Dynamic Structural Equation Modeling of Intensive Longitudinal Data

Oisín Ryan
Utrecht University
o.ryan@uu.nl

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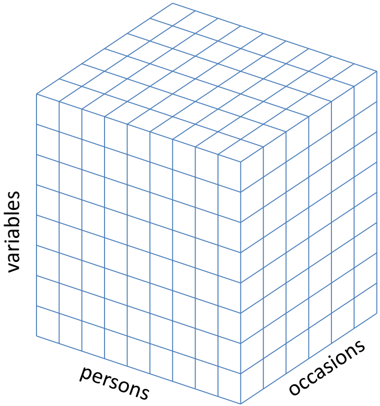
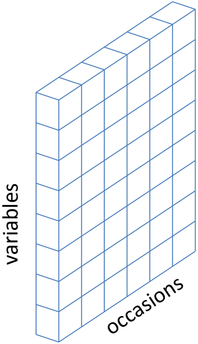
Slides from Ellen L. Hamaker

Cattell's data box



Time series data: N=1 and T is large



Idiographic (N=1) research in psychology

N=1 research has included:

- Cattell's P-technique: factor analysis of N=1 data
- Dynamic factor analysis: considering lagged relationships
- Measurement burst design: multiple waves of intensive measurements
- Intervention research: ABAB design etc.

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Critique of this kind of research:

- within-person fluctuations are just **noise**
- results are **not generalizable**
- no one has these data

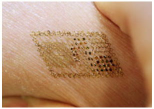
New technology

Smart phones

Smart glasses

Implants

Smart tattoo



Secure continuous remote alcohol monitor (SCRAM)

Smart watches

Activity trackers

Intensive longitudinal data

Different forms of intensive longitudinal data:

- daily diary (DD); self-report end-of-day
- experience sampling method (ESM); self-report of subjective experience
- ecological momentary assessment (EMA); healthcare related self-report
- ambulatory assessment (AA); physiological measurements
- event-based measurements; self-report after a particular event
- observational measurements; expert rater

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For more info on **methodology**, check out:

- Seminar of Tamlin Conner and Joshua Smyth on YouTube (https://www.youtube.com/watch?v=nQBBVp9vBIQ)
- Society for Ambulatory Assessment (http://www.saa2009.org/)
- Life Data (https://www.lifedatacorp.com/)
- Quantified Self (http://quantifiedself.com/)

Characteristics of these kind of data

Data structure:

- one or more measurements per day
- typically for multiple days
- sometimes multiple waves (i.e., Nesselroade's measurement-burst design)

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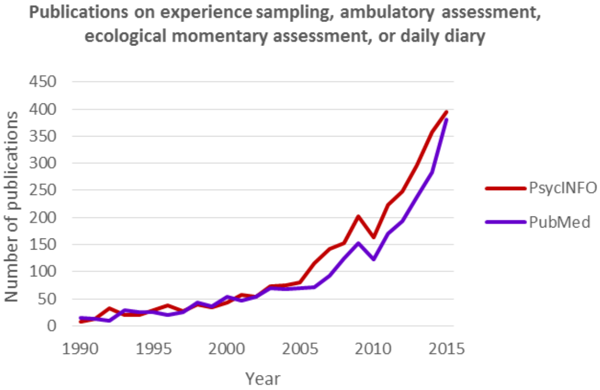
Advantages of ESM, EMA and AA

- no recall bias
- high ecological validity
- physiological measures over a large time span
- monitoring of symptoms and behavior, with new possibilities for feedback and intervention (e-Health and m-Health)
- window into the dynamics of processes

A paradigm shift



Taken from Hamaker and Wichers (2017)

Outline

- **Time series analysis**
- Multilevel time series analysis
- DSEM application 1: Multilevel VAR(1) model
- DSEM application 2: Mediation
- Discussion

What is time series analysis?

Time series analysis is a class of techniques that is used in econometrics, seismology, meteorology, control engineering, and signal processing.

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Main characteristics:

- N=1 technique
- T is large (say $>$50)
- concerned with *trends*, *cycles* and *autocorrelation structure* (i.e., serial dependency)
- goal: forecasting ($\neq$ prediction)

Y

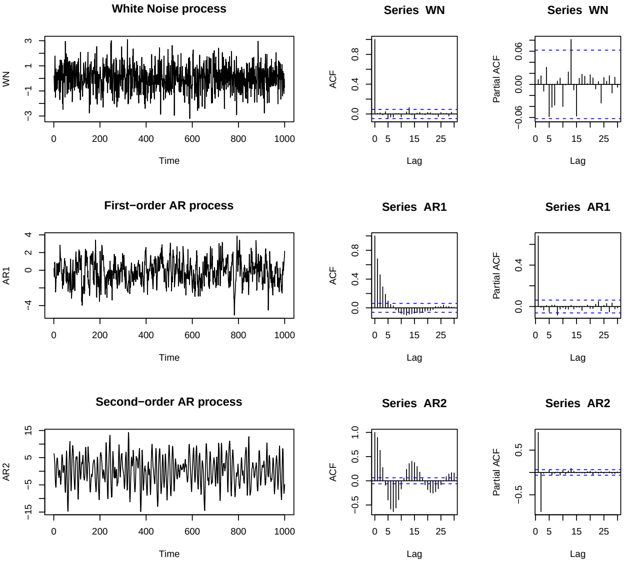
y_1
y_2
y_3
y_4
y_5
y_6
y_7
y_8
$\ldots$
y_T

Y	Y at lag 1
y_1	
y_2	y_1
y_3	y_2
y_4	y_3
y_5	y_4
y_6	y_5
y_7	y_6
y_8	y_7
$\dots$	$\dots$
y_T	y_{T-1}
	y_T

Lags

Y	Y at lag 1	Y at lag 2
y_1		
y_2	y_1	
y_3	y_2	y_1
y_4	y_3	y_2
y_5	y_4	y_3
y_6	y_5	y_4
y_7	y_6	y_5
y_8	y_7	y_6
$\ldots$	$\ldots$	$\ldots$
y_T	y_{T-1}	y_{T-2}
	y_T	y_{T-1}
		y_T

Sequence, ACF and PACF



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Lagged relationships in multilevel data

If we have t**ime series data from multiple individuals**, we may want to study:

- individual differences in lagged relationships between a variable and itself: **autoregression**
- individual differences in lagged relationship between different variables: **cross-lagged relationships**

Lagged relationships in multilevel data

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If we use multilevel modeling for this, we could refer to it as **multilevel time series analysis**, or **dynamic multilevel modeling**.

