MOMENTUM AND THERMAL BOUNDARY LAYER APPROXIMATION ON AN AIRFOIL

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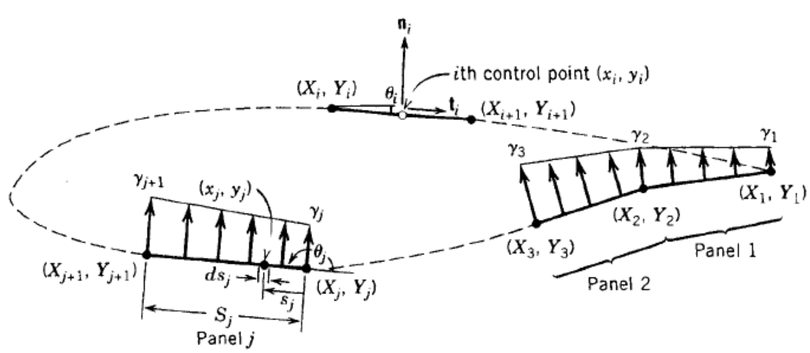
1. Inviscid Solver

If the flow is far enough from the airfoil wall, it can be modeled as potential flow. Potential ow is an assumption in which the ow can be treated as inviscid (i.e. frictionless) and irrotational (i.e. fluid particles are not rotating). One of the implementation of this assumption is for calculating the aerodynamic properties of an airfoil using two-dimensional panel method analysis. In this case, the flow has to follow the Laplace equation.

$$\nabla \cdot \nabla \nu = 0$$

$$\nabla^2 \varphi = 0$$

There are many methods that can be used for solving Laplace equation. One of them is the vortex panel method. In this method, the airfoil geometry is split into numbers of panels with finite length. Later, a vortex will be given into each panel [1].



With n is the number of the panel, $(\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_n)$ are the corresponding vortex strength on a panel, P is an arbitrary point, r_{pj} is a distance between panel j and point P, and θ_{pj} is an angle between r_{pj} and the x axis. The velocity potential can be written as:

$$\Delta\varphi_j = -\frac{1}{2}\int_j \theta_{pj}\gamma_{pj}\, ds_j$$

Where $\theta_{pj}$is obtained from

$$\theta_{pj} = \tan^{-1}\frac{y_i - y_j}{x_i - x_j}$$

If P is placed on a control point at the first panel with coordinate (x_i, x_i) therefore, the equation can be written as.

$$\theta_{ij} = \tan^{-1}\frac{y_i - y_j}{x_i - x_j}$$

$$\theta(x_i, y_i) = -\sum_{j=1}^{n}\frac{\gamma_j}{2\pi}\int_j \theta_{ij}\, ds_j$$

Because the normal velocity at the control point is 0, therefore the superposition between the normal component and freestream velocity V∞ is.

$$V_\infty \cos \beta_i - \sum_{j=1}^{n}\frac{\gamma_j}{2\pi}\int_j \frac{\partial\theta_{ij}}{\partial n_i} ds_j = 0$$

This can be rewritten as:

$$V_\infty \cos \beta_i - \sum_{j=1}^{n}\frac{\gamma_j}{2\pi} J_{i,j} = 0$$

To solve the potential equation, Kutta condition also has to be satisfied. Mathematically, the Kutta condition can be written as:

$$\gamma_{TE_{upper}} - \gamma_{TE_{Lower}} = 0$$

To determine the tangential velocity on the airfoil surface, vortex sheet theory declare that the vortex strength is depend on the difference between the velocity above and below the vortex panel. In airfoil, the velocity below the vortex panel is 0, therefore:

$$\gamma = u_1 - u_2 = u_1 - 0 = u_1$$

From this correlation, it can be concluded that the tangential velocity at each control point is proportional to the vortex strength at the panel. Therefore the lift force on the airfoil can be defined as:

$$L = \rho_\infty V_\infty \sum_{j=1}^{n} \gamma_j s_j$$

Where ρ_∞ is the fluid density, V_∞ is freestream velocity, and $s_j$is the length of each panel.

