Institut Teknologi Bandung

AE6001

Continuum Mechanics 2

Single and Double Layer Potential Flow Solver using Python

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1 Introduction

Potential flow is an assumption in which the flow can be treated as inviscid (i.e. frictionless) and irrotational (i.e. fluid particles are not rotating). One of the implementation of this assumption is for calculating the aerodynamic properties of an airfoil using two dimensional panel method analysis. There are two types of panel method that can be used for analyzing airfoil. The first type is the single layer, or widely known as source panel method. The second type is the double layer, or known as vortex panel method [1].

In contrast with finite volume or finite element method solver where the whole domain has to be discretized [3], the potential flow solver needs only the domain boundary to be discretized, this method also known as the boundary element method. The method approximate the velocity and pressure around the body by computing N number of panels. Then, after the velocity and pressure at each panel is obtained, the value of lift coefficient is calculated. In order to obtain this, Kutta condition has to be imposed on the airfoil.

In this assignment, Python language is used to solve the numerical scheme of boundary element method. Python language is chosen because of its simplicity and its ability to process scientific program.

2 Assumption

The assumptions that are assigned in order to do the simulation :

- Inviscid flow (no friction between flow and object)
- Irrotational (fluid particles are not rotating)
- Incompressible (no change in density)
- Steady (values of properties is constant with respect to time)
- Laminar (the boundary layer is assumed to be fully laminar)

3 Workflow

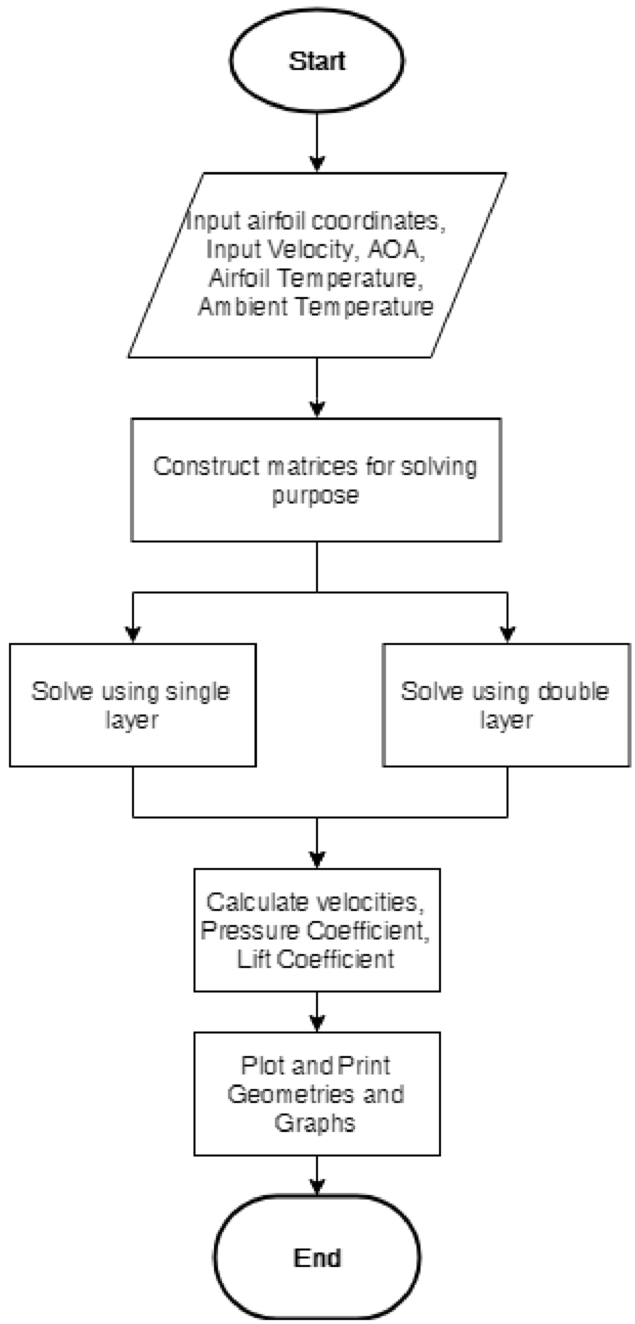

Figure 1: Program workflow.

