Properties of Regular Languages

* اَللّٰهُمَّ إِنَّا نَسْأَلُكَ لِسَانًا رَطِبًا بِذِكْرِكَ وَقَلْبًا مَفْعُمًا بِشُكْرِكَ وَبَدَنًا هَيِّنًا لَيِّنًا بِطَاعَتِكَ اَللّٰهُمَّ إِنَّا نَسْأَلُكَ إِيْمَانًا كَامِلاً وَنَسْأَلُكَ قَلْبًا خَاشِعًا وَنَسْأَلُكَ عِلْمًا نَافِعًا وَنَسْأَلُكَ يَقِيْنًا صَادِقًا وَنَسْأَلُكَ دِيْنًا قَيِّمًا وَنَسْأَلُكَ الْعَافِيَةَ مِنْ كُلِّ بَلِيَّةٍ وَنَسْأَلُكَ تَمَامَ الْغِنَى عَنِ النَّاسِ وَهَبْ لَنَا حَقِيْقَةَ الْإِيْمَانِ بِكَ حَتَّى لَا نَخَافَ وَلَا نَرْجُوْ غَيْرَكَ وَلَا نَعْبُدَ شَيْئًا سِوَاكَ وَاجْعَلْ يَدَكَ مَبْسُوْطَةً عَلَيْنَا وَعَلَى أَهْلِيْنَا وَأَوْلَادِنَا وَمَنْ مَعَنَا بِرَحْمَتِكَ وَلَا تَكِلْنَا إِلَى أَنْفُسِنَا طَرْفَةَ عَيْنٍ وَلَا أَقَلَّ مِنْ ذٰلِكَ يَا نِعْمَ الْمُجِيْبُ.

Ya Allah kurniakanlah kami lisan yang lembut basah mengingati dan menyebut (nama)-Mu, hati yang penuh segar mensyukuri (nikmat)-Mu, serta badan yang ringan menyempurnakan ketaatan kepada (perintah)Mu. Ya Allah, kurniakanlah kami iman yang sempurna,

hati yang khusyuk, ilmu yang berguna, keyakinan yang benar-benar mantap. (Ya Allah) kurniakanlah kami (din) cara hidup yang jitu dan unggul, selamat dari segala mara bahaya dan petaka. Kami mohon (Ya Allah) kecukupan yang tidak sampai kami tepaksa meminta jasa orang lain. Berikanlah kami (Ya Allah) iman yang sebenarnya sehingga kami tidak lagi gentar atau mengharap orang lain selain dari Engkau sendiri. Kembangkanlah lembayung rahmatMu kepada kami, keluarga dan anak-anak kami serta sesiapa sahaja yang bersama-sama kami. Jangan (Ya Allah) Engkau biarkan nasib kami ditentukan oleh diri kami sendiri; walaupun kadar sekelip mata atau kadar masa yang lebih pendek dari itu. Wahai Tuhan yang paling mudah dan cepat memperkenankan pinta (perkenankanlah).

Properties of Regular Languages

- Regular languages are closed under:

  Union

  Concatenation

  Star operation

  Reverse

Properties of Regular Languages

- Namely, for regular languages L_1 and L_2:

Union	$L_1 \cup L_2$	Regular Languages
Concatenation	$L_1 L_2$	
Star operation	L_1^*	
Reverse	L_1^R	

Properties of Regular Languages

- Regular languages are closed under:

Complement

Intersection

Properties of Regular Languages

- Namely, for regular languages L_1 and L_2:

$$\left.\begin{array}{ll} \text{Complement} & \overline{L_1} \\ \\ \\ \text{Intersection} & L_1 \cap L_2 \end{array}\right\} \text{Regular Languages}$$

Complement

- Theorem:

  For regular language L the complement $\overline{L}$ is regular.

