



American International University-Bangladesh

Aiub_Eclipse

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1	Setup	
1.1	Sublime Build	
<pre>"shell_cmd": "g++ -std=c++17 -o \"\$file_base_name\" \"\$file\" && timeout 2.5s ./\"\$file_base_name\" < input.txt > output.txt", "file_regex": "^(.*[:.]*):([0-9]+):?([0-9]+)?:(? ([*]\$", "working_dir": "\${file_path}", "selector": "source.c, source.c++"</pre>		
1.2	Template	
<pre>#include<bits/stdc++.h> #include<ext/pb_ds/assoc_container.hpp> #include<ext/pb_ds/tree_policy.hpp> using namespace __gnu_pbds; using namespace std;</pre>		

```
template <typename T> using o_set =
    tree<T, null_type, less<T>,
    rb_tree_tag,
    tree_order_statistics_node_update>;
typedef long long ll;
int32_t main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
}

2 Stress Testing
2.1 Input Gen
mt19937_64 rnd(chrono::steady_clock::now_()
    .time_since_epoch().count());
ll get_rand(ll l, ll r) {
    assert(l <= r);
    return l + rnd() % (r - l + 1);
}

2.2 Bash Script
// run -> bash script.sh
set -e
g++ code.cpp -o code
g++ gen.cpp -o gen
g++ brute.cpp -o brute
for(i = 1; ; ++i); do
    ./gen $i > input_file
    ./code < input_file > myAnswer
    ./brute < input_file > correctAnswer
    diff -Z myAnswer correctAnswer >
        /dev/null || break
    echo "Passed test: " $i
done
echo "WA on the following test:"
cat input_file
echo "Your answer is:"
cat myAnswer
echo "Correct answer is:"
cat correctAnswer

3 Number Theory
3.1 Euler Totient Function
// Time: O(sqrt(N))
map<int, int> dp; // memo
int phi(int n) {
    if (dp.count(n)) return dp[n];
    int ans = n, m = n;
    for (int i = 2; i * i <= m; i++) {
        if (m % i == 0) {
            while (m % i == 0) m /= i;
            ans = ans / i * (i - 1);
        }
    }
    if (m > 1) ans = ans / m * (m - 1);
    return dp[n] = ans;
}

3.2 Phi 1 to N
void phi_1_to_n(int n) {
    vector<int> phi(n + 1);
    for (int i = 0; i <= n; i++)
        phi[i] = i;
    for (int i = 2; i <= n; i++) {
        if (phi[i] == i) {
            for (int j = i; j <= n; j += i)
                phi[j] -= phi[j] / i;
        }
    }
}

3.3 Segmented Sieve
vector<char> segmentedSieve(ll L, ll R) {
    // generate all primes up to sqrt(R)
    ll lim = sqrt(R);
    vector<char> mark(lim + 1, false);
    vector<ll> primes;
    for (ll i = 2; i <= lim; ++i) {
        if (!mark[i]) {
            primes.emplace_back(i);
            for (ll j = i * i; j <= lim; j += i)
                mark[j] = true;
        }
    }
    vector<char> isPrime(R - L + 1, true);
    for (ll i : primes)
        for (ll j = max(i * i, (L + i - 1) /
            i * i); j <= R; j += i)
            isPrime[j - L] = false;
    if (L == 1) isPrime[0] = false;
    return isPrime;
}

3.4 Extended GCD
// ax + by = gcd(a,b)
int egcd(int a, int b, int& x, int& y) {
    if (b == 0) {
        x = 1, y = 0;
        return a;
    }
    int x1, y1;
    int d = egcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}

3.5 Linear Diophantine Equation
// ax + by = c, find any x and y
bool find_any_solution(int a, int b, int
    c, int &x0, int &y0, int &g) {
    g = egcd(abs(a), abs(b), x0, y0);
    if (c % g) return false;
    x0 *= c / g;
    y0 *= c / g;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    return true;
}
void shift_solution(int &x, int &y,
    int a, int b, int cnt) {
    x += cnt * b;
    y -= cnt * a;
}
int find_all_solutions(int a, int b, int
    c, int minx, int maxx, int miny, int
    maxy) {
    int x, y, g;
    if (!find_any_solution(a, b, c, x, y,
        g)) return 0;
```

```
a /= g, b /= g;
int sign_a = a > 0 ? +1 : -1;
int sign_b = b > 0 ? +1 : -1;
shift_solution(x, y, a, b, (minx - x) / b);
if (x < minx) shift_solution(x, y, a, b, sign_b);
if (x > maxx) return 0;
int lx1 = x;
shift_solution(x, y, a, b, (maxx - x) / b);
if (x > maxx) shift_solution(x, y, a, b, -sign_b);
int rx1 = x;
shift_solution(x, y, a, b, -(miny - y) / a);
if (y < miny) shift_solution(x, y, a, b, -sign_a);
if (y > maxy) return 0;
int lx2 = x;
shift_solution(x, y, a, b, -(maxy - y) / a);
if (y > maxy) shift_solution(x, y, a, b, sign_a);
int rx2 = x;
if (lx2 > rx2) swap(lx2, rx2);
int lx = max(lx1, lx2);
int rx = min(rx1, rx2);
if (lx > rx) return 0;
return (rx - lx) / abs(b) + 1;
}
```

3.6 Modular Inverse using EGCD

```
// finding inverse(a) modulo m
int x, y;
int g = extended_euclidean(a, m, x, y);
if (g != 1) cout << "No solution!";
else {
    x = (x % m + m) % m;
    cout << x << endl;
}
```

3.7 Exclusion DP

```
ll f[N], g[N];
for (int i = N - 1; i >= 1; i--) {
    f[i] = nc4(div_cnt[i]);
    g[i] = f[i];
    for (int j = i + i; j < N; j += i) {
        g[i] -= g[j];
    }
}
```

Here, $f[i]$ = how many pairs/k-tuple such that their gcd is i or it's multiple (count of pairs those are divisible by i).

$g[i]$ = how many pairs/k-tuple such that their gcd is i .

$g[i] = f[i] - \sum_{j|i} g[j]$.

Sum of all pair gcd:

We know, how many pairs are there such that their gcd is i for every i (1 to n). So now, $\sum_{i=1}^n g[i]*i$.

Sum of all pair lcm ($i = 1, j = 1$):
We know, $\text{lcm}(a, b) = \frac{a*b}{\gcd(a, b)}$.

Now, $f[i]$ = All pair product sum of those, whose gcd is i or it's multiple.

$g[i] = \text{All pair product sum of those, whose gcd is } i.$
 $\text{Ans} = \sum_{i=1}^n \frac{g[i]}{i}.$

All pair product sum = $(a_1 + a_2 + \dots + a_n) * (a_1 + a_2 + \dots + a_n)$

3.8 Legendres Formula

```
// n! / p^x - you will get the largest x
int legendre(int n, int p) {
    int ex = 0;
    while(n) {
        ex += (n / p);
        n /= p;
    }
    return ex;
}
```

3.9 Binary Expo

```
int power(int x, long long n, int mod) {
    int ans = 1 % mod;
    while (n > 0) {
        if (n & 1) {
            ans = 1LL * ans * x % mod;
        }
        x = 1LL * x * x % mod;
        n >>= 1;
    }
    return ans;
}
```

3.10 Digit Sum of 1 to N

```
// for n=10, ans = 1+2+...+9+1+0
ll solve(ll n) {
    ll res = 0, p = 1;
    while (n / p > 0) {
        ll left = n / (p * 10);
        ll cur = (n / p) % 10;
        ll right = n % p;
        res += left * 45 * p;
        res += (cur * (cur - 1) / 2) * p;
        res += cur * (right + 1);
        p *= 10;
    }
    return res;
}
```

3.11 Pollard Rho

```
namespace PollardRho {
mt19937 rnd(chrono::steady_clock::now());
time_since_epoch().count());
const int P = 1e6 + 9;
ll seq[P];
int primes[P], spf[P];
inline ll add_mod(ll x, ll y, ll m) {
    return (x + y) < m ? x : x - m;
}
inline ll mul_mod(ll x, ll y, ll m) {
    ll res = __int128(x) * y % m;
    return res;
}
// ll res = x * y - (ll)((long double)x * y / m + 0.5) * m;
// return res < 0 ? res + m : res;
}
inline ll pow_mod(ll x, ll n, ll m) {
    ll res = 1 % m;
    for (; n; n >>= 1) {
        if (n & 1) res = mul_mod(res, x, m);
    }
}
```

```
x = mul_mod(x, x, m);
}
return res;
}
// O(it * (log n)^3), it = number of rounds performed
inline bool miller_rabin(ll n) {
    if (n <= 2 || (n & 1 == 1)) return (n == 2);
    if (n < P) return spf[n] == n;
    ll c, d, s = 0, r = n - 1;
    for (; !(r & 1); r >>= 1, s++) {}
    // each iteration is a round
    for (int i = 0; primes[i] < n && primes[i] < 32; i++) {
        c = pow_mod(primes[i], r, n);
        for (int j = 0; j < s; j++) {
            d = mul_mod(c, c, n);
            if (d == 1 && c != 1 && c != (n - 1)) return false;
            c = d;
        }
        if (c != 1) return false;
    }
    return true;
}
void init() {
    int cnt = 0;
    for (int i = 2; i < P; i++) {
        if (!spf[i]) primes[cnt++] = spf[i] = i;
        for (int j = 0, k; (k = i * primes[j]) < P; j++) {
            spf[k] = primes[j];
            if (spf[i] == spf[k]) break;
        }
    }
}
// returns O(n^(1/4))
ll pollard_rho(ll n) {
while (1) {
    ll x = rnd() % n, y = x, c = rnd() % n,
        u = 1, v, t = 0;
    ll *px = seq, *py = seq;
    while (1) {
        *py++ = y = add_mod(mul_mod(y, y, n), c, n);
        *py++ = y = add_mod(mul_mod(y, y, n), c, n);
        if ((x = *px++) == y) break;
        v = u;
        u = mul_mod(u, abs(y - x), n);
        if (!u) return __gcd(v, n);
        if (++t == 32) {
            t = 0;
            if ((u = __gcd(u, n)) > 1 && u < n) return u;
        }
        if (t && (u = __gcd(u, n)) > 1 && u < n) return u;
    }
    vector<ll> factorize(ll n) {

```

```
if (n == 1) return vector<ll>();
if (miller_rabin(n)) return vector<ll>{n};
vector<ll> v, w;
while (n > 1 && n < P) {
    v.push_back(spf[n]);
    n /= spf[n];
}
if (n >= P) {
    ll x = pollard_rho(n);
    v = factorize(x);
    w = factorize(n / x);
    v.insert(v.end(), w.begin(), w.end());
}
return v;
}
}
```