3.1 Single Layer for Laplace Equation

The following steps are used to solve the laplace equation using single layer [2]. First consider the Neumann problems, then the *Interior Neumann Problem* will be,

$$\begin{cases} \Delta u = 0 & x \in \Omega \\ \dfrac{\partial u}{\partial \nu} = g & x \in \partial\Omega \end{cases}$$

The compatability condition on the boundary is.

$$\int_{\Omega} \Delta u = \int_{\partial\Omega} \frac{\partial u}{\partial \nu} dS(y) \tag{1}$$

In order for a solution exist,

$$\int_{\partial\Omega} g(y) dS(y) = 0 \tag{2}$$

For a continuous function h, define the single-layer potential

$$\overline{u}(x) = -\int_{\partial\Omega} h(y)\Phi(y-x)dy \tag{3}$$

that is harmonic with Ω. By defining the "normal derivative" of $\overline{u}$ away from $\partial\Omega$, we can show that.

$$\lim_{t \to 0^-} i^{(x_0)}(t) = -\frac{1}{2}h(x_0) + \frac{\partial \overline{u}}{\partial \nu}(x_0) \tag{4}$$

Hence, for continuous function h

$$g(x_0) = -\frac{1}{2}h(x_0) - \int_{\partial\Omega} h(y)\frac{\partial \Phi}{\partial \nu_x}(x_0 - y)dS(y) \tag{5}$$

For *Exterior Neumann Problem*, the problem will be:

$$\begin{cases} \Delta u = 0 & x \in \Omega^c \\ \dfrac{\partial u}{\partial \nu} = g & x \in \partial\Omega \end{cases}$$

Again, for any continuous function h, the single layer potential

$$\overline{u}(x) = -\int_{\partial\Omega} h(y)\Phi(x-y)dS(y) \tag{6}$$

is harmonic in Ω^c. In addition, by defining the "normal derivative" of $\overline{u}$ away from $\partial\Omega$, we can show that.

$$\lim_{t\to 0^+} \partial_{\nu_x}\overline{u}^{(x_0+t\nu(x_0))}(t) = \frac{1}{2}h(x_0) + \partial_{\nu_x}\overline{u}(x_0) \tag{7}$$

Hence, for continuous function h

$$g(x_0) = \frac{1}{2}h(x_0) - \int_{\partial\Omega} h(y)\frac{\partial\Phi}{\partial\nu_x}(x_0-y)dS(y) \tag{8}$$

3.2 Double Layer

The following steps are used to solve the laplace equation using double layer [2]. For double layer, let's begin with considering the *Interior Dirichlet Problem.*

$$\begin{cases} \Delta u = 0 & x \in \Omega \\ u = g & x \in \partial\Omega \end{cases}$$

For a given function h, define double-layer potential $\overline{\overline{u}}$ associated with h as

$$\overline{\overline{u}}(x) = -\int h(y)\frac{\partial\Phi}{\partial\nu_x}(x_0-y)dS(y) \tag{9}$$

$$\lim_{x\in\Omega\to x_0} \overline{\overline{u}}(x) = \frac{1}{2}h(x_0) + \overline{\overline{u}}(x_0) \tag{10}$$

Therefore, if we can find a continuous function h such that for all $x_0 \in \partial\Omega$

$$g(x_0) = \frac{1}{2}h(x_0) - \int_{\partial\Omega} h(y)\frac{\partial \Phi}{\partial \nu_y}(x_0 - y)dS(y) \tag{11}$$

then defining

$$\overline{\overline{u}}(x) = -\int_{\partial\Omega} h(y)\frac{\partial \Phi}{\partial \nu_y}(y)dS(y) \tag{12}$$

Next, consider the *Exterior Dirichlet Problem*,

$$\begin{cases} \Delta u = 0 & x \in \Omega^c \\ u = g & x \in \partial\Omega^c \end{cases}$$

for any continuous function h,

$$\overline{\overline{u}}(x) = -\int h(y)\frac{\partial \Phi}{\partial \nu_x}(x_0 - y)dS(y) \tag{13}$$

is harmonic in Ω^c and satisfies

$$\lim_{x\in\Omega^c \to x_0} \overline{\overline{u}}(x) = -\frac{1}{2}h(x_0) + \overline{\overline{u}}(x_0) \tag{14}$$

