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<b>1 Setup</b>	
<b>1.1 Sublime Build</b>	
"shell_cmd": "g++ -std=c++17 -o \"\$file_base_name\" \"\$file\" && timeout 2.5s ./\"\$file_base_name\" < input.txt > output.txt",	
"file_regex":	
"^([^.[:]]*)([0-9]+)?([0-9]+)?\$",	
(.*)\$",	
"working_dir": "\${file_path}",	
"selector": "source.c, source.c++"	

1.2 Template

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
using namespace std;
template <typename T> using o_set =
    tree<T, null_type, less<T>,
        rb_tree_tag,
        tree_order_statistics_node_update>;
typedef long long ll;
int32_t main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
}

2 Stress Testing
2.1 Input Gen
mt19937_64 rnd(chrono::steady_clock::now()
    ().time_since_epoch().count());
ll get_rand(ll l, ll r) {
    assert(l <= r);
    return l + rnd() % (r - l + 1);
}
2.2 Bash Script
// run -> bash script.sh

set -e
g++ code.cpp -o code
g++ gen.cpp -o gen
g++ brute.cpp -o brute
for((i = 1; ; ++i)); do
    ./gen $i > input_file
    ./code < input_file > myAnswer
    ./brute < input_file > correctAnswer
    diff -Z myAnswer correctAnswer >
        /dev/null || break
    echo "Passed test: " $i
done
echo "WA on the following test:"
cat input_file
echo "Your answer is:"
cat myAnswer
echo "Correct answer is:"
cat correctAnswer

3 Number Theory
3.1 Euler Totient Function
// Time: O(√N)
map<int, int> dp; // memo
int phi(int n) {
    if (dp.count(n)) return dp[n];
    int ans = n, m = n;
    for (int i = 2; i * i <= m; i++) {
        if (m % i == 0) {
            while (m % i == 0) m /= i;
            ans = ans / i * (i - 1);
        }
    }
    if (m > 1) ans = ans / m * (m - 1);
    return dp[n] = ans;
}
```

3.2 Phi 1 to N

```
void phi_1_to_n(int n) {
    vector<int> phi(n + 1);
    for (int i = 0; i <= n; i++)
        phi[i] = i;
    for (int i = 2; i <= n; i++) {
        if (phi[i] == i) {
            for (int j = i; j <= n; j += i)
                phi[j] -= phi[j] / i;
        }
    }
}

3.3 Segmented Sieve
vector<char> segmentedSieve(ll L, ll R) {
    // generate all primes up to √R
    ll lim = sqrt(R);
    vector<char> mark(lim + 1, false);
    vector<ll> primes;
    for (ll i = 2; i <= lim; ++i) {
        if (!mark[i]) {
            primes.emplace_back(i);
            for (ll j = i * i; j <= lim; j += i)
                mark[j] = true;
        }
    }
    vector<char> isPrime(R - L + 1, true);
    for (ll i : primes)
        for (ll j = max(i * i, (L + i - 1) / i * i); j <= R; j += i)
            isPrime[j - L] = false;
    if (L == 1) isPrime[0] = false;
    return isPrime;
}

3.4 Extended GCD
// ax + by = gcd(a,b)
int egcd(int a, int b, int& x, int& y) {
    if (b == 0) {
        x = 1, y = 0;
        return a;
    }
    int x1, y1;
    int d = egcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}

3.5 Linear Diophantine Equation
// ax + by = c, find any x and y
bool find_any_solution(int a, int b, int c, int &x0, int &y0, int &g) {
    g = egcd(abs(a), abs(b), x0, y0);
    if (c % g) return false;
    x0 *= c / g;
    y0 *= c / g;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    return true;
}
void shift_solution(int &x, int &y, int a, int b, int cnt) {
    x += cnt * b;
    y -= cnt * a;
}
```

```

}
int find_all_solutions(int a, int b, int
c, int minx, int maxx, int miny, int
maxy) {
    int x, y, g;
    if (!find_any_solution(a, b, c, x, y,
        g)) return 0;
    a /= g, b /= g;
    int sign_a = a > 0 ? +1 : -1;
    int sign_b = b > 0 ? +1 : -1;
    shift_solution(x, y, a, b, (minx - x)
        / b);
    if (x < minx) shift_solution(x, y, a,
        b, sign_b);
    if (x > maxx) return 0;
    int lx1 = x;
    shift_solution(x, y, a, b, (maxx - x)
        / b);
    if (x > maxx) shift_solution(x, y, a,
        b, -sign_b);
    int rx1 = x;
    shift_solution(x, y, a, b, -(miny - y)
        / a);
    if (y < miny) shift_solution(x, y, a,
        b, -sign_a);
    if (y > maxy) return 0;
    int lx2 = x;
    shift_solution(x, y, a, b, -(maxy - y)
        / a);
    if (y > maxy) shift_solution(x, y, a,
        b, sign_a);
    int rx2 = x;
    if (lx2 > rx2) swap(lx2, rx2);
    int lx = max(lx1, lx2);
    int rx = min(rx1, rx2);
    if (lx > rx) return 0;
    return (rx - lx) / abs(b) + 1;
}

```

### 3.6 Modular Inverse using EGCD

```

// finding inverse(a) modulo m
int x, y;
int g = extended_euclidean(a, m, x, y);
if (g != 1) cout << "No solution!";
else {
    x = (x % m + m) % m;
    cout << x << endl;
}

```

### 3.7 Exclusion DP

```

ll f[N], g[N];
for (int i = N - 1; i >= 1; i--) {
    f[i] = nC4(div_cnt[i]);
    g[i] = f[i];
    for (int j = i + 1; j < N; j += i) {
        g[i] -= g[j];
    }
}

```

Here,  $f[i]$  = how many pairs/k-tuple such that their gcd is  $i$  or it's multiple (count of pairs those are divisible by  $i$ ).

$g[i]$  = how many pairs/k-tuple such that their gcd is  $i$ .

$g[i] = f[i] - \sum_{i|j} g[j]$ .

Sum of all pair gcd:

We know, how many pairs are there such that their gcd is  $i$  for every  $i$  (1 to  $n$ ). So now,  $\sum_{i=1}^n g[i] * i$ .

Sum of all pair lcm ( $i = 1, j = 1$ ): We know,  $\text{lcm}(a, b) = \frac{a * b}{\text{gcd}(a, b)}$ .

Now,  $f[i]$  = All pair product sum of those, whose gcd is  $i$  or it's multiple.

$g[i]$  = All pair product sum of those, whose gcd is  $i$ .

$\text{Ans} = \sum_{i=1}^n \frac{g[i]}{i}$ .

All pair product sum =  $(a_1 + a_2 + \dots + a_n) * (a_1 + a_2 + \dots + a_n)$

### 3.8 Legendres Formula

//  $\frac{n!}{p^x}$  - you will get the largest  $x$

```

int legendre(int n, int p) {
    int ex = 0;
    while(n) {
        ex += (n / p);
        n /= p;
    }
    return ex;
}

```

### 3.9 Binary Expo

```

int power(int x, long long n, int mod) {
    int ans = 1 % mod;
    while (n > 0) {
        if (n & 1) {
            ans = 1LL * ans * x % mod;
        }
        x = 1LL * x * x % mod;
        n >>= 1;
    }
    return ans;
}

```

### 3.10 Digit Sum of 1 to N

// for  $n=10$ ,  $\text{ans} = 1+2+\dots+9+1+0$

```

ll solve(ll n) {
    ll res = 0, p = 1;
    while (n / p > 0) {
        ll left = n / (p * 10);
        ll cur = (n / p) % 10;
        ll right = n % p;
        res += left * 45 * p;
        res += (cur * (cur - 1) / 2) * p;
        res += cur * (right + 1);
        p *= 10;
    }
    return res;
}

```

### 3.11 Pollard Rho

```

namespace PollardRho {
mt19937 rnd(chrono::steady_clock::now().
    time_since_epoch().count());
const int P = 1e6 + 9;
ll seq[P];
int primes[P], spf[P];
inline ll add_mod(ll x, ll y, ll m) {
    return (x += y) < m ? x : x - m;
}
inline ll mul_mod(ll x, ll y, ll m) {
    ll res = __int128(x) * y % m;
    return res;
}
}

```

```

// ll res = x * y - (ll)((long double)x
    * y / m + 0.5) * m;
// return res < 0 ? res + m : res;
}
inline ll pow_mod(ll x, ll n, ll m) {
    ll res = 1 % m;
    for (; n; n >>= 1) {
        if (n & 1) res = mul_mod(res, x, m);
        x = mul_mod(x, x, m);
    }
    return res;
}
//  $O(it * (\log n)^3)$ , it = number of rounds
    performed
inline bool miller_rabin(ll n) {
    if (n <= 2 || (n & 1 ^ 1)) return (n
        == 2);
    if (n < P) return spf[n] == n;
    ll c, d, s = 0, r = n - 1;
    for (; !(r & 1); r >>= 1, s++) {}
    // each iteration is a round
    for (int i = 0; primes[i] < n &&
        primes[i] < 32; i++) {
        c = pow_mod(primes[i], r, n);
        for (int j = 0; j < s; j++) {
            d = mul_mod(c, c, n);
            if (d == 1 && c != 1 && c != (n -
                1)) return false;
            c = d;
        }
        if (c != 1) return false;
    }
    return true;
}
void init() {
    int cnt = 0;
    for (int i = 2; i < P; i++) {
        if (!spf[i]) primes[cnt++] = spf[i]
            = i;
        for (int j = 0, k; (k = i *
            primes[j]) < P; j++) {
            spf[k] = primes[j];
            if (spf[i] == spf[k]) break;
        }
    }
}
// returns  $O(n^{\frac{1}{4}})$ 
ll pollard_rho(ll n) {
    while (1) {
        ll x = rnd() % n, y = x, c = rnd() %
            n, u = 1, v, t = 0;
        ll *px = seq, *py = seq;
        while (1) {
            *py++ = y = add_mod(mul_mod(y, y,
                n), c, n);
            *py++ = y = add_mod(mul_mod(y, y,
                n), c, n);
            if ((x = *px++) == y) break;
            v = u;
            u = mul_mod(u, abs(y - x), n);
            if (!u) return __gcd(v, n);
            if (++t == 32) {
                t = 0;

```

```

                if ((u = __gcd(u, n)) > 1 && u <
                    n) return u;
            }
        }
    }
    vector<ll> factorize(ll n) {
        if (n == 1) return vector<ll>();
        if (miller_rabin(n)) return vector<ll>
            {n};
        vector<ll> v, w;
        while (n > 1 && n < P) {
            v.push_back(spf[n]);
            n /= spf[n];
        }
        if (n >= P) {
            ll x = pollard_rho(n);
            v = factorize(x);
            w = factorize(n / x);
            v.insert(v.end(), w.begin(), w.end());
        }
        return v;
    }
}

```

### 3.12 [Problem] How Many Bases - UVA

// Given a number  $N^M$ , find out the number of integer bases in which it has exactly  $T$  trailing zeroes.