3.12 [Problem] How Many Bases - UVa

```
// Given a number N^M, find out the number of integer bases in which it has exactly T trailing zeroes.
int solve_greater_or_equal(vector<int> e, int t) {
    int ans = 1;
    for (auto i : e) {
        ans = 1LL * ans * (i / t + 1) % mod;
    }
    return ans;
}
// e contains e1, e2 -> p1^{e1}, p2^{e2}
int solve_equal(vector<int> e, int t) {
    return (solve_greater_or_equal(e, t) -
            solve_greater_or_equal(e, t + 1) +
            mod) % mod;
}
```

3.13 [Problem] Power Tower - CF

```
// A sequence w1, w2,...,wn and Q queries, l and r will be given.
Calculate w_l^{(w_r^{(w_r)})} mod m
// n^x mod m = n^{\phi(m)+x \mod \phi(m)} \mod m
inline int MOD(int x, int m) {
    if (x < m) return x;
    return x % m + m;
}
int power(int n, int k, int mod) {
    int ans = MOD(1, mod);
    while (k) {
        if (k & 1) ans = MOD(ans * n, mod);
        n = MOD(n * n, mod);
        k >>= 1;
    }
    return ans;
}
int f(int l, int r, int m) {
    if (l == r) return MOD(a[l], m);
    if (m == 1) return 1;
    return power(a[l], f(l + 1, r, phi(m)), m);
}
```

3.14 Formula and Properties

- $\phi(n) = n \cdot \frac{p_1-1}{p_1} \cdot \frac{p_2-1}{p_2} \dots$
- $\phi(p^e) = p^e - \frac{p^e}{p} = p^e \cdot \frac{p-1}{p}$
- For $n > 2$, $\phi(n)$ is always even.
- $\sum_{d|n} \phi(d) = n$
- NOD: $(e_1 + 1) \cdot (e_2 + 1) \dots$
- SOD: $\frac{p_1^{e_1+1}-1}{p_1-1} \cdot \frac{p_2^{e_2+1}-1}{p_2-1} \dots$
- $\log(a \cdot b) = \log(a) + \log(b)$
- $\log(a^x) = x \cdot \log(a)$
- $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$
- Digit Count of n:** $\lfloor \log_{10}(n) \rfloor + 1$
- Arithmetic Progression Sum:** $\frac{n}{2} \cdot (a + p), \quad \frac{n}{2} \cdot (2a + (n-1)d)$
- Geometric Sum:** $S_n = a \cdot \frac{r^n - 1}{r-1}$
- $(1^2 + 2^2 + \dots + n^2) = \frac{n(n+1)(2n+1)}{6}$
- $(1^3 + 2^3 + \dots + n^3) = \frac{n^2(n+1)^2}{4}$
- $(2^2 + 4^2 + \dots + (2n)^2) = \frac{2n(n+1)(2n+1)}{3}$
- $(1^2 + 3^2 + \dots + (2n-1)^2) = \frac{n(2n-1)(2n+1)}{3}$
- $(2^3 + 4^3 + \dots + (2n)^3) = 2n^2(n+1)^2$
- $(1^3 + 3^3 + \dots + (2n-1)^3) = n^2(2n^2 - 1)$
- For any number n and bases $> \sqrt{n}$, there will be no representation where the number contains 0 at its second least significant digit. So it is enough to check for bases $\leq \sqrt{n}$.
- For some x and y , let's try to find all m such that $x \bmod m \equiv y \bmod m$. We can rearrange the equation into $(x - y) \equiv 0 \pmod{m}$. Thus, if m is a factor of $|x - y|$, then x and y will be equal modulo m .

4 Combinatorics and Probability

4.1 Combinations

```
// Prime Mod in O(n)
void prec() {
    fact[0] = 1;
    for (int i = 1; i < N; i++) {
        fact[i] = 1ll * fact[i - 1] * i % mod;
    }
    ifact[N - 1] = inverse(fact[N - 1]);
    for (int i = N - 2; i >= 0; i--) {
        ifact[i] = 1ll * ifact[i + 1] * (i + 1) % mod;
    }
}

int nCr(int n, int r) {
    if (r > n) return 0;
    return 1ll * fact[n] * ifact[r] % mod
        * ifact[n - r] % mod;
}

int nPr(int n, int r) {
    if (r > n) return 0;
    return 1ll * fact[n] * ifact[n - r] %
        mod;
}
```

4.2 nCr for any mod

```
// Time: O(n^2)
// nCr = (n-1)C(r-1) + (n-1)Cr
for (int i = 0; i < N; i++) {
    C[i][i] = 1;
    for (int j = 0; j < i; j++) {
        C[i][j] = (C[i - 1][j] + C[i - 1][j - 1]) % mod;
    }
}
```

4.3 nCk without mod in O(r)

```
ll nCk(ll n, ll k) {
    double res = 1;
    for (ll i = 1; i <= k; ++i)
        res = res * (n - k + i) / i;
    return (ll)(res + 0.01);
}
```

4.4 Lucas Theorem

```
// returns nCr modulo mod where mod is a prime
// Complexity: ?
ll Lucas(ll n, ll r) {
    if (r < 0 || r > n) return 0;
    if (r == 0 || r == n) return 1;
    if (n >= MOD) {
        return (Lucas(n / MOD, r / MOD) %
            MOD * Lucas(n % MOD, r % MOD) %
            MOD) % MOD;
    }
    return (((fact[n] * invFact[r]) % MOD) *
        * invFact[n - r]) % MOD;
}
```

4.5 Catalan Number

```
const int MOD = 1e9 + 7, int MAX = 1e7;
int catalan[MAX];
void init(ll n) {
    catalan[0] = catalan[1] = 1;
    for (ll i = 2; i <= n; i++) {
        catalan[i] = 0;
        for (ll j = 0; j < i; j++) {
            catalan[i] += (catalan[j] *
                catalan[i - j - 1]) % MOD;
            if (catalan[i] >= MOD) {
                catalan[i] -= MOD;
            }
        }
    }
}
```

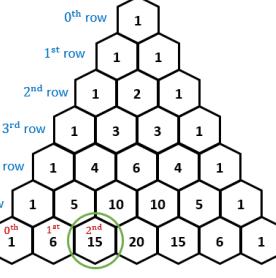
4.6 Derangement

```
// number of combinations such that
// a_i != i of a permutation a
const int N = 1e6 + 100, int p = 1e9 + 7;
ll der[N];
void countDer() {
    der[1] = 0;
    der[2] = 1;
    for (ll i = 3; i <= N; ++i) {
        der[i] = (i - 1) % p * (der[i - 1] %
            p + der[i - 2] % p);
        der[i] %= p;
    }
}
```

4.7 Stars and Bars Theorem

- Find the number of k -tuples of non-negative integers whose sum is n . $\binom{n+k-1}{n}$
- Find the number of k -tuples of non-negative integers whose sum is $\leq n$. $\binom{n+k}{k}$
- Combination with Repetition (choose k elements from n objects, same element can be chosen multiple times). $\binom{n+k-1}{k}$
- How many ways to go from $(0,0)$ to (n,m) . $\binom{n+m}{m}$

Pascals Triangle is equivalent to nCr:



4.8 Properties of Pascal's Triangle

- $(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$
- $(k+1)^n = \sum_{i=0}^n k^i \cdot \binom{n}{i}$
- $\sum_{i=0}^n \binom{n}{i} = 2^n$
- $\binom{k}{n} = \frac{k}{n} \binom{k-1}{n-1}$
- $\sum_{k=0}^m \binom{n+k}{k} = \binom{n+m+1}{m}$
- $\binom{n}{0}^2 + \binom{n}{1}^2 + \dots + \binom{n}{n}^2 = \binom{2n}{n}$
- $1 \binom{n}{1} + 2 \binom{n}{2} + \dots + n \binom{n}{n} = n 2^{n-1}$

4.9 Contribution Technique

- Sum of all pair sums: $\sum_{i=1}^n \sum_{j=1}^n (a_i + a_j)$
Every element will be added $2n$ times. $\sum_{i=1}^n (2 \times n \times a_i) = 2 \times n \times \sum_{i=1}^n a_i$.
- Sum of all subarray sums — $\sum_{i=1}^n (a_i \times i \times (n - i + 1))$.
- Sum of all Subsets sums — $\sum_{i=1}^n (2^{n-1} \times a_i)$.
- Product of all pair product — $\prod_{i=1}^n (a_i^{2 \times n})$.
- XOR of subarray XORS — How many subarrays does an element have? $(i \cdot (n - i + 1))$ times. If subarray count is odd then this element can contribute in total XORs.
- Sum of max minus min over all subset — Sort the array. $Min = 2^{n-i}, Max = 2^{i-1}$. $\sum_{i=1}^n (a_i \cdot 2^{i-1} - a_i \cdot 2^{n-i})$
- Sum using bits — $\sum_{k=0}^{30} (cnt_k[1] \times 2^k)$.

- Sum of Pair XORs — XOR will 1 if two bits are different $\sum_{k=0}^{30} (cnt_k[0] \times cnt_k[1] \times 2^k)$.
- Sum of Pair ANDs — $\sum_{k=0}^{30} (cnt_k[1]^2 \times 2^k)$.
- Sum of Pair ORs — $\sum_{k=0}^{30} ((cnt_k[1]^2 + 2 \times cnt_k[1] \times cnt_k[0]) \times 2^k)$.
- Sum of Subset XORs — where $cnt0! = 0$ $\sum_{k=0}^{30} (2^{cnt_k[1]} + cnt_k[0] - 1 \times 2^k)$.
- Sum of Subset ANDs — $\sum_{k=0}^{30} ((2^{cnt_k[1]} - 1) \times 2^k)$.
- Sum of Subset ORs — $\sum_{k=0}^{30} ((2^n - 2^{cnt_k[0]}) \times 2^k)$.
- Sum of subarray XORs — Convert to prefix xor, then solve for pairs.
- Sum of product of all subsequence — $\prod_{i=1}^n (a_i + 1) - 1$. Example array — $[a, b]$ the subsequences are $\{a\}, \{b\}, \{a, b\}$ so ans is $a + b + (a \cdot b)$

4.10 Probability and Expected Value

- Expected Value: $E = \frac{\text{Sum of all possible values}}{\text{Total number of outcomes}}$
- Expected Value: $E = \sum_{i=1}^n P_i \cdot i$
- Variance: $V(x) = E(x^2) - \{E(x)\}^2$
- Expected Value with DP: $E(i) = \sum P(i \rightarrow j) \times (R(i \rightarrow j) + E(j))$ Where $R()$ is Immediate Reward(cost/count)
- Linearity of Expectation:
 - $E[X + Y] = E[X] + E[Y]$
 - $E[\text{Total}] = E[I_1] + E[I_2] + \dots = \sum P(I_i = 1)$

We need to define the Indicator Random Variable (I) correctly. Some Examples below —

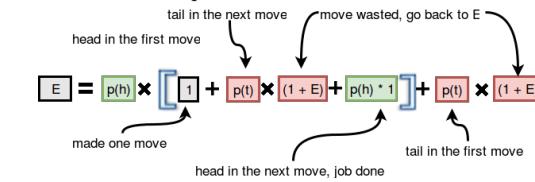
Problem-1: Find $E[\text{correct hats}]$ in a random permutation. Indicator I_i : "Does person i get their own hat?"

