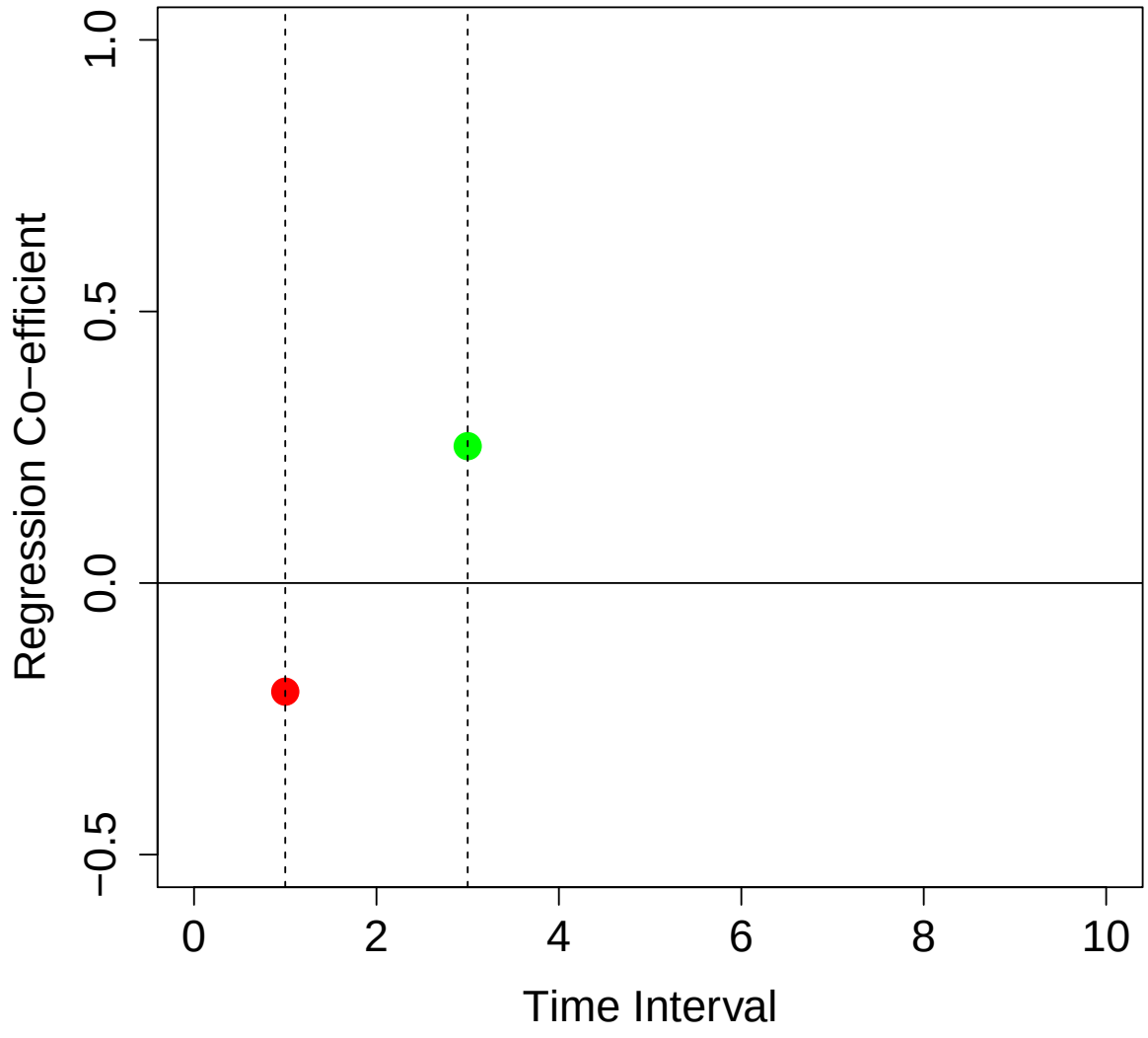

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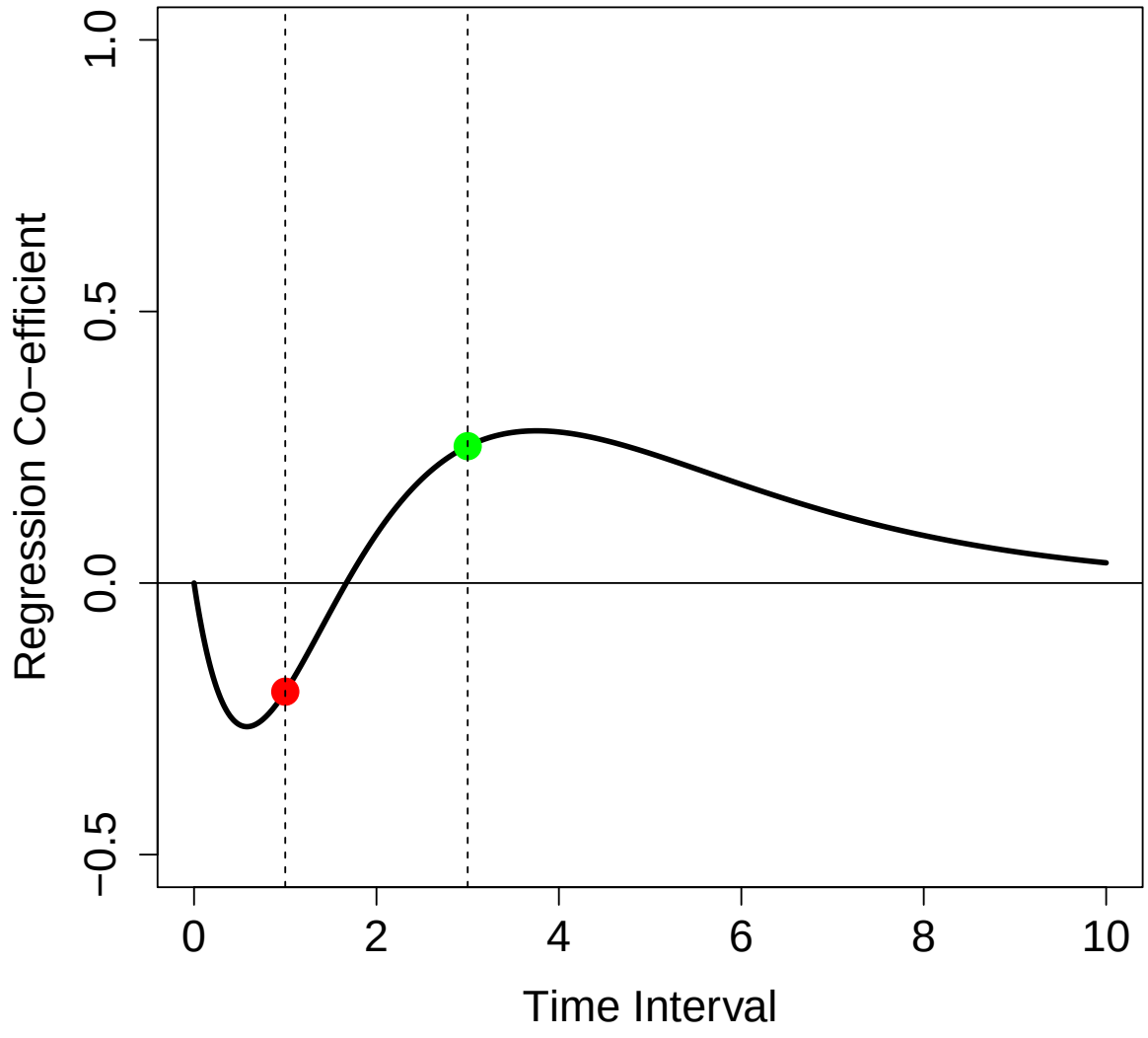

Implications for Causal Interpretation

For model parameters to be **causally meaningful** we need assumptions about the underlying processes

- Discrete Time processes evolve in discrete steps according to measurement occasions
- Continuous Time processes continue to effect each other in between and after measurement occasions

