

Mediation and Path-Specific effects in Continuous-Time VAR(1) Models

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Overview

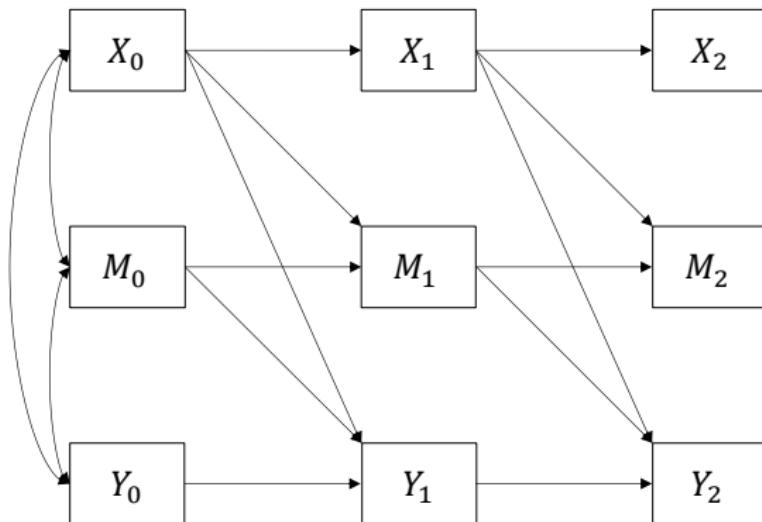
- ▶ Current commonly used **discrete time models** may lead to **misleading conclusions** regarding causal relationships

Overview

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- ▶ How time is treated determines what we interpret as **total, direct and indirect** effects

Longitudinal Mediation

Cole & Maxwell (2003, 2007)



Causal Interpretation of Model Parameters

For model parameters to be **causally meaningful** we need assumptions about the underlying processes

Much of the **counterfactual** causality literature (e.g. Rubin, Pearl) is focused on identifying these assumptions

Granger causality places an emphasis on **time-ordering**

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Treatment of time determines what assumptions must be made for causal interpretation of model parameters

VAR(1) Models and Time

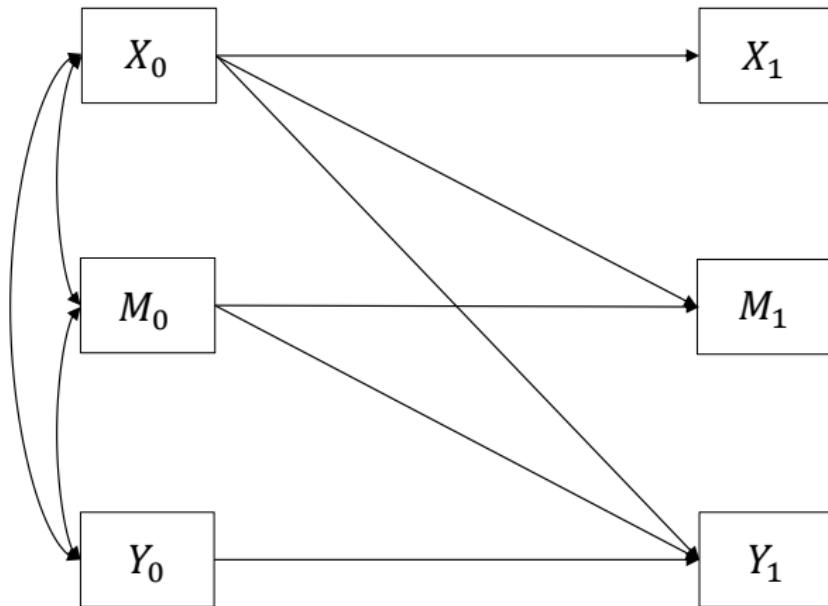
The VAR(1) model is a **Discrete Time** model

$$\mathbf{Z}_m = \mathbf{B} \mathbf{Z}_{m-1} + \mathbf{w}_m$$

- ▶ Time is taken account of w.r.t order of measurement
- ▶ Amount of time between measurements Δt_m ignored