Therefore, if we can find a continuous function h such that for all $x_0 \in \partial\Omega^c$

$$g(x_0) = -\frac{1}{2}h(x_0) - \int_{\partial\Omega} h(y)\frac{\partial \Phi}{\partial \nu_y}(x_0 - y)dS(y) \tag{15}$$

then defining

$$\overline{\overline{u}}(x) = -\int_{\partial\Omega} h(y)\frac{\partial \Phi}{\partial \nu_y}(x - y)dS(y) \tag{16}$$

4 Results and Analysis

By inputting air velocity, angle of attack, and airfoil coordinates results can be obtained. The outputs of the program are airfoil plot, pressure coefficient plot, lift coefficient, and flow visualization. For the present case, air velocity of 10 m/s is imposed, therefore the Reynolds number will be around 700,000. By comparing the result to Javafoil, the following results are obtained.

Table 1: Lift coefficient comparison.

AoA	Javafoil	Code
-2	0.016	0.0182
0	0.253	0.2533
2	0.490	0.4880
4	0.727	0.7198
6	0.962	0.9470
8	1.197	1.1680
10	1.430	1.3800

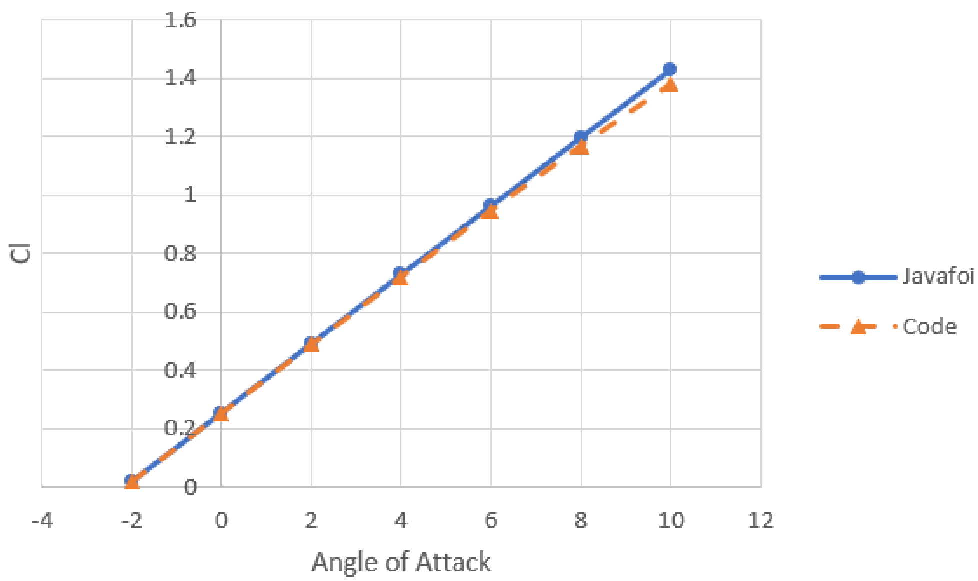

Figure 2: Lift coefficient vs angle of attack graph.

