

Variations of the Turing Machine

A Standard Turing Machine

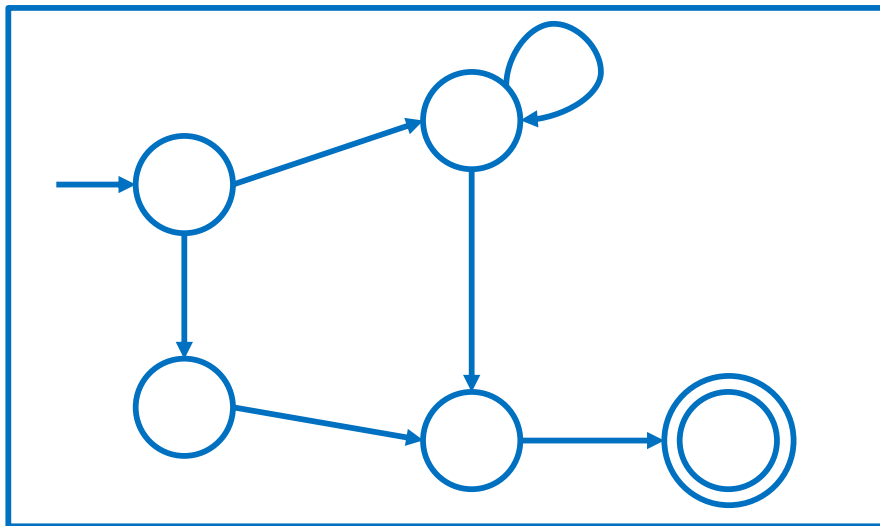
Tape



Read-Write head (Left or Right)



Control Unit



Deterministic

Variations of the Standard Model

- Turing machines with:
 - Stay-Option
 - Semi-Infinite Tape
 - Off-Line
 - Multitape
 - Multidimensional
 - Nondeterministic

Variations of the Standard Model

- The variations form different **Turing Machine Classes**

BUT

- Each class has the **same power** with the **Standard Model**

Variations of the Standard Model

- Same power of two classes means:

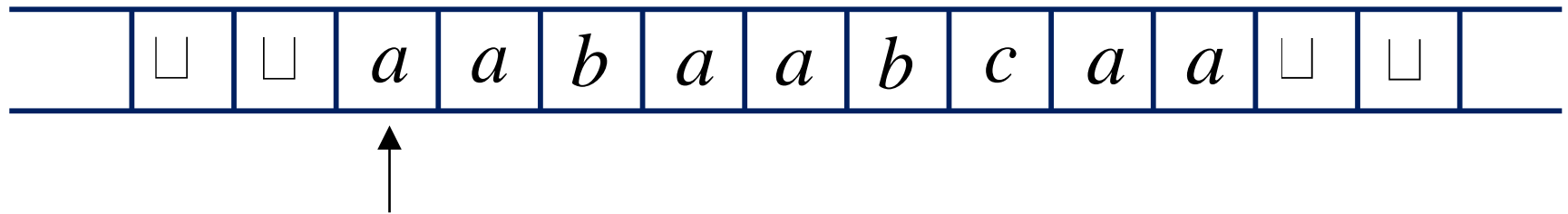
For any machine M_1 of first class, there is a machine M_2 such that

$$L(M_1) = L(M_2)$$

and vice versa.

Stay-Option

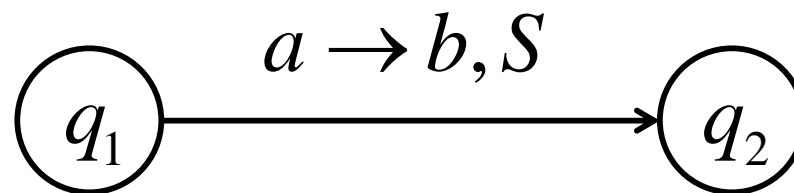
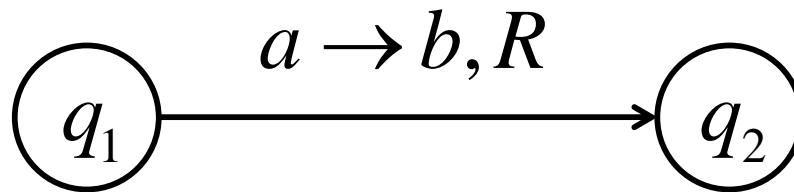
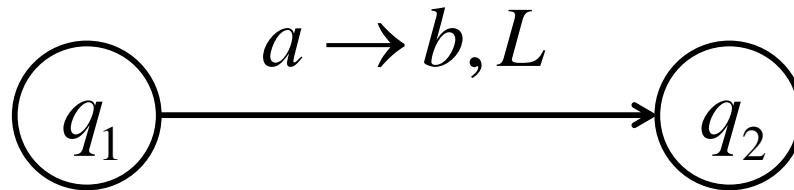
The head can stay in the same position



