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# Aiub_Eclipse

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# Contents

## 1 Setup

### 1.1 Sublime Build

```
"shell_cmd": "g++ -std=c++17 -o
    \"$file_base_name\" \"$file\" &&
    timeout 2.5s ./\"$file_base_name\" <
    input.txt > output.txt",
"file_regex":
    "^(..[^:]*):([0-9]+):?([0-9]+)?:?
    (.*)$",
"working_dir": "${file_path}",
"selector": "source.c, source.c++"
```

### 1.2 Template

```
#include<bits/stdc++.h>
#include<ext/pb_ds/assoc_container.hpp>
#include<ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
using namespace std;
template <typename T> using o_set =
    tree<T, null_type, less<T>,
    rb_tree_tag,
    tree_order_statistics_node_update>;
typedef long long ll;
int32_t main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);
}
```

## 2 Stress Testing

### 2.1 Input Gen

```
mt19937_64 rnd(chrono::steady_clock::now
    ().time_since_epoch().count());
ll get_rand(ll l, ll r) {
  assert(l <= r);
  return l + rnd() % (r - l + 1);
}
```

### 2.2 Bash Script

```
// run -> bash script.sh

set -e
g++ code.cpp -o code
g++ gen.cpp -o gen
g++ brute.cpp -o brute
for((i = 1; ; ++i)); do
    ./gen $i > input_file
    ./code < input_file > myAnswer
    ./brute < input_file > correctAnswer
    diff -Z myAnswer correctAnswer >
        /dev/null || break
    echo "Passed test: "  $i
done
echo "WA on the following test:"
cat input_file
echo "Your answer is:"
cat myAnswer
echo "Correct answer is:"
cat correctAnswer
```

## 3 Number Theory

### 3.1 Euler Totient Function

```
// Time: O(√N)
map<int, int> dp; // memo
int phi(int n) {
  if (dp.count(n)) return dp[n];
  int ans = n, m = n;
  for (int i = 2; i * i <= m; i++) {
    if (m % i == 0) {
      while (m % i == 0) m /= i;
      ans = ans / i * (i - 1);
    }
  }
  if (m > 1) ans = ans / m * (m - 1);
  return dp[n] = ans;
}
```

### 3.2 Phi 1 to N

```
void phi_1_to_n(int n) {
  vector<int> phi(n + 1);
  for (int i = 0; i <= n; i++)
    phi[i] = i;
  for (int i = 2; i <= n; i++) {
    if (phi[i] == i) {
      for (int j = i; j <= n; j += i)
        phi[j] -= phi[j] / i;
    }
  }
}
```

### 3.3 Segmented Sieve

```
vector<char> segmentedSieve(ll L, ll R) {
  // generate all primes up to √R
  ll lim = sqrt(R);
  vector<char> mark(lim + 1, false);
  vector<ll> primes;
  for (ll i = 2; i <= lim; ++i) {
    if (!mark[i]) {
      primes.emplace_back(i);
      for (ll j = i * i; j <= lim; j +=
          i) mark[j] = true;
    }
  }
  vector<char> isPrime(R - L + 1, true);
  for (ll i : primes)
    for (ll j = max(i * i, (L + i - 1) /
        i * i); j <= R; j += i)
      isPrime[j - L] = false;
  if (L == 1) isPrime[0] = false;
  return isPrime;
}
```

### 3.4 Extended GCD

```
// ax + by = gcd(a, b)
int egcd(int a, int b, int& x, int& y) {
  if (b == 0) {
    x = 1, y = 0;
    return a;
  }
  int x1, y1;
  int d = egcd(b, a % b, x1, y1);
  x = y1;
  y = x1 - y1 * (a / b);
  return d;
}
```

### 3.5 Linear Diophantine Equation

```
// ax + by = c, find any x and y
bool find_any_solution(int a, int b, int
    c, int &x0, int &y0, int &g) {
  g = egcd(abs(a), abs(b), x0, y0);
  if (c % g) return false;
  x0 *= c / g;
  y0 *= c / g;
  if (a < 0) x0 = -x0;
  if (b < 0) y0 = -y0;
  return true;
}
void shift_solution(int & x, int & y,
    int a, int b, int cnt) {
  x += cnt * b;
  y -= cnt * a;
}
int find_all_solutions(int a, int b, int
    c, int minx, int maxx, int miny, int
    maxy) {
  int x, y, g;
  if (!find_any_solution(a, b, c, x, y,
      g)) return 0;
  a /= g, b /= g;
  int sign_a = a > 0 ? +1 : -1;
  int sign_b = b > 0 ? +1 : -1;
  shift_solution(x, y, a, b, (minx - x)
      / b);
  if (x < minx) shift_solution(x, y, a,
      b, sign_b);
  if (x > maxx) return 0;
  int lx1 = x;
  shift_solution(x, y, a, b, (maxx - x)
      / b);
  if (x > maxx) shift_solution(x, y, a,
      b, -sign_b);
  int rx1 = x;
  shift_solution(x, y, a, b, -(miny - y)
      / a);
  if (y < miny) shift_solution(x, y, a,
      b, -sign_a);
  if (y > maxy) return 0;
  int lx2 = x;
  shift_solution(x, y, a, b, -(maxy - y)
      / a);
  if (y > maxy) shift_solution(x, y, a,
      b, sign_a);
  int rx2 = x;
  if (lx2 > rx2) swap(lx2, rx2);
  int lx = max(lx1, lx2);
  int rx = min(rx1, rx2);
  if (lx > rx) return 0;
  return (rx - lx) / abs(b) + 1;
}
```

## 3.6 Modular Inverse using EGCD

```
// finding inverse(a) modulo m
int x, y;
int g = extended_euclidean(a, m, x, y);
if (g != 1) cout << "No solution!";
else {
  x = (x % m + m) % m;
  cout << x << endl;
}
```

## 3.7 Exclusion DP

```
ll f[N], g[N];
for (int i = N - 1; i >= 1; i--) {
  f[i] = nC4(div_cnt[i]);
  g[i] = f[i];
  for (int j = i + i; j < N; j += i) {
    g[i] -= g[j];
  }
}
```

Here, $f[i]$ = how many pairs/k-tuple such that their gcd is $i$ or it's multiple (count of pairs those are divisible by $i$).
$g[i]$ = how many pairs/k-tuple such that their gcd is $i$.
$g[i] = f[i] - \sum_{i|j} g[j]$.

**Sum of all pair gcd:**
We know, how many pairs are there such that their gcd is $i$ for every $i$ (1 to $n$). So now, $\sum_{i=1}^{n} g[i] * i$.

**Sum of all pair lcm (i = 1, j = 1):**
We know, $\text{lcm}(a, b) = \frac{a*b}{\text{gcd}(a,b)}$.

Now, $f[i]$ = All pair product sum of those, whose gcd is $i$ or it's multiple.
$g[i]$ = All pair product sum of those, whose gcd is $i$.
Ans = $\sum_{i=1}^{n} \frac{g[i]}{i}$.

All pair product sum = $(a_1 + a_2 + \cdots + a_n) * (a_1 + a_2 + \cdots + a_n)$

## 3.8 Legendres Formula

```
// $\frac{n!}{p^x}$ - you will get the largest $x$
int legendre(int n, int p) {
  int ex = 0;
  while(n) {
    ex += (n / p);
    n /= p;
  }
  return ex;
}
```

## 3.9 Binary Expo

```
int power(int x, long long n, int mod) {
  int ans = 1 % mod;
  while (n > 0) {
    if (n & 1) {
      ans = 1LL * ans * x % mod;
    }
    x = 1LL * x * x % mod;
    n >>= 1;
  }
  return ans;
}
```

## 3.10 Digit Sum of 1 to N

```
// for n=10, ans = 1+2+...+9+1+0
ll solve(ll n) {
  ll res = 0, p = 1;
  while (n / p > 0) {
    ll left = n / (p * 10);
    ll cur  = (n / p) % 10;
    ll right = n % p;
    res += left * 45 * p;
    res += (cur * (cur - 1) / 2) * p;
    res += cur * (right + 1);
    p *= 10;
  } return res;
}
```

## 3.11 Pollard Rho

```
namespace PollardRho {
mt19937 rnd(chrono::steady_clock::now().
    time_since_epoch().count());
const int P = 1e6 + 9;
ll seq[P];
int primes[P], spf[P];
inline ll add_mod(ll x, ll y, ll m) {
  return (x += y) < m ? x : x - m;
}
inline ll mul_mod(ll x, ll y, ll m) {
  ll res = __int128(x) * y % m;
  return res;
  // ll res = x * y - (ll)((long double)x
      * y / m + 0.5) * m;
  // return res < 0 ? res + m : res;
}
inline ll pow_mod(ll x, ll n, ll m) {
  ll res = 1 % m;
  for (; n; n >>= 1) {
    if (n & 1) res = mul_mod(res, x, m);
    x = mul_mod(x, x, m);
  }
  return res;
}
  // $O(it * (logn)^3)$, it = number of rounds
      performed
inline bool miller_rabin(ll n) {
  if (n <= 2 || (n & 1 ^ 1)) return (n
      == 2);
  if (n < P) return spf[n] == n;
  ll c, d, s = 0, r = n - 1;
  for (; !(r & 1); r >>= 1, s++) {}
  // each iteration is a round
  for (int i = 0; primes[i] < n &&
      primes[i] < 32; i++) {
    c = pow_mod(primes[i], r, n);
    for (int j = 0; j < s; j++) {
      d = mul_mod(c, c, n);
      if (d == 1 && c != 1 && c != (n -
          1)) return false;
      c = d;
    }
    if (c != 1) return false;
  }
  return true;
}
void init() {
  int cnt = 0;
  for (int i = 2; i < P; i++) {
    if (!spf[i]) primes[cnt++] = spf[i]
        = i;
    for (int j = 0, k; (k = i *
        primes[j]) < P; j++) {
      spf[k] = primes[j];
      if (spf[i] == spf[k]) break;
    }
  }
}
// returns O(n^(1/4))
ll pollard_rho(ll n) {
  while (1) {
    ll x = rnd() % n, y = x, c = rnd() %
        n, u = 1, v, t = 0;
    ll *px = seq, *py = seq;
    while (1) {
      *py++ = y = add_mod(mul_mod(y, y,
          n), c, n);
      *py++ = y = add_mod(mul_mod(y, y,
          n), c, n);
      if ((x = *px++) == y) break;
      v = u;
      u = mul_mod(u, abs(y - x), n);
      if (!u) return __gcd(v, n);
      if (++t == 32) {
        t = 0;
        if ((u = __gcd(u, n)) > 1 && u <
            n) return u;
      }
    }
    if (t && (u = __gcd(u, n)) > 1 && u
        < n) return u;
  }
}
vector<ll> factorize(ll n) {
  if (n == 1) return vector <ll>();
  if (miller_rabin(n)) return vector<ll>
      {n};
  vector <ll> v, w;
  while (n > 1 && n < P) {
    v.push_back(spf[n]);
    n /= spf[n];
  }
  if (n >= P) {
    ll x = pollard_rho(n);
    v = factorize(x);
    w = factorize(n / x);
    v.insert(v.end(), w.begin(), w.end());
  }
  return v;
}
}
```

