Cocke–Younger–Kasami (CYK)

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Grammar denotes the syntactical rules for conversation in natural language. But in the theory of formal language, grammar is defined as a set of rules that can generate strings. The set of all strings that can be generated from a grammar is called the language of the grammar.

Context Free Grammar:

We are given a Context Free Grammar G = (V, X, R, S) and a string w, where:

- *V* is a finite set of variables or non-terminal symbols,
- *X* is a finite set of terminal symbols,
- *R* is a finite set of rules,
- *S* is the start symbol, a distinct element of *V*, and
- *V* and *X* are assumed to be disjoint sets.

The **Membership problem** is defined as: Grammar G generates a language L(G). Is the given string a member of L(G)?

Chomsky Normal Form:

A Context Free Grammar G is in Chomsky Normal Form (CNF) if each rule if each rule of *G* is of the form:

- *A –> BC*, [with at most two non-terminal symbols on the RHS]
- *A –> a*, or [one terminal symbol on the RHS]
- *S –>* nullstring, [null string]

Cocke-Younger-Kasami Algorithm

It is used to solves the membership problem using a dynamic programming approach. The algorithm is based on the principle that the solution to problem *[i, j]* can constructed from solution to subproblem *[i, k]* and solution to sub problem *[k, j].* The algorithm requires the Grammar *G* to be in Chomsky Normal Form (CNF). Note that any Context-Free Grammar can be systematically converted to CNF. This restriction is employed so that each problem can only be divided into two subproblems and not more – to bound the time complexity.

How does the CYK Algorithm work?

For a string of length **N**, construct a table **T** of size **N x N**. Each cell in the table **T[i, j]** is the set of all constituents that can produce the substring spanning from position i to j. The process involves filling the table with the solutions to the subproblems encountered in the bottom-up parsing process. Therefore, cells will be filled from left to right and bottom to top.

	1	**2**	**3**	**4**	**5**
1	[1, 1]	[1, 2]	[1, 3]	[1, 4]	[1, 5]
2		[2, 2]	[2, 3]	[2, 4]	[2, 5]
3			[3, 3]	[3, 4]	[3, 5]
4				[4, 4]	[4, 5]

In T[i, j], the row number *i* denotes the start index and the column number *j* denotes the end index.

$A \in T[i,j]$ if and only if $B \in T[i,k], C \in T[k,j]$ and $A \rightarrow BC$ is a rule of G

The algorithm considers every possible subsequence of letters and adds **K** to *T[i, j]* if the sequence of letters starting from *i* to *j* can be generated from the non-terminal *K*. For subsequences of length 2 and greater, it considers every possible partition of the subsequence into two parts, and checks if there is a rule of the form *A* ? *BC* in the grammar where B and C can generate the two parts respectively, based on already existing entries in *T*. The sentence can be produced by the grammar only if the entire string is matched by the start symbol, i.e, if *S* is a member of *T[1, n]*.

Consider a sample grammar in Chomsky Normal Form:

```
NP   -->  Det | Nom
Nom  -->  AP | Nom
AP  -->  Adv | A
Det  -->  a | an
Adv  -->  very | extremely
AP   -->  heavy | orange | tall
A   -->  heavy | orange | tall | muscular
Nom -->  book | orange | man
```

Now consider the phrase, "**a very heavy orange book**":

```
a(1) very(2) heavy (3) orange(4) book(5)
```

Let us start filling up the table from left to right and bottom to top, according to the rules described above:

	1 a	**2 very**	**3 heavy**	**4 orange**	**5 book**
1 a	Det	–	–	NP	NP
2 very		Adv	AP	Nom	Nom
3 heavy			A, AP	Nom	Nom
4 orange				Nom, A, AP	Nom
5 book					Nom