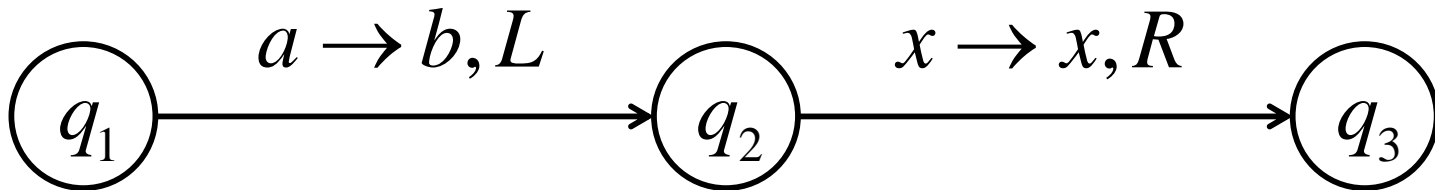
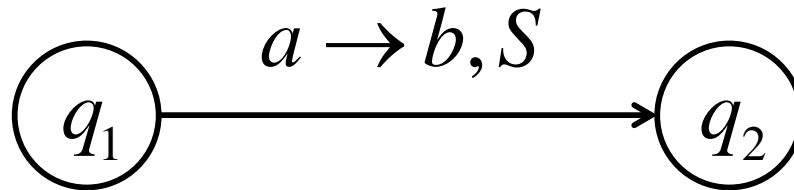
Left, Right, Stay