## 3.12 [Problem] How Many Bases - UVa

```
  // Given a number $N^M$, find out the
    number of integer bases in which it
    has exactly T trailing zeroes.
int solve_greater_or_equal(vector<int>
    e, int t) {
  int ans = 1;
  for (auto i : e) {
    ans = 1LL * ans * (i / t + 1) % mod;
  }
  return ans;
}
// e contains $e_1$, $e_2$ -> ${p_1}^{e_1}$, ${p_2}^{e_2}$
int solve_equal(vector<int> e, int t) {
  return (solve_greater_or_equal(e, t) -
      solve_greater_or_equal(e, t + 1) +
      mod) % mod;
}
```

## 3.13 [Problem] Power Tower - CF

```
    // A sequence $w_1, w_2, ..., w_n$ and Q
    queries, l and r will be given.
    Calculate $w_l^{(w_{l+1}^{\cdots^{(w_r)}})}$
// $n^x \bmod m = n^{\phi(m)+x \bmod \phi(m)} \bmod m$
inline int MOD(int x, int m) {
  if (x < m) return x;
  return x % m + m;
}
int power(int n, int k, int mod) {
  int ans = MOD(1, mod);
  while (k) {
    if (k & 1) ans = MOD(ans * n, mod);
    n = MOD(n * n, mod);
    k >>= 1;
  }
  return ans;
}
int f(int l, int r, int m) {
  if (l == r) return MOD(a[l], m);
  if (m == 1) return 1;
  return power(a[l], f(l + 1, r,
      phi(m)), m);
}
```

## 3.14 Formula and Properties

- $\phi(n) = n \cdot \frac{p_1-1}{p_1} \cdot \frac{p_2-1}{p_2} \cdots$
- $\phi(p^e) = p^e - \frac{p^e}{p} = p^e \cdot \frac{p-1}{p}$
- For $n > 2$, $\phi(n)$ is always even.
- $\sum_{d|n} \phi(d) = n$
- NOD: $(e_1+1) \cdot (e_2+1) \cdots$
- SOD: $\frac{p_1^{e_1+1}-1}{p_1-1} \cdot \frac{p_2^{e_2+1}-1}{p_2-1} \cdots$
- $\log(a \cdot b) = \log(a) + \log(b)$
- $\log(a^x) = x \cdot \log(a)$
- $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$
- **Digit Count of n:** $\lfloor \log_{10}(n) \rfloor + 1$
- **Arithmetic Progression Sum:** $\frac{n}{2} \cdot (a+p)$, $\frac{n}{2} \cdot (2a + (n-1)d)$
- **Geometric Sum:** $S_n = a \cdot \frac{r^n - 1}{r-1}$
- $(1^2 + 2^2 + \cdots + n^2) = \frac{n(n+1)(2n+1)}{6}$
- $(1^3 + 2^3 + \cdots + n^3) = \frac{n^2(n+1)^2}{4}$
- $(2^2 + 4^2 + \cdots + (2n)^2) = \frac{2n(n+1)(2n+1)}{3}$
- $(1^2 + 3^2 + \cdots + (2n-1)^2) = \frac{n(2n-1)(2n+1)}{3}$
- $(2^3 + 4^3 + \cdots + (2n)^3) = 2n^2(n+1)^2$
- $(1^3 + 3^3 + \cdots + (2n-1)^3) = n^2(2n^2-1)$
- For any number $n$ and bases $> \sqrt{n}$, there will be no representation where the number contains 0 at its second least significant digit. So it is enough to check for bases $\leq \sqrt{n}$.
- For some $x$ and $y$, let's try to find all $m$ such that $x \bmod m \equiv y \bmod m$. We can rearrange the equation into $(x-y) \equiv 0 \pmod m$. Thus, if $m$ is a factor of $|x-y|$, then $x$ and $y$ will be equal modulo $m$.

# 4 Combinatorics and Probability

## 4.1 Combinations

```
// Prime Mod in O(n)
void prec() {
  fact[0] = 1;
  for (int i = 1; i < N; i++) {
    fact[i] = 1ll * fact[i - 1] * i % mod;
  }
  ifact[N - 1] = inverse(fact[N - 1]);
  for (int i = N - 2; i >= 0; i--) {
    ifact[i] = 1ll * ifact[i + 1] * (i +
        1) % mod;
  }
}
int nCr(int n, int r) {
  if (r > n) return 0;
  return 1ll * fact[n] * ifact[r] % mod
      * ifact[n - r] % mod;
}
int nPr(int n, int r) {
  if (r > n) return 0;
  return 1ll * fact[n] * ifact[n - r] %
      mod;
}
```

## 4.2 nCr for any mod

```
// Time: O(n^2)
// nCr = (n-1)C(r-1) + (n-1)Cr
for (int i = 0; i < N; i++) {
  C[i][i] = 1;
  for (int j = 0; j < i; j++) {
    C[i][j] = (C[i - 1][j] + C[i - 1][j
        - 1]) % mod;
  }
}
```

## 4.3 nCk without mod in O(r)

```
ll nCk(ll n, ll k) {
  double res = 1;
  for (ll i = 1; i <= k; ++i)
    res = res * (n - k + i) / i;
  return (ll)(res + 0.01);
}
```

## 4.4 Lucas Theorem

```
// returns nCr modulo mod where mod is a
   prime
// Complexity: ?
ll Lucas(ll n, ll r) {
  if (r < 0 || r > n) return 0;
  if (r == 0 || r == n) return 1;
  if (n >= MOD) {
    return (Lucas(n / MOD, r / MOD) %
        MOD * Lucas(n % MOD, r % MOD) %
        MOD) % MOD;
  }
  return (((fact[n] * invFact[r]) % MOD)
      * invFact[n - r]) % MOD;
}
```

## 4.5 Catalan Number

```
const int MOD = 1e9 + 7, int MAX = 1e7;
int catalan[MAX];
void init(ll n) {
  catalan[0] = catalan[1] = 1;
  for ( ll i = 2; i <= n; i++ ) {
    catalan[i] = 0;
    for ( ll j = 0; j < i; j++ ) {
      catalan[i] += ( catalan[j] *
          catalan[i - j - 1] ) % MOD;
      if ( catalan[i] >= MOD ) {
        catalan[i] -= MOD;
      }
    }
  }
}
```

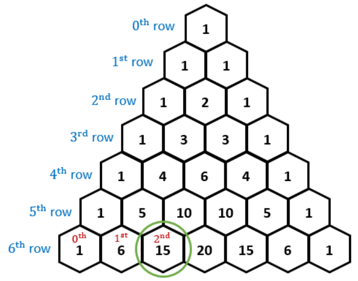
## 4.6 Derangement

```
    // number of combinations such that
    a_i! = i of a permuation a
const int N = 1e6 + 100, int p = 1e9 + 7;
ll der[N];
void countDer() {
  der[1] = 0;
  der[2] = 1;
  for (ll i = 3; i <= N; ++i) {
    der[i] = (i - 1) % p * (der[i - 1] %
        p + der[i - 2] % p);
    der[i] %= p;
  }
}
```

## 4.7 Stars and Bars Theorem

- Find the number of $k$-tuples of non-negative integers whose sum is $n$. $\binom{n+k-1}{n}$
- Find the number of $k$-tuples of non-negative integers whose sum is $\leq n$. $\binom{n+k}{k}$
- Combination with Repetition (choose $k$ elements from $n$ objects, same element can be chosen multiple times). $\binom{n+k-1}{k}$
- How many ways to go from $(0,0)$ to $(n,m)$. $\binom{n+m}{m}$

**Pascals Triangle is equivalent to nCr:**



## 4.8 Properties of Pascal's Triangle

- $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$
- $(k+1)^n = \sum_{i=0}^{n} k^i \cdot \binom{n}{i}$
- $$\sum_{i=0}^{n} \binom{n}{i} = 2^n$$
- $\binom{k}{n} = \frac{k}{n}\binom{k-1}{n-1}$
- $$\sum_{k=0}^{m} \binom{n+k}{k} = \binom{n+m+1}{m}$$
- $\binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}$
- $1\binom{n}{1} + 2\binom{n}{2} + \cdots + n\binom{n}{n} = n\,2^{n-1}$

## 4.9 Contribution Technique

- Sum of all pair sums: $\sum_{i=1}^{n} \sum_{j=1}^{n} (a_i + a_j)$ Every element will be added $2n$ times. $\sum_{i=1}^{n} (2 \times n \times a_i) = 2 \times n \times \sum_{i=1}^{n} a_i$.
- Sum of all subarray sums — $\sum_{i=1}^{n} (a_i \times i \times (n-i+1))$.
- Sum of all Subsets sums — $\sum_{i=1}^{n} (2^{n-1} \times a_i)$.
- Product of all pair product — $\prod_{i=1}^{n} \left(a_i^{2 \times n}\right)$.
- XOR of subarray XORS — How many subarrays does an element have? $(i \cdot (n-i+1)$ times). If subarray count is odd then this element can contribute in total XORs.
- Sum of max minus min over all subset — Sort the array. $Min = 2^{n-i}$, $Max = 2^{i-1}$. $\sum_{i=1}^{n} (a_i \cdot 2^{i-1} - a_i \cdot 2^{n-i})$
- Sum using bits — $\sum_{k=0}^{30} (cnt_k[1] \times 2^k)$.
- Sum of Pair XORs — XOR will 1 if two bits are different $\sum_{k=0}^{30} (cnt_k[0] \times cnt_k[1] \times 2^k)$.
- Sum of Pair ANDs — $\sum_{k=0}^{30} (cnt_k[1]^2 \times 2^k)$.
- Sum of Pair ORs — $\sum_{k=0}^{30} ((cnt_k[1]^2 + 2 \times cnt_k[1] \times cnt_k[0]) \times 2^k)$.
- Sum of Subset XORs — where $cnt0! = 0$ $\sum_{k=0}^{30} (2^{cnt_k[1]+cnt_k[0]-1} \times 2^k)$.
- Sum of Subset ANDs — $\sum_{k=0}^{30} ((2^{cnt_k[1]} - 1) \times 2^k)$.
- Sum of Subset ORs — $\sum_{k=0}^{30} ((2^n - 2^{cnt_k[0]}) \times 2^k)$.
- Sum of subarray XORs — Convert to prefix xor, then solve for pairs.
- Sum of product of all subsequence — $\prod_{i=1}^{n}(a_i + 1) - 1$. Example array — $[a,b]$ the subsequences are $\{a\}, \{b\}, \{a,b\}$ so ans is $a + b + (a \cdot b)$