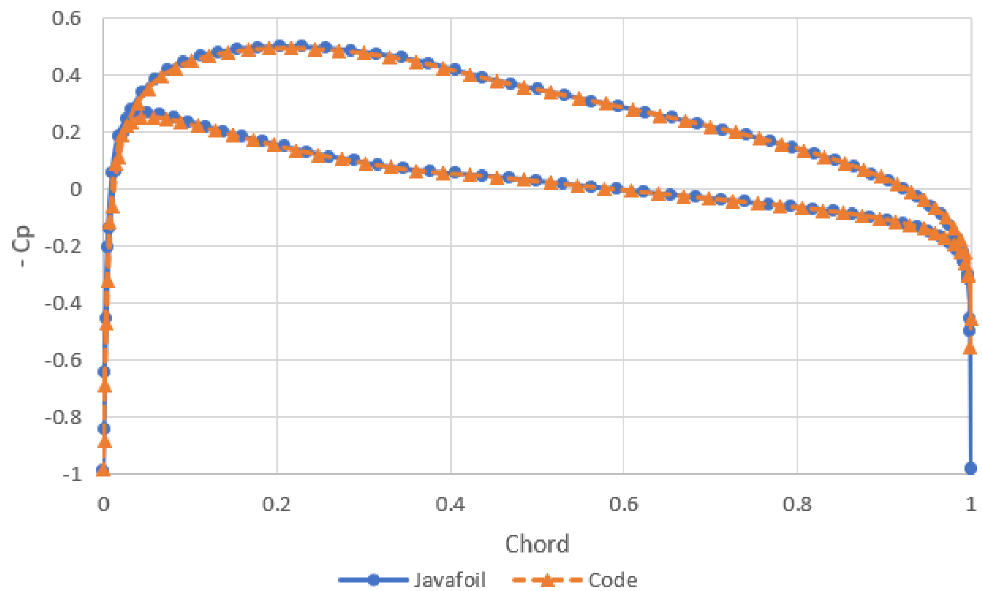

Figure 3: Pressure coefficient at angle of attack = 0.

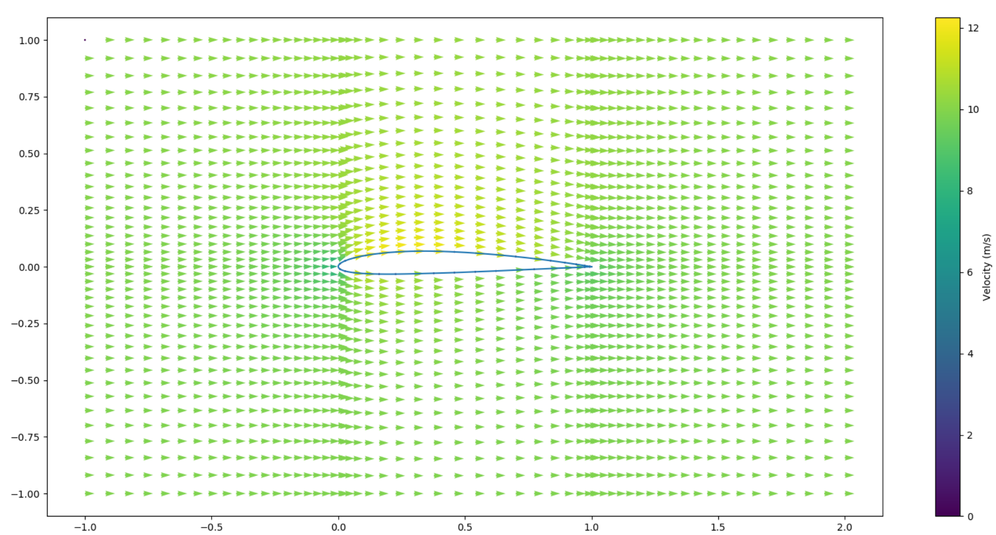

Figure 4: Velocity field around airfoil

Mainly, the error of lift coefficient is caused by the assumption of numerical method. For instance, the numerical integration or differentiation method, or even the control point selection. However, this error is not significant and can be ignored.

5 Conclusion

The program made to solve the boundary value problem for potential flow shows great agreement with JavaFoil. For the lift coefficient, the program overshoots in lower angle (as much as 13% at $\alpha = -2^o$) and slightly undershoots in higher angle (by 3.5% at $\alpha = 10^o$). Taking the Cp distribution at $\alpha = 0^o$, it can be seen that the problem rose in the trailing edge section.

The velocity field generated directly from the potential value at the airfoil boundary. It is successful in depicting the velocity arising from the potential field in the flow domain.

References

[1] Arnold M. Kuethe and Chuen-Yen Chow. *Foundations of Aerodynamics: Bases of Aerodynamic Design, 5th Edition.* John Wiley & Sons, 5 edition, December 1997.

[2] Gerald B. Folland. *Introduction to Partial Differential Equations.* Princeton University Press, 2 edition, October 1995.

[3] John D. Anderson. *Fundamentals of Aerodynamics.* McGraw-Hill Education, 5 edition, February 2010.