Creating lagged predictors

ID
1
1
1
1
1
2
2
2
2
2
$\ldots$
N
N
N
N
N

ID	y_{it}
1	y_{11}
1	y_{12}
1	y_{13}
1	$\dots$
1	y_{1T}
2	y_{21}
2	y_{22}
2	y_{23}
2	$\dots$
2	y_{2T}
$\dots$	$\dots$
N	y_{N1}
N	y_{N2}
N	y_{N3}
N	$\dots$
N	y_{NT}

Creating lagged predictors

ID	y_{it}	y_{it-1}
1	y_{11}	
1	y_{12}	y_{11}
1	y_{13}	y_{12}
1	$\dots$	$\dots$
1	y_{1T}	y_{1T-1}
2	y_{21}	
2	y_{22}	y_{21}
2	y_{23}	y_{22}
2	$\dots$	$\dots$
2	y_{2T}	y_{2T-1}
$\dots$	$\dots$	$\dots$
N	y_{N1}	
N	y_{N2}	y_{N1}
N	y_{N3}	y_{N2}
N	$\dots$	$\dots$
N	y_{NT}	y_{NT-1}

Creating lagged predictors

ID	y_{it}	y_{it-1}	x_{it-1}
1	y_{11}		
1	y_{12}	y_{11}	x_{11}
1	y_{13}	y_{12}	x_{12}
1	$\dots$	$\dots$	$\dots$
1	y_{1T}	y_{1T-1}	x_{1T-1}
2	y_{21}		
2	y_{22}	y_{21}	x_{21}
2	y_{23}	y_{22}	x_{22}
2	$\dots$	$\dots$	$\dots$
2	y_{2T}	y_{2T-1}	x_{2T-1}
$\dots$	$\dots$	$\dots$	$\dots$
N	y_{N1}		
N	y_{N2}	y_{N1}	x_{N1}
N	y_{N3}	y_{N2}	x_{N2}
N	$\dots$	$\dots$	$\dots$
N	y_{NT}	y_{NT-1}	x_{NT-1}

Inertia research based on multilevel AR(1) models

Level 1 model:

$$NA_{it} = c_i + \phi_i NA_{i,t-1} + \zeta_{it}$$

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Level 2 model:

$$c_i = \gamma_{00} + u_{0i}$$
$$\phi_i = \gamma_{01} + u_{1i}$$

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Level 2 model:

$$c_i = \gamma_{00} + u_{0i}$$
$$\phi_i = \gamma_{01} + u_{1i}$$

This research line was initiated by **Suls, Green and Hillis (1998)**, and continued by the group of **Kuppens**.

The focus is on individual differences in the **autoregressive parameter** ϕ_i (=inertia, carry-over, regulatory weakness), which is shown to be:

- positively related to current depression, neuroticism, and being female
- predictive of later depression (Kuppens and Koval)

Dynamic networks based on multilevel VAR(1) models

Level 1 model:

$$
\begin{aligned}
&y_{1it} = c_{1i} + \phi_{11i} y_{1it-1} + \cdots + \phi_{1ki} y_{kit-1} + \zeta_{1it} \\
&y_{2it} = c_{2i} + \phi_{21i} y_{1it-1} + \cdots + \phi_{2ki} y_{kit-1} + \zeta_{2it} \\
&\dots \\
&y_{kit} = c_{ki} + \phi_{k1i} y_{1it-1} + \cdots + \phi_{kki} y_{kit-1} + \zeta_{kit}
\end{aligned}
$$

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\end{aligned}$$

Initiated by **Bringmann et al. (2013)**, and further popularized by the software from **Sacha Epskamp.**

The focus is on **cross-lagged parameters** between variables (=nodes; typically symptoms), and on measures based on these (e.g., centrality).

Main idea is that **stronger connections** lead to an **increased risk** of developing and maintaining psychopathology.

A fundamental problem in a nutshell

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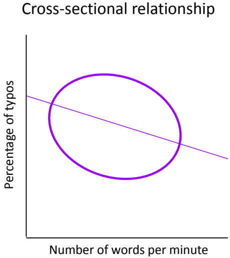

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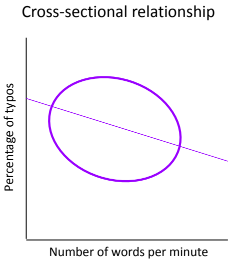

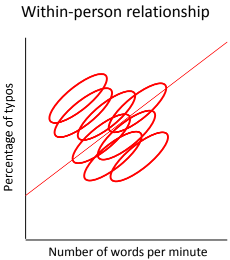

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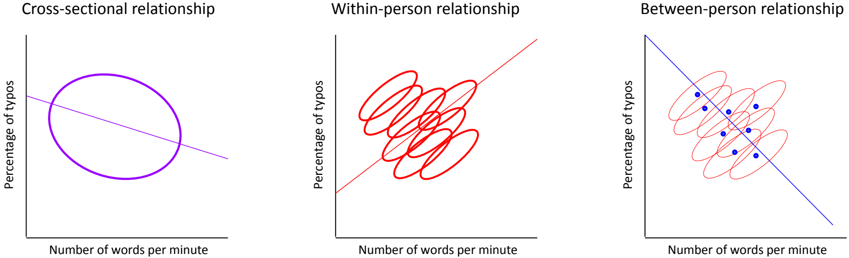
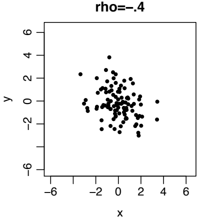


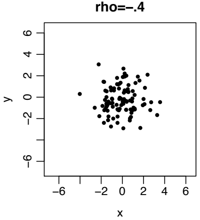
Taken from Hamaker (2012).