## 4.10 Probability and Expected Value

- Expected Value: $E = \frac{\text{Sum of all possible values}}{\text{Total number of outcomes}}$
- Expected Value: $E = \sum_{i=1}^{n} P_i \cdot i$
- Variance: $V(x) = E(x^2) - \{E(x)\}^2$
- Expected Value with DP: $E(i) = \sum P(i \to j) \times (R(i \to j) + E(j))$ Where $R()$ is Immediate Reward(cost/count)
- Linearity of Expectation:
  - $E[X+Y] = E[X] + E[Y]$
  - $E[\text{Total}] = E[I_1] + E[I_2] + \cdots = \sum P(I_i = 1)$

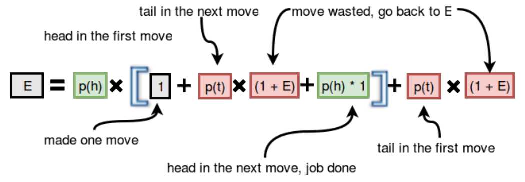
  We need to define the Indicator Random Variable (I) correctly. Some Examples below —

  **Problem-1:** Find $E$[correct hats] in a random permutation. Indicator $I_i$: "Does person $i$ get their own hat?"

  **Problem-2:** Find $E$[total inversions] in a random permutation. Indicator $I_{ij}$: "Is pair $(i,j)$ an inversion?"

  **Problem-3:** Given a string $S$, delete a random index until it becomes empty. Find the expected count of palindromes seen (exclude $S$, include empty string). Indicator $I_L$: "Is the string of length $L$ a palindrome?"

  **Problem-4:** Pick $N$ random integers from $[1,k]$. Merge consecutive equal values (e.g., $1,1,2,2,1,1 \to 1,2,1$). Find the expected final length. Indicator $I_i$: "Is item $i$ different from item $i-1$?"
- To get two consecutive heads, what is the expected number of tosses?



tail in the next move
move wasted, go back to E
head in the first move
E = p(h) × [ 1 + p(t) × (1 + E) + p(h) * 1 ] + p(t) × (1 + E)
made one move
tail in the first move
head in the next move, job done

- To get $n$ heads, what is the expected number of tosses? Let's define: to get $n$ heads, we need to toss $E(n)$ times. Now — I can get a head; I need to toss $E(n-1)$ more times, or if I get a tail; I need to toss $E(n)$ times. So, the recurrence is: $E(n) = 0.5 \cdot (1 + E(n-1)) + 0.5 \cdot (1 + E(n))$

- You have $n$ bulbs, all of which are initially off. In each move, you randomly select one bulb. If the selected bulb is **off**, you toss a coin:
  - If you get head, you turn it on.
  - If you get tail, you do nothing.

  If the bulb is already **on**, you skip that move (nothing happens).

  Now, what is the expected number of moves required to turn all bulbs on?

  The coin is not fair — the probability of getting tail is $p$. This problem can also be solved recursively.

  Let's assume at some moment, $x$ bulbs are already on, and the expected number of moves needed from here is $e(x)$.

  The probability of picking an already on bulb is $\frac{x}{n}$. In that case, the expected number of moves is $\frac{x}{n} \times (1 + e(x))$.

  The probability of picking an off bulb is $\frac{n-x}{n}$.

  Now two things can happen:

  - With probability $p$, you get tail, so you stay at the same state ($e(x)$ more moves).
  - With probability $(1-p)$, you get head, so one more bulb turns on ($e(x+1)$ moves from there).

  So, the recurrence relation is:

$$e(x) = \frac{x}{n}(1 + e(x)) + \frac{n-x}{n}\left(p(1 + e(x)) + (1-p)(1 + e(x+1))\right)$$

## 5 Data Structure

### 5.1 Trie for bit

```cpp
struct Trie {
  static const int B = 31;
  struct node {
    node* nxt[2];
    int sz;
    node() {
      nxt[0] = nxt[1] = NULL;
      sz = 0;
    }
  }*root;
  Trie() {
    root = new node();
  }
  void insert(int val) {
    node* cur = root;
    cur -> sz++;
    for (int i = B - 1; i >= 0; i--) {
      int b = val >> i & 1;
      if (cur -> nxt[b] == NULL) cur ->
          nxt[b] = new node();
      cur = cur -> nxt[b];
      cur -> sz++;
    }
  }
  int query(int x, int k) {  // number
      of values s.t. val ⊕ x < k
    node* cur = root;
    int ans = 0;
    for (int i = B - 1; i >= 0; i--) {
      if (cur == NULL) break;
      int b1 = x >> i & 1, b2 = k >> i &
          1;
      if (b2 == 1) {
        if (cur -> nxt[b1]) ans += cur
            -> nxt[b1] -> sz;
        cur = cur -> nxt[!b1];
      } else cur = cur -> nxt[b1];
    }
    return ans;
  }
  int get_max(int x) {  // returns
      maximum of val ⊕ x
    node* cur = root;
    int ans = 0;
    for (int i = B - 1; i >= 0; i--) {
      int k = x >> i & 1;
      if (cur -> nxt[!k]) cur = cur ->
          nxt[!k], ans <<= 1, ans++;
      else cur = cur -> nxt[k], ans <<= 1;
    }
    return ans;
  }
  int get_min(int x) {  // returns
      minimum of val ⊕ x
    node* cur = root;
    int ans = 0;
    for (int i = B - 1; i >= 0; i--) {
      int k = x >> i & 1;
      if (cur -> nxt[k]) cur = cur ->
          nxt[k], ans <<= 1;
      else cur = cur -> nxt[!k], ans <<=
          1, ans++;
    }
    return ans;
  }
  void del(node* cur) {
    for (int i = 0; i < 2; i++) if (cur
        -> nxt[i]) del(cur -> nxt[i]);
    delete(cur);
  }
} t;
```

## 6 Dynamic Programming

### 6.1 Knapsack 2

```
Constraints: N <= 100, W <= 1e9, val[i]
    <= 1000
dp[i][cur_val] = min weight needed to
    achieve cur_val from i to n. if
    dp[1][val] <= W then val is a
    candidate ans. where val is
    all_possible_val (1 to 100*1000)
```

### 6.2 LIS using Segment Tree

```cpp
int32_t main() {
  int n; cin >> n;
  for (int i = 1; i <= n; i++) {
    cin >> a[i]; // a[i] must be >= 2
  }
  // dp[i] = LIS ending at pos i
  st.build(1, 1, M); // range max query,
      and upd idx with max(cur_val,
      new_val)
  for (int i = 1; i <= n; i++) {
    dp[i] = 1;
    if (a[i] != 1) {
      int mx = st.query(1, 1, M, 1, a[i]
          - 1);
      mx++;
      dp[i] = max(dp[i], mx);
    }
    st.upd(1, 1, M, a[i], dp[i]);
  }
  int ans = 0;
  for (int i = 1; i <= n; i++) ans =
      max(ans, dp[i]);
  cout << ans << '\n';
}
```

### 6.3 Digit DP

```cpp
// Problem: How Many Zeroes? - LightOJ
  // How many zeroes between n to m
      (n <= m). 100 to 102 ans 4
ll fun2(int i, bool is_small) {
  if (i == sz) return 1;
  int l = 0, r = s[i] - '0';
  if (is_small) r = 9;
  ll &ans = dp2[i][is_small];
  if (ans != -1) return ans;
  ans = 0;
  for (int x = l; x <= r; x++) {
    ans += fun2(i + 1, (is_small | (x <
        r)));
  }
  return ans;
}
ll fun(int i, bool is_small, bool
    has_started) {
  if (i == sz) return 0;
  int l = 0, r = s[i] - '0';
  if (is_small) r = 9;
  ll &ans = dp1[i][is_small][has_started];
  if (ans != -1) return ans;
  ans = 0;
  for (int x = l; x <= r; x++) {
    bool new_has_started = has_started |
        (x != 0);
    ans += fun(i + 1, (is_small | (x <
        r)), new_has_started);
    if (x == 0 and has_started) {
      ans += fun2(i + 1, (is_small | (x
          < r)));
    }
  }
  return ans;
}
void get(long long x) {
  if (x < 0) return; s = "";
  while (x > 0) {
    char c = (x % 10) + '0';
    s += c; x /= 10;
  }
  reverse(s.begin(), s.end());
  sz = s.size();
  memset(dp1, -1, sizeof(dp1));
  memset(dp2, -1, sizeof(dp2));
}
void solve() {
  ll n, m; cin >> n >> m;
  get(n - 1);
  memset(dp1, -1, sizeof(dp1));
  memset(dp2, -1, sizeof(dp2));
  ll ans1 = (n == 0) ? 0 : fun(0, false,
      false);
  get(m);
  ll ans2 = fun(0, false, false);
  cout << ans2 - ans1 + (n == 0) << '\n';
}
```