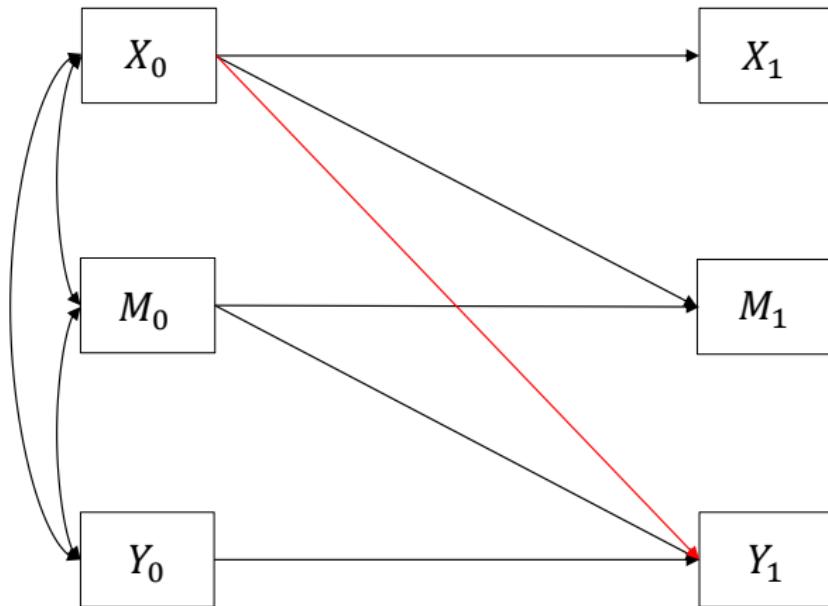
Path-Specific Effects: VAR(1)

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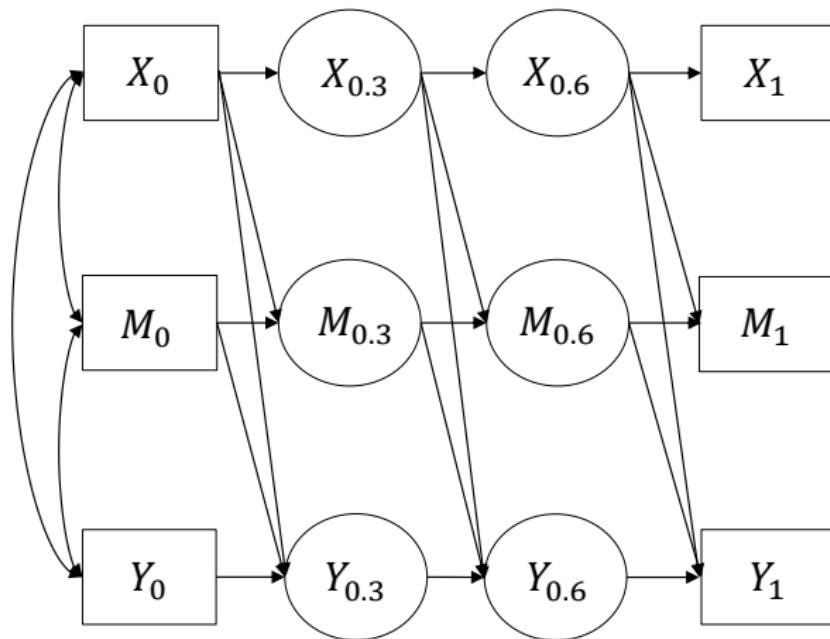
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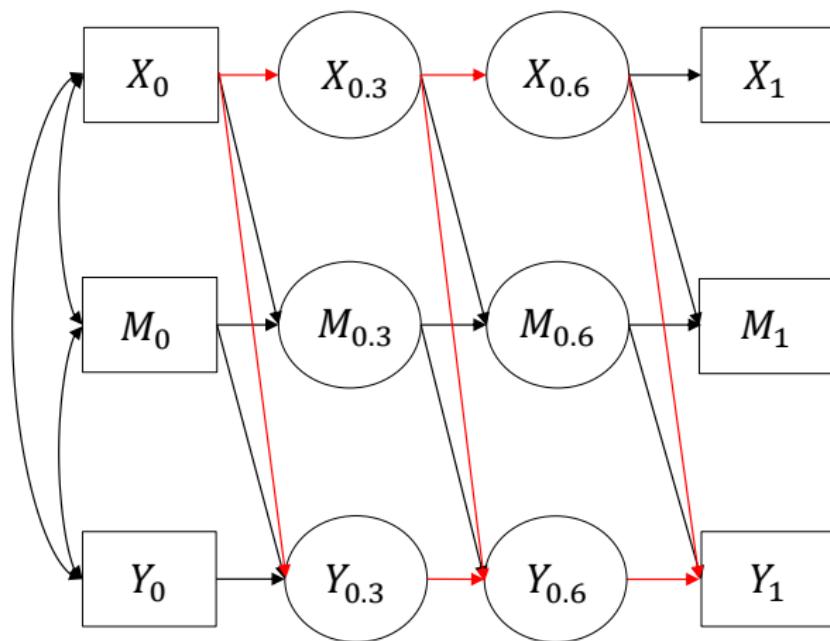
Path-Specific Effects: VAR(1)