- Proof:

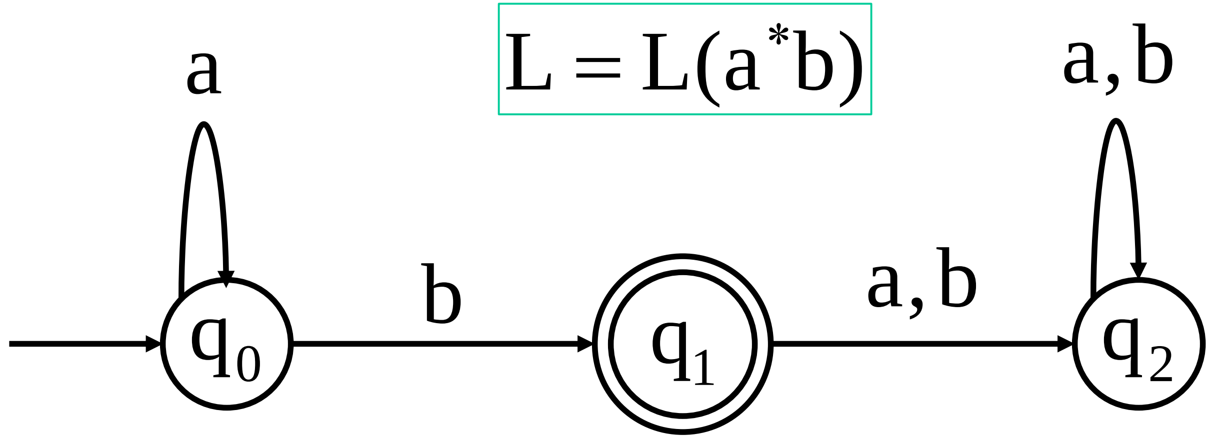
  Take DFA that accepts L and make

  - non-final states final
  - final states non-final

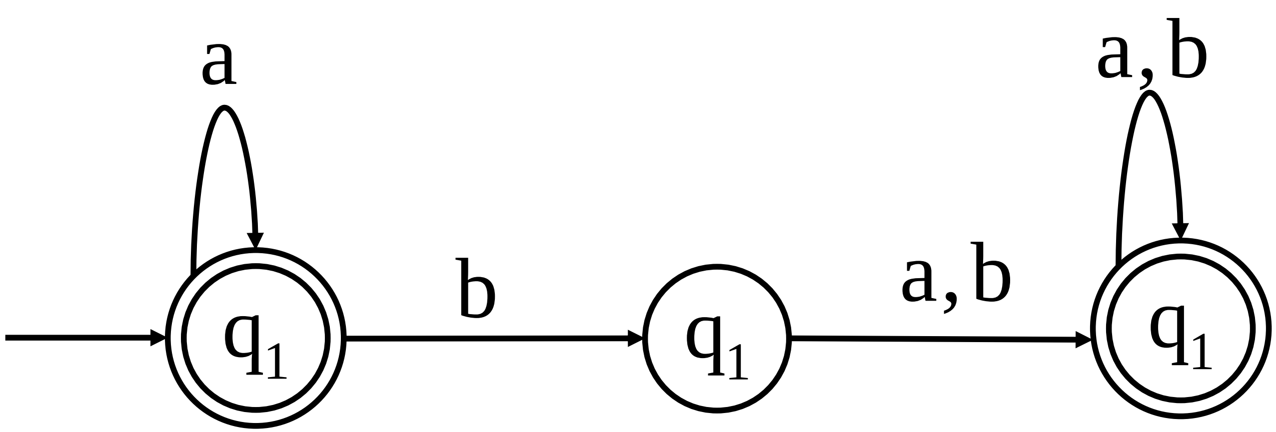
  Resulting DFA accepts $\overline{L}$

Example

$$\boxed{L = L(a^* b)}$$



$$\boxed{\overline{L} = L(a^* + a^* b(a+b)(a+b)^*)}$$



Intersection

- Theorem:

  For regular languages L_1 and L_2 the intersection $L_1 \cap L_2$ is regular.

- Proof:

  Apply DeMorgan's Law:

$$\boxed{L_1 \cap L_2 = \overline{\overline{L_1} \cap \overline{L_2}}}$$

Intersection

L_1, L_2 regular

$\overline{L_1}, \overline{L_2}$ regular

$\overline{L_1} \cup \overline{L_2}$ regular

$\overline{\overline{L_1} \cup \overline{L_2}}$ regular

$L_1 \cap L_2$ regular

Reverse of a Regular Language

Theorem

- The reverse L^R of a regular language is a regular language L.