### 6.4 Bitmask DP

```cpp
// Problem: DNA Sequence - LOJ
// Given n strings, find the shortest
    string that contains all the given
    strings as substrings and
    lexicographically smallest.
const int N = 17;
int n, tail[N][N], dp[(1 << N) + 2][N +
    2];
string s[N + 2];
bool cmp(string a, string b) {
  if (a.size() < b.size()) {
    return true;
  }
  return false;
}
int fun(int mask, int last) {
  if (__builtin_popcount(mask) >= n)
      return 0;
  int &ans = dp[mask][last];
  if (ans != -1) return ans;
  ans = 1e9;
  for (int j = 1; j <= n; j++) {
    if (!(mask & (1 << j))) {
      if (last == n + 1) {
        int x = s[j].length() + fun(mask
            | (1 << j), j);
        ans = min(ans, x);
      }
      else {
        int x = (s[j].length() -
            tail[last][j]) + fun(mask |
            (1 << j), j);
        ans = min(ans, x);
      }
    }
  }
  return ans;
}
void print(int mask, int last) {
  if (__builtin_popcount(mask) >= n)
      return;
  int ans = fun(mask, last), idx = -1;
  string str = "{";
  for (int j = 1; j <= n; j++) {
    if (!(mask & (1 << j))) {
      if (last == n + 1) {
        int x = s[j].length() + fun(mask
            | (1 << j), j);
        if (x == ans) {
          string d = s[j];
          if (d <= str) str = d; idx = j;
        }
      }
      else {
        int x = (s[j].length() -
            tail[last][j]) + fun(mask |
            (1 << j), j);
        if (x == ans) {
          string d =
              s[j].substr(tail[last][j]);
          if (d <= str) str = d; idx = j;
        }
      }
    }
  }
  cout << s[idx].substr(tail[last][idx]);
  print(mask | (1 << idx), idx);
}
void solve() {
  cin >> n;
  string str[n + 1];
  for (int i = 1; i <= n; i++) {
    cin >> str[i]; // len <= 100
  }
  // remove duplicates and strings which
      is a subarray of others
  sort(str + 1, str + n + 1, cmp);
  vector<string> vec;
  for (int i = 1; i <= n; i++) {
    bool ok = true;
    for (int j = i + 1; j <= n; j++) {
      if (str[j].find(str[i]) !=
          string::npos) {
        ok = false;
        break;
      }
    }
    if (ok) vec.push_back(str[i]);
  }
  n = vec.size();
  int idx = 1;
  for (auto x : vec) s[idx++] = x;
  // maximum length that suffix of a[i] =
      prefix of a[j]
  memset(tail, 0, sizeof(tail));
  for (int i = 1; i <= n; i++) {
    string x = s[i];
    for (int j = 1; j <= n; j++) {
      string y = s[j];
      int cnt = 0;
      for (int k = min(x.length(),
          y.length()); k >= 1; k--) {
        if (x.substr(x.length() - k) ==
            y.substr(0, k)) {
          cnt = k;
          break;
        }
      }
      tail[i][j] = cnt;
    }
  }
  memset(dp, -1, sizeof(dp));
  fun(0, n + 1);
  print(0, n + 1);
}
```

## 6.5 MCM DP

```
// Problem: Slimes - Atcoder DP Contest
// Given n slimes. Choose two adjacent
    slimes, and combine them into a new
    slime. The new slime has a size of
    x+y, where x and y are the sizes of
    the slimes before combining them.
    Here, a cost of x+y is incurred.
    Example-
// (10, 20, 30, 40) → (30, 30, 40)
// (30, 30, 40) → (60, 40)
// (60, 40) → (100) ans = 190
        // Solution: Think reverse. We
    are given the final sum, from i to j.
    Now we will cut any point between i to
    j and calculate the cost
// Time: O(n^3)
ll fun(int i, int j) {
  if (i == j) return 0;
  ll &ans = dp[i][j];
  if (ans != -1) return ans;
  ll cur = 0;
  for (int x = i; x <= j; x++) {
    cur += a[x];
  }
  ans = inf;
  for (int x = i; x < j; x++) {
    ans = min(ans, cur + fun(i, x) +
        fun(x + 1, j));
  }
  return ans;
}
cout << fun(1, n) << '\n';
```

# 7 Graph Theory

## 7.1 Binary Lifting and LCA

```
const int N = 2e5 + 9, LOG = 20;
vector<int> g[N];
int par[N][LOG], depth[N];
void dfs(int u, int p) {
  par[u][0] = p;
  depth[u] = depth[p] + 1;
  for (int i = 1; i < LOG; i++) {
    par[u][i] = par[par[u][i - 1]][i - 1];
  }
  for (auto v : g[u]) {
    if (v != p) {
      dfs(v, u);
    }
  }
}
int lca(int u, int v) {
  if (depth[u] < depth[v]) {
    swap(u, v);
  }
  int k = depth[u] - depth[v];
  for (int i = 0; i < LOG; i++) {
    if (CHECK(k, i)) u = par[u][i];
  }
  if (u == v) return u;
  for (int i = LOG - 1; i >= 0; i--) {
    if (par[u][i] != par[v][i]) {
      u = par[u][i];
      v = par[v][i];
    }
  }
  return par[u][0];
}
int kth(int u, int k) { // kth parent of
    u
  assert(k >= 0);
  for (int i = 0; i < LOG; i++) {
    if (CHECK(k, i)) u = par[u][i];
  }
  return u;
}
int dist(int u, int v) { // distance from
    u to v
  int l = lca(u, v);
  return (depth[u] - depth[l]) +
      (depth[v] - depth[l]);
}
// kth node from u to v, 0th node is u
int kth(int u, int v, int k) {
  int l = lca(u, v);
  int d = dist(u, v);
  assert(k <= d);
  if (depth[l] + k <= depth[u]) {
    return kth(u, k);
  }
  k -= depth[u] - depth[l];
  return kth(v, depth[v] - depth[l] - k);
}
```

## 7.2 LCA and Sparse Table on Tree

```
// max and min weights of a path
const int N = 1e5 + 9, LOG = 20, inf =
    1e9; // change here
vector<array<int, 2>> g[N];
int par[N][LOG], tree_mx[N][LOG],
    depth[N];
void dfs(int u, int p, int dis) {
  par[u][0] = p;
  tree_mx[u][0] = dis;
  depth[u] = depth[p] + 1;
  for (int i = 1; i < LOG; i++) {
    par[u][i] = par[par[u][i - 1]][i - 1];
    tree_mx[u][i] = max(tree_mx[u][i -
        1], tree_mx[par[u][i - 1]][i -
        1]);
  }
  for (auto [v, w] : g[u]) {
    if (v != p) {
      dfs(v, u, w);
    }
  }
}
int query_max(int u, int v) { // max
    weight on path u to v
  int l = lca(u, v);
  int d = dist(l, u);
  int ans = 0;
  for (int i = 0; i < LOG; i++) {
    if (CHECK(d, i)) {
      ans = max(ans, tree_mx[u][i]);
      u = par[u][i];
    }
  }
  d = dist(l, v);
  for (int i = 0; i < LOG; i++) {
    if (CHECK(d, i)) {
      ans = max(ans, tree_mx[v][i]);
      v = par[v][i];
    }
  }
  return ans;
}
```

## 7.3 Dijkstra

```
vector<int> dijkstra(int s) {
  vector<int> dis(n + 1, inf);
  vector<bool> vis(n + 1, false);
  dis[s] = 0;
  priority_queue<array<int, 2>,
      vector<array<int, 2>>,
      greater<int, 2>>> pq;
  pq.push({0, s});
  while (!pq.empty()) {
    auto [d, u] = pq.top(); pq.pop();
    if (vis[u]) continue;
    vis[u] = true;
    for (auto [v, w] : g[u]) {
      if (dis[v] > d + w) {
        dis[v] = d + w;
        pq.push({dis[v], v});
      }
    }
  }
  return dis;
}
```

## 7.4 Bellman Ford

```
// works for neg edge, can detect neg
    cycle
// Time: O(n^2)
const ll inf = 1e18;
vector<ll> dis(N, inf);
bool bellman_ford(int s) {
  dis[s] = 0;
  bool has_cycle = false;
  for (int i = 1; i <= n; i++) {
    for (int u = 1; u <= n; u++) {
      for (auto [v, w] : g[u]) {
        if (dis[v] > dis[u] + w) {
          if (i == n) has_cycle = true;
          dis[v] = dis[u] + w;
        }
      }
    }
  }
  return has_cycle;
}
```

## 7.5 Floyd Warshall

```
// dis[i][j] = min distance to reach i to
    j, works for neg edge (no neg cycle)
// Time: O(n^3)
vector<int> construct_path(int u, int v)
    { // O(n)
  if (nxt[u][v] == -1) return {};
  vector<int> path = { u };
  while (u != v) {
    u = nxt[u][v];
    path.push_back(u);
  }
  return path;
}
void floyd_warshall() {
  for (int i = 1; i <= n; i++) {
    for (int j = 1; j <= n; j++) {
      if (i == j) dis[i][j] = 0;
      else if (g[i][j] == 0) dis[i][j] =
          inf;
      else dis[i][j] = g[i][j];
    }
  }
  for (int k = 1; k <= n; ++k) {
    for (int i = 1; i <= n; ++i) {
      for (int j = 1; j <= n; ++j) {
        if (dis[i][k] < inf and
            dis[k][j] < inf)
          dis[i][j] = min(dis[i][j],
              dis[i][k] + dis[k][j]);
          nxt[i][j] = nxt[i][k];
      }
    }
  }
}
int32_t main() {
  int q; cin >> n >> m >> q;
  memset(nxt, -1, sizeof nxt);
  while (m--) {
    int u, v, w; cin >> u >> v >> w;
    g[u][v] = (g[u][v] != 0 ?
        min(g[u][v], w) : w);
    g[v][u] = (g[v][u] != 0 ?
        min(g[v][u], w) : w);
    nxt[u][v] = v;
    nxt[v][u] = u;
  }
  floyd_warshall();
  while (q--) {
    int u, v; cin >> u >> v;
    cout << (dis[u][v] == inf ? -1 :
        dis[u][v]) << '\n';
  }
  return 0;
}
```