Deboeck & Preacher (2015)



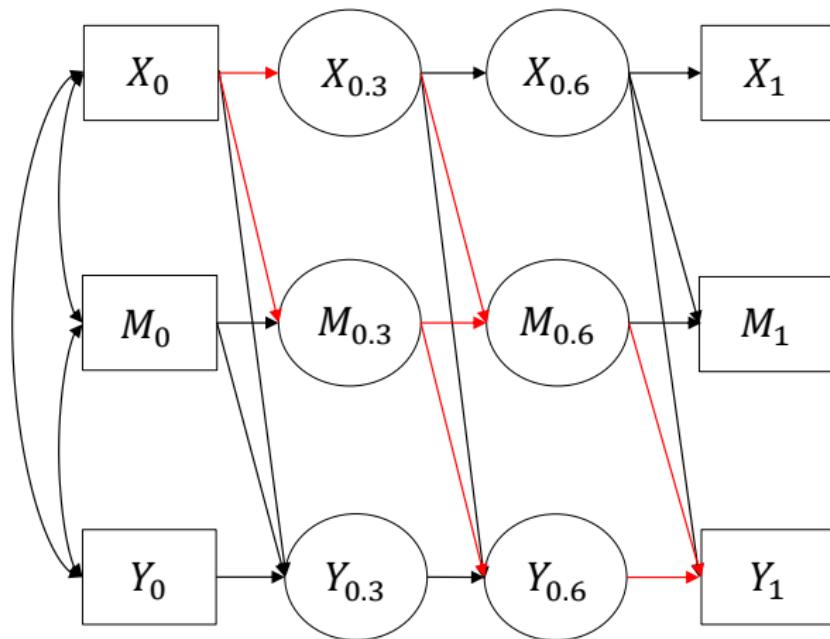
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Path-Specific Effects: VAR(1)

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Continuous Time Model

Based on a First-Order Stochastic Differential Equation
(cf. Oud & Jansen, 2000; Oravecz et al., 2011)

$$\frac{d\mathbf{Z}(t)}{dt} = \mathbf{A}\mathbf{Z}(t) + G \frac{d\mathbf{W}(t)}{dt}$$

CT VAR(1) Model

$$\mathbf{B} = e^{\mathbf{A}\Delta t_m}$$

Causality and Continuous Time

Pros:

- ▶ Models processes which evolve continuously over time
- ▶ A more “mechanistically correct” picture of how processes function (Aalen, Roysland & Gran, 2012)
- ▶ Calculation of path-specific effects for simple CT mediation model defined by Deboeck & Preacher (2015)

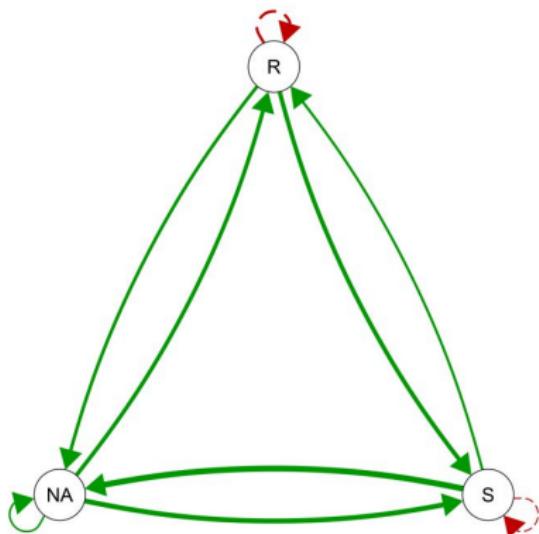
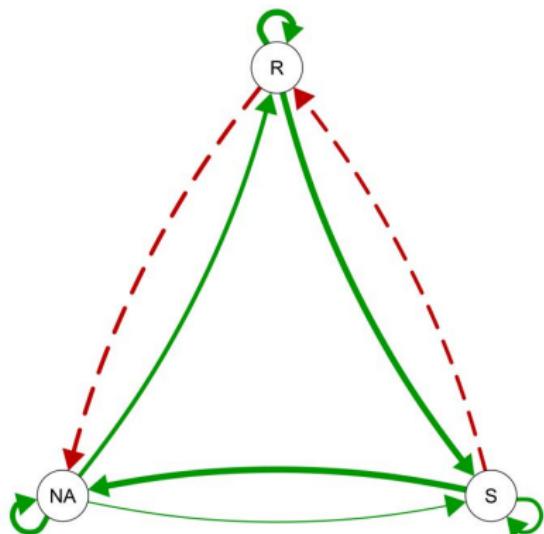
Causality and Continuous Time