Three perspectives on data

Three perspectives on data

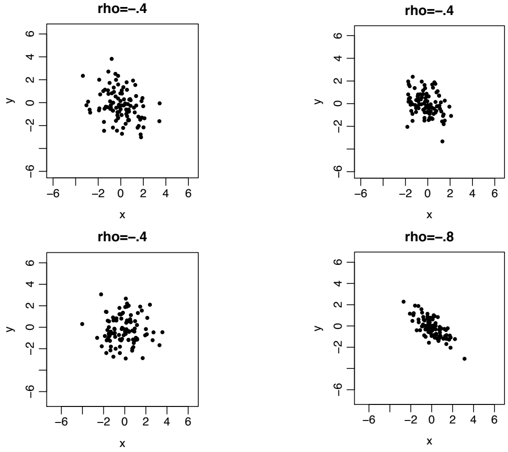
Cross-sectional





Three perspectives on data

Cross-sectional — Within



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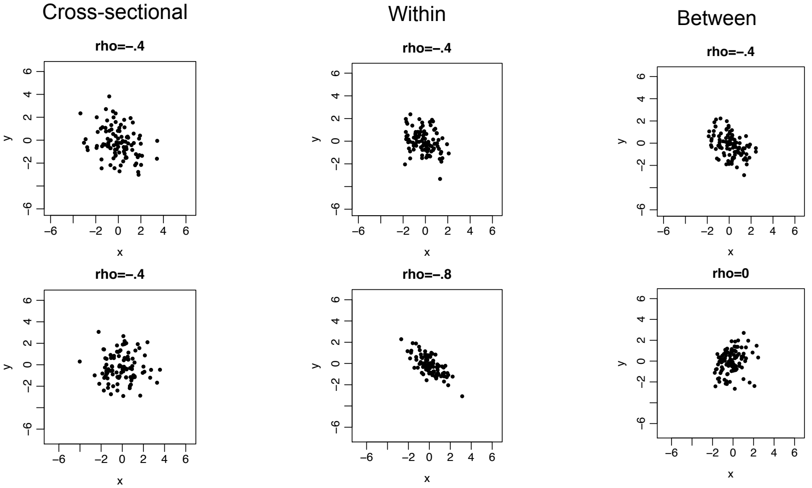

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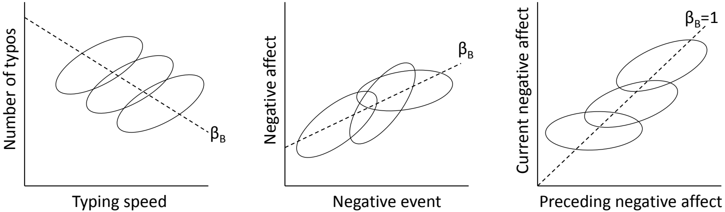
Between-person differences in within-person slopes



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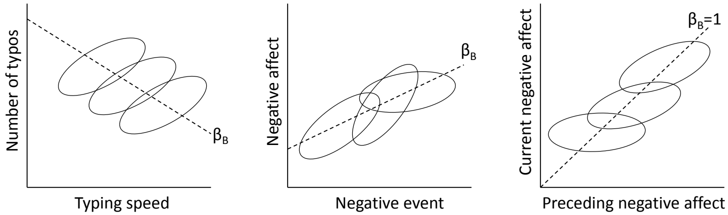

Taken from Hamaker and Grasman (2014).

In conclusion: To study within-person processes we need

- (intensive) **longitudinal** data
- to **decompose** observed variance into within and between
- to consider **individual differences** in within-person dynamics

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Dynamic structural equation modeling (DSEM) in Mplus tackles all these problems.

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Data: Daily measurements affect

Data come from the **COGITO study** of the MPI in Berlin; goal is to study aging using a younger and older sample.

Analyses here are based on Hamaker et al. (under revision).

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Characteristics of the **younger** and **older sample**:

- aged 20-31; aged 65-80
- 101 individuals; 103 individuals
- about 100 daily measurements of positive affect (PA) and negative affect (NA)

Decomposition

Decomposition into a between part and a within part

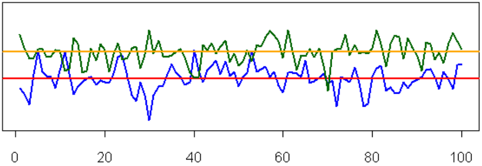
$$PA_{it} = \mu_{PA,i} + PA^*_{it}$$
$$NA_{it} = \mu_{NA,i} + NA^*_{it}$$

Decomposition into a between part and a within part

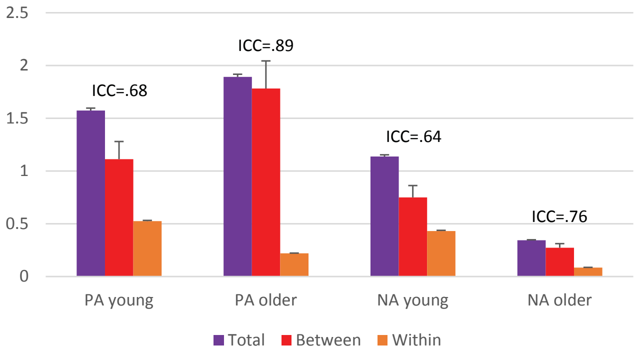
$$PA_{it} = \mu_{PA,i} + PA^*_{it}$$
$$NA_{it} = \mu_{NA,i} + NA^*_{it}$$

where

- $\mu_{PA,i}$ and $\mu_{NA,i}$ are the individual's **means** on PA and NA (i.e., baseline, trait, or equilibrium scores) $\Rightarrow$ between-person part
- PA^*_{it} and NA^*_{it} are the **within-person centered** (cluster-mean centered) scores $\Rightarrow$ within-person part



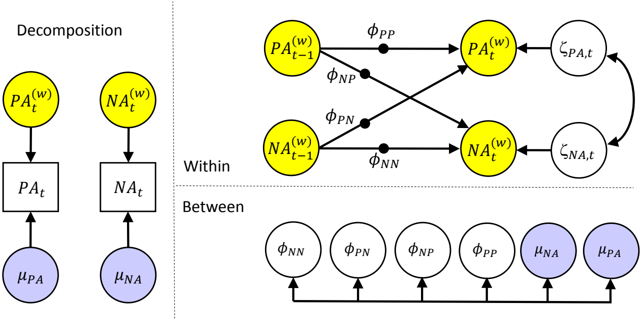
Total, between-, and within-person variance