Problem-2: Find $E[\text{total inversions}]$ in a random permutation. Indicator I_{ij} : "Is pair (i, j) an inversion?"

Problem-3: Given a string S , delete a random index until it becomes empty. Find the expected count of palindromes seen (exclude S , include empty string). Indicator I_L : "Is the string of length L a palindrome?"

Problem-4: Pick N random integers from $[1, k]$. Merge consecutive equal values (e.g., $1, 1, 2, 2, 1, 1 \rightarrow 1, 2, 1$). Find the expected final length. Indicator I_i : "Is item i different from item $i - 1$?"

- To get two consecutive heads, what is the expected number of tosses?



- To get n heads, what is the expected number of tosses? Let's define: to get n heads, we need to toss $E(n)$ times. Now — I can get a head; I need

to toss $E(n - 1)$ more times, or if I get a tail; I need to toss $E(n)$ times. So, the recurrence is: $E(n) = 0.5 \cdot (1 + E(n - 1)) + 0.5 \cdot (1 + E(n))$

- You have n bulbs, all of which are initially off. In each move, you randomly select one bulb. If the selected bulb is off, you toss a coin:

- If you get head, you turn it on.
- If you get tail, you do nothing.

If the bulb is already on, you skip that move (nothing happens).

Now, what is the expected number of moves required to turn all bulbs on?

The coin is not fair — the probability of getting tail is p . This problem can also be solved recursively.

Let's assume at some moment, x bulbs are already on, and the expected number of moves needed from here is $e(x)$.

The probability of picking an already on bulb is $\frac{x}{n}$. In that case, the expected number of moves is $\frac{x}{n} \times (1 + e(x))$.

The probability of picking an off bulb is $\frac{n-x}{n}$.

Now two things can happen:

- With probability p , you get tail, so you stay at the same state ($e(x)$ more moves).
- With probability $(1 - p)$, you get head, so one more bulb turns on ($e(x + 1)$ moves from there).

So, the recurrence relation is:

$$\begin{aligned} e(x) &= \frac{x}{n}(1 + e(x)) + \\ &\quad \frac{n-x}{n}(p(1 + e(x)) + (1-p)(1 + e(x+1))) \end{aligned}$$

5 Data Structure

5.1 Trie for bit

```
struct Trie {
    static const int B = 31;
    struct node {
        node* nxt[2];
        int sz;
        node() {
            nxt[0] = nxt[1] = NULL;
            sz = 0;
        }
    }*root;
    Trie() {
        root = new node();
    }
    void insert(int val) {
        node* cur = root;
        cur->sz++;
        for (int i = B - 1; i >= 0; i--) {
            int b = val >> i & 1;
            if (cur->nxt[b] == NULL) cur->nxt[b] = new node();
            cur = cur->nxt[b];
            cur->sz++;
        }
    }
}
```

```
int query(int x, int k) { // number
    of values s.t. val ⊕ x < k
    node* cur = root;
    int ans = 0;
    for (int i = B - 1; i >= 0; i--) {
        if (cur == NULL) break;
        int b1 = x >> i & 1, b2 = k >> i &
        1;
        if (b2 == 1) {
            if (cur->nxt[b1]) ans += cur
            ->nxt[b1] -> sz;
            cur = cur->nxt[!b1];
        } else cur = cur->nxt[b1];
    }
    return ans;
}

int get_max(int x) { // returns
    maximum of val ⊕ x
    node* cur = root;
    int ans = 0;
    for (int i = B - 1; i >= 0; i--) {
        int k = x >> i & 1;
        if (cur->nxt[!k]) cur = cur->
        nxt[!k], ans <= 1, ans++;
        else cur = cur->nxt[k], ans <= 1;
    }
    return ans;
}

int get_min(int x) { // returns
    minimum of val ⊕ x
    node* cur = root;
    int ans = 0;
    for (int i = B - 1; i >= 0; i--) {
        int k = x >> i & 1;
        if (cur->nxt[k]) cur = cur->
        nxt[k], ans <= 1;
        else cur = cur->nxt[!k], ans <=
        1, ans++;
    }
    return ans;
}

void del(node* cur) {
    for (int i = 0; i < 2; i++) if (cur
    ->nxt[i]) del(cur->nxt[i]);
    delete(cur);
}
}
```

6 Dynamic Programming

6.1 Knapsack 2

Constraints: $N \leq 100$, $W \leq 1e9$, $val[i] \leq 1000$
 $dp[i][cur_val] = \min$ weight needed to achieve cur_val from i to n . **if** $dp[i][val] \leq W$ then val is a candidate ans . where val is all_possible_val (1 to 100×1000)

6.2 LIS using Segment Tree

```
int32_t main() {
    int n; cin >> n;
    for (int i = 1; i <= n; i++) {
        cin >> a[i]; // a[i] must be >= 2
    }
}
```

```
// dp[i] = LIS ending at pos i
st.build(1, 1, M); // range max query,
and upd idx with max(cur_val,
new_val)
for (int i = 1; i <= n; i++) {
    dp[i] = 1;
    if (a[i] != 1) {
        int mx = st.query(1, 1, M, 1, a[i]
        - 1);
        mx++;
        dp[i] = max(dp[i], mx);
    }
    st.upd(1, 1, M, a[i], dp[i]);
}
int ans = 0;
for (int i = 1; i <= n; i++) ans =
max(ans, dp[i]);
cout << ans << '\n';
}

6.3 Digit DP
// Problem: How Many Zeroes? - LightOJ
// How many zeroes between n to m
(n <= m). 100 to 102 ans 4
11 fun2(int i, bool is_small) {
    if (i == sz) return 1;
    int l = 0, r = s[i] - '0';
    if (is_small) r = 9;
    ll &ans = dp2[i][is_small];
    if (ans != -1) return ans;
    ans = 0;
    for (int x = l; x <= r; x++) {
        ans += fun2(i + 1, (is_small | (x <
        r)));
    }
    return ans;
}
11 fun(int i, bool is_small, bool
has_started) {
    if (i == sz) return 0;
    int l = 0, r = s[i] - '0';
    if (is_small) r = 9;
    ll &ans = dp1[i][is_small][has_started];
    if (ans != -1) return ans;
    ans = 0;
    for (int x = l; x <= r; x++) {
        bool new_has_started = has_started |
        (x != 0);
        ans += fun(i + 1, (is_small | (x <
        r)), new_has_started);
        if (x == 0 and has_started) {
            ans += fun2(i + 1, (is_small | (x
            < r)));
        }
    }
    return ans;
}
void get(long long x) {
    if (x < 0) return; s = "";
    while (x > 0) {
        char c = (x % 10) + '0';
        s += c; x /= 10;
    }
    reverse(s.begin(), s.end());
    sz = s.size();
}
```

```
memset(dp1, -1, sizeof(dp1));
memset(dp2, -1, sizeof(dp2));
}
void solve() {
    ll n, m; cin >> n >> m;
    get(n - 1);
    memset(dp1, -1, sizeof(dp1));
    memset(dp2, -1, sizeof(dp2));
    ll ans1 = (n == 0) ? 0 : fun(0, false,
    false);
    get(m);
    ll ans2 = fun(0, false, false);
    cout << ans2 - ans1 + (n == 0) << '\n';
}

6.4 Bitmask DP
// Problem: DNA Sequence - LOJ
// Given n strings, find the shortest
string that contains all the given
strings as substrings and
lexicographically smallest.
const int N = 17;
int n, tail[N][N], dp[(1 << N) + 2][N +
2];
string s[N + 2];
bool cmp(string a, string b) {
    if (a.size() < b.size()) {
        return true;
    }
    return false;
}
int fun(int mask, int last) {
    if (_builtin_popcount(mask) >= n)
        return 0;
    int &ans = dp[mask][last];
    if (ans != -1) return ans;
    ans = 1e9;
    for (int j = 1; j <= n; j++) {
        if (!(mask & (1 << j))) {
            if (last == n + 1) {
                int x = s[j].length() + fun(mask
                | (1 << j), j);
                ans = min(ans, x);
            }
            else {
                int x = (s[j].length() -
                tail[last][j]) + fun(mask |
                (1 << j), j);
                ans = min(ans, x);
            }
        }
    }
    return ans;
}
void print(int mask, int last) {
    if (_builtin_popcount(mask) >= n)
        return;
    int ans = fun(mask, last), idx = -1;
    string str = "{";
    for (int j = 1; j <= n; j++) {
        if (!(mask & (1 << j))) {
            if (last == n + 1) {
                int x = s[j].length() + fun(mask
                | (1 << j), j);
                if (x == ans) {
```

```

        string d = s[j];
        if (d <= str) str = d; idx = j;
    }
    else {
        int x = (s[j].length() -
                  tail[last][j]) + fun(mask | (1 << j), j);
        if (x == ans) {
            string d =
                s[j].substr(tail[last][j]);
            if (d <= str) str = d; idx = j;
        }
    }
}
cout << s[idx].substr(tail[last][idx]);
print(mask | (1 << idx), idx);
}

void solve() {
    cin >> n;
    string str[n + 1];
    for (int i = 1; i <= n; i++) {
        cin >> str[i]; // len <= 100
    }
    // remove duplicates and strings which
    // is a subarray of others
    sort(str + 1, str + n + 1, cmp);
    vector<string> vec;
    for (int i = 1; i <= n; i++) {
        bool ok = true;
        for (int j = i + 1; j <= n; j++) {
            if (str[j].find(str[i]) !=
                string::npos) {
                ok = false;
                break;
            }
        }
        if (ok) vec.push_back(str[i]);
    }
    n = vec.size();
    int idx = 1;
    for (auto x : vec) s[idx++] = x;
    // maximum length that suffix of a[i] =
    // prefix of a[j]
    memset(tail, 0, sizeof(tail));
    for (int i = 1; i <= n; i++) {
        string x = s[i];
        for (int j = 1; j <= n; j++) {
            string y = s[j];
            int cnt = 0;
            for (int k = min(x.length(),
                              y.length()); k >= 1; k--) {
                if (x.substr(x.length() - k) ==
                    y.substr(0, k)) {
                    cnt = k;
                    break;
                }
            }
            tail[i][j] = cnt;
        }
    }
    memset(dp, -1, sizeof(dp));
    fun(0, n + 1);
    print(0, n + 1);
}