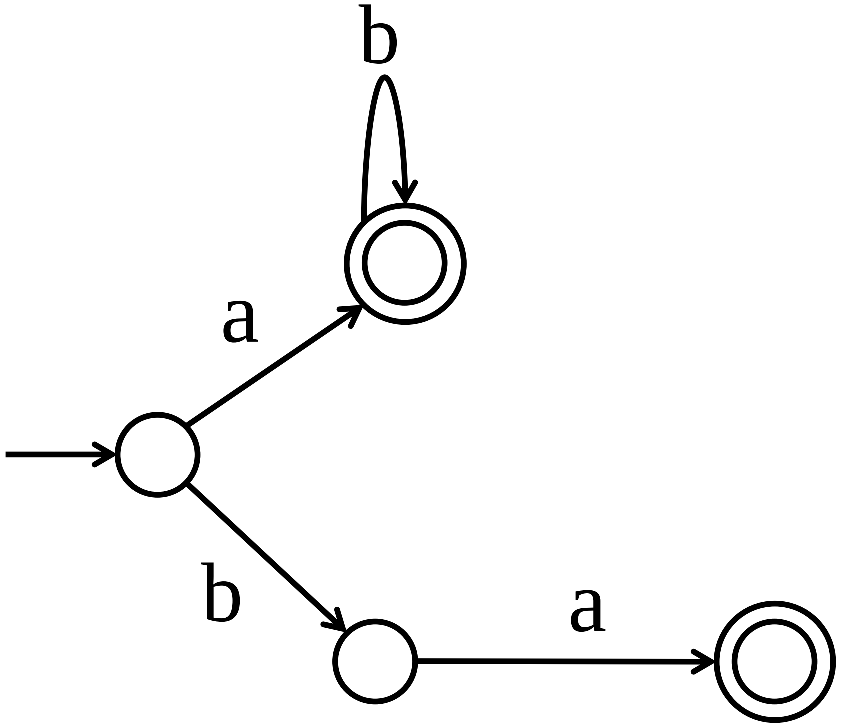
- **Proof idea:**
- Construct NFA that accepts L^R:

  invert the transitions of the NFA that accepts L.

Proof

- Since L is regular, there is NFA that accepts L.

- **Example**

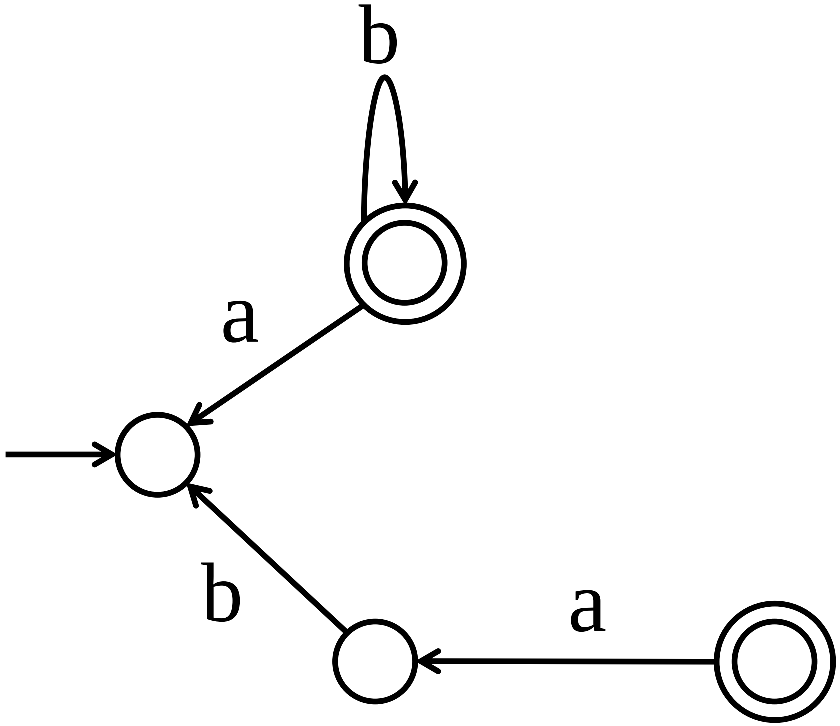


$$L = ab^* + ba$$

Proof

- Since L is regular, there is NFA that accepts L.