Intraclass correlation:

$$\frac{\sigma^2_{between}}{\sigma^2_{between}+\sigma^2_{within}} = \frac{\sigma^2_{between}}{\sigma^2_{total}}$$

Bivariate model: Multilevel vector AR(1) model



Within-person level model

Lagged within-person model:

$$PA^*_{it} = \phi_{PP,i} PA^*_{i,t-1} + \phi_{PN,i} NA^*_{i,t-1} + \zeta_{PA,it}$$
$$NA^*_{it} = \phi_{NN,i} NA^*_{i,t-1} + \phi_{NP,i} PA^*_{i,t-1} + \zeta_{NA,it}$$

where

- $\phi_{PP,i}$ is the **autoregressive parameter** for PA (i.e., inertia, carry-over)
- $\phi_{NN,i}$ is the **autoregressive parameter** for NA (i.e., inertia, carry-over)

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- $\phi_{PN,i}$ is the **cross-lagged parameter** for NA to PA (i.e., spill-over)
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- $\zeta_{PA,it}$ is the **innovation** for PA (residual, disturbance, dynamic error)
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- $\zeta_{PA,it}$ is the **innovation** for PA (residual, disturbance, dynamic error)
- $\zeta_{NA,it}$ is the **innovation** for NA (residual, disturbance, dynamic error)

Parameters estimated at this level are the residual variances and covariance:

$$\begin{bmatrix} \zeta_{PA,it} \\ \zeta_{NA,it} \end{bmatrix} \sim MN \left[\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \theta_{11} & \\ \theta_{21} & \theta_{22} \end{bmatrix} \right]$$

Between-person level model

Between level: fixed and random effects

$$\begin{bmatrix} \mu_{PA,i} \\ \mu_{NA,i} \\ \phi_{PP,i} \\ \phi_{PN,i} \\ \phi_{NP,i} \\ \phi_{NN,i} \end{bmatrix} = \begin{bmatrix} \gamma_P \\ \gamma_N \\ \gamma_{PP} \\ \gamma_{PN} \\ \gamma_{NP} \\ \gamma_{NN} \end{bmatrix} + \begin{bmatrix} u_{P,i} \\ u_{N,i} \\ u_{PP,i} \\ u_{PN,i} \\ u_{NP,i} \\ u_{NN,i} \end{bmatrix} \quad \boldsymbol{u}_i \sim MN(\mathbf{0}, \mathbf{\Psi})$$

Where:

- γ_P to γ_{NN} $\Rightarrow$ fixed effects
- $u_{P,i}$ to $u_{NN,i}$ $\Rightarrow$ random effects

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$$\begin{bmatrix} \mu_{PA,i} \\ \mu_{NA,i} \\ \phi_{PP,i} \\ \phi_{PN,i} \\ \phi_{NP,i} \\ \phi_{NN,i} \end{bmatrix} = \begin{bmatrix} \gamma_{P} \\ \gamma_{N} \\ \gamma_{PP} \\ \gamma_{PN} \\ \gamma_{NP} \\ \gamma_{NN} \end{bmatrix} + \begin{bmatrix} u_{P,i} \\ u_{N,i} \\ u_{PP,i} \\ u_{PN,i} \\ u_{NP,i} \\ u_{NN,i} \end{bmatrix} \qquad \boldsymbol{u}_i \sim MN(\mathbf{0}, \boldsymbol{\Psi})$$

Where:

- γ_P to γ_{NN} $\Rightarrow$ fixed effects
- $u_{P,i}$ to $u_{NN,i}$ $\Rightarrow$ random effects

Parameters estimated at this level are:

- 6 fixed effects (i.e., γ's)
- 6 variances for random effects (i.e., diagonal elements of $\boldsymbol{\Psi}$)
- 15 covariances between the random effects (i.e., off-diagonal elements in $\boldsymbol{\Psi}$)

Bivariate model: Mplus code

```
VARIABLE:    NAMES ARE id sessdate
             na1 na2 na3 na4 na5 na6 na7 na8 na9 na10
             pa1 pa2 pa3 pa4 pa5 pa6 pa7 pa8 pa9 pa10
             sessionNr age_pre sex CESDpre CESDpost dayNA dayPA older;

             CLUSTER = id; ! Specify the person id variable
             USEVAR = dayPA dayNA; ! Specify which variables are used in the model
             MISSING = ALL(-999);

             LAGGED = dayPA(1) dayNA(1); ! This creates lagged variables
             TINTERVAL = sessdate(1); ! This is to account for unequal intervals

ANALYSIS:    TYPE IS TWOLEVEL RANDOM; ! This allows for random slopes
             ESTIMATOR = BAYES; ! DSEM requires Bayesian estimation
             PROC = 2; ! Using 2 processors makes it faster
             BITER = (5000); ! This implies at least 5000 iterations are used
             THIN = 10; ! Thinning helps with getting more stable results
```

Bivariate model: Mplus code

```
MODEL:      %WITHIN% ! Specify the random lagged relationships
            p_pp | dayPA ON dayPA&1;
            p_pn | dayPA ON dayNA&1;
            p_np | dayNA ON dayPA&1;
            p_nn | dayNA ON dayNA&1;

            %BETWEEN% ! Allow all 6 random effects to be correlated
            p_pp WITH p_pn-p_nn dayPA dayNA;
            p_pn WITH p_np-p_nn dayPA dayNA;
            p_np WITH p_nn dayPA dayNA;
            p_nn WITH dayPA dayNA;
            dayPA WITH dayNA;

OUTPUT:     TECH1 TECH8 STDYX;

PLOT:       TYPE = PLOT3;
            FACTORS =ALL;
```

Mplus results: Within-person (younger sample)

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%	Significance
Within Level						
DAYNA WITH						
DAYPA	-0.069	0.004	0.000	-0.076	-0.061	*
Residual Variances						
DAYPA	0.414	0.006	0.000	0.403	0.426	*
DAYNA	0.302	0.004	0.000	0.294	0.311	*