```

6.5 MCM DP

// Problem: Slimes - Atcoder DP Contest
// Given n slimes. Choose two adjacent slimes, and combine them into a new slime. The new slime has a size of $x+y$, where x and y are the sizes of the slimes before combining them. Here, a cost of $x+y$ is incurred.
Example-
 $(10, 20, 30, 40) \rightarrow (30, 30, 40)$
 $(30, 30, 40) \rightarrow (60, 40)$
 $(60, 40) \rightarrow (100)$ ans = 190
Solution: Think reverse. We are given the final sum, from i to j . Now we will cut any point between i to j and calculate the cost
Time: $O(n^3)$

```

ll fun(int i, int j) {
    if (i == j) return 0;
    ll &ans = dp[i][j];
    if (ans != -1) return ans;
    ll cur = 0;
    for (int x = i; x <= j; x++) {
        cur += a[x];
    }
    ans = inf;
    for (int x = i; x < j; x++) {
        ans = min(ans, cur + fun(i, x) +
                   fun(x + 1, j));
    }
    return ans;
}
cout << fun(1, n) << '\n';

```

6.6 Number of Unique Subsequence

```

int nxt[N][26], dp[N];
int f(int i) {
    if (i > n) return 0;
    int &ans = dp[i];
    if (ans != -1) return ans;
    ans = 1;
    for (int c = 0; c < 26; c++) {
        ans = (ans + f(nxt[i][c])) % mod;
    }
    return ans;
}
// nxt[i][c] = nxt c strictly after i
void solve() {
    cin >> s; s = '.' + s;
    cout << f(0) << '\n';
}

```

7 Graph Theory

7.1 Binary Lifting and LCA

```

const int N = 2e5 + 9, LOG = 20;
vector<int> g[N];
int par[N][LOG], depth[N];
void dfs(int u, int p) {
    par[u][0] = p;
    depth[u] = depth[p] + 1;
    for (int i = 1; i < LOG; i++) {
        par[u][i] = par[par[u][i - 1]][i - 1];
        depth[u] = depth[par[u][i - 1]] + 1;
    }
}

```

```

for (auto v : g[u]) {
    if (v != p) {
        dfs(v, u);
    }
}

```

```

int lca(int u, int v) {
    if (depth[u] < depth[v]) {
        swap(u, v);
    }
    int k = depth[u] - depth[v];
    for (int i = 0; i < LOG; i++) {
        if (CHECK(k, i)) u = par[u][i];
    }
}

```

```

if (u == v) return u;
for (int i = LOG - 1; i >= 0; i--) {
    if (par[u][i] != par[v][i]) {
        u = par[u][i];
        v = par[v][i];
    }
}
return par[u][0];
}

```

```

int kth(int u, int k) { // kth parent of u
    assert(k >= 0);
    for (int i = 0; i < LOG; i++) {
        if (CHECK(k, i)) u = par[u][i];
    }
    return u;
}

```

```

int dist(int u, int v) { // distance from u to v
    int l = lca(u, v);
    return (depth[u] - depth[l]) +
           (depth[v] - depth[l]);
}

```

```

// kth node from u to v, 0th node is u
int kth(int u, int v, int k) {
    int l = lca(u, v);
    int d = dist(u, v);
    assert(k <= d);
    if (depth[l] + k <= depth[u]) {
        return kth(u, k);
    }
    k -= depth[u] - depth[l];
    return kth(v, depth[v] - depth[l] - k);
}

```

7.2 LCA and Sparse Table on Tree

```

// max and min weights of a path
const int N = 1e5 + 9, LOG = 20, inf =
    1e9; // change here
vector<array<int, 2>> g[N];

```

```
int par[N][LOG], tree_mx[N][LOG],
    depth[N];

```

```

void dfs(int u, int p, int dis) {
    par[u][0] = p;
    tree_mx[u][0] = dis;
    depth[u] = depth[p] + 1;
    for (int i = 1; i < LOG; i++) {
        par[u][i] = par[par[u][i - 1]][i - 1];
        tree_mx[u][i] = max(tree_mx[u][i - 1],
                           tree_mx[par[u][i - 1]][i - 1]);
    }
}

```

```

} for (auto [v, w] : g[u]) {
    if (v != p) {
        dfs(v, u, w);
    }
}

```

```

int query_max(int u, int v) { // max weight on path u to v
    int l = lca(u, v);
    int d = dist(l, u);
    int ans = 0;
    for (int i = 0; i < LOG; i++) {
        if (CHECK(d, i)) {
            ans = max(ans, tree_mx[u][i]);
            u = par[u][i];
        }
    }
    d = dist(l, v);
    for (int i = 0; i < LOG; i++) {
        if (CHECK(d, i)) {
            ans = max(ans, tree_mx[v][i]);
            v = par[v][i];
        }
    }
    return ans;
}

```

7.3 Dijkstra

```

vector<int> dijkstra(int s) {
    vector<int> dis(n + 1, inf);
    vector<bool> vis(n + 1, false);
    dis[s] = 0;
    priority_queue<array<int, 2>, vector<array<int, 2>>, greater<array<int, 2>>> pq;
    pq.push({0, s});
    while (!pq.empty()) {
        auto [d, u] = pq.top(); pq.pop();
        if (vis[u]) continue;
        vis[u] = true;
        for (auto [v, w] : g[u]) {
            if (dis[v] > d + w) {
                dis[v] = d + w;
                pq.push({dis[v], v});
            }
        }
    }
    return dis;
}

```

7.4 Bellman Ford

```

// works for neg edge, can detect neg cycle
// Time:  $O(n^2)$ 
const ll inf = 1e18;
vector<ll> dis(N, inf);
bool bellman_ford(int s) {
    dis[s] = 0;
    bool has_cycle = false;
    for (int i = 1; i <= n; i++) {
        for (int u = 1; u <= n; u++) {
            for (auto [v, w] : g[u]) {

```

```

    if (dis[v] > dis[u] + w) {
        if (i == n) has_cycle = true;
        dis[v] = dis[u] + w;
    }
}
return has_cycle;
}

7.5 Floyd Warshall
// dis[i][j] = min distance to reach i to
// j, works for neg edge (no neg cycle)
// Time: O(n3)
vector<int> construct_path(int u, int v)
{ // O(n)
    if (nxt[u][v] == -1) return {};
    vector<int> path = { u };
    while (u != v) {
        u = nxt[u][v];
        path.push_back(u);
    }
    return path;
}
void floyd_marshall() {
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= n; j++) {
            if (i == j) dis[i][j] = 0;
            else if (g[i][j] == 0) dis[i][j] =
                inf;
            else dis[i][j] = g[i][j];
        }
    }
    for (int k = 1; k <= n; ++k) {
        for (int i = 1; i <= n; ++i) {
            for (int j = 1; j <= n; ++j) {
                if (dis[i][k] < inf and
                    dis[k][j] < inf)
                    dis[i][j] = min(dis[i][j],
                        dis[i][k] + dis[k][j]);
                nxt[i][j] = nxt[i][k];
            }
        }
    }
}
int32_t main() {
    int q; cin >> n >> m >> q;
    memset(nxt, -1, sizeof nxt);
    while (m--) {
        int u, v, w; cin >> u >> v >> w;
        g[u][v] = (g[u][v] != 0 ?
            min(g[u][v], w) : w);
        g[v][u] = (g[v][u] != 0 ?
            min(g[v][u], w) : w);
        nxt[u][v] = v;
        nxt[v][u] = u;
    }
    floyd_marshall();
    while (q--) {
        int u, v; cin >> u >> v;
        cout << (dis[u][v] == inf ? -1 :
            dis[u][v]) << '\n';
    }
    return 0;
}

```

7.6 Strongly Connected Components

// Time: $O(n+m)$

```

const int N = 1e5 + 9;
vector<int> g[N], gT[N], G[N];
vector<bool> vis(N, false);
vector<vector<int>> components;
vector<int> order;
int n, roots[N], sz[N];
void dfs(int u) {
    vis[u] = true;
    for (auto v : g[u]) {
        if (!vis[v]) dfs(v);
    }
    order.push_back(u);
}
void dfs2(int u, vector<int> &component) {
    vis[u] = true;
    component.push_back(u);
    for (auto v : gT[u]) {
        if (!vis[v]) dfs2(v, component);
    }
}
void scc() {
    // get order sorted by end time
    order.clear();
    for (int u = 1; u <= n; u++) {
        if (!vis[u]) dfs(u);
    }
    reverse(order.begin(), order.end());
    // transpose the graph
    for (int u = 1; u <= n; u++) {
        for (auto v : g[u]) {
            gT[v].push_back(u);
        }
    }
    // get all components
    components.clear();
    for (int i = 1; i <= n; i++) vis[i] =
        false;
    for (auto u : order) {
        if (!vis[u]) {
            vector<int> component;
            dfs2(u, component);
            sort(component.begin(),
                  component.end());
            components.push_back(component);
            for (auto v : component) {
                roots[v] = component.front();
                sz[v] = component.size();
            }
        }
    }
    // add edges to condensation graph
    for (int u = 1; u <= n; u++) {
        for (auto v : g[u]) {
            if (roots[u] != roots[v]) {
                G[roots[u]].push_back(roots[v]);
            }
        }
    }
}
// when you need to use condensed graph,
// use it carefully (Specially g->G,
// i->roots[i])

```

7.7 Articulation Points

```

int disc[N], low[N], timer, n;
vector<bool> vis(N, false), is_ap(N,
    false);
void ap_dfs(int u, int p) {
    disc[u] = low[u] = ++timer;
    vis[u] = true;
    int children_cnt = 0;
    for (auto v : g[u]) {
        if (v == p) continue;
        if (vis[v]) low[u] = min(low[u],
            disc[v]);
        else {
            ap_dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (disc[u] <= low[v] and p != -1)
                is_ap[u] = true;
            children_cnt++;
        }
    }
    if (p == -1 and children_cnt > 1)
        is_ap[u] = true;
}
void find_articulation_points() {
    for (int u = 1; u <= n; u++) {
        if (!vis[u]) {
            timer = 0;
            ap_dfs(u, -1);
        }
    }
}

```

7.8 Find Bridges

```

map<pair<int, int>, int> bridges;
void bridges_dfs(int u, int p) { // find
    bridges
    disc[u] = low[u] = ++timer;
    vis[u] = true;
    for (auto v : g[u]) {
        if (v == p) continue;
        if (vis[v]) low[u] = min(low[u],
            disc[v]);
        else {
            bridges_dfs(v, u);
            low[u] = min(low[u], low[v]);
            if (disc[u] < low[v]) {
                bridges[{make_pair(min(u, v),
                    max(u, v))}]++;
            }
        }
    }
}
void find_bridges() {
    for (int u = 1; u <= n; u++) {
        if (!vis[u]) {
            timer = 0;
            bridges_dfs(u, -1);
        }
    }
}

```

7.9 DSU on Tree

```

// Problem: Distinct Colors CSES (Number
// of distinct color in a subtree)
// Complexity: O(n(logn)2)