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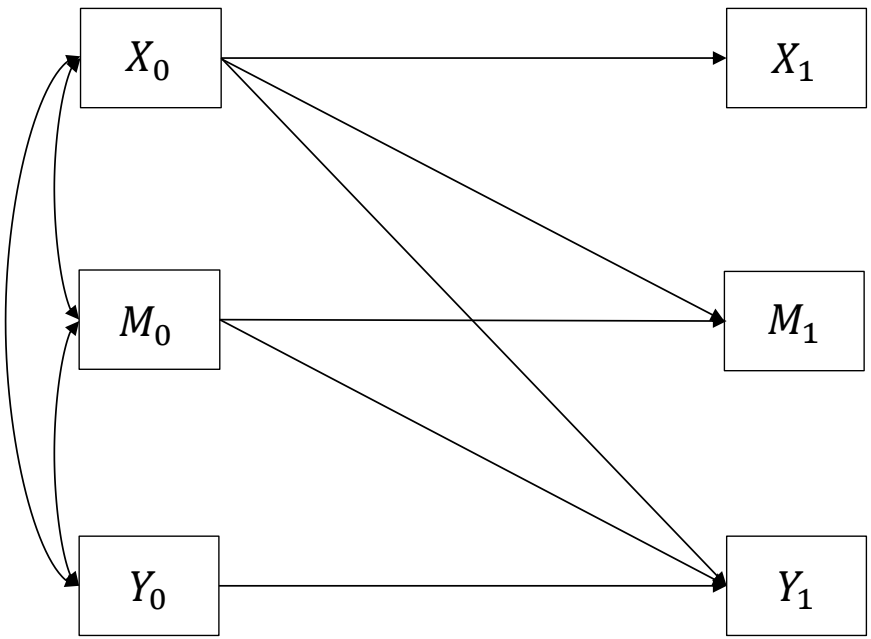
DT model **direct** effects at best represent **total** effects (Deboeck & Preacher, 2015)

Indirect, Direct and Total Effects

Discrete Time Framework



$$\boldsymbol{B}(\Delta t_m) = \begin{bmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

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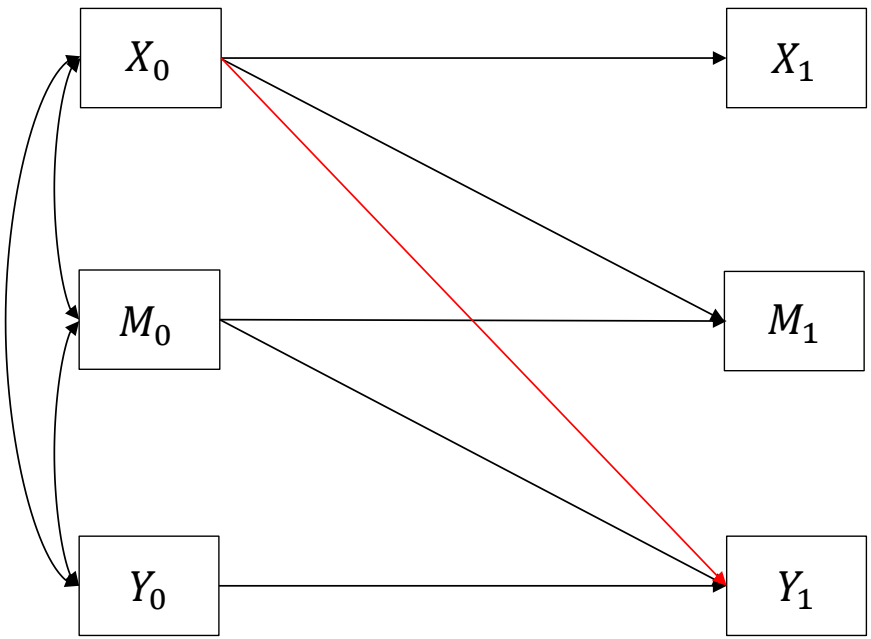

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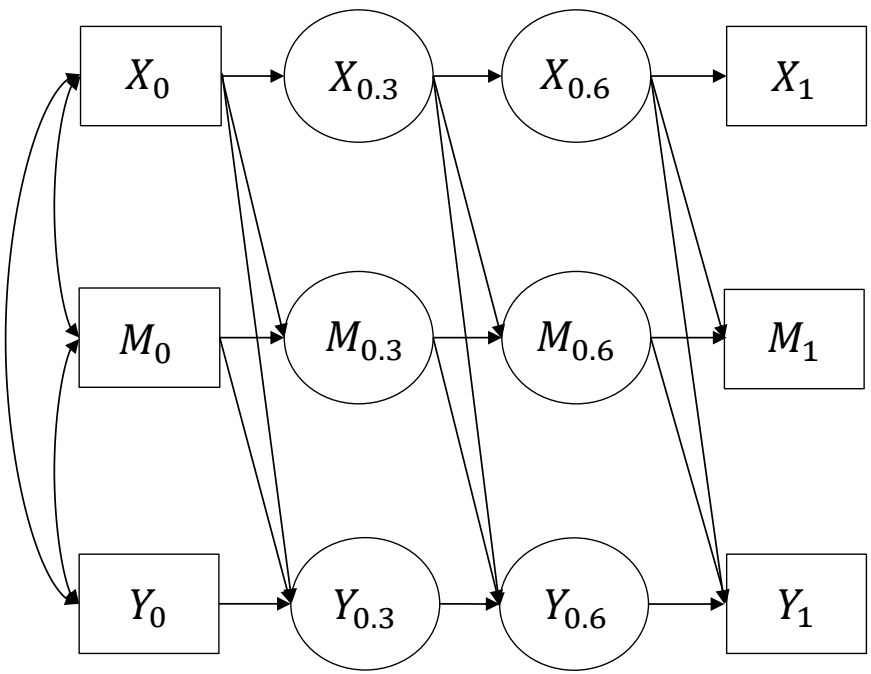

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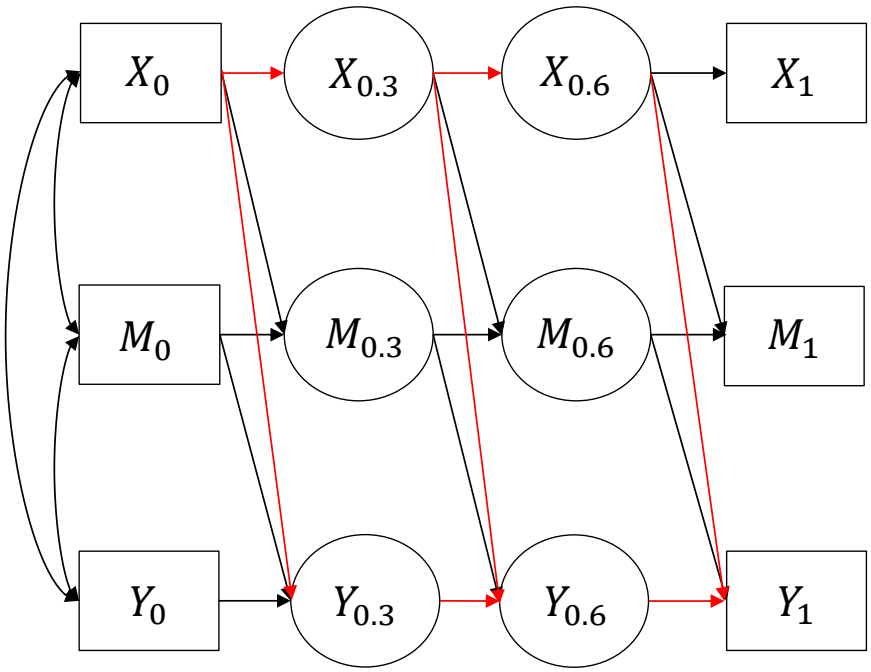