- **Example:** Invert transitions

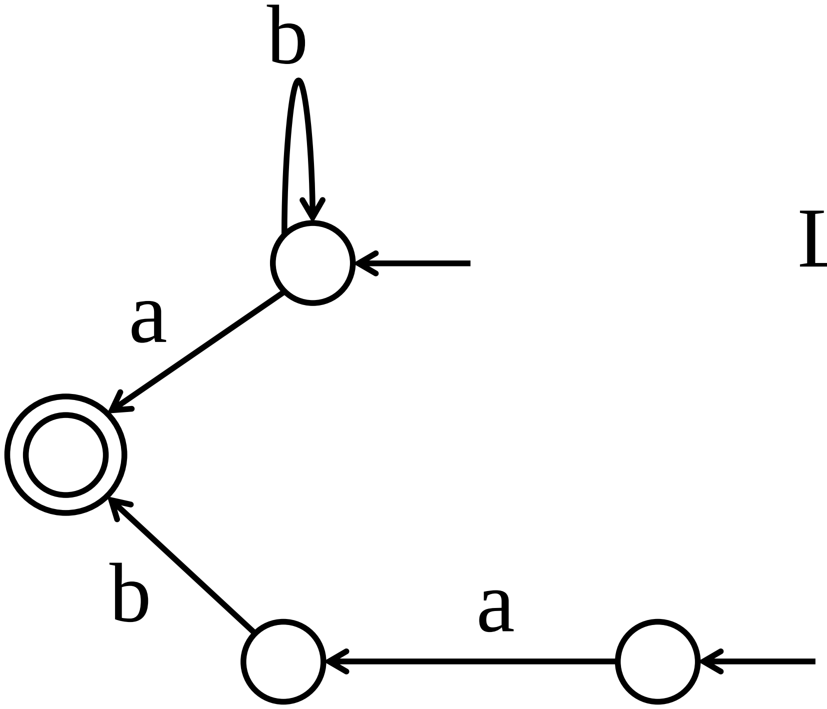


$$L = ab^* + ba$$

Proof

- Since L is regular, there is NFA that accepts L.

- **Example:** Make old initial state a final state

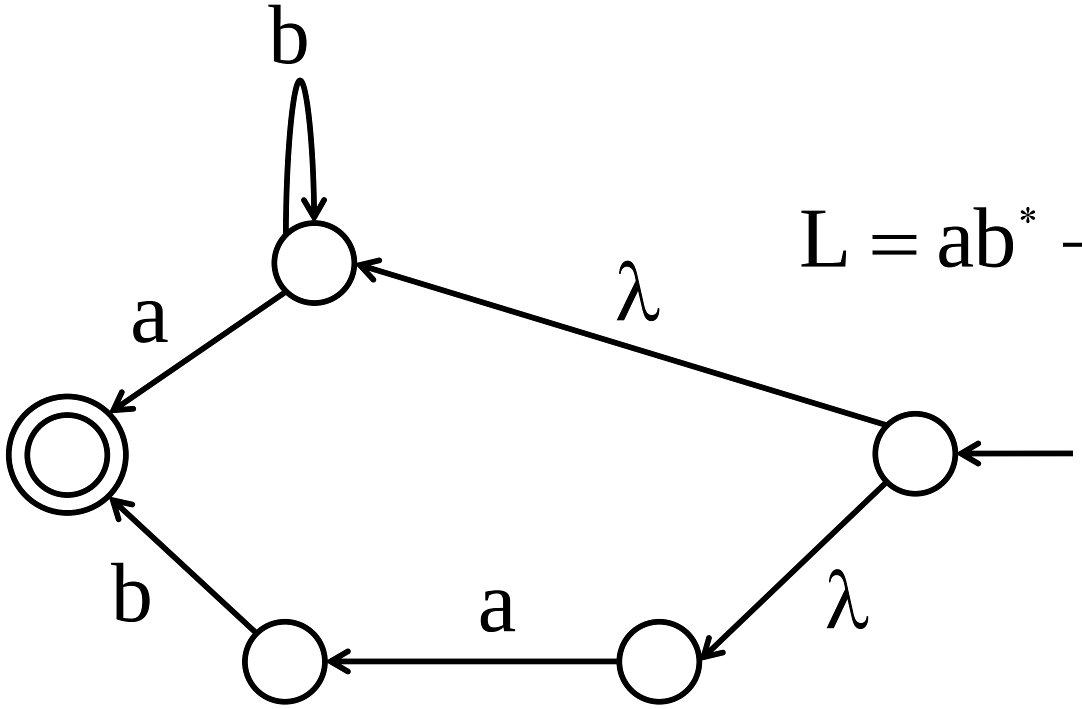


$$L = ab^* + ba$$

Proof

- Since L is regular, there is NFA that accepts L.