```

```

set<int> se[N];
int col[N], ans[N], par[N]; // par[i] = i
initially
int find(int i) {
    return (i == par[i] ? i : par[i] =
        find(par[i]));
}
void merge(int u, int v) {
    if ((u = find(u)) == (v = find(v)))
        return;
    if (se[u].size() > se[v].size())
        swap(u, v);
    for (auto x : se[u]) {
        se[v].insert(x);
    }
    se[u].clear();
    par[u] = v;
}
void dfs(int u, int p) {
    se[find(u)].insert(col[u]);
    for (auto v : g[u]) {
        if (v != p) {
            dfs(v, u);
            merge(u, v);
        }
    }
    ans[u] = se[find(u)].size();
}

```

7.10 Heavy-Light Decomposition

```

// Per Query Complexity: O(logn2)
// Path and subtree updates and queries.
const int N = 2e5 + 9, LOG = 20, inf =
    1e9; // change here
vector<int> g[N];
int par[N][LOG], depth[N], sz[N];
int disc[N], finish[N], timer, head[N];
int n;
void dfs(int u, int p = 0) {
    par[u][0] = p;
    depth[u] = depth[p] + 1;
    sz[u] = 1;
    for (int i = 1; i < LOG; i++) {
        par[u][i] = par[par[u][i - 1]][i - 1];
    }
    if (p) g[u].erase(find(g[u].begin(),
        g[u].end(), p));
    for (auto &v : g[u]) {
        if (v != p) {
            dfs(v, u);
            sz[u] += sz[v];
            if (sz[v] > sz[g[u][0]]) swap(v,
                g[u][0]);
        }
    }
}
void dfs_hld(int u) {
    disc[u] = ++timer;
    for (auto v : g[u]) {
        head[v] = (v == g[u][0] ? head[u] :
            v);
        dfs_hld(v);
    }
}
```

```

        }
        finish[u] = timer;
    }

int kth(int u, int k) {
    assert(k >= 0);
    for (int i = 0; i < LOG; i++) {
        if (CHECK(k, i)) u = par[u][i];
    }
    return u;
}

int query_up(int u, int v) {
    int ans = -inf; // change here
    while (head[u] != head[v]) {
        ans = max(ans, st.query(1, 1, n,
            disc[head[u]], disc[u])); // change here
        u = par[head[u]][0];
    }
    ans = max(ans, st.query(1, 1, n,
        disc[v], disc[u])); // change here
    return ans;
}

int query(int u, int v) {
    int lc = lca(u, v);
    int ans = query_up(u, lc);
    if (v != lc) {
        ans = max(ans, query_up(v, kth(v,
            depth[v] - depth[lc] - 1))); // change here
    }
    return ans;
}

void solve() {
    cin >> n;
    for (int i = 2; i <= n; i++) {
        int u, v; cin >> u >> v;
        g[u].push_back(v);
        g[v].push_back(u);
    }
    dfs(1);
    head[1] = 1;
    dfs_hld(1);
    st.build(1, 1, n);
}

```

7.11 Dinic

```

const int N = 105, inf = 1e9;
int n, m, st, en;
// Edges should be added in both direction separately if the graph is undirected
const int INF = 2000000000;
struct Edge {
    int from, to, cap, flow, index;
    Edge(int from, int to, int cap, int flow, int index) :
        from(from), to(to), cap(cap),
        flow(flow), index(index) {}
};
struct Dinic {
    int N;
    vector<vector<Edge> > G;
    vector<Edge*> dad;
    vector<int> Q;

```

```

Dinic(int N) : N(N), G(N), dad(N),
             Q(N) {}
void AddEdge(int from, int to, int cap) {
    G[from].emplace_back(from, to, cap,
                          0, G[to].size());
    if (from == to)
        G[from].back().index++;
    G[to].emplace_back(to, from, 0, 0,
                       G[from].size() - 1);
}
long long BlockingFlow(int s, int t) {
    fill(dad.begin(), dad.end(), (Edge*) NULL);
    dad[s] = &G[0][0] - 1;
    int head = 0, tail = 0;
    Q[tail++] = s;
    while (head < tail) {
        int x = Q[head++];
        for (int i = 0; i < G[x].size(); i++) {
            Edge &e = G[x][i];
            if (!dad[e.to] && e.cap - e.flow > 0) {
                dad[e.to] = &G[x][i];
                Q[tail++] = e.to;
            }
        }
        if (!dad[t]) return 0;
        long long totflow = 0;
        for (int i = 0; i < G[t].size(); i++) {
            Edge *start =
                &G[G[t][i].to][G[t][i].index];
            int amt = INF;
            for (Edge *e = start; amt && e != dad[s]; e = dad[e->from]) {
                if (!e) { amt = 0; break; }
                amt = min(amt, e->cap - e->flow);
            }
            if (amt == 0) continue;
            for (Edge *e = start; amt && e != dad[s]; e = dad[e->from]) {
                e->flow += amt;
                G[e->to][e->index].flow -= amt;
            }
            totflow += amt;
        }
        return totflow;
    }
    long long GetMaxFlow(int s, int t) {
        long long totflow = 0;
        while (long long flow =
               BlockingFlow(s, t))
            totflow += flow;
        return totflow;
    }
}

void solve() {
    cin >> n >> st >> en >> m;
    Dinic *dinic = new Dinic(n + 1);
    while (m--) {
        int u, v, w; cin >> u >> v >> w;

```

```

        dinic->AddEdge(u, v, w);
        dinic->AddEdge(v, u, w);
    }
    cout << dinic->GetMaxFlow(st, en) << '\n';
    delete(dinic);
}

```

8 String

8.1 Hashing

```

const int N = 1e6 + 9; // change here
const int MOD1 = 127657753, MOD2 =
    987654319;
const int p1 = 137, p2 = 277; // change here
int ip1, ip2;
pair<int, int> pw[N], ipw[N];
void prec() {
    pw[0] = {1, 1};
    for (int i = 1; i < N; i++) {
        pw[i].first = 1ll * pw[i - 1].first *
                      p1 % MOD1;
        pw[i].second = 1ll * pw[i - 1].second *
                      p2 % MOD2;
    }
    ip1 = power(p1, MOD1 - 2, MOD1);
    ip2 = power(p2, MOD2 - 2, MOD2);
    ipw[0] = {1, 1};
    for (int i = 1; i < N; i++) {
        ipw[i].first = 1ll * ipw[i - 1].first *
                      ip1 % MOD1;
        ipw[i].second = 1ll * ipw[i - 1].second *
                      ip2 % MOD2;
    }
}

```

struct Hashing {

```

    int n;
    string s;
    vector<pair<int, int>> hash_val;
    vector<pair<int, int>> rev_hash_val;
    Hashing() {}
    Hashing(string _s) {
        s = _s;
        n = s.size();
        hash_val.emplace_back(0, 0);
        for (int i = 0; i < n; i++) {
            pair<int, int> p;
            p.first = (hash_val[i].first + 1ll *
                       s[i] * pw[i].first % MOD1) %
                       MOD1;
            p.second = (hash_val[i].second + 1ll *
                        s[i] * pw[i].second % MOD2) %
                        MOD2;
            hash_val.push_back(p);
        }
        rev_hash_val.emplace_back(0, 0);
        for (int i = 0, j = n - 1; i < n;
             i++, j--) {
            pair<int, int> p;
            p.first = (rev_hash_val[i].first + 1ll *
                       s[i] * pw[j].first % MOD1) %
                       MOD1;
            p.second = (rev_hash_val[i].second + 1ll *
                        s[i] * pw[j].second % MOD2) %
                        MOD2;
            rev_hash_val.push_back(p);
        }
    }
}
```

```

    rev_hash_val.push_back(p);
}
pair<int, int> get_hash(int l, int r) {
    // 1 indexed
    pair<int, int> ans;
    ans.first = (hash_val[r].first -
                 hash_val[l - 1].first + MOD1) *
    1ll * ipw[l - 1].first % MOD1;
    ans.second = (hash_val[r].second -
                 hash_val[l - 1].second + MOD2) *
    1ll * ipw[l - 1].second % MOD2;
    return ans;
}
pair<int, int> rev_hash(int l, int r) {
    // 1 indexed
    pair<int, int> ans;
    ans.first = (rev_hash_val[r].first -
                 rev_hash_val[l - 1].first + MOD1) *
    1ll * ipw[n - r].first % MOD1;
    ans.second = (rev_hash_val[r].second -
                 rev_hash_val[l - 1].second + MOD2) *
    1ll * ipw[n - r].second % MOD2;
    return ans;
}
pair<int, int> get_hash() { // 1 indexed
    return get_hash(1, n);
}
bool is_palindrome(int l, int r) {
    return get_hash(l, r) == rev_hash(l, r);
}
};


```

8.2 Hashing with Updates

```

using T = array<int, 2>;
const T MOD = {127657753, 987654319};
const T p = {137, 277}; // change here
T operator + (T a, int x) {return {(a[0] +
    x) % MOD[0], (a[1] + x) % MOD[1]};}
T operator - (T a, int x) {return {(a[0] -
    x + MOD[0]) % MOD[0], (a[1] - x +
    MOD[1]) % MOD[1]};}
T operator * (T a, int x) {return {(int)((long long) a[0] * x %
    MOD[0]), (int)((long long) a[1] * x %
    MOD[1])};}
T operator + (T a, T x) {return {(a[0] +
    x[0]) % MOD[0], (a[1] + x[1]) %
    MOD[1]};}
T operator - (T a, T x) {return {(a[0] -
    x[0] + MOD[0]) % MOD[0], (a[1] - x[1] -
    MOD[1]) % MOD[1]};}
T operator * (T a, T x) {return {(int)((long long) a[0] * x[0] %
    MOD[0]), (int)((long long) a[1] * x[1] %
    MOD[1])};}
ostream &operator << (ostream& os, T hash) {return os << "(" << hash[0]
    << ", " << hash[1] << ")";}