Current Project:

- ▶ Adapting tools from the counterfactual approach for use with SDEs?
 - ▶ Eichler & Didelez (2007, 2010) - merging graphical approaches and Granger causality
- ▶ Calculating and defining direct and indirect effects in the general case
 - ▶ What can these tell us about the effects of interventions?

Indirect, Direct and Total Effects

Issue is exacerbated as model size increases



Application to Empirical Data

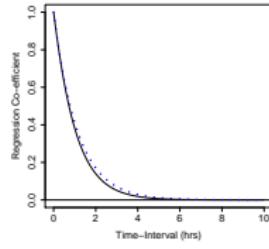
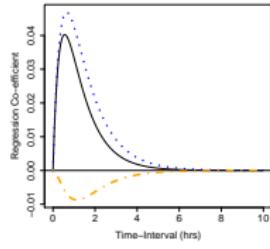
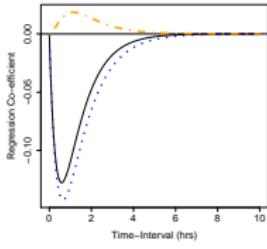
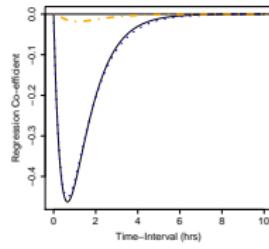
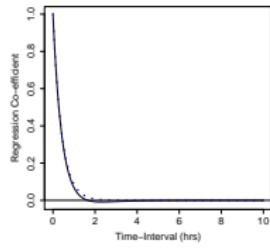
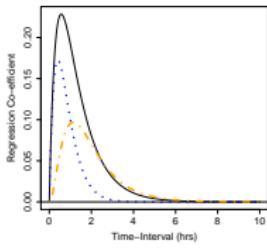
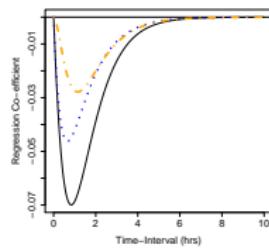
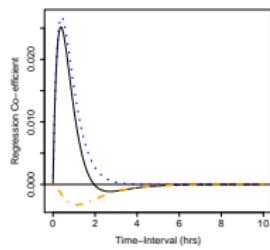
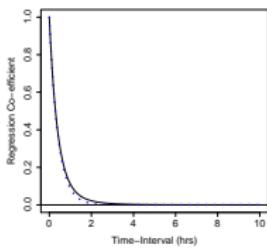
- ▶ N=1 Experience Sampling Data
- ▶ Geschwind et al. (2011)
- ▶ 115 repeated measurements
 - ▶ *Perceived Unpleasantness* (PU)
 - ▶ *Worry* (W)
 - ▶ *Relaxation* (Re)

