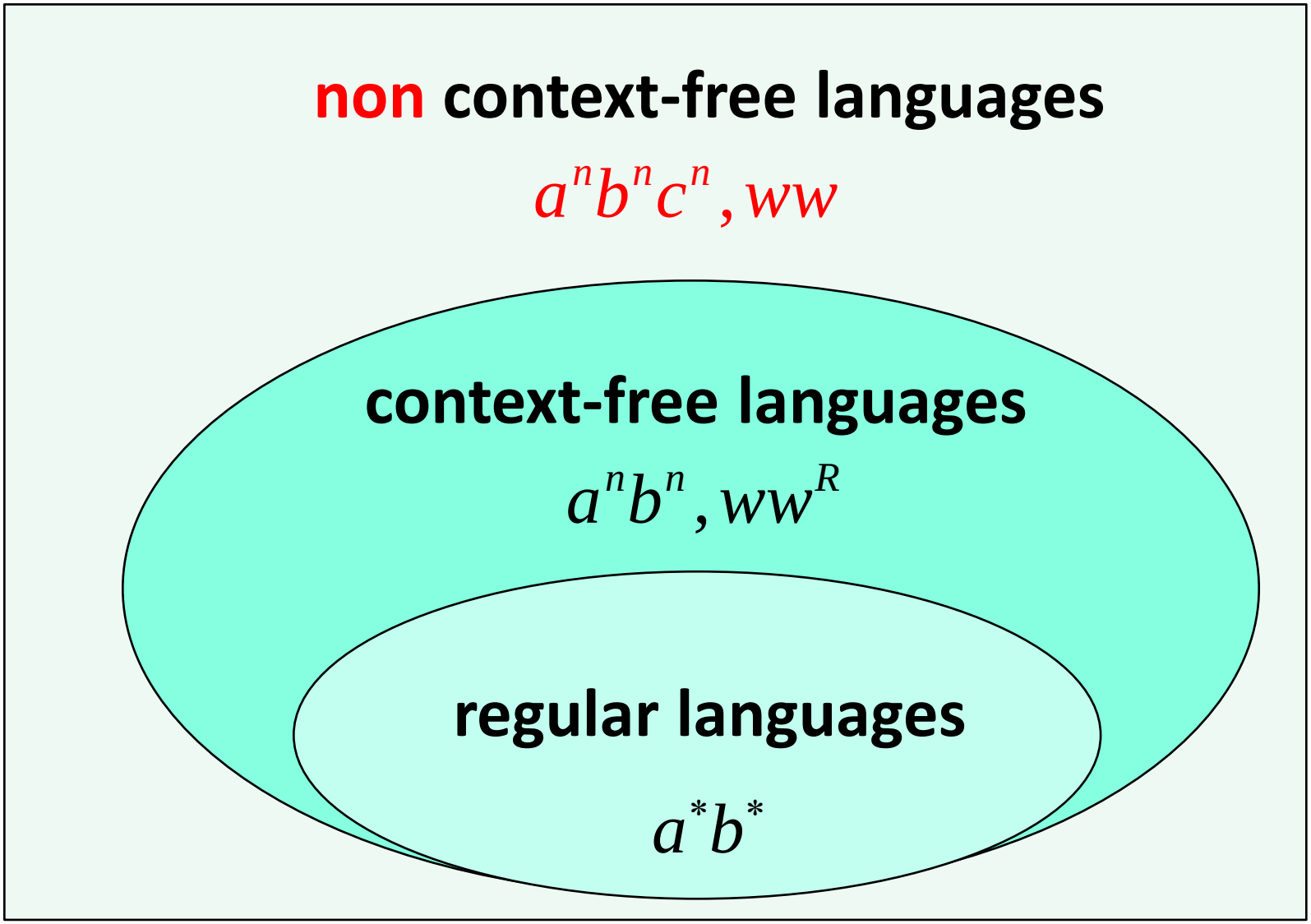
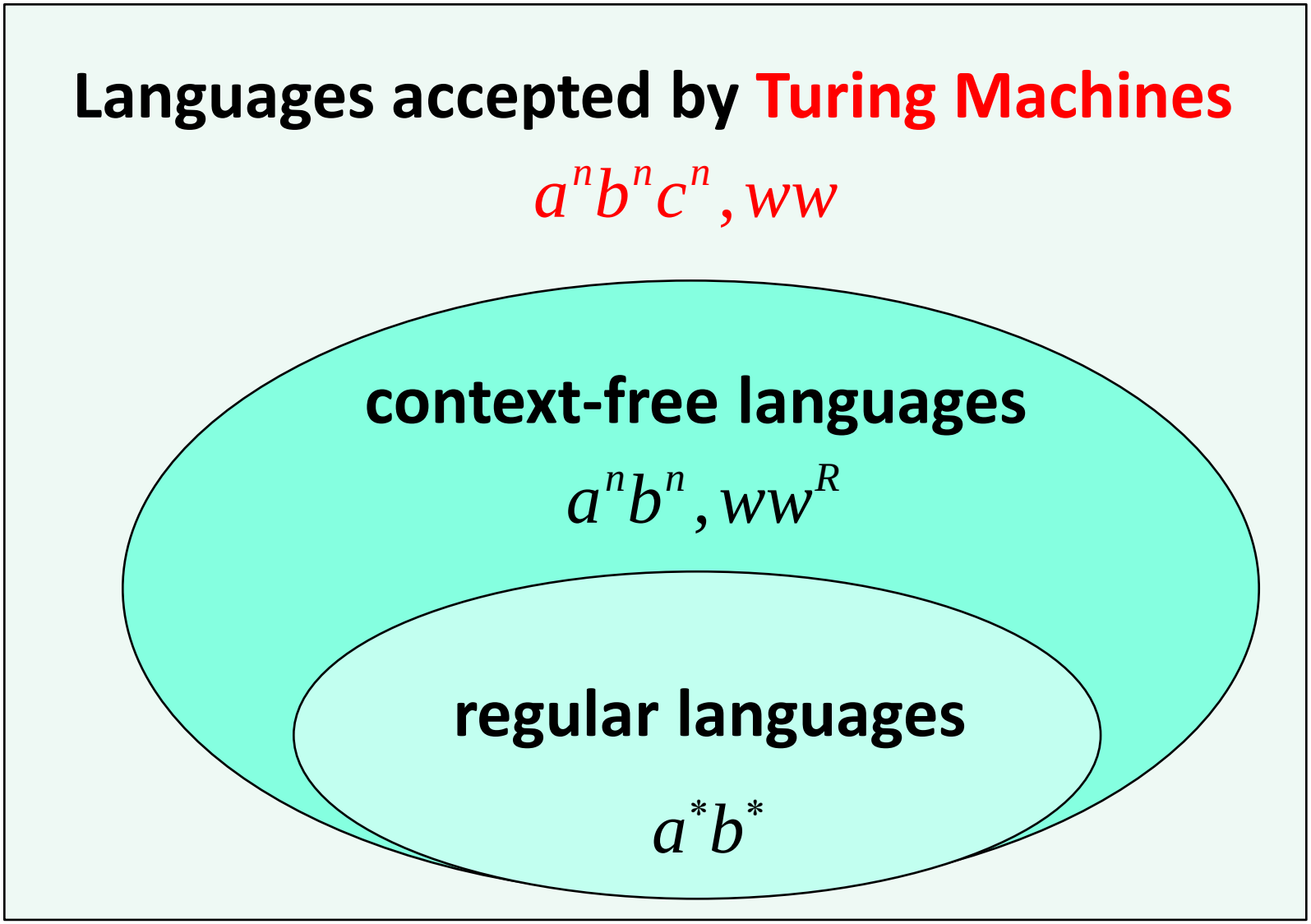
Turing Machines

The Language Hierarchy



The Language Hierarchy

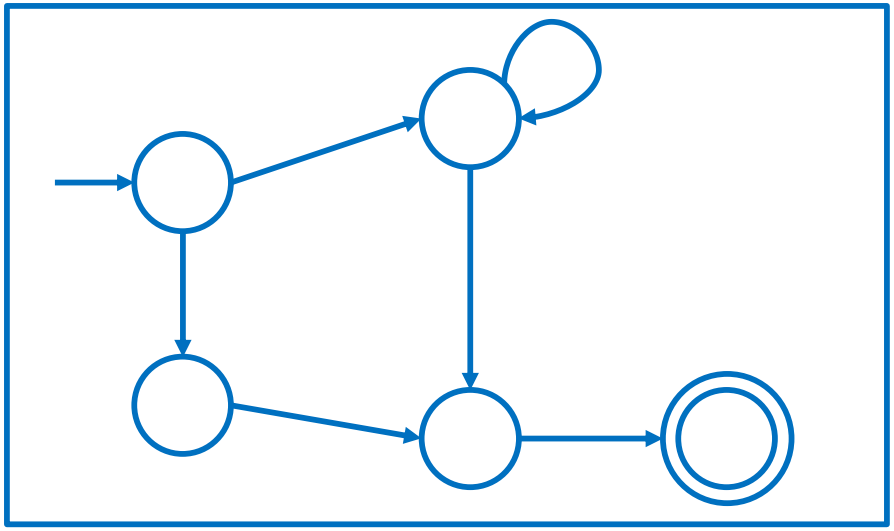


A Turing Machine

Tape

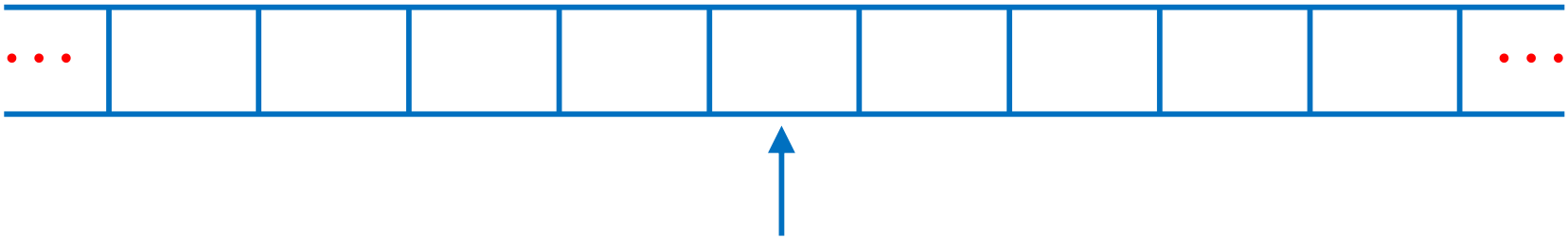
Read-Write head

Control Unit



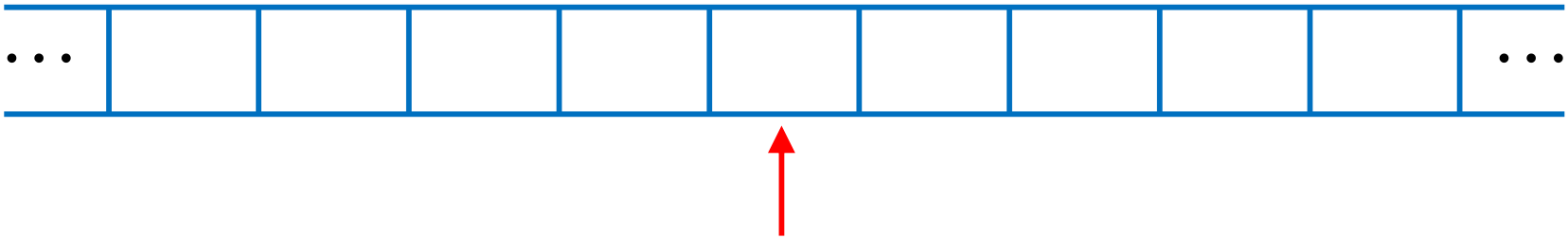
A Turing Machine: Tape

No boundaries - infinite length



Read-Write head

A Turing Machine: Read-Write Head

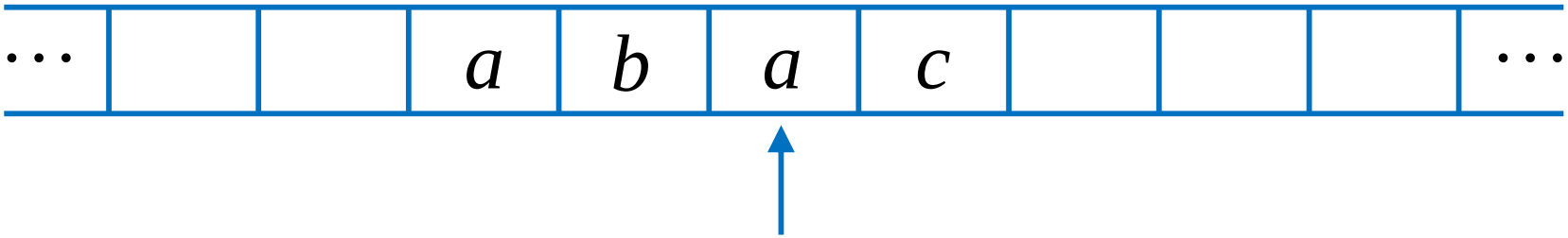


The head at <u>each step</u>:

1. Reads a symbol
2. Writes a symbol
3. Moves Left or Right

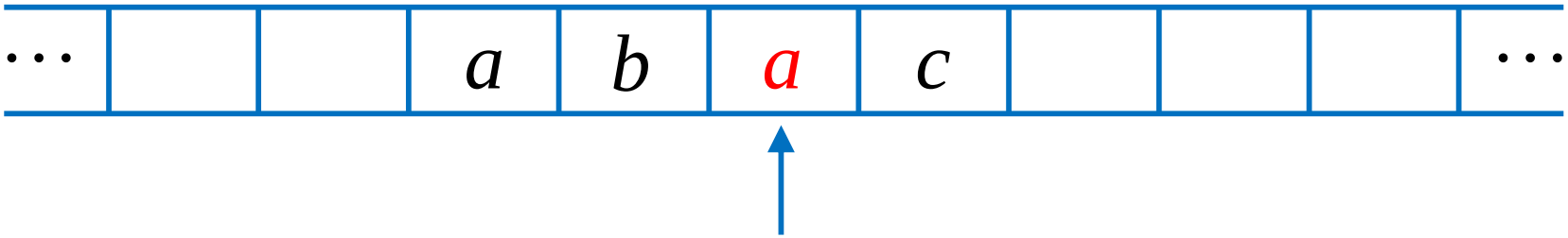
A Turing Machine: Read-Write Head

Example: **Time 0**



A Turing Machine: Read-Write Head

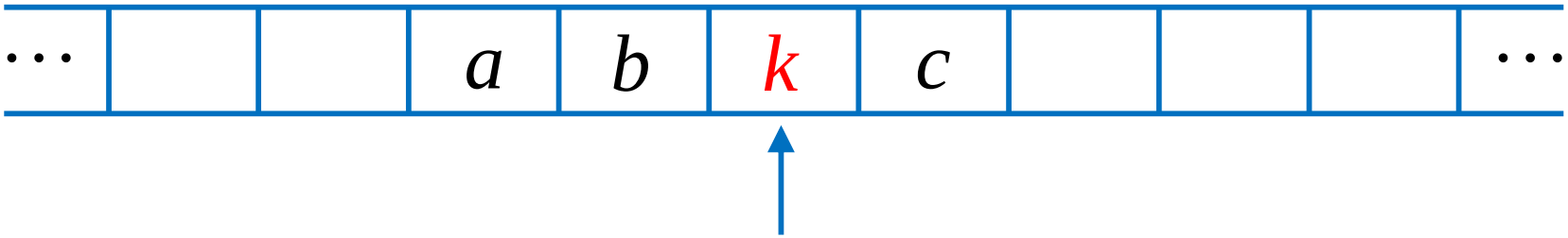
Example: **Time 1**



1. Reads a

A Turing Machine: Read-Write Head

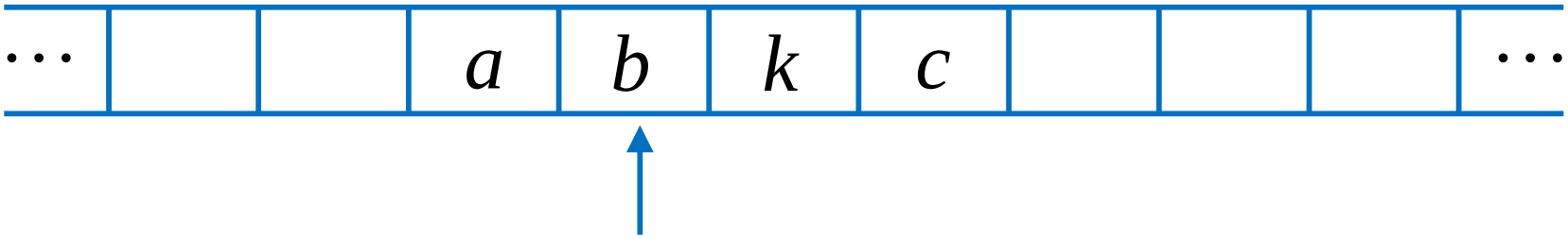
Example: **Time 1**



1. Reads a
2. Writes k

A Turing Machine: Read-Write Head

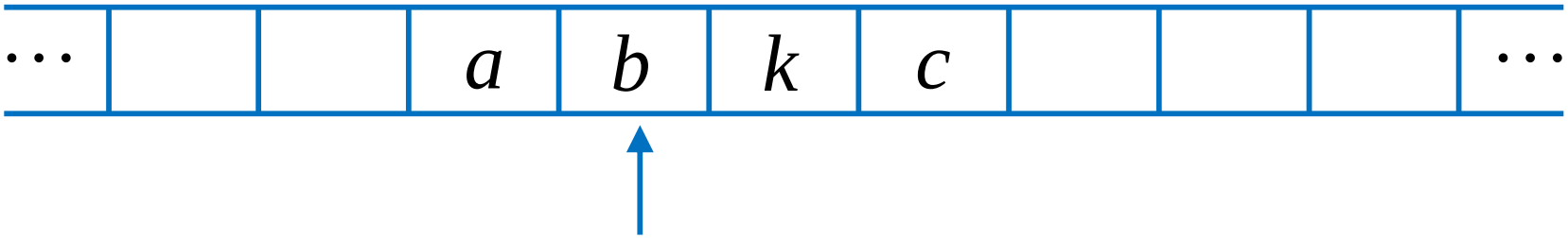
Example: **Time 1**



1. Reads a
2. Writes k
3. Moves Left

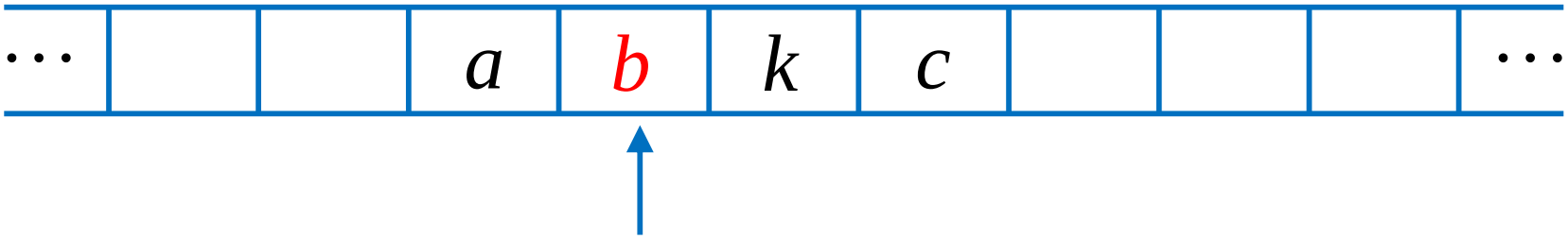
A Turing Machine: Read-Write Head

Example: **Time 1**



A Turing Machine: Read-Write Head

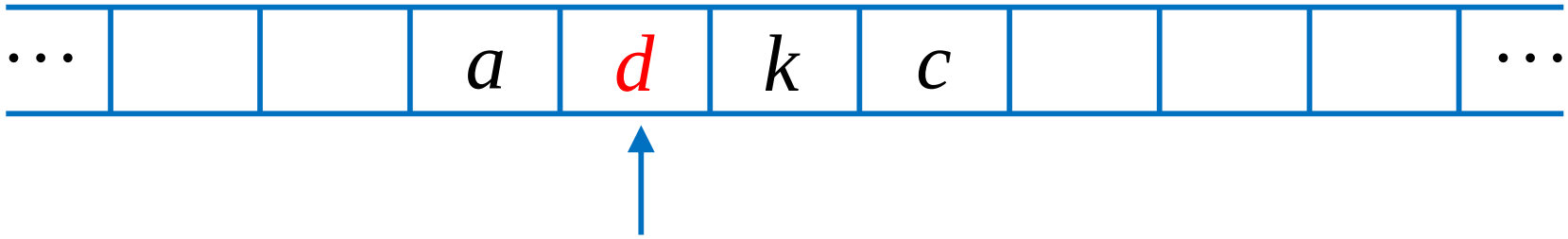
Example: **Time 2**



1. Reads b

A Turing Machine: Read-Write Head

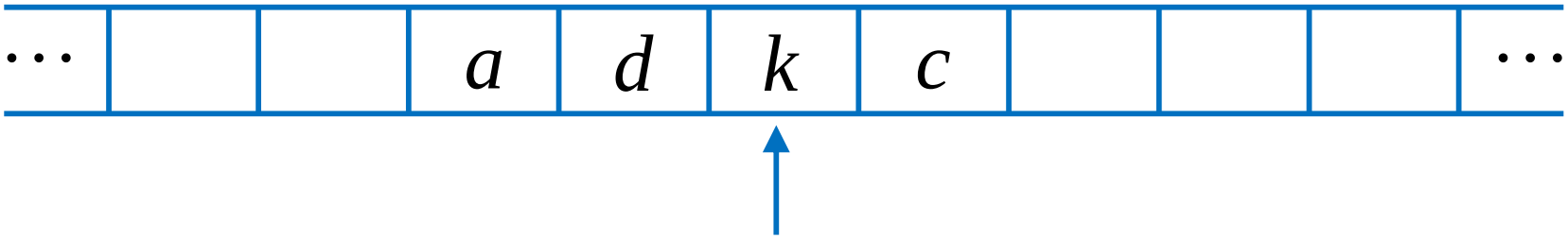
Example: **Time 2**



1. Reads b
2. Writes d

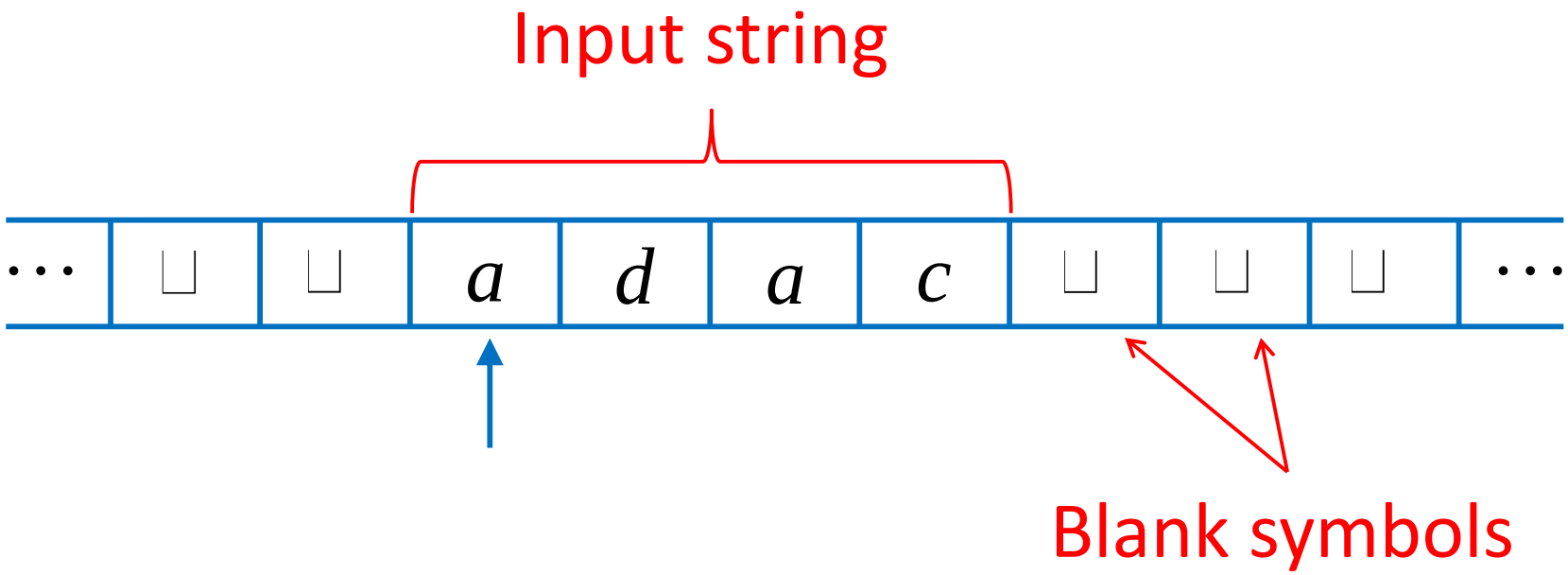
A Turing Machine: Read-Write Head

Example: **Time 2**



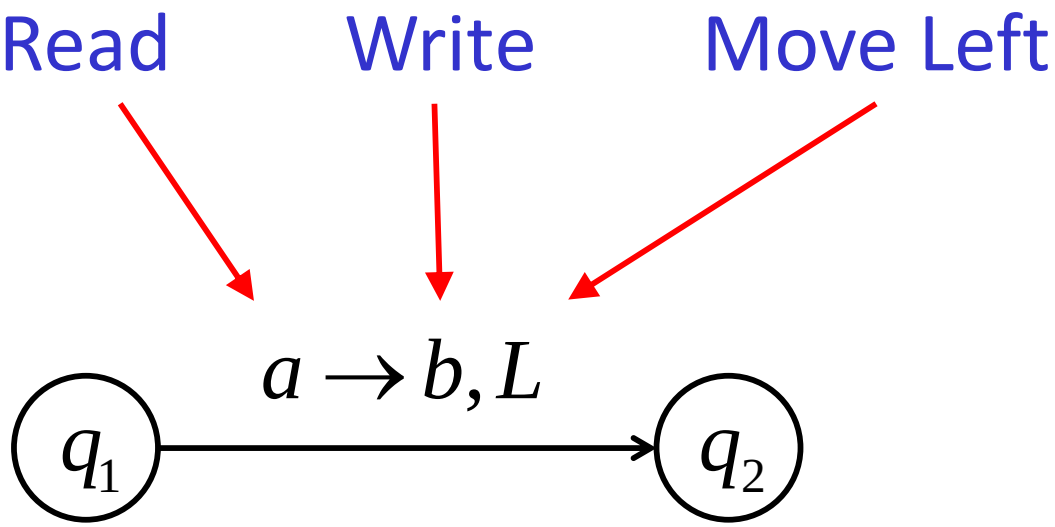
1. Reads b
2. Writes d
3. Moves Right

A Turing Machine: Input

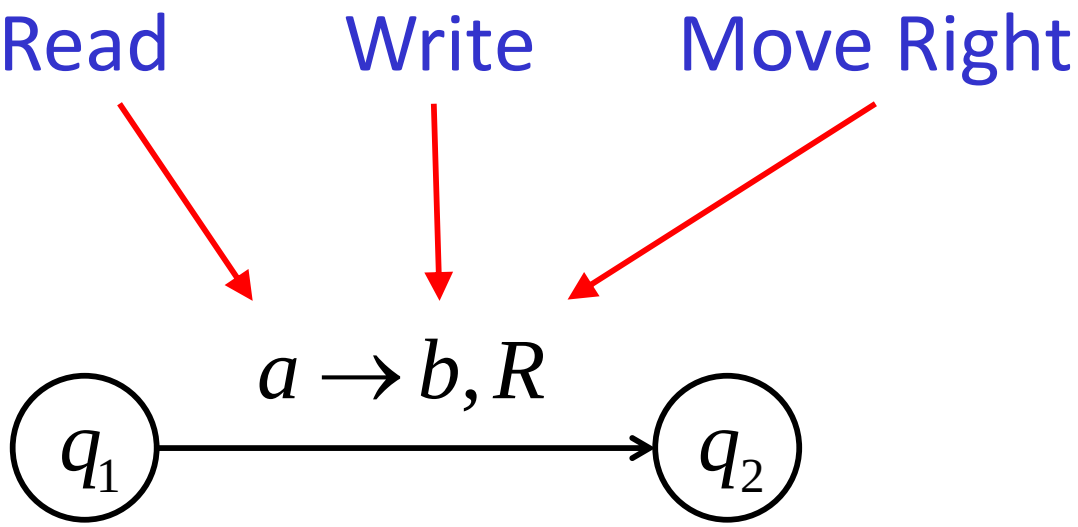


The head starts at the leftmost position of the input string

States & Transitions

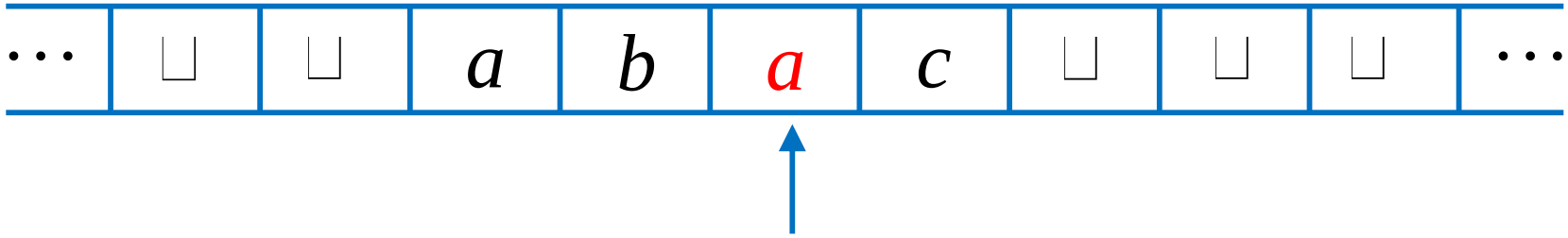


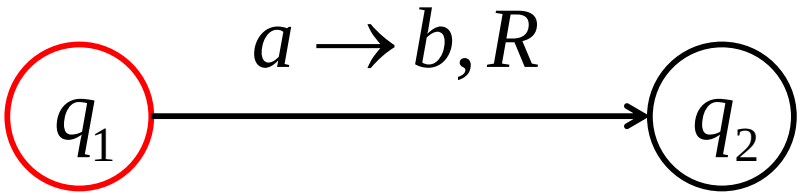
States & Transitions



Example: States & Transitions

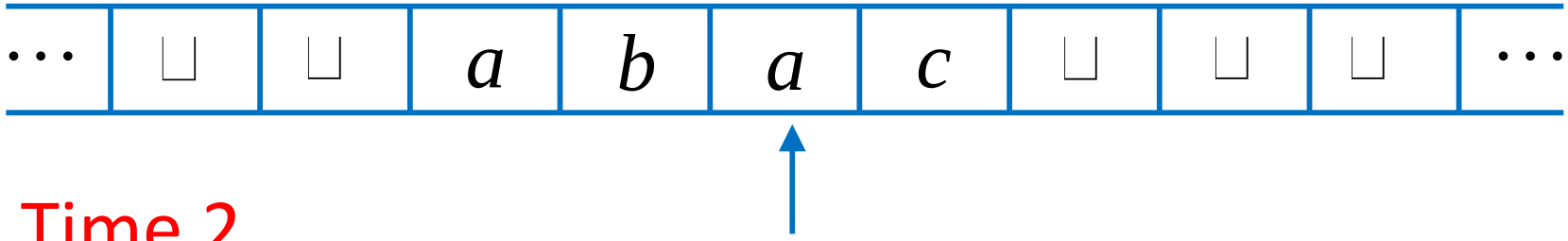
Time 1



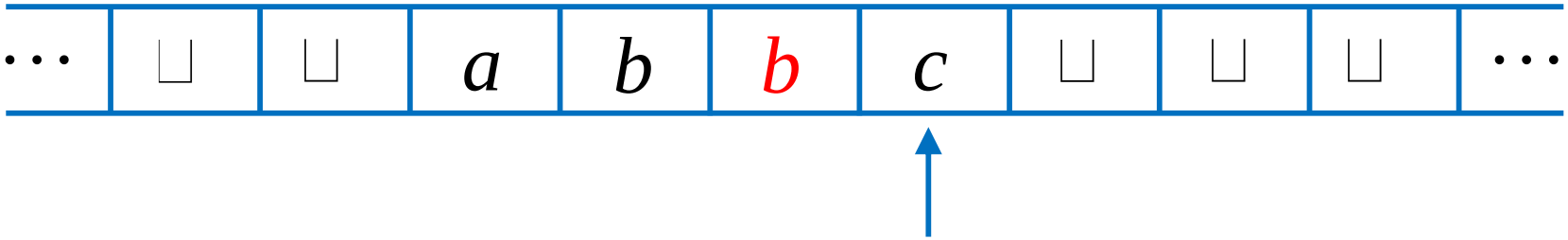


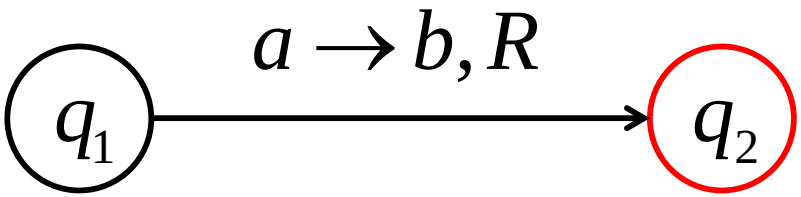
Example: States & Transitions

Time 1



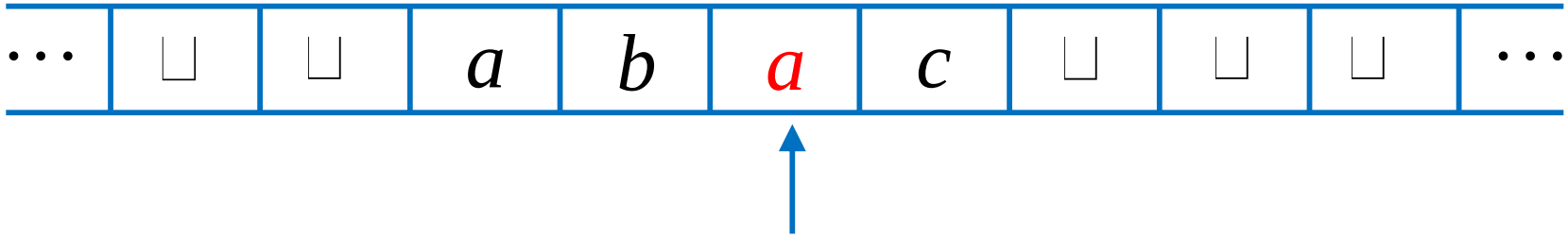
Time 2

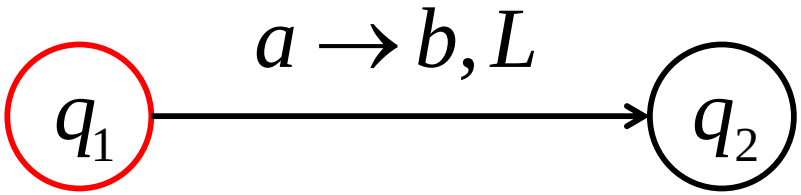




Example: States & Transitions

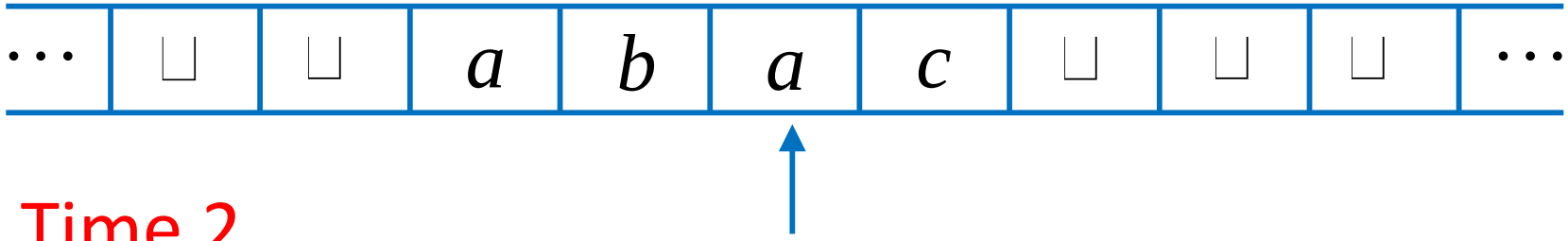
Time 1



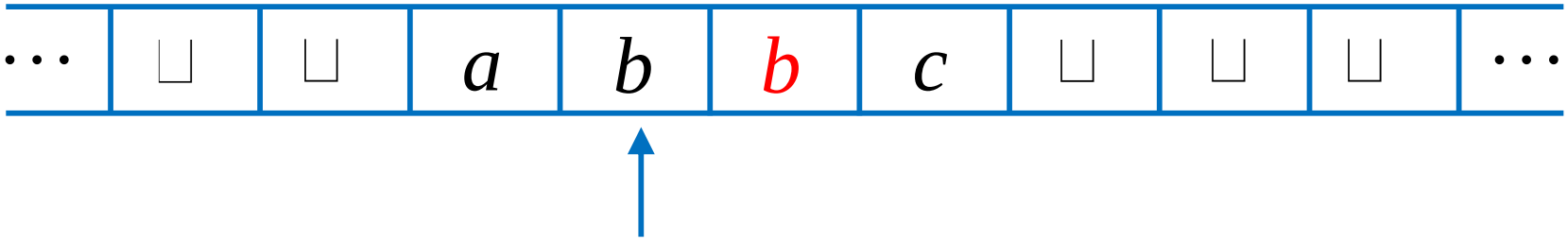


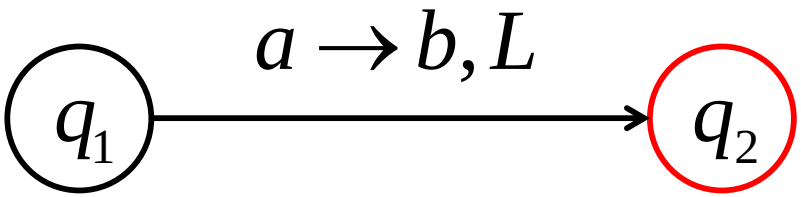
Example: States & Transitions

Time 1



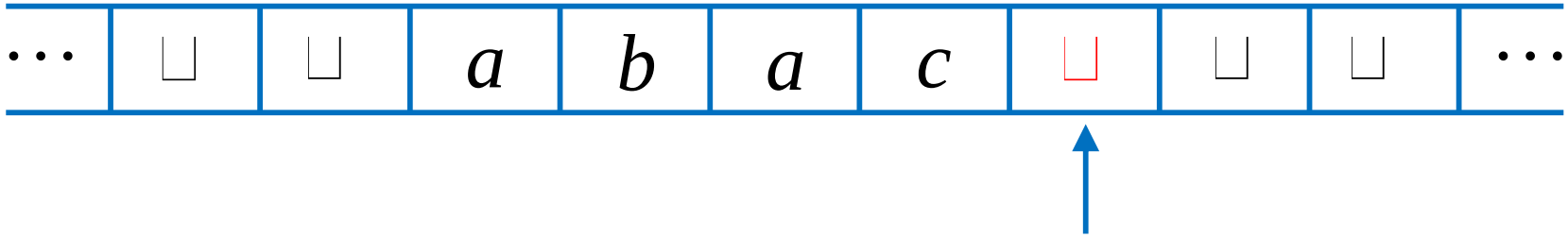
Time 2

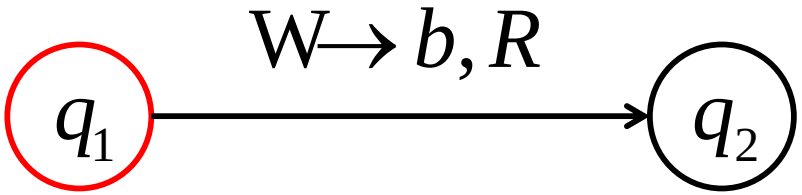




Example: States & Transitions

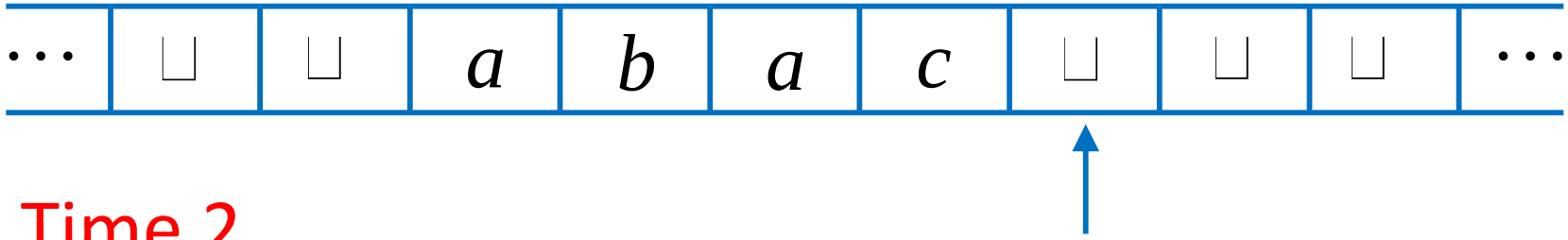
Time 1



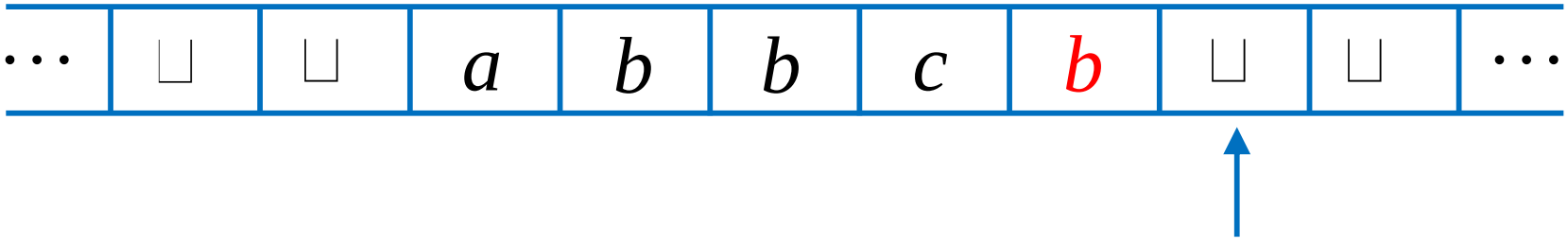


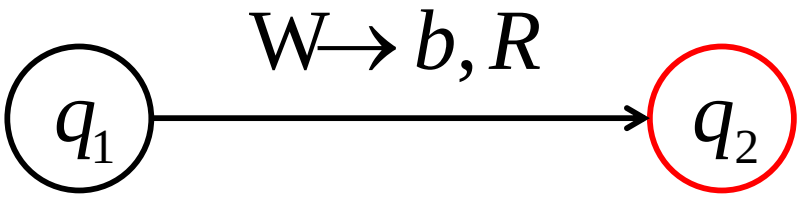
Example: States & Transitions

Time 1



Time 2

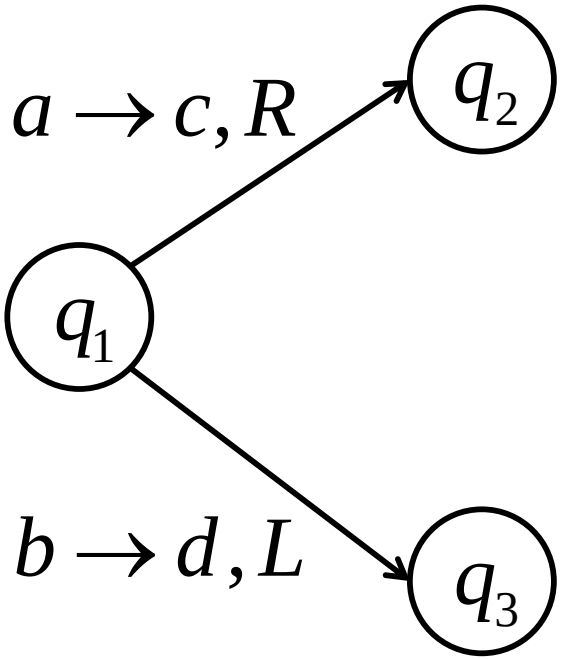




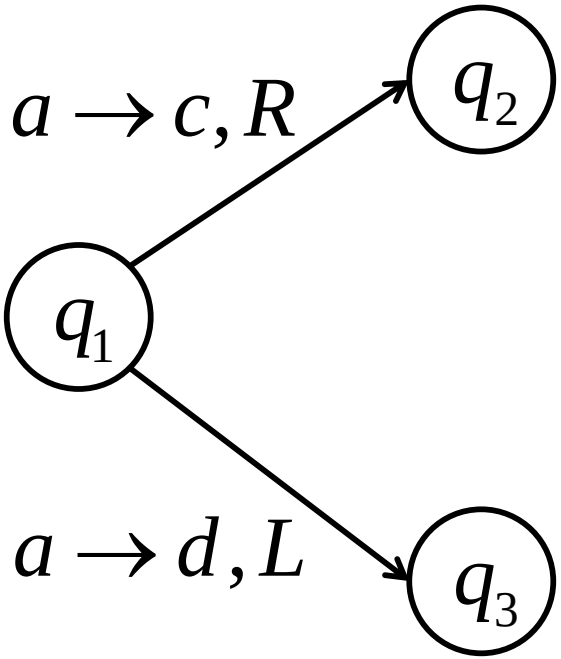
Determinism

- Turing Machines are deterministic:

Allowed

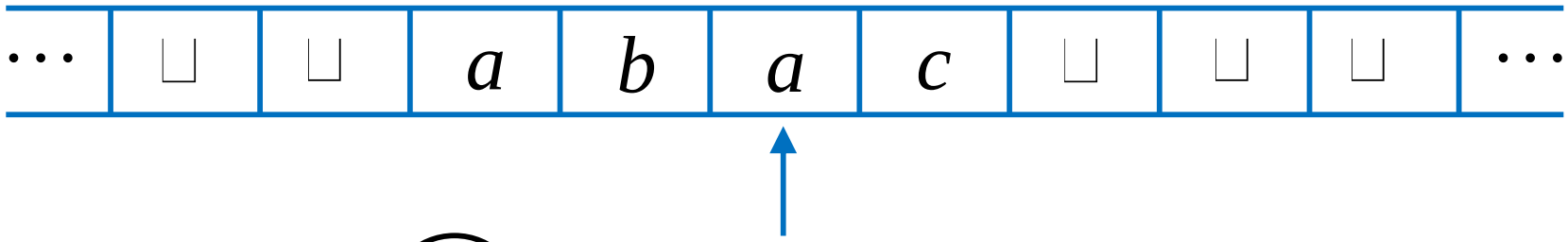


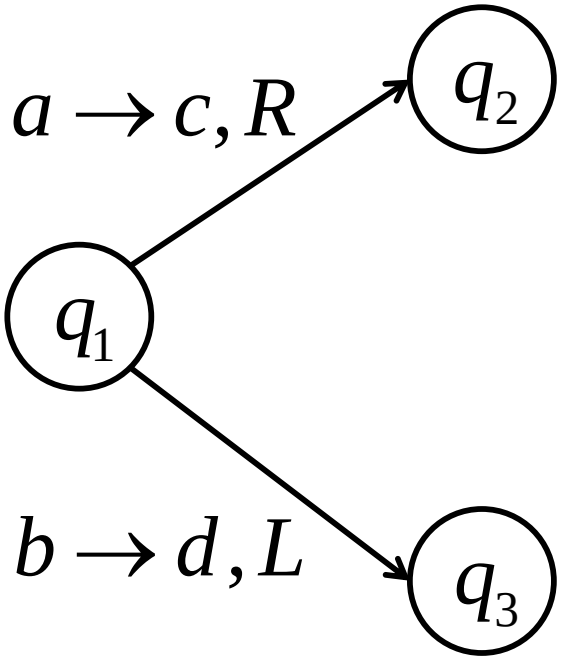
Not allowed



No lambda transitions allowed

Determinism: Partial Transition Function





Allowed

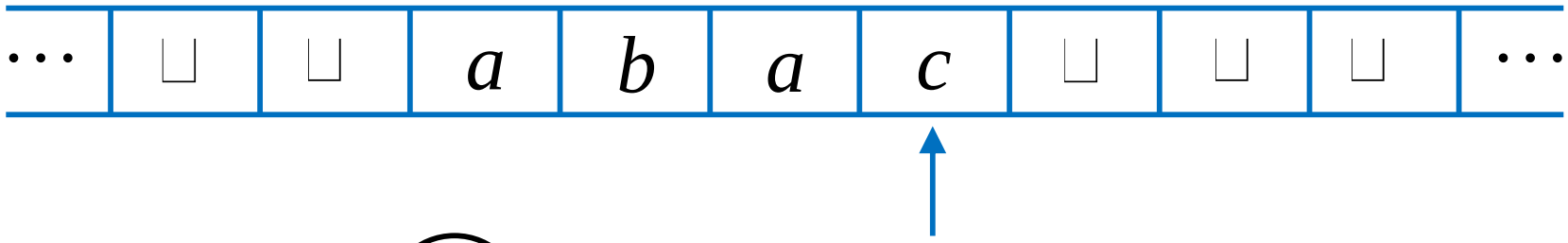
No transition

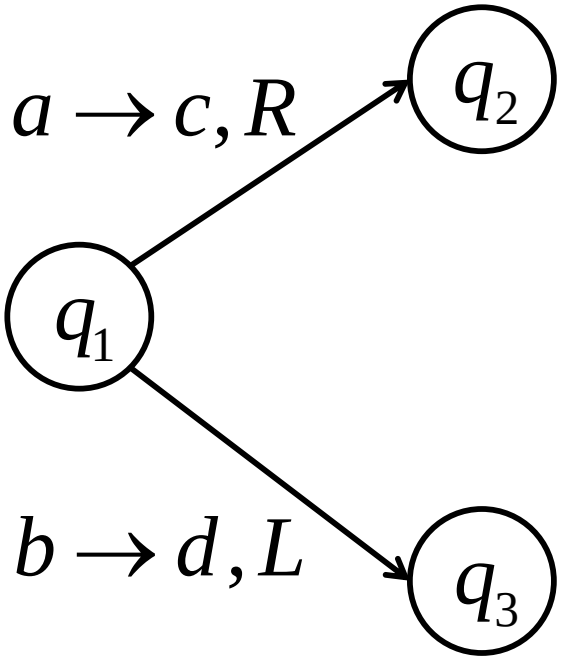
for input symbol c

Halting

The machine **halts** if there are no possible transitions to follow.

Halting

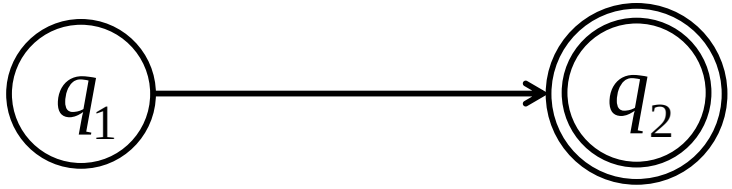




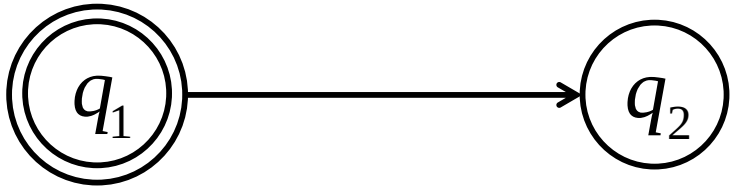
No possible transition

Halt!

Final States



Allowed



Not allowed

- Final states have no outgoing transitions
- In a final state the machine halts

Acceptance

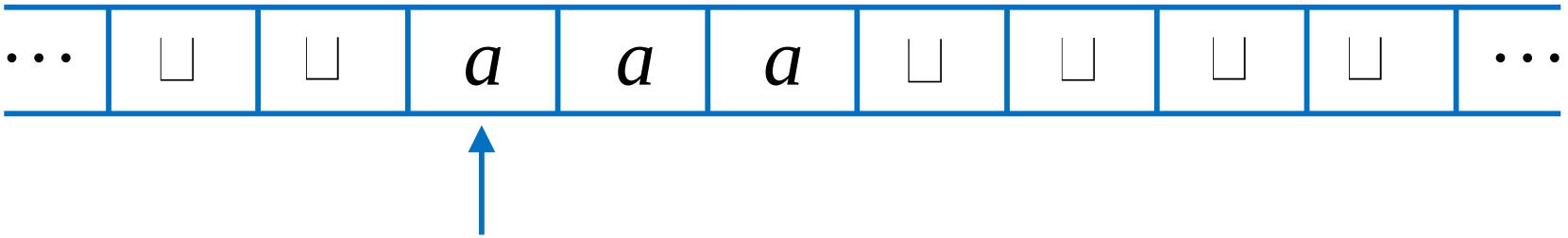
Accept Input → If machine halts in a **final state**

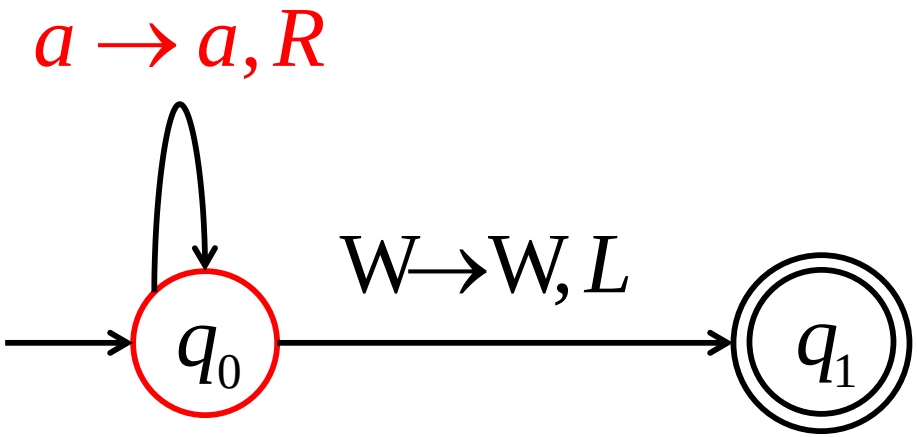
Reject Input → If machine halts in a **non-final state** or If machine enters an **infinite loop**

