

```
for (i=0; i<n) {
```

```
  for (j=i; j<n) {
```

```
    sum = 0;
```

```
    → for (k=i; k<=j) {
```

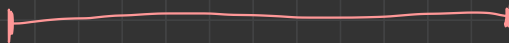
```
      sum += a[k];
```

```
    }
```

```
    if ( sum % n == 0 ) count++;
```

```
  }
```

sum = csum[j]
- csum[i-1]



1 5 4 6 8 9 10



csum

1	6	10	16	24	33	43
0	1	2	3	4	5	6

at index i , stores sum of all elements from 0 to i

prefix sum



can you find sum b/w any two indices efficiently

$i \rightarrow$ $2 \rightarrow 5$

$$\left(\underline{csum[5]} - \underline{csum[2-1]} \right) \quad \text{sum of elements b/w } \underline{2} \text{ and } \underline{5}$$

$\quad \quad \quad \underline{csum[i]}$

1 5 4 6 8 9 10

~~1~~

1	6	10	16	24	33	43
---	---	----	----	----	----	----

⑦

$$csum[i] = csum[i-1] + a[i]$$

⑧

2	4	5	6	7	18
---	---	---	---	---	----

$n = 6$

pre

0	2	6	11	17	24	42
---	---	---	----	----	----	----

suffix

42	40	36	31	25	18	0
----	----	----	----	----	----	---

csum : sum from $0 \rightarrow i$ ↙

prefix sum : sum from $0 \rightarrow i-1$

suffix : sum from $i+1 \rightarrow n-1$

2	4	5	6	7	12
---	---	---	---	---	----

$$n=6$$

sum

2	6	11	17	24	36
---	---	----	----	----	----

remainder
of csum

2	0	5	5	0	0
---	---	---	---	---	---

these sum upto a good array

$$csum[3] - csum[2]$$

$$\div 2$$

rem

0

1

2

3

4

5

index

1, 4, 5

—

0

—

11

—

1

—

2, 3

$$\begin{aligned} 2 \rightarrow 4 &\Rightarrow 5+6+7 = 18 \\ 2 \rightarrow 5 &\Rightarrow 5+6+7+12 = 30 \\ 5 \rightarrow 5 &\Rightarrow 12 \end{aligned}$$

not original arr

csum →

10	12	15	17	18
----	----	----	----	----

%8 %8 %8 %8 %8

2	4	7	1	2
---	---	---	---	---



divisibility test

m divide

remainder → {0, 1, 2, ..., m-1}

$$(a + b) \% m = ((\underline{a \% m}) + (b \% m)) \% \underline{m}$$

17/8 = 2 quotient
→ 1 rem

it is a multiple
of 8

$n=6$

2	4	5	6	7	12
---	---	---	---	---	----

cum

0	2	6	11	17	24	36
---	---	---	----	----	----	----

rem

0	2	0	5	5	0	0
---	---	---	---	---	---	---

$$5+6+7+12 = 30$$

$$\begin{array}{r} 36 \\ -6 \\ \hline 30 \end{array}$$

$$6 \% 6 = 0$$

$$24 \% 6 = 0$$

$$5+6+7 = 18$$

$$24 - 6 = 18 \% 6 = 0$$

→ we simply need to count the frequency of remainders
in csum[]

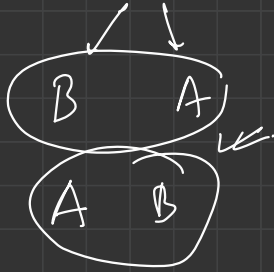
→ then we pick any two of them and it would
give a required subarray

→ we know frequency of each remainder,
and want to count no of ways we can pick
2 of those

n items

and i need to pick two
of them

A B C D E



$$nC_2$$

$$= \frac{n \times (n-1)}{2}$$

0, 0, 0, 0

arr → 2 4 5 6 7 12

sum 0 2 6 11 17 24 36.

rem 0 2 0 5 5 0 0

0-i

we can pick any two same values in rem[] and they will represent a required array.

given a freq of a remainder, we need to know no. of unique ways to choose any two of them.

$$\binom{n}{2} \rightarrow \frac{n \times (n-1)}{2}$$

4 items

A B C D

(2) $\frac{4 \times 3}{2} = 6$

1st $\frac{n}{1}$ 2nd $\frac{n-1}{3}$

$\begin{array}{cc} A & B \\ \hline & () \\ B & A \\ \hline \end{array}$

$\left\{ \begin{array}{l} \text{A B} \\ \text{A C} \\ \text{A D} \end{array} \right.$
 $\left\{ \begin{array}{l} \text{B A} \\ \text{B C} \\ \text{B D} \end{array} \right.$
 $\left\{ \begin{array}{l} \text{C A} \\ \text{C B} \\ \text{C D} \end{array} \right.$
 $\left\{ \begin{array}{l} \text{D A} \\ \text{D B} \\ \text{D C} \end{array} \right.$

generate all substrings and check

↓
TLE

$p \rightarrow$ anagram

anagram: substring(s) \rightarrow len(sub(s)) = len(p).