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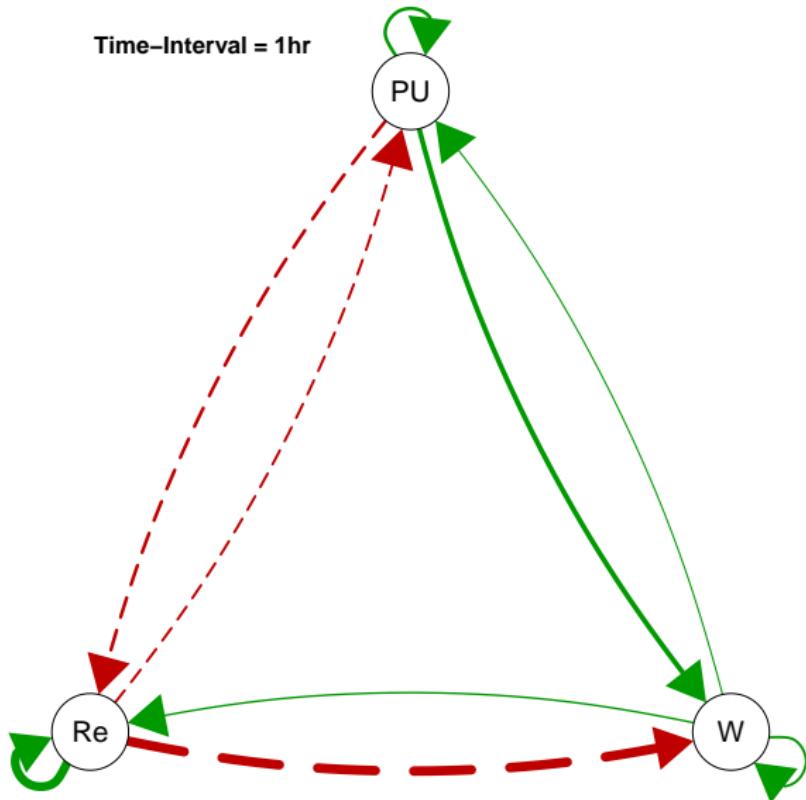
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$$\mathbf{A} = \begin{bmatrix} -2.423 & 0.177 & -0.200 \\ 1.140 & -2.445 & -1.964 \\ -0.616 & 0.204 & -0.884 \end{bmatrix}$$