```

```

T pw[N], ipw[N];
void prec() {
    pw[0] = {1, 1};
    for (int i = 1; i < N; i++) {
        pw[i] = pw[i - 1] * p;
    }
    ipw[0] = {1, 1};
    T ip = {power(p[0], MOD[0] - 2,
                  MOD[0]), power(p[1], MOD[1] - 2,
                  MOD[1])};
    for (int i = 1; i < N; i++) {
        ipw[i] = ipw[i - 1] * ip;
    }
}

struct Hashing {
    int n;
    string s; // 1 - indexed
    vector<array<T, 2>> t; // (normal, rev)
    hash
    array<T, 2> merge(array<T, 2> l,
                      array<T, 2> r) {
        l[0] = l[0] + r[0];
        l[1] = l[1] + r[1];
        return l;
    }
    void build(int node, int b, int e) {
        if (b == e) {
            t[node][0] = pw[b] * s[b];
            t[node][1] = pw[n - b + 1] * s[b];
            return;
        }
        int mid = (b + e) >> 1, l = node <<
            1, r = l | 1;
        build(l, b, mid);
        build(r, mid + 1, e);
        t[node] = merge(t[l], t[r]);
    }
    void upd(int node, int b, int e, int
             i, char x) {
        if (b > i || e < i) return;
        if (b == e && b == i) {
            t[node][0] = pw[b] * x;
            t[node][1] = pw[n - b + 1] * x;
            return;
        }
        int mid = (b + e) >> 1, l = node <<
            1, r = l | 1;
        upd(l, b, mid, i, x);
        upd(r, mid + 1, e, i, x);
        t[node] = merge(t[l], t[r]);
    }
    array<T, 2> query(int node, int b, int
                      e, int i, int j) {
        if (b > j || e < i) return {T({0,
                                         0}), T({0, 0})};
        if (b >= i && e <= j) return t[node];
        int mid = (b + e) >> 1, l = node <<
            1, r = l | 1;
        return merge(query(l, b, mid, i, j),
                     query(r, mid + 1, e, i, j));
    }
}
Hashing() {}
Hashing(string _s) {
    n = _s.size();
}

```

```

    s = "." + _s;
    t.resize(4 * n + 1);
    build(1, 1, n);
}
void upd(int i, char c) {
    upd(1, 1, n, i, c);
    s[i] = c;
}
T get_hash(int l, int r) { // 1 -
    indexed
    return query(1, 1, n, l, r)[0] *
        ipw[l - 1];
}
T rev_hash(int l, int r) { // 1 -
    indexed
    return query(1, 1, n, l, r)[1] *
        ipw[n - r];
}
T get_hash() {
    return get_hash(1, n);
}
bool is_palindrome(int l, int r) {
    return get_hash(l, r) == rev_hash(l,
                                     r);
}

```

8.3 Hashing with Upd and Deletes

```

// update or delete a char in the string
// or check whether a range [l,r] is a
// palindrome or not (Palindromic Query I
// - Toph)
#define int long long
const int N = 1e5 + 9;
int en;
struct ST {
    pair<int, int> tree[4 * (N + N)];
    void build(int n, int b, int e) {
        if (b == e) {
            tree[n].first = b;
            tree[n].second = 1;
            return;
        }
        int mid = (b + e) >> 1, l = n << 1,
            r = l + 1;
        build(l, b, mid);
        build(r, mid + 1, e);
        tree[n].second = tree[l].second +
            tree[r].second;
    }
    void upd(int n, int b, int e, int i,
             int x1, int x2) {
        if (b > i || e < i) return;
        if (b == e && b == i) {
            tree[n].first = x1;
            tree[n].second = x2;
            return;
        }
        int mid = (b + e) >> 1, l = n << 1,
            r = l + 1;
        upd(l, b, mid, i, x1, x2);
        upd(r, mid + 1, e, i, x1, x2);
        tree[n].second = tree[l].second +
            tree[r].second;
    }
}

```

```

pair<int, int> query(int n, int b, int
                     e, int x) {
    if (b > e) return {-1, -1};
    if (tree[n].second < x) return
        {tree[n].second, -1};
    if (b == e) return tree[n];
    int mid = (b + e) >> 1, l = n << 1,
        r = l + 1;
    pair<int, int> L = query(l, b, mid,
                             x);
    if (L.second != -1) return L;
    pair<int, int> R = query(r, mid + 1,
                             e, x - L.first);
    return R;
}
} st, st2;
using T = array<int, 2>;
const T MOD = {127657753, 987654319};
const T p = {137, 277};
// add operators overloading of T (from
// only upd) + prec()
int get(int i, int n) {
    return n - i + 1;
}
struct Hashing {
    int n; string s;
    vector<T> tree, lazy;
    void push(int node, int b, int e) {
        if (lazy[node][0] == 1) return;
        tree[node] = tree[node] * lazy[node];
        if (b != e) {
            int l = node << 1, r = l + 1;
            lazy[l] = lazy[l] * lazy[node];
            lazy[r] = lazy[r] * lazy[node];
        }
        lazy[node] = T{1, 1};
    }
    void build(int node, int b, int e) {
        lazy[node] = T{1, 1};
        if (b == e) {
            tree[node] = pw[b] * s[b];
            return;
        }
        int mid = (b + e) >> 1, l = node <<
            1, r = l | 1;
        build(l, b, mid);
        build(r, mid + 1, e);
        tree[node] = tree[l] + tree[r];
    }
    void upd(int node, int b, int e, int
             i, T x) {
        push(node, b, e);
        if (b > i || e < i) return;
        if (b == e && b == i) {
            tree[node] = x;
            return;
        }
        int mid = (b + e) >> 1, l = node <<
            1, r = l + 1;
        upd(l, b, mid, i, x);
        upd(r, mid + 1, e, i, x);
        tree[node] = tree[l] + tree[r];
    }
    void del(int node, int b, int e, int
             i, int j) {

```

```

        push(node, b, e);
        if (b > j || e < i) return;
        if (b >= i && e <= j) {
            lazy[node] = lazy[node] * ipw[1];
            push(node, b, e);
            return;
        }
        int mid = (b + e) >> 1, l = node <<
            1, r = l + 1;
        del(l, b, mid, i, j);
        del(r, mid + 1, e, i, j);
        tree[node] = tree[l] + tree[r];
    }
    T query(int node, int b, int e, int i,
            int j) {
        push(node, b, e);
        if (b > j || e < i) return {0, 0};
        if (b >= i && e <= j) return
            tree[node];
        int mid = (b + e) >> 1, l = node <<
            1, r = l + 1;
        T L = query(l, b, mid, i, j);
        T R = query(r, mid + 1, e, i, j);
        return L + R;
    }
}
Hashing() {}
Hashing(string _s) {
    s = _s;
    n = s.size();
    s = '.' + s;
    tree.resize(4 * n + 1);
    lazy.resize(4 * n + 1);
    build(1, 1, n);
}
void upd(int i, char c, int cur) {
    T x = pw[i] * c;
    if (cur == 1) i = st.query(1, 1, en,
                               i).first;
    else i = st2.query(1, 1, en, i).first;
    upd(1, 1, n, i, x);
}
void del(int i, int cur) {
    int orgi = i;
    T x = pw[i] * 0ll;
    if (cur == 1) i = st.query(1, 1, en,
                               i).first;
    else i = st2.query(1, 1, en, i).first;
    upd(1, 1, n, i, x);
    del(1, 1, n, i + 1, n);
    if (cur == 1) st.upd(1, 1, en, i, i,
                        0);
    else st2.upd(1, 1, en, i, i, 0);
}
T get_hash(int l, int r, int cur) { // 1 -
    indexed
    int ll = st.query(1, 1, en, l).first;
    int rr = st.query(1, 1, en, r).first;
    if (cur == 2) {
        ll = st2.query(1, 1, en, l).first;
        rr = st2.query(1, 1, en, r).first;
    }
    return query(1, 1, n, ll, rr) *
        ipw[l - 1];
}

```

```

}
};

int32_t main() {
    prec(); // must include
    string s; cin >> s;
    int n = s.size();
    int q; cin >> q;
    string t = s;
    reverse(t.begin(), t.end());
    Hashing hs(s), hs2(t);
    en = n + q + 5;
    st.build(1, 1, en);
    st2.build(1, 1, en);
    while (q--) {
        char c; cin >> c;
        if (c == 'C') {
            int l, r; cin >> l >> r;
            int l2 = get(l, n);
            int r2 = get(r, n);
            if (hs.get_hash(l, r, 1) ==
                hs2.get_hash(r2, l2, 2)) cout
                << "Yes!\n";
            else cout << "No!\n";
        } else if (c == 'U') {
            int i; char x; cin >> i >> x;
            int i2 = get(i, n);
            hs.upd(i, x, 1);
            hs2.upd(i2, x, 2);
        } else {
            int i; cin >> i;
            int i2 = get(i, n);
            hs.del(i, 1);
            hs2.del(i2, 2);
            --n;
        }
    }
}



### 8.4 Hashing on Tree



---



```
// Given a tree, Check whether it is
// symmetrical or not. Problem - CF G.
// Symmetree
```



---



```
// The value for each node is it's
// subtree size and position is the
// level (ordered). But the order of
// childs doesn't matter (unordered)
```



```
const int N = 2e5 + 9;
vector<int> g[N];
vector<array<int, 3>> hassh[N]; // hash1,
// hash2, node
int n, sz[N];
const int MOD1 = 1e9 + 9, MOD2 = 1e9 + 21;
const int p1 = 1e5 + 19, p2 = 1e5 + 43;
void dfs2(int u, int p, int lvl) {
 array<int, 3> my_hash;
 my_hash[0] = 1ll * sz[u] *
 pw[lvl].first % MOD1;
 my_hash[1] = 1ll * sz[u] *
 pw[lvl].second % MOD2;
 my_hash[2] = u;
 bool leaf = true;
 for (auto v : g[u]) {
 if (v != p) {

```


```

```

            dfs2(v, u, lvl + 1);
            leaf = false;
        }
    }
    if (!leaf) {
        int sum1 = 1, sum2 = 1;
        for (auto here : hassh[u]) {
            auto [x, y, _] = here;
            sum1 = (sum1 * x) % MOD1;
            sum2 = (sum2 * y) % MOD2;
        }
        my_hash[0] = power(my_hash[0], sum1,
                           MOD1);
        my_hash[1] = power(my_hash[1], sum2,
                           MOD2);
    }
    hassh[p].push_back(my_hash);
}
bool ok(int u) {
    map<pair<int, int>, int> mp;
    for (auto [x, y, who] : hassh[u]) {
        mp[{x, y}]++;
    }
    int odd = 0;
    pair<int, int> val;
    for (auto [here, cnt] : mp) {
        odd += cnt & 1;
        if (cnt & 1) val = here;
    }
    if (odd == 0) return true;
    if (odd > 1) return false;
    int node;
    for (auto [x, y, who] : hassh[u]) {
        pair<int, int> here = {x, y};
        if (here == val) node = who;
    }
    return ok(node);
}
void solve() {
    cin >> n; clr(n);
    for (int i = 2; i <= n; i++) {
        int u, v; cin >> u >> v;
        g[u].push_back(v);
        g[v].push_back(u);
    }
    dfs1(1, 0); // calc. subtree size
    dfs2(1, 0, 1);
    if (ok(0)) cout << "YES\n";
    else cout << "NO\n";
}
```