```

int solve_greater_or_equal(vector<int>
e, int t) {
    int ans = 1;
    for (auto i : e) {
        ans = 1LL * ans * (i / t + 1) % mod;
    }
    return ans;
}
// e contains  $e_1, e_2 \rightarrow p_1^{e_1}, p_2^{e_2}$ 
int solve_equal(vector<int> e, int t) {
    return (solve_greater_or_equal(e, t) -
        solve_greater_or_equal(e, t + 1) +
        mod) % mod;
}

```

### 3.13 [Problem] Power Tower - CF

// A sequence  $w_1, w_2, \dots, w_n$  and  $Q$  queries,  $l$  and  $r$  will be given.

Calculate  $w_l^{(w_{l+1}^{(w_{l+2}^{(w_r)})})}$

//  $n^x \bmod m = n^{\phi(m) + x \bmod \phi(m)} \bmod m$

```

inline int MOD(int x, int m) {
    if (x < m) return x;
    return x % m + m;
}
int power(int n, int k, int mod) {
    int ans = MOD(1, mod);
    while (k) {
        if (k & 1) ans = MOD(ans * n, mod);
        n = MOD(n * n, mod);
        k >>= 1;
    }
    return ans;
}

```

```
int f(int l, int r, int m) {
    if (l == r) return MOD(a[l], m);
    if (m == 1) return 1;
    return power(a[l], f(l + 1, r,
        phi(m)), m);
}
```

### 3.14 Formula and Properties

- $\phi(n) = n \cdot \frac{p_1-1}{p_1} \cdot \frac{p_2-1}{p_2} \dots$
- $\phi(p^e) = p^e - \frac{p^e}{p} = p^e \cdot \frac{p-1}{p}$
- For  $n > 2$ ,  $\phi(n)$  is always even.
- $\sum_{d|n} \phi(d) = n$
- NOD:  $(e_1 + 1) \cdot (e_2 + 1) \dots$
- SOD:  $\frac{p_1^{e_1+1}-1}{p_1-1} \cdot \frac{p_2^{e_2+1}-1}{p_2-1} \dots$
- $\log(a \cdot b) = \log(a) + \log(b)$
- $\log(a^x) = x \cdot \log(a)$
- $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$
- Digit Count of n:**  $\lfloor \log_{10}(n) \rfloor + 1$
- Arithmetic Progression Sum:**  $\frac{n}{2} \cdot (a + p)$ ,  $\frac{n}{2} \cdot (2a + (n - 1)d)$
- Geometric Sum:**  $S_n = a \cdot \frac{r^n-1}{r-1}$
- $(1^2 + 2^2 + \dots + n^2) = \frac{n(n+1)(2n+1)}{6}$
- $(1^3 + 2^3 + \dots + n^3) = \frac{n^2(n+1)^2}{4}$
- $(2^2 + 4^2 + \dots + (2n)^2) = \frac{2n(n+1)(2n+1)}{3}$
- $(1^2 + 3^2 + \dots + (2n - 1)^2) = \frac{n(2n-1)(2n+1)}{3}$
- $(2^3 + 4^3 + \dots + (2n)^3) = 2n^2(n+1)^2$
- $(1^3 + 3^3 + \dots + (2n - 1)^3) = n^2(2n^2 - 1)$
- For any number  $n$  and bases  $> \sqrt{n}$ , there will be no representation where the number contains 0 at its second least significant digit. So it is enough to check for bases  $\leq \sqrt{n}$ .
- For some  $x$  and  $y$ , let's try to find all  $m$  such that  $x \bmod m \equiv y \bmod m$ . We can rearrange the equation into  $(x - y) \equiv 0 \pmod m$ . Thus, if  $m$  is a factor of  $|x - y|$ , then  $x$  and  $y$  will be equal modulo  $m$ .

## 4 Combinatorics and Probability

### 4.1 Combinations

```
// Prime Mod in O(n)
void prec() {
    fact[0] = 1;
    for (int i = 1; i < N; i++) {
        fact[i] = 1ll * fact[i - 1] * i % mod;
    }
    ifact[N - 1] = inverse(fact[N - 1]);
    for (int i = N - 2; i >= 0; i--) {
        ifact[i] = 1ll * ifact[i + 1] * (i +
            1) % mod;
    }
}
```

```
int nCr(int n, int r) {
    if (r > n) return 0;
```

```
    return 1ll * fact[n] * ifact[r] % mod
        * ifact[n - r] % mod;
}
int nPr(int n, int r) {
    if (r > n) return 0;
    return 1ll * fact[n] * ifact[n - r] %
        mod;
}
```

### 4.2 nCr for any mod

```
// Time: O(n^2)
// nCr = (n-1)C(r-1) + (n-1)Cr
for (int i = 0; i < N; i++) {
    C[i][i] = 1;
    for (int j = 0; j < i; j++) {
        C[i][j] = (C[i - 1][j] + C[i - 1][j
            - 1]) % mod;
    }
}
```

### 4.3 nCk without mod in O(r)

```
ll nCk(ll n, ll k) {
    double res = 1;
    for (ll i = 1; i <= k; ++i)
        res = res * (n - k + i) / i;
    return (ll)(res + 0.01);
}
```

### 4.4 Lucas Theorem

```
// returns nCr modulo mod where mod is a
// prime
// Complexity: ?
ll Lucas(ll n, ll r) {
    if (r < 0 || r > n) return 0;
    if (r == 0 || r == n) return 1;
    if (n >= MOD) {
        return (Lucas(n / MOD, r / MOD) %
            MOD * Lucas(n % MOD, r % MOD) %
            MOD) % MOD;
    }
    return (((fact[n] * invFact[r]) % MOD)
        * invFact[n - r]) % MOD;
}
```

### 4.5 Catalan Number

```
const int MOD = 1e9 + 7, int MAX = 1e7;
int catalan[MAX];
void init(ll n) {
    catalan[0] = catalan[1] = 1;
    for ( ll i = 2; i <= n; i++ ) {
        catalan[i] = 0;
        for ( ll j = 0; j < i; j++ ) {
            catalan[i] += ( catalan[j] *
                catalan[i - j - 1] ) % MOD;
            if ( catalan[i] >= MOD ) {
                catalan[i] -= MOD;
            }
        }
    }
}
```

### 4.6 Derangement

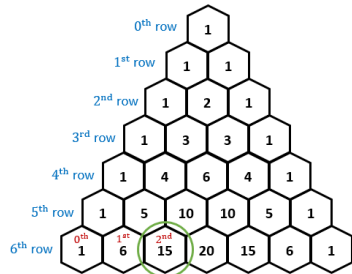
```
// number of combinations such that
// a_i! = i of a permutation a
const int N = 1e6 + 100, int p = 1e9 + 7;
ll der[N];
```

```
void countDer() {
    der[1] = 0;
    der[2] = 1;
    for (ll i = 3; i <= N; ++i) {
        der[i] = (i - 1) % p * (der[i - 1] %
            p + der[i - 2] % p);
        der[i] %= p;
    }
}
```

### 4.7 Stars and Bars Theorem

- Find the number of  $k$ -tuples of non-negative integers whose sum is  $n$ .  $\binom{n+k-1}{n}$
- Find the number of  $k$ -tuples of non-negative integers whose sum is  $\leq n$ .  $\binom{n+k}{k}$
- Combination with Repetition (choose  $k$  elements from  $n$  objects, same element can be chosen multiple times).  $\binom{n+k-1}{k}$
- How many ways to go from  $(0, 0)$  to  $(n, m)$ .  $\binom{n+m}{m}$

Pascals Triangle is equivalent to nCr:



### 4.8 Properties of Pascal's Triangle

- $(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$
- $(k + 1)^n = \sum_{i=0}^n k^i \cdot \binom{n}{i}$
- $\sum_{i=0}^n \binom{n}{i} = 2^n$
- $\binom{k}{n} = \frac{k}{n} \binom{k-1}{n-1}$
- $\sum_{k=0}^m \binom{n+k}{k} = \binom{n+m+1}{m}$
- $\binom{n}{0}^2 + \binom{n}{1}^2 + \dots + \binom{n}{n}^2 = \binom{2n}{n}$
- $1\binom{n}{1} + 2\binom{n}{2} + \dots + n\binom{n}{n} = n \cdot 2^{n-1}$

### 4.9 Contribution Technique

- Sum of all pair sums:  $\sum_{i=1}^n \sum_{j=1}^n (a_i + a_j)$   
Every element will be added  $2n$  times.  $\sum_{i=1}^n (2 \times n \times a_i) = 2 \times n \times \sum_{i=1}^n a_i$ .
- Sum of all subarray sums —  $\sum_{i=1}^n (a_i \times i \times (n - i + 1))$ .
- Sum of all Subsets sums —  $\sum_{i=1}^n (2^{n-1} \times a_i)$ .
- Product of all pair product —  $\prod_{i=1}^n \left(a_i^{2 \times n}\right)$ .

- XOR of subarray XORS — How many subarrays does an element have?  $(i \cdot (n - i + 1))$  times). If subarray count is odd then this element can contribute in total XORS.
- Sum of max minus min over all subset — Sort the array.  $Min = 2^{n-i}$ ,  $Max = 2^{i-1}$ .  $\sum_{i=1}^n (a_i \cdot 2^{i-1} - a_i \cdot 2^{n-i})$
- Sum using bits —  $\sum_{k=0}^{30} (cnt_k[1] \times 2^k)$ .
- Sum of Pair XORS — XOR will 1 if two bits are different  $\sum_{k=0}^{30} (cnt_k[0] \times cnt_k[1] \times 2^k)$ .
- Sum of Pair ANDs —  $\sum_{k=0}^{30} (cnt_k[1]^2 \times 2^k)$ .
- Sum of Pair ORs —  $\sum_{k=0}^{30} ((cnt_k[1]^2 + 2 \times cnt_k[1] \times cnt_k[0]) \times 2^k)$ .
- Sum of Subset XORS — where  $cnt0! = 0$   $\sum_{k=0}^{30} (2^{cnt_k[1]+cnt_k[0]-1} \times 2^k)$ .
- Sum of Subset ANDs —  $\sum_{k=0}^{30} ((2^{cnt_k[1]} - 1) \times 2^k)$ .
- Sum of Subset ORs —  $\sum_{k=0}^{30} ((2^n - 2^{cnt_k[0]}) \times 2^k)$ .
- Sum of subarray XORS — Convert to prefix xor, then solve for pairs.
- Sum of product of all subsequence —  $\prod_{i=1}^n (a_i + 1) - 1$ . Example array —  $[a, b]$  the subsequences are  $\{a\}$ ,  $\{b\}$ ,  $\{a, b\}$  so ans is  $a + b + (a \cdot b)$

### 4.10 Probability and Expected Value

- Expected Value:  $E = \frac{\text{Sum of all possible values}}{\text{Total number of outcomes}}$
- Expected Value:  $E = \sum_{i=1}^n P_i \cdot i$
- Variance:  $V(x) = E(x^2) - \{E(x)\}^2$
- Expected Value with DP:  $E(i) = \sum P(i \rightarrow j) \times (R(i \rightarrow j) + E(j))$  Where  $R()$  is Immediate Reward(cost/count)
- Linearity of Expectation:
  - $E[X + Y] = E[X] + E[Y]$
  - $E[\text{Total}] = E[I_1] + E[I_2] + \dots = \sum P(I_i = 1)$

We need to define the Indicator Random Variable (I) correctly. Some Examples below —

**Problem-1:** Find  $E[\text{correct hats}]$  in a random permutation. Indicator  $I_i$ : "Does person  $i$  get their own hat?"

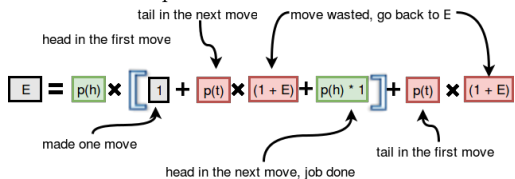
**Problem-2:** Find  $E[\text{total inversions}]$  in a random permutation. Indicator  $I_{ij}$ : "Is pair  $(i, j)$  an inversion?"

**Problem-3:** Given a string  $S$ , delete a random index until it becomes empty. Find the expected count of palindromes seen (exclude  $S$ , include empty string). Indicator  $I_L$ : "Is the string of length  $L$  a palindrome?"

**Problem-4:** Pick  $N$  random integers from  $[1, k]$ . Merge consecutive equal values (e.g.,  $1, 1, 2, 2, 1, 1 \rightarrow 1, 2, 1$ ). Find the expected final length. Indicator  $I_i$ : "Is item  $i$  different from item  $i - 1$ ?"



- To get two consecutive heads, what is the expected number of tosses?



- To get  $n$  heads, what is the expected number of tosses? Let's define: to get  $n$  heads, we need to toss  $E(n)$  times. Now — I can get a head; I need to toss  $E(n - 1)$  more times, or if I get a tail; I need to toss  $E(n)$  times. So, the recurrence is:  $E(n) = 0.5 \cdot (1 + E(n - 1)) + 0.5 \cdot (1 + E(n))$
- You have  $n$  bulbs, all of which are initially off. In each move, you randomly select one bulb. If the selected bulb is **off**, you toss a coin:
  - If you get head, you turn it on.
  - If you get tail, you do nothing.

If the bulb is already **on**, you skip that move (nothing happens).

Now, what is the expected number of moves required to turn all bulbs on?

The coin is not fair — the probability of getting tail is  $p$ . This problem can also be solved recursively.

Let's assume at some moment,  $x$  bulbs are already on, and the expected number of moves needed from here is  $e(x)$ .

The probability of picking an already on bulb is  $\frac{x}{n}$ . In that case, the expected number of moves is  $\frac{x}{n} \times (1 + e(x))$ .

The probability of picking an off bulb is  $\frac{n-x}{n}$ .

Now two things can happen:

- With probability  $p$ , you get tail, so you stay at the same state ( $e(x)$  more moves).
- With probability  $(1 - p)$ , you get head, so one more bulb turns on ( $e(x + 1)$  moves from there).

So, the recurrence relation is:

$$e(x) = \frac{x}{n}(1 + e(x)) + \frac{n-x}{n}(p(1 + e(x)) + (1-p)(1 + e(x+1)))$$

## 5 Data Structure

### 5.1 Trie for bit