Mplus results: Between-person (younger sample)

```
                                Posterior  One-Tailed         95% C.I.
                    Estimate       S.D.      P-Value   Lower 2.5%  Upper 2.5%  Significance

[...]

Between Level

[...]

 Means
    DAYPA              3.090       0.110      0.000       2.875       3.308      *
    DAYNA              0.977       0.077      0.000       0.826       1.128      *
    P_PP               0.334       0.026      0.000       0.283       0.387      *
    P_PN               0.050       0.022      0.016       0.006       0.093      *
    P_NP               0.038       0.015      0.006       0.008       0.068      *
    P_NN               0.370       0.027      0.000       0.315       0.423      *

 Variances
    DAYPA              1.178       0.189      0.000       0.886       1.618      *
    DAYNA              0.595       0.101      0.000       0.443       0.832      *
    P_PP               0.055       0.010      0.000       0.039       0.079      *
    P_PN               0.024       0.006      0.000       0.014       0.039      *
    P_NP               0.013       0.003      0.000       0.008       0.021      *
    P_NN               0.062       0.012      0.000       0.044       0.089      *
```

Comparing cross-lagged parameters

Standardization in multilevel models is a **tricky issue**.

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Schuurman, Ferrer, Boer-Sonnenschein and Hamaker (2016) discuss four forms of **standardization in multilevel models**, using:

- total variance (i.e., grand standardization)
- between-person variance (i.e., between standardization)
- average within-person variance
- within-person variance (i.e., within standardization)

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Conclusion: last form is most meaningful, as it **parallels standardizing when N=1.**

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- average within-person variance
- within-person variance (i.e., within standardization)

Conclusion: last form is most meaningful, as it **parallels standardizing when N=1.**

Standardized fixed effect should be the **average standardized within-person effect.**

Mplus standardized results (younger sample)

```
STDYX Standardization

                                Posterior  One-Tailed         95% C.I.
                    Estimate       S.D.      P-Value   Lower 2.5%  Upper 2.5%  Significance

Within-Level Standardized Estimates Averaged Over Clusters

 P_PP | DAYPA ON
    DAYPA&1            0.335       0.011      0.000       0.312       0.358      *

 P_PN | DAYPA ON
    DAYNA&1            0.034       0.013      0.006       0.008       0.059      *

 P_NP | DAYNA ON
    DAYPA&1            0.038       0.011      0.000       0.017       0.059      *

 P_NN | DAYNA ON
    DAYNA&1            0.370       0.012      0.000       0.347       0.394      *

 DAYNA    WITH
    DAYPA             -0.194       0.010      0.000      -0.213      -0.175      *

 Residual Variances
    DAYPA              0.816       0.008      0.000       0.799       0.832      *
    DAYNA              0.792       0.008      0.000       0.775       0.808      *
```

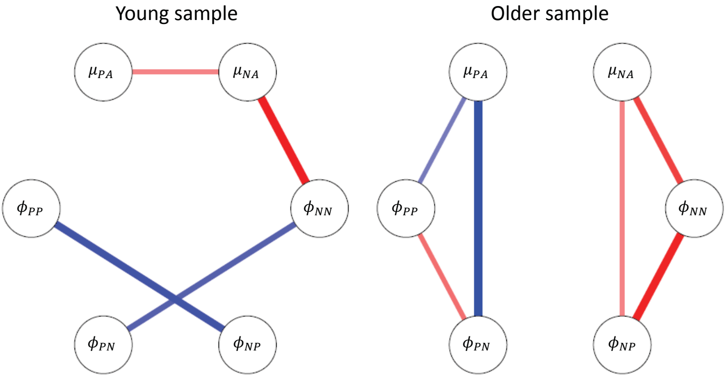
Mplus standardized results (younger sample)

R-SQUARE

Within-Level R-Square Averaged Across Clusters

Variable	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%
DAYPA	0.184	0.008	0.000	0.168	0.201
DAYNA	0.208	0.008	0.000	0.192	0.225

To **represent the correlation matrices** of the 6 random effects in each group, we can use the network representation (with qgraph from Sacha Epskamp in R):



Outline

- Time series analysis
- Multilevel time series analysis
- DSEM application 1: Multilevel VAR(1) model
- **DSEM application 2: Mediation**
- Discussion

Including level 2 predictor and outcome

Depression was measured prior to the ILD phase and afterwards, using the CESD; we include these measures at the between-person level as a **predictor** and an **outcome.**

Between level: Including a level 2 predictor

$$\mu_{PA,i} = \gamma_{00} + \gamma_{01} CESDpre_i + u_{0i}$$
$$\mu_{NA,i} = \gamma_{10} + \gamma_{11} CESDpre_i + u_{1i}$$
$$\phi_{PP,i} = \gamma_{20} + \gamma_{21} CESDpre_i + u_{2i}$$
$$\phi_{PN,i} = \gamma_{30} + \gamma_{31} CESDpre_i + u_{3i}$$
$$\phi_{NN,i} = \gamma_{40} + \gamma_{41} CESDpre_i + u_{4i}$$
$$\phi_{NP,i} = \gamma_{50} + \gamma_{51} CESDpre_i + u_{5i}$$

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$$
\begin{aligned}
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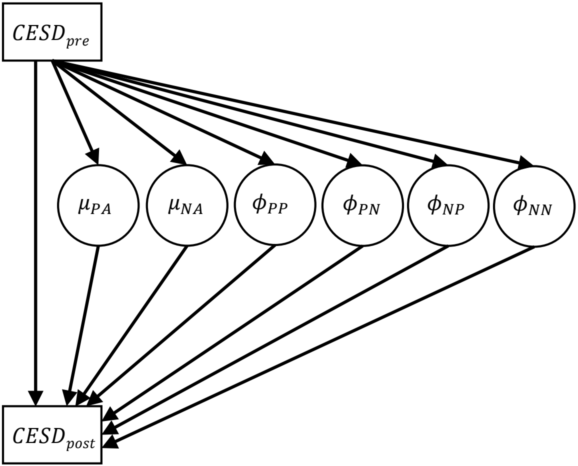
\mu_{NA,i} &= \gamma_{10} + \gamma_{11} CESDpre_i + u_{1i} \\
\phi_{PP,i} &= \gamma_{20} + \gamma_{21} CESDpre_i + u_{2i} \\
\phi_{PN,i} &= \gamma_{30} + \gamma_{31} CESDpre_i + u_{3i} \\
\phi_{NN,i} &= \gamma_{40} + \gamma_{41} CESDpre_i + u_{4i} \\
\phi_{NP,i} &= \gamma_{50} + \gamma_{51} CESDpre_i + u_{5i}
\end{aligned}
$$