L,R,S: moves

Stay-Option

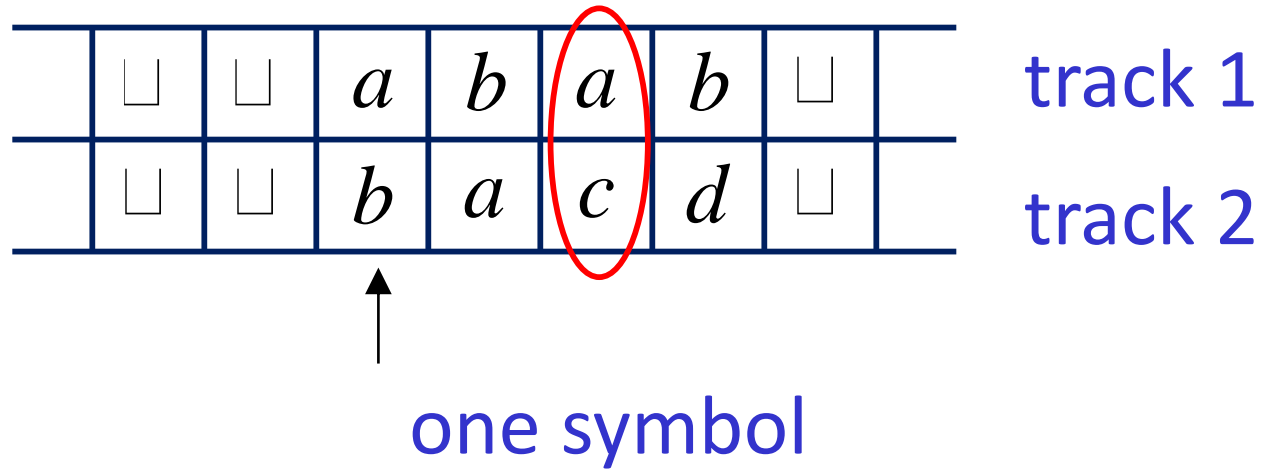


Stay-Option

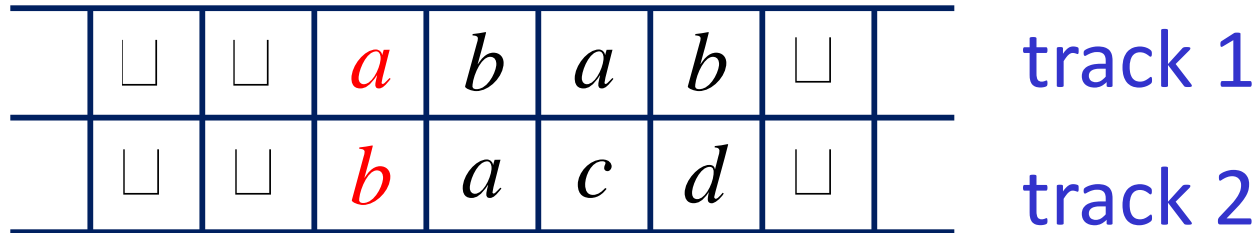


For every symbol $x \in \Sigma$

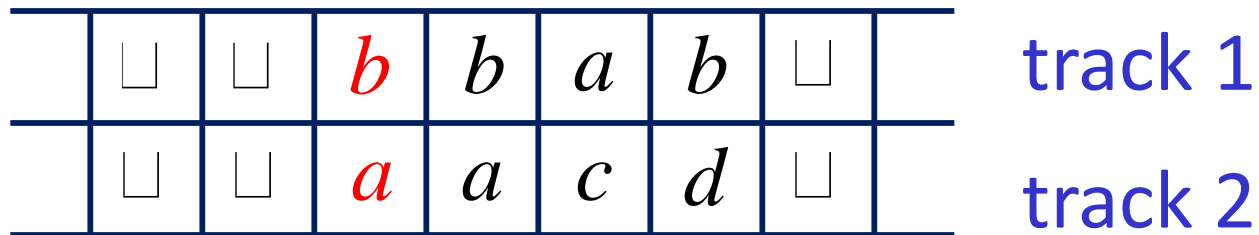
Multiple Track Tape



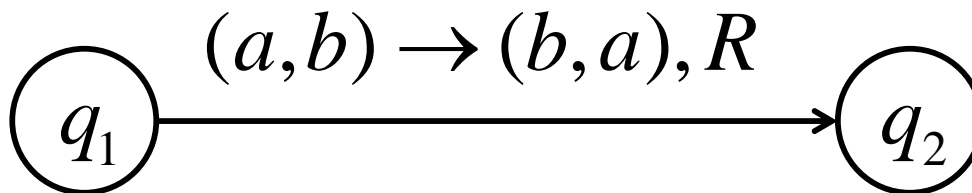
Multiple Track Tape



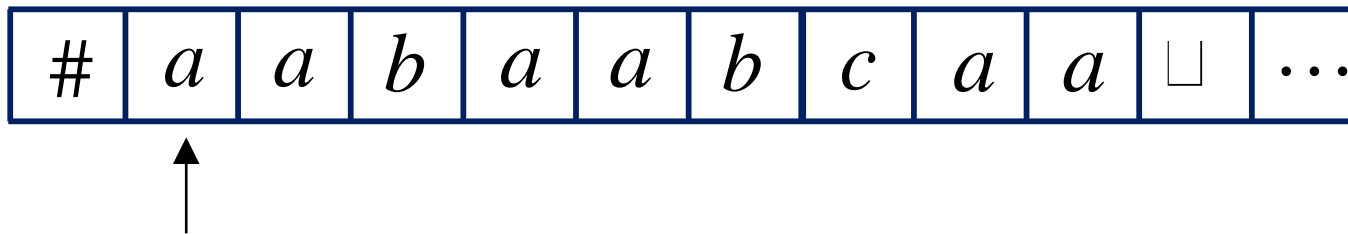
↑ q_1