- **Example:** Add the new initial state



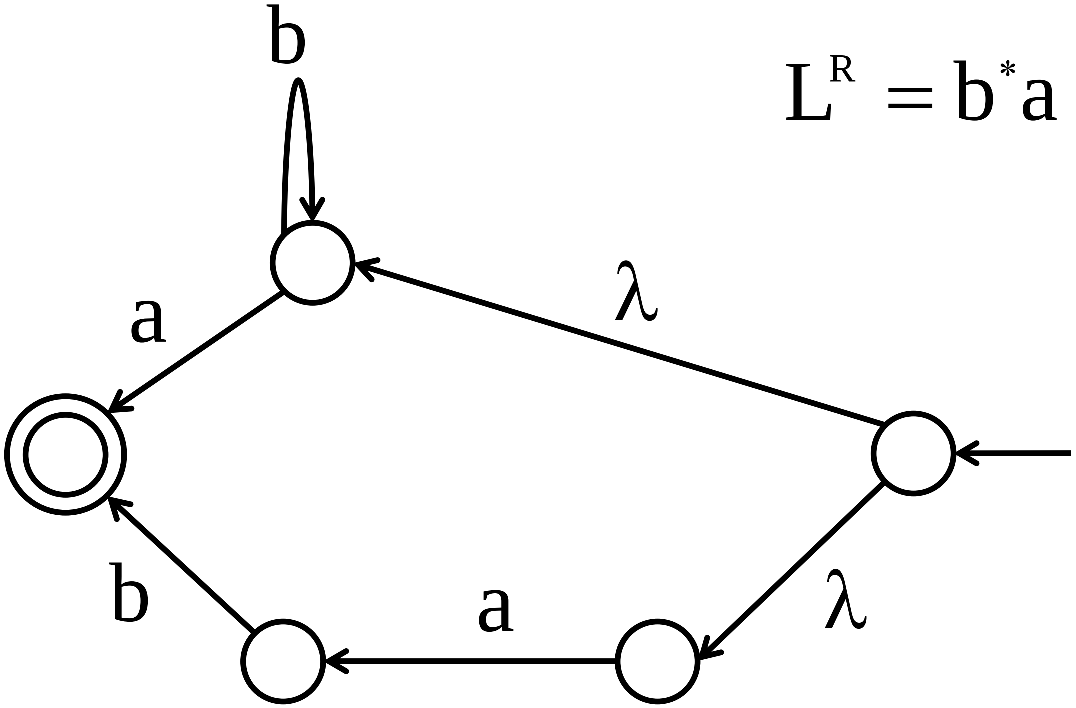
$$L = ab^* + ba$$

Proof

- Resulting machine accepts L^R.
- L^R is regular

$$L = ab^* + ba$$

$$L^R = b^*a + ab$$



Exercise

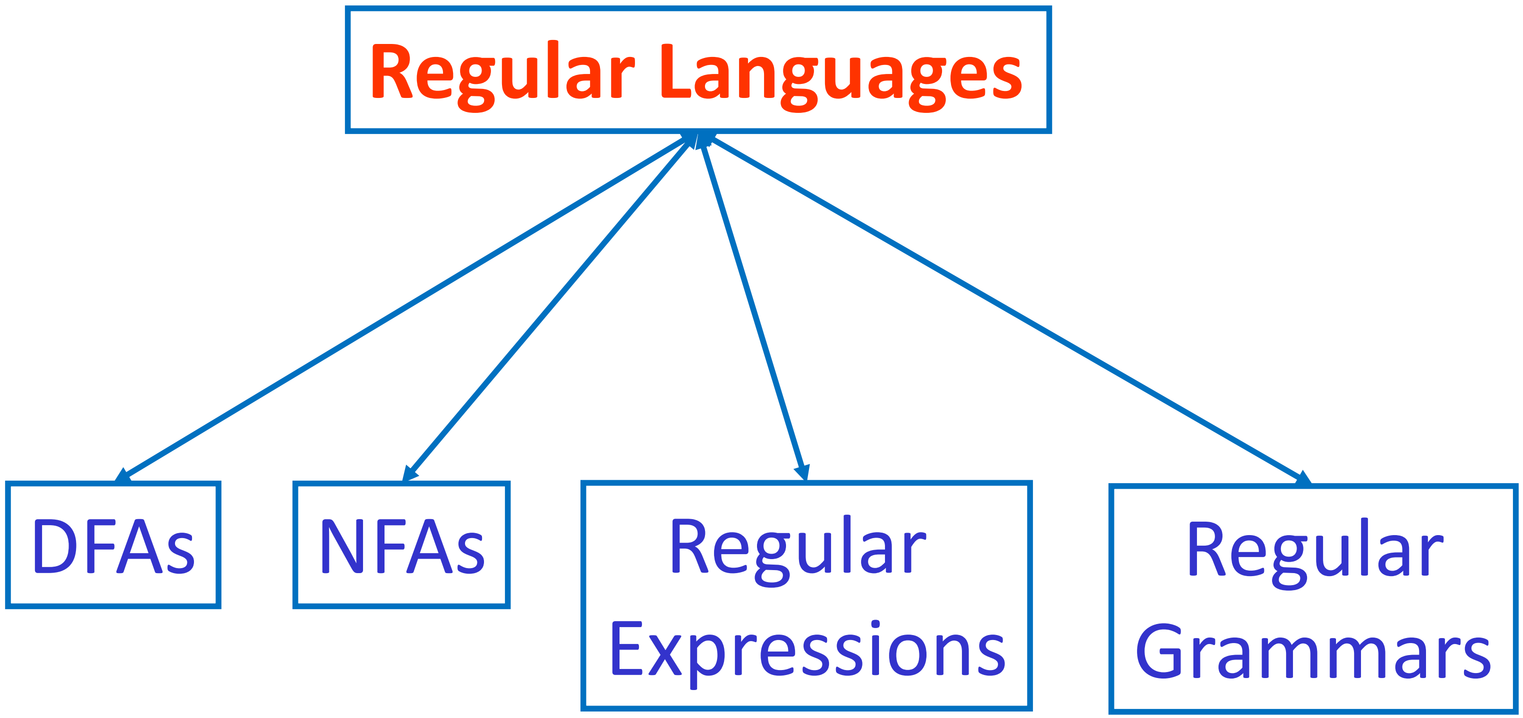
- Find NFAs that accept

a. $L\left((a+b)a^*\right) \cap L(bba^*)$

b. $L(ab^*a^*) \cap L(a^*b^*a)$

Standard Representations of Regular Languages

Standard Representations



Standard Representations

- When we say:

  We are given a regular language L

- We mean:

  Language L is in a standard representation

Elementary Questions About Regular Languages

Membership Question

- **Question:**

  Given regular language L and string w how can we check if $w \in L$**?**

Membership Question

- **Question:**

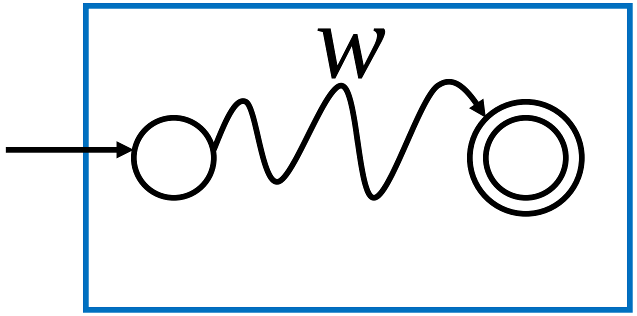
  Given regular language L and string w how can we check if $w \in L$?

- **Answer:**

  Take the DFA that accepts L and check if w is accepted.