```
struct Trie {
    static const int B = 31;
    struct node {
        node* nxt[2];
        int sz;
        node() {
            nxt[0] = nxt[1] = NULL;
            sz = 0;
        }
    };
    node* root;
    Trie() {
```

```
    root = new node();
}
void insert(int val) {
    node* cur = root;
    cur->sz++;
    for (int i = B - 1; i >= 0; i--) {
        int b = val >> i & 1;
        if (cur->nxt[b] == NULL) cur->
            nxt[b] = new node();
        cur = cur->nxt[b];
        cur->sz++;
    }
}
int query(int x, int k) { // number
    of values s.t. val ⊕ x < k
    node* cur = root;
    int ans = 0;
    for (int i = B - 1; i >= 0; i--) {
        if (cur == NULL) break;
        int b1 = x >> i & 1, b2 = k >> i &
            1;
        if (b2 == 1) {
            if (cur->nxt[b1]) ans += cur->
                sz;
            cur = cur->nxt[b1];
        } else cur = cur->nxt[b1];
    }
    return ans;
}
int get_max(int x) { // returns
    maximum of val ⊕ x
    node* cur = root;
    int ans = 0;
    for (int i = B - 1; i >= 0; i--) {
        int k = x >> i & 1;
        if (cur->nxt[!k]) cur = cur->
            nxt[!k], ans <= 1, ans++;
        else cur = cur->nxt[k], ans <= 1;
    }
    return ans;
}
int get_min(int x) { // returns
    minimum of val ⊕ x
    node* cur = root;
    int ans = 0;
    for (int i = B - 1; i >= 0; i--) {
        int k = x >> i & 1;
        if (cur->nxt[k]) cur = cur->
            nxt[k], ans <= 1;
        else cur = cur->nxt[!k], ans <=
            1, ans++;
    }
    return ans;
}
} t;
void del(node* cur) {
    for (int i = 0; i < 2; i++) if (cur->
        nxt[i]) del(cur->nxt[i]);
    delete(cur);
}
} t;
```

## 6 Dynamic Programming

### 6.1 Knapsack 2

Constraints:  $N \leq 100$ ,  $W \leq 1e9$ ,  $val[i] \leq 1000$

$dp[i][cur\_val]$  = min weight needed to achieve  $cur\_val$  from  $i$  to  $n$ . if  $dp[1][val] \leq W$  then  $val$  is a candidate ans. where  $val$  is all\_possible\_val (1 to  $100 \cdot 1000$ )

### 6.2 LIS using Segment Tree

```
int32_t main() {
    int n; cin >> n;
    for (int i = 1; i <= n; i++) {
        cin >> a[i]; // a[i] must be >= 2
    }
    // dp[i] = LIS ending at pos i
    st.build(1, 1, M); // range max query,
        and upd idx with max(cur_val,
        new_val)
    for (int i = 1; i <= n; i++) {
        dp[i] = 1;
        if (a[i] != 1) {
            int mx = st.query(1, 1, M, 1, a[i]
                - 1);
            mx++;
            dp[i] = max(dp[i], mx);
        }
        st.upd(1, 1, M, a[i], dp[i]);
    }
    int ans = 0;
    for (int i = 1; i <= n; i++) ans =
        max(ans, dp[i]);
    cout << ans << '\n';
}
```

### 6.3 Digit DP

// Problem: How Many Zeroes? - LightOJ  
// How many zeroes between  $n$  to  $m$   
( $n \leq m$ ). 100 to 102 ans 4

```
ll fun2(int i, bool is_small) {
    if (i == sz) return 1;
    int l = 0, r = s[i] - '0';
    if (is_small) r = 9;
    ll &ans = dp2[i][is_small];
    if (ans != -1) return ans;
    ans = 0;
    for (int x = 1; x <= r; x++) {
        ans += fun2(i + 1, (is_small | (x <
            r)));
    }
    return ans;
}
ll fun(int i, bool is_small, bool
    has_started) {
    if (i == sz) return 0;
    int l = 0, r = s[i] - '0';
    if (is_small) r = 9;
    ll &ans = dp1[i][is_small][has_started];
    if (ans != -1) return ans;
    ans = 0;
    for (int x = 1; x <= r; x++) {
        bool new_has_started = has_started |
            (x != 0);
        ans += fun(i + 1, (is_small | (x <
            r)), new_has_started);
    }
    if (x == 0 and has_started) {
        ans += fun2(i + 1, (is_small | (x
            < r)));
    }
}
```

```
    }
}
return ans;
}
void get(long long x) {
    if (x < 0) return; s = "";
    while (x > 0) {
        char c = (x % 10) + '0';
        s += c; x /= 10;
    }
    reverse(s.begin(), s.end());
    sz = s.size();
    memset(dp1, -1, sizeof(dp1));
    memset(dp2, -1, sizeof(dp2));
}
void solve() {
    ll n, m; cin >> n >> m;
    get(n - 1);
    memset(dp1, -1, sizeof(dp1));
    memset(dp2, -1, sizeof(dp2));
    ll ans1 = (n == 0) ? 0 : fun(0, false,
        false);
    get(m);
    ll ans2 = fun(0, false, false);
    cout << ans2 - ans1 + (n == 0) << '\n';
}
```

### 6.4 Bitmask DP

// Problem: DNA Sequence - LOJ  
// Given  $n$  strings, find the shortest string that contains all the given strings as substrings and lexicographically smallest.

```
const int N = 17;
int n, tail[N][N], dp[(1 << N) + 2][N +
    2];
string s[N + 2];
bool cmp(string a, string b) {
    if (a.size() < b.size()) {
        return true;
    }
    return false;
}
int fun(int mask, int last) {
    if (__builtin_popcount(mask) >= n)
        return 0;
    int &ans = dp[mask][last];
    if (ans != -1) return ans;
    ans = 1e9;
    for (int j = 1; j <= n; j++) {
        if (!(mask & (1 << j))) {
            if (last == n + 1) {
                int x = s[j].length() + fun(mask
                    | (1 << j), j);
                ans = min(ans, x);
            }
            else {
                int x = (s[j].length() -
                    tail[last][j]) + fun(mask |
                    (1 << j), j);
                ans = min(ans, x);
            }
        }
    }
    return ans;
}
```

```

}
void print(int mask, int last) {
    if (__builtin_popcount(mask) >= n)
        return;
    int ans = fun(mask, last), idx = -1;
    string str = "";
    for (int j = 1; j <= n; j++) {
        if (!(mask & (1 << j))) {
            if (last == n + 1) {
                int x = s[j].length() + fun(mask | (1 << j), j);
                if (x == ans) {
                    string d = s[j];
                    if (d <= str) str = d; idx = j;
                }
            }
            else {
                int x = (s[j].length() - tail[last][j]) + fun(mask | (1 << j), j);
                if (x == ans) {
                    string d = s[j].substr(tail[last][j]);
                    if (d <= str) str = d; idx = j;
                }
            }
        }
    }
    cout << s[idx].substr(tail[last][idx]);
    print(mask | (1 << idx), idx);
}

void solve() {
    cin >> n;
    string str[n + 1];
    for (int i = 1; i <= n; i++) {
        cin >> str[i]; // len <= 100
    }
    // remove duplicates and strings which
    // is a subarray of others
    sort(str + 1, str + n + 1, cmp);
    vector<string> vec;
    for (int i = 1; i <= n; i++) {
        bool ok = true;
        for (int j = i + 1; j <= n; j++) {
            if (str[j].find(str[i]) != string::npos) {
                ok = false;
                break;
            }
        }
        if (ok) vec.push_back(str[i]);
    }
    n = vec.size();
    int idx = 1;
    for (auto x : vec) s[idx++] = x;
    // maximum length that suffix of a[i] =
    // prefix of a[j]
    memset(tail, 0, sizeof(tail));
    for (int i = 1; i <= n; i++) {
        string x = s[i];
        for (int j = 1; j <= n; j++) {
            string y = s[j];
            int cnt = 0;
            for (int k = min(x.length(), y.length()); k >= 1; k--) {

```

```

                if (x.substr(x.length() - k) ==
                    y.substr(0, k)) {
                    cnt = k;
                    break;
                }
            }
            tail[i][j] = cnt;
        }
    }
    memset(dp, -1, sizeof(dp));
    fun(0, n + 1);
    print(0, n + 1);
}

```

## 6.5 MCM DP

*// Problem: Slimes - Atcoder DP Contest*  
*// Given n slimes. Choose two adjacent*  
*slimes, and combine them into a new*  
*slime. The new slime has a size of*  
*x+y, where x and y are the sizes of*  
*the slimes before combining them.*  
*Here, a cost of x+y is incurred.*  
*Example-*  
*// (10, 20, 30, 40) + (30, 30, 40)*  
*// (30, 30, 40) + (60, 40)*  
*// (60, 40) + (100) ans = 190*  
*// Solution: Think reverse. We*  
*are given the final sum, from i to j.*  
*Now we will cut any point between i to*  
*j and calculate the cost*  
*// Time: O(n<sup>3</sup>)*

```

ll fun(int i, int j) {
    if (i == j) return 0;
    ll &ans = dp[i][j];
    if (ans != -1) return ans;
    ll cur = 0;
    for (int x = i; x <= j; x++) {
        cur += a[x];
    }
    ans = inf;
    for (int x = i; x < j; x++) {
        ans = min(ans, cur + fun(i, x) +
            fun(x + 1, j));
    }
    return ans;
}
cout << fun(1, n) << '\n';

```

## 6.6 Number of Unique Subsequence

```

int nxt[N][26], dp[N];
int f(int i) {
    if (i > n) return 0;
    int &ans = dp[i];
    if (ans != -1) return ans;
    ans = 1;
    for (int c = 0; c < 26; c++) {
        ans = (ans + f(nxt[i][c])) % mod;
    }
    return ans;
}
// nxt[i][c] = nxt c strictly after i
void solve() {
    cin >> s; s = '.' + s;
    cout << f(0) << '\n';
}

```

## 6.7 [Problem] Rose Land - SUST 24

*// Given a complete*  
*binary tree, i-th node grows a<sub>i</sub> roses*  
*per day and deliver to u in dis(u, i)*  
*days. Alice at node u, How many roses*  
*he will get within t days? u and t for*  
*Q queries.*  
*// Idea: as it is a complete binary*  
*tree, max dis is 2\*logn. We can do DP*  
*on Tree for ~40 days.*