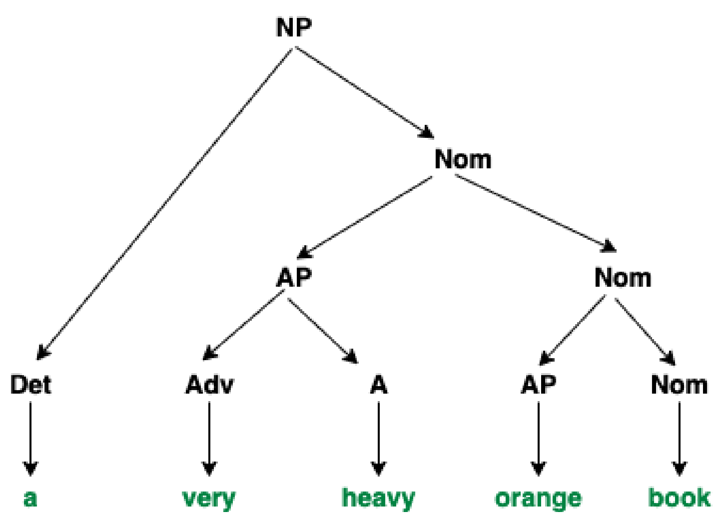
The table is filled in the following manner:

1. T[1, 1] = {Det} as Det –> a is one of the rules of the grammar.
2. T[2, 2] = {Adv} as Adv –> very is one of the rules of the grammar.
3. T[1, 2] = {} as no matching rule is observed.
4. T[3, 3] = {A, AP} as A –> very and AP –> very are rules of the grammar.
5. T[2, 3] = {AP} as AP –> Adv (T[2, 2]) A (T[3, 3]) is a rule of the grammar.
6. T[1, 3] = {} as no matching rule is observed.
7. T[4, 4] = {Nom, A, AP} as Nom –> orange and A –> orange and AP –> orange are rules

9. T[2, 4] = {Nom} as Nom –> AP (T[2, 3]) Nom (T[4, 4]) is a rule of the grammar.
10. T[1, 4] = {NP} as NP –> Det (T[1, 1]) Nom (T[2, 4]) is a rule of the grammar.
11. T[5, 5] = {Nom} as Nom –> book is a rule of the grammar.
12. T[4, 5] = {Nom} as Nom –> AP (T[4, 4]) Nom (T[5, 5]) is a rule of the grammar.
13. T[3, 5] = {Nom} as Nom –> AP (T[3, 3]) Nom (T[4, 5]) is a rule of the grammar.
14. T[2, 5] = {Nom} as Nom –> AP (T[2, 3]) Nom (T[4, 5]) is a rule of the grammar.
15. T[1, 5] = {NP} as NP –> Det (T[1, 1]) Nom (T[2, 5]) is a rule of the grammar.

We see that T[1][5] has **NP**, the start symbol, which means that this phrase is a member of the language of the grammar *G*.

The parse tree of this phrase would look like this:



Let us look at another example phrase, "**a very tall extremely muscular man":**

```
a(1) very(2) tall(3) extremely(4) muscular(5) man(6)
```

We will now use the CYK algorithm to find if this string is a member of the grammar *G:*

	1 a	2 very	3 tall	4 extremely	5 muscular	6 man
1 a	Det	–	–	–	–	NP
2 very		Adv	AP	–	–	Nom
3 tall			AP, A	–	–	Nom
4 extremely				Adv	AP	Nom
5 muscular					A	–
6 man						Nom

We see that T[1][6] has **NP**, the start symbol, which means that this phrase is a member of the language of the grammar G.

Below is the implementation of the above algorithm:

C++

```
// C++ implementation for the
// CYK Algorithm

#include<bits/stdc++.h>
using namespace std;

// Non-terminals symbols
vector<string> terminals,non_terminals;
```

```
// function to perform the CYK Algorithm
void cykParse(vector<string> w)
{
  int n = (int)w.size();

  // Initialize the table
  map<int,map<int,set<string>>> T;

  // Filling in the table
  for(int j=0;j<n;j++)
  {

    // Iterate over the rules
    for(auto x:R)
    {
      string lhs = x.first;
      vector<vector<string>> rule = x.second;

      for(auto rhs:rule)
      {

        // If a terminal is found
        if(rhs.size() == 1 && rhs[0] == w[j])
          T[j][j].insert(lhs);
      }

    }
    for(int i=j;i>=0;i--)
    {

      // Iterate over the range from i to j
      for(int k = i;k<=j;k++)
      {

        // Iterate over the rules
        for(auto x:R)
        {
          string lhs = x.first;
          vector<vector<string>> rule = x.second;

          for(auto rhs:rule)
          {
          // If a terminal is found
            if(rhs.size()==2 && T[i][k].find(rhs[0])!=T[i][k].end() && T[k+1][j].find(rhs[1]
              T[i][j].insert(lhs);
          }

        }
      }
    }
  }
```

```
  // of given grammar
  if(T[0][n-1].size()!=0)
    cout << "True\n";
  else
    cout << "False\n";
}

// Driver Code
int main()
{
  // terminal symbols
  terminals = {"book", "orange", "man",
              "tall", "heavy",
              "very", "muscular"};

  // non terminal symbols
  non_terminals = {"NP", "Nom", "Det", "AP",
                   "Adv", "A"};

  // Rules
  R["NP"]={{"Det", "Nom"}};
  R["Nom"]= {{"AP", "Nom"}, {"book"},
             {"orange"}, {"man"}};
  R["AP"] = {{"Adv", "A"}, {"heavy"},
            {"orange"}, {"tall"}};
  R["Det"] = {{"a"}};
  R["Adv"] = {{"very"}, {"extremely"}};
  R["A"] = {{"heavy"}, {"orange"}, {"tall"},
           {"muscular"}};

  // Given String
  vector<string> w = {"a", "very", "heavy", "orange", "book"};

  // Function Call
  cykParse(w);

  return 0;
}
```

Java

```
import java.util.*;

class GFG
{

  // Non-terminals symbols
  static List<String> terminals = new ArrayList<>();
  static List<String> non_terminals = new ArrayList<>();
```

```
        = new HashMap<>();

    // function to perform the CYK Algorithm
    static void cykParse(List<String> w)
    {
      int n = w.size();

      // Initialize the table
      Map<Integer, Map<Integer, Set<String> > > T
        = new HashMap<>();

      // Filling in the table
      for (int j = 0; j < n; j++) {

        // Iterate over the rules
        for (Map.Entry<String, List<List<String> > > x :
             R.entrySet()) {
          String lhs = x.getKey();
          List<List<String> > rule = x.getValue();

          for (List<String> rhs : rule) {

            // If a terminal is found
            if (rhs.size() == 1
                && rhs.get(0).equals(w.get(j))) {
              if (T.get(j) == null)
                T.put(j, new HashMap<>());
              T.get(j)
                .computeIfAbsent(
                j, k -> new HashSet<>())
                .add(lhs);
            }
          }
        }
        for (int i = j; i >= 0; i--) {

          // Iterate over the range from i to j
          for (int k = i; k <= j; k++) {

            // Iterate over the rules
            for (Map.Entry<String,
                 List<List<String> > > x :
                 R.entrySet()) {
              String lhs = x.getKey();
              List<List<String> > rule
                = x.getValue();

              for (List<String> rhs : rule) {
                // If a terminal is found
                if (rhs.size() == 2
                    && T.get(i) != null
                    && T.get(i).get(k) != null
```

```
                && T.get(k + 1) != null
                && T.get(k + 1).get(j)
                != null
                && T.get(k + 1)
                .get(j)
                .contains(
                  rhs.get(1))) {
              if (T.get(i) == null)
                T.put(i,
                      new HashMap<>());
              if (T.get(i).get(j) == null)
                T.get(i).put(
                j, new HashSet<>());
              T.get(i).get(j).add(lhs);
            }
          }
        }
      }
    }
  }

  // If word can be formed by rules
  // of given grammar
  if (T.get(0) != null && T.get(0).get(n - 1) != null
      && T.get(0).get(n - 1).size() != 0)
    System.out.println("True");
  else
    System.out.println("False");
}

// Driver Code
public static void main(String[] args)
{
  // terminal symbols
  terminals
    = Arrays.asList("book", "orange", "man", "tall",
                    "heavy", "very", "muscular");

  // non terminal symbols
  non_terminals = Arrays.asList("NP", "Nom", "Det",
                                "AP", "Adv", "A");

  // Rules
  R.put("NP",
        Arrays.asList(Arrays.asList("Det", "Nom")));
  R.put("Nom",
        Arrays.asList(Arrays.asList("AP", "Nom"),
                      Arrays.asList("book"),
                      Arrays.asList("orange"),
                      Arrays.asList("man")));
  R.put("AP", Arrays.asList(Arrays.asList("Adv", "A"),
                            Arrays.asList("heavy"),
```

```
    R.put("Det", Arrays.asList(Arrays.asList("a")));
    R.put("Adv",
          Arrays.asList(Arrays.asList("very"),
                        Arrays.asList("extremely")));
    R.put("A",
          Arrays.asList(Arrays.asList("heavy"),
                        Arrays.asList("orange"),
                        Arrays.asList("tall"),
                        Arrays.asList("muscular")));

    // Given String
    List<String> w = Arrays.asList("a", "very", "heavy",
                                   "orange", "book");

    // Function Call
    cykParse(w);
  }
}

// This code is contributed by lokeshpotta20
```