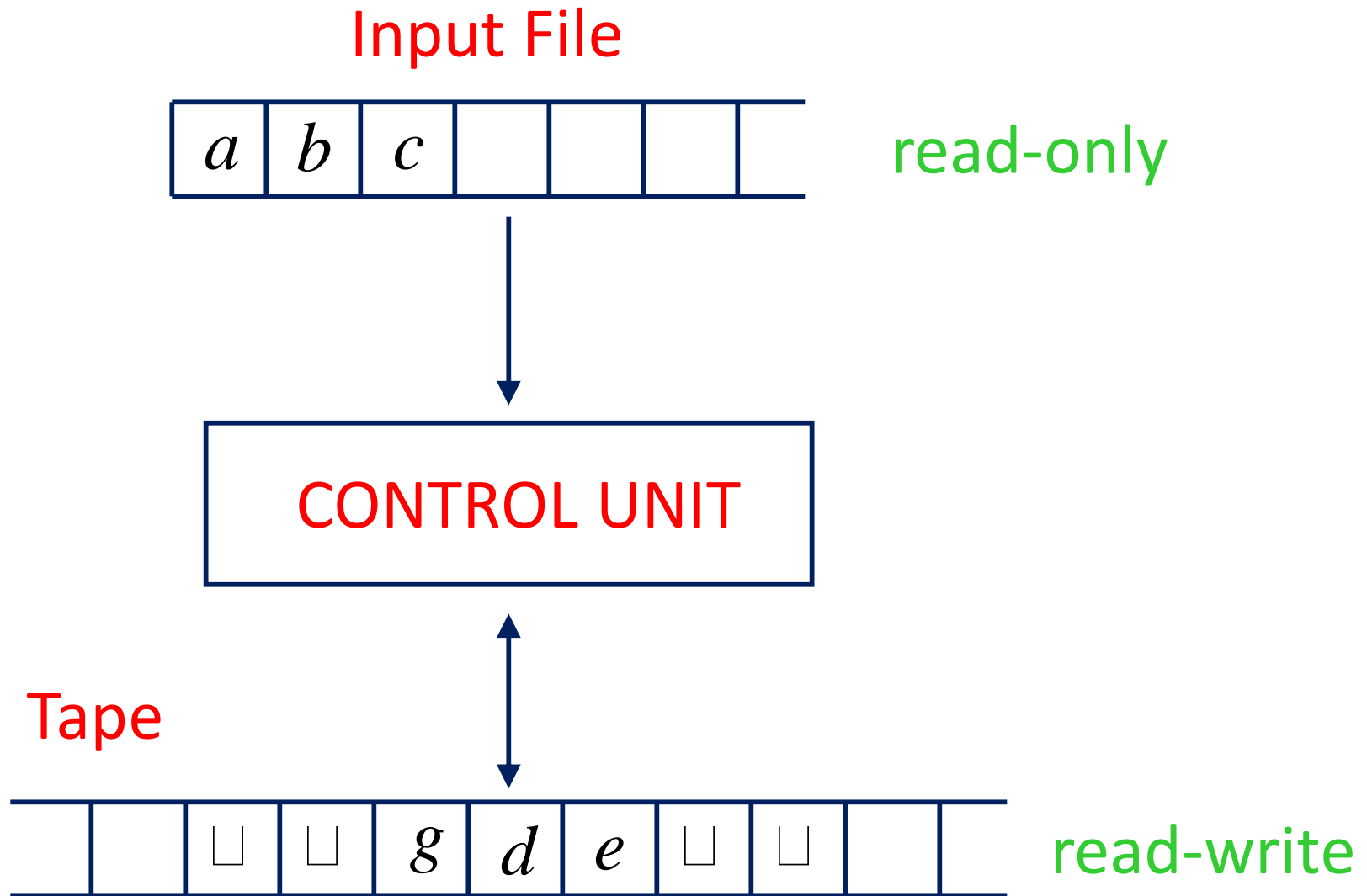
↑ q_2



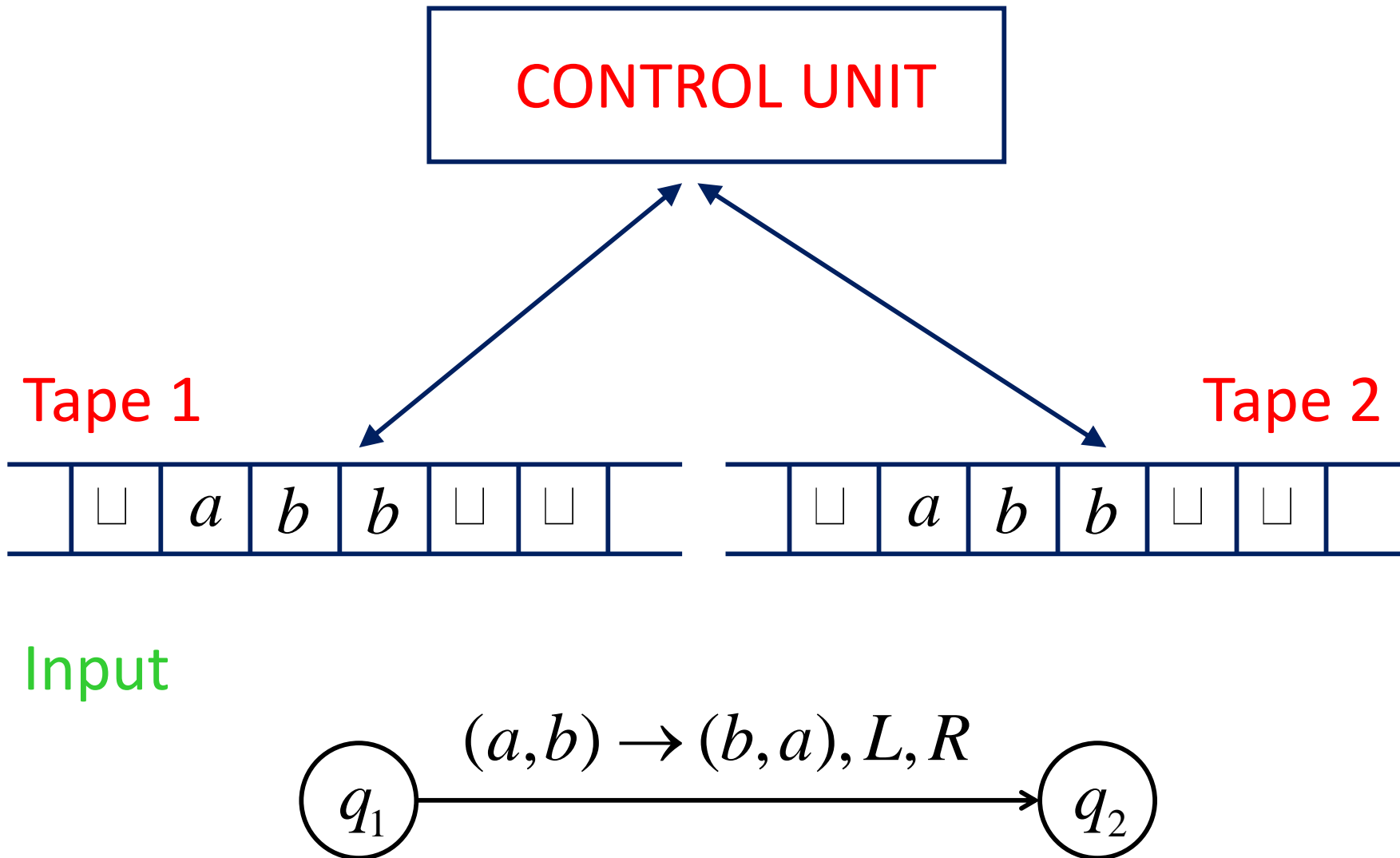
Semi-Infinite Tape



The Off-Line Machine



Multitape Turing Machines



Multitape Turing Machines

Same power doesn't imply same speed:

$$L = \{a^n b^n\}$$

Acceptance Time

Standard machine

$$n^2$$

Two-tape machine

$$n$$

Multitape Turing Machines

Standard machine:

Go back and forth n^2 times

Two-tape machine:

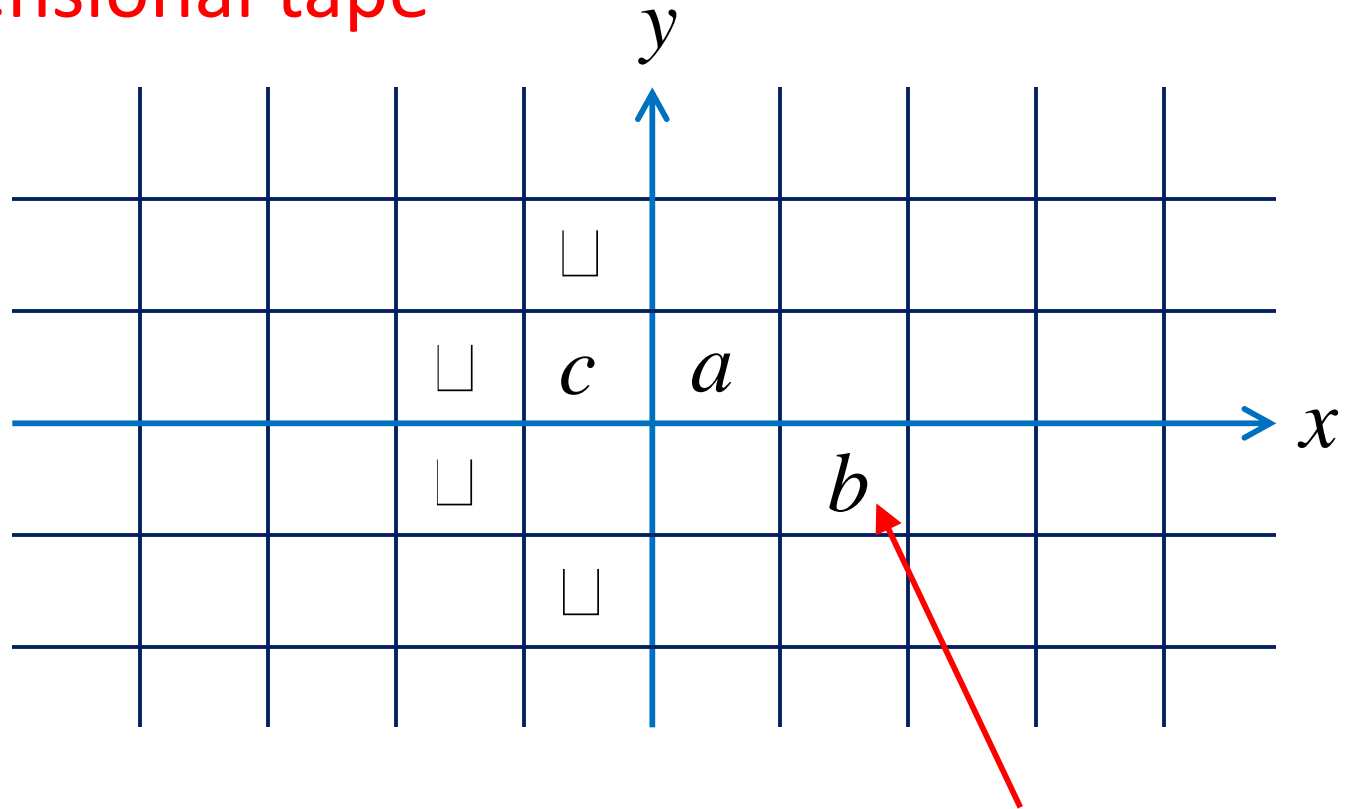
Copy b^n to tape 2 (n steps)

Leave a^n on tape 1 (n steps)

Compare tape 1 and tape 2 (n steps)