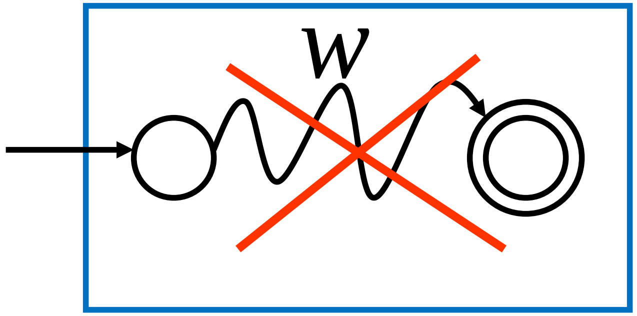
Membership Question

DFA



$w \in L$

DFA



$w \notin L$

Emptiness Question

- **Question:**

  Given regular language L how can we check if L is empty: $(L = \emptyset)$ **?**

Emptiness Question

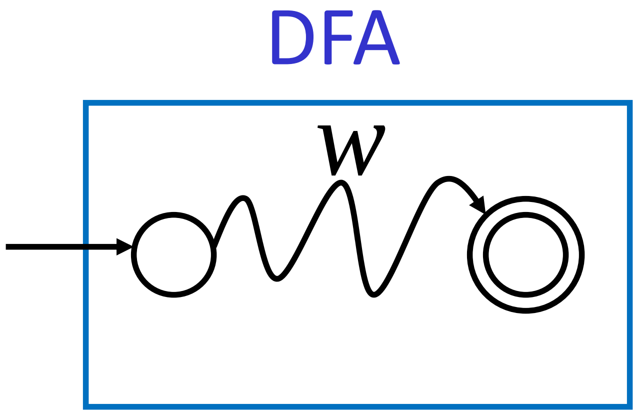
- **Question:**

  Given regular language L how can we check if L is empty: $(L = \varnothing)$?

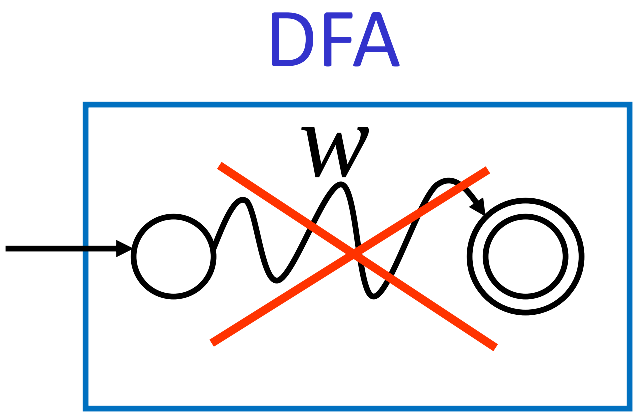
- **Answer:**

  Take the DFA that accepts L. Check if there is a path from the initial state to a final state.

Emptiness Question



$L \neq \varnothing$



$L = \varnothing$

Finiteness Question

- **Question:**

  Given regular language L, how can we check if L is finite?

Finiteness Question

- **Question:**

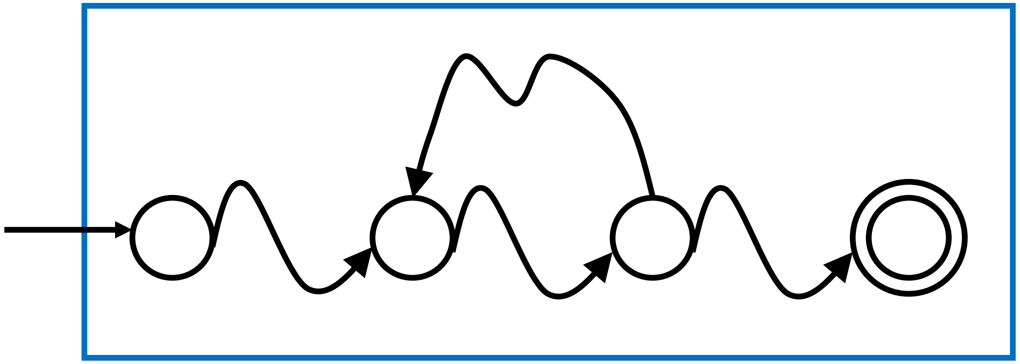
  Given regular language L, how can we check if L is finite?

- **Answer:**

  Take the DFA that accepts L. Check if there is a walk with a cycle from the initial state to a final state.