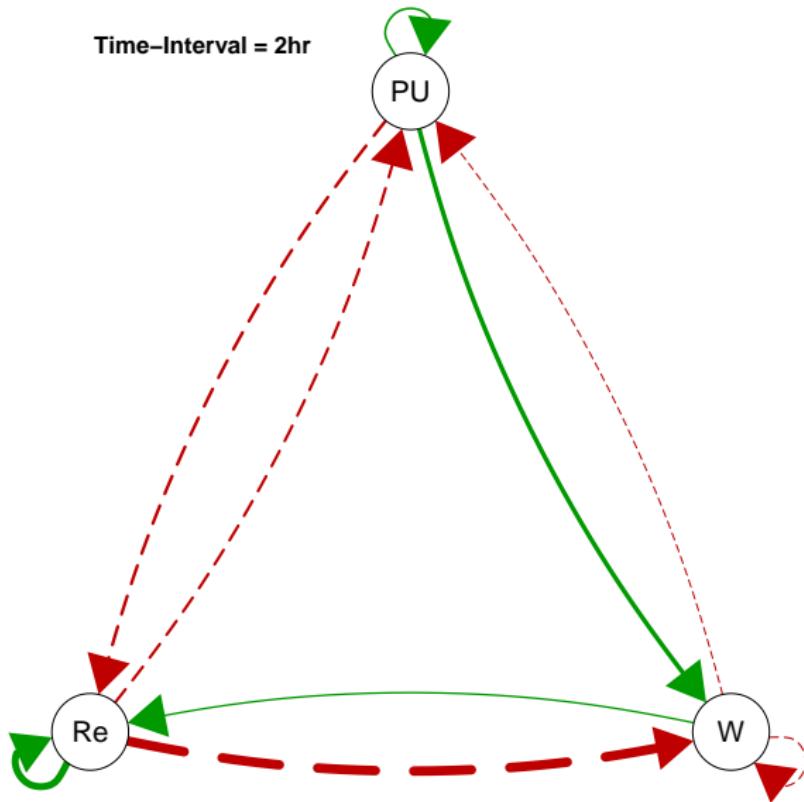
Results Empirical Data



Results - DT Network

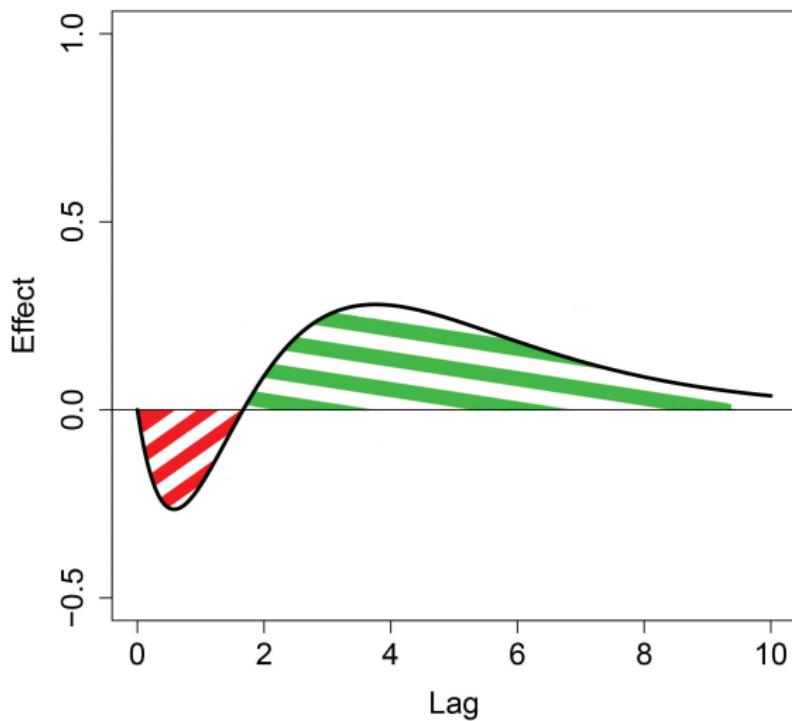


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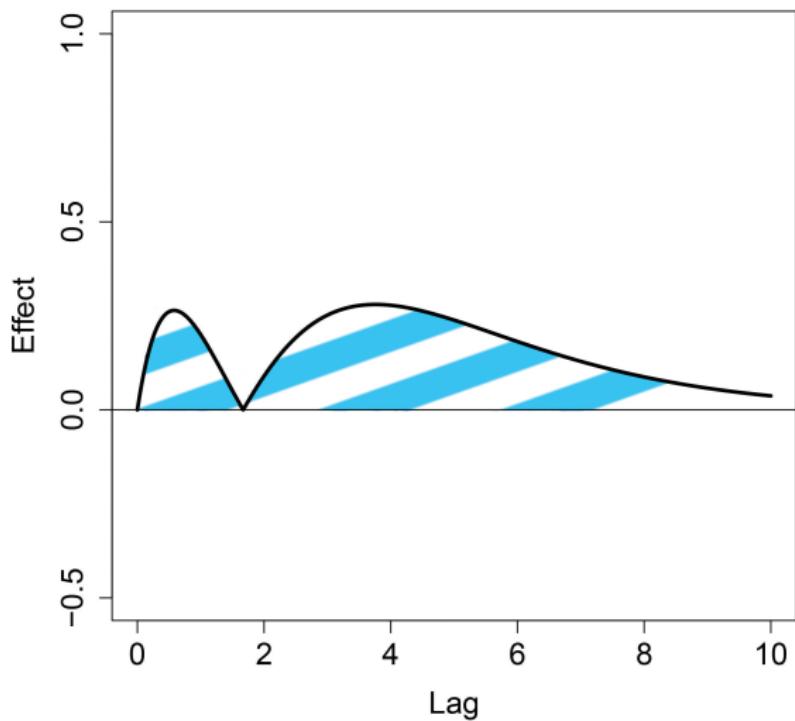
Summary Measures