Example: Turing Machine

A Turing machine that accepts the language: a^*

Time 1

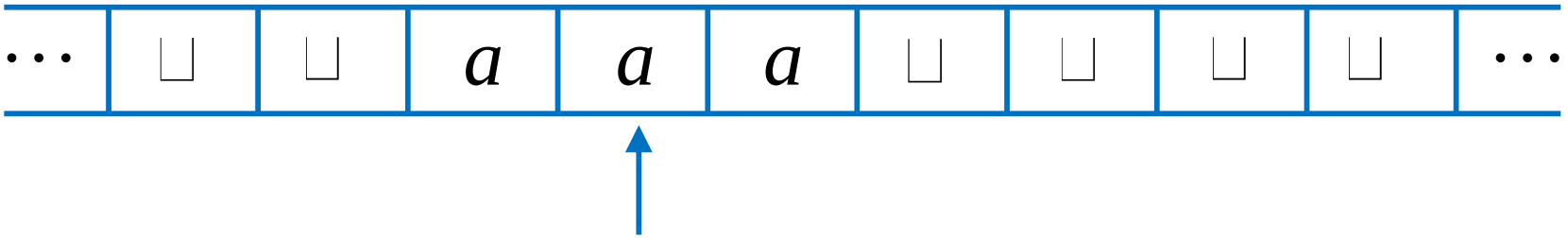


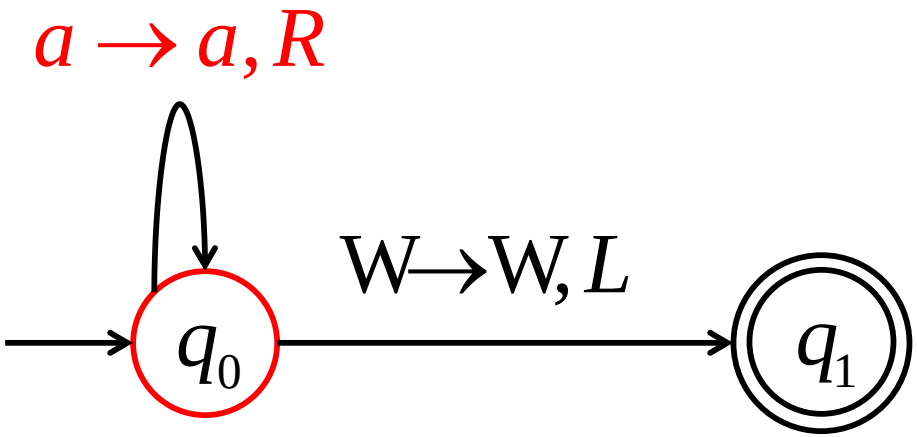


Example: Turing Machine

A Turing machine that accepts the language: a^*

Time 2

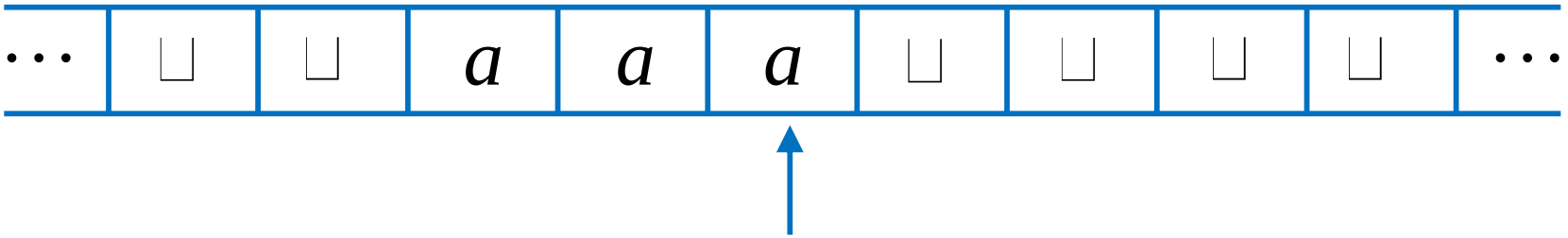


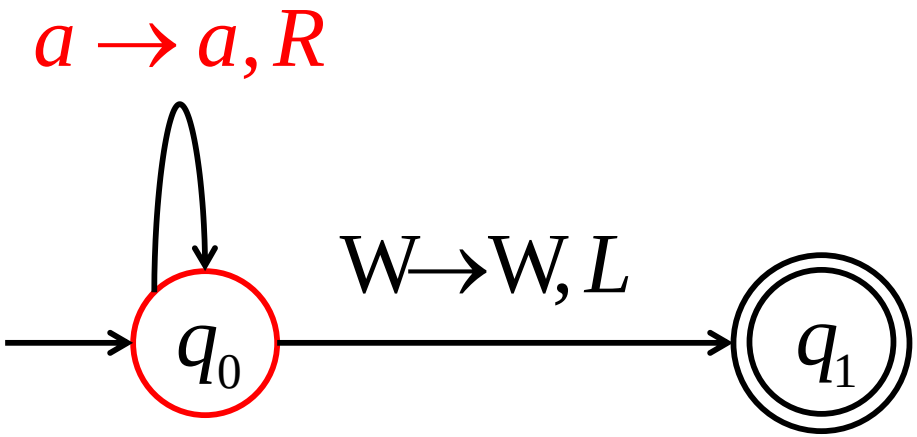


Example: Turing Machine

A Turing machine that accepts the language: a^*

Time 3

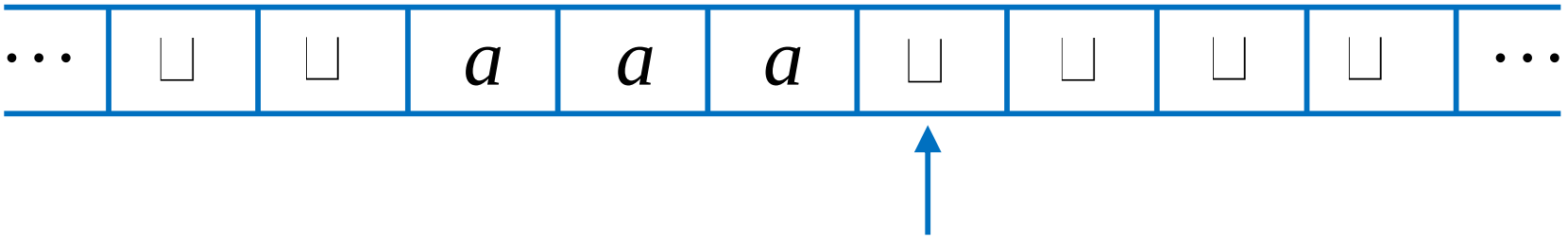


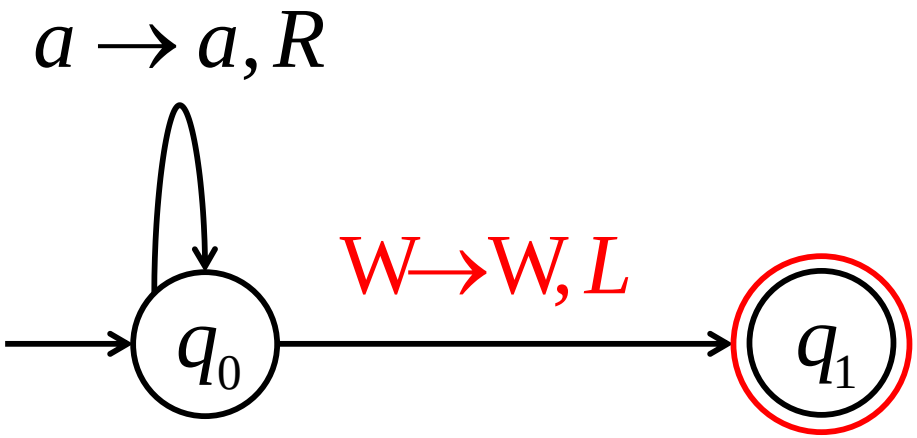


Example: Turing Machine

A Turing machine that accepts the language: a^*

Time 4

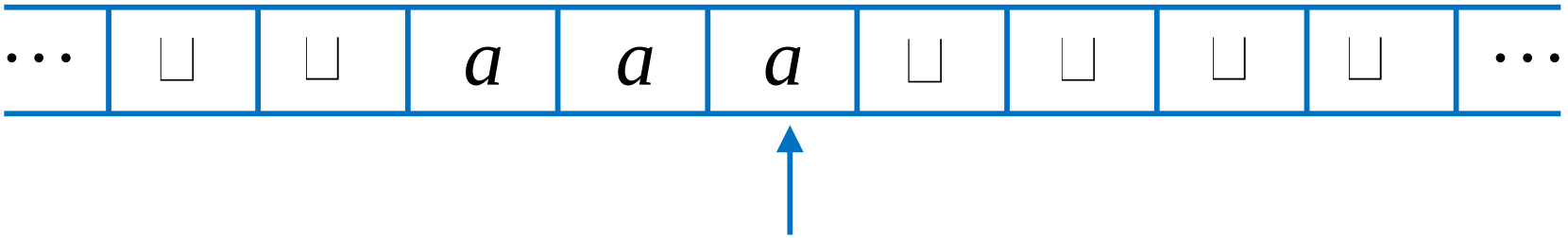


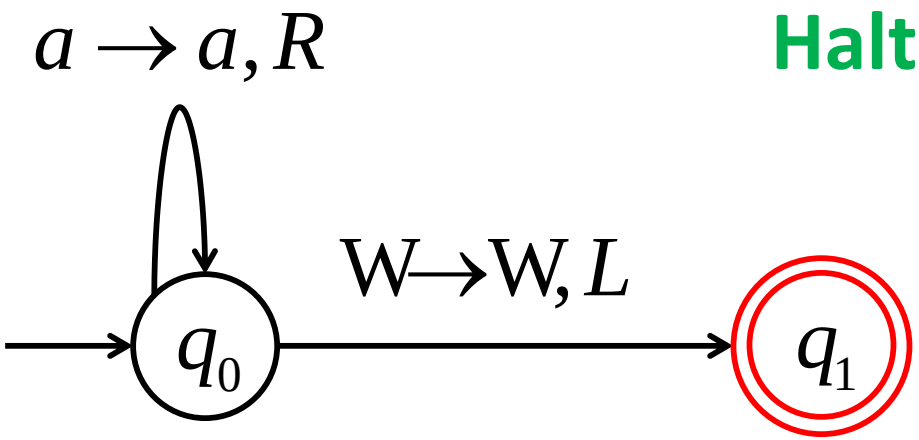


Example: Turing Machine

A Turing machine that accepts the language: a^*

Time 5



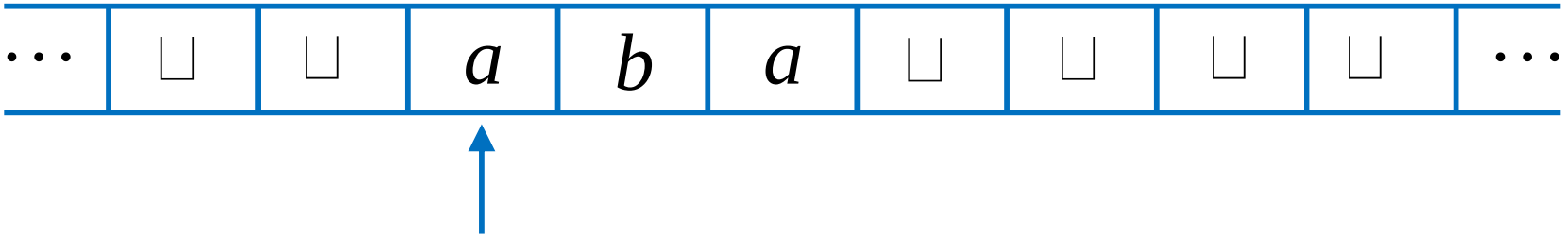


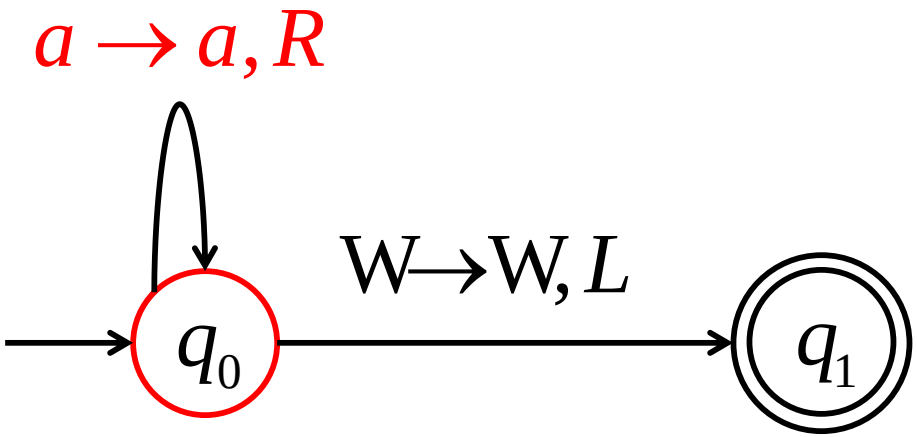
Halt & Accept

Example: Turing Machine

A Turing machine that accepts the language: a^*

Time 1

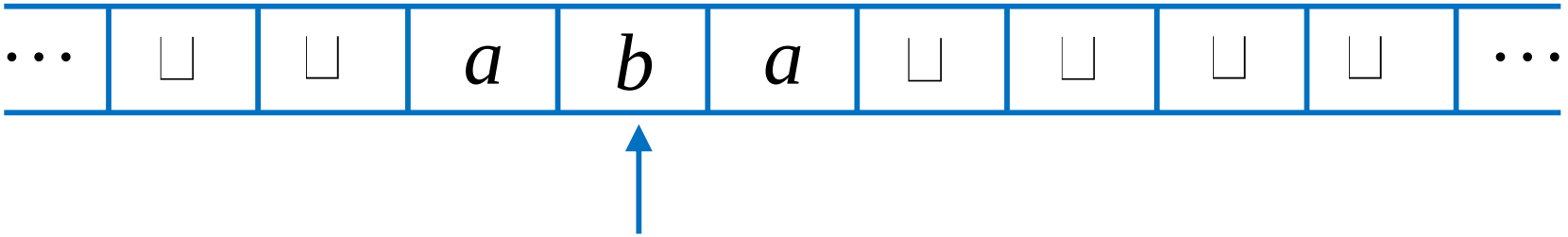




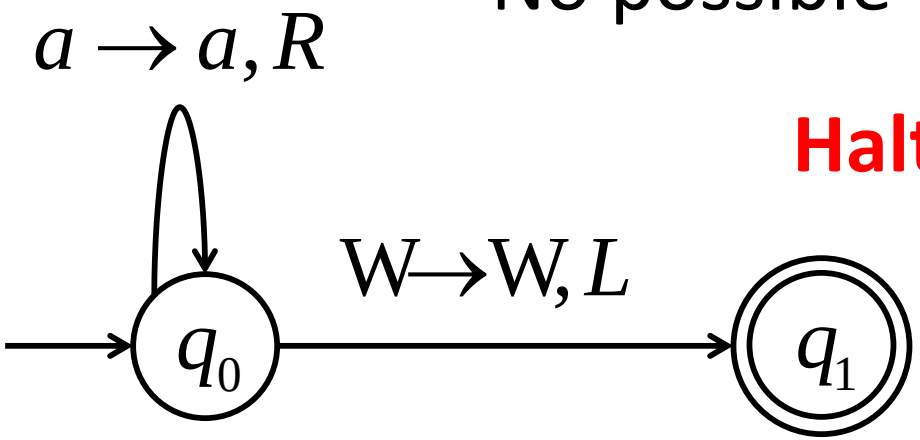
Example: Turing Machine

A Turing machine that accepts the language: a^*

Time 2



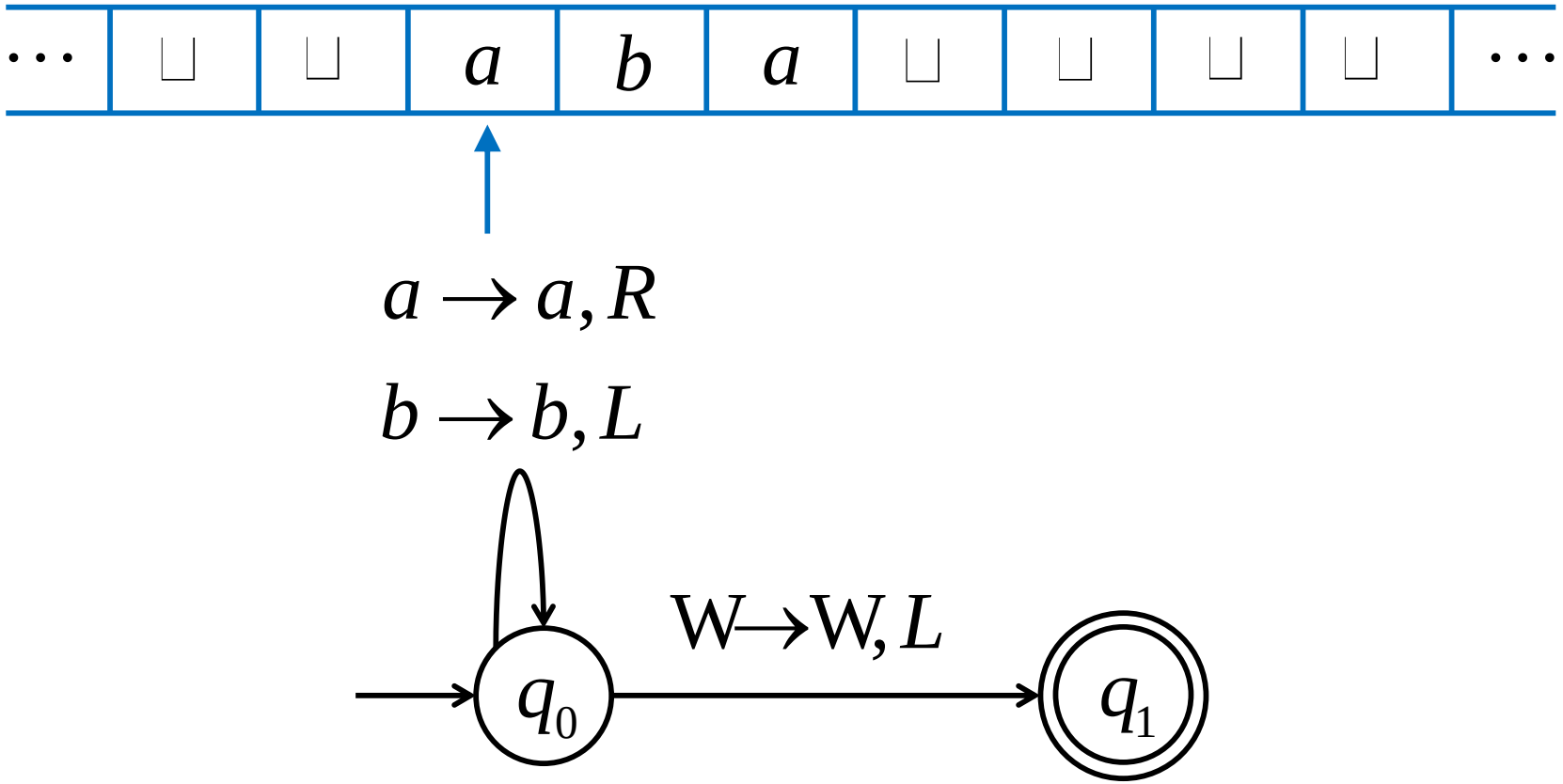
No possible Transition



Halt & Reject

Another Example: Infinite Loop

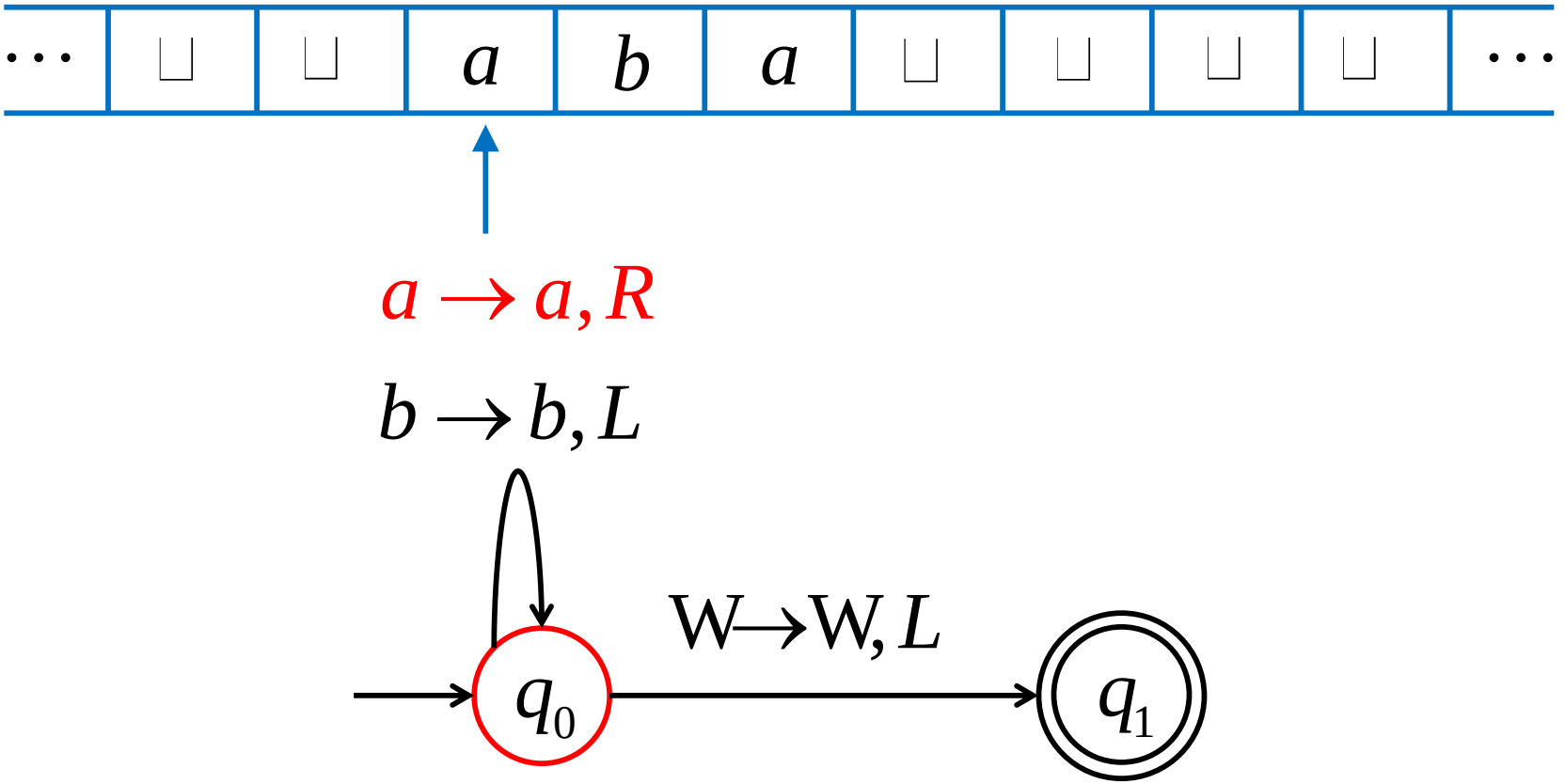
A Turing machine that accepts the language: a^*