Between level: Including a level 2 outcome

$$
\begin{aligned}
CESDpost_i = {} & \gamma_{60} + \gamma_{61} CESDpre_i + \gamma_{62} \mu_{PA,i} + \gamma_{63} \mu_{NA,i} \\
& + \gamma_{64} \phi_{PP,i} + \gamma_{65} \phi_{PN,i} + \gamma_{66} \phi_{NN,i} + \gamma_{67} \phi_{NP,i} + u_{6i}
\end{aligned}
$$

Dynamic mediation model



Mplus input mediation model

```
VARIABLE:   NAMES ARE id sessdate
            na1 na2 na3 na4 na5 na6 na7 na8 na9 na10
            pa1 pa2 pa3 pa4 pa5 pa6 pa7 pa8 pa9 pa10
            sessionNr age_pre sex CESDpre CESDpost dayNA dayPA older;
            CLUSTER = id;
            USEVAR = dayPA dayNA CESDpre CESDpost; ! Plus level 2 variables
            BETWEEN = CESDpre CESDpost; ! Specify these as level 2 variables
            LAGGED = dayPA(1) dayNA(1);
            TINTERVAL = sessdate(1);
            MISSING = ALL(-999);

DEFINE:     CENTER CESDpre CESDpost (GRANDMEAN);! Grand mean centering

ANALYSIS:   TYPE IS TWOLEVEL RANDOM;
            ESTIMATOR = BAYES;
            PROCESSORS = 2;
            BITER = (5000);
            THIN = 10;
```

Bivariate model: Mplus code

```
MODEL:                  %WITHIN% ! Same as before
                        p_pp | dayPA ON dayPA&1;
                        p_pn | dayPA ON dayNA&1;
                        p_np | dayNA ON dayPA&1;
                        p_nn | dayNA ON dayNA&1;

                        %BETWEEN% ! Mediation model with parameter names
                        p_pp-p_nn dayPA dayNA ON CESDpre (a1-a6);
                        CESDpost ON p_pp-p_nn dayPA dayNA CESDpre (b1-b7);

MODEL CONSTRAINT:       ! Compute the indirect effects
                        new (ab_p_pp); ab_p_pp=a1*b1;
                        new (ab_p_pn); ab_p_pn=a2*b2;
                        new (ab_p_np); ab_p_np=a3*b3;
                        new (ab_p_nn); ab_p_nn=a4*b4;
                        new (ab_dayPA); ab_dayPA=a5*b5;
                        new (ab_dayNA); ab_dayNA=a6*b6;

OUTPUT:                 TECH1 TECH8 STDYX;

PLOT:                   TYPE = PLOT3;
                        FACTOR =ALL;
```

Mplus output mediation model (younger sample)

```
                              Posterior  One-Tailed         95% C.I.
                    Estimate     S.D.      P-Value   Lower 2.5%  Upper 2.5%  Significance
[...]
Between Level
[...]

 Intercepts
    CESDPOST           0.104       0.136      0.223      -0.167       0.365
    DAYPA              3.088       0.103      0.000       2.888       3.293      *
    DAYNA              0.989       0.076      0.000       0.844       1.146      *
    P_PP               0.338       0.024      0.000       0.289       0.386      *
    P_PN               0.031       0.020      0.057      -0.008       0.071
    P_NP               0.035       0.014      0.006       0.007       0.062      *
    P_NN               0.376       0.024      0.000       0.329       0.423      *

 Residual Variances
    CESDPOST           0.067       0.012      0.000       0.048       0.095      *
    DAYPA              1.049       0.158      0.000       0.798       1.416      *
    DAYNA              0.517       0.091      0.000       0.377       0.729      *
    P_PP               0.045       0.008      0.000       0.032       0.064      *
    P_PN               0.019       0.005      0.000       0.011       0.030      *
    P_NP               0.010       0.003      0.000       0.005       0.016      *
    P_NN               0.043       0.008      0.000       0.031       0.062      *

New/Additional Parameters
    AB_P_PP            0.010       0.025      0.266      -0.028       0.076
    AB_P_PN           -0.002       0.032      0.439      -0.074       0.062
    AB_P_NP           -0.004       0.037      0.401      -0.089       0.067
    AB_P_NN            0.195       0.070      0.000       0.081       0.359      *
    AB_DAYPA           0.049       0.035      0.029      -0.001       0.135
    AB_DAYNA           0.028       0.043      0.234      -0.052       0.119
```

Mplus output mediation model (older sample)

```
                                Posterior  One-Tailed         95% C.I.
                    Estimate       S.D.      P-Value   Lower 2.5%  Upper 2.5%  Significance
[...]
Between Level
[...]

 Intercepts
    CESDPOST           0.015       0.113      0.448      -0.210       0.236
    DAYPA              4.566       0.120      0.000       4.336       4.796      *
    DAYNA              0.313       0.052      0.000       0.210       0.417      *
    P_PP               0.421       0.026      0.000       0.370       0.472      *
    P_PN               0.133       0.039      0.000       0.057       0.212      *
    P_NP               0.016       0.017      0.167      -0.018       0.051
    P_NN               0.239       0.027      0.000       0.185       0.291      *

 Residual Variances
    CESDPOST           0.039       0.006      0.000       0.029       0.053      *
    DAYPA              1.416       0.221      0.000       1.079       1.918      *
    DAYNA              0.269       0.041      0.000       0.203       0.365      *
    P_PP               0.056       0.010      0.000       0.039       0.079      *
    P_PN               0.083       0.021      0.000       0.051       0.131      *
    P_NP               0.024       0.004      0.000       0.018       0.035      *
    P_NN               0.051       0.009      0.000       0.037       0.072      *

New/Additional Parameters
    AB_P_PP            0.005       0.016      0.302      -0.018       0.049
    AB_P_PN           -0.004       0.025      0.396      -0.061       0.045
    AB_P_NP            0.012       0.027      0.268      -0.035       0.076
    AB_P_NN           -0.036       0.038      0.112      -0.130       0.025
    AB_DAYPA           0.028       0.038      0.209      -0.042       0.110
    AB_DAYNA           0.027       0.036      0.194      -0.040       0.108
```