```

ll dp[N][45], dp2[N][45];
void dfs(int u, int p) {
    for (int i = 1; i <= 40; i++) {
        dp[u][i] = i * a[u];
    }
    for (auto v : g[u]) {
        if (v != p) {
            dfs(v, u);
            for (int i = 1; i <= 40; i++) {
                dp[u][i] += dp[v][i - 1];
            }
        }
    }
}

void dfs2(int u, int p) {
    for (int i = 2; u != 1 and i <= 40; i++) {
        dp2[u][i] = (dp[p][i - 1] + dp2[p][i - 1]) - dp[u][i - 2];
    }
    for (auto v : g[u]) {
        if (v != p) {
            dfs2(v, u);
        }
    }
}

int32_t main() {
    cin >> n; ll sum = 0;
    for (int i = 1; i <= n; i++) {
        cin >> a[i]; sum += a[i];
    }
    dfs(1, 0); // rose from subtree
    dfs2(1, 0); // rose from par
    int q; cin >> q;
    while (q--) {
        int u, t; cin >> u >> t;
        int extra = t - 40;
        t = min(40, t);
        ll ans = dp[u][t] + dp2[u][t];
        if (extra > 0) ans += 1ll * extra * sum;
        cout << ans << '\n';
    }
}

```

## 6.8 [Problem] Sub-Palindromic Tree

*// Given a tree, each node has a char.*  
*Print max len subsequence any path*  
*which is a palindrome. (CF/1771/D)*  
*// Time: O(N<sup>2</sup>)*

```

int nxt[N][N], dp[N][N];
vector<int> vec;
void dfs(int u, int p) {
    vec.push_back(u);
    for (auto v : g[u]) {

```

```

        if (v != p) dfs(v, u);
    }
}

int f(int u, int v) {
    if (v == u) return 1;
    int &ans = dp[u][v];
    if (ans != -1) return ans;
    ans = 0;
    if (s[u] == s[v]) ans = 2 + (nxt[u][v] == v ? 0 : f(nxt[u][v],
        nxt[v][u]));
    else ans = max(f(nxt[u][v], v), f(u,
        nxt[v][u]));
    return ans;
}

void solve() {
    cin >> n >> s; s = '.' + s;
    for (int i = 1; i < n; i++) {
        int u, v; cin >> u >> v;
        g[u].push_back(v);
        g[v].push_back(u);
    }
    // nxt[u][v] = next node after u
    // if I want to go from u to v
    for (int u = 1; u <= n; u++) {
        for (auto x : g[u]) {
            vec.clear();
            dfs(x, u);
            for (auto v : vec) nxt[u][v] = x;
        }
    }
    memset(dp, -1, sizeof dp);
    int ans = 0;
    for (int u = 1; u <= n; u++) {
        for (int v = 1; v <= n; v++) {
            ans = max(ans, f(u, v));
        }
    }
    cout << ans << '\n';
}

```

## 7 Graph Theory

### 7.1 Binary Lifting and LCA

```

const int N = 2e5 + 9, LOG = 20;
vector<int> g[N];
int par[N][LOG], depth[N];
void dfs(int u, int p) {
    par[u][0] = p;
    depth[u] = depth[p] + 1;
    for (int i = 1; i < LOG; i++) {
        par[u][i] = par[par[u][i - 1]][i - 1];
    }
    for (auto v : g[u]) {
        if (v != p) {
            dfs(v, u);
        }
    }
}

int lca(int u, int v) {
    if (depth[u] < depth[v]) {
        swap(u, v);
    }
}

```

```

}
int k = depth[u] - depth[v];
for (int i = 0; i < LOG; i++) {
    if (CHECK(k, i)) u = par[u][i];
}
if (u == v) return u;
for (int i = LOG - 1; i >= 0; i--) {
    if (par[u][i] != par[v][i]) {
        u = par[u][i];
        v = par[v][i];
    }
}
return par[u][0];
}
int kth(int u, int k) { // kth parent of
    u
    assert(k >= 0);
    for (int i = 0; i < LOG; i++) {
        if (CHECK(k, i)) u = par[u][i];
    }
    return u;
}
int dist(int u, int v) { // distance from
    u to v
    int l = lca(u, v);
    return (depth[u] - depth[l]) +
        (depth[v] - depth[l]);
}
// kth node from u to v, 0th node is u
int kth(int u, int v, int k) {
    int l = lca(u, v);
    int d = dist(u, v);
    assert(k <= d);
    if (depth[l] + k <= depth[u]) {
        return kth(u, k);
    }
    k -= depth[u] - depth[l];
    return kth(v, depth[v] - depth[l] - k);
}

```

## 7.2 LCA and Sparse Table on Tree

```

// max and min weights of a path
const int N = 1e5 + 9, LOG = 20, inf =
    1e9; // change here
vector<array<int, 2>> g[N];
int par[N][LOG], tree_mx[N][LOG],
    depth[N];
void dfs(int u, int p, int dis) {
    par[u][0] = p;
    tree_mx[u][0] = dis;
    depth[u] = depth[p] + 1;
    for (int i = 1; i < LOG; i++) {
        par[u][i] = par[par[u][i - 1]][i - 1];
        tree_mx[u][i] = max(tree_mx[u][i - 1],
            tree_mx[par[u][i - 1]][i - 1]);
    }
    for (auto [v, w] : g[u]) {
        if (v != p) {
            dfs(v, u, w);
        }
    }
}
int query_max(int u, int v) { // max
    weight on path u to v

```

```

int l = lca(u, v);
int d = dist(l, u);
int ans = 0;
for (int i = 0; i < LOG; i++) {
    if (CHECK(d, i)) {
        ans = max(ans, tree_mx[u][i]);
        u = par[u][i];
    }
}
d = dist(l, v);
for (int i = 0; i < LOG; i++) {
    if (CHECK(d, i)) {
        ans = max(ans, tree_mx[v][i]);
        v = par[v][i];
    }
}
return ans;
}

```

## 7.3 Dijkstra

```

vector<int> dijkstra(int s) {
    vector<int> dis(n + 1, inf);
    vector<bool> vis(n + 1, false);
    dis[s] = 0;
    priority_queue<array<int, 2>,
        vector<array<int, 2>>,
        greater<array<int, 2>>> pq;
    pq.push({0, s});
    while (!pq.empty()) {
        auto [d, u] = pq.top(); pq.pop();
        if (vis[u]) continue;
        vis[u] = true;
        for (auto [v, w] : g[u]) {
            if (dis[v] > d + w) {
                dis[v] = d + w;
                pq.push({dis[v], v});
            }
        }
    }
    return dis;
}

```

## 7.4 Bellman Ford

```

// works for neg edge, can detect neg
    cycle
// Time: O(n^2)
const ll inf = 1e18;
vector<ll> dis(N, inf);
bool bellman_ford(int s) {
    dis[s] = 0;
    bool has_cycle = false;
    for (int i = 1; i <= n; i++) {
        for (int u = 1; u <= n; u++) {
            for (auto [v, w] : g[u]) {
                if (dis[v] > dis[u] + w) {
                    if (i == n) has_cycle = true;
                    dis[v] = dis[u] + w;
                }
            }
        }
    }
    return has_cycle;
}

```

## 7.5 Floyd Warshall

```

// dis[i][j] = min distance to reach i to
    j, works for neg edge (no neg cycle)
// Time: O(n^3)
vector<int> construct_path(int u, int v)
    { // O(n)
    if (nxt[u][v] == -1) return {};
    vector<int> path = { u };
    while (u != v) {
        u = nxt[u][v];
        path.push_back(u);
    }
    return path;
}
void floyd_warshall() {
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= n; j++) {
            if (i == j) dis[i][j] = 0;
            else if (g[i][j] == 0) dis[i][j] =
                inf;
            else dis[i][j] = g[i][j];
        }
    }
    for (int k = 1; k <= n; ++k) {
        for (int i = 1; i <= n; ++i) {
            for (int j = 1; j <= n; ++j) {
                if (dis[i][k] < inf and
                    dis[k][j] < inf)
                    dis[i][j] = min(dis[i][j],
                        dis[i][k] + dis[k][j]);
                nxt[i][j] = nxt[i][k];
            }
        }
    }
}
int32_t main() {
    int q; cin >> n >> m >> q;
    memset(nxt, -1, sizeof nxt);
    while (m--) {
        int u, v, w; cin >> u >> v >> w;
        g[u][v] = (g[u][v] != 0 ?
            min(g[u][v], w) : w);
        g[v][u] = (g[v][u] != 0 ?
            min(g[v][u], w) : w);
        nxt[u][v] = v;
        nxt[v][u] = u;
    }
    floyd_warshall();
    while (q--) {
        int u, v; cin >> u >> v;
        cout << (dis[u][v] == inf ? -1 :
            dis[u][v]) << '\n';
    }
    return 0;
}

```

## 7.6 Strongly Connected Components

```

// Time: O(n + m)
const int N = 1e5 + 9;
vector<int> g[N], gT[N], G[N];
vector<bool> vis(N, false);
vector<vector<int>> components;
vector<int> order;
int n, roots[N], sz[N];
void dfs(int u) {

```

```

    vis[u] = true;
    for (auto v : g[u]) {
        if (!vis[v]) dfs(v);
    }
    order.push_back(u);
}
void dfs2(int u, vector<int> &component) {
    vis[u] = true;
    component.push_back(u);
    for (auto v : gT[u]) {
        if (!vis[v]) dfs2(v, component);
    }
}
void scc() {
    // get order sorted by end time
    order.clear();
    for (int u = 1; u <= n; u++) {
        if (!vis[u]) dfs(u);
    }
    reverse(order.begin(), order.end());
    // transpose the graph
    for (int u = 1; u <= n; u++) {
        for (auto v : g[u]) {
            gT[v].push_back(u);
        }
    }
    // get all components
    components.clear();
    for (int i = 1; i <= n; i++) vis[i] =
        false;
    for (auto u : order) {
        if (!vis[u]) {
            vector<int> component;
            dfs2(u, component);
            sort(component.begin(),
                component.end());
            components.push_back(component);
            for (auto v : component) {
                roots[v] = component.front();
                sz[v] = component.size();
            }
        }
    }
    // add edges to condensation graph
    for (int u = 1; u <= n; u++) {
        for (auto v : g[u]) {
            if (roots[u] != roots[v]) {
                G[roots[u]].push_back(roots[v]);
            }
        }
    }
}
// when you need to use condensed graph,
    use it carefully (Specially g->G,
    i->roots[i])

```

## 7.7 Articulation Points

```

int disc[N], low[N], timer, n;
vector<bool> vis(N, false), is_ap(N,
    false);
void ap_dfs(int u, int p) {
    disc[u] = low[u] = ++timer;
    vis[u] = true;
    int children_cnt = 0;

```

```

for (auto v : g[u]) {
    if (v == p) continue;
    if (vis[v]) low[u] = min(low[u],
        disc[v]);
    else {
        ap_dfs(v, u);
        low[u] = min(low[u], low[v]);
        if (disc[u] <= low[v] and p != -1)
            is_ap[u] = true;
        children_cnt++;
    }
}
if (p == -1 and children_cnt > 1)
    is_ap[u] = true;
}
void find_articulation_points() {
    for (int u = 1; u <= n; u++) {
        if (!vis[u]) {
            timer = 0;
            ap_dfs(u, -1);
        }
    }
}

```

## 7.8 Find Bridges

```

map<pair<int, int>, int> bridges;
void bridges_dfs(int u, int p) { // find
    bridges
    disc[u] = low[u] = ++timer;
    vis[u] = true;
    for (auto v : g[u]) {
        if (v == p) continue;
        if (vis[v]) low[u] = min(low[u],
            disc[v]);
        else {
            bridges_dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (disc[u] < low[v]) {
                bridges[make_pair(min(u, v),
                    max(u, v))]++;
            }
        }
    }
}
void find_bridges() {
    for (int u = 1; u <= n; u++) {
        if (!vis[u]) {
            timer = 0;
            bridges_dfs(u, -1);
        }
    }
}

```

## 7.9 DSU on Tree