## 7.6 Strongly Connected Components

```
// Time: O(n + m)
const int N = 1e5 + 9;
vector<int> g[N], gT[N], G[N];
vector<bool> vis(N, false);
vector<vector<int>> components;
vector<int> order;
int n, roots[N], sz[N];
void dfs(int u) {
  vis[u] = true;
  for (auto v : g[u]) {
    if (!vis[v]) dfs(v);
  }
  order.push_back(u);
}
void dfs2(int u, vector<int> &component) {
  vis[u] = true;
  component.push_back(u);
  for (auto v : gT[u]) {
    if (!vis[v]) dfs2(v, component);
  }
}
void scc() {
  // get order sorted by end time
  order.clear();
  for (int u = 1; u <= n; u++) {
    if (!vis[u]) dfs(u);
  }
  reverse(order.begin(), order.end());
  // transpose the graph
  for (int u = 1; u <= n; u++) {
    for (auto v : g[u]) {
      gT[v].push_back(u);
    }
  }
  // get all components
  components.clear();
  for (int i = 1; i <= n; i++) vis[i] =
      false;
  for (auto u : order) {
    if (!vis[u]) {
      vector<int> component;
      dfs2(u, component);
      sort(component.begin(),
          component.end());
      components.push_back(component);
      for (auto v : component) {
        roots[v] = component.front();
        sz[v] = component.size();
      }
    }
  }
  // add edges to condensation graph
  for (int u = 1; u <= n; u++) {
    for (auto v : g[u]) {
      if (roots[u] != roots[v]) {
        G[roots[u]].push_back(roots[v]);
      }
    }
  }
}
// when you need to use condensed graph,
  use it carefully (Specially g->G,
    i->roots[i])
```

## 7.7 Articulation Points

```
int disc[N], low[N], timer, n;
vector<bool> vis(N, false), is_ap(N,
      false);
void ap_dfs(int u, int p) {
  disc[u] = low[u] = ++timer;
  vis[u] = true;
  int children_cnt = 0;
  for (auto v : g[u]) {
    if (v == p) continue;
    if (vis[v]) low[u] = min(low[u],
        disc[v]);
    else {
      ap_dfs(v, u);
      low[u] = min(low[u], low[v]);
      if (disc[u] <= low[v] and p != -1)
          is_ap[u] = true;
      children_cnt++;
    }
  }
  if (p == -1 and children_cnt > 1)
      is_ap[u] = true;
}
void find_articulation_points() {
  for (int u = 1; u <= n; u++) {
    if (!vis[u]) {
      timer = 0;
      ap_dfs(u, -1);
    }
  }
}
```

## 7.8 Find Bridges

```
map<pair<int, int>, int> bridges;
void bridges_dfs(int u, int p) { // find
    bridges
  disc[u] = low[u] = ++timer;
  vis[u] = true;
  for (auto v : g[u]) {
    if (v == p) continue;
    if (vis[v]) low[u] = min(low[u],
        disc[v]);
    else {
      bridges_dfs(v, u);
      low[u] = min(low[u], low[v]);
      if (disc[u] < low[v]) {
        bridges[(make_pair(min(u, v),
            max(u, v)))]++;
      }
    }
  }
}
void find_bridges() {
  for (int u = 1; u <= n; u++) {
    if (!vis[u]) {
      timer = 0;
      bridges_dfs(u, -1);
    }
  }
}
```

## 7.9 DSU on Tree

```
// Problem: Distinct Colors CSES (Number
    of distinct color in a subtree)
// Complexity: O(n(logn)^2)
set<int> se[N];
 int col[N], ans[N], par[N];  // par[i] = i
    initially
int find(int i) {
  return (i == par[i] ? i : par[i] =
      find(par[i]));
}
void merge(int u, int v) {
  if ((u = find(u)) == (v = find(v)))
      return;
  if (se[u].size() > se[v].size())
      swap(u, v);
  for (auto x : se[u]) {
    se[v].insert(x);
  }
  se[u].clear();
  par[u] = v;
}
void dfs(int u, int p) {
  se[find(u)].insert(col[u]);
  for (auto v : g[u]) {
    if (v != p) {
      dfs(v, u);
      merge(u, v);
    }
  }
  ans[u] = se[find(u)].size();
}
```

## 7.10 Heavy-Light Decomposition

```
// Per Query Complexity: O(logn^2)
// Path and subtree updates and queries.
const int N = 2e5 + 9, LOG = 20, inf =
    1e9; // change here
vector<int> g[N];
int par[N][LOG], depth[N], sz[N];
int disc[N], finish[N], timer, head[N];
int n;
void dfs(int u, int p = 0) {
  par[u][0] = p;
  depth[u] = depth[p] + 1;
  sz[u] = 1;
  for (int i = 1; i < LOG; i++) {
    par[u][i] = par[par[u][i - 1]][i - 1];
  }
  if (p) g[u].erase(find(g[u].begin(),
      g[u].end(), p));
  for (auto &v : g[u]) {
    if (v != p) {
      dfs(v, u);
      sz[u] += sz[v];
      if (sz[v] > sz[g[u][0]]) swap(v,
          g[u][0]);
    }
  }
}
void dfs_hld(int u) {
  disc[u] = ++timer;
  for (auto v : g[u]) {
    head[v] = (v == g[u][0] ? head[u] :
        v);
    dfs_hld(v);
  }
  finish[u] = timer;
}
int kth(int u, int k) {
  assert(k >= 0);
  for (int i = 0; i < LOG; i++) {
    if (CHECK(k, i)) u = par[u][i];
  }
  return u;
}
int query_up(int u, int v) {
  int ans = -inf; // change here
  while (head[u] != head[v]) {
    ans = max(ans, st.query(1, 1, n,
        disc[head[u]], disc[u])); //
        change here
    u = par[head[u]][0];
  }
  ans = max(ans, st.query(1, 1, n,
      disc[v], disc[u])); // change here
  return ans;
}
int query(int u, int v) {
  int lc = lca(u, v);
  int ans = query_up(u, lc);
  if (v != lc) {
    ans = max(ans, query_up(v, kth(v,
        depth[v] - depth[lc] - 1))); //
        change here
  }
  return ans;
}
void solve() {
  cin >> n;
  for (int i = 2; i <= n; i++) {
    int u, v; cin >> u >> v;
    g[u].push_back(v);
    g[v].push_back(u);
  }
  dfs(1);
  head[1] = 1;
  dfs_hld(1);
  st.build(1, 1, n);
}
```

### 7.11 Dinic

```
const int N = 105, inf = 1e9;
int n, m, st, en;
// Edges should be added in both
    direction separately if the graph is
    undirected
const int INF = 2000000000;
struct Edge {
  int from, to, cap, flow, index;
  Edge(int from, int to, int cap, int
      flow, int index) :
    from(from), to(to), cap(cap),
        flow(flow), index(index) {}
};
struct Dinic {
  int N;
  vector<vector<Edge> > G;
  vector<Edge*> dad;
  vector<int> Q;
  Dinic(int N) : N(N), G(N), dad(N),
      Q(N) {}
  void AddEdge(int from, int to, int
      cap) {
    G[from].emplace_back(from, to, cap,
        0, G[to].size());
    if (from == to)
        G[from].back().index++;
    G[to].emplace_back(to, from, 0, 0,
        G[from].size() - 1);
  }
  long long BlockingFlow(int s, int t) {
    fill(dad.begin(), dad.end(), (Edge
        *) NULL);
    dad[s] = &G[0][0] - 1;
    int head = 0, tail = 0;
    Q[tail++] = s;
    while (head < tail) {
      int x = Q[head++];
      for (int i = 0; i < G[x].size();
          i++) {
        Edge &e = G[x][i];
        if (!dad[e.to] && e.cap - e.flow
            > 0) {
          dad[e.to] = &G[x][i];
          Q[tail++] = e.to;
        }
      }
    }
    if (!dad[t]) return 0;
    long long totflow = 0;
    for (int i = 0; i < G[t].size();
        i++) {
      Edge *start =
          &G[G[t][i].to][G[t][i].index];
      int amt = INF;
      for (Edge *e = start; amt && e !=
          dad[s]; e = dad[e->from]) {
        if (!e) { amt = 0; break; }
        amt = min(amt, e->cap - e->flow);
      }
      if (amt == 0) continue;
      for (Edge *e = start; amt && e !=
          dad[s]; e = dad[e->from]) {
        e->flow += amt;
        G[e->to][e->index].flow -= amt;
      }
      totflow += amt;
    }
    return totflow;
  }
  long long GetMaxFlow(int s, int t) {
    long long totflow = 0;
    while (long long flow =
        BlockingFlow(s, t))
      totflow += flow;
    return totflow;
  }
};
void solve() {
  cin >> n >> st >> en >> m;
  Dinic *dinic = new Dinic(n + 1);
  while (m--) {
    int u, v, w; cin >> u >> v >> w;
    dinic->AddEdge(u, v, w);
    dinic->AddEdge(v, u, w);
  }
  cout << dinic->GetMaxFlow(st, en) <<
      '\n';
  delete(dinic);
}
```

## 8 String

### 8.1 Hashing

```
const int N = 1e6 + 9; // change here
const int MOD1 = 127657753, MOD2 =
    987654319;
const int p1 = 137, p2 = 277; // change
    here
int ip1, ip2;
pair<int, int> pw[N], ipw[N];
void prec() {
  pw[0] = {1, 1};
  for (int i = 1; i < N; i++) {
    pw[i].first = 1ll * pw[i - 1].first
        * p1 % MOD1;
    pw[i].second = 1ll * pw[i -
        1].second * p2 % MOD2;
  }
  ip1 = power(p1, MOD1 - 2, MOD1);
  ip2 = power(p2, MOD2 - 2, MOD2);
  ipw[0] = {1, 1};
  for (int i = 1; i < N; i++) {
    ipw[i].first = 1ll * ipw[i -
        1].first * ip1 % MOD1;
    ipw[i].second = 1ll * ipw[i -
        1].second * ip2 % MOD2;
  }
}
struct Hashing {
  int n;
  string s;
  vector<pair<int, int>> hash_val;
  vector<pair<int, int>> rev_hash_val;
  Hashing() {}
  Hashing(string _s) {
    s = _s;
    n = s.size();
    hash_val.emplace_back(0, 0);
    for (int i = 0; i < n; i++) {
      pair<int, int> p;
      p.first = (hash_val[i].first + 1ll
          * s[i] * pw[i].first % MOD1) %
          MOD1;
      p.second = (hash_val[i].second +
          1ll * s[i] * pw[i].second %
          MOD2) % MOD2;
      hash_val.push_back(p);
    }
    rev_hash_val.emplace_back(0, 0);
    for (int i = 0, j = n - 1; i < n;
        i++, j--) {
      pair<int, int> p;
      p.first = (rev_hash_val[i].first +
          1ll * s[i] * pw[j].first %
          MOD1) % MOD1;
      p.second = (rev_hash_val[i].second
          + 1ll * s[i] * pw[j].second %
          MOD2) % MOD2;
      rev_hash_val.push_back(p);
    }
  }
  pair<int, int> get_hash(int l, int r)
      { // 1 indexed
    pair<int, int> ans;
    ans.first = (hash_val[r].first -
        hash_val[l - 1].first + MOD1) *
        1ll * ipw[l - 1].first % MOD1;
    ans.second = (hash_val[r].second -
        hash_val[l - 1].second + MOD2) *
        1ll * ipw[l - 1].second % MOD2;
    return ans;
  }
  pair<int, int> rev_hash(int l, int r)
      { // 1 indexed
    pair<int, int> ans;
    ans.first = (rev_hash_val[r].first -
        rev_hash_val[l - 1].first +
        MOD1) * 1ll * ipw[n - r].first %
        MOD1;
    ans.second = (rev_hash_val[r].second
        - rev_hash_val[l - 1].second +
        MOD2) * 1ll * ipw[n - r].second
        % MOD2;
    return ans;
  }
  pair<int, int> get_hash() { // 1
      indexed
    return get_hash(1, n);
  }
  bool is_palindrome(int l, int r) {
    return get_hash(l, r) == rev_hash(l,
        r);
  }
};
```