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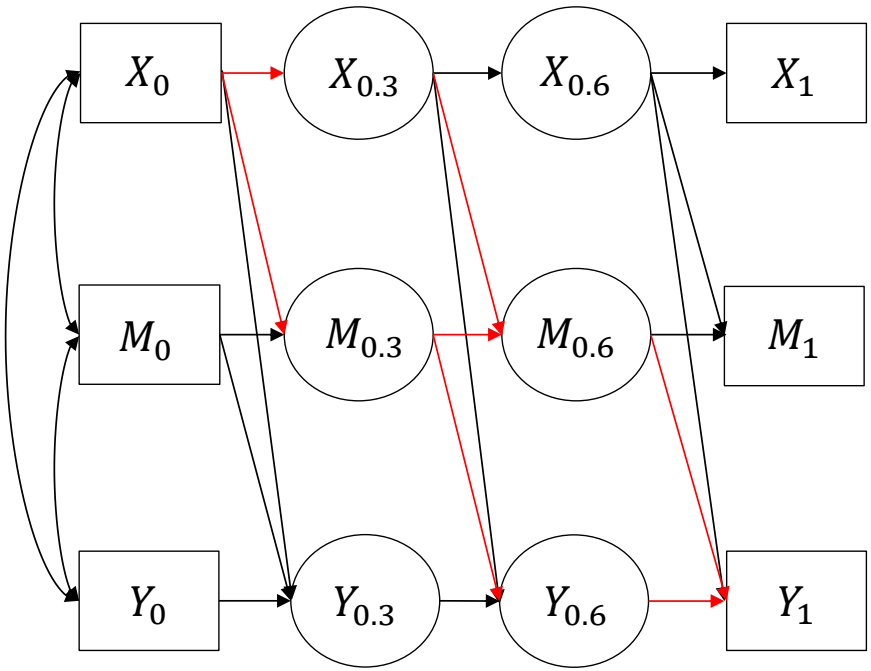

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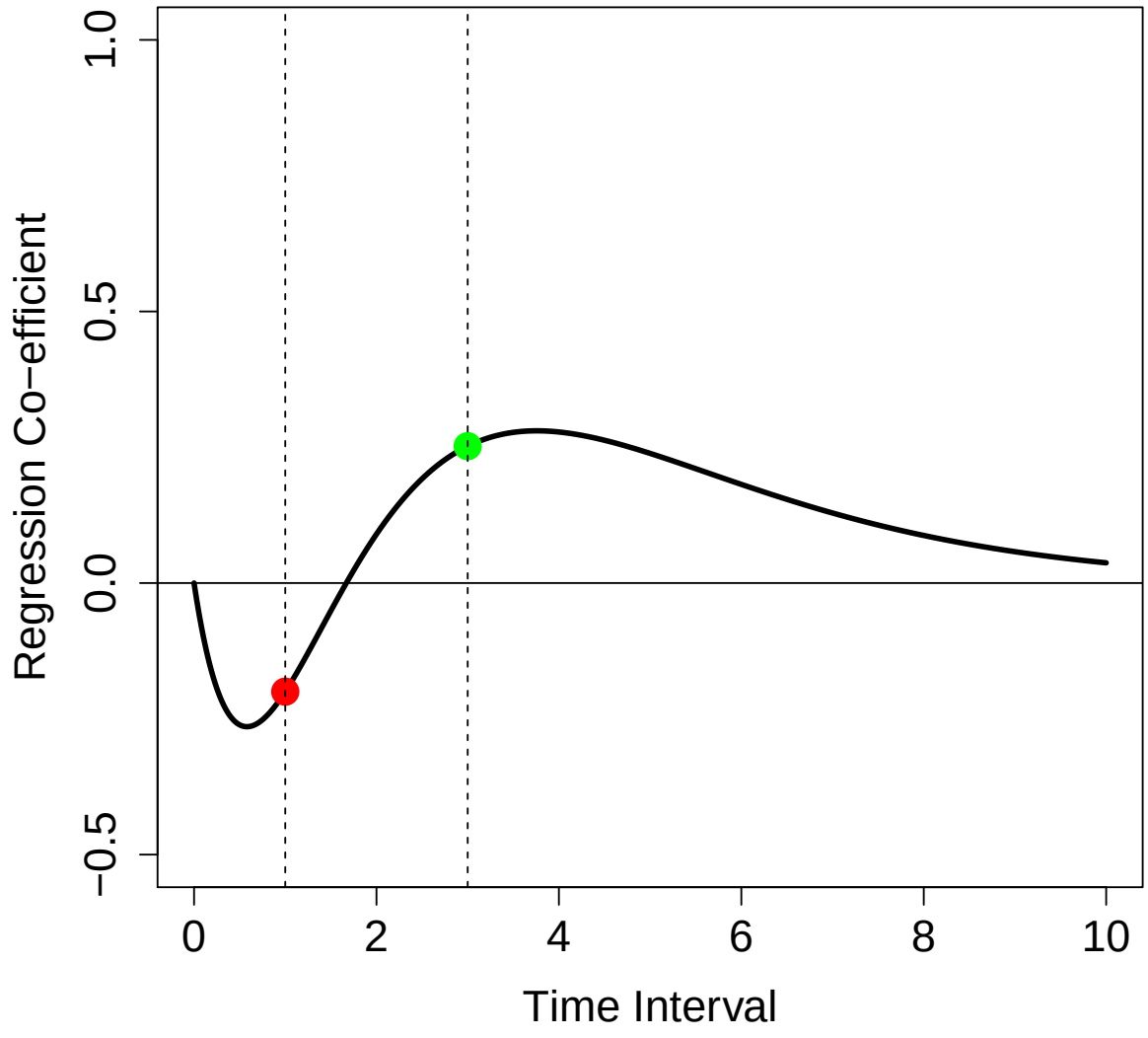