```

// Problem: Distinct Colors CSES (Number
// of distinct color in a subtree)
// Complexity: O(n(logn)^2)
set<int> se[N];
int col[N], ans[N], par[N]; // par[i] = i
// initially
int find(int i) {
    return (i == par[i] ? i : par[i] =
        find(par[i]));
}
void merge(int u, int v) {

```

```

    if ((u = find(u)) == (v = find(v)))
        return;
    if (se[u].size() > se[v].size())
        swap(u, v);
    for (auto x : se[u]) {
        se[v].insert(x);
    }
    se[u].clear();
    par[u] = v;
}
void dfs(int u, int p) {
    se[find(u)].insert(col[u]);
    for (auto v : g[u]) {
        if (v != p) {
            dfs(v, u);
            merge(u, v);
        }
    }
    ans[u] = se[find(u)].size();
}

```

## 7.10 Heavy-Light Decomposition

```

// Per Query Complexity: O(logn^2)
// Path and subtree updates and queries.
const int N = 2e5 + 9, LOG = 20, inf =
    1e9; // change here
vector<int> g[N];
int par[N][LOG], depth[N], sz[N];
int disc[N], finish[N], timer, head[N];
int n;
void dfs(int u, int p = 0) {
    par[u][0] = p;
    depth[u] = depth[p] + 1;
    sz[u] = 1;
    for (int i = 1; i < LOG; i++) {
        par[u][i] = par[par[u][i - 1]][i - 1];
    }
    if (p) g[u].erase(find(g[u].begin(),
        g[u].end(), p));
    for (auto &v : g[u]) {
        if (v != p) {
            dfs(v, u);
            sz[u] += sz[v];
            if (sz[v] > sz[g[u][0]]) swap(v,
                g[u][0]);
        }
    }
}
void dfs_hld(int u) {
    disc[u] = ++timer;
    for (auto v : g[u]) {
        head[v] = (v == g[u][0] ? head[u] :
            v);
        dfs_hld(v);
    }
    finish[u] = timer;
}
int kth(int u, int k) {
    assert(k >= 0);
    for (int i = 0; i < LOG; i++) {
        if (CHECK(k, i)) u = par[u][i];
    }
    return u;
}
int query_up(int u, int v) {

```

```

    int ans = -inf; // change here
    while (head[u] != head[v]) {
        ans = max(ans, st.query(1, 1, n,
            disc[head[u]], disc[u])); //
        // change here
        u = par[head[u]][0];
    }
    ans = max(ans, st.query(1, 1, n,
        disc[v], disc[u])); // change here
    return ans;
}
int query(int u, int v) {
    int lc = lca(u, v);
    int ans = query_up(u, lc);
    if (v != lc) {
        ans = max(ans, query_up(v, kth(v,
            depth[v] - depth[lc] - 1))); //
        // change here
    }
    return ans;
}
void solve() {
    cin >> n;
    for (int i = 2; i <= n; i++) {
        int u, v; cin >> u >> v;
        g[u].push_back(v);
        g[v].push_back(u);
    }
    dfs(1);
    head[1] = 1;
    dfs_hld(1);
    st.build(1, 1, n);
}

```

## 7.11 Inverse Graph

```

// Problem: CF/0-1 MST
// A Complete Graph. m edges have
// weight 1, rest have 0. MST?
const int N = 200000 + 9;
set<int> g[N], non_vis;
void dfs(int u) {
    non_vis.erase(u);
    for (int v = 1; v <= n; v++) {
        if (v == u) continue;
        auto it = non_vis.lower_bound(v);
        if (it == non_vis.end()) break;
        v = *it;
        if (g[u].find(v) != g[u].end())
            continue;
        dfs(v);
    }
}
int32_t main() {
    int m; cin >> n >> m;
    while (m--) {
        int u, v; cin >> u >> v;
        g[u].insert(v);
        g[v].insert(u);
    }
    for (int i = 1; i <= n; i++)
        non_vis.insert(i);
    int component_cnt = 0;
    for (int i = 1; i <= n; i++) {
        if (non_vis.find(i) !=
            non_vis.end()) {

```

```

            component_cnt++;
            dfs(i);
        }
    }
    cout << component_cnt - 1 << '\n';
}

```

## 7.12 [Problem] Rooted Tree - Hackerrank

Given a rooted tree, Upd: Add  $d * k$  on  $v$  (subtree nodes of  $u$ ), where  $k$  will be given and  $d$  is distance of  $v$  from  $u$ . Query: Node Value/Subtree Sum (using ETT) or Path Value Sum (using HLD). **Idea:**  $d_i$  = dis from root to  $i$ . Then,  $ans = k \cdot \sum_{v \in subtree(u)} d_v$ . It's hard to find  $d_v$  for each  $u$ , rather we can find  $d_v$  = dis of  $v$  from root. Although some extra value added on each  $v$  of subtree  $u$  because of  $d_v$  from root (not  $u$ ), we can easily subtract them ( $depth[u] * k$ ).

## 7.13 Dinic

```

const int N = 105, inf = 1e9;
int n, m, st, en;
// Edges should be added in both
// direction separately if the graph is
// undirected
const int INF = 2000000000;
struct Edge {
    int from, to, cap, flow, index;
    Edge(int from, int to, int cap, int
        flow, int index) :
        from(from), to(to), cap(cap),
        flow(flow), index(index) {}
};
struct Dinic {
    int N;
    vector<vector<Edge>> G;
    vector<Edge*> dad;
    vector<int> Q;
    Dinic(int N) : N(N), G(N), dad(N),
        Q(N) {}
    void AddEdge(int from, int to, int
        cap) {
        G[from].emplace_back(from, to, cap,
            0, G[to].size());
        if (from == to)
            G[from].back().index++;
        G[to].emplace_back(to, from, 0, 0,
            G[from].size() - 1);
    }
}
long long BlockingFlow(int s, int t) {
    fill(dad.begin(), dad.end(), (Edge
        *) NULL);
    dad[s] = &G[0][0] - 1;
    int head = 0, tail = 0;
    Q[tail++] = s;
    while (head < tail) {
        int x = Q[head++];
        for (int i = 0; i < G[x].size();
            i++) {

```



```

Edge &e = G[x][i];
if (!dad[e.to] && e.cap - e.flow
    > 0) {
    dad[e.to] = &G[x][i];
    Q[tail++] = e.to;
}
}
if (!dad[t]) return 0;
long long totflow = 0;
for (int i = 0; i < G[t].size();
    i++) {
    Edge *start =
        &G[G[t][i].to][G[t][i].index];
    int amt = INF;
    for (Edge *e = start; amt && e !=
        dad[s]; e = dad[e->from]) {
        if (!e) { amt = 0; break; }
        amt = min(amt, e->cap - e->flow);
    }
    if (amt == 0) continue;
    for (Edge *e = start; amt && e !=
        dad[s]; e = dad[e->from]) {
        e->flow += amt;
        G[e->to][e->index].flow -= amt;
    }
    totflow += amt;
}
return totflow;
}
long long GetMaxFlow(int s, int t) {
    long long totflow = 0;
    while (long long flow =
        BlockingFlow(s, t))
        totflow += flow;
    return totflow;
}
}
void solve() {
    cin >> n >> st >> en >> m;
    Dinic *dinic = new Dinic(n + 1);
    while (m--) {
        int u, v, w; cin >> u >> v >> w;
        dinic->AddEdge(u, v, w);
        dinic->AddEdge(v, u, w);
    }
    cout << dinic->GetMaxFlow(st, en) <<
        '\n';
    delete(dinic);
}

```

## 8 String

### 8.1 Hashing

```

const int N = 1e6 + 9; // change here
const int MOD1 = 127657753, MOD2 =
    987654319;
const int p1 = 137, p2 = 277; // change
    here
int ip1, ip2;
pair<int, int> pw[N], ipw[N];
void prec() {
    pw[0] = {1, 1};
    for (int i = 1; i < N; i++) {
        pw[i].first = 111 * pw[i-1].first
            * p1 % MOD1;
    }
}

```

```

pw[i].second = 111 * pw[i-1].second * p2 % MOD2;
}
ip1 = power(p1, MOD1 - 2, MOD1);
ip2 = power(p2, MOD2 - 2, MOD2);
ipw[0] = {1, 1};
for (int i = 1; i < N; i++) {
    ipw[i].first = 111 * ipw[i-1].first * ip1 % MOD1;
    ipw[i].second = 111 * ipw[i-1].second * ip2 % MOD2;
}
}
struct Hashing {
    int n;
    string s;
    vector<pair<int, int>> hash_val;
    vector<pair<int, int>> rev_hash_val;
    Hashing(string _s) {
        s = _s;
        n = s.size();
        hash_val.emplace_back(0, 0);
        for (int i = 0; i < n; i++) {
            pair<int, int> p;
            p.first = (hash_val[i].first + 111
                * s[i] * pw[i].first % MOD1) %
                MOD1;
            p.second = (hash_val[i].second +
                111 * s[i] * pw[i].second %
                MOD2) % MOD2;
            hash_val.push_back(p);
        }
        rev_hash_val.emplace_back(0, 0);
        for (int i = 0, j = n - 1; i < n;
            i++, j--) {
            pair<int, int> p;
            p.first = (rev_hash_val[i].first +
                111 * s[i] * pw[j].first %
                MOD1) % MOD1;
            p.second = (rev_hash_val[i].second
                + 111 * s[i] * pw[j].second %
                MOD2) % MOD2;
            rev_hash_val.push_back(p);
        }
    }
    pair<int, int> get_hash(int l, int r)
    { // 1 indexed
        pair<int, int> ans;
        ans.first = (hash_val[r].first -
            hash_val[l-1].first + MOD1) *
            111 * ipw[l-1].first % MOD1;
        ans.second = (hash_val[r].second -
            hash_val[l-1].second + MOD2) *
            111 * ipw[l-1].second % MOD2;
        return ans;
    }
    pair<int, int> rev_hash(int l, int r)
    { // 1 indexed
        pair<int, int> ans;
        ans.first = (rev_hash_val[r].first -
            rev_hash_val[l-1].first +
            MOD1) * 111 * ipw[n-r].first %
            MOD1;
        ans.second = (rev_hash_val[r].second -
            rev_hash_val[l-1].second +
            MOD2) * 111 * ipw[n-r].second %
            MOD2;
        return ans;
    }
}

```

```

ans.second = (rev_hash_val[r].second -
    rev_hash_val[l-1].second +
    MOD2) * 111 * ipw[n-r].second
    % MOD2;
return ans;
}
pair<int, int> get_hash() { // 1
    indexed
    return get_hash(1, n);
}
bool is_palindrome(int l, int r) {
    return get_hash(l, r) == rev_hash(l,
        r);
}
}
};

```

### 8.2 Hashing with Updates

```

using T = array<int, 2>;
const T MOD = {127657753, 987654319};
const T p = {137, 277}; // change here
T operator + (T a, int x) {return {(a[0]
    + x) % MOD[0], (a[1] + x) % MOD[1]};}
T operator - (T a, int x) {return {(a[0]
    - x + MOD[0]) % MOD[0], (a[1] - x +
    MOD[1]) % MOD[1]};}
T operator * (T a, int x) {return
    {(int)((long long) a[0] * x %
    MOD[0]), (int)((long long) a[1] * x
    % MOD[1])};}
T operator + (T a, T x) {return {(a[0] +
    x[0]) % MOD[0], (a[1] + x[1]) %
    MOD[1]};}
T operator - (T a, T x) {return {(a[0] -
    x[0] + MOD[0]) % MOD[0], (a[1] -
    x[1] + MOD[1]) % MOD[1]};}
T operator * (T a, T x) {return
    {(int)((long long) a[0] * x[0] %
    MOD[0]), (int)((long long) a[1] *
    x[1] % MOD[1])};}
ostream& operator << (ostream& os, T
    hash) {return os << "(" << hash[0]
    << ", " << hash[1] << ")";}
T pw[N], ipw[N];
void prec() {
    pw[0] = {1, 1};
    for (int i = 1; i < N; i++) {
        pw[i] = pw[i-1] * p;
    }
    ipw[0] = {1, 1};
    T ip = {power(p[0], MOD[0] - 2,
        MOD[0]), power(p[1], MOD[1] - 2,
        MOD[1])};
    for (int i = 1; i < N; i++) {
        ipw[i] = ipw[i-1] * ip;
    }
}
struct Hashing {
    int n;
    string s; // 1 - indexed
    vector<array<T, 2>> t; // (normal, rev)
    array<T, 2> merge(array<T, 2> l,
        array<T, 2> r) {
        l[0] = l[0] + r[0];
        l[1] = l[1] + r[1];
    }
}

```