### 8.2 Hashing with Updates

```
using T = array<int, 2>;
const T MOD = {127657753, 987654319};
const T p = {137, 277}; // change here
T operator + (T a, int x) {return {(a[0]
    + x) % MOD[0], (a[1] + x) % MOD[1]};}
T operator - (T a, int x) {return {(a[0]
    - x + MOD[0]) % MOD[0], (a[1] - x +
    MOD[1]) % MOD[1]};}
T operator * (T a, int x) {return
    {(int)((long long) a[0] * x %
    MOD[0]), (int)((long long) a[1] * x
    % MOD[1])};}
T operator + (T a, T x) {return {(a[0] +
    x[0]) % MOD[0], (a[1] + x[1]) %
    MOD[1]};}
T operator - (T a, T x) {return {(a[0] -
    x[0] + MOD[0]) % MOD[0], (a[1] -
    x[1] + MOD[1]) % MOD[1]};}
T operator * (T a, T x) {return
    {(int)((long long) a[0] * x[0] %
    MOD[0]), (int)((long long) a[1] *
    x[1] % MOD[1])};}
ostream& operator << (ostream& os, T
    hash) {return os << "(" << hash[0]
    << ", " << hash[1] << ")";}
T pw[N], ipw[N];
void prec() {
  pw[0] =  {1, 1};
  for (int i = 1; i < N; i++) {
    pw[i] = pw[i - 1] * p;
  }
  ipw[0] =  {1, 1};
  T ip = {power(p[0], MOD[0] - 2,
      MOD[0]), power(p[1], MOD[1] - 2,
      MOD[1])};
  for (int i = 1; i < N; i++) {
    ipw[i] = ipw[i - 1] * ip;
  }
}
struct Hashing {
  int n;
  string s; // 1 - indexed
  vector<array<T, 2>> t; // (normal, rev)
      hash
  array<T, 2> merge(array<T, 2> l,
      array<T, 2> r) {
    l[0] = l[0] + r[0];
    l[1] = l[1] + r[1];
    return l;
  }
  void build(int node, int b, int e) {
    if (b == e) {
      t[node][0] = pw[b] * s[b];
      t[node][1] = pw[n - b + 1] * s[b];
      return;
    }
    int mid = (b + e) >> 1, l = node <<
        1, r = l | 1;
    build(l, b, mid);
    build(r, mid + 1, e);
    t[node] = merge(t[l], t[r]);
  }
  void upd(int node, int b, int e, int
      i, char x) {
    if (b > i || e < i) return;
    if (b == e && b == i) {
      t[node][0] = pw[b] * x;
      t[node][1] = pw[n - b + 1] * x;
      return;
    }
    int mid = (b + e) >> 1, l = node <<
        1, r = l | 1;
    upd(l, b, mid, i, x);
    upd(r, mid + 1, e, i, x);
    t[node] = merge(t[l], t[r]);
  }
  array<T, 2> query(int node, int b, int
      e, int i, int j) {
    if (b > j || e < i) return {T({0,
        0}), T({0, 0})};
    if (b >= i && e <= j) return t[node];
    int mid = (b + e) >> 1, l = node <<
        1, r = l | 1;
    return merge(query(l, b, mid, i, j),
        query(r, mid + 1, e, i, j));
  }
  Hashing() {}
  Hashing(string _s) {
    n = _s.size();
    s = "." + _s;
    t.resize(4 * n + 1);
    build(1, 1, n);
  }
  void upd(int i, char c) {
    upd(1, 1, n, i, c);
    s[i] = c;
  }
  T get_hash(int l, int r) { // 1 -
      indexed
    return query(1, 1, n, l, r)[0] *
        ipw[l - 1];
  }
  T rev_hash(int l, int r) { // 1 -
      indexed
    return query(1, 1, n, l, r)[1] *
        ipw[n - r];
    }
  T get_hash() {
    return get_hash(1, n);
  }
  bool is_palindrome(int l, int r) {
    return get_hash(l, r) == rev_hash(l,
        r);
  }
};
```

### 8.3 Hashing with Upd and Deletes

```
// update or delete a char in the string
    or check whether a range [l,r] is a
    palindrome or not (Palindromic Query I
    - Toph)
#define int  long long
const int N = 1e5 + 9;
int en;
struct ST {
  pair<int, int> tree[4 * (N + N)];
  void build(int n, int b, int e) {
    if (b == e) {
      tree[n].first = b;
      tree[n].second = 1;
      return;
    }
    int mid = (b + e) >> 1, l = n << 1,
        r = l + 1;
    build(l, b, mid);
    build(r, mid + 1, e);
    tree[n].second = tree[l].second +
        tree[r].second;
  }
  void upd(int n, int b, int e, int i,
      int x1, int x2) {
    if (b > i || e < i) return;
    if (b == e && b == i) {
      tree[n].first = x1;
      tree[n].second = x2;
      return;
    }
    int mid = (b + e) >> 1, l = n << 1,
        r = l + 1;
    upd(l, b, mid, i, x1, x2);
    upd(r, mid + 1, e, i, x1, x2);
    tree[n].second = tree[l].second +
        tree[r].second;
  }
  pair<int, int> query(int n, int b, int
      e, int x) {
    if (b > e) return { -1, -1};
    if (tree[n].second < x) return
        {tree[n].second, -1};
    if (b == e) return tree[n];
    int mid = (b + e) >> 1, l = n << 1,
        r = l + 1;
    pair<int, int> L = query(l, b, mid,
        x);
    if (L.second != -1) return L;
    pair<int, int> R = query(r, mid + 1,
        e, x - L.first);
    return R;
  }
} st, st2;
using T = array<int, 2>;
const T MOD = {127657753, 987654319};
const T p = {137, 277};
// add operators overloading of T (from
    only upd) + prec()
int get(int i, int n) {
  return n - i + 1;
}
struct Hashing {
  int n; string s;
  vector<T> tree, lazy;
  void push(int node, int b, int e) {
    if (lazy[node][0] == 1) return;
    tree[node] = tree[node] * lazy[node];
    if (b != e) {
      int l = node << 1, r = l + 1;
      lazy[l] = lazy[l] * lazy[node];
      lazy[r] = lazy[r] * lazy[node];
    }
    lazy[node] = T{1, 1};
  }
  void build(int node, int b, int e) {
    lazy[node] = T{1, 1};
    if (b == e) {
      tree[node] = pw[b] * s[b];
      return;
    }
    int mid = (b + e) >> 1, l = node <<
        1, r = l | 1;
    build(l, b, mid);
    build(r, mid + 1, e);
    tree[node] = tree[l] + tree[r];
  }
  void upd(int node, int b, int e, int
      i, T x) {
    push(node, b, e);
    if (b > i || e < i) return;
    if (b == e && b == i) {
      tree[node] = x;
      return;
    }
    int mid = (b + e) >> 1, l = node <<
        1, r = l + 1;
    upd(l, b, mid, i, x);
    upd(r, mid + 1, e, i, x);
    tree[node] = tree[l] + tree[r];
  }
  void del(int node, int b, int e, int
      i, int j) {
    push(node, b, e);
    if (b > j || e < i) return;
    if (b >= i && e <= j) {
      lazy[node] = lazy[node] * ipw[1];
      push(node, b, e);
      return;
    }
    int mid = (b + e) >> 1, l = node <<
        1, r = l + 1;
    del(l, b, mid, i, j);
    del(r, mid + 1, e, i, j);
    tree[node] = tree[l] + tree[r];
  }
  T query(int node, int b, int e, int i,
      int j) {
    push(node, b, e);
    if (b > j || e < i) return {0, 0};
    if (b >= i && e <= j) return
        tree[node];
    int mid = (b + e) >> 1, l = node <<
        1, r = l + 1;
    T L = query(l, b, mid, i, j);
    T R = query(r, mid + 1, e, i, j);
    return L + R;
  }
  Hashing() {}
  Hashing(string _s) {
    s = _s;
    n = s.size();
    s = '.' + s;
    tree.resize(4 * n + 1);
    lazy.resize(4 * n + 1);
    build(1, 1, n);
  }
  void upd(int i, char c, int cur) {
    T x = pw[i] * c;
    if (cur == 1) i = st.query(1, 1, en,
        i).first;
    else i = st2.query(1, 1, en, i).first;
    upd(1, 1, n, i, x);
  }
  void del(int i, int cur) {
    int orgi = i;
    T x = pw[i] * 0ll;
    if (cur == 1) i = st.query(1, 1, en,
        i).first;
    else i = st2.query(1, 1, en, i).first;
    upd(1, 1, n, i, x);
    del(1, 1, n, i + 1, n);
    if (cur == 1) st.upd(1, 1, en, i, i,
        0);
    else st2.upd(1, 1, en, i, i, 0);
  }
  T get_hash(int l, int r, int cur) { // 1
      - indexed
    int ll = st.query(1, 1, en, l).first;
    int rr = st.query(1, 1, en, r).first;
    if (cur == 2) {
      ll = st2.query(1, 1, en, l).first;
      rr = st2.query(1, 1, en, r).first;
    }
    return query(1, 1, n, ll, rr) *
        ipw[l - 1];
  }
};
int32_t main() {
  prec(); // must include
  string s; cin >> s;
  int n = s.size();
  int q; cin >> q;
  string t = s;
  reverse(t.begin(), t.end());
  Hashing hs(s), hs2(t);
  en = n + q + 5;
  st.build(1, 1, en);
  st2.build(1, 1, en);
  while (q--) {
    char c; cin >> c;
    if (c == 'C') {
      int l, r; cin >> l >> r;
      int l2 = get(l, n);
      int r2 = get(r, n);
      if (hs.get_hash(l, r, 1) ==
          hs2.get_hash(r2, l2, 2)) cout
          << "Yes!\n";
      else cout << "No!\n";
    }
    else if (c == 'U') {
      int i; char x; cin >> i >> x;
      int i2 = get(i, n);
      hs.upd(i, x, 1);
      hs2.upd(i2, x, 2);
    }
    else {
      int i; cin >> i;
      int i2 = get(i, n);
      hs.del(i, 1);
      hs2.del(i2, 2);
      --n;
    }
  }
}
```