Another Example: Infinite Loop

A Turing machine that accepts the language: a^*

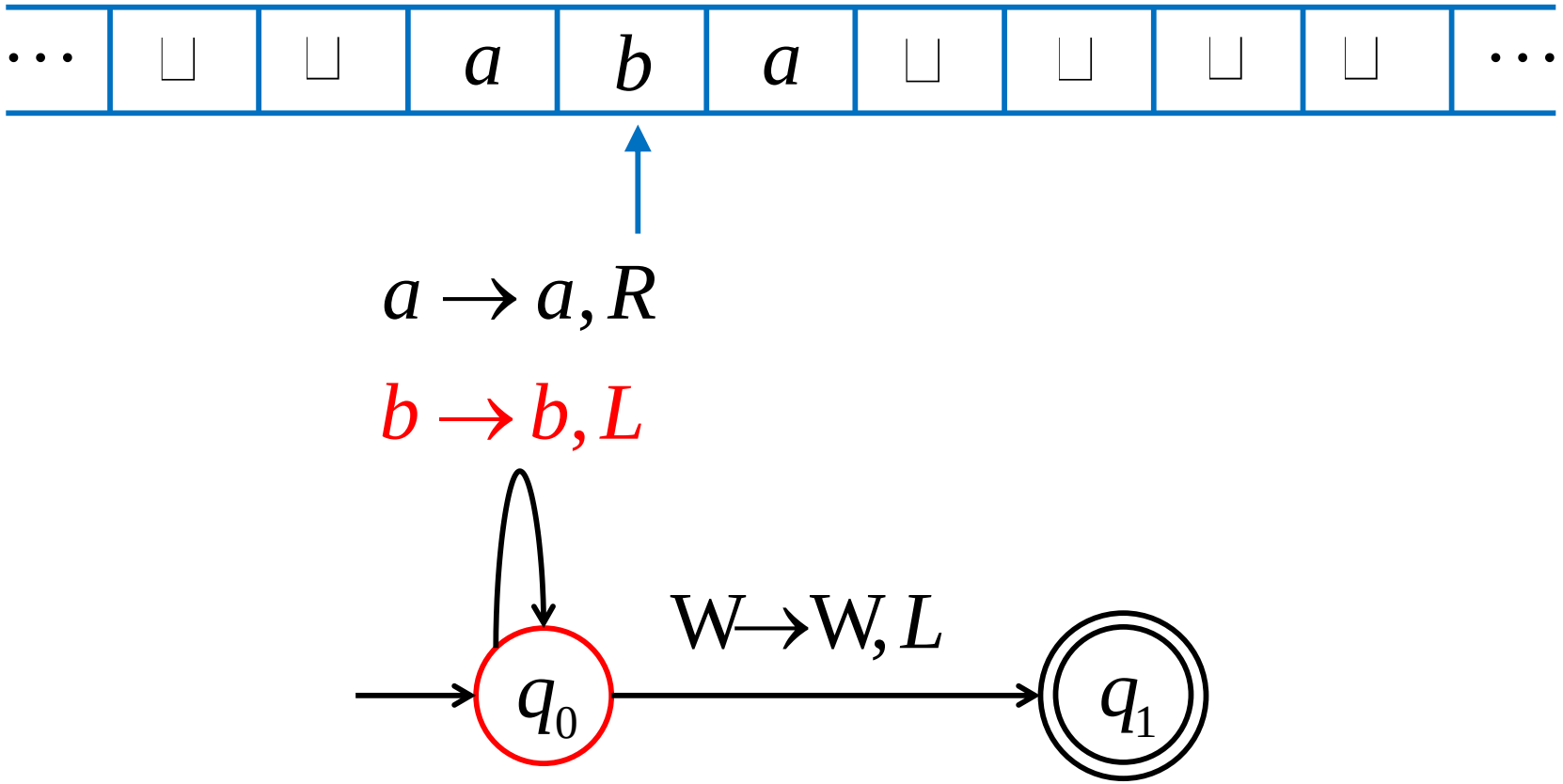
Time 1



Another Example: Infinite Loop

A Turing machine that accepts the language: a^*

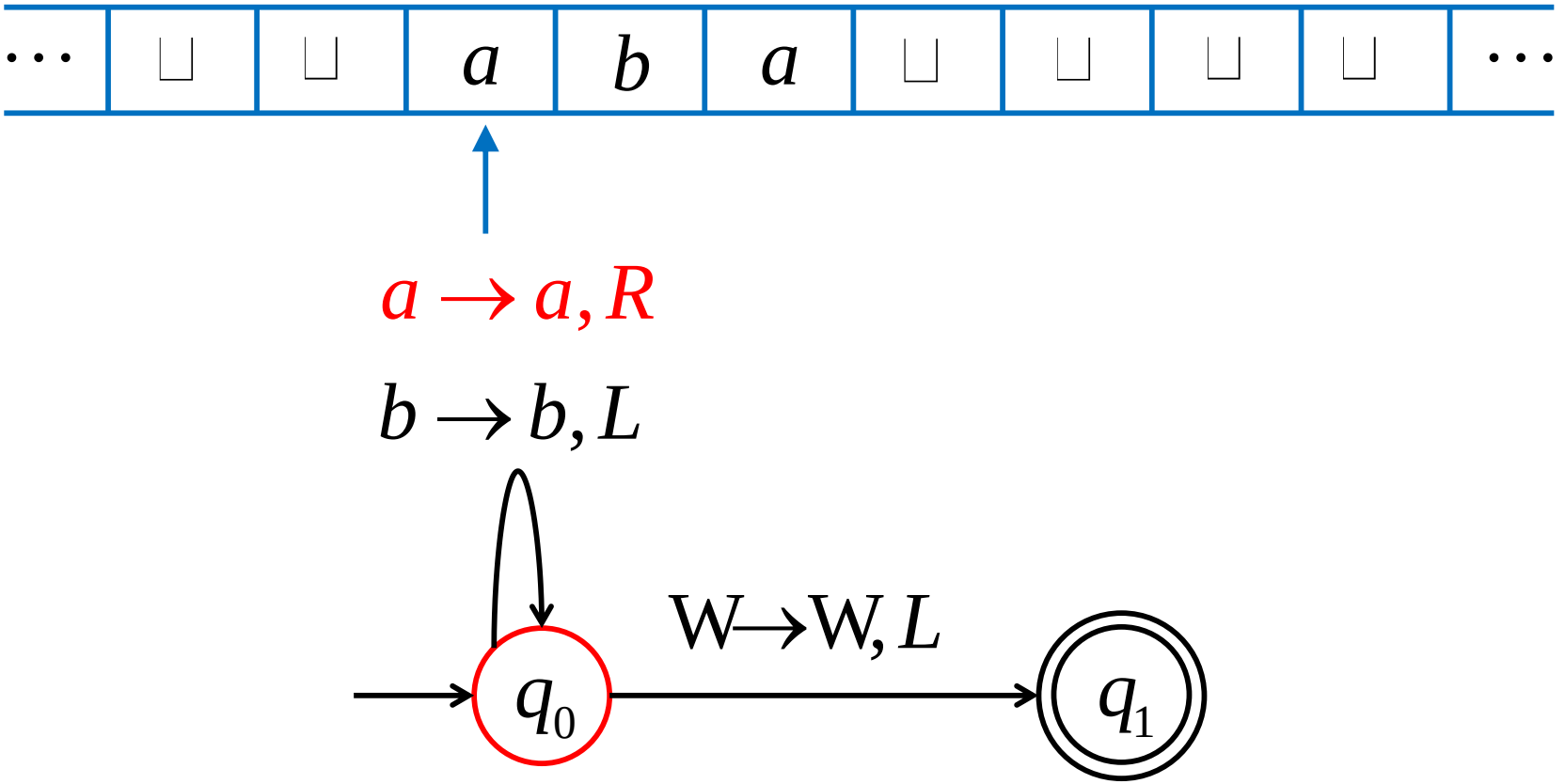
Time 2



Another Example: Infinite Loop

A Turing machine that accepts the language: a^*

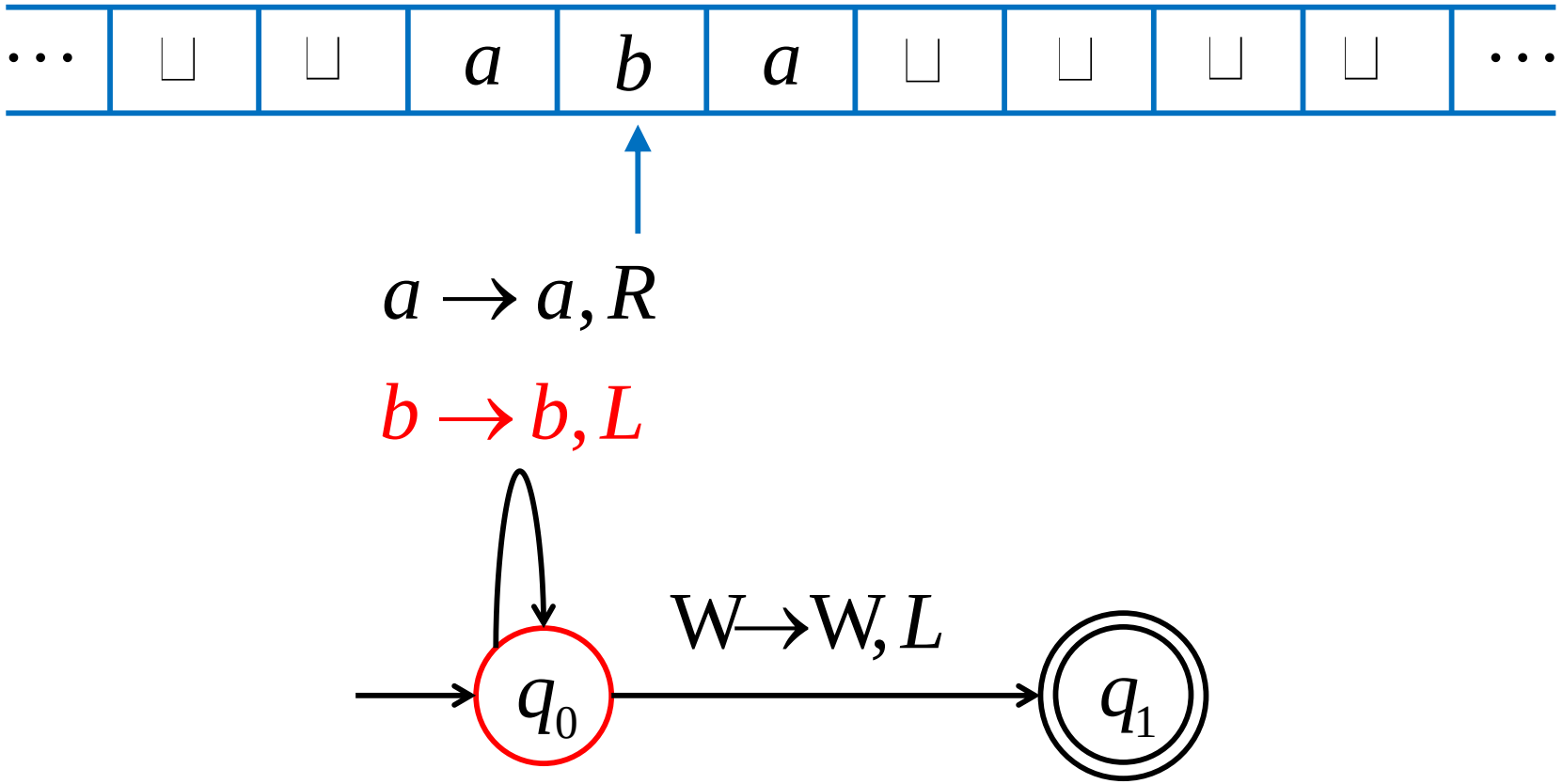
Time 3



Another Example: Infinite Loop

A Turing machine that accepts the language: a^*

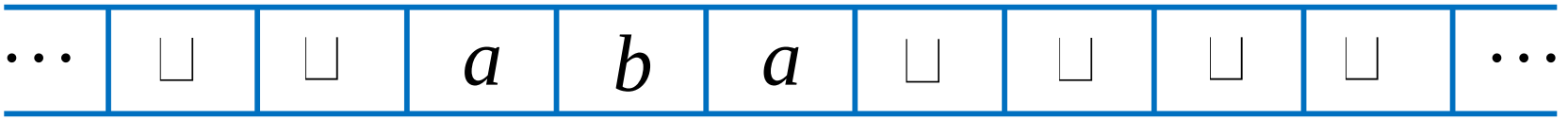
Time 4



Another Example: Infinite Loop

A Turing machine that accepts the language: a^*

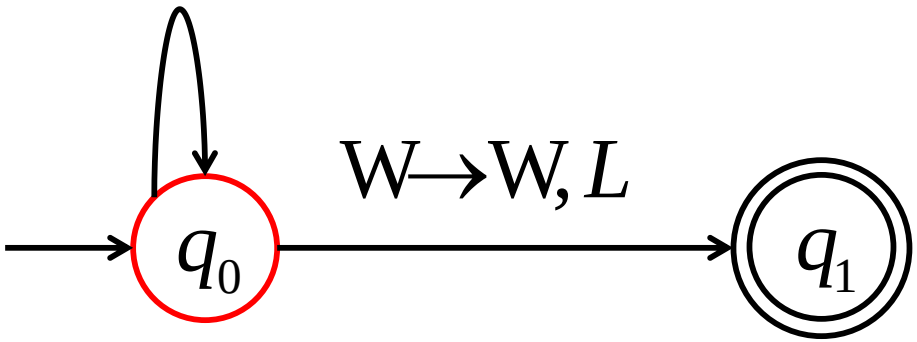
Time 5…



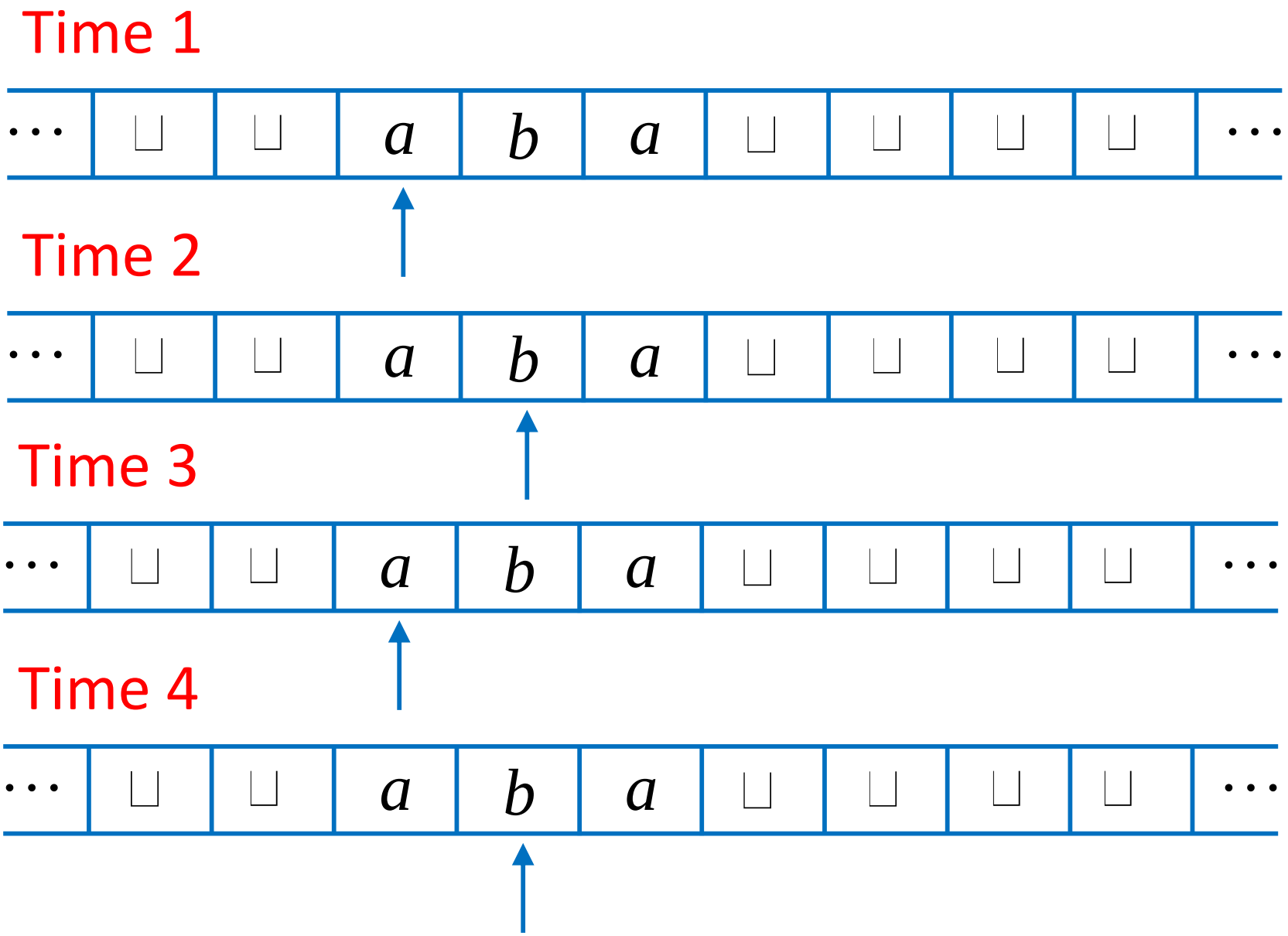
$a \to a, R$

$b \to b, L$

Infinite Loop



Another Example: Infinite Loop



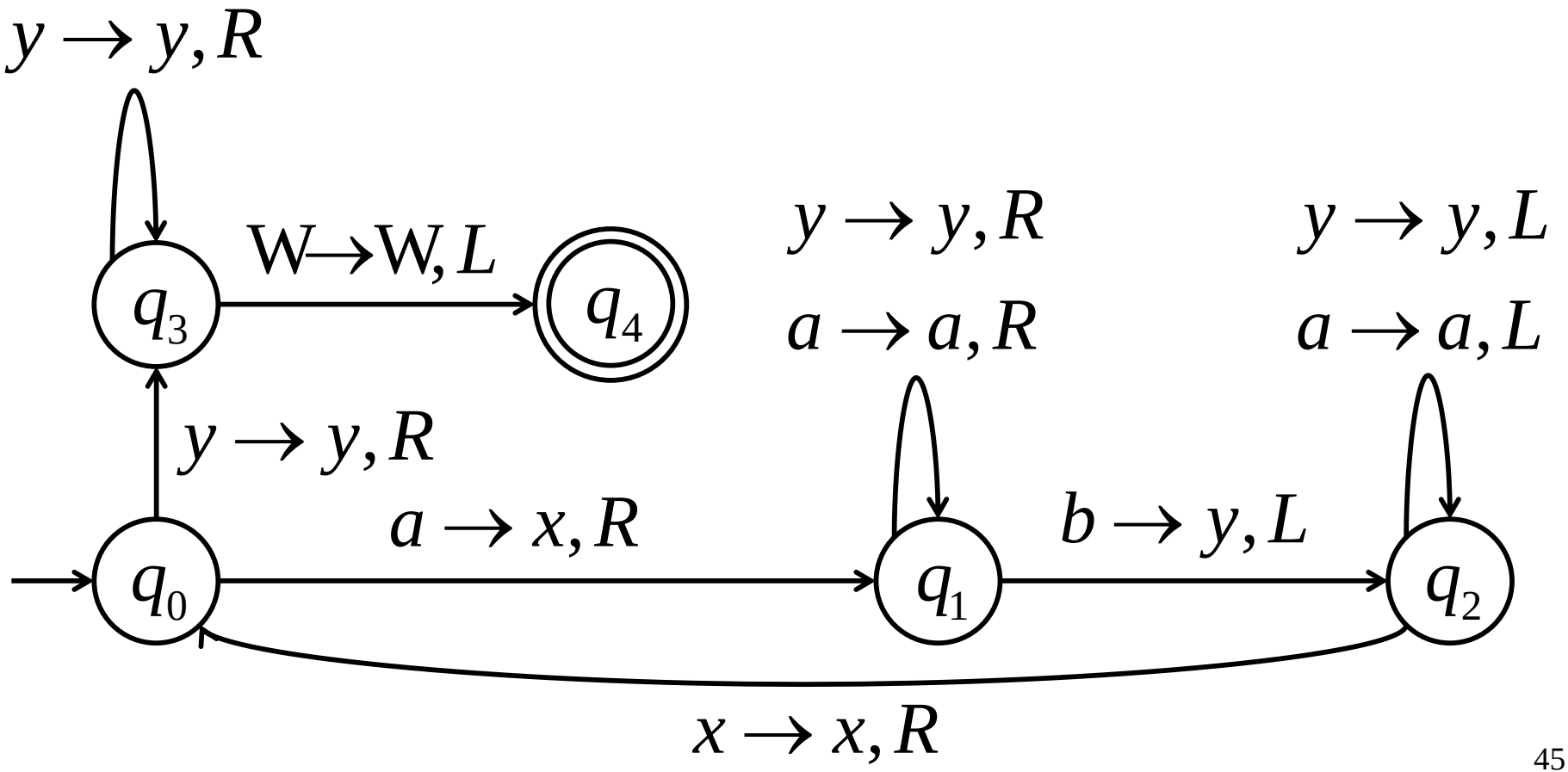
Infinite Loop

Because of the **infinite loop**:

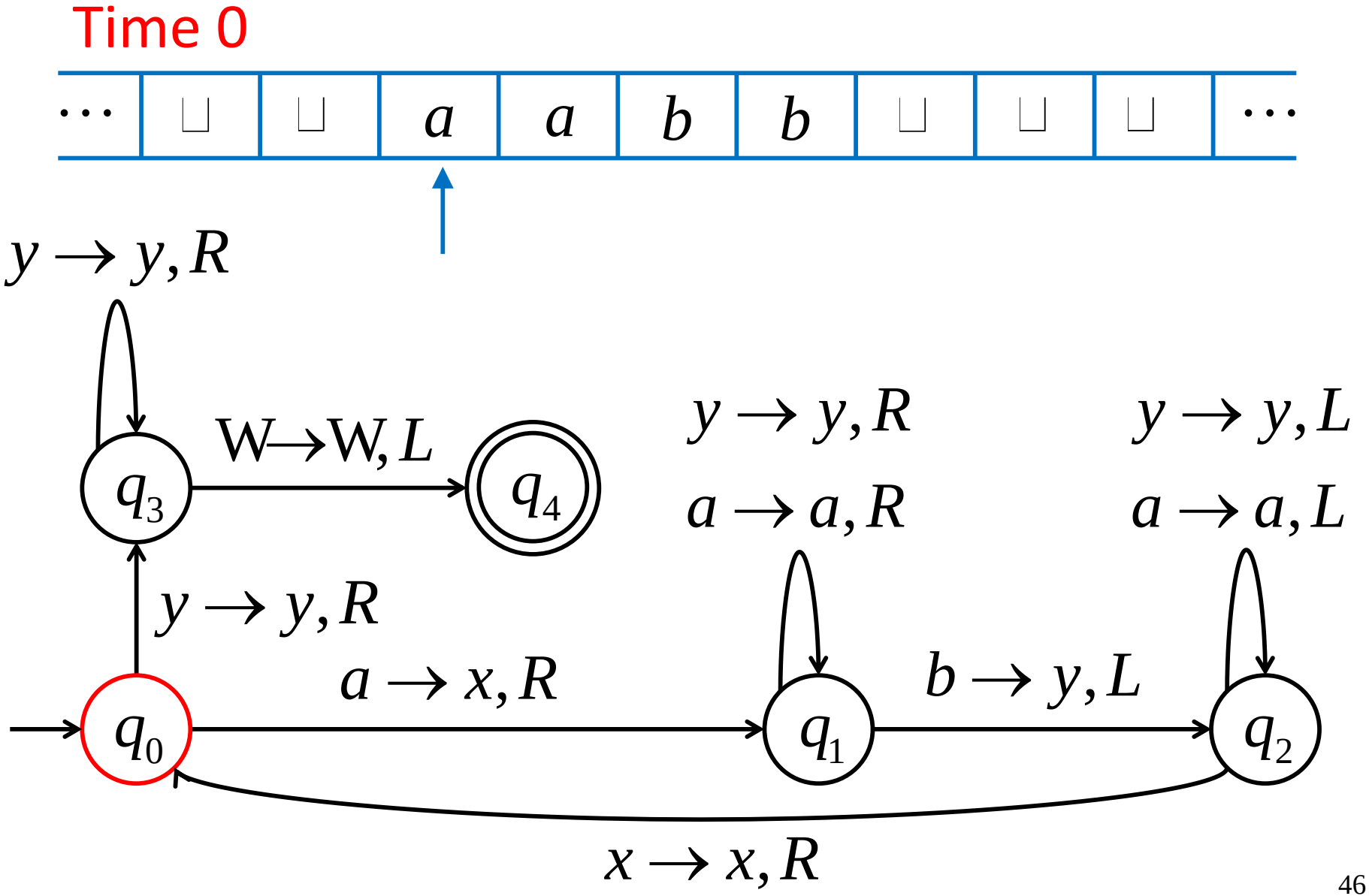
- The final state **cannot be reached**
- The machine **never halts**
- The input is **not accepted**

Another Example: Turing Machine

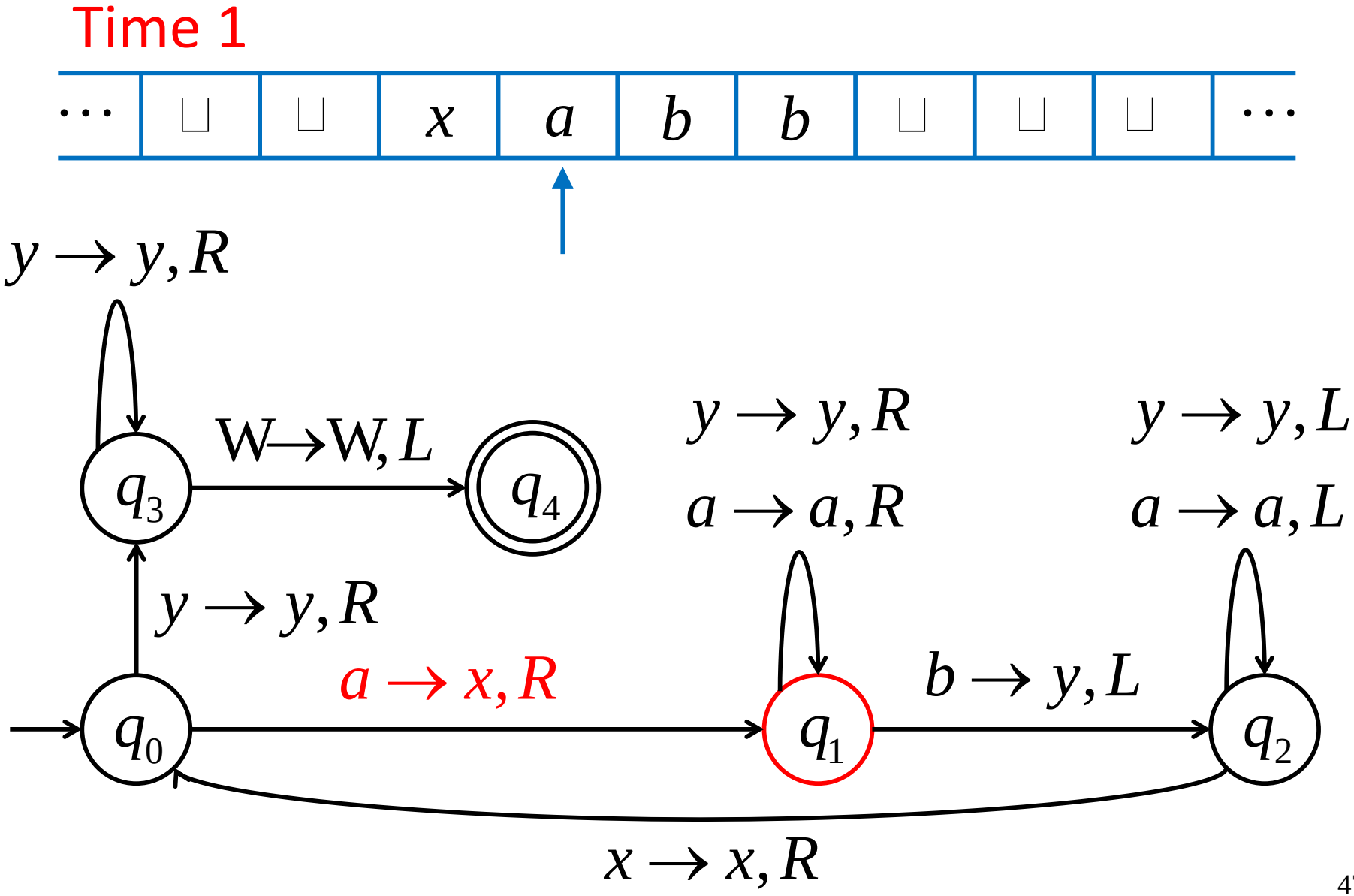
Turing machine for the language $\{a^n b^n\}$