1. Proportional Area Under the CT-Curve (PACT)



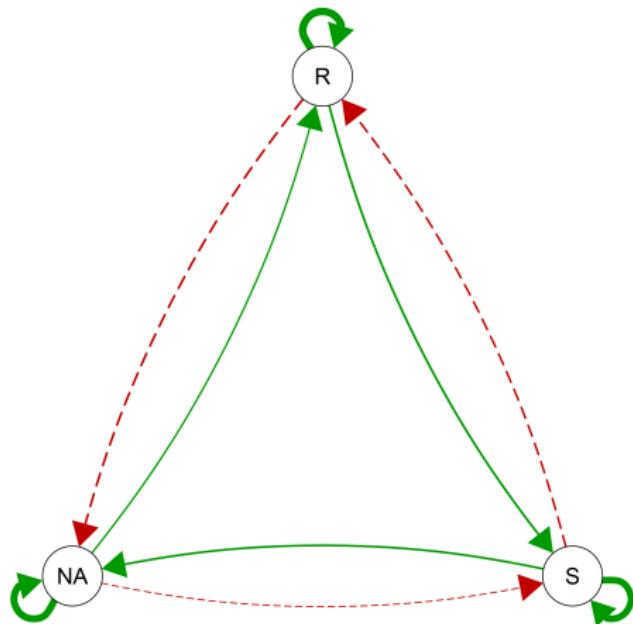
Summary Measures

2. Absolute Proportional Area Under the CT-Curve (APACT)



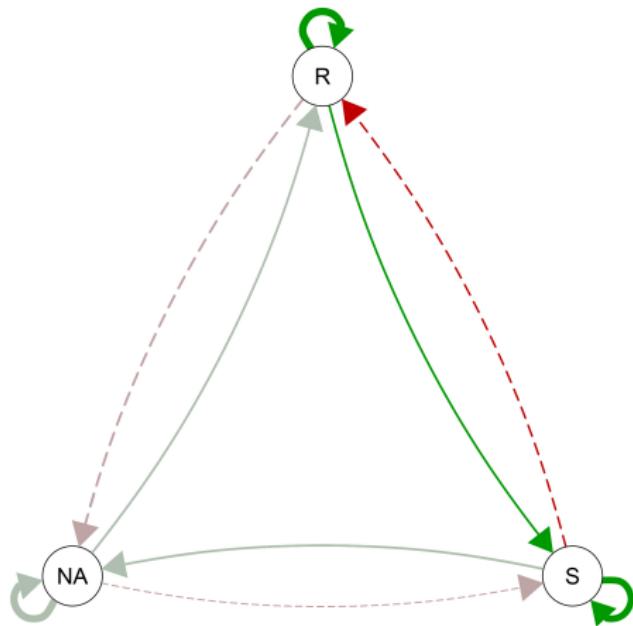
Summary Measures

3. Overall Flow Betweenness Centrality (ofb_w)



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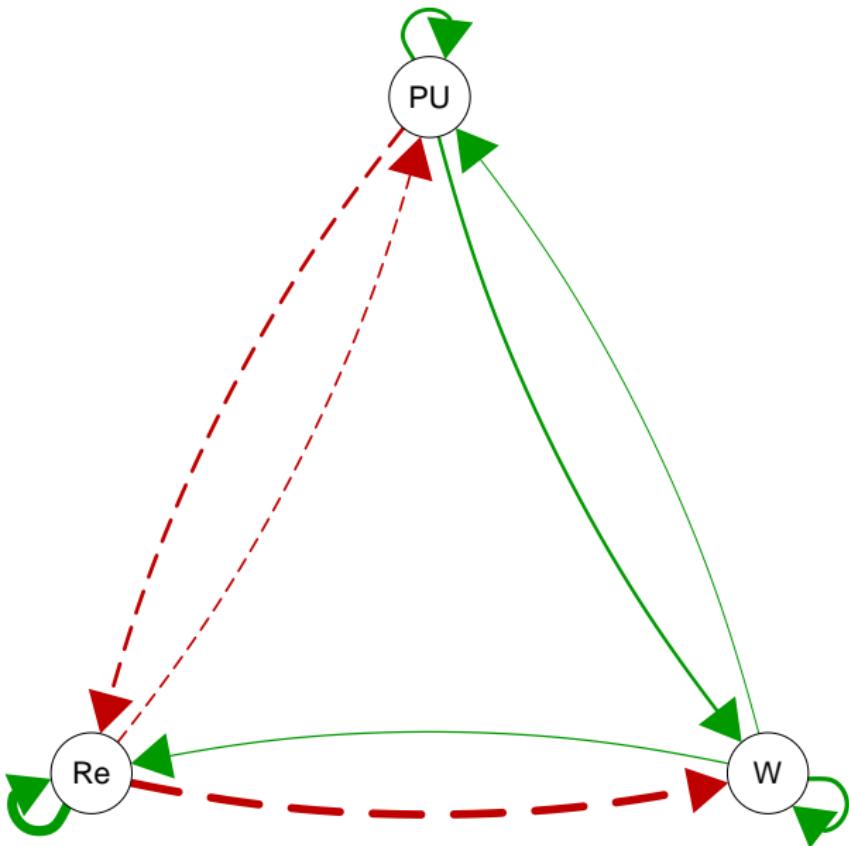
Empirical Data - PACT and APACT

Effect	PACT			APACT Total
	Total	Direct	Indirect	
$PU \rightarrow PU$	β_{11}	0.046	0.041	-
$W \rightarrow PU$	β_{12}	0.002	0.003	-0.001
$Re \rightarrow PU$	β_{13}	-0.015	-0.009	-0.007
$PU \rightarrow W$	β_{21}	0.040	0.019	0.023
$W \rightarrow W$	β_{22}	0.036	0.041	-
$Re \rightarrow W$	β_{23}	-0.090	-0.091	-0.004
$PU \rightarrow Re$	β_{31}	-0.023	-0.029	0.004
$W \rightarrow Re$	β_{32}	0.007	0.009	-0.002
$Re \rightarrow Re$	β_{33}	0.103	0.113	-