## 8.4 Hashing on Tree

```
// Given a tree, Check whether it is
    symmetrical or not. Problem - CF G.
    Symmetree
// The value for each node is it's
    subtree size and position is the
    level (ordered). But the order of
    childs doesn't matter (unordered)
const int N = 2e5 + 9;
vector<int> g[N];
vector<array<int, 3>> hassh[N]; // hash1,
    hash2, node
int n, sz[N];
const int MOD1 = 1e9 + 9, MOD2 = 1e9 + 21;
const int p1 = 1e5 + 19, p2 = 1e5 + 43;
void dfs2(int u, int p, int lvl) {
  array<int, 3> my_hash;
  my_hash[0] = 1ll * sz[u] *
      pw[lvl].first % MOD1;
  my_hash[1] = 1ll * sz[u] *
      pw[lvl].second % MOD2;
  my_hash[2] = u;
  bool leaf = true;
  for (auto v : g[u]) {
    if (v != p) {
      dfs2(v, u, lvl + 1);
      leaf = false;
    }
  }
  if (!leaf) {
    int sum1 = 1, sum2 = 1;
    for (auto here : hassh[u]) {
      auto [x, y, _] = here;
      sum1 = (sum1 * x) % MOD1;
      sum2 = (sum2 * y) % MOD2;
    }
    my_hash[0] = power(my_hash[0], sum1,
        MOD1);
    my_hash[1] = power(my_hash[1], sum2,
        MOD2);
  }
  hassh[p].push_back(my_hash);
}
bool ok(int u) {
  map<pair<int, int>, int> mp;
  for (auto [x, y, who] : hassh[u]) {
    mp[{x, y}]++;
  }
  int odd = 0;
  pair<int, int> val;
  for (auto [here, cnt] : mp) {
    odd += cnt & 1;
    if (cnt & 1) val = here;
  }
  if (odd == 0) return true;
  if (odd > 1) return false;
  int node;
  for (auto [x, y, who] : hassh[u]) {
    pair<int, int> here = {x, y};
    if (here == val) node = who;
  }
  return ok(node);
}
void solve() {
  cin >> n; clr(n);
  for (int i = 2; i <= n; i++) {
    int u, v; cin >> u >> v;
    g[u].push_back(v);
    g[v].push_back(u);
  }
  dfs(1, 0); // calc. subtree size
  dfs2(1, 0, 1);
  if (ok(0)) cout << "YES\n";
  else cout << "NO\n";
}
```

## 8.5 Compare 2 strings Lexicographically

```
// Time: O(logn)
string s;
Hashing hs;
// return 0 if both equal
// return 1 if first substring greater
// return -1 if second substring greater
// here lcp() provides the len of longest
    common prefix
int compare(int i, int j, int x, int y) {
  int common_prefix = lcp(i, j, x, y);
  int len1 = j - i + 1, len2 = y - x + 1;
  if (common_prefix == len1 and len1 ==
      len2) return 0;
  else if (common_prefix == len1) return
      -1;
  else if (common_prefix == len2) return
      1;
  else return (s[i + common_prefix - 1]
      < s[x + common_prefix - 1] ? -1 :
      1);
}
```

## 8.6 KMP

```
vector<int> build_lps(string &pat) {
  int n = pat.size();
  vector<int> lps(n, 0);
  for (int i = 1; i < n; i++) {
    int j = lps[i - 1];
    while (j > 0 and pat[i] != pat[j]) {
      j = lps[j - 1];
    }
    if (pat[i] == pat[j]) j++;
    lps[i] = j;
  }
  return lps;
}
int kmp(string &txt, string &pat) {
  string s = pat + '#' + txt;
  vector<int> lps = build_lps(s);
  int ans = 0;
  for (auto x : lps) {
    if (x == pat.size()) ans++;
  }
  return ans;
}
int kmp(string &txt, string &pat) {
  vector<int> lps = build_lps(pat);
  int n = txt.size(), m = pat.size();
  int ans = 0;
  int j = 0;
  for (int i = 0; i < n; i++) {
    while (j > 0 and txt[i] != pat[j]) {
      j = lps[j - 1];
    }
    if (txt[i] == pat[j]) j++;
    if (j == m) {
      ans++;
      j = lps[j - 1];
    }
  }
  return ans;
}
```

## 8.7 KMP Automata

```
// like DFA. if string is "abcdeabg",
    aut[7]['c'] = 3. Means 7th index e
    'c' bosaile LPS koto, aut[7]['g'] = 8
void compute_automaton(string s,
    vector<vector<int>>& aut) {
  s += '#';
  int n = s.size();
  vector<int> pi = build_lps(s);
  aut.assign(n, vector<int>(26));
  for (int i = 0; i < n; i++) {
    for (int c = 0; c < 26; c++) {
      if (i > 0 && 'a' + c != s[i])
        aut[i][c] = aut[pi[i - 1]][c];
      else
        aut[i][c] = i + ('a' + c == s[i]);
    }
  }
}
```

## 8.8 Prefix Occurance Count

```
// Count the number of occurances of each
    prefix
vector<int> ans(n + 1);
for (int i = 0; i < n; i++) ans[lps[i]]++;
for (int i = n - 1; i > 0; i--)
    ans[lps[i - 1]] += ans[i];
for (int i = 0; i <= n; i++) ans[i]++;
```

## 8.9 Number of palindormic substring in L to R using Wavelet Tree

```
// Problem - Kattis palindromes
ll f(int x) {
  return (1ll * x * (x + 1)) / 2;
}
ll f(int l, int r) {
  if (l > r) return 0;
  return f(r) - f(l - 1);
}
bool ok(int l, int r) {
  return hash_s.is_palindrome(l, r);
}
int32_t main() {
  cin >> s;
  n = s.size();
  hash_s = Hashing(s);
  for (int i = 1; i <= n; i++) {
    int l = 0, r = min(n - i, i - 1),
        cnt = 1;
    while (l <= r) {
      int mid = (l + r) >> 1;
      if (ok(i - mid, i + mid)) {
        cnt = mid;
        l = mid + 1;
      }
      else r = mid - 1;
    }
    pi1[i] = cnt + 1;
    pi1_left[i] = pi1[i] - i;
    pi1_right[i] = i + pi1[i];
  }
  for (int i = 2; i <= n; i++) {
    if (s[i - 1] == s[i - 2]) {
      int l = 0, r = min(n - i, i - 1),
          cnt = 2;
      while (l <= r) {
        int mid = (l + r) >> 1;
        if (ok(i - 1 - mid, i + mid)) {
          cnt = mid;
          l = mid + 1;
        }
        else r = mid - 1;
      }
      pi2[i] = cnt + 1;
    }
    else pi2[i] = 0;

    pi2_left[i] = pi2[i] - i;
    pi2_right[i] = i + pi2[i];
  }
  // wavelet trees (odd_len_left,
      odd_len_right, even_len_left,
      even_len_right)
  t1.init(pi1_left + 1, pi1_left + n +
      1, -N, N);
  t2.init(pi1_right + 1, pi1_right + n +
      1, -N, N);
  t3.init(pi2_left + 1, pi2_left + n +
      1, -N, N);
  t4.init(pi2_right + 1, pi2_right + n +
      1, -N, N);

  int q; cin >> q;
  while (q--) {
    int l, r; cin >> l >> r;
    // define k, find cnt > k and
        summation whose are <= k;
    int m = (l + r) / 2;
    int k = 1 - l;
    ll ans = f(l, m);
    ans += t1.sum(l, m, k);
    int cnt = t1.GT(l, m, k);
    ans += 1ll * k * cnt;
    k = 1 + r;
    ans += -f(m + 1, r);
    ans += t2.sum(m + 1, r, k);
    cnt = t2.GT(m + 1, r, k);
    ans += 1ll * k * cnt;
    if (l + 1 <= m) { // a bit different
        than others
      k = -l;
      ans += f(l + 1, m);
      ans += t3.sum(l + 1, m, k);
      cnt = t3.GT(l + 1, m, k);
      ans += 1ll * k * cnt;
    }
    k = 1 + r;
    ans += -f(m + 1, r);
    ans += t4.sum(m + 1, r, k);
    cnt = t4.GT(m + 1, r, k);
    ans += 1ll * k * cnt;
    cout << ans << '\n';
  }
}
```

It is easier to explain by considering only palindromes centered at indicies (so, odd length), the idea is the same anyway. For each index $i$, $r_i$ will be the longest radius of a palindrome centered there (in other words, the amount of palindromes centered at index $i$). Directly from manacher, this takes $\mathcal{O}(n)$ to calculate.

For a query $[l, r]$, we first compute $m = \frac{l+r}{2}$. Now we want to calculate

$$\sum_{i=l}^{m}\min(i-l+1,r_i)+\sum_{i=m+1}^{r}\min(r-i+1,r_i)$$

$$\sum_{i=l}^{m}\min(i-l+1,r_i)=\sum_{i=l}^{m} i+\min(1-l,r_i-i).$$

The sum over $i$ can be found in constant time. As for the other term, if we create some array $r'_i = r_i - i$ during the preprocessing, then the queries are asking for some over range of $\min(C, r'_i)$ where $C$ is constant. You can solve this in $\mathcal{O}(\log n)$ per query using wavelet tree.