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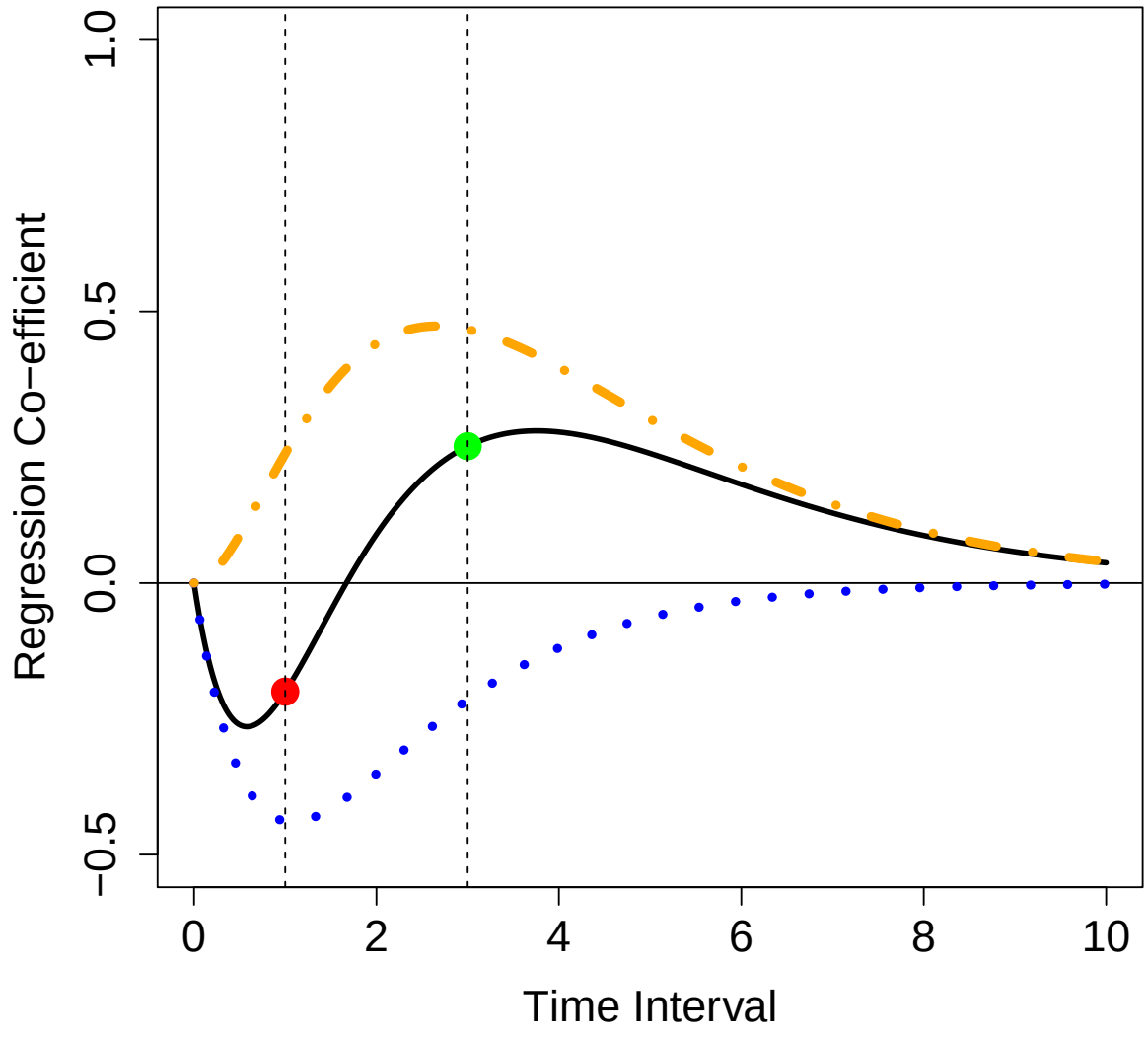
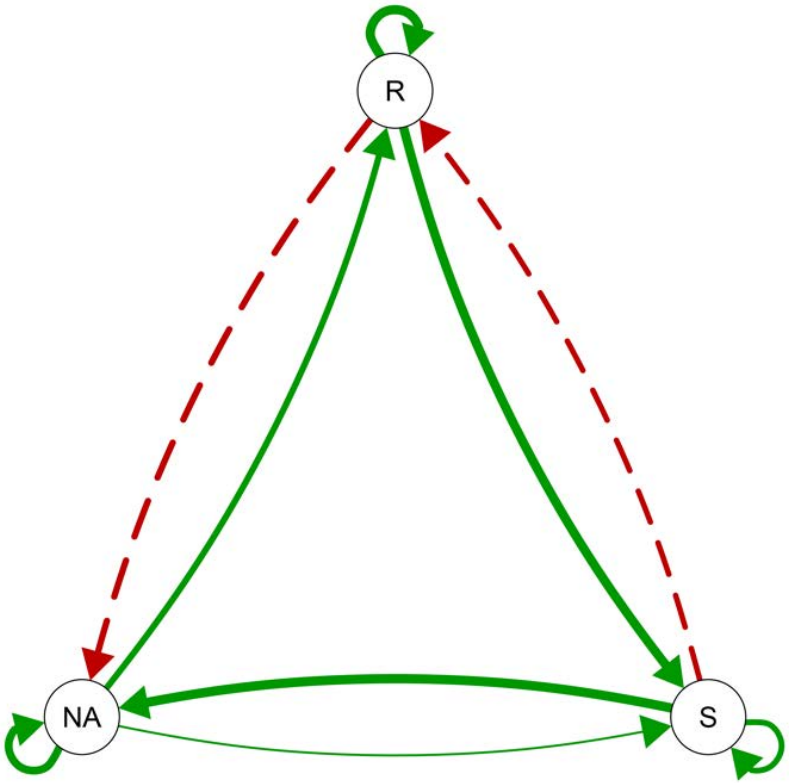
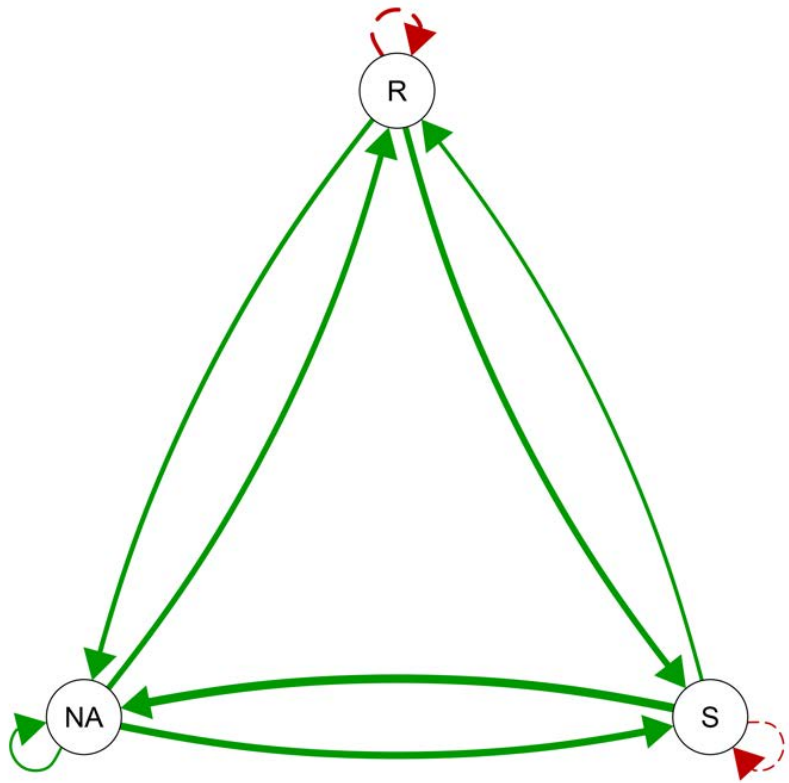


Indirect, Direct and Total Effects

Issue is exacerbated as model size increases





Conclusion

Current **Discrete-Time** longitudinal models can lead to **misleading conclusions** regarding causal relationships

Misleading estimates of **total, direct** and **indirect** effects

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Misleading estimates of **total, direct** and **indirect** effects

Processes which evolve **continuously over time**

CT models may offer a less problematic model of reality

Current Research Projects

- How can this viewpoint be linked to counterfactual causality?
- How can we condense rich CT information for ease of interpretation?
- How to define and calculate direct and indirect effects in the more general VAR(1) model?

Application to Empirical Data

- N=1 Experience Sampling Data
- Geschwind et al. (2011)
- 115 repeated measurements
  - *Perceived Unpleasantness* (PU)
  - *Worry* (W)
  - *Relaxation* (Re)