Multi-Dimensional Turing Machines

Two-dimensional tape



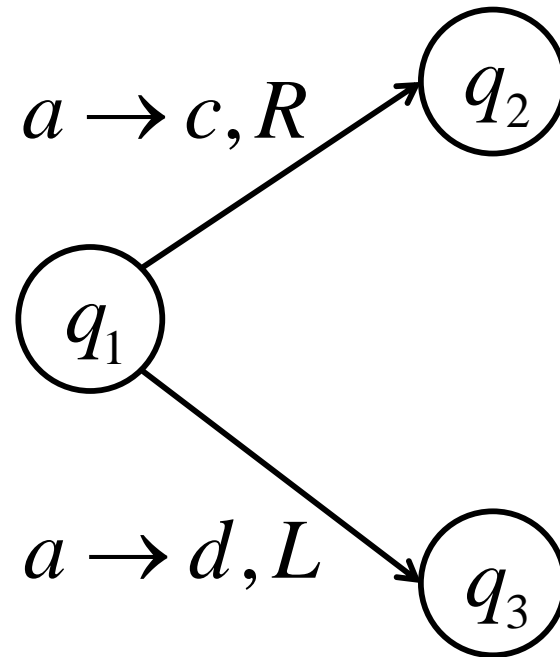
MOVES: L,R,U,D

U: up D: down

HEAD

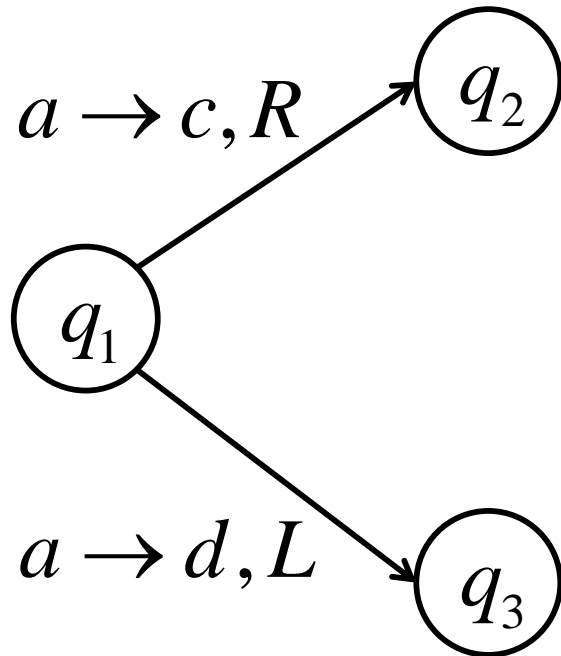
Position: (+2,-1)

Non-deterministic Turing Machines

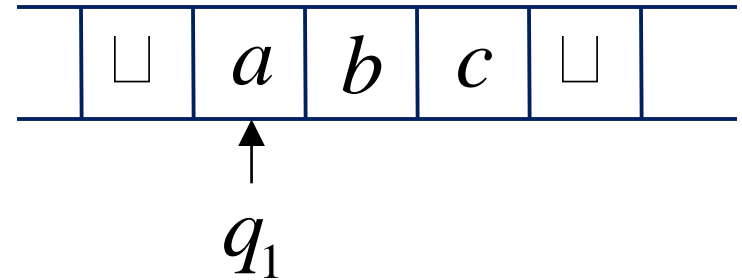


Non-deterministic Choice

Non-deterministic Turing Machines



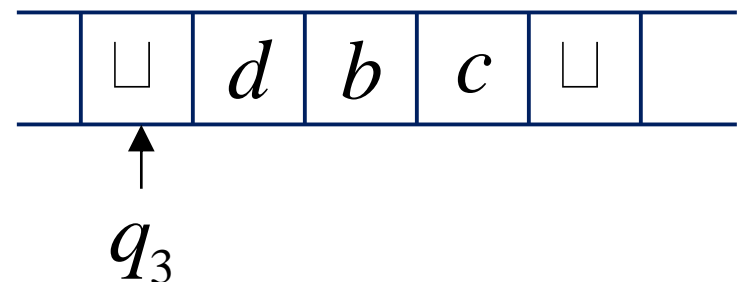
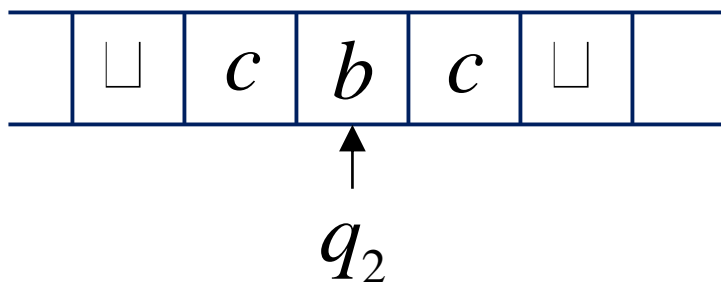
Time 0