```

return l;
}
void build(int node, int b, int e) {
    if (b == e) {
        t[node][0] = pw[b] * s[b];
        t[node][1] = pw[n-b+1] * s[b];
        return;
    }
    int mid = (b + e) >> 1, l = node <<
        1, r = l | 1;
    build(l, b, mid);
    build(r, mid+1, e);
    t[node] = merge(t[l], t[r]);
}
void upd(int node, int b, int e, int
    i, char x) {
    if (b > i || e < i) return;
    if (b == e && b == i) {
        t[node][0] = pw[b] * x;
        t[node][1] = pw[n-b+1] * x;
        return;
    }
    int mid = (b + e) >> 1, l = node <<
        1, r = l | 1;
    upd(l, b, mid, i, x);
    upd(r, mid+1, e, i, x);
    t[node] = merge(t[l], t[r]);
}
array<T, 2> query(int node, int b, int
    e, int i, int j) {
    if (b > j || e < i) return {T{0,
        0}, T{0, 0}};
    if (b >= i && e <= j) return t[node];
    int mid = (b + e) >> 1, l = node <<
        1, r = l | 1;
    return merge(query(l, b, mid, i, j),
        query(r, mid+1, e, i, j));
}
Hashing() {}
Hashing(string _s) {
    n = _s.size();
    s = "." + _s;
    t.resize(4 * n + 1);
    build(1, 1, n);
}
void upd(int i, char c) {
    upd(1, 1, n, i, c);
    s[i] = c;
}
T get_hash(int l, int r) { // 1 -
    indexed
    return query(1, 1, n, l, r)[0] *
        ipw[l-1];
}
T rev_hash(int l, int r) { // 1 -
    indexed
    return query(1, 1, n, l, r)[1] *
        ipw[n-r];
}
T get_hash() {
    return get_hash(1, n);
}
}

```

```

bool is_palindrome(int l, int r) {
    return get_hash(l, r) == rev_hash(l, r);
}
};

8.3 Hashing with Upd and Deletes
// update or delete a char in the string
// or check whether a range [l,r] is a
// palindrome or not (Palindromic Query I
// - Toph)
#define int long long
const int N = 1e5 + 9;
int en;
struct ST {
    pair<int, int> tree[4 * (N + N)];
    void build(int n, int b, int e) {
        if (b == e) {
            tree[n].first = b;
            tree[n].second = 1;
            return;
        }
        int mid = (b + e) >> 1, l = n << 1,
            r = l + 1;
        build(l, b, mid);
        build(r, mid + 1, e);
        tree[n].second = tree[l].second +
            tree[r].second;
    }
    void upd(int n, int b, int e, int i,
        int x1, int x2) {
        int x1, int x2) {
            if (b > i || e < i) return;
            if (b == e && b == i) {
                tree[n].first = x1;
                tree[n].second = x2;
                return;
            }
            int mid = (b + e) >> 1, l = n << 1,
                r = l + 1;
            upd(l, b, mid, i, x1, x2);
            upd(r, mid + 1, e, i, x1, x2);
            tree[n].second = tree[l].second +
                tree[r].second;
        }
    }
    pair<int, int> query(int n, int b, int
        e, int x) {
        if (b > e) return { -1, -1 };
        if (tree[n].second < x) return
            { tree[n].second, -1 };
        if (b == e) return tree[n];
        int mid = (b + e) >> 1, l = n << 1,
            r = l + 1;
        pair<int, int> L = query(l, b, mid,
            x);
        if (L.second != -1) return L;
        pair<int, int> R = query(r, mid + 1,
            e, x - L.first);
        return R;
    }
} st, st2;
using T = array<int, 2>;
const T MOD = {127657753, 987654319};
const T p = {137, 277};
// add operators overloading of T (from
// only upd) + prec()

```

```

int get(int i, int n) {
    return n - i + 1;
}
struct Hashing {
    int n; string s;
    vector<T> tree, lazy;
    void push(int node, int b, int e) {
        if (lazy[node][0] == 1) return;
        tree[node] = tree[node] * lazy[node];
        if (b != e) {
            int l = node << 1, r = l + 1;
            lazy[l] = lazy[l] * lazy[node];
            lazy[r] = lazy[r] * lazy[node];
        }
        lazy[node] = T{1, 1};
    }
    void build(int node, int b, int e) {
        lazy[node] = T{1, 1};
        if (b == e) {
            tree[node] = pw[b] * s[b];
            return;
        }
        int mid = (b + e) >> 1, l = node <<
            1, r = l + 1;
        build(l, b, mid);
        build(r, mid + 1, e);
        tree[node] = tree[l] + tree[r];
    }
    void upd(int node, int b, int e, int
        i, T x) {
        push(node, b, e);
        if (b > i || e < i) return;
        if (b == e && b == i) {
            tree[node] = x;
            return;
        }
        int mid = (b + e) >> 1, l = node <<
            1, r = l + 1;
        upd(l, b, mid, i, x);
        upd(r, mid + 1, e, i, x);
        tree[node] = tree[l] + tree[r];
    }
    void del(int node, int b, int e, int
        i, int j) {
        push(node, b, e);
        if (b > j || e < i) return;
        if (b >= i && e <= j) {
            lazy[node] = lazy[node] * ipw[1];
            push(node, b, e);
            return;
        }
        int mid = (b + e) >> 1, l = node <<
            1, r = l + 1;
        del(l, b, mid, i, j);
        del(r, mid + 1, e, i, j);
        tree[node] = tree[l] + tree[r];
    }
    T query(int node, int b, int e, int i,
        int j) {
        push(node, b, e);
        if (b > j || e < i) return {0, 0};
        if (b >= i && e <= j) return
            tree[node];
        int mid = (b + e) >> 1, l = node <<
            1, r = l + 1;
    }
}

```

```

    T L = query(l, b, mid, i, j);
    T R = query(r, mid + 1, e, i, j);
    return L + R;
}
Hashing() {}
Hashing(string _s) {
    s = _s;
    n = s.size();
    s = '.' + s;
    tree.resize(4 * n + 1);
    lazy.resize(4 * n + 1);
    build(1, 1, n);
}
void upd(int i, char c, int cur) {
    T x = pw[i] * c;
    if (cur == 1) i = st.query(1, 1, en,
        i).first;
    else i = st2.query(1, 1, en, i).first;
    upd(1, 1, n, i, x);
}
void del(int i, int cur) {
    int orgi = i;
    T x = pw[i] * 011;
    if (cur == 1) i = st.query(1, 1, en,
        i).first;
    else i = st2.query(1, 1, en, i).first;
    upd(1, 1, n, i, x);
    del(1, 1, n, i + 1, n);
    if (cur == 1) st.upd(1, 1, en, i, i,
        0);
    else st2.upd(1, 1, en, i, i, 0);
}
T get_hash(int l, int r, int cur) { // 1
    - indexed
    int ll = st.query(1, 1, en, l).first;
    int rr = st2.query(1, 1, en, r).first;
    if (cur == 2) {
        ll = st2.query(1, 1, en, l).first;
        rr = st2.query(1, 1, en, r).first;
    }
    return query(1, 1, n, ll, rr) *
        ipw[l - 1];
}
};
int32_t main() {
    prec(); // must include
    string s; cin >> s;
    int n = s.size();
    int q; cin >> q;
    string t = s;
    reverse(t.begin(), t.end());
    Hashing hs(s), hs2(t);
    en = n + q + 5;
    st.build(1, 1, en);
    st2.build(1, 1, en);
    while (q--) {
        char c; cin >> c;
        if (c == 'C') {
            int l, r; cin >> l >> r;
            int l2 = get(1, n);
            int r2 = get(r, n);
            if (hs.get_hash(l, r, 1) ==
                hs2.get_hash(r2, l2, 2)) cout
                << "Yes!\n";
            else cout << "No!\n";
        }
    }
}

```

```

}
else if (c == 'U') {
    int i; char x; cin >> i >> x;
    int i2 = get(i, n);
    hs.upd(i, x, 1);
    hs2.upd(i2, x, 2);
}
else {
    int i; cin >> i;
    int i2 = get(i, n);
    hs.del(i, 1);
    hs2.del(i2, 2);
    --n;
}
}
}

8.4 Hashing on Tree
// Given a tree, Check whether it is
// symmetrical or not. Problem - CF G.
// Symmtree
// The value for each node is it's
// subtree size and position is the
// level (ordered). But the order of
// childs doesn't matter (unordered)
const int N = 2e5 + 9;
vector<int> g[N];
vector<array<int, 3>> hashh[N]; // hash1,
// hash2, node
int n, sz[N];
const int MOD1 = 1e9 + 9, MOD2 = 1e9 + 21;
const int p1 = 1e5 + 19, p2 = 1e5 + 43;
void dfs2(int u, int p, int lvl) {
    array<int, 3> my_hash;
    my_hash[0] = ll1 * sz[u] *
        pw[lvl].first % MOD1;
    my_hash[1] = ll1 * sz[u] *
        pw[lvl].second % MOD2;
    my_hash[2] = u;
    bool leaf = true;
    for (auto v : g[u]) {
        if (v != p) {
            dfs2(v, u, lvl + 1);
            leaf = false;
        }
    }
    if (!leaf) {
        int sum1 = 1, sum2 = 1;
        for (auto here : hashh[u]) {
            auto [x, y, _] = here;
            sum1 = (sum1 * x) % MOD1;
            sum2 = (sum2 * y) % MOD2;
        }
        my_hash[0] = power(my_hash[0], sum1,
            MOD1);
        my_hash[1] = power(my_hash[1], sum2,
            MOD2);
    }
    hashh[p].push_back(my_hash);
}
bool ok(int u) {
    map<pair<int, int>, int> mp;
}

```

```

for (auto [x, y, who] : hassh[u]) {
    mp[{x, y}]++;
}
int odd = 0;
pair<int, int> val;
for (auto [here, cnt] : mp) {
    odd += cnt & 1;
    if (cnt & 1) val = here;
}
if (odd == 0) return true;
if (odd > 1) return false;
int node;
for (auto [x, y, who] : hassh[u]) {
    pair<int, int> here = {x, y};
    if (here == val) node = who;
}
return ok(node);
}
void solve() {
    cin >> n; clr(n);
    for (int i = 2; i <= n; i++) {
        int u, v; cin >> u >> v;
        g[u].push_back(v);
        g[v].push_back(u);
    }
    dfs(1, 0); // calc. subtree size
    dfs2(1, 0, 1);
    if (ok(0)) cout << "YES\n";
    else cout << "NO\n";
}

```

## 8.5 Compare 2 strings Lexicographically

```

// Time: O(logn)
string s;
Hashing hs;
// return 0 if both equal
// return 1 if first substring greater
// return -1 if second substring greater
// here lcp() provides the len of longest
// common prefix
int compare(int i, int j, int x, int y) {
    int common_prefix = lcp(i, j, x, y);
    int len1 = j - i + 1, len2 = y - x + 1;
    if (common_prefix == len1 and len1 ==
        len2) return 0;
    else if (common_prefix == len1) return
        -1;
    else if (common_prefix == len2) return
        1;
    else return (s[i + common_prefix - 1]
        < s[x + common_prefix - 1] ? -1 :
        1);
}

```

## 8.6 KMP