Another Example: Turing Machine

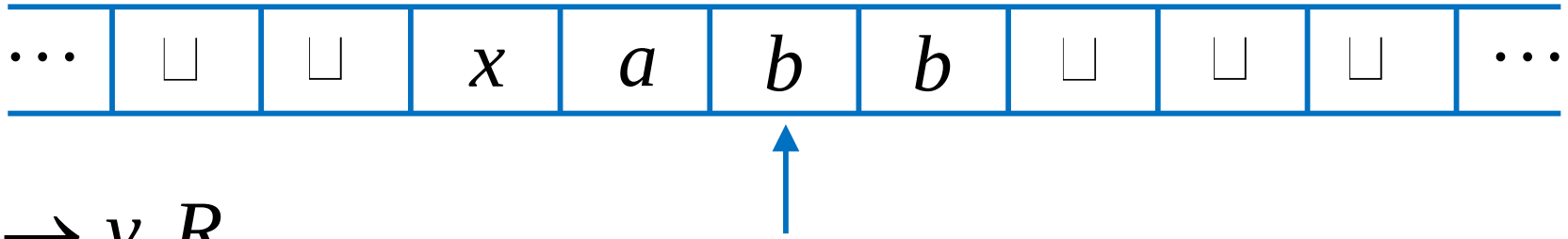


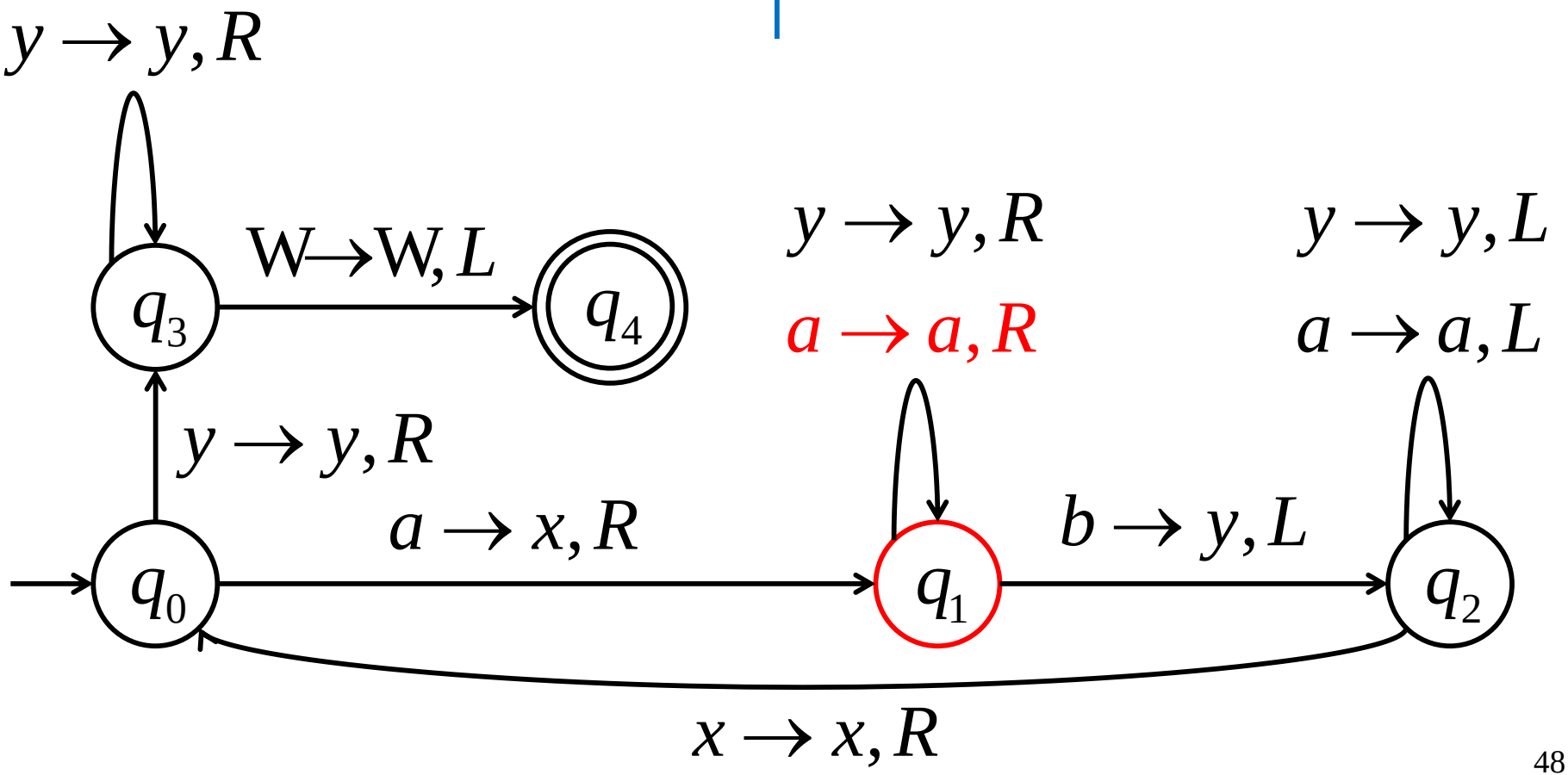
Another Example: Turing Machine



Another Example: Turing Machine

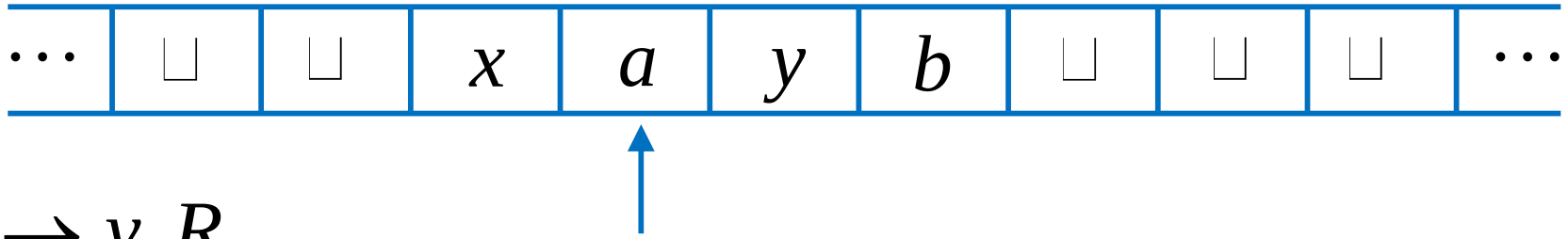
Time 2

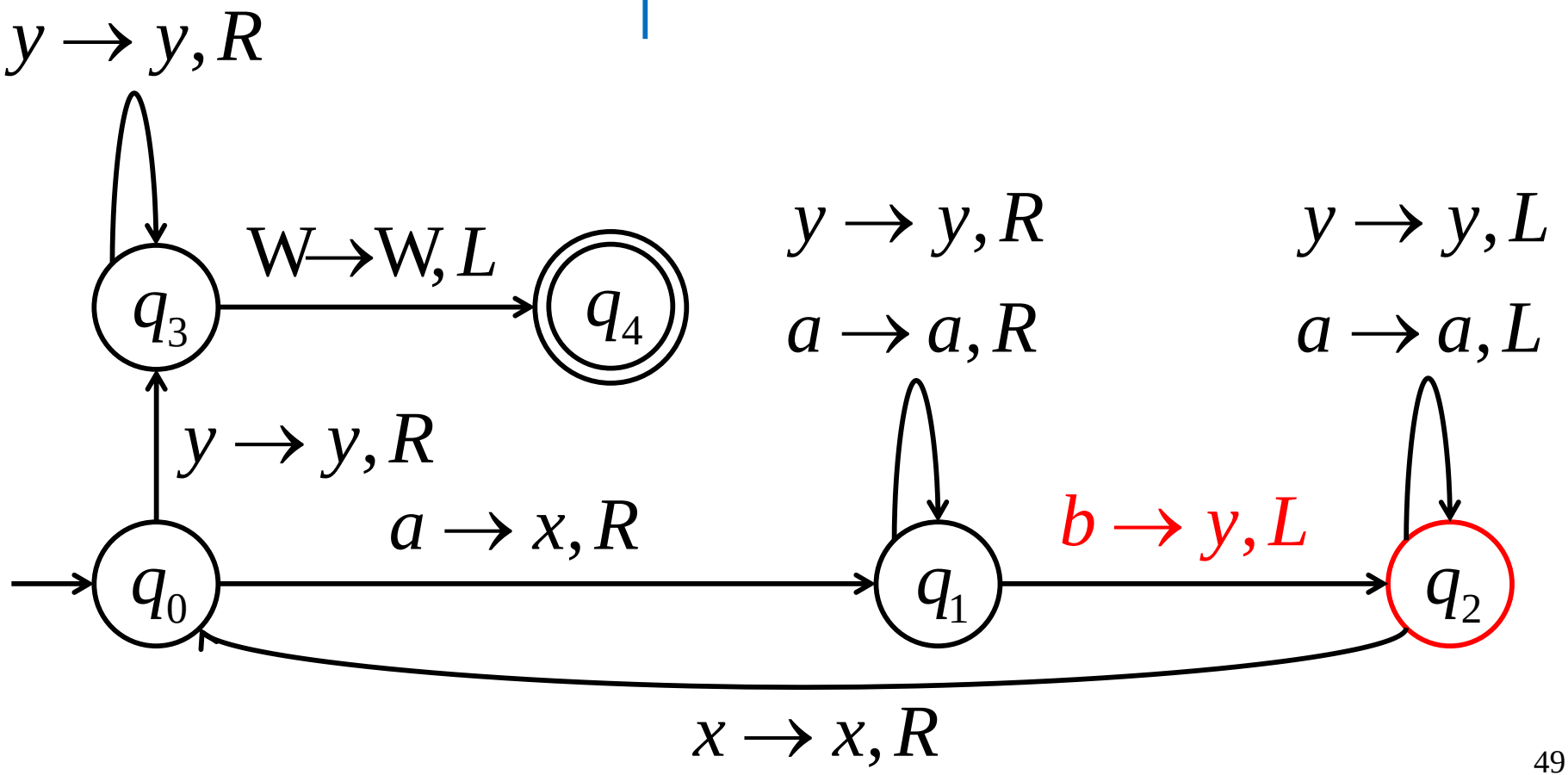




Another Example: Turing Machine

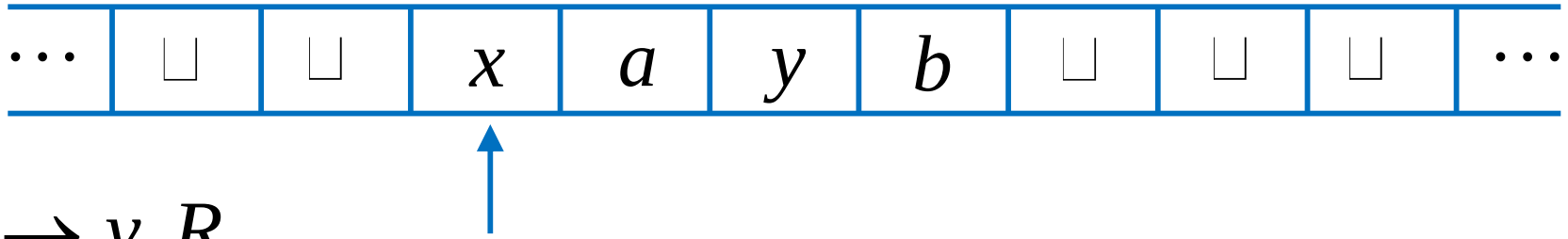
Time 3

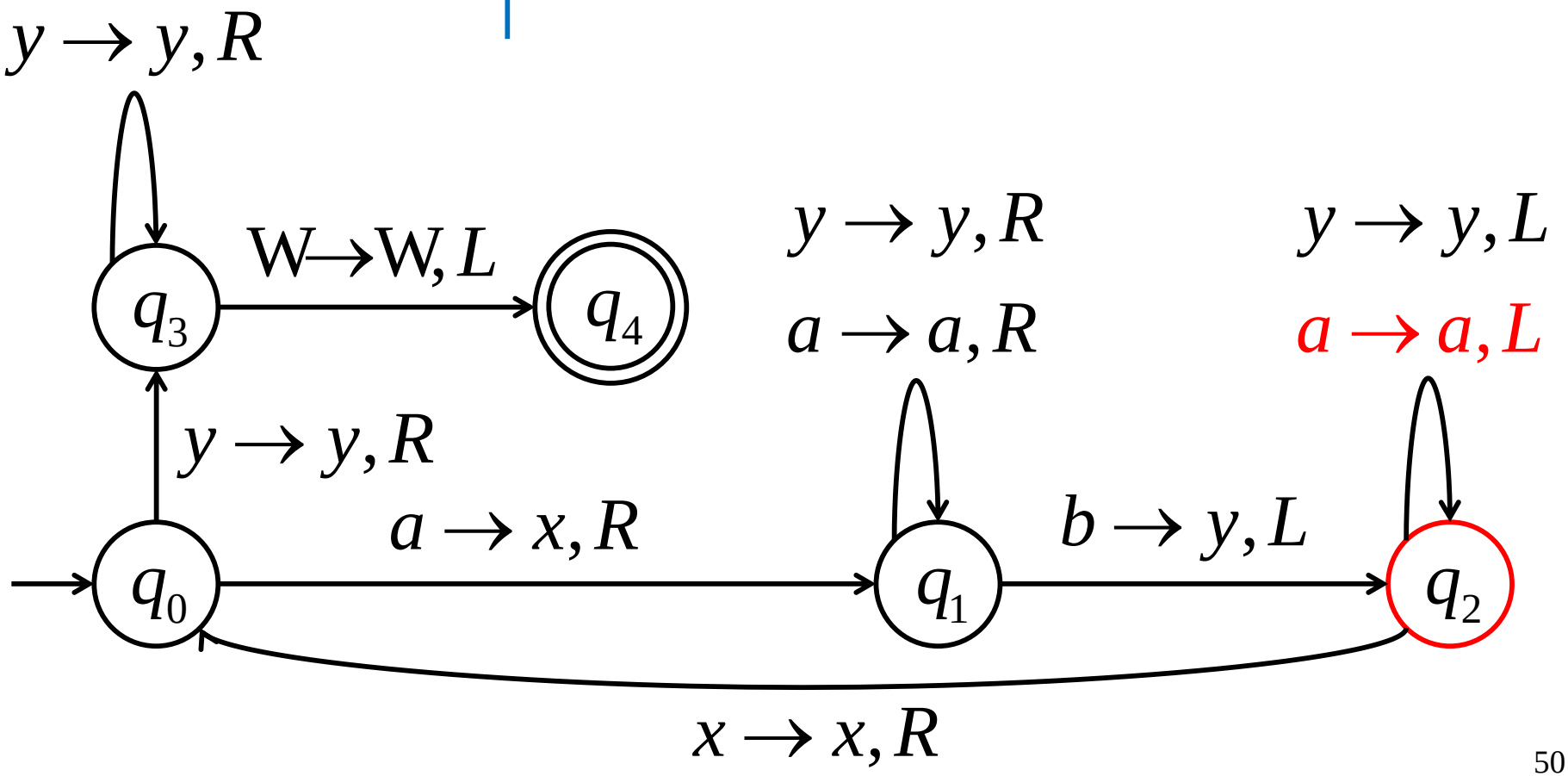




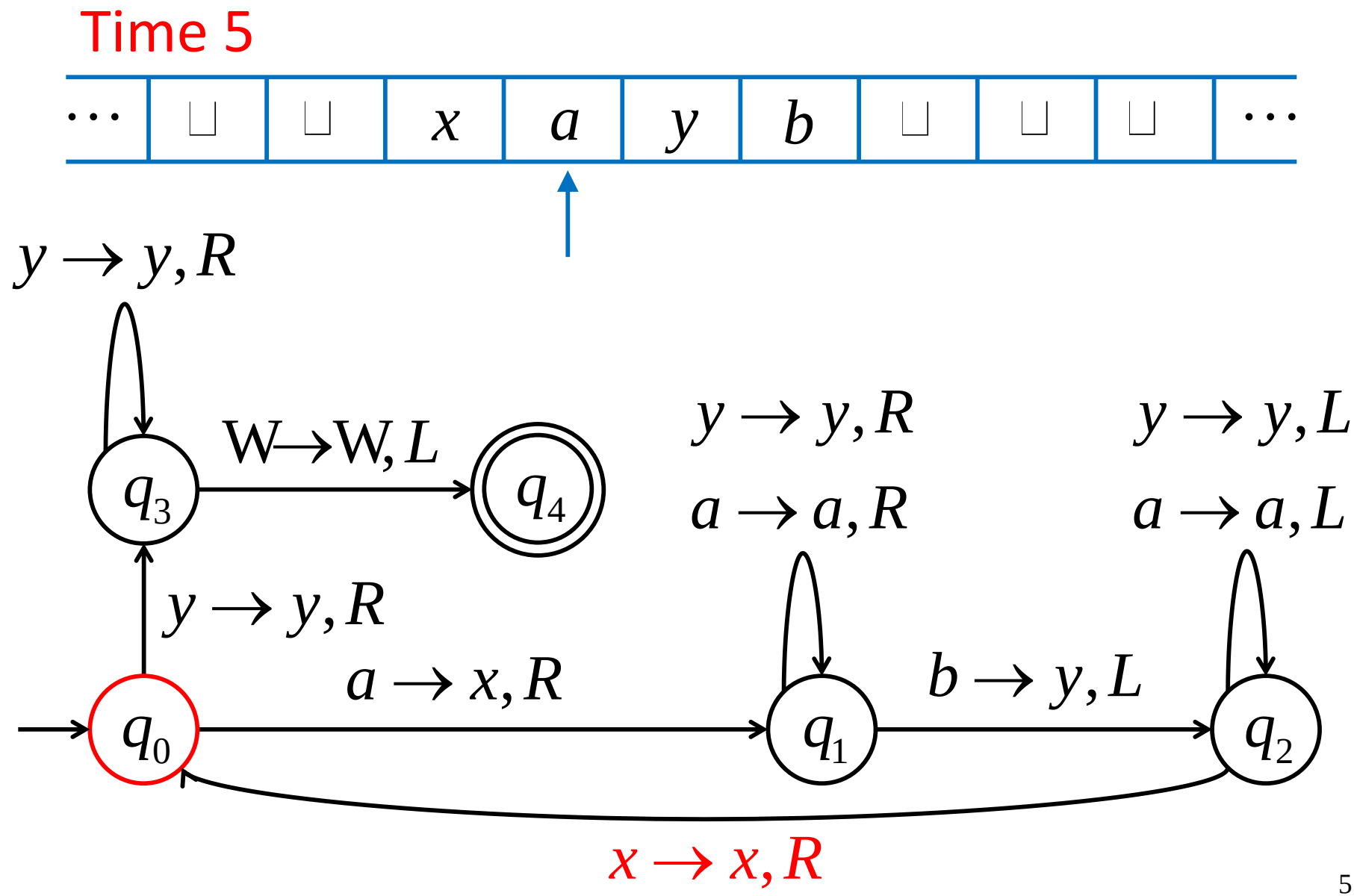
Another Example: Turing Machine

Time 4



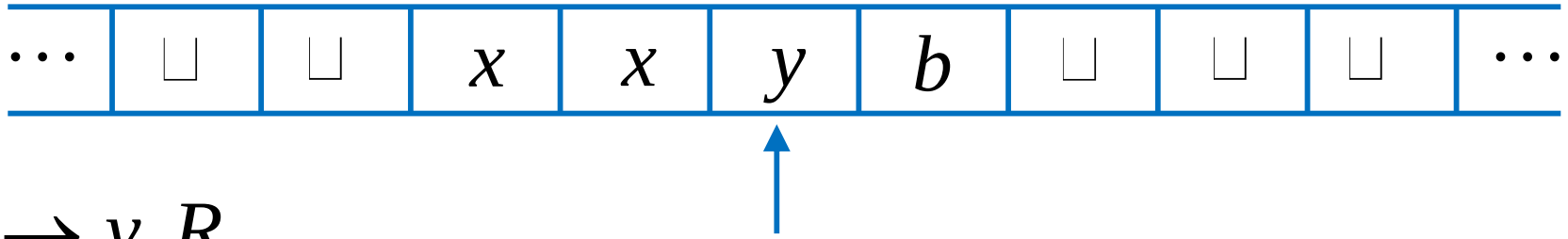


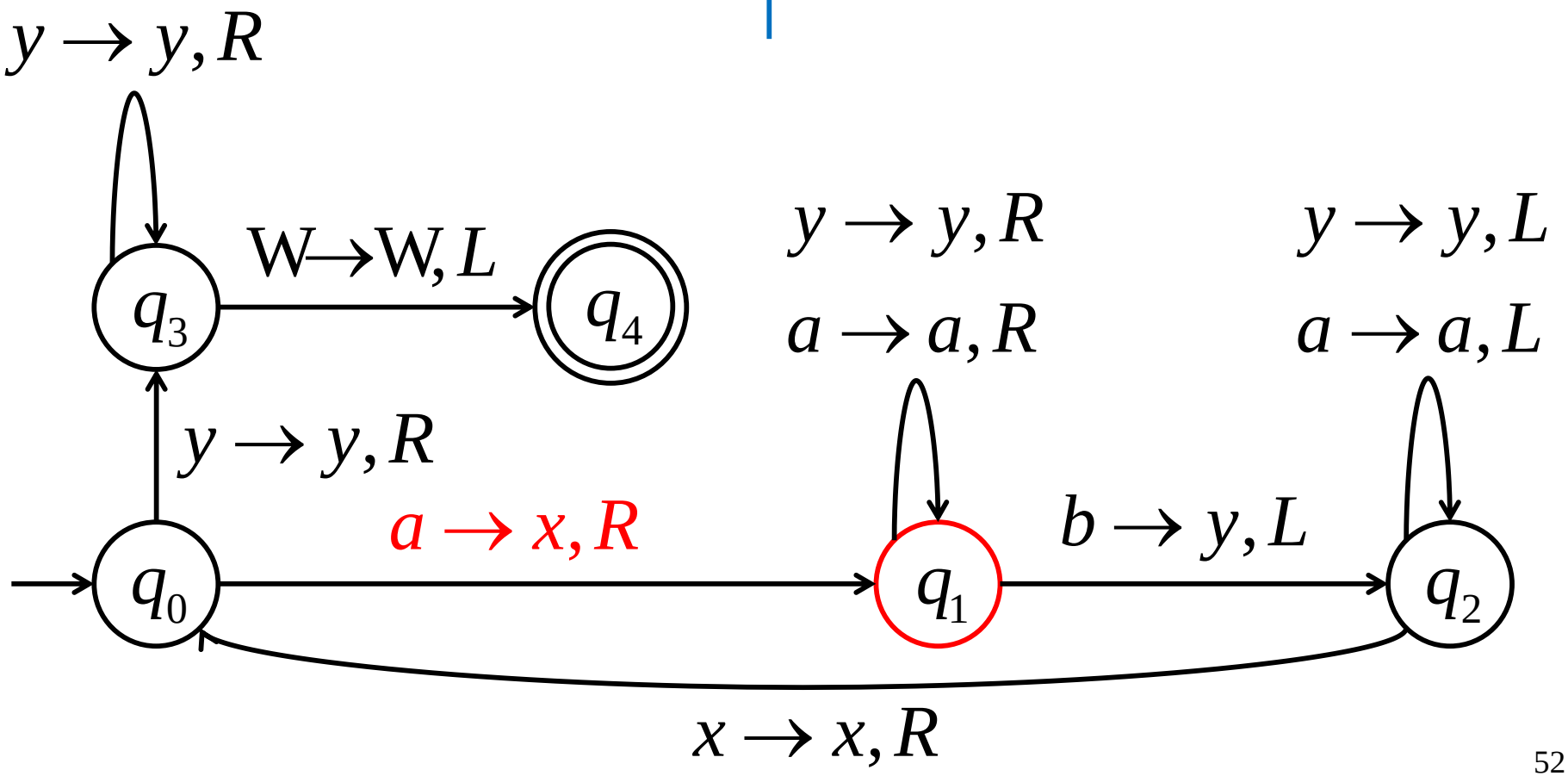
Another Example: Turing Machine



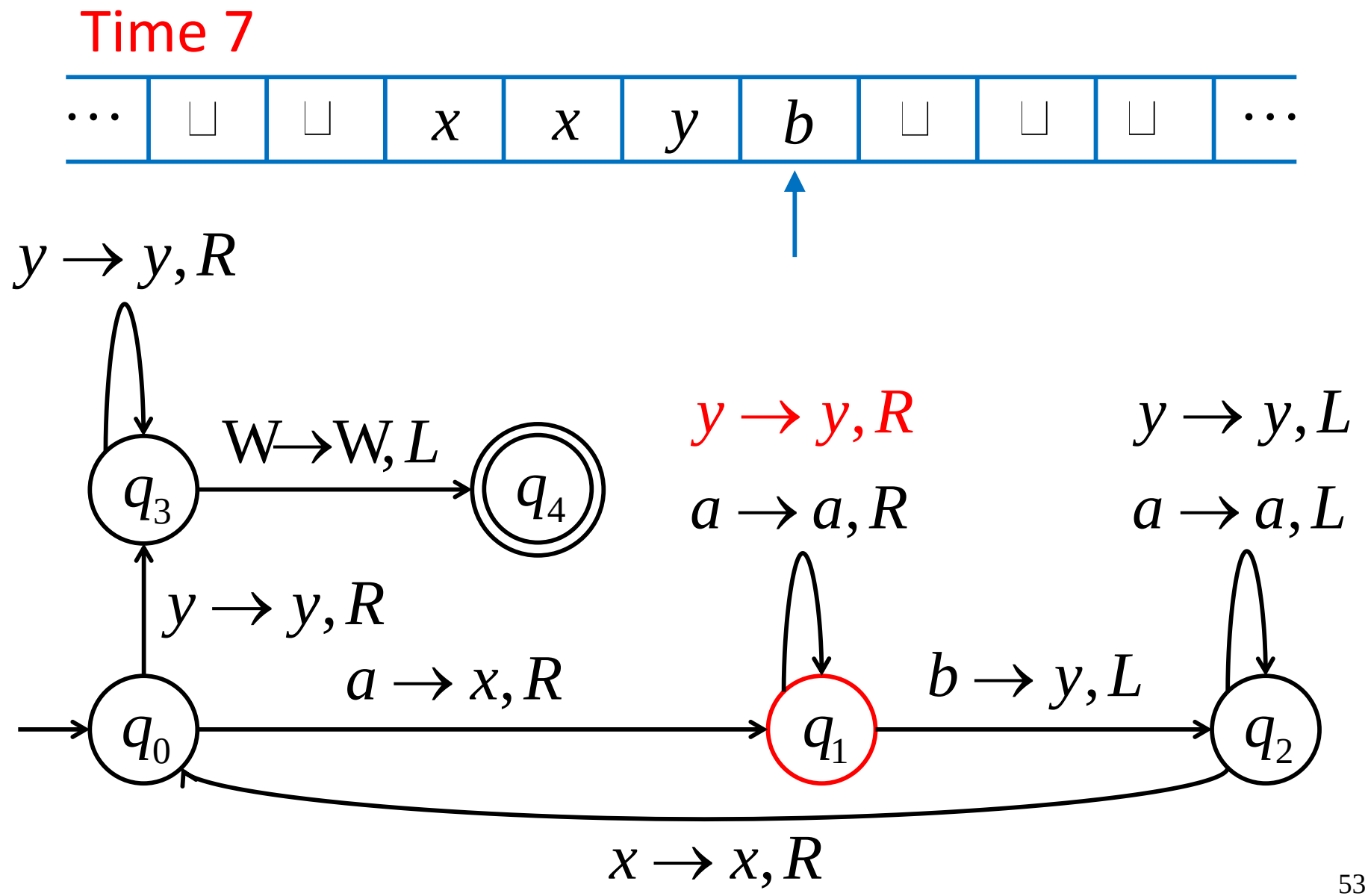
Another Example: Turing Machine

Time 6



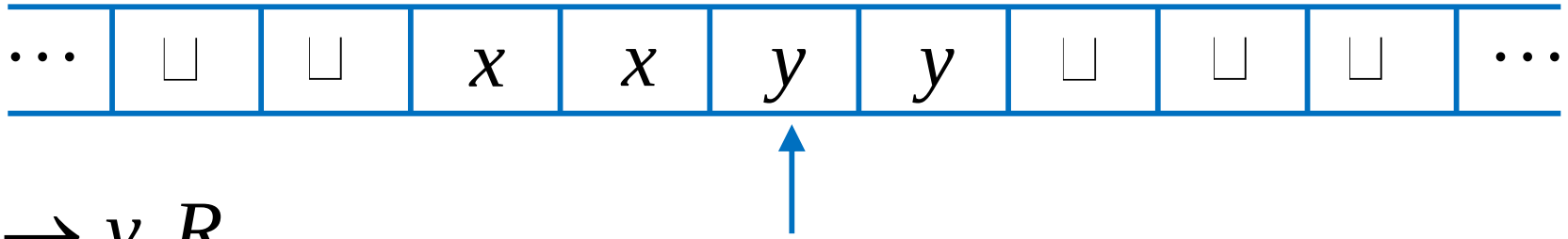


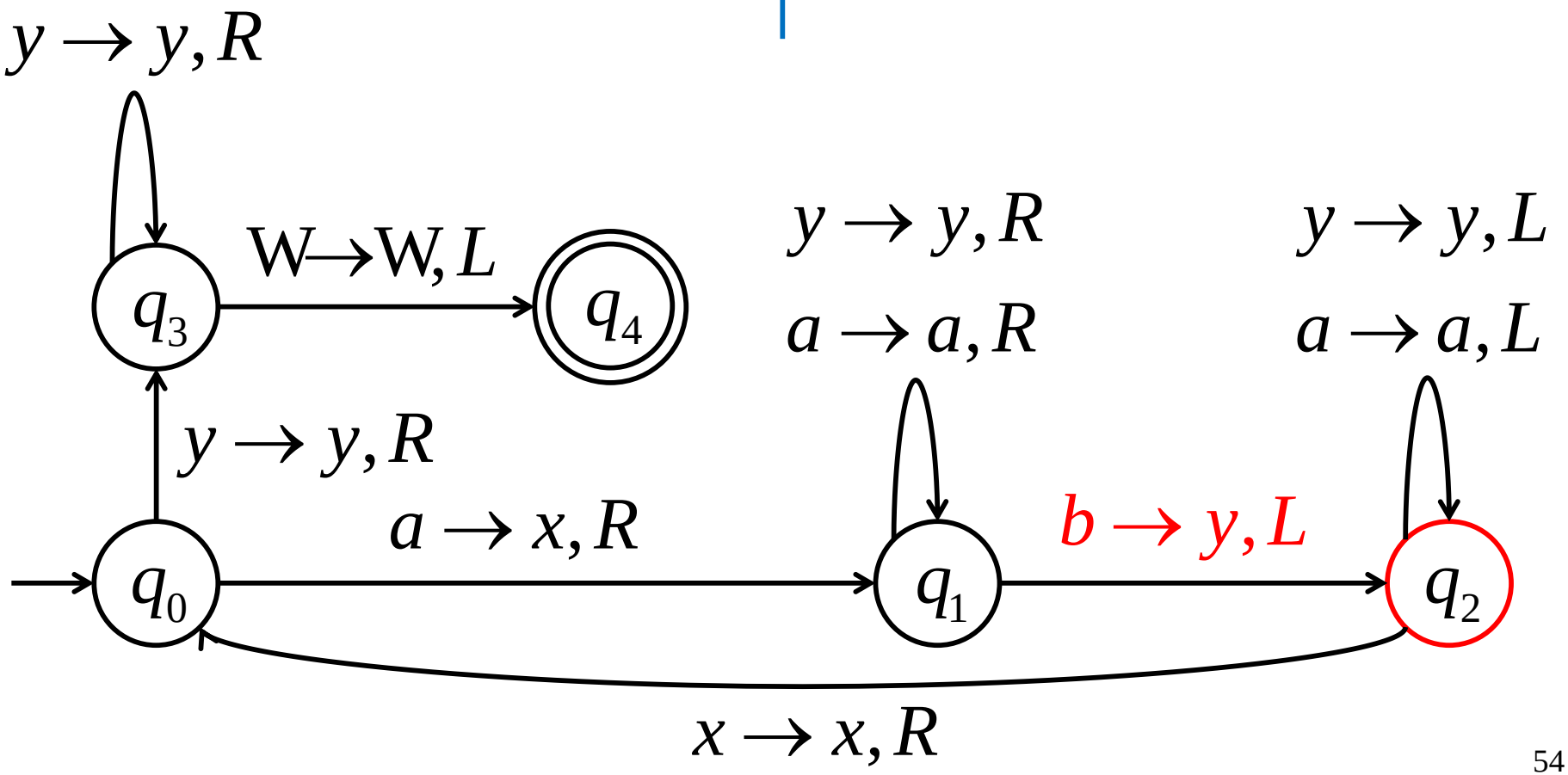
Another Example: Turing Machine



Another Example: Turing Machine

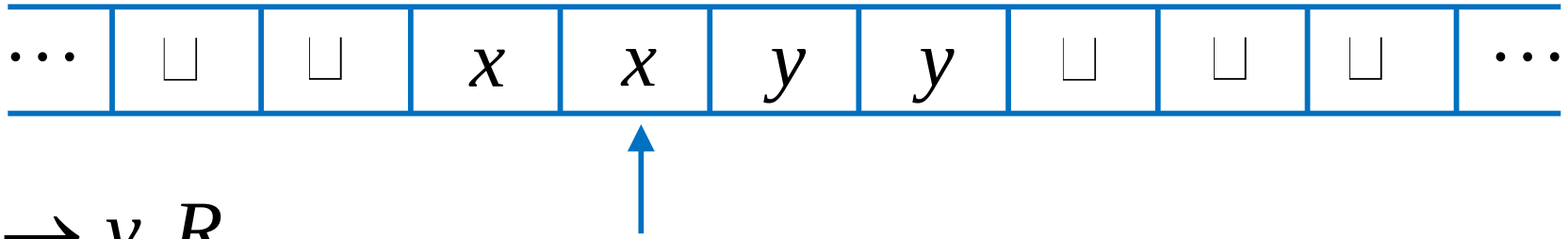
Time 8

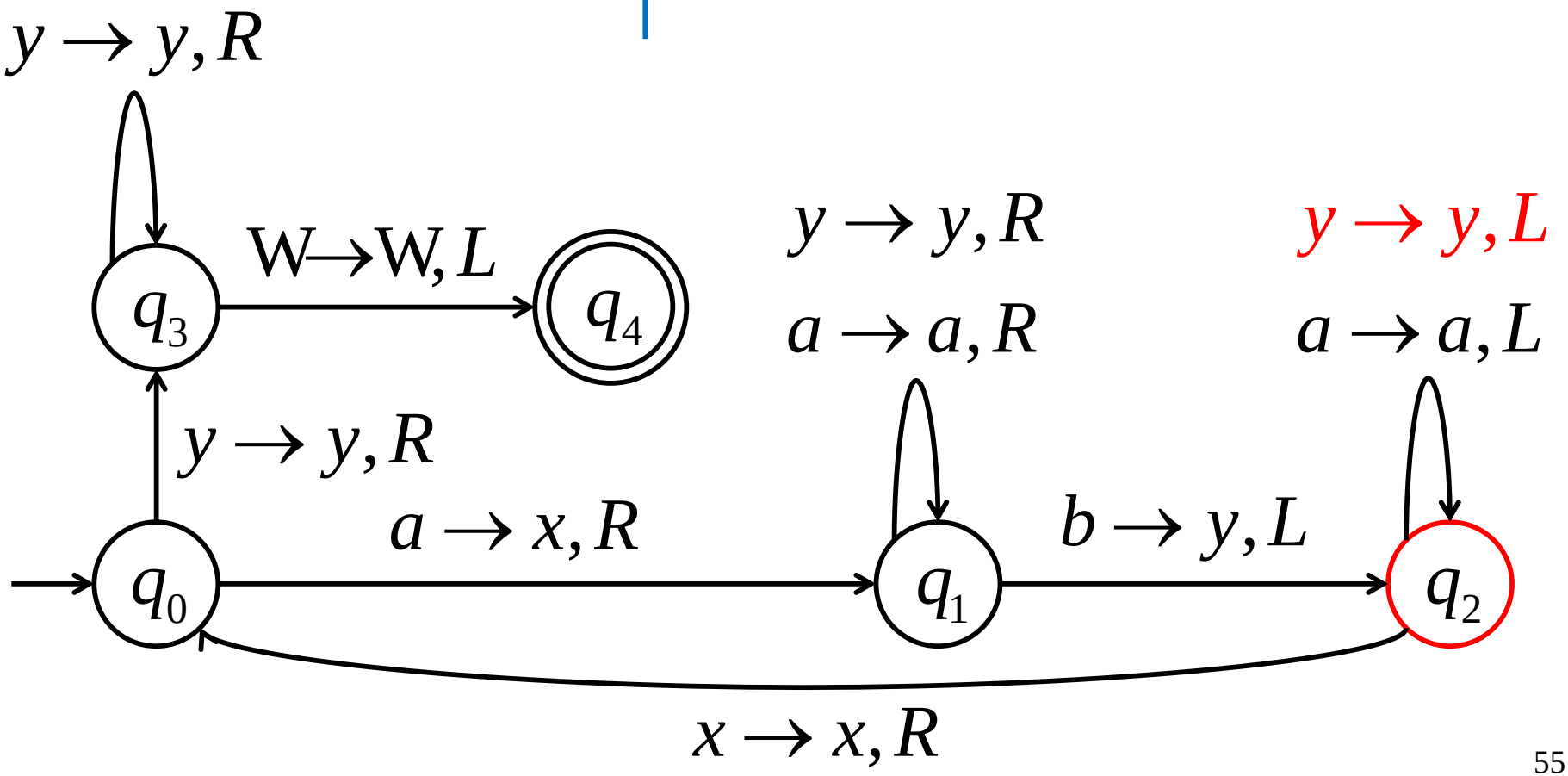




Another Example: Turing Machine

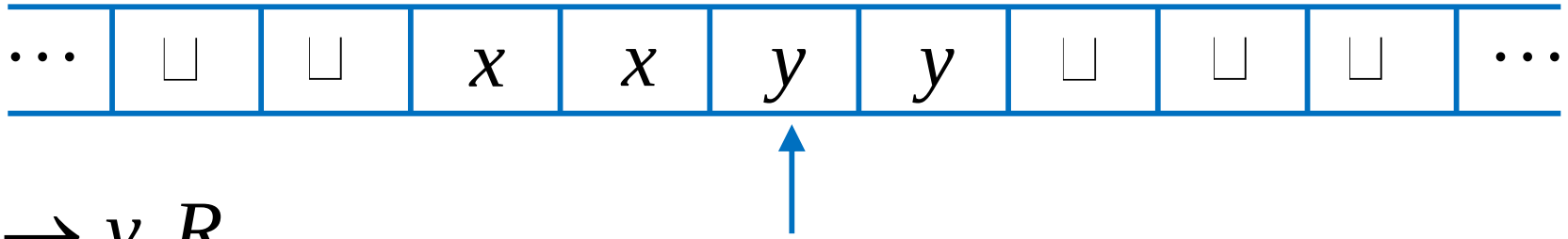
Time 9