Python3

```
# Python implementation for the
# CYK Algorithm

# Non-terminal symbols
non_terminals = ["NP", "Nom", "Det", "AP",
                  "Adv", "A"]
terminals = ["book", "orange", "man",
             "tall", "heavy",
             "very", "muscular"]

# Rules of the grammar
R = {
     "NP": [["Det", "Nom"]],
     "Nom": [["AP", "Nom"], ["book"],
             ["orange"], ["man"]],
     "AP": [["Adv", "A"], ["heavy"],
            ["orange"], ["tall"]],
     "Det": [["a"]],
     "Adv": [["very"], ["extremely"]],
     "A": [["heavy"], ["orange"], ["tall"],
           ["muscular"]]
    }

# Function to perform the CYK Algorithm
def cykParse(w):
    n = len(w)
```

```
    # Filling in the table
    for j in range(0, n):

        # Iterate over the rules
        for lhs, rule in R.items():
            for rhs in rule:

                # If a terminal is found
                if len(rhs) == 1 and \
                rhs[0] == w[j]:
                    T[j][j].add(lhs)

        for i in range(j, -1, -1):

            # Iterate over the range i to j + 1
            for k in range(i, j + 1):

                # Iterate over the rules
                for lhs, rule in R.items():
                    for rhs in rule:

                        # If a terminal is found
                        if len(rhs) == 2 and \
                        rhs[0] in T[i][k] and \
                        rhs[1] in T[k + 1][j]:
                            T[i][j].add(lhs)

    # If word can be formed by rules
    # of given grammar
    if len(T[0][n-1]) != 0:
        print("True")
    else:
        print("False")

# Driver Code

# Given string
w = "a very heavy orange book".split()

# Function Call
cykParse(w)
```