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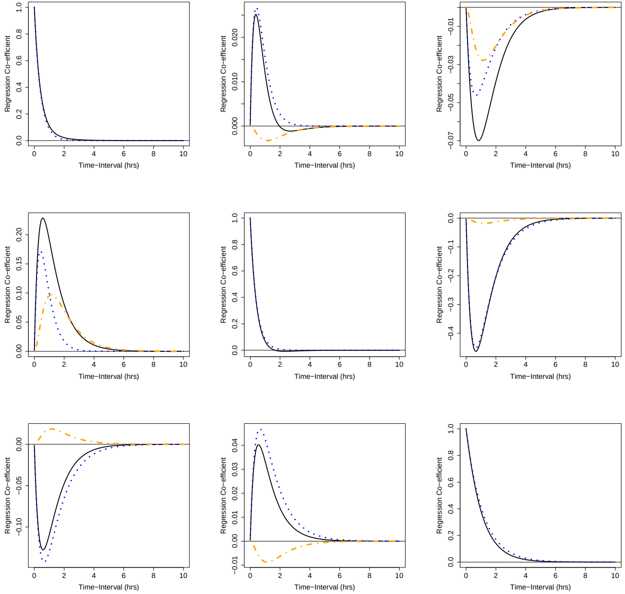
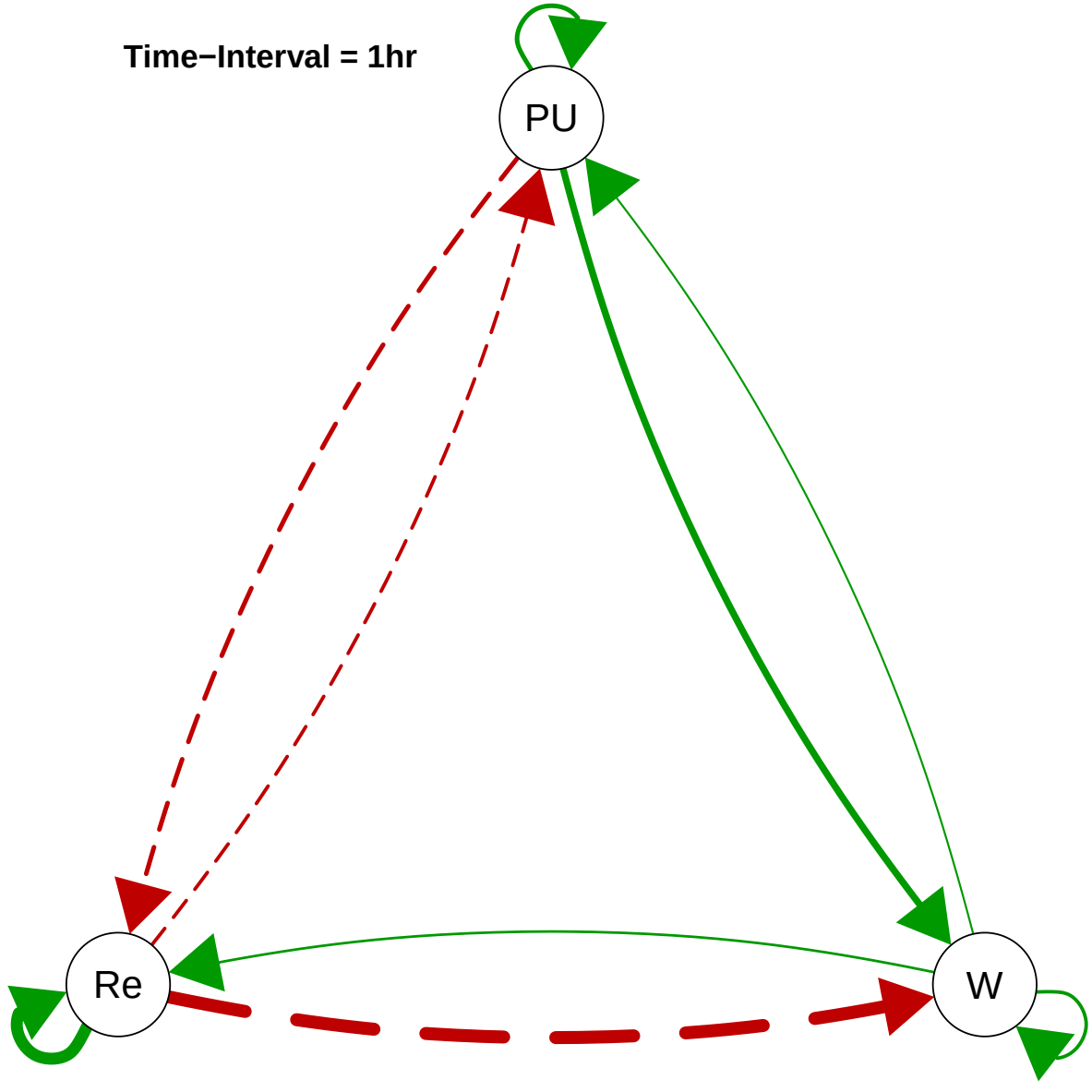
$$\boldsymbol{A} = \begin{bmatrix} -2.423 & 0.177 & -0.200 \\ 1.140 & -2.445 & -1.964 \\ -0.616 & 0.204 & -0.884 \end{bmatrix}$$

Results Empirical Data



Results - DT Network



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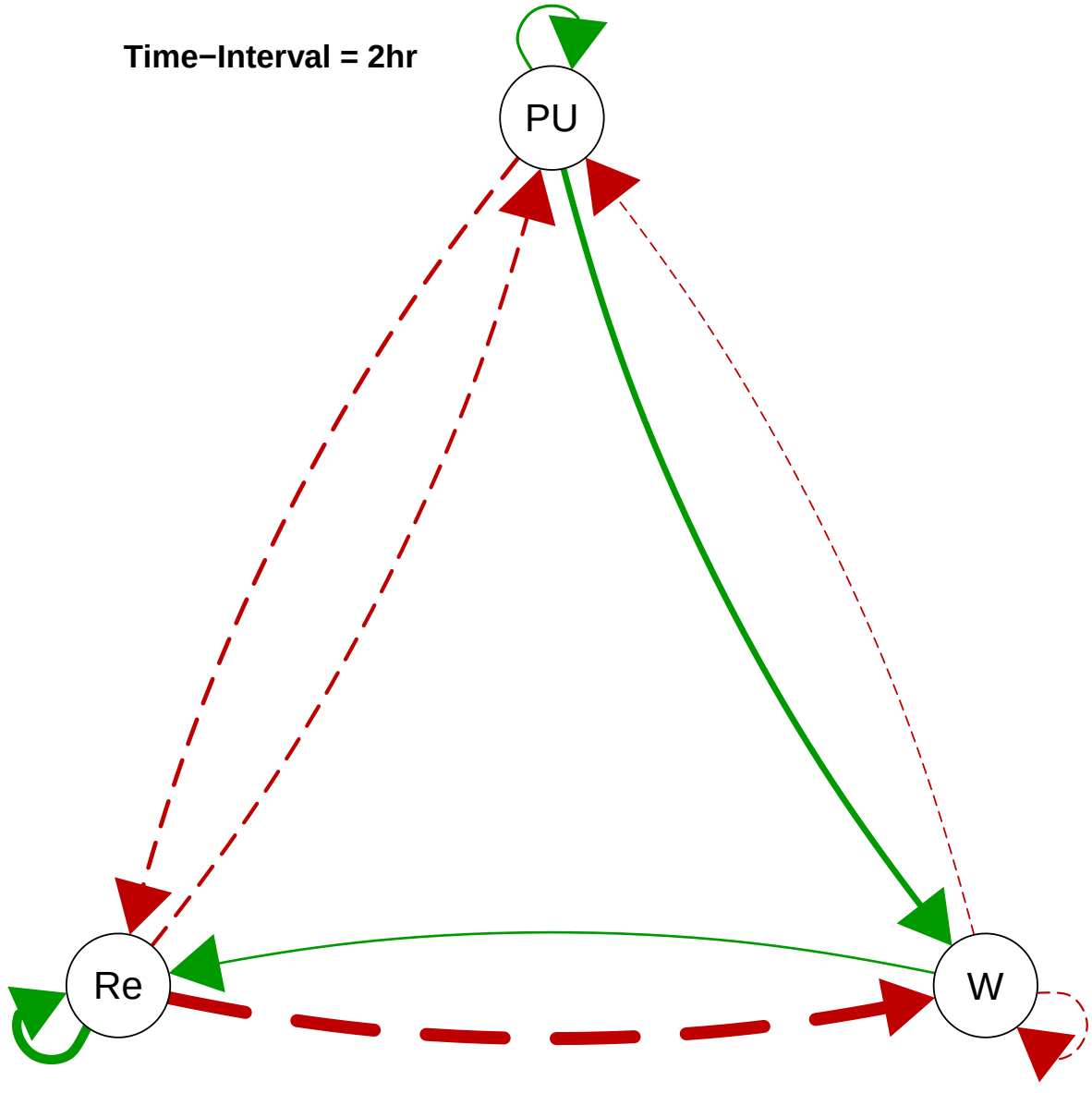
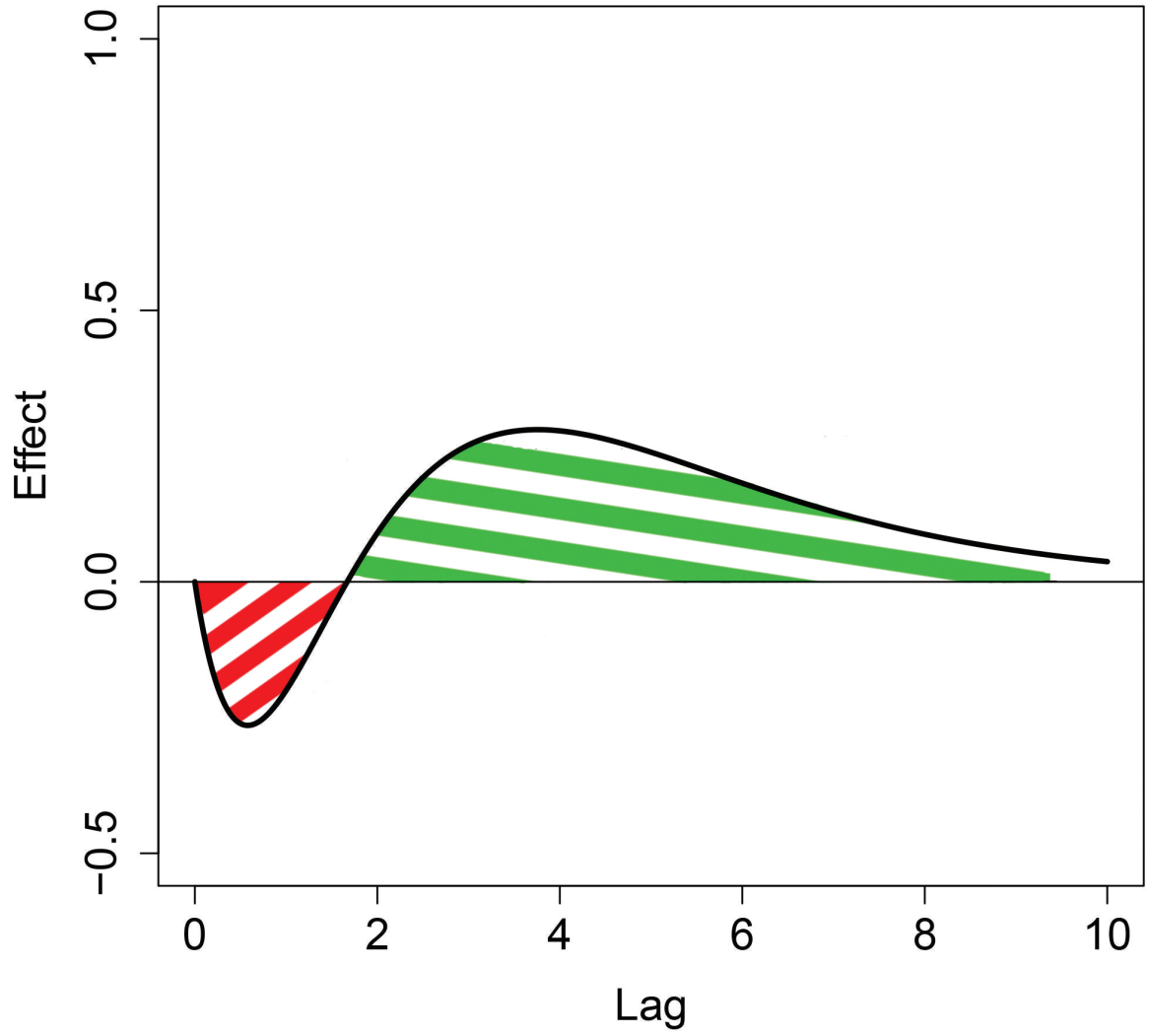