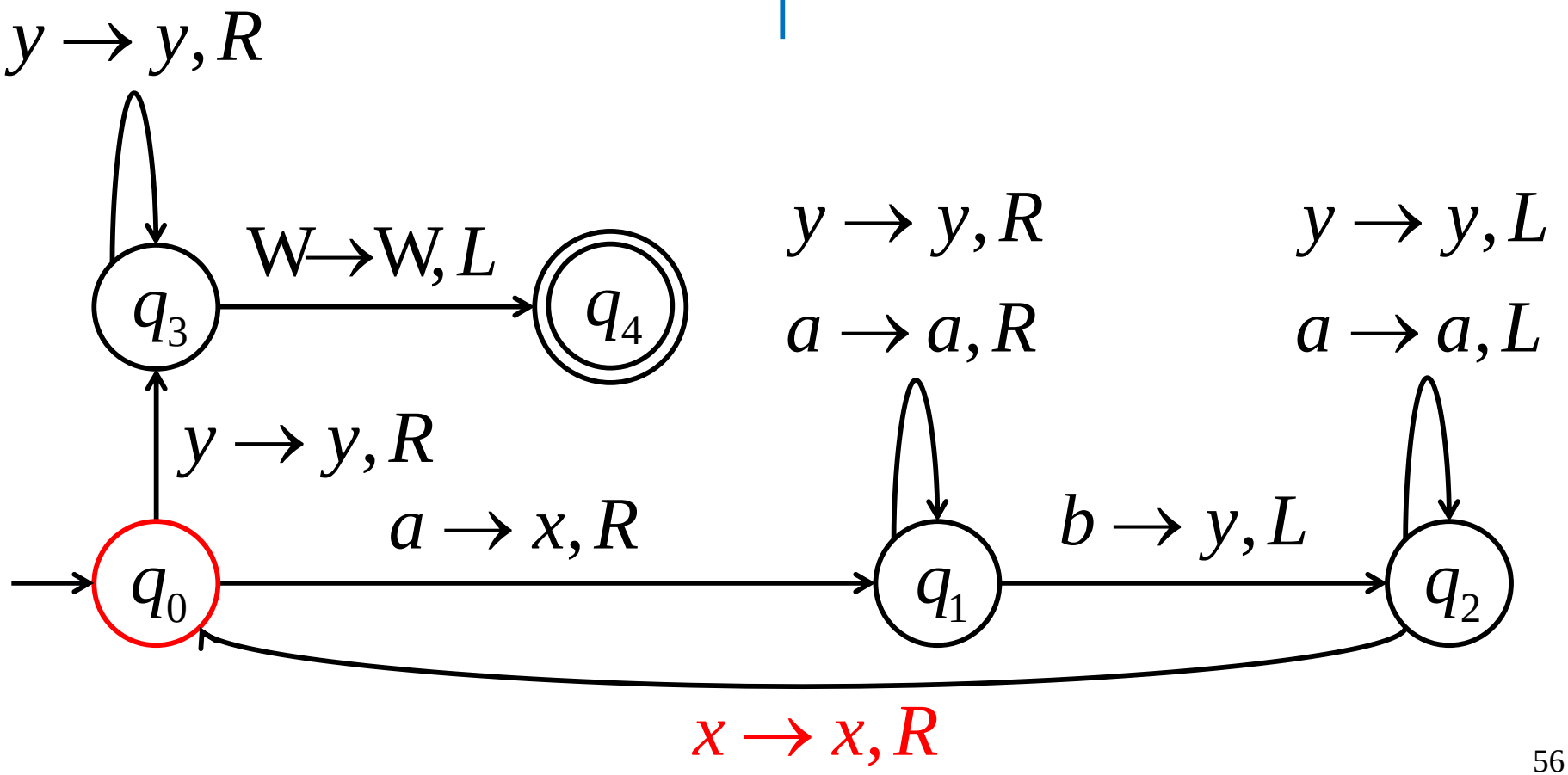




Another Example: Turing Machine

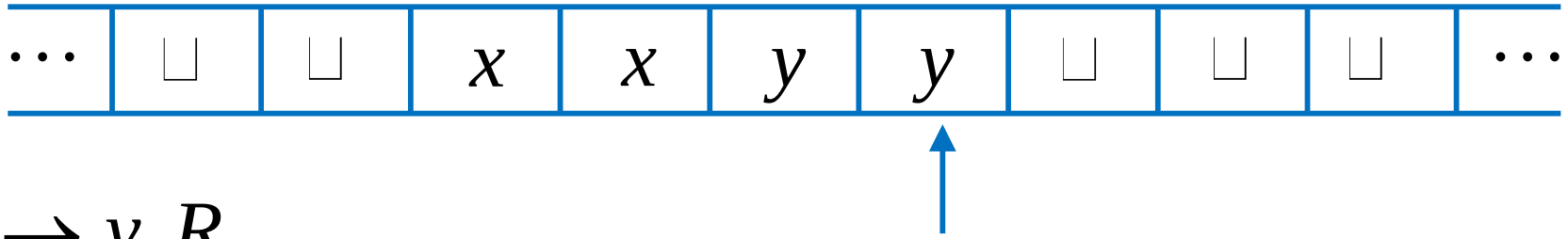
Time 10

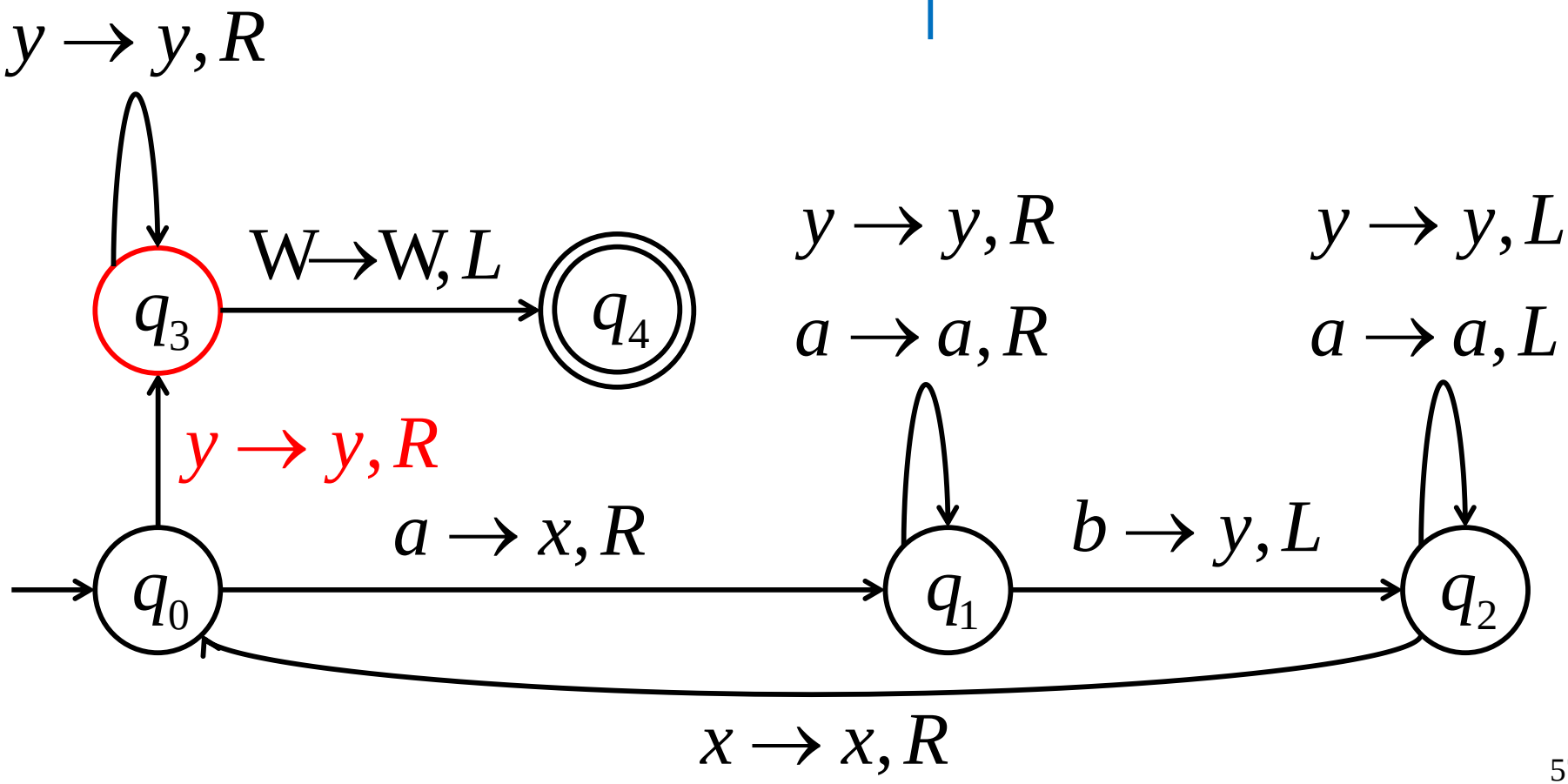




Another Example: Turing Machine

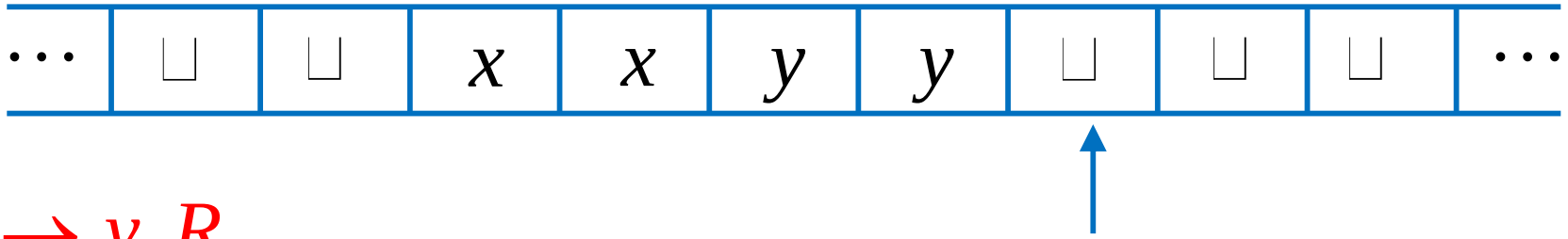
Time 11

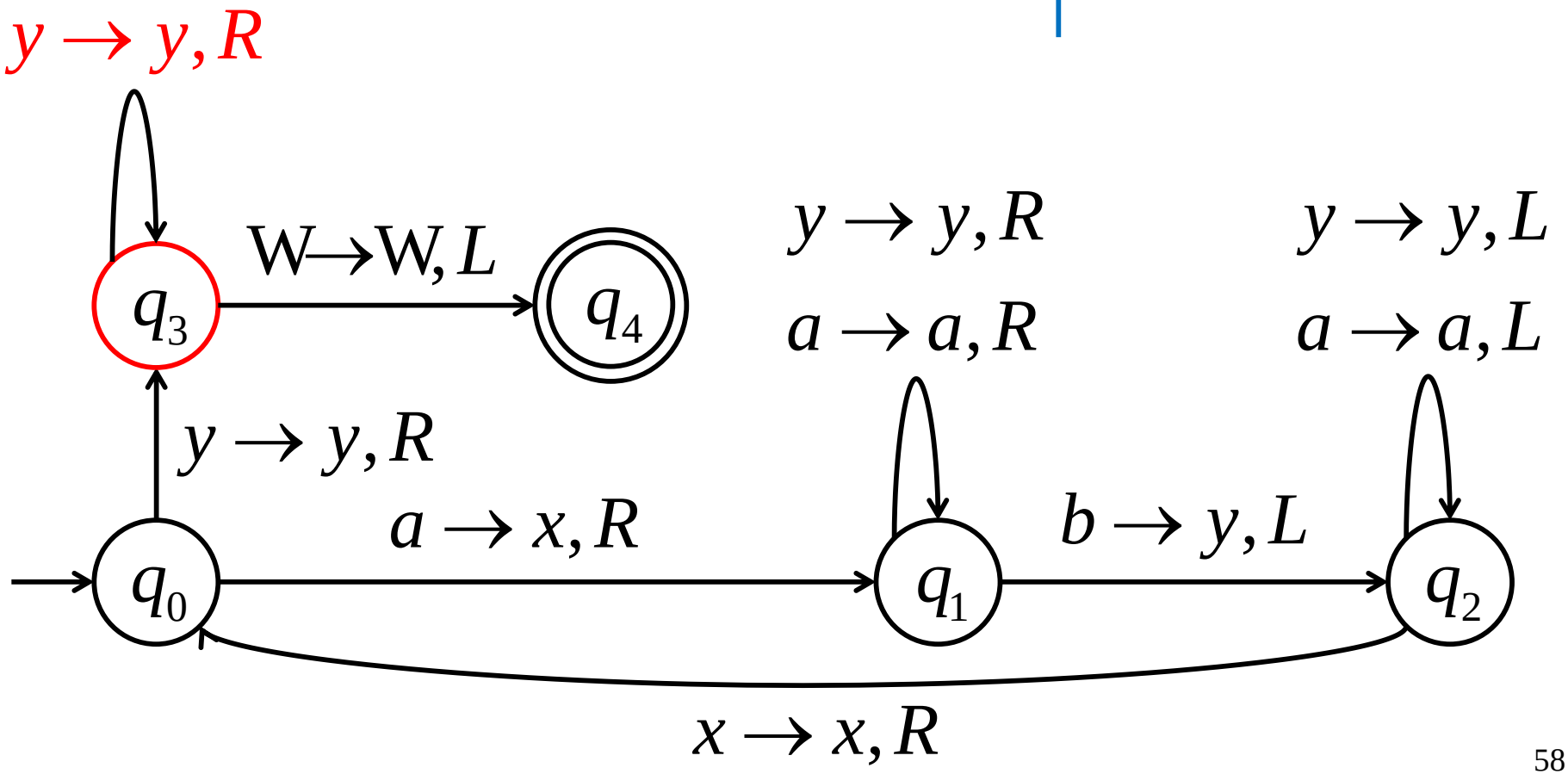




Another Example: Turing Machine

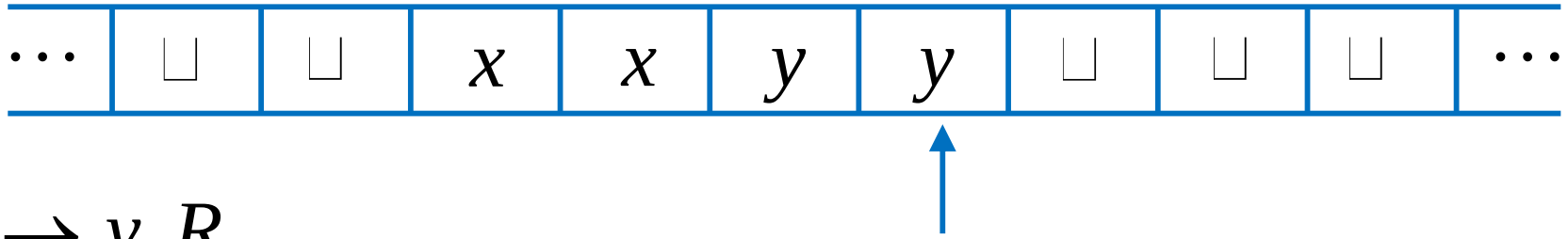
Time 12

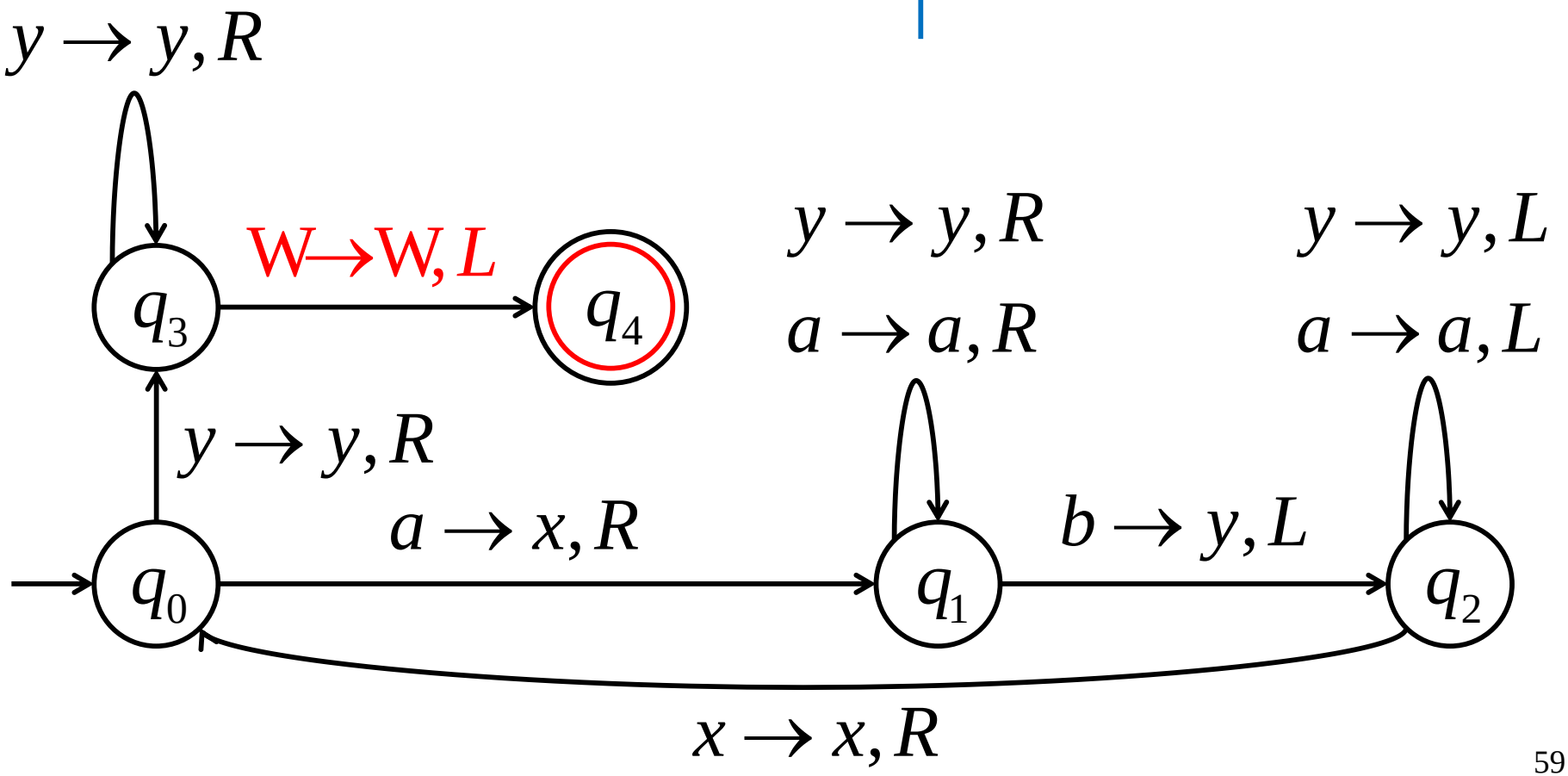




Another Example: Turing Machine

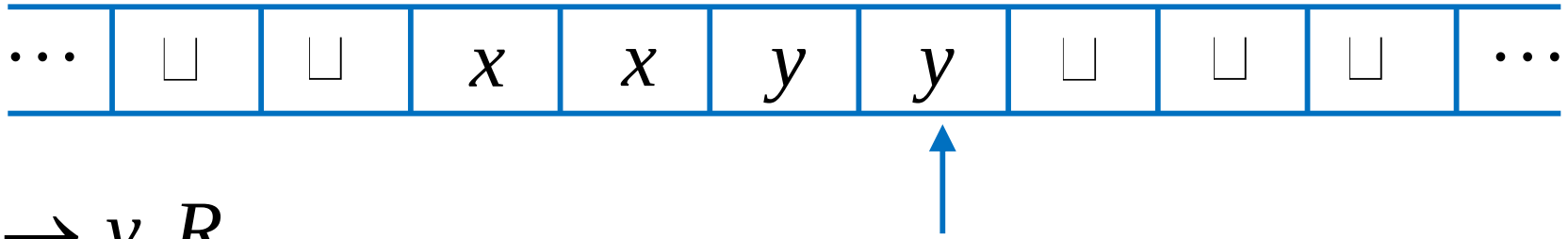
Time 13

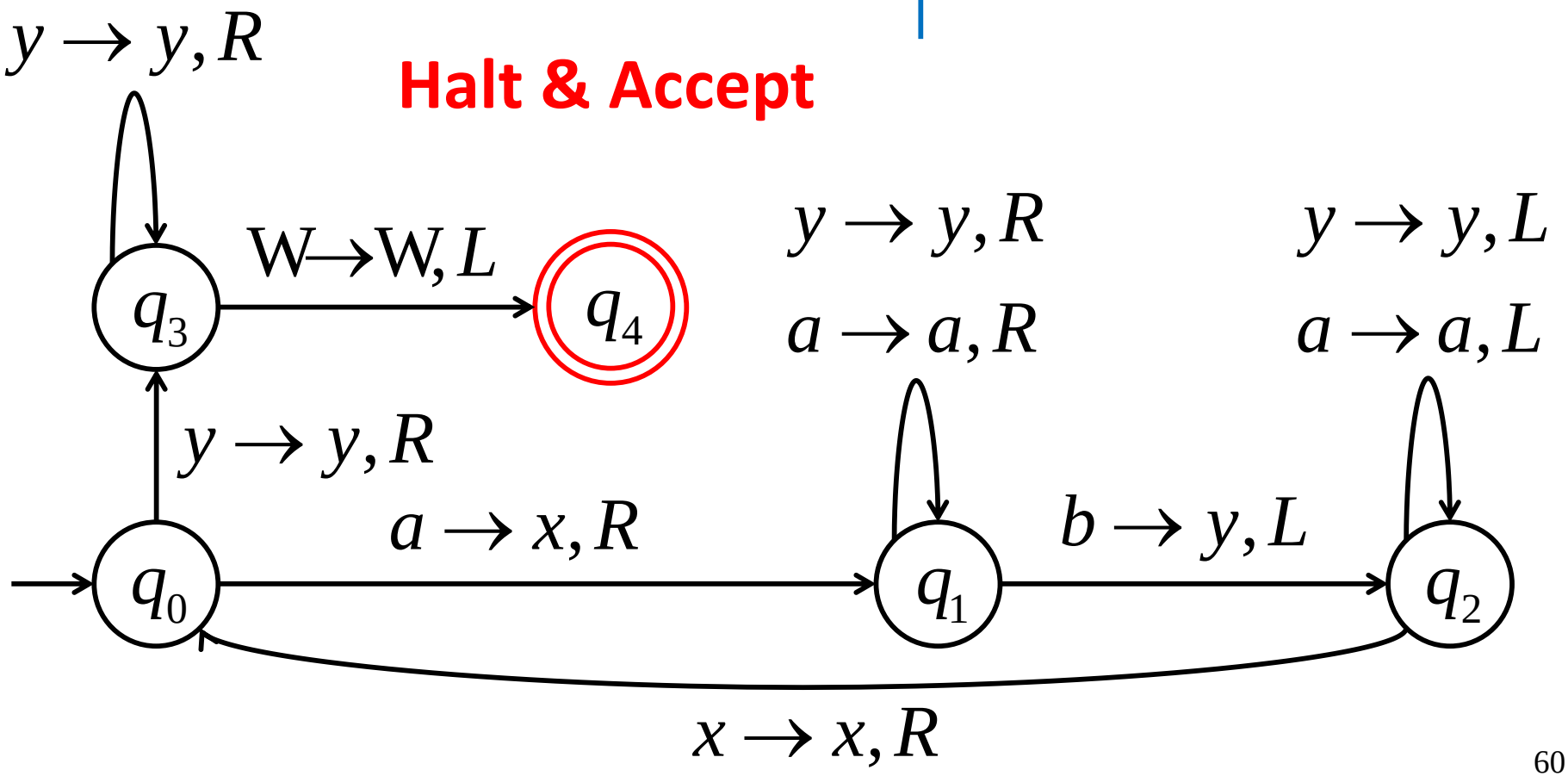




Another Example: Turing Machine

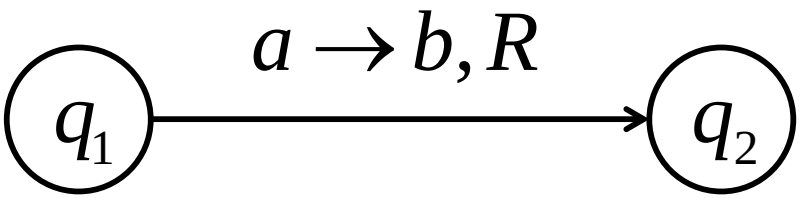
Time 14





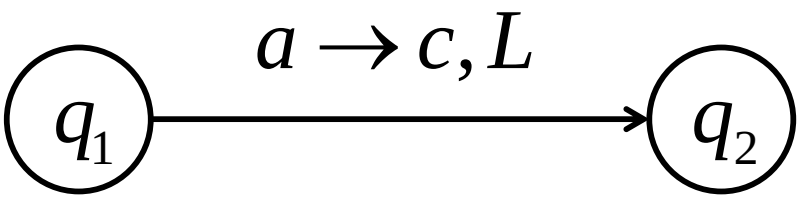
Formal Definitions for Turing Machines

Transition Function



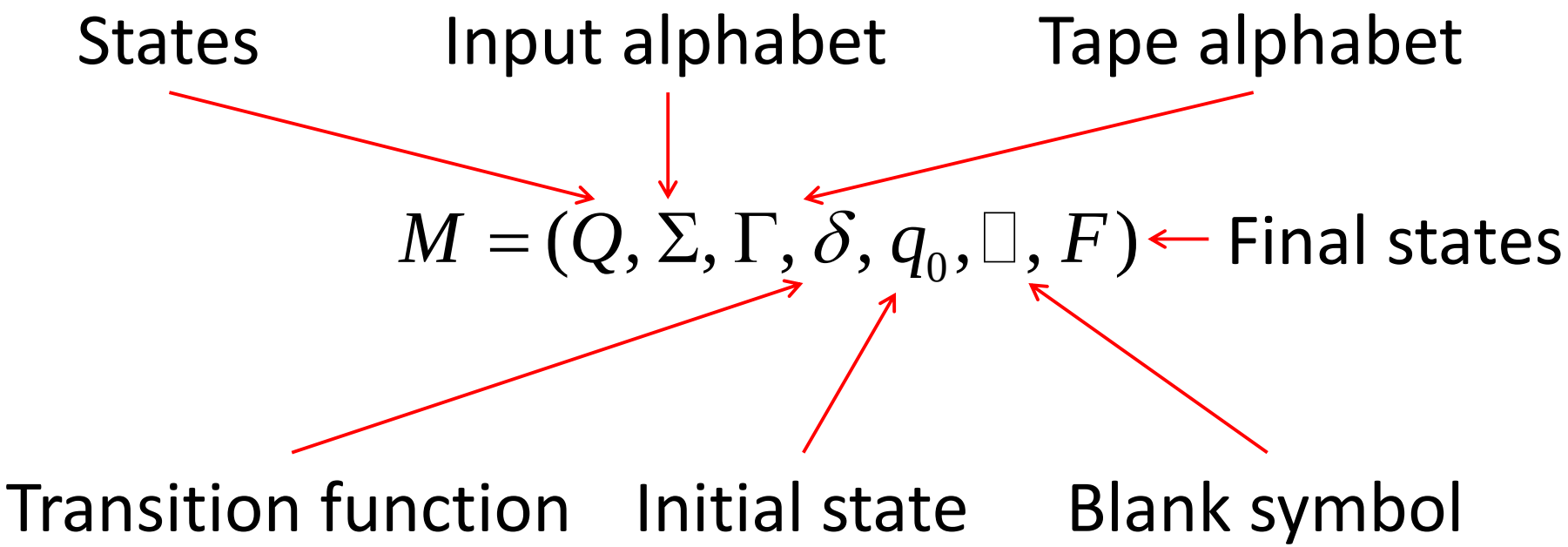
$$\delta(q_1, a) = (q_2, b, R)$$

Transition Function

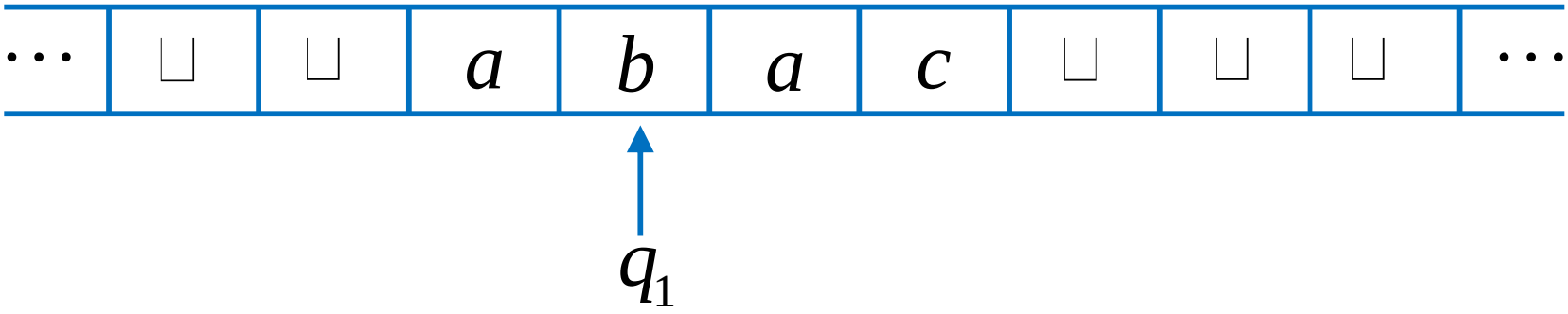


$$\delta(q_1, a) = (q_2, c, L)$$

Turing Machine

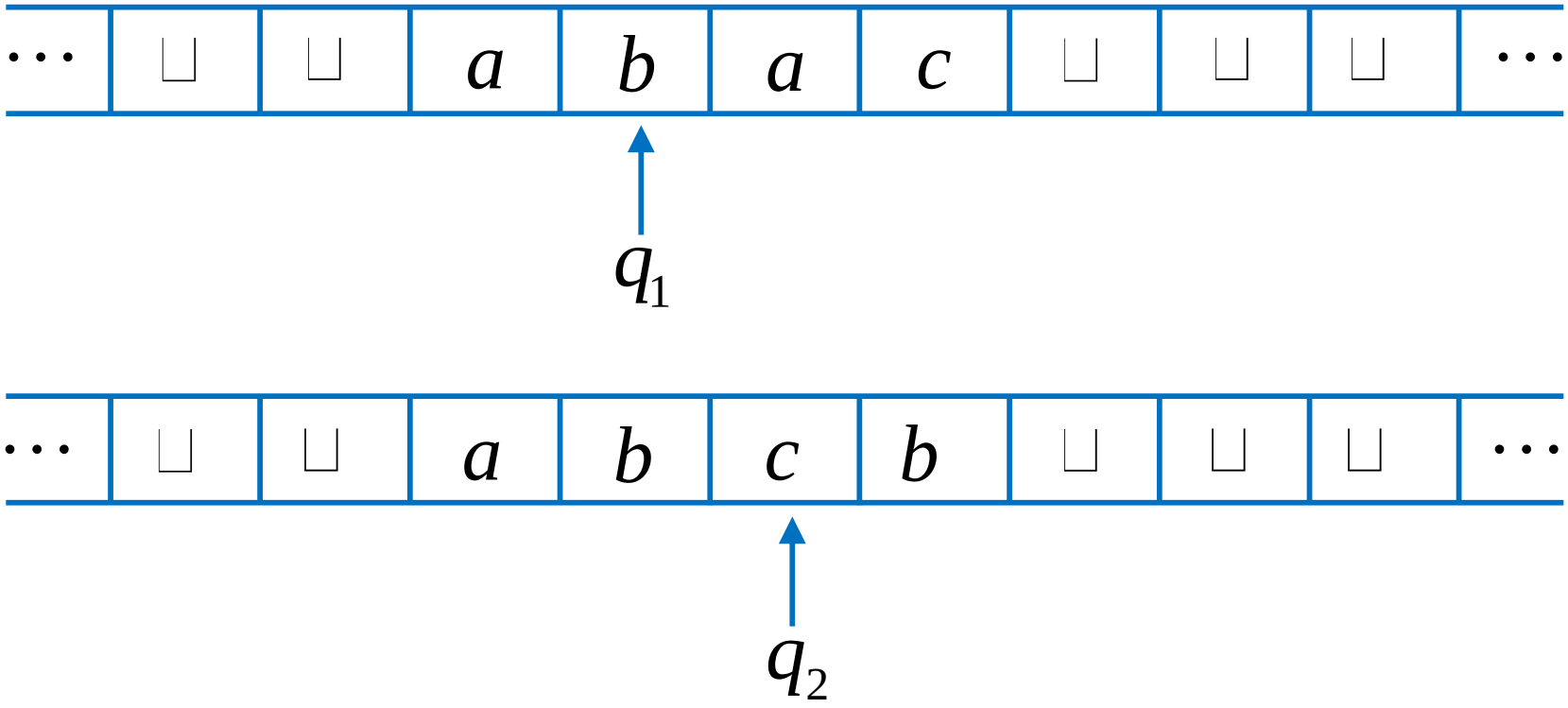


Configuration



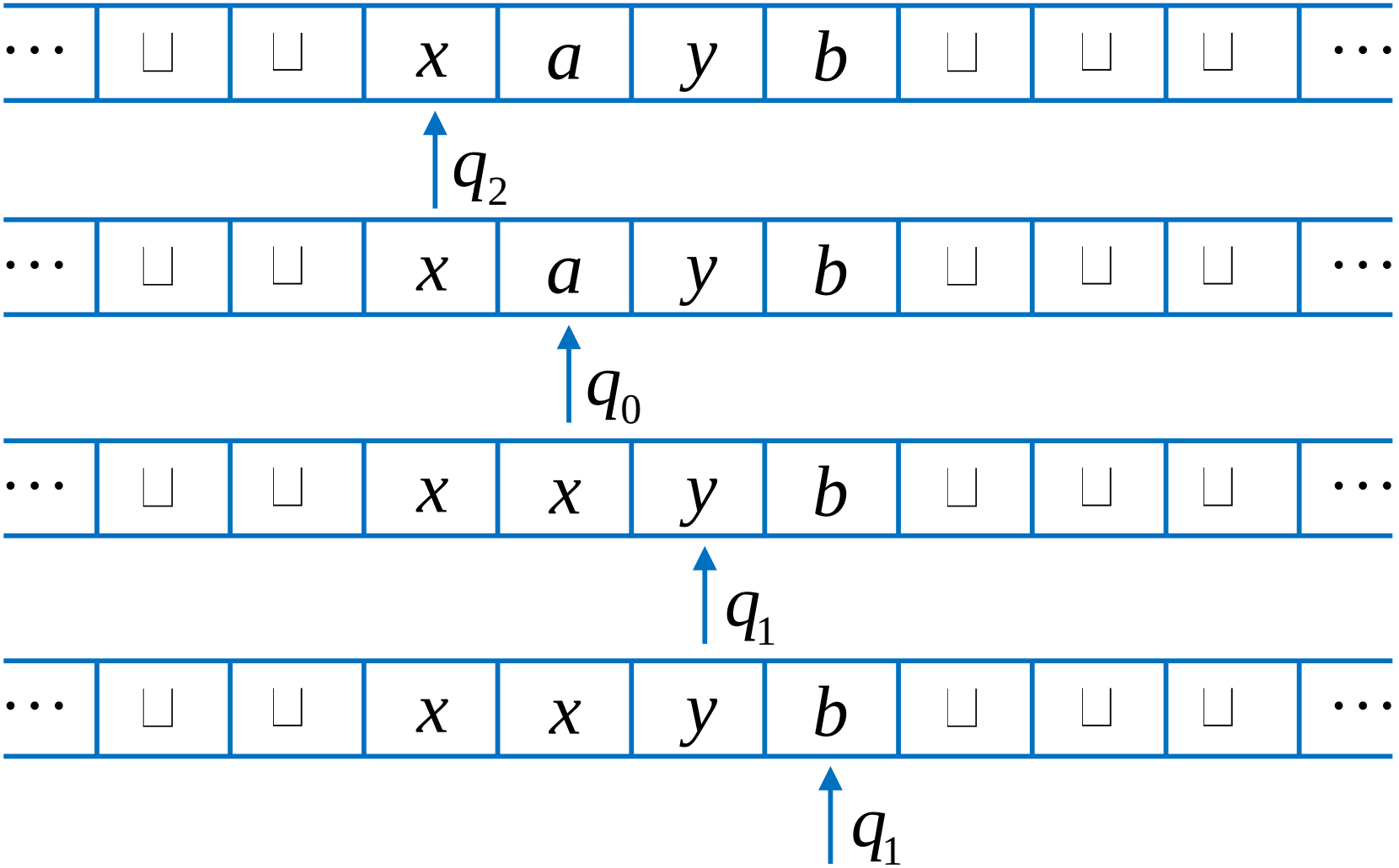