Empirical Data- Overall Flow Betweenness Centrality

Variable	Label	ofb_w
<i>Perceived Unpleasantness</i>	PU	.085
<i>Worry</i>	W	-.092
<i>Relaxation</i>	Re	.156

Empirical Data - PACT Direct Effects Network Structure



Empirical Data - Overall Network Structures

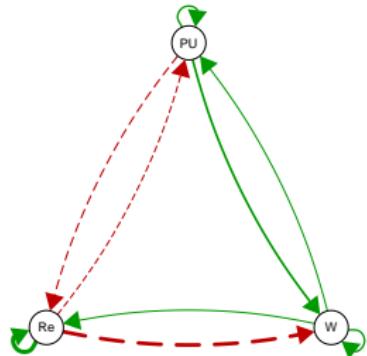


Figure: Total

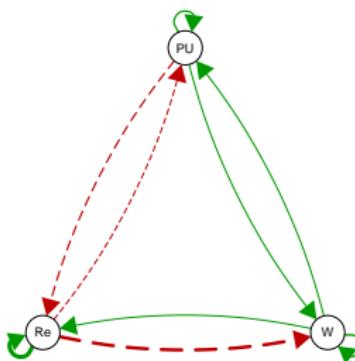


Figure: Direct

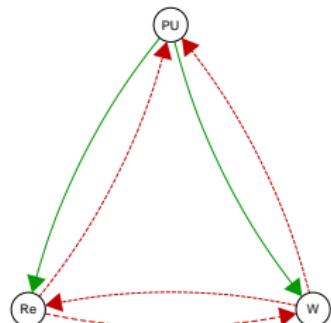
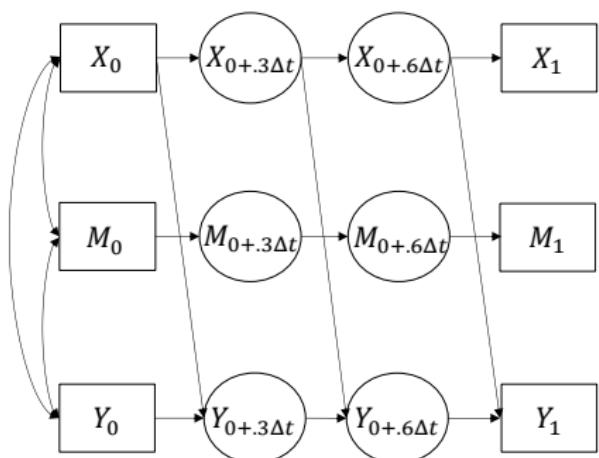


Figure: Indirect

Indirect, Direct and Total Effects

Continuous Time Framework

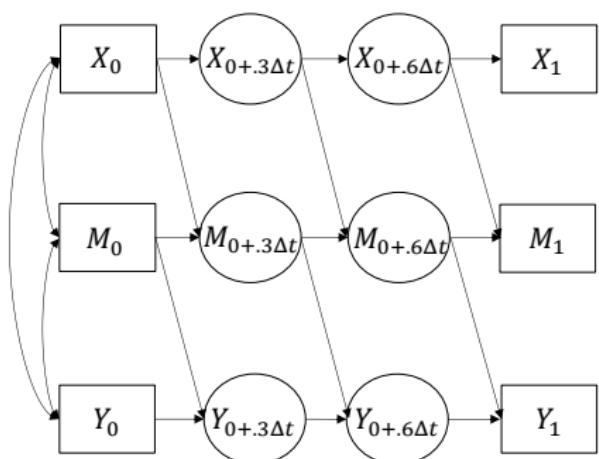


$$B(\Delta t_m) = \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ DE & 0 & b_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ a_{31} & 0 & a_{33} \end{bmatrix}$$

Indirect, Direct and Total Effects

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Key References

- Deboeck, P. R., & Preacher, K. J. (2015). No need to be discrete: A method for continuous time mediation analysis. *Structural Equation Modeling*. Available at http://quantpsy.org/pubs/deboeck_preacher_%28in.press%29.pdf
- Geschwind, N., Peeters, F., Drukker, M., van Os, J., & Wichers, M. (2011). Mindfulness training increases momentary positive emotions and reward experience in adults vulnerable to depression: a randomized controlled trial. *Journal of Consulting and Clinical Psychology*, 79(5), 618-628.