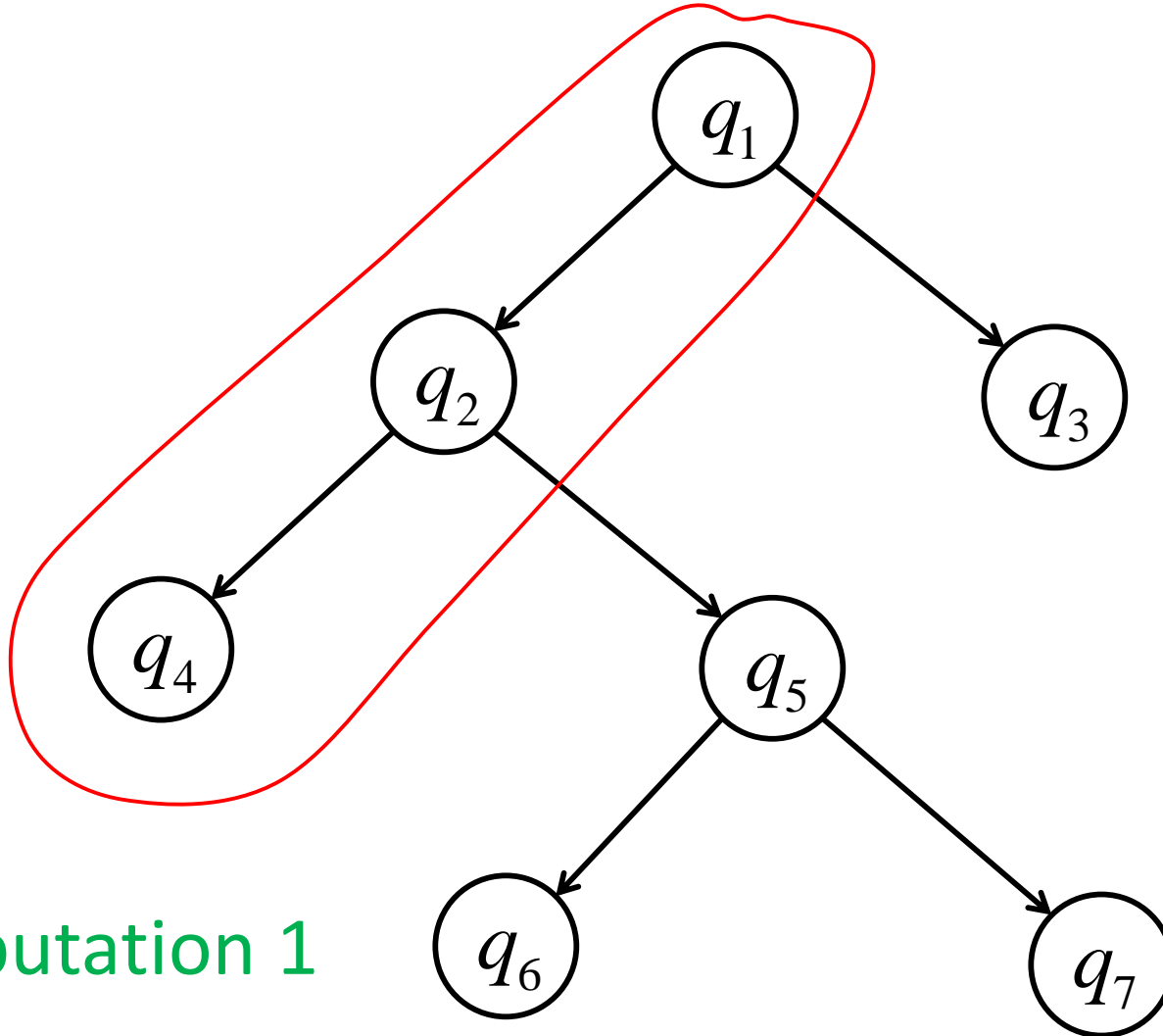
Choice 1

Time 1

Choice 2

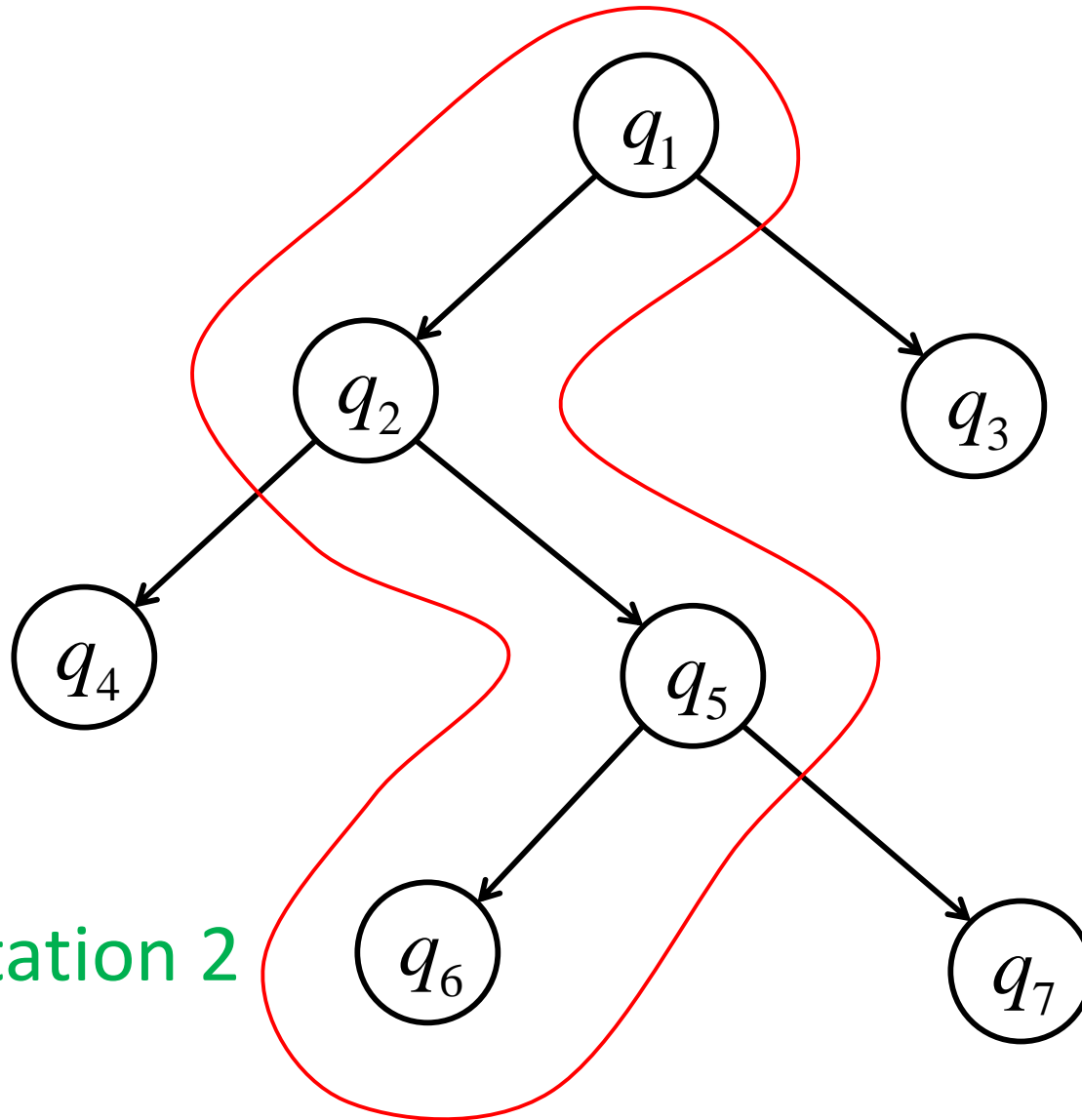


Non-deterministic Turing Machines



Computation 1

Non-deterministic Turing Machines



Computation 2

Non-deterministic Turing Machines

Input string w is accepted if
this a possible computation

$$q_0 w \succ^* x q_f y$$

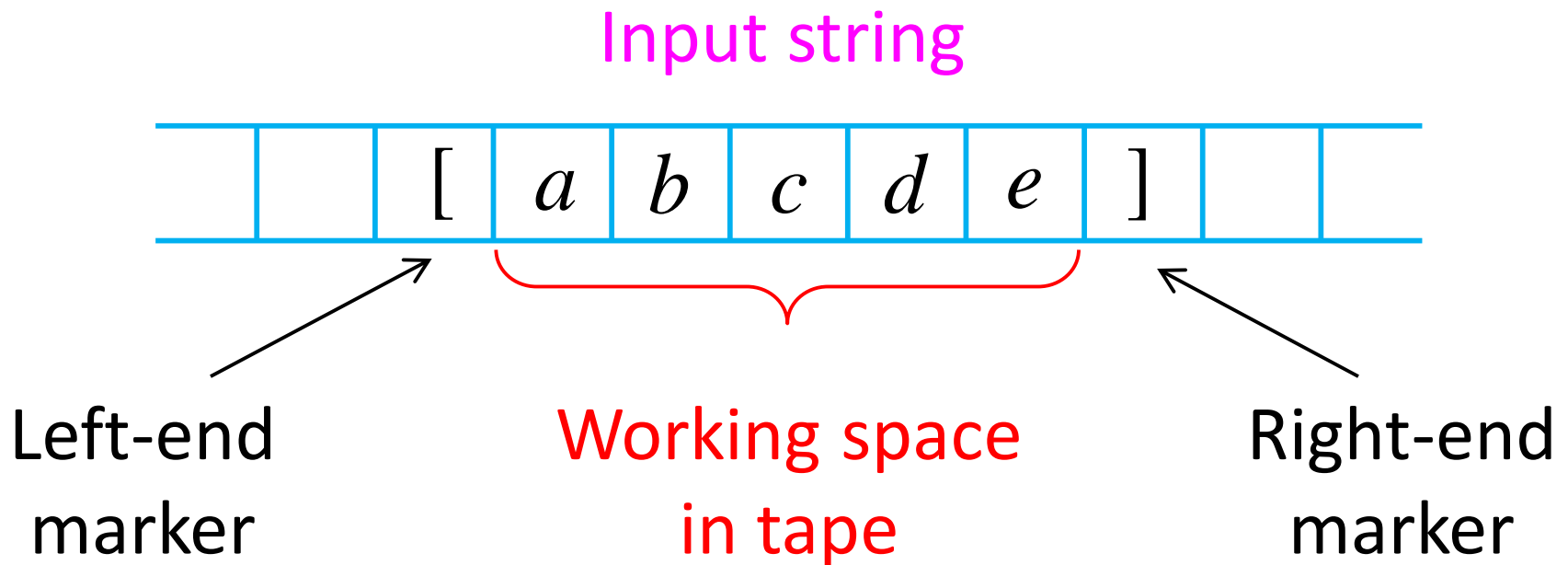
Initial configuration

Final Configuration

Final state

Linear Bounded Automata

- **Linear Bounded Automata (LBAs)** are the same as Turing Machines with **one difference**: the input string tape space is the only tape space allowed to use.



Linear Bounded Automata

- Example languages accepted by LBAs:

$$L = \{a^n b^n c^n\} \qquad L = \{a^{n!}\}$$

- LBA have less power than Turing Machines