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Advantages of using DSEM in Mplus (thus far)

Compared to standard multilevel software:

- **Multiple outcome variables**: this allows for correlated residuals and correlated random effects
- **Unequal time interval**: can be handled by choosing a grid for inserting missings
- **Outcomes** at between-person level
- **Person-mean centering** integral part of model estimation (solves Nickell's bias)

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Compared to other Bayesian software (e.g., WinBUGS, jags, Stan):

- **Easy to use** due to tailor-made code
- **Default uninformative priors** for parameters (even for small variances)
- **Fast** (which makes a difference in case of Bayes)

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Other recent developments: mlVAR, ctsem and open Mx (in R); Bayesian Ornstein-Uhlenbeck Model (BOUM); GIMME.

More advantages of using DSEM in Mplus

Other options offered by DSEM in Mplus version 8:

- **Diverse plotting options**: allows for inspection of data and results
- **Latent variables**: allows for measurement error to be split off and for moving average terms
- **Cross-classified models**: allows for random effects of time
- **Random variance**: allows for individual difference in variability

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- **Random variance**: allows for individual difference in variability

Future options Mplus will offer:

- **Regime-switching models**: allows for a process to switch between distinct states
- **Residual dynamic modeling**: allows for easy combination of time trends and residual lagged relationships

Random innovation variance (univariately)

Within level: AR(1) with random ϕ_i

$$NA^*_{it} = \phi_i NA^*_{i,t-1} + \zeta_{it} \qquad \zeta_{it} \sim N(0, \sigma_i^2)$$

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$$NA^*_{it} = \phi_i NA^*_{i,t-1} + \zeta_{it} \qquad \zeta_{it} \sim N(0, \sigma_i^2)$$

Between level: fixed and random effects

$$\begin{aligned} \mu_i &= \gamma_\mu + u_{0i} \\ \phi_i &= \gamma_\phi + u_{1i} \\ \log(\sigma_i^2) &= \gamma_{\log(\sigma^2)} + u_{2i} \end{aligned} \qquad \begin{bmatrix} u_{0i} \\ u_{1i} \\ u_{2i} \end{bmatrix} \sim MN \left[\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{11} & & \\ \psi_{21} & \psi_{22} & \\ \psi_{31} & \psi_{32} & \psi_{33} \end{bmatrix} \right]$$

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$$NA^*_{it} = \phi_i NA^*_{i,t-1} + \zeta_{it} \qquad \zeta_{it} \sim N(0, \sigma_i^2)$$

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Reasons to assume **individual differences** for σ^2:

- individuals may differ with respect to the **variability in exposure** to external factors
- individuals may differ with respect to their **reactivity** to external influences (see reward experience and stress sensitivity research)

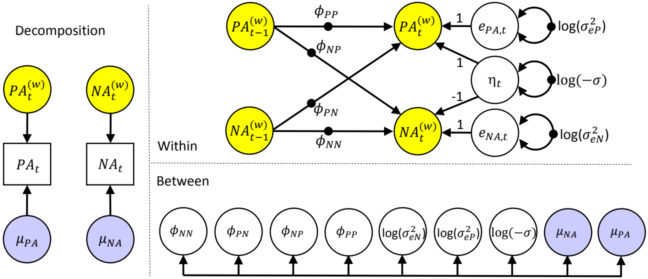
Random innovation variances and covariance

In the bivariate case, we want **random innovation variances** AND **random innovation covariance.**

Random innovation variances and covariance

In the bivariate case, we want **random innovation variances** AND **random innovation covariance.**

The latter is modeled with an additional factor η_t:



Where:

- -η_t is the shared part (we assume a negative covariance)
- $e_{PA,t}$ and $e_{NA,t}$ are the unique parts

Mplus code: Within model

```
MODEL:     %WITHIN%
           p_pp | dayPA ON dayPA&1;
           p_pn | dayPA ON dayNA&1;
           p_np | dayNA ON dayPA&1;
           p_nn | dayNA ON dayNA&1;

           ! Create latent variable that represents negative covariance
           Cov BY dayPA1 dayNA-1;

           ! Create random (log) variances
           logvarPA | dayPA;
           logvarNA | dayNA;
           logCov | Cov;

           %BETWEEN%
           p_pp-p_nn WITH p_pn-p_nn logvarPA logvarNA logCov dayPA dayNA;
           logvarPA WITH logvarNA logCov dayPA dayNA;
           logvarNA WITH logCov dayPA dayNA;
           logCov WITH dayPA dayNA;
           dayPA WITH dayNA;

OUTPUT:    TECH1 TECH8 STDYX FSCOMPARISON;
```

What about many variables?

Emilio Ferrer obtained data from **193 dyads** for **52-108 days** on **8 variables** (i.e., general and relationship specific PA and NA).

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Within level: Vector autoregressive model