(len p = m)

len
 $1 \rightarrow n$

$2 \rightarrow n-1 = n-2+1$

$3 \rightarrow n-2 = n-3+1$

m len

c b a e b a w a c d

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

↓ ↓

$[n - m + 1]$

$n - (m - 1)$

x x x

abc

ans \rightarrow 0, 6

p = abc

freq p

freq subs

a	1	1
b	1	0
c	1	1
d	0	1
e	0	0

S = c b a e b a b a c d

a = 1 2 3 4

b = 9 5 1 2

$$f(0, 3) = (a[0] \cdot b[0]) + (a[1] \cdot b[1]) + (a[2] \cdot b[2]) + (a[3] \cdot b[3])$$

can i reorder a[] and b[] so that f(0,3) is minimized

$$= (1 \cdot 9) + (2 \cdot 5) + (3 \cdot 1) + (4 \cdot 2) \\ = 9 + 10 + 3 + 8 = \underline{\underline{30}}$$

$$\begin{cases} a \rightarrow 1 & 2 & 3 & 4 & (\text{inc}) \\ b \rightarrow 9 & 5 & 2 & 1 & (\text{dec}) \end{cases}$$

$$(3 \cdot 1) + (4 \cdot 2)$$

$$3 + 8 = 11$$

$$(3 \cdot 2) + (4 \cdot 1)$$

$$6 + 4 = \underline{10}$$

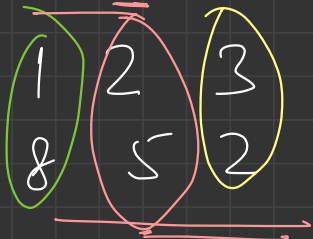
$$\Sigma \quad \underline{\underline{f(0,3) = 29}}$$

$$f(0,0) \quad f(0,1) \quad f(0,2) \quad f(0,3)$$

$$f(1,1) \quad f(1,2) \quad f(1,3)$$

$$f(2,2) \quad f(2,3)$$

$$f(3,3)$$



$\rightarrow i$ 0 1 2
3 times 4 times 3 times
 3 4 3

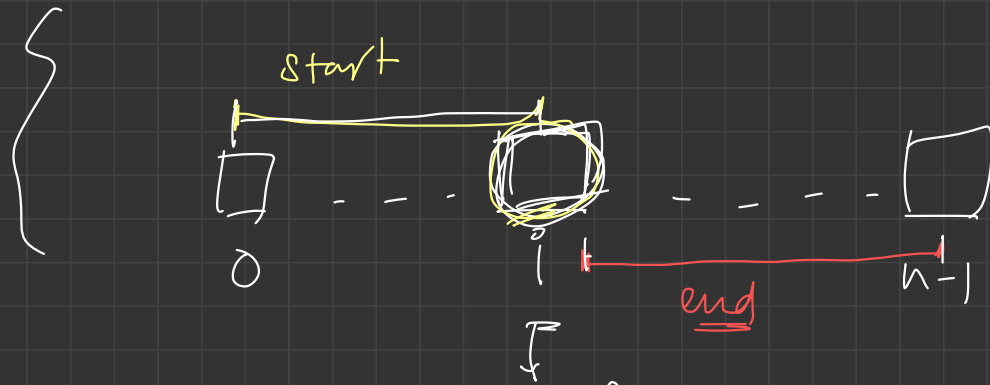
$(i+1)(n-i)$

$a \rightarrow$ 1 8 7 2 4
 $b \rightarrow$ 9 7 2 9 3
 $i =$ 0 1 2 3 4

$f(0,0) \rightarrow 8$
 $f(0,1) \rightarrow 8+10$
 $f(0,2) \rightarrow 8+10+6$
 $f(1,1) \rightarrow 10$
 $f(1,2) \rightarrow 10+6$
 $f(2,2) \rightarrow 6$

$(i+1)(n-i)$
 $i=0 : 1 \times 5$
 $i=1 : 2 \times 4$
 $i=2 : 3 \times 3$
 $i=3 : 4 \times 2$
 $i=4 : 5 \times 1$

$a \rightarrow$ 1 8 7 2 4
 $b \rightarrow$ 9 7 2 9 3



part of how many subarray?

start
 $(i+1) \times (n-i)$

every index i will be present $(i+1) \times (n-i)$ times

$$n=5$$

$$(i+1)(n-i)$$

	0	1	2	3	4
<u>contri</u>	5	8	9	8	5
	x	x	x	x	x
a →	7	5	4	6	2

{ b → 9 7 2 9 3 } reorder

contri[i] × a[i] · b[i] ^{change} for i = 0 to n-1

??

answer

$$\sum_{1 \leq l \leq r \leq n} f(l, r)$$

