# PRESENTATION ON
# LU DECOMPOSITION

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**SE 341**

**Batch: 33 A**

**Submission Date: 30/11/2023**

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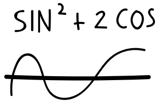
# LU DECOMPOSITION




LU decomposition, or LU factorization, is a method used in linear algebra to decompose a square matrix into the product of a lower triangular matrix (L) and an upper triangular matrix (U). This decomposition is particularly useful for solving systems of linear equations and in numerical analysis.

LU decomposition, short for Lower-Upper decomposition, is a factorization of a square matrix into the product of a lower triangular matrix (L) and an upper triangular matrix (U). Mathematically, for a given square matrix $A$, the LU decomposition can be expressed as: $A = LU$

$$\left.\begin{aligned} x+2y+3z &= 9 \\ 4x+5y+6z &= 24 \\ 3x+y-2z &= 4 \end{aligned}\right\} \text{— (i)}$$

$$AX = B \quad \text{— (ii)}$$

Here,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 1 & -1 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}. \quad B = \begin{bmatrix} 9 \\ 24 \\ 4 \end{bmatrix}$$

$$LU = A \quad \text{— (iii)}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$LUX = B \quad \text{——(iv)}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ l_{21} \cdot U_{11} & l_{21} U_{12} + U_{22} & l_{21} U_{13} + U_{23} \\ l_{31} \; U_{11} & l_{31} U_{12} + l_{32} \cdot U_{22} & l_{31} U_{13} + l_{32} U_{23} + U_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 1 & -2 \end{bmatrix}$$

$U_{11} = 1$

$l_{21} \cdot U_{11} = 4$

$\Rightarrow l_{21} = 4$

$U_{12} = 2$

$l_{21} \cdot U_{12} + U_{22} = 5$

$\Rightarrow 4 \times 2 + U_{22} = 5$

$\Rightarrow U_{22} = -3$

$U_{13} = 3$

$l_{21} \cdot U_{13} + U_{23} = 6$

$\Rightarrow 4 \times 3 + U_{23} = 6$

$\Rightarrow U_{23} = -6$

$l_{31} \cdot U_{11} = 3$

$\Rightarrow l_{31} = 3$

$l_{31} \cdot U_{12} + l_{32} l_{22} = 1$

$\Rightarrow 3 \times 2 + l_{32}(-3) = 1$

$\Rightarrow 6 - 3 \cdot l_{32} = 1$

$l_{32} = +5/3$

$l_{31} \cdot U_{13} + l_{32} \cdot U_{23} + U_{33} = -2$

$\Rightarrow 3 \times 3 + 5/3 \cdot (-6) + U_{33} = -2$

$\Rightarrow 9 + 5/3(-6) + U_{33} = -2$

$\Rightarrow -1 + U_{33} = -2$

$\Rightarrow U_{33} = -1$

$Ux = y$ —— (V)

where, $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

$LUx = B$

$Ly = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 24 \\ 4 \end{bmatrix}$$

$\Rightarrow y_1 = 9$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & 5/3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 24 \\ 4 \end{bmatrix}$$

$\Rightarrow y_1 = 9$

$4y_1 + y_2 = 24$

$\Rightarrow 4\times 9 + y_2 = 24$

$\Rightarrow y_2 = -12$

$3y_1 + \frac{5}{3}\cdot y_2 + y_3 = 4$

$\Rightarrow 3\times 9 + \frac{5}{3} \times -12 + y_3 = 4$

$\Rightarrow 27 - 20 + y_3 = 4$

$\Rightarrow y_3 = -3$

Now,

$Ux = y$

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -12 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 9 \\ -12 \\ -3 \end{bmatrix}$$

$$= x + 2y + 3z = 9$$

$$\Rightarrow x = 9 - 2y - 3z$$

$$= 9 - 2 \times (-2) - 3 \times 3$$

$$= 9 + 4 - 9$$

$$\Rightarrow 4$$

$$-3y - 6z = -12$$

$$\Rightarrow -3y \; 18 = -12$$

$$\Rightarrow -3y = 6$$

$$\Rightarrow y = -2$$

$$-z = -3$$

$$\therefore z = 3$$

# USAGE OF LU DECOMPOSITION

**Matrix decomposition methods, such as LU decomposition, are used for several reasons in linear algebra and numerical computations:**

- **Solving Systems of Linear Equations**
- **Matrix Inversion**
- **Numerical Stability**
- **Efficiency**
- **Eigenvalue and Singular Value Decomposition**
- **Parallel Computing**
- **Conditioning Analysis**
- **Sparse Matrix Operations**

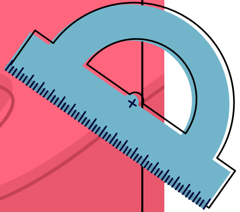
$C = \sin^2\left(\frac{2}{3}\right)$

$= \sin^3 \times 0{,}747$

$= 7{,}38$

$A^3 C^2 4^B = 9^3 + 5^8 + 7^C$

$5^C = 54718{,}32.$



# THANK YOU