```

vector<int> build_lps(string &pat) {
    int n = pat.size();
    vector<int> lps(n, 0);
    for (int i = 1; i < n; i++) {
        int j = lps[i - 1];
        while (j > 0 and pat[i] != pat[j]) {
            j = lps[j - 1];
        }
        if (pat[i] == pat[j]) j++;
        lps[i] = j;
    }
}

```

```

}
return lps;
}
int kmp(string &txt, string &pat) {
    string s = pat + '#' + txt;
    vector<int> lps = build_lps(s);
    int ans = 0;
    for (auto x : lps) {
        if (x == pat.size()) ans++;
    }
    return ans;
}
int kmp(string &txt, string &pat) {
    vector<int> lps = build_lps(pat);
    int n = txt.size(), m = pat.size();
    int ans = 0;
    int j = 0;
    for (int i = 0; i < n; i++) {
        while (j > 0 and txt[i] != pat[j]) {
            j = lps[j - 1];
        }
        if (txt[i] == pat[j]) j++;
        if (j == m) {
            ans++;
            j = lps[j - 1];
        }
    }
    return ans;
}

```

## 8.7 KMP Automata

```

// like DFA. if string is "abcdeabg",
// aut[7]['c'] = 3. Means 7th index e
// 'c' bosale LPS koto, aut[7]['g'] = 8
void compute_automaton(string s,
    vector<vector<int>>& aut) {
    s += '#';
    int n = s.size();
    vector<int> pi = build_lps(s);
    aut.assign(n, vector<int>(26));
    for (int i = 0; i < n; i++) {
        for (int c = 0; c < 26; c++) {
            if (i > 0 && 'a' + c != s[i])
                aut[i][c] = aut[pi[i - 1]][c];
            else
                aut[i][c] = i + ('a' + c == s[i]);
        }
    }
}

```

## 8.8 Prefix Occurance Count

```

// Count the number of occurrences of each
// prefix
vector<int> ans(n + 1);
for (int i = 0; i < n; i++) ans[lps[i]]++;
for (int i = n - 1; i > 0; i--)
    ans[lps[i - 1]] += ans[i];
for (int i = 0; i <= n; i++) ans[i]++;

```

## 8.9 Number of palindromic substring in L to R using Wavelet Tree

```

// Problem - Kattis palindromes
ll f(int x) {
    return (1ll * x * (x + 1)) / 2;
}

```

```

ll f(int l, int r) {
    if (l > r) return 0;
    return f(r) - f(l - 1);
}
bool ok(int l, int r) {
    return hash_s.is_palindrome(l, r);
}
int32_t main() {
    cin >> s;
    n = s.size();
    hash_s = Hashing(s);
    for (int i = 1; i <= n; i++) {
        int l = 0, r = min(n - i, i - 1),
            cnt = 1;
        while (l <= r) {
            int mid = (l + r) >> 1;
            if (ok(i - mid, i + mid)) {
                cnt = mid;
                l = mid + 1;
            }
            else r = mid - 1;
        }
        pi1[i] = cnt + 1;
        pi1_left[i] = pi1[i] - i;
        pi1_right[i] = i + pi1[i];
    }
    for (int i = 2; i <= n; i++) {
        if (s[i - 1] == s[i - 2]) {
            int l = 0, r = min(n - i, i - 1),
                cnt = 2;
            while (l <= r) {
                int mid = (l + r) >> 1;
                if (ok(i - 1 - mid, i + mid)) {
                    cnt = mid;
                    l = mid + 1;
                }
                else r = mid - 1;
            }
            pi2[i] = cnt + 1;
        }
        else pi2[i] = 0;

        pi2_left[i] = pi2[i] - i;
        pi2_right[i] = i + pi2[i];
    }
    // wavelet trees (odd_len_left,
    // odd_len_right, even_len_left,
    // even_len_right)
    t1.init(pi1_left + 1, pi1_left + n +
        1, -N, N);
    t2.init(pi1_right + 1, pi1_right + n +
        1, -N, N);
    t3.init(pi2_left + 1, pi2_left + n +
        1, -N, N);
    t4.init(pi2_right + 1, pi2_right + n +
        1, -N, N);

    int q; cin >> q;
    while (q--) {
        int l, r; cin >> l >> r;
        // define k, find cnt > k and
        // summation whose are <= k;
        int m = (l + r) / 2;
        int k = 1 - l;
    }
}

```

```

ll ans = f(1, m);
ans += t1.sum(1, m, k);
int cnt = t1.GT(1, m, k);
ans += 1ll * k * cnt;
k = 1 + r;
ans += -f(m + 1, r);
ans += t2.sum(m + 1, r, k);
cnt = t2.GT(m + 1, r, k);
ans += 1ll * k * cnt;
if (l + 1 <= m) { // a bit different
    than others
        k = -1;
        ans += f(1 + 1, m);
        ans += t3.sum(1 + 1, m, k);
        cnt = t3.GT(1 + 1, m, k);
        ans += 1ll * k * cnt;
    }
    k = 1 + r;
    ans += -f(m + 1, r);
    ans += t4.sum(m + 1, r, k);
    cnt = t4.GT(m + 1, r, k);
    ans += 1ll * k * cnt;
    cout << ans << '\n';
}
}

```

It is easier to explain by considering only palindromes centered at indices (so, odd length), the idea is the same anyway. For each index  $i$ ,  $r_i$  will be the longest radius of a palindrome centered there (in other words, the amount of palindromes centered at index  $i$ ). Directly from manacher, this takes  $\mathcal{O}(n)$  to calculate.

For a query  $[l, r]$ , we first compute  $m = \frac{l+r}{2}$ . Now we want to calculate

$$\sum_{i=l}^m \min(i - l + 1, r_i) + \sum_{i=m+1}^r \min(r - i + 1, r_i)$$

$$\sum_{i=l}^m \min(i - l + 1, r_i) = \sum_{i=l}^m i + \min(1 - l, r_i - i).$$

The sum over  $i$  can be found in constant time. As for the other term, if we create some array  $r'_i = r_i - i$  during the preprocessing, then the queries are asking for some over range of  $\min(C, r'_i)$  where  $C$  is constant. You can solve this in  $\mathcal{O}(\log n)$  per query using wavelet tree.

## 8.10 Trie

```

const int N = 10; // change here
const char base_char = '0'; // change
here
struct TrieNode {
    int cnt;
    TrieNode * nxt[N];
    TrieNode() {
        cnt = 0;
        for (int i = 0; i < N; i++) nxt[i] =
            NULL;
    }
} *root;
void insert(const string &s) {
}

```



```

TrieNode *cur = root;
int n = (int)s.size();
for (int i = 0; i < n; i++) {
    int idx = s[i] - base_char;
    if (cur -> nxt[idx] == NULL) cur ->
        nxt[idx] = new TrieNode();
    cur = cur -> nxt[idx];
    cur -> cnt++;
}
}
void rem(TrieNode *cur, string &s, int
pos) { // free :: De Allocates Memory
if (pos == s.size()) return;
int idx = s[pos] - base_char;
rem(cur -> nxt[idx], s, pos + 1);
cur -> nxt[idx] -> cnt--;
if (cur -> nxt[idx] -> cnt == 0) {
    free(cur -> nxt[idx]);
    cur -> nxt[idx] = NULL;
}
}
int query(const string &s) { // "s" is a
    prefix of some element or not
    int n = (int)s.size();
    TrieNode *cur = root;
    for (int i = 0; i < n; i++) {
        int idx = s[i] - base_char;
        if (cur -> nxt[idx] == NULL) return 0;
        cur = cur -> nxt[idx];
    }
    return cur -> cnt;
}
}
void del(TrieNode *cur) {
    for (int i = 0; i < N; i++) if (cur ->
        nxt[i]) del(cur -> nxt[i]);
    delete(cur);
}
int32_t main() {
    root = new TrieNode(); // init new trie
    del(root); // clear trie
}

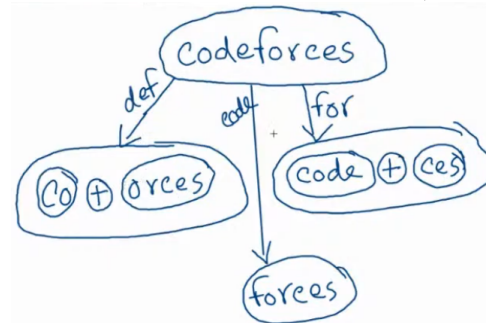
```

## 9 Game Theory

### 9.1 Notes

- First Write a Bruteforce Solution
- If game is not impartial. Greedy, DP may work
- Nim = All XOR
- Misere Nim (Last player who took a stone loses) = Nim + (Corner Case: if all (odd) piles are 1 you lose)
- Bogus Nim = Nim
- Staircase Nim (Given an array where  $a[i]$  is the number of stones at index  $i$ . Move: Choose any stones ( $> 1$ ) from index  $i$  ( $i > 1$ ) and move them to index  $i-1$ . ) = Odd indexed pile Nim (Even indexed pile doesn't matter, as one player can give bogus moves to drop all even piles to ground)
- Sprague-Grundy:** Every impartial game under the normal play convention is equivalent to a ONE-HEAP GAME of NIM.

Grundy value can be calculate by DP/Pattern.



## 10 Misc.

### 10.1 [Trick] Median of Medians

**Problem:** Median of All Sub-Array Medians

**Idea:** Total Subarray,  $k = \frac{n(n+1)}{2}$ . We need all subarray medians, sort them and get  $\frac{k}{2} + 1$ . Now, Binary Search on Answer. `ok()` will return true if  $cnt \geq \frac{k}{2} + 1$ , where  $cnt$  = count of subarrays whose median is  $\leq mid$ . If true,  $r = mid - 1$

### 10.2 Ternary Search

// Problem: Pyramid-ICPC Dhaka 2023  
// Given the surface area of a square base pyramid, Need to maximize volume.

```

double surface_area;
double fun(double square_area) {
    double base = sqrt(square_area);
    double triangle_area = surface_area -
        square_area;
    double per_triangle_area =
        triangle_area / 4;
    double triangle_height =
        (per_triangle_area * 2) / base;
    double x = base / 2;
    double height = sqrt((triangle_height
        * triangle_height) - (x * x));
    double volume = (base * base * height)
        / 3;
    if (x > triangle_height) volume = 0;
    return volume;
}
int32_t main() {
    cin >> surface_area;
    double l = 0, r = surface_area, ans = -1;
    int it = 100;
    while (it-- > 0) {
        double mid1 = l + (r - l) / 3;
        double mid2 = r - (r - l) / 3;
        double x = fun(mid1);
        double y = fun(mid2);
        if (x > y) {
            ans = x;
            r = mid2;
        }
        else l = mid1;
    }
}

```

## 10.3 Pigeonhole Principle

- At least 1 subarray of an array of length  $N$  must be divisible by  $N$ .
- Build all possible sequences of length 10 whose value is between 1 to 100. At least any two sequences will be same.
- For  $N \geq 100$ , at least one 4-tuples XOR is 0.
- For  $N > 20$  and  $a_i \leq 10^6$ , at least two subsets has same XOR. Because distinct XOR count can be  $10^6$  and Total Subset  $2^{20} > 10^6$ . Means there is a subset whose XOR is 0.

## 10.4 Matrix Expo