$$\begin{bmatrix} GPAM^*_{it} \\ GNAM^*_{it} \\ RSPAM^*_{it} \\ RSNAM^*_{it} \\ GPAF^*_{it} \\ GNAF^*_{it} \\ RSPAF^*_{it} \\ RSNAF^*_{it} \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16} & \phi_{17} & \phi_{18} \\ \phi_{21} & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \phi_{26} & \phi_{27} & \phi_{28} \\ \phi_{31} & \phi_{32} & \phi_{33} & \phi_{34} & \phi_{35} & \phi_{36} & \phi_{37} & \phi_{38} \\ \phi_{41} & \phi_{42} & \phi_{43} & \phi_{44} & \phi_{45} & \phi_{46} & \phi_{47} & \phi_{48} \\ \phi_{51} & \phi_{52} & \phi_{53} & \phi_{54} & \phi_{55} & \phi_{56} & \phi_{57} & \phi_{58} \\ \phi_{61} & \phi_{62} & \phi_{63} & \phi_{64} & \phi_{65} & \phi_{66} & \phi_{67} & \phi_{68} \\ \phi_{71} & \phi_{72} & \phi_{73} & \phi_{74} & \phi_{75} & \phi_{76} & \phi_{77} & \phi_{78} \\ \phi_{81} & \phi_{82} & \phi_{73} & \phi_{84} & \phi_{85} & \phi_{86} & \phi_{87} & \phi_{88} \end{bmatrix} \begin{bmatrix} GPAM^*_{it-1} \\ GNAM^*_{it-1} \\ RSPAM^*_{it-1} \\ RSNAM^*_{it-1} \\ GPAF^*_{it-1} \\ GNAF^*_{it-1} \\ RSPAF^*_{it-1} \\ RSNAF^*_{it-1} \end{bmatrix} + \begin{bmatrix} \zeta_{1it} \\ \zeta_{2it} \\ \zeta_{3it} \\ \zeta_{4it} \\ \zeta_{5it} \\ \zeta_{6it} \\ \zeta_{7it} \\ \zeta_{8it} \end{bmatrix}$$

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$$\begin{bmatrix} GPAM^*_{it} \\ GNAM^*_{it} \\ RSPAM^*_{it} \\ RSNAM^*_{it} \\ GPAF^*_{it} \\ GNAF^*_{it} \\ RSPAF^*_{it} \\ RSNAF^*_{it} \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16} & \phi_{17} & \phi_{18} \\ \phi_{21} & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \phi_{26} & \phi_{27} & \phi_{28} \\ \phi_{31} & \phi_{32} & \phi_{33} & \phi_{34} & \phi_{35} & \phi_{36} & \phi_{37} & \phi_{38} \\ \phi_{41} & \phi_{42} & \phi_{43} & \phi_{44} & \phi_{45} & \phi_{46} & \phi_{47} & \phi_{48} \\ \phi_{51} & \phi_{52} & \phi_{53} & \phi_{54} & \phi_{55} & \phi_{56} & \phi_{57} & \phi_{58} \\ \phi_{61} & \phi_{62} & \phi_{63} & \phi_{64} & \phi_{65} & \phi_{66} & \phi_{67} & \phi_{68} \\ \phi_{71} & \phi_{72} & \phi_{73} & \phi_{74} & \phi_{75} & \phi_{76} & \phi_{77} & \phi_{78} \\ \phi_{81} & \phi_{82} & \phi_{73} & \phi_{84} & \phi_{85} & \phi_{86} & \phi_{87} & \phi_{88} \end{bmatrix} \begin{bmatrix} GPAM^*_{it-1} \\ GNAM^*_{it-1} \\ RSPAM^*_{it-1} \\ RSNAM^*_{it-1} \\ GPAF^*_{it-1} \\ GNAF^*_{it-1} \\ RSPAF^*_{it-1} \\ RSNAF^*_{it-1} \end{bmatrix} + \begin{bmatrix} \zeta_{1it} \\ \zeta_{2it} \\ \zeta_{3it} \\ \zeta_{4it} \\ \zeta_{5it} \\ \zeta_{6it} \\ \zeta_{7it} \\ \zeta_{8it} \end{bmatrix}$$

which gives:

$$GPAM^*_{it} = \phi_{11} GPAM^*_{it-1} + \phi_{12} GNAM^*_{it-1} + \cdots + \phi_{18} RSNAF^*_{it-1} + \zeta_{1it}$$

$\ldots$

$$RSNAF^*_{it} = \phi_{81} GPAM^*_{it-1} + \phi_{82} GNAM^*_{it-1} + \cdots + \phi_{88} RSNAF^*_{it-1} + \zeta_{8it}$$

Within level: Residual covariance matrix

$$\begin{bmatrix} \zeta_{1it} \\ \zeta_{2it} \\ \dots \\ \zeta_{8it} \end{bmatrix} \sim MN(\mathbf{0}, \mathbf{\Theta}^{*})$$

Hence, we estimate $8 \times 8 = 64$ lagged parameters, and $8 \times 9/2 = 36$ variances and covariances at the within-person level.

Multilevel VAR(1)

Within level: Residual covariance matrix

$$\begin{bmatrix} \zeta_{1it} \\ \zeta_{2it} \\ \dots \\ \zeta_{8it} \end{bmatrix} \sim MN(\mathbf{0}, \mathbf{\Theta}^*)$$

Hence, we estimate $8 \times 8 = 64$ lagged parameters, and $8 \times 9/2 = 36$ variances and covariances at the within-person level.

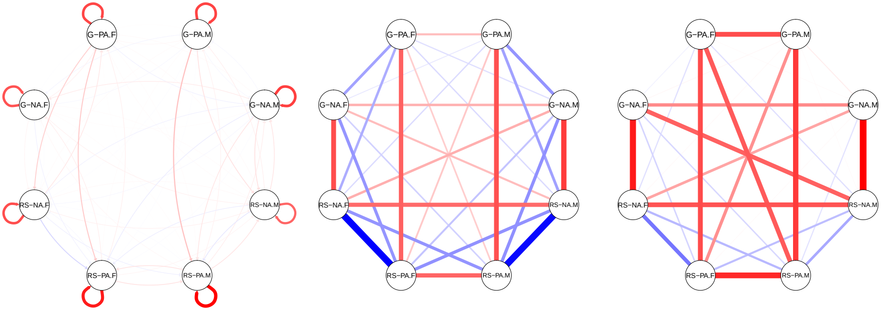
Between level: Fixed and random effects

$$\begin{bmatrix} \mu_{1i} \\ \mu_{2i} \\ \dots \\ \mu_{8i} \end{bmatrix} \sim MN(\boldsymbol{\gamma}, \mathbf{\Psi})$$

Hence, we estimate 8 grand means, and $8 \times 9/2 = 36$ variances and covariances at the between-person level. In total: 144 parameters.

Three networks

Lagged, within-person (residual), and between-person:



Note:

- lagged network = within-person standardized lagged relationships
- within-person residual network = correlations of within-person residuals
- between-person network = correlations of within-person means

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