C#

```
// C# program to implement above approach
using System;
using System.Collections;
using System.Collections.Generic;

class GFG
```

```
    // static List<string> terminals = new List<string>();
    // static List<string> non_terminals = new List<string>();

    // Rules of the grammar
    static Dictionary<string,List<List<string>>> R = new Dictionary<string,List<List<string

    // function to perform the CYK Algorithm
    static void cykParse(List<string> w)
    {
        int n = w.Count;

        // Initialize the table
        SortedDictionary<int, SortedDictionary<int, SortedSet<string>>> T = new SortedDicti

        // Filling in the table
        for (int j = 0 ; j < n ; j++)
        {

            // Iterate over the rules
            foreach (KeyValuePair<string,List<List<string>>> x in R)
            {
                string lhs = x.Key;
                List<List<string>> rule = x.Value;

                foreach (List<string> rhs in rule)
                {

                    // If a terminal is found
                    if(rhs.Count == 1 && rhs[0] == w[j]){
                        if(!T.ContainsKey(j)){
                            T.Add(j, new SortedDictionary<int, SortedSet<string>>());
                        }
                        if(!T[j].ContainsKey(j)){
                            T[j].Add(j, new SortedSet<string>());
                        }
                        T[j][j].Add(lhs);
                    }
                }

            }
            for(int i = j ; i >= 0 ; i--)
            {

                // Iterate over the range from i to j
                for(int k = i ; k <= j ; k++)
                {

                    // Iterate over the rules
                    foreach (KeyValuePair<string,List<List<string>>> x in R)
                    {
                        string lhs = x.Key;
                        List<List<string>> rule = x.Value;
```

```
                    {
                    // If a terminal is found
                        if(rhs.Count == 2 &&
                            T.ContainsKey(i) &&
                            T[i].ContainsKey(k) &&
                            T[i][k].Contains(rhs[0]) &&
                            T.ContainsKey(k + 1) &&
                            T[k + 1].ContainsKey(j) &&
                            T[k + 1][j].Contains(rhs[1]))
                        {
                            if(!T.ContainsKey(i)){
                                T.Add(i, new SortedDictionary<int, SortedSet<string>>()
                            }
                            if(!T[i].ContainsKey(j)){
                                T[i].Add(j, new SortedSet<string>());
                            }
                            T[i][j].Add(lhs);
                        }
                    }

                }
            }
        }
    }

    // If word can be formed by rules
    // of given grammar
    if(T.ContainsKey(0) && T[0].ContainsKey(n - 1) && T[0][n - 1].Count != 0){
        Console.Write("True\n");
    }else{
        Console.Write("False\n");
    }
}


// Driver code
public static void Main(string[] args){

    // terminal symbols
    // terminals = new List<string>{
    //     "book",
    //     "orange", "man",
    //    "tall", "heavy",
    //    "very", "muscular"
    // };

    // non terminal symbols
    // non_terminals = new List<string>{
    //     "NP", "Nom", "Det",
    //     "AP", "Adv", "A"
    // };
```

```
            new List<string>{"Det", "Nom"}
        });

        R["Nom"]= new List<List<string>>{
            new List<string>{"AP", "Nom"},
            new List<string>{"book"},
            new List<string>{"orange"},
            new List<string>{"man"}
        };

        R["AP"] = new List<List<string>>{
            new List<string>{"Adv", "A"},
            new List<string>{"heavy"},
            new List<string>{"orange"},
            new List<string>{"tall"}
        };

        R["Det"] = new List<List<string>>{
            new List<string>{"a"}
        };

        R["Adv"] = new List<List<string>>{
            new List<string>{"very"},
            new List<string>{"extremely"}
        };

        R["A"] = new List<List<string>>{
            new List<string>{"heavy"},
            new List<string>{"orange"},
            new List<string>{"tall"},
            new List<string>{"muscular"}
        };

        // Given String
        List<string> w = new List<string>{"a", "very", "heavy", "orange", "book"};

        // Function Call
        cykParse(w);

    }
}

// This code is contributed by subhamgoyal2014.
```

Javascript

```
// CYK Algorithm

// Non-terminal symbols
```

```
// Rules of the grammar
const R = {
  "NP": [["Det", "Nom"]],
  "Nom": [["AP", "Nom"], ["book"],
          ["orange"], ["man"]],
  "AP": [["Adv", "A"], ["heavy"],
         ["orange"], ["tall"]],
  "Det": [["a"]],
  "Adv": [["very"], ["extremely"]],
  "A": [["heavy"], ["orange"], ["tall"],
        ["muscular"]]
};

// function to perform the CYK Algorithm
function cykParse(w) {
  let n = w.length;

  // Initialize the table
  let T = [];
  for (let i = 0; i < n; i++) {
    T[i] = [];
    for (let j = 0; j < n; j++) {
      T[i][j] = new Set();
    }
  }

  // Filling in the table
  for (let j = 0; j < n; j++) {
    // Iterate over the rules
    for (let lhs in R) {
      let rule = R[lhs];
      for (let rhs of rule) {
        // If a terminal is found
        if (rhs.length == 1 && rhs[0] == w[j]) {
          T[j][j].add(lhs);
        }
      }
    }
    for (let i = j; i >= 0; i--) {
      // Iterate over the range from i to j
      for (let k = i; k <= j; k++) {
        // Iterate over the rules
        for (let lhs in R) {
          let rule = R[lhs];
          for (let rhs of rule) {
            // If a terminal is found
            if (rhs.length == 2 && T[i][k].has(rhs[0]) && T[k + 1][j].has(rhs[1])) {
              T[i][j].add(lhs);
            }
          }
        }
```

```
    }

    // If word can be formed by rules
    // of given grammar
    if (T[0][n - 1].size !== 0) {
      console.log("True");
    } else {
      console.log("False");
    }
  }

  // Given String
  const w = ["a", "very", "heavy", "orange", "book"];

  // Function Call
  cykParse(w);
```

Output:

```
True
```

Time Complexity: O(N^3)
Auxiliary Space:O(N^2)

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