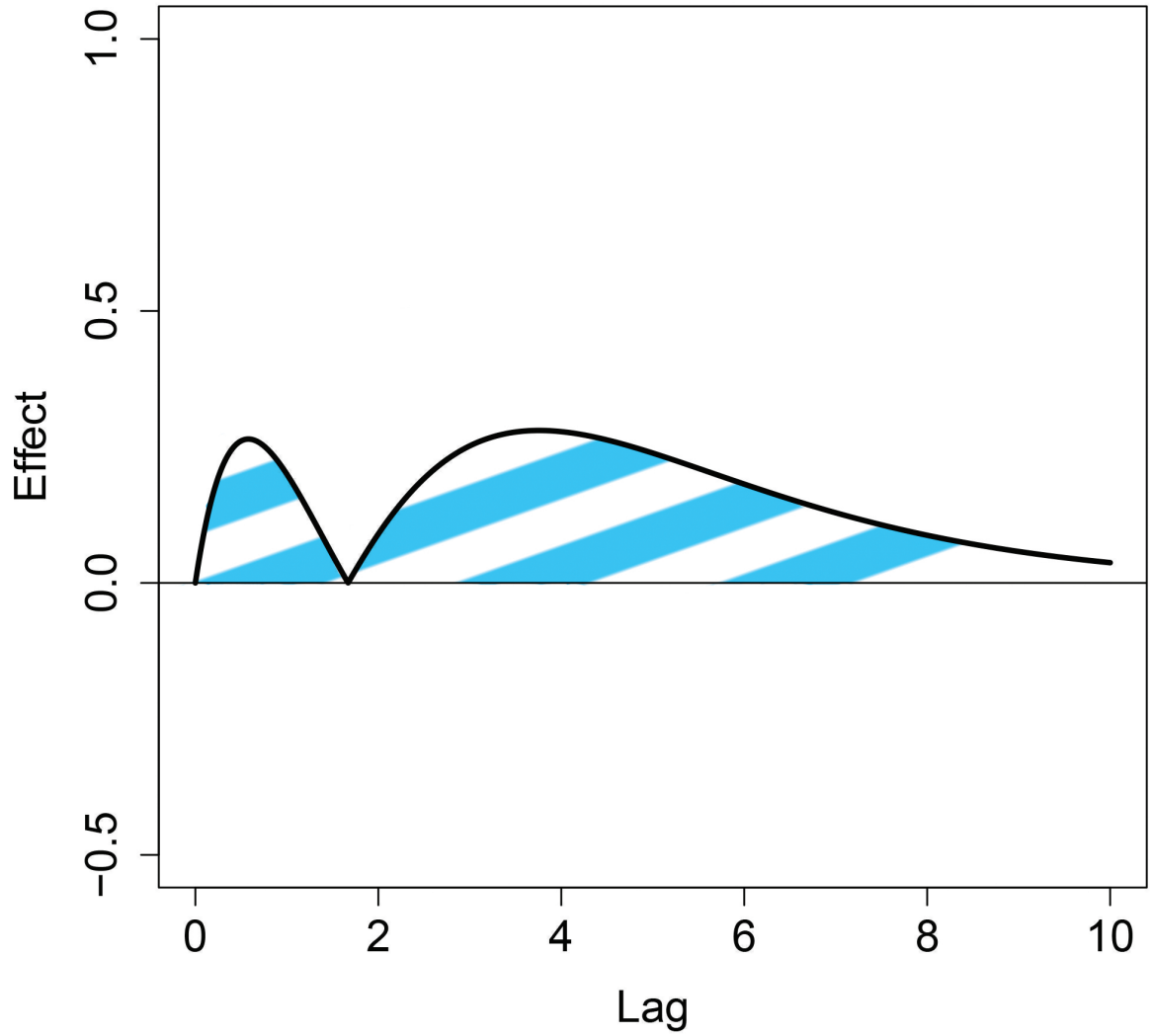
Summary Measures

1. Proportional Area Under the CT-Curve (PACT)



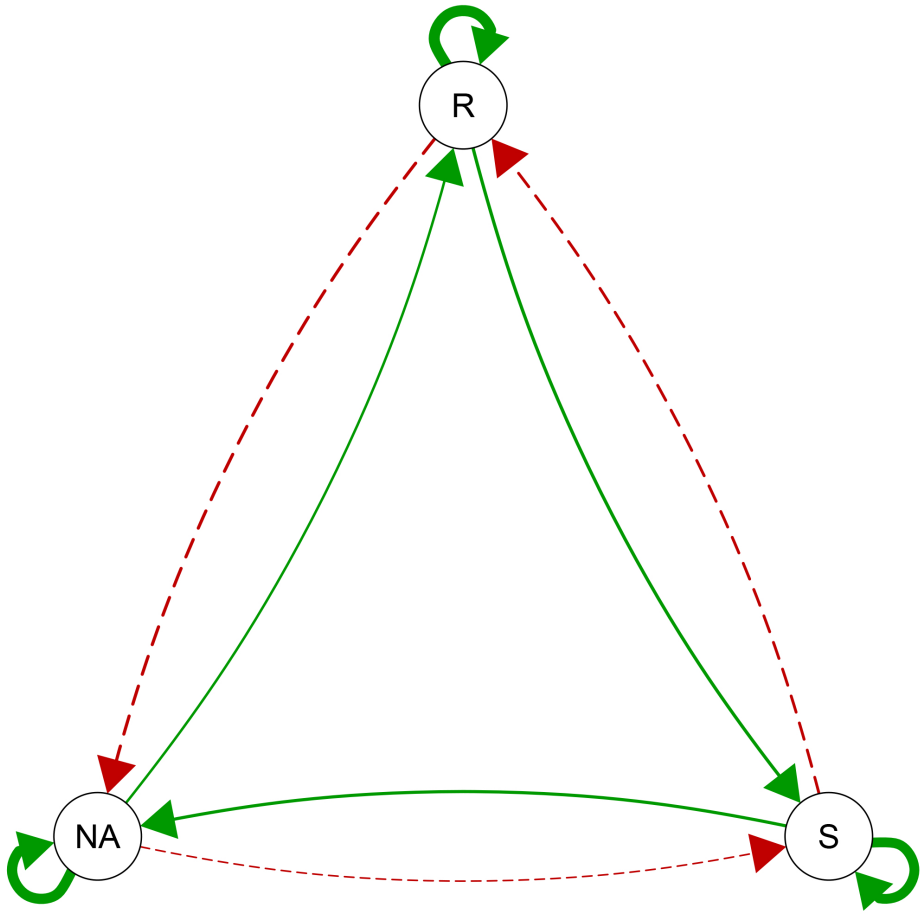
Summary Measures

2. Absolute Proportional Area Under the CT-Curve (APACT)



Summary Measures

3. Overall Flow Betweenness Centrality (ofb_w)



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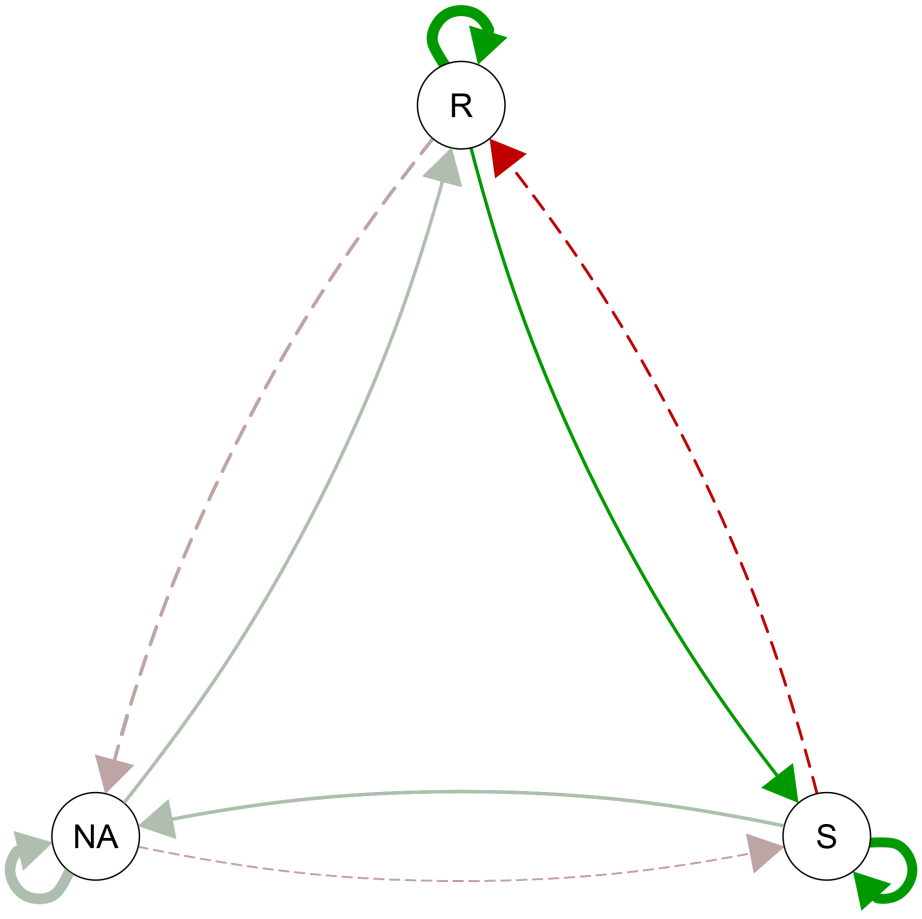


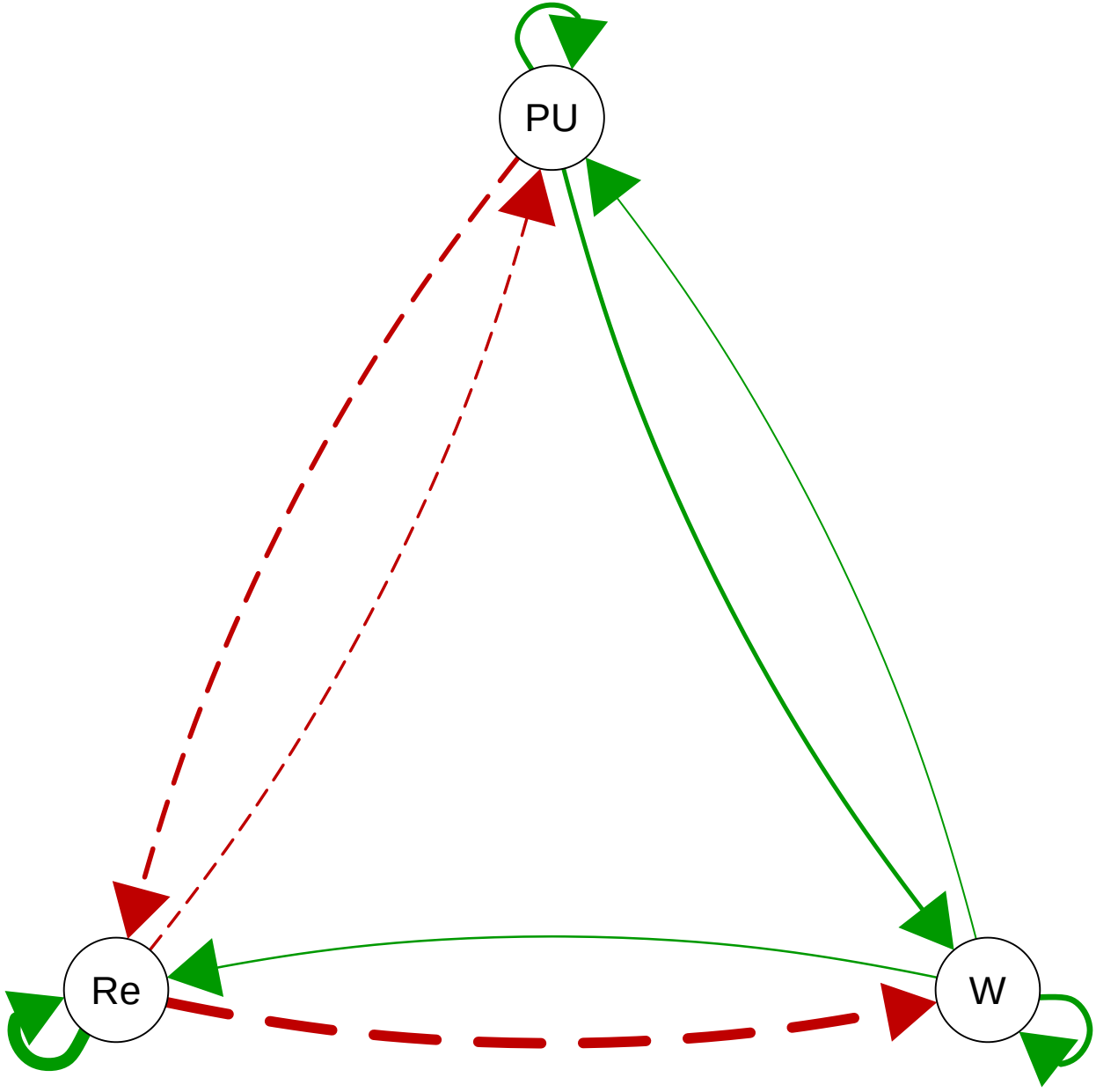
Empirical Data - PACT and APACT

		PACT			*APACT*
Effect		Total	Direct	Indirect	Total
$PU \longrightarrow PU$	β_{11}	0.046	0.041	-	0.046
$W \longrightarrow PU$	β_{12}	0.002	0.003	-0.001	0.003
$Re \longrightarrow PU$	β_{13}	-0.015	-0.009	-0.007	0.015
$PU \longrightarrow W$	β_{21}	0.040	0.019	0.023	0.040
$W \longrightarrow W$	β_{22}	0.036	0.041	-	0.040
$Re \longrightarrow W$	β_{23}	-0.090	-0.091	-0.004	0.090
$PU \longrightarrow Re$	β_{31}	-0.023	-0.029	0.004	0.023
$W \longrightarrow Re$	β_{32}	0.007	0.009	-0.002	0.007
$Re \longrightarrow Re$	β_{33}	0.103	0.113	-	0.103