2. Thwaites Method

To calculate the boundary layer thickness, Thwaites correlation method was used [2]. Thwaites defined a parameter λ to rewrite the momentum integral equation. It is defined as

$$\lambda := \frac{\theta^2}{\nu}\frac{\partial U_{inv}}{\partial x}$$

By inserting this parameter into the momentum integral equation and defining the shear (S) and shape (H) correlation,

$$\tau_w = \frac{\mu U}{\theta} S(\lambda) \qquad \delta^* = \theta H(\lambda)$$

Thwaites was able to derive a relation between momentum thickness θ and U_{inv},

$$\theta^2 = \frac{0.45\nu}{U_{inv}^6}\int_0^x U_{inv}^5 dx$$

From experiments, the following relations were also obtained

$$S(\lambda) = (\lambda + 0.09)^{0.62}$$

$$H(\lambda) = 2 + 4.14z - 83.5z^2 + 854z^3 - 3337z^4 + 4576z^5$$

$$z = 0.25 - \lambda$$

These sets of equation were then solved to obtain τ_{wall} according to these steps:

i. Calculate U_{inv} from inviscid solver
ii. Calculate θ^2 from the integral equation
iii. Obtain λ
iv. Obtain S(λ) and H(λ)
v. Obtain δ^* and τ_{wall}

3. Viscous Inviscid Interaction

By the presence of boundary layer, the inviscid flow will be deflected as if the airfoil changes its shape. Subsequently, because the inviscid flow is changing, the thickness of boundary layer also changes. Therefore, to resolve the problem, an interaction between viscous and inviscid flow has to be determined. The coupling strategy are:

1. The outer flow is calculated with boundary condition $v_n = 0$ on the airfoil body.

2. The boundary layer is calculated, which provides the displacement thickness distribution of $\delta^*(x)$.
3. The outer flow is recomputed with boundary condition of $v_n = \frac{d}{dx}(u_e\delta^*)$.
4. Return to point 2 until convergence

Where the value of $u_e = 0.03$ in which it physically represents the normal velocity at the panel. The value of 0.03 is proposed by Yousefi [3]. By using this procedure, the final boundary layer thickness and friction can be obtained.

4. One Parameter Integral Method

The one parameter integral method [2] is very similar to Thwaites correlation method. However, instead of θ^2 what was calculated is the conduction thickness δ_c defined as,

$$\delta_c \coloneqq \frac{k\left(T_{wall} - T_{freestream}\right)}{q_{wall}}$$

To obtain δ_c use the experimental fit as a function of U_{inv}

$$\frac{\delta_c^2}{\nu} = \frac{a}{U_{inv}^b}\int_{x_0}^{x} U_{inv}^{b-1}dx$$

Where a and b are a function of Prandtl's number

$$a^{-1/2} = 0.332Pr^{0.35}$$

$$b = 2.95Pr^{0.07}$$

Again these sets of equation are simple to numerically solve with this steps:

i. Calculate U_{inv} from inviscid solver
ii. Calculate the Prandtl number, Pr.
iii. Obtain a and b
iv. Calculate δ_c using the integral equation
v. Obtain q_{wall}

5. Results and Analysis

In the present assignment, a NACA 2410 airfoil was used as a test case for calculating the laminar boundary layer. The airfoil coordinates were obtained by using Javafoil. The inputs that were used to run the program are:

$U_\infty = 10\ m/s$

$\alpha = 0^o$

$\mu = 0.0000181206$

$\rho = 1.225\ kg/m^3$

$T_\infty = 273\ K$

$T_{wall} = 350\ K$

Then, the following results are obtained

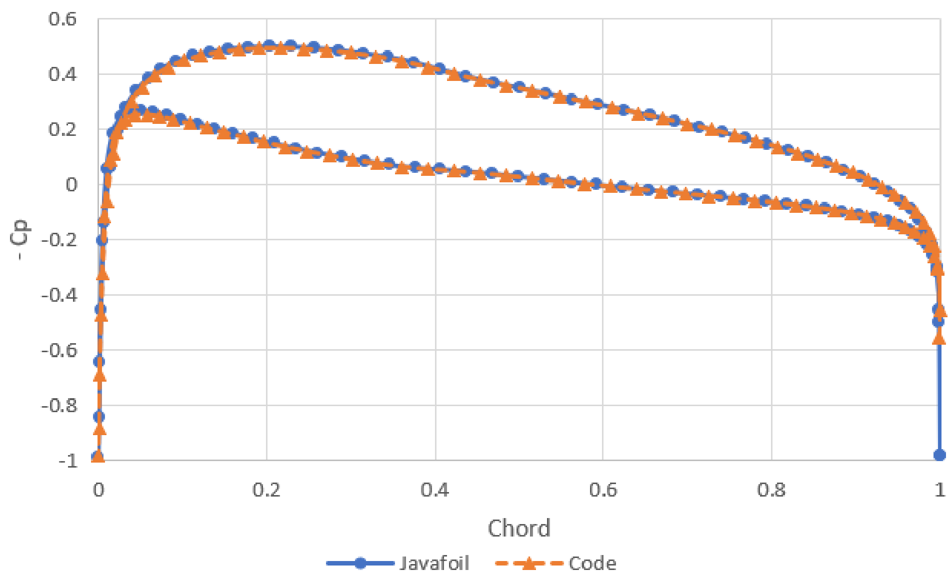


Figure 1. Comparison of pressure coefficient

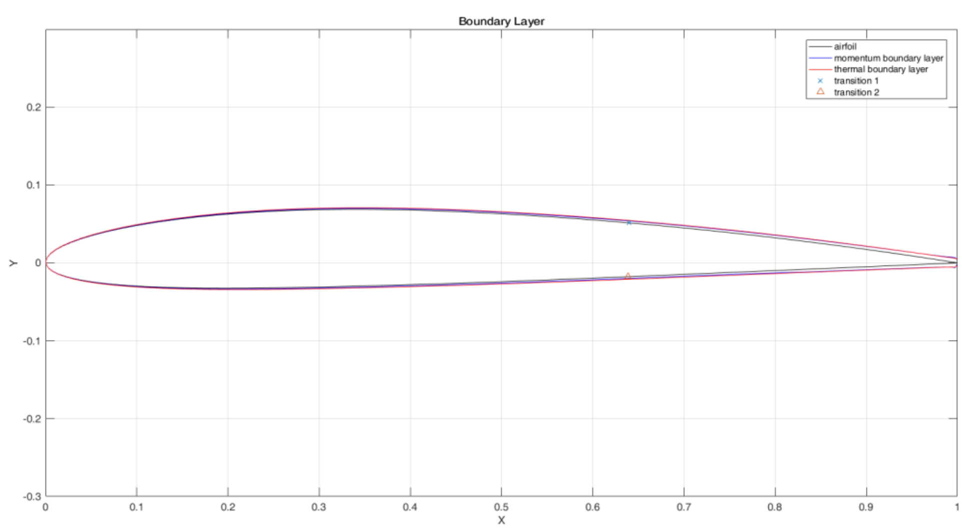


Figure 2. Boundary layer on airfoil

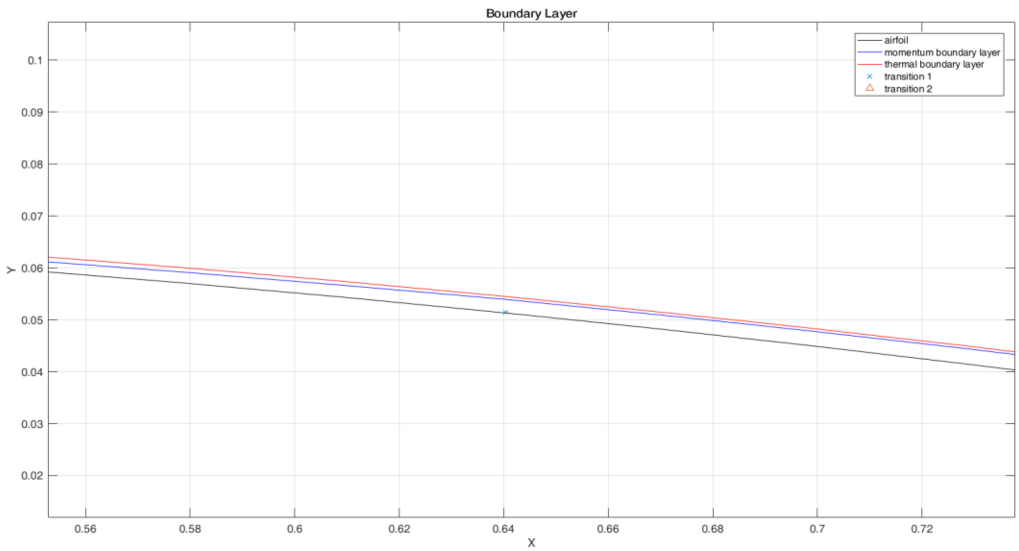


Figure 3. Zoomed view of boundary layer

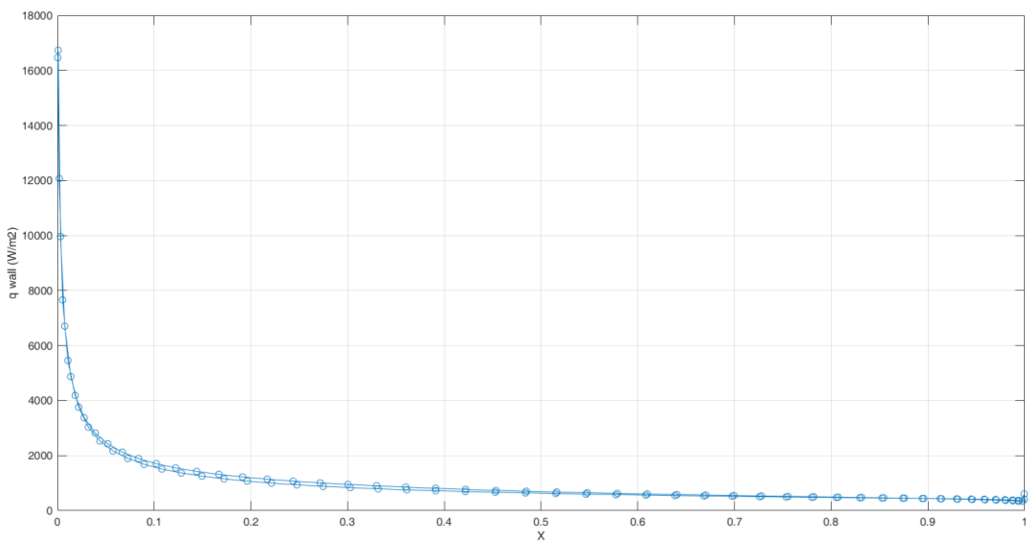


Figure 4 Heat transfer on airfoil wall

Meanwhile, from the code, the value of lift coefficient and drag coefficient also obtained.

$C_l = 0.2496$ $C_d = 0.0032$

Compared to Javafoil at Reynolds number = 700,000.

$C_l = 0.253$ $C_d = 0.00613$

Therefore, the value of lift coefficient has error of 1.34% and the error of drag coefficient is 47.8%.

Mainly, the error of lift coefficient is caused by the assumption of numerical method. For instance, the numerical integration or differentiation method, or even the control point selection. However, this error is not significant and can be ignored. Meanwhile, the drag coefficient has a significant value of error, in which the error is caused by the Thwaites method assumption, and numerical method calculation. But mainly, the error in the drag coefficient is caused by ignoring the pressure drag and the physical assumption where the boundary layer is fully laminar. However, this assumption is physically invalid. The program shows us that transitions were occurred on the airfoil. Therefore, the effects of turbulent boundary layer cannot be ignored.

6. Conclusion

- The program was able to calculate lift coefficient for the given airfoil as accurate as 1.34 % difference from Javafoil.
- The drag coefficient calculated is a laminar approximation and is not advisable to be used on a turbulent case. For the given case, the program introduces almost 50% difference from Javafoil.
- The program was able to show the location of transition to turbulence

References

[1] A. M. Kuethe and C.-Y. Chow, Foundations of Aerodynamics, New York: John Wiley & Sons, 1998.

[2] F. M. White, Viscous Fluid Flow, New York: McGraw-Hill, 1991.

[3] K. Yousefi, R. Saleh and P. Zahedi, "Numerical study of blowing and suction slot geometry optimization on NACA 0012 airfoil," *Journal of Mechanical Science and Technology,* vol. 4, no. 28, pp. 1297-1310, 2014.