```

struct Mat {
    int n, m;
    vector<vector<int>> a;
    Mat() {}
    Mat(int _n, int _m) {n = _n; m = _m;
        a.assign(n, vector<int>(m, 0)); }
    Mat(vector<vector<int>> v) {n =
        v.size(); m = n ? v[0].size() : 0;
        a = v; }
    inline void make_unit() {
        assert(n == m);
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < n; j++)
                a[i][j] = i == j;
        }
    }
    inline Mat operator + (const Mat &b) {
        assert(n == b.n && m == b.m);
        Mat ans = Mat(n, m);
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < m; j++) {
                ans.a[i][j] = (a[i][j] +
                    b.a[i][j]) % mod;
            }
        }
        return ans;
    }
    inline Mat operator - (const Mat &b) {
        assert(n == b.n && m == b.m);
        Mat ans = Mat(n, m);
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < m; j++) {
                ans.a[i][j] = (a[i][j] -
                    b.a[i][j] + mod) % mod;
            }
        }
        return ans;
    }
    inline Mat operator * (const Mat &b) {
        assert(m == b.n);
        Mat ans = Mat(n, b.m);
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < b.m; j++) {
                for (int k = 0; k < m; k++) {
                    ans.a[i][j] = (ans.a[i][j] +
                        1LL * a[i][k] * b.a[k][j]
                        % mod) % mod;
                }
            }
        }
    }
}

```

```

}
return ans;
}
inline Mat pow(long long k) {
    assert(n == m);
    Mat ans(n, n), t = a; ans.make_unit();
    while (k) {
        if (k & 1) ans = ans * t;
        t = t * t;
        k >>= 1;
    }
    return ans;
}
inline Mat& operator += (const Mat& b)
{ return *this = (*this) + b; }
inline Mat& operator -= (const Mat& b)
{ return *this = (*this) - b; }
inline Mat& operator *= (const Mat& b)
{ return *this = (*this) * b; }
inline bool operator == (const Mat& b)
{ return a == b.a; }
inline bool operator != (const Mat& b)
{ return a != b.a; }
};
int32_t main() {
    int n; long long k; cin >> n >> k;
    Mat a(n, n); // then assign value
    Mat ans = a.pow(k);
}

```

## 10.5 2D Prefix Sum

```

const int N = 1e3 + 9;
int a[N][N];
ll pref[N][N];
void query(int x1, int y1, int x2, int
y2) {
    return pref[x2][y2] - pref[x1 -
        1][y2] - pref[x2][y1 - 1] +
        pref[x1 - 1][y1 - 1]
}
int32_t main() {
    int n, m; cin >> n >> m;
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= m; j++) {
            pref[i][j] = pref[i - 1][j] +
                pref[i][j - 1] - pref[i - 1][j
                    - 1] + a[i][j];
        }
    }
    int q; cin >> q;
    while (q--) {
        int x1, y1, x2, y2; cin >> x1 >> y1
            >> x2 >> y2;
        cout << query(x1, y1, x2, y2) << '\n';
    }
}

```



## 10.6 2D Static Range Update

```
// Add x on a rectangle q times,
// Finally print the array
const int N = 1e3 + 9;
int a[N][N];
ll d[N][N]; // difference array
int32_t main() {
    int n, m; cin >> n >> m;
    int q; cin >> q;
    while (q--) {
        int x1, y1, x2, y2, x; cin >> x1 >>
            y1 >> x2 >> y2 >> x; // add x on
            this rectangle
        d[x1][y1] += x;
        d[x1][y2 + 1] -= x;
        d[x2 + 1][y1] -= x;
        d[x2 + 1][y2 + 1] += x;
    }
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= m; j++) {
            d[i][j] += d[i - 1][j] + d[i][j -
                1] - d[i - 1][j - 1];
        }
    }
    // new updated array
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= m; j++) {
            cout << d[i][j] + a[i][j] << ' ';
        }
        cout << '\n';
    }
}
```

## 10.7 CHECK SET CLR

```
bool CHECK(int N, int pos) { return
    (bool)(N & (1ll << pos)); }
void SET(int &N, int pos) { N |= (1ll
    << pos); }
void CLR(int &N, int pos) { N &= ~(1ll
    << pos); }
```

## 10.8 Graph Directions

```
int dx[] = {+0, +0, +1, -1, -1, +1, -1,
    +1};
int dy[] = {-1, +1, +0, +0, +1, +1, -1,
    -1};
```

## 10.9 Custom Hash

```
struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) *
            0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) *
            0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM =
            chrono::steady_clock::now().time
                _since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
};
```

## 10.10 Coordinate Compression

```
int32_t main() {
    vector<int> a({100, 9, 10, 10, 9});
    vector<int> v = a;
    sort(v.begin(), v.end());
    v.resize(unique(v.begin(), v.end()) -
        v.begin());
    for (int i = 0; i < a.size(); i++) {
        a[i] = lower_bound(v.begin(),
            v.end(), a[i]) - v.begin() + 1;
    }
}
```

## 10.11 Submask Generator

```
int mask = 0; // any value
for (int submask = mask; submask;
    submask = (submask - 1) & mask) {}
```

## 10.12 Custom Comparators

```
bool cmp(pair<int, int> a, pair<int,
    int> b) { // for vector, arr,..
    if (a.first != b.first) return a.first
        > b.first;
    return a.second < b.second; // must
        return false for equal elements
}
struct cmp { // for set, map, pq,..
    bool operator()(const int& a, const
        int& b) const {
        return a > b;
    }
};
```

## 10.13 MOD Operations

```
inline ll modAdd(ll a, ll b, ll MOD) {ll
    res = a + b; res += ((res >> 63) &
        MOD); if (res >= MOD) res -= MOD;
    return res;}
inline ll modSub(ll a, ll b, ll MOD) {ll
    res = a - b; res += ((res >> 63) &
        MOD); if (res >= MOD) res -= MOD;
    return res;}
inline ll modMul(ll a, ll b, ll MOD) {a
    %= MOD; b %= MOD; a += ((a >> 63) &
        MOD); b += ((b >> 63) & MOD); return
        (a * b) % MOD;}
inline ll modInverse(ll a, ll MOD)
    {return modPow(a, MOD - 2, MOD);}
inline ll modDiv(ll a, ll b, ll MOD)
    {return modMul(a, modInverse(b,
        MOD), MOD);}
inline ll modMulBigMod(ll a, ll b, ll
    MOD) {a %= MOD; b %= MOD; a += ((a
        >> 63) & MOD); b += ((b >> 63) &
        MOD); return (ll)((__int128)a * b %
        MOD);}
```

## 10.14 Mex with Array Updates

```
int32_t main() {
    int n, q; cin >> n >> q;
    set<int> missing_numbers;
    for (int i = 0; i <= n + 200; i++) {
        missing_numbers.insert(i);
    }
    int a[n + 1];
```

```
map<int, int> freq;
for (int i = 1; i <= n; i++) {
    cin >> a[i];
    freq[a[i]]++;
    missing_numbers.erase(a[i]);
}
while (q--) {
    int i, x; cin >> i >> x; // Set
        a[i] = x
    int y = a[i]; a[i] = x;
    missing_numbers.erase(x);
    freq[y]--;
    freq[x]++;
    if (freq[y] == 0) {
        freq.erase(y);
        missing_numbers.insert(y);
    }
    cout << *missing_numbers.begin() <<
        '\n'; // mex after upd
}
}
```

## 11 Geometry

### 11.1 Convex Hull

```
typedef vector<long long> vll;
typedef long double ld;
#define int long long
struct point {
    int x, y;
    point() {}
    point(int x, int y) { this->x = x;
        this->y = y; }
    bool operator<(const point &other) {
        if (x == other.x) return y <
            other.y;
        return x < other.x;
    }
};
int crossProduct(point &a, point &b,
    point &c) {
    return (b.x - a.x) * (c.y - a.y) -
        (b.y - a.y) * (c.x - a.x);
}
vector<point> hull(vector<point> a) {
    vector<point> v = a;
    sort(v.begin(), v.end());
    vector<point> lower, upper;
    for (auto& p : v) {
        while (lower.size() >= 2 &&
            crossProduct(lower[lower.size()
                - 2], lower.back(), p) < 0) //
            <= for coleniar
            lower.pop_back();
        lower.push_back(p);
    }
    for (int i = v.size() - 1; i >= 0;
        i--) {
        point p = v[i];
        while (upper.size() >= 2 &&
            crossProduct(upper[upper.size()
                - 2], upper.back(), p) < 0) //
            <= for coleniar
            upper.pop_back();
        upper.push_back(p);
```

```
}
    lower.pop_back();
    upper.pop_back();
    lower.insert(lower.end(),
        upper.begin(), upper.end());
    return lower;
}
void senritsu() {
    int n; cin >> n;
    vector<point> a(n); for (int i = 0; i
        < n; i++) cin >> a[i].x >> a[i].y;
    a = hull(a);
    n = a.size();
    cout << n << endl;
    for (auto c : a) cout << c.x << ' '
        << c.y << endl;
}
```

### 11.2 Area of 2 polygon intersection

```
typedef vector<long long> vll;
typedef long double ld;
#define int long long
struct point {
    ld x, y;
    point() {}
    point(ld x, ld y) { this->x = x;
        this->y = y; }
    bool operator<(const point &other) {
        if (x == other.x) return y <
            other.y;
        return x < other.x;
    }
    bool operator==(const point &other) {
        return (x == other.x && y ==
            other.y);
    }
    bool operator!=(const point &other) {
        return (x != other.x || y !=
            other.y);
    }
    point operator-(const point &other) {
        return {x - other.x, y - other.y};
    }
};
ld crossProduct(point &a, point &b,
    point &c) {
    return (b.x - a.x) * (c.y - a.y) -
        (b.y - a.y) * (c.x - a.x);
}
ld crossProduct(point &a, point &b) {
    return a.x * b.y - a.y * b.x;
}
bool inside(point &a, point &b, point
    &c) {
    return crossProduct(a, b, c) >= 0;
}
bool intersection(point &a, point &b,
    point &c, point &d, point &ans) { //
    2 line intersection point
    point ab = b - a;
    point cd = d - c;
    ld det = crossProduct(ab, cd);
    if (det == 0) return false; //
        parallel or collinear
```

```

    ld z1 = crossProduct( a, b );
    ld z2 = crossProduct( c, d );
    ans.x = (ld)(z1 * (c.x - d.x) - z2 *
        (a.x - b.x)) / det;
    ans.y = (ld)(z1 * (c.y - d.y) - z2 *
        (a.y - b.y)) / det;
    return true;
}
vector<point> innerhull( vector<point>
    a, vector<point> b ) {
    int n = a.size();
    vector<point> ans = b;
    for ( int i = 0; i < n; i++ ) {
        point x = a[i], y = a[(i + 1) % n];
        vector<point> temp = ans; ans.clear();
        int m = temp.size();
        if ( m == 0 ) break;
        for ( int j = 0; j < m; j++ ) {
            point p = temp[j], q = temp[(j +
                1) % m];
            int pIn = inside(x, y, p);
            int qIn = inside(x, y, q);
            if ( pIn ) ans.push_back(p);
            if ( pIn != qIn ) {
                point cur;
                if ( intersection(x, y, p, q,
                    cur) ) {
                    ans.push_back(cur);
                }
            }
        }
        if ( !ans.empty() ) {
            vector<point> temp;
            temp.push_back(ans[0]);
            for ( int j = 1; j < ans.size();
                j++ ) {
                if ( ans[j] != ans[j - 1] ) {
                    temp.push_back(ans[j]);
                }
            }
            ans = temp;
        }
    }
    return ans;
}
ld areaOfPolygon( vector<point> a ) {
    if ( a.size() < 3 ) return 0.0;
    a.push_back(a[0]);
    int n = a.size();
    ld ans = 0;
    for ( int i = 0; i < n - 1; i++ ) {
        ans += a[i].x * a[i + 1].y;
        ans -= a[i].y * a[i + 1].x;
    }
    if ( ans < 0 ) ans = -ans;
    return ans / 2.0;
}
void senritsu() {
    int n, m; cin >> n >> m;
    vector<point> a(n), b(m);
    for ( int i = 0; i < n; i++ ) {
        cin >> a[i].x >> a[i].y;
    }
    for ( int i = 0; i < m; i++ ) {
        cin >> b[i].x >> b[i].y;

```

```

    }
    vector<point> in = innerhull(a, b);
    cout << fixed << setprecision(4) <<
        areaOfPolygon(in) << endl;
}

```