8.5 Compare 2 strings Lexicographically

```
// Time: O(logn)
string s;
Hashing hs;
// return 0 if both equal
// return 1 if first substring greater
// return -1 if second substring greater
// here lcp() provides the len of longest
// common prefix
```

```
int compare(int i, int j, int x, int y) {
    int common_prefix = lcp(i, j, x, y);
    int len1 = j - i + 1, len2 = y - x + 1;
    if (common_prefix == len1 and len1 ==
        len2) return 0;
```

```

    else if (common_prefix == len1) return
        -1;
    else if (common_prefix == len2) return
        1;
    else return (s[i + common_prefix - 1]
        < s[x + common_prefix - 1] ? -1 :
        1);
}



### 8.6 KMP



---



```
vector<int> build_lps(string &pat) {
 int n = pat.size();
 vector<int> lps(n, 0);
 for (int i = 1; i < n; i++) {
 int j = lps[i - 1];
 while (j > 0 and pat[i] != pat[j]) {
 j = lps[j - 1];
 }
 if (pat[i] == pat[j]) j++;
 lps[i] = j;
 }
 return lps;
}
int kmp(string &txt, string &pat) {
 string s = pat + '#' + txt;
 vector<int> lps = build_lps(s);
 int ans = 0;
 for (auto x : lps) {
 if (x == pat.size()) ans++;
 }
 return ans;
}
int kmp(string &txt, string &pat) {
 vector<int> lps = build_lps(pat);
 int n = txt.size(), m = pat.size();
 int ans = 0;
 int j = 0;
 for (int i = 0; i < n; i++) {
 while (j > 0 and txt[i] != pat[j]) {
 j = lps[j - 1];
 }
 if (txt[i] == pat[j]) j++;
 if (j == m) {
 ans++;
 j = lps[j - 1];
 }
 }
 return ans;
}
```


```

8.8 Prefix Occurance Count

```
// Count the number of occurrences of each
// prefix
vector<int> ans(n + 1);
for (int i = 0; i < n; i++) ans[lps[i]]++;
for (int i = n - 1; i > 0; i--)
    ans[lps[i - 1]] += ans[i];
for (int i = 0; i <= n; i++) ans[i]++;

```

8.9 Number of palindromic substring in L to R using Wavelet Tree

```
// Problem - Kattis palindromes
11 f(int x) {
    return (1ll * x * (x + 1)) / 2;
}
11 f(int l, int r) {
    if (l > r) return 0;
    return f(r) - f(l - 1);
}
bool ok(int l, int r) {
    return hash_s.is_palindrome(l, r);
}
int32_t main() {
    cin >> s;
    n = s.size();
    hash_s = Hashing(s);
    for (int i = 1; i <= n; i++) {
        int l = 0, r = min(n - i, i - 1),
            cnt = 1;
        while (l <= r) {
            int mid = (l + r) >> 1;
            if (ok(i - mid, i + mid)) {
                cnt = mid;
                l = mid + 1;
            } else r = mid - 1;
        }
        pi1[i] = cnt + 1;
        pi1_left[i] = pi1[i] - i;
        pi1_right[i] = i + pi1[i];
    }
    for (int i = 2; i <= n; i++) {
        if (s[i - 1] == s[i - 2]) {
            int l = 0, r = min(n - i, i - 1),
                cnt = 2;
            while (l <= r) {
                int mid = (l + r) >> 1;
                if (ok(i - 1 - mid, i + mid)) {
                    cnt = mid;
                    l = mid + 1;
                } else r = mid - 1;
            }
            pi2[i] = cnt + 1;
        } else pi2[i] = 0;
    }
    pi2_left[i] = pi2[i] - i;
```

```

    pi2_right[i] = i + pi2[i];
}

// wavelet trees (odd_len_left,
// odd_len_right, even_len_left,
// even_len_right)
t1.init(pi1_left + 1, pi1_left + n +
    1, -N, N);
t2.init(pi1_right + 1, pi1_right + n +
    1, -N, N);
t3.init(pi2_left + 1, pi2_left + n +
    1, -N, N);
t4.init(pi2_right + 1, pi2_right + n +
    1, -N, N);

int q; cin >> q;
while (q--) {
    int l, r; cin >> l >> r;
    // define k, find cnt > k and
    // summation whose are <= k;
    int m = (l + r) / 2;
    int k = 1 - 1;
    ll ans = f(l, m);
    ans += t1.sum(l, m, k);
    int cnt = t1.GT(l, m, k);
    ans += 1ll * k * cnt;
    k = 1 + r;
    ans += -f(m + 1, r);
    ans += t2.sum(m + 1, r, k);
    cnt = t2.GT(m + 1, r, k);
    ans += 1ll * k * cnt;
    if (l + 1 <= m) { // a bit different
        than others
        k = -1;
        ans += f(l + 1, m);
        ans += t3.sum(l + 1, m, k);
        cnt = t3.GT(l + 1, m, k);
        ans += 1ll * k * cnt;
    }
    k = 1 + r;
    ans += -f(m + 1, r);
    ans += t4.sum(m + 1, r, k);
    cnt = t4.GT(m + 1, r, k);
    ans += 1ll * k * cnt;
    cout << ans << '\n';
}

```

It is easier to explain by considering only palindromes centered at indices (so, odd length), the idea is the same anyway. For each index i , r_i will be the longest radius of a palindrome centered there (in other words, the amount of palindromes centered at index i). Directly from manacher, this takes $\mathcal{O}(n)$ to calculate.

For a query $[l, r]$, we first compute $m = \frac{l+r}{2}$. Now we want to calculate

$$\sum_{i=l}^m \min(i - l + 1, r_i) + \sum_{i=m+1}^r \min(r - i + 1, r_i)$$

$$\sum_{i=l}^m \min(i - l + 1, r_i) = \sum_{i=l}^m i + \min(1 - l, r_i - i).$$

The sum over i can be found in constant time. As for the other term, if we create some array $r'_i = r_i - i$

during the preprocessing, then the queries are asking for some over range of $\min(C, r'_i)$ where C is constant. You can solve this in $\mathcal{O}(\log n)$ per query using wavelet tree.

8.10 Trie

```

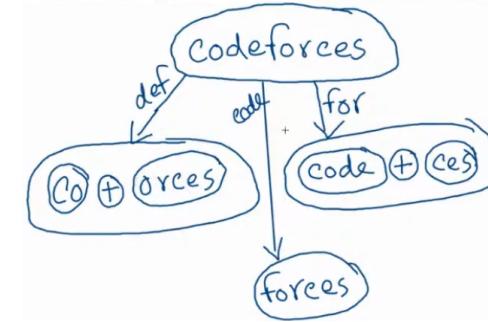
const int N = 10; // change here
const char base_char = '0'; // change
here
struct TrieNode {
    int cnt;
    TrieNode *nxt[N];
    TrieNode() {
        cnt = 0;
        for (int i = 0; i < N; i++) nxt[i] =
            NULL;
    }
} *root;
void insert(const string &s) {
    TrieNode *cur = root;
    int n = (int)s.size();
    for (int i = 0; i < n; i++) {
        int idx = s[i] - base_char;
        if (cur -> nxt[idx] == NULL) cur ->
            nxt[idx] = new TrieNode();
        cur = cur -> nxt[idx];
        cur -> cnt++;
    }
}
void rem(TrieNode *cur, string &s, int
pos) { // free :: De Allocates Memory
if (pos == s.size()) return;
int idx = s[pos] - base_char;
rem(cur -> nxt[idx], s, pos + 1);
cur -> nxt[idx] -> cnt--;
if (cur -> nxt[idx] -> cnt == 0) {
    free(cur -> nxt[idx]);
    cur -> nxt[idx] = NULL;
}
}
int query(const string &s) { // "s" is a
prefix of some element or not
int n = (int)s.size();
TrieNode *cur = root;
for (int i = 0; i < n; i++) {
    int idx = s[i] - base_char;
    if (cur -> nxt[idx] == NULL) return 0;
    cur = cur -> nxt[idx];
}
return cur -> cnt;
}
void del(TrieNode *cur) {
    for (int i = 0; i < N; i++) if (cur ->
        nxt[i]) del(cur -> nxt[i]);
    delete(cur);
}
int32_t main() {
    root = new TrieNode(); // init new trie
    del(root); // clear trie
}

```

9 Game Theory

9.1 Notes

- First Write a Bruteforce Solution
- If game is not impartial. Greedy, DP may work
- Nim = All XOR
- Misere Nim (Last player who took a stone loses) = Nim + (Corner Case: if all (odd) piles are 1 you lose)
- Bogus Nim = Nim
- Staircase Nim (Given an array where $a[i]$ is the number of stones at index i . Move: Choose any stones (> 1) from index i ($i > 1$) and move them to index $i - 1$.) = Odd indexed pile Nim (Even indexed pile doesn't matter, as one player can give bogus moves to drop all even piles to ground)
- Sprague-Grundy:** Every impartial game under the normal play convention is equivalent to a ONE-HEAP GAME of NIM.
Grundy value can be calculate by DP/Pattern.



10 Misc.

10.1 Matrix Expo

```

struct Mat {
    int n, m;
    vector<vector<int>> a;
    Mat() {}
    Mat(int _n, int _m) {n = _n; m = _m;
        a.assign(n, vector<int>(m, 0)); }
    Mat(vector<vector<int>> v) {n =
        v.size(); m = n ? v[0].size() : 0;
        a = v; }
    inline void make_unit() {
        assert(n == m);
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < n; j++)
                a[i][j] = i == j;
        }
    }
    inline Mat operator + (const Mat &b) {
        assert(n == b.n && m == b.m);
        Mat ans = Mat(n, m);
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < m; j++) {
                ans.a[i][j] = (a[i][j] +
                    b.a[i][j]) % mod;
            }
        }
        return ans;
    }
    inline Mat operator - (const Mat &b) {

```

```

        assert(n == b.n && m == b.m);
        Mat ans = Mat(n, m);
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < m; j++) {
                ans.a[i][j] = (a[i][j] -
                    b.a[i][j] + mod) % mod;
            }
        }
        return ans;
    }
    inline Mat operator * (const Mat &b) {
        assert(m == b.n);
        Mat ans = Mat(n, b.m);
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < b.m; j++) {
                for (int k = 0; k < m; k++) {
                    ans.a[i][j] = (ans.a[i][j] +
                        1LL * a[i][k] * b.a[k][j] %
                        mod) % mod;
                }
            }
        }
        return ans;
    }
    inline Mat pow(long long k) {
        assert(n == m);
        Mat ans(n, n), t = a; ans.make_unit();
        while (k) {
            if (k & 1) ans = ans * t;
            t = t * t;
            k >>= 1;
        }
        return ans;
    }
    inline Mat& operator += (const Mat& b)
        { return *this = (*this) + b; }
    inline Mat& operator -= (const Mat& b)
        { return *this = (*this) - b; }
    inline Mat& operator *= (const Mat& b)
        { return *this = (*this) * b; }
    inline bool operator == (const Mat& b)
        { return a == b.a; }
    inline bool operator != (const Mat& b)
        { return a != b.a; }
}
int32_t main() {
    int n; long long k; cin >> n >> k;
    Mat a(n, n); // then assign value
    Mat ans = a.pow(k);
}