### 8.10 Trie

```
const int N = 10; // change here
const char base_char = '0';  // change
    here
struct TrieNode {
  int cnt;
  TrieNode * nxt[N];
  TrieNode() {
    cnt = 0;
    for (int i = 0; i < N; i++) nxt[i] =
        NULL;
  }
} *root;
void insert(const string &s) {
  TrieNode *cur = root;
  int n = (int)s.size();
  for (int i = 0; i < n; i++) {
    int idx = s[i] - base_char;
    if (cur -> nxt[idx] == NULL) cur ->
        nxt[idx] = new TrieNode();
    cur = cur -> nxt[idx];
    cur -> cnt++;
  }
}
void rem(TrieNode *cur, string &s, int
    pos) { // free :: De Alloactes Memory
  if (pos == s.size()) return;
  int idx = s[pos] - base_char;
  rem(cur -> nxt[idx], s, pos + 1);
  cur -> nxt[idx] -> cnt--;
  if (cur -> nxt[idx] -> cnt == 0) {
    free(cur -> nxt[idx]);
    cur -> nxt[idx] = NULL;
  }
}
int query(const string &s) { // "s" is a
    prefix of some element or not
  int n = (int)s.size();
  TrieNode *cur = root;
  for (int i = 0; i < n; i++) {
    int idx = s[i] - base_char;
    if (cur -> nxt[idx] == NULL) return 0;
    cur = cur -> nxt[idx];
  }
  return cur -> cnt;
}
void del(TrieNode *cur) {
  for (int i = 0; i < N; i++) if (cur ->
      nxt[i]) del(cur -> nxt[i]);
  delete(cur);
}
int32_t main() {
  root = new TrieNode(); // init new trie
  del(root); // clear trie
}
```

## 9 Misc.

### 9.1 Matrix Expo

```
struct Mat {
  int n, m;
  vector<vector<int>> a;
  Mat() { }
  Mat(int _n, int _m) {n = _n; m = _m;
      a.assign(n, vector<int>(m, 0)); }
  Mat(vector< vector<int> > v) { n =
      v.size(); m = n ? v[0].size() : 0;
      a = v; }
  inline void make_unit() {
    assert(n == m);
    for (int i = 0; i < n; i++)  {
      for (int j = 0; j < n; j++)
          a[i][j] = i == j;
    }
  }
  inline Mat operator + (const Mat &b) {
    assert(n == b.n && m == b.m);
    Mat ans = Mat(n, m);
    for(int i = 0; i < n; i++) {
      for(int j = 0; j < m; j++) {
        ans.a[i][j] = (a[i][j] +
            b.a[i][j]) % mod;
      }
    }
    return ans;
  }
  inline Mat operator - (const Mat &b) {
    assert(n == b.n && m == b.m);
    Mat ans = Mat(n, m);
    for(int i = 0; i < n; i++) {
      for(int j = 0; j < m; j++) {
        ans.a[i][j] = (a[i][j] -
            b.a[i][j] + mod) % mod;
      }
    }
    return ans;
  }
  inline Mat operator * (const Mat &b) {
    assert(m == b.n);
    Mat ans = Mat(n, b.m);
    for(int i = 0; i < n; i++) {
      for(int j = 0; j < b.m; j++) {
        for(int k = 0; k < m; k++) {
          ans.a[i][j] = (ans.a[i][j] +
              1LL * a[i][k] * b.a[k][j]
              % mod) % mod;
        }
      }
    }
    return ans;
  }
  inline Mat pow(long long k) {
    assert(n == m);
    Mat ans(n, n), t = a; ans.make_unit();
    while (k) {
      if (k & 1) ans = ans * t;
      t = t * t;
      k >>= 1;
    }
    return ans;
  }
  inline Mat& operator += (const Mat& b)
      { return *this = (*this) + b; }
  inline Mat& operator -= (const Mat& b)
      { return *this = (*this) - b; }
  inline Mat& operator *= (const Mat& b)
      { return *this = (*this) * b; }
  inline bool operator == (const Mat& b)
      { return a == b.a; }
  inline bool operator != (const Mat& b)
      { return a != b.a; }
};
int32_t main() {
  int n; long long k; cin >> n >> k;
  Mat a(n, n); // then assign value
  Mat ans = a.pow(k);
}
```

### 9.2 2D Prefix Sum

```
const int N = 1e3 + 9;
int a[N][N];
ll pref[N][N];
void query(int x1, int y1, int x2, int
    y2) {
  return pref[x2][y2] - pref[x1 -
      1][y2] - pref[x2][y1 - 1] +
      pref[x1 - 1][y1 - 1]
}
int32_t main() {
  int n, m; cin >> n >> m;
  for (int i = 1; i <= n; i++) {
    for (int j = 1; j <= m; j++) {
      pref[i][j] = pref[i - 1][j] +
          pref[i][j - 1] - pref[i - 1][j
          - 1] + a[i][j];
    }
  }
  int q; cin >> q;
  while (q--) {
    int x1, y1, x2, y2; cin >> x1 >> y1
        >> x2 >> y2;
    cout << query(x1, y1, x2, y2) << '\n';
  }
}
```

### 9.3 2D Static Range Update

```
  // Add x on a rectangle q times,
    Finally print the array
const int N = 1e3 + 9;
int a[N][N];
ll d[N][N]; // difference array
int32_t main() {
  int n, m; cin >> n >> m;
  int q; cin >> q;
  while (q--) {
    int x1, y1, x2, y2, x; cin >> x1 >>
        y1 >> x2 >> y2 >> x; // add x on
        this rectangle
    d[x1][y1] += x;
    d[x1][y2 + 1] -= x;
    d[x2 + 1][y1] -= x;
    d[x2 + 1][y2 + 1] += x;
  }
  for (int i = 1; i <= n; i++) {
    for (int j = 1; j <= m; j++) {
      d[i][j] += d[i - 1][j] + d[i][j -
          1] - d[i - 1][j - 1];
    }
  }
  // new updated array
  for (int i = 1; i <= n; i++) {
    for (int j = 1; j <= m; j++) {
      cout << d[i][j] + a[i][j] << ' ';
    }
    cout << '\n';
  }
}
```

### 9.4 CHECK SET CLR

```
bool CHECK(int N, int pos) { return
    (bool)(N & (1ll << pos)); }
void SET(int &N, int pos) { (N |= (1ll
    << pos)); }
void CLR(int &N, int pos) { (N &= ~(1ll
    << pos)); }
```

## 9.5 Graph Directions

```
int dx[] = {+0, +0, +1, -1, -1, +1, -1,
    +1};
int dy[] = {-1, +1, +0, +0, +1, +1, -1,
    -1};
```

## 9.6 Custom Hash

```
struct custom_hash {
  static uint64_t splitmix64(uint64_t x) {
    x += 0x9e3779b97f4a7c15;
    x = (x ^ (x >> 30)) *
        0xbf58476d1ce4e5b9;
    x = (x ^ (x >> 27)) *
        0x94d049bb133111eb;
    return x ^ (x >> 31);
  }
  size_t operator()(uint64_t x) const {
    static const uint64_t FIXED_RANDOM =
        chrono::steady_clock::now().time
        _since_epoch().count();
    return splitmix64(x + FIXED_RANDOM);
  }
};
```

## 9.7 Coordinate Compression

```
int32_t main() {
  vector<int> a({100, 9, 10, 10, 9});
  vector<int> v = a;
  sort(v.begin(), v.end());
  v.resize(unique(v.begin(), v.end()) -
      v.begin());
  for (int i = 0; i < a.size(); i++) {
    a[i] = lower_bound(v.begin(),
        v.end(), a[i]) - v.begin() + 1;
  }
}
```

## 9.8 Submask Generator

```
int mask = 0; // any value
for (int submask = mask; submask;
    submask = (submask - 1) & mask) {}
```

## 9.9 Custom Comparators

```
bool cmp(pair<int, int> a, pair<int,
    int> b) { // for vector, arr,..
  if (a.first != b.first) return a.first
      > b.first;
  return a.second < b.second; // must
      return false for equal elements
}
struct cmp { // for set, map, pq,..
  bool operator()(const int& a, const
      int&b) const {
    return a > b;
  }
};
```

## 9.10 MOD Operations

```
inline ll modAdd(ll a, ll b, ll MOD) {ll
    res = a + b; res += ((res >> 63) &
    MOD); if (res >= MOD) res -= MOD;
    return res;}
inline ll modSub(ll a, ll b, ll MOD) {ll
    res = a - b; res += ((res >> 63) &
    MOD); if (res >= MOD) res -= MOD;
    return res;}
inline ll modMul(ll a, ll b, ll MOD) {a
    %= MOD; b %= MOD; a += ((a >> 63) &
    MOD); b += ((b >> 63) & MOD); return
    (a * b) % MOD; }
inline ll modInverse(ll a, ll MOD)
    {return modPow(a, MOD - 2, MOD);}
inline ll modDiv(ll a, ll b, ll MOD)
    {return modMul(a, modInverse(b,
    MOD), MOD);}
inline ll modMulBigMod(ll a, ll b, ll
    MOD) {a %= MOD; b %= MOD; a += ((a
    >> 63) & MOD); b += ((b >> 63) &
    MOD); return (ll)((__int128)a * b %
    MOD);}
```

## 9.11 Mex with Array Updates

```
int32_t main() {
  int n, q; cin >> n >> q;
  set<int> missing_numbers;
  for (int i = 0; i <= n + 200; i++) {
    missing_numbers.insert(i);
  }
  int a[n + 1];
  map<int, int> freq;
  for (int i = 1; i <= n; i++) {
    cin >> a[i];
    freq[a[i]]++;
    missing_numbers.erase(a[i]);
  }
  while (q--) {
     int i, x; cin >> i >> x;  // Set
        a[i] = x
    int y = a[i]; a[i] = x;
    missing_numbers.erase(x);
    freq[y]--;
    freq[x]++;
    if (freq[y] == 0) {
      freq.erase(y);
      missing_numbers.insert(y);
    }
    cout << *missing_numbers.begin() <<
        '\n'; // mex after upd
  }
}
```