Empirical Data- Overall Flow Betweenness Centrality

Variable	**Label**	$\boldsymbol{ofb_w}$
Perceived Unpleasantness	PU	.085
Worry	W	-.092
Relaxation	Re	.156

Empirical Data - PACT Direct Effects Network Structure



Empirical Data - Overall Network Structures

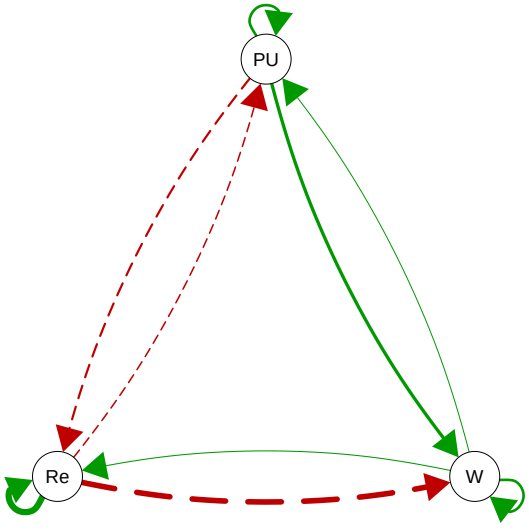


Figure: Total

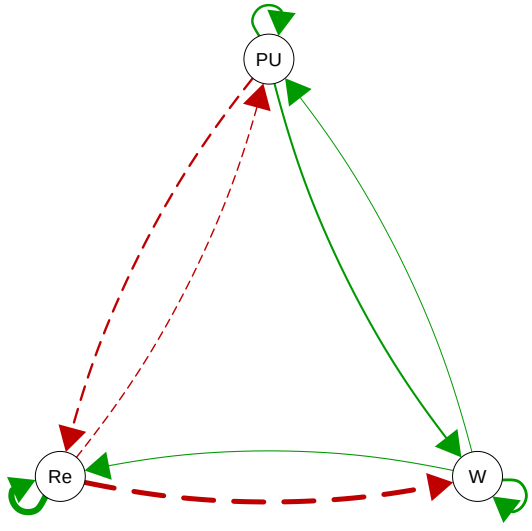


Figure: Direct

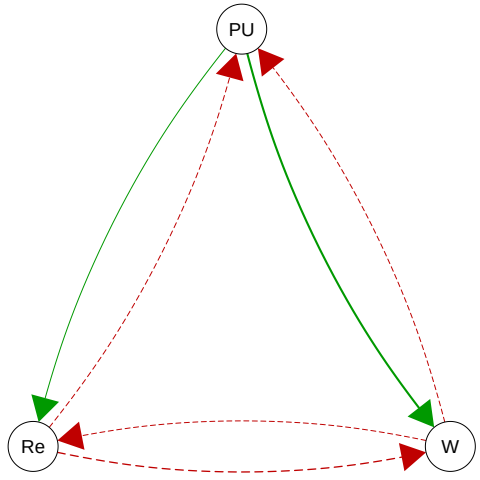
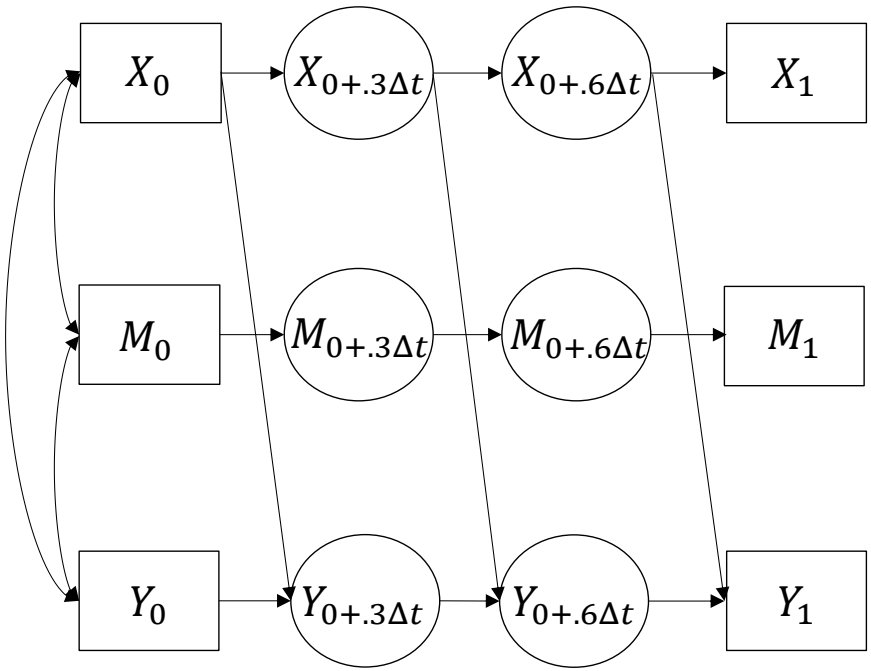


Figure: Indirect

Indirect, Direct and Total Effects

Continuous Time Framework



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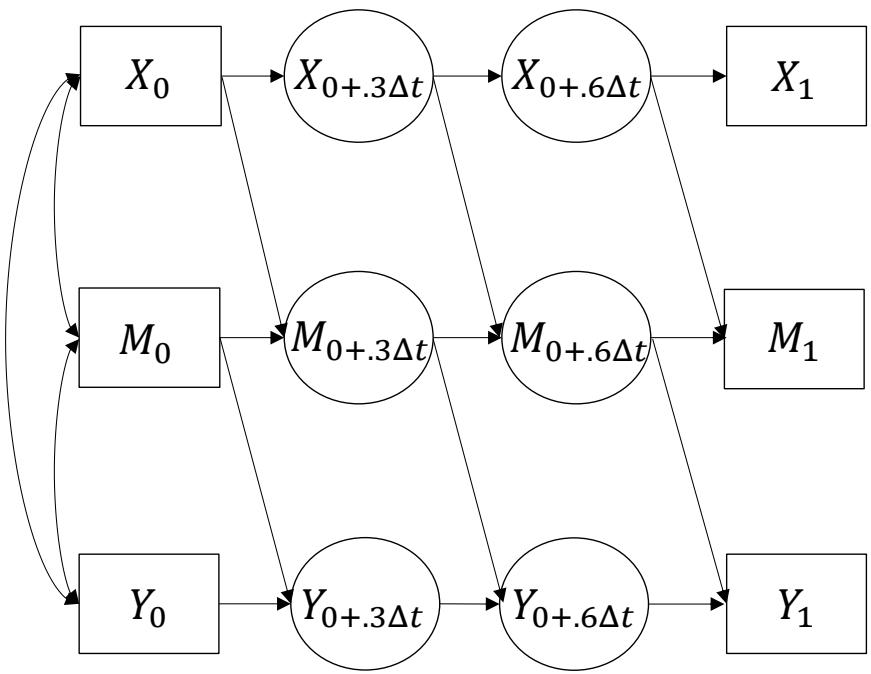

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Key References

Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. *Structural Equation Modeling*. Available at `http://quantpsy.org/pubs/deboeck_preacher_%28in.press%29.pdf`

Geschwind, N., Peeters, F., Drukker, M., van Os, J., & Wichers, M. (2011). Mindfulness training increases momentary positive emotions and reward experience in adults vulnerable to depression: a randomized controlled trial. *Journal of Consulting and Clinical Psychology, 79*(5), 618-628.