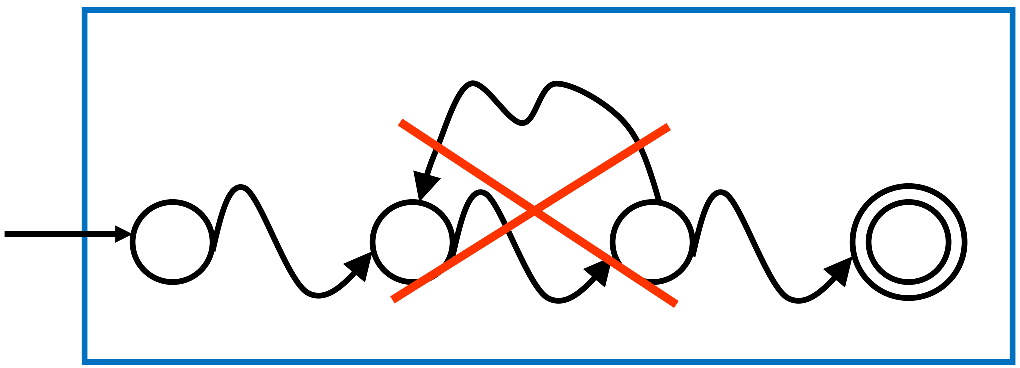
Finiteness Question

DFA



L is infinite

DFA



L is finite

Equivalence Question

- **Question:**

  Given regular language L_1 and L_2 how can we check if $L_1 = L_2$**?**

Equivalence Question

- **Question:**

  Given regular language L_1 and L_2 how can we check if $L_1 = L_2$**?**

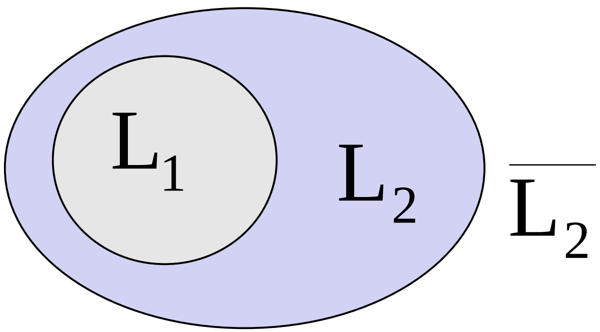
- **Answer:**

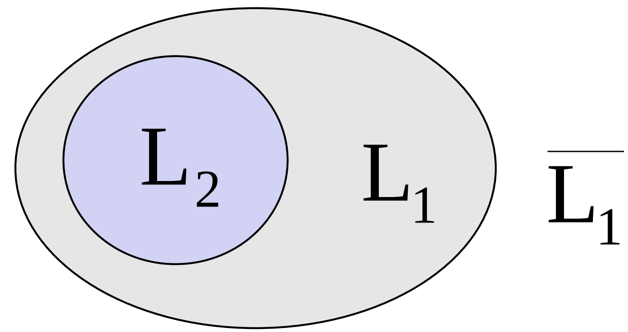
  Find if $(L_1 \cap \overline{L_2}) \cup (\overline{L_1} \cap L_2) = \varnothing$

Equivalence Question

$$(L_1 \cap \overline{L_2}) \cup (\overline{L_1} \cap L_2) = \varnothing$$

$$L_1 \cap \overline{L_2} = \varnothing \qquad \text{and} \qquad \overline{L_1} \cap L_2 = \varnothing$$





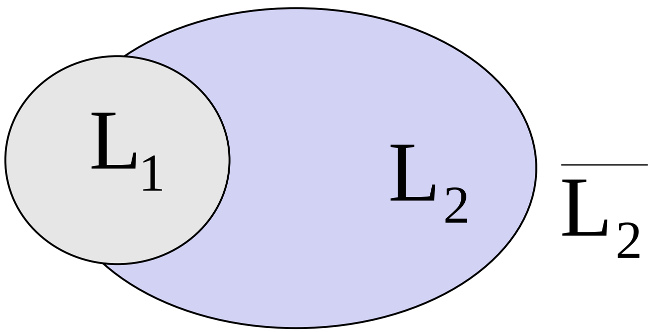
$$L_1 \subseteq L_2 \qquad\qquad L_2 \subseteq L_1$$

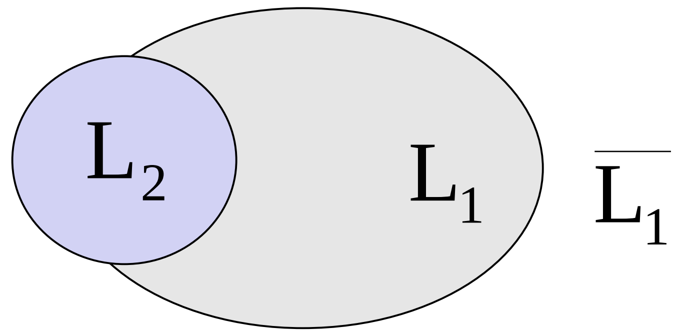
$$L_1 = L_2$$

Equivalence Question

$$(L_1 \cap \overline{L_2}) \cup (\overline{L_1} \cap L_2) \neq \varnothing$$

$L_1 \cap \overline{L_2} \neq \varnothing$ and $\overline{L_1} \cap L_2 \neq \varnothing$





$L_1 \not\subset S L_2$

$L_2 \not\subset L_1$

$$L_1 \neq L_2$$