Instantaneous description: $a\ q_1\ bac$

Configuration



A move: $a\, q_1\, bac \succ ab\, q_2\, cb$

Configuration



$$q_2\ xayb \succ x\ q_0\ ayb \succ xx\ q_1\ yb \succ xxy\ q_1\ b$$

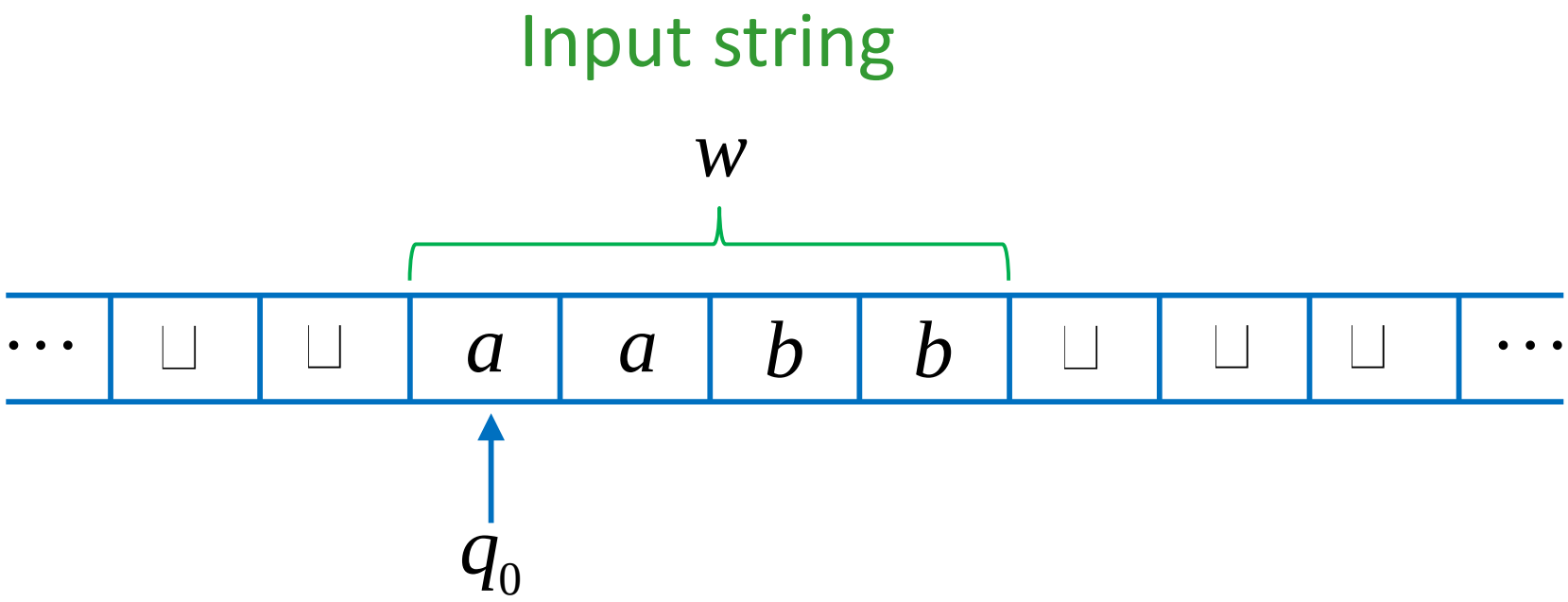
Configuration

$$q_2\ xayb \succ x\ q_0\ ayb \succ xx\ q_1\ yb \succ xxy\ q_1\ b$$

- Equivalent notation: $q_2\ xayb \succ^{*} xxy\ q_1\ b$

Configuration

- Initial configuration: $q_0\ w$



The Accepted Language

For any Turing Machine M

$$L(M) = \{w : q_0\ w \succ^* x_1\ q_f\ x_2\}$$

Initial state

Final state

Standard Turing Machine

The machine we described is the standard:

- Deterministic
- Infinite tape in both directions
- Tape is the input/output file

Exercises

Construct Turing machines that will accept the following languages on $\{a, b\}$.

a. $L = L(aba^*b)$

b. $L = \{a^n b^m a^{n+m} : n \geq 0, m \geq 1\}$

c. $L = \{a^n b^n a^n b^n : n \geq 0\}$

d. $L = \{a^n b^{2n} : n \geq 0\}$

e. $L = \{ww^R : w \in \{a, b\}^*\}$

f. $L = \{wcw : w \in \{a, b\}^*\}$