```

10.2 2D Prefix Sum

```

const int N = 1e3 + 9;
int a[N][N];
ll pref[N][N];
void query(int x1, int y1, int x2, int
y2) {
    return pref[x2][y2] - pref[x1 - 1][y2] -
        pref[x2][y1 - 1] + pref[x1 - 1][y1 - 1]
}
int32_t main() {

```

```

int n, m; cin >> n >> m;
for (int i = 1; i <= n; i++) {
    for (int j = 1; j <= m; j++) {
        pref[i][j] = pref[i - 1][j] +
            pref[i][j - 1] - pref[i - 1][j -
            1] + a[i][j];
    }
}
int q; cin >> q;
while (q--) {
    int x1, y1, x2, y2; cin >> x1 >> y1
        >> x2 >> y2;
    cout << query(x1, y1, x2, y2) << '\n';
}

```

10.3 2D Static Range Update

```

// Add x on a rectangle q times,
// Finally print the array
const int N = 1e3 + 9;
int a[N][N];
ll d[N][N]; // difference array
int32_t main() {
    int n, m; cin >> n >> m;
    int q; cin >> q;
    while (q--) {
        int x1, y1, x2, y2, x; cin >> x1 >>
            y1 >> x2 >> y2 >> x; // add x on
            this rectangle
        d[x1][y1] += x;
        d[x1][y2 + 1] -= x;
        d[x2 + 1][y1] -= x;
        d[x2 + 1][y2 + 1] += x;
    }
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= m; j++) {
            d[i][j] += d[i - 1][j] + d[i][j - 1] -
                d[i - 1][j - 1];
        }
    }
    // new updated array
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= m; j++) {
            cout << d[i][j] + a[i][j] << ' ';
        }
        cout << '\n';
    }
}

```

10.4 CHECK SET CLR

```

bool CHECK(int N, int pos) { return
    (bool)(N & (1ll << pos)); }
void SET(int &N, int pos) { (N |= (1ll
    << pos)); }
void CLR(int &N, int pos) { (N &= ~(1ll
    << pos)); }

```

10.5 Graph Directions

```

int dx[] = {+0, +0, +1, -1, -1, +1, -1,
            +1};
int dy[] = {-1, +1, +0, +0, +1, +1, -1,
            -1};

```

10.6 Custom Hash

```

struct custom_hash {
    static uint64_t splitmix64(uint64_t x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) *
            0xbfb58476d1ce4e5b9;
        x = (x ^ (x >> 27)) *
            0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM =
            chrono::steady_clock::now().time_
            _since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }
}

```

10.7 Coordinate Compression

```

int32_t main() {
    vector<int> a({100, 9, 10, 10, 9});
    vector<int> v = a;
    sort(v.begin(), v.end());
    v.resize(unique(v.begin(), v.end()) -
        v.begin());
    for (int i = 0; i < a.size(); i++) {
        a[i] = lower_bound(v.begin(),
                           v.end(), a[i]) -
            v.begin() + 1;
    }
}

```

10.8 Submask Generator

```

int mask = 0; // any value
for (int submask = mask; submask;
    submask = (submask - 1) & mask) {}

```

10.9 Custom Comparators

```

bool cmp(pair<int, int> a, pair<int,
         int> b) { // for vector, arr, ..
    if (a.first != b.first) return a.first
        > b.first;
    return a.second < b.second; // must
        return false for equal elements
}

```

```

struct cmp { // for set, map, pq, ..
    bool operator()(const int& a, const
        int& b) const {
        return a > b;
    }
}

```

10.10 MOD Operations

```

inline ll modAdd(ll a, ll b, ll MOD) {ll
    res = a + b; res += ((res >> 63) &
    MOD); if (res >= MOD) res -= MOD;
    return res;}
inline ll modSub(ll a, ll b, ll MOD) {ll
    res = a - b; res += ((res >> 63) &
    MOD); if (res >= MOD) res -= MOD;
    return res;}
inline ll modMul(ll a, ll b, ll MOD) {a
    %= MOD; b %= MOD; a += ((a >> 63) &
    MOD); b += ((b >> 63) & MOD); return
    (a * b) % MOD; }
inline ll modInverse(ll a, ll MOD)
    {return modPow(a, MOD - 2, MOD);}

```

```

inline ll modDiv(ll a, ll b, ll MOD)
    {return modMul(a, modInverse(b,
    MOD), MOD);}
inline ll modMulBigMod(ll a, ll b, ll
    MOD) {a %= MOD; b %= MOD; a += ((a
    >> 63) & MOD); b += ((b >> 63) &
    MOD); return ((ll)((__int128)a * b %
    MOD));}

```

10.11 Mex with Array Updates

```

int32_t main() {
    int n, q; cin >> n >> q;
    set<int> missing_numbers;
    for (int i = 0; i <= n + 200; i++) {
        missing_numbers.insert(i);
    }
    int a[n + 1];
    map<int, int> freq;
    for (int i = 1; i <= n; i++) {
        cin >> a[i];
        freq[a[i]]++;
        missing_numbers.erase(a[i]);
    }
    while (q--) {
        int i, x; cin >> i >> x; // Set
        a[i] = x;
        int y = a[i]; a[i] = x;
        missing_numbers.erase(x);
        freq[y]--;
        freq[x]++;
        if (freq[y] == 0) {
            freq.erase(y);
            missing_numbers.insert(y);
        }
        cout << *missing_numbers.begin() <<
            '\n'; // mex after upd
    }
}

```

11 Geometry

11.1 Convex Hull

```

typedef vector<long long> vll;
typedef long double ld;
#define int long long
struct point {
    ld x, y;
    point() {}
    point( ld x, ld y ) { this->x = x;
        this->y = y; }
    bool operator<( const point &other ) {
        if ( x == other.x ) return y <
            other.y;
        return x < other.x;
    }
    int crossProduct( point &a, point &b,
        point &c ) {
        return (b.x - a.x) * (c.y - a.y) -
            (b.y - a.y) * (c.x - a.x);
    }
    vector<point> hull( vector<point> a ) {
        vector<point> v = a;
        sort(v.begin(), v.end());
        vector<point> lower, upper;

```

```

        for ( auto& p : v ) {
            while ( lower.size() >= 2 &&
                crossProduct(lower[lower.size() -
                2], lower.back(), p) < 0 ) //
                    <= for colinear
                lower.pop_back();
            lower.push_back(p);
        }
        for ( int i = v.size() - 1; i >= 0;
            i-- ) {
            point p = v[i];
            while ( upper.size() >= 2 &&
                crossProduct(upper[upper.size() -
                2], upper.back(), p) < 0 ) //
                    <= for colinear
                upper.pop_back();
            upper.push_back(p);
        }
        lower.pop_back();
        upper.pop_back();
        lower.insert(lower.end(),
            upper.begin(), upper.end());
        return lower;
    }
    void senritsu() {
        int n; cin >> n;
        vector<point> a(n); for ( int i = 0; i
            < n; i++ ) cin >> a[i].x >> a[i].y;
        a = hull(a);
        n = a.size();
        cout << n << endl;
        for ( auto c : a ) cout << c.x << ' '
            << c.y << endl;
    }
}

```

11.2 Area of 2 polygon intersection

```

typedef vector<long long> vll;
typedef long double ld;
#define int long long
struct point {
    ld x, y;
    point() {}
    point( ld x, ld y ) { this->x = x;
        this->y = y; }
    bool operator<( const point &other ) {
        if ( x == other.x ) return y <
            other.y;
        return x < other.x;
    }
    bool operator==( const point &other ) {
        return ( x == other.x && y ==
            other.y );
    }
    bool operator!=( const point &other ) {
        return ( x != other.x || y !=
            other.y );
    }
    point operator-( const point &other ) {
        return {x - other.x, y - other.y};
    }
    ld crossProduct( point &a, point &b,
        point &c ) {
        return (b.x - a.x) * (c.y - a.y) -
            (b.y - a.y) * (c.x - a.x);
    }
}

```

```

}
ld crossProduct( point &a, point &b ) {
    return a.x * b.y - a.y * b.x;
}
bool inside( point &a, point &b, point
    &c ) {
    return crossProduct(a, b, c) >= 0;
}
bool intersection( point &a, point &b,
    point &c, point &d, point &ans ) { // 
    2 line intersection point
point ab = b - a;
point cd = d - c;
ld det = crossProduct( ab, cd );
if ( det == 0 ) return false; // 
    parallel or collinear
ld z1 = crossProduct( a, b );
ld z2 = crossProduct( c, d );
ans.x = (ld)(z1 * (c.x - d.x) - z2 *
    (a.x - b.x)) / det;
ans.y = (ld)(z1 * (c.y - d.y) - z2 *
    (a.y - b.y)) / det;
return true;
}
vector<point> innerhull( vector<point>
    a, vector<point> b ) {
int n = a.size();
vector<point> ans = b;
for ( int i = 0; i < n; i++ ) {
    point x = a[i], y = a[(i + 1) % n];
    vector<point> temp = ans; ans.clear();
    int m = temp.size();
    if ( m == 0 ) break;
    for ( int j = 0; j < m; j++ ) {
        point p = temp[j], q = temp[(j +
            1) % m];
        int pIn = inside(x, y, p);
        int qIn = inside(x, y, q);
        if ( pIn ) ans.push_back(p);
        if ( pIn != qIn ) {
            point cur;
            if ( intersection(x, y, p, q,
                cur) ) {
                ans.push_back(cur);
            }
        }
    }
    if ( !ans.empty() ) {
        vector<point> temp;
        temp.push_back(ans[0]);
        for ( int j = 1; j < ans.size();
            j++ ) {
            if ( ans[j] != ans[j - 1] ) {
                temp.push_back(ans[j]);
            }
        }
        ans = temp;
    }
}
return ans;
}
ld areaOfPolygon( vector<point> a ) {
    if ( a.size() < 3 ) return 0.0;
    a.push_back(a[0]);
}

```

```

int n = a.size();
ld ans = 0;
for ( int i = 0; i < n - 1; i++ ) {
    ans += a[i].x * a[i + 1].y;
    ans -= a[i].y * a[i + 1].x;
}
if ( ans < 0 ) ans = -ans;
return ans / 2.0;
}
void senritsu() {
    int n, m; cin >> n >> m;
    vector<point> a(n), b(m);
    for ( int i = 0; i < n; i++ ) {
        cin >> a[i].x >> a[i].y;
    }
    for ( int i = 0; i < m; i++ ) {
        cin >> b[i].x >> b[i].y;
    }
    vector<point> in = innerhull(a, b);
    cout << fixed << setprecision(4) <<
        areaOfPolygon(in) << endl;
}

```