Image Sensing and Acquisition:-

Most of the images in which we are interested are generated by the combination of an "illumination" source and the reflection or absorption of energy from that source by the elements of the "Scene" being imaged.

→ Three principal sensor arrangements used to transform incident energy into digital images

→ Incoming energy is transformed into a voltage by a combination of the input electrical power and sensor material that is responsive to the type of energy being detected.

→ The o/p V/g waveform is the response of the sensor, and a digital quantity is obtained by digitizing that response.

1. Image Acquisition using a single sensing element.
2. Image Acquisition using sensor strips.
3. Image Acquisition using sensor Arrays.

Image Acquisition using a Single Sensing element:-

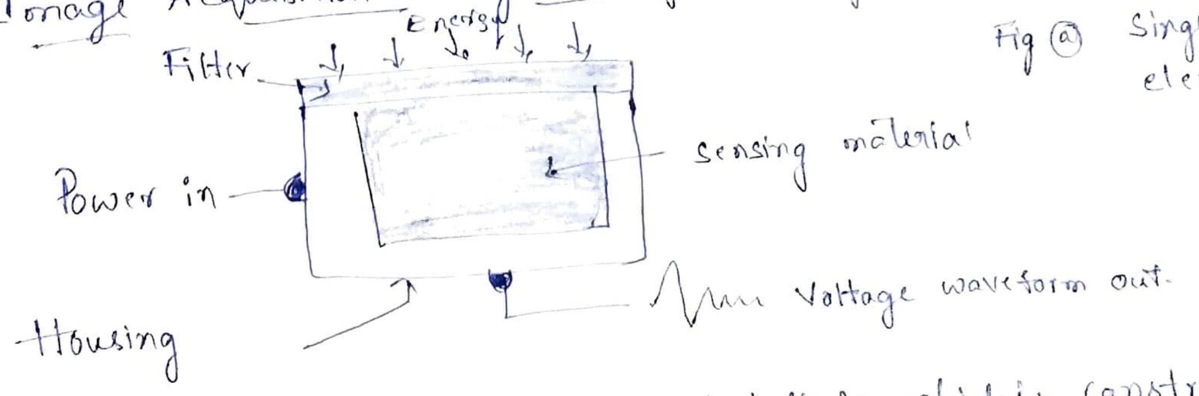
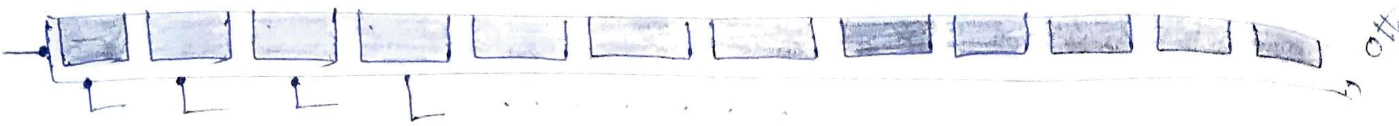


Fig (a) Single Sensing element

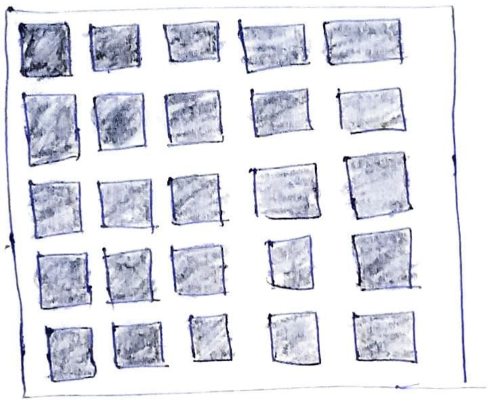
→ Sensor of this type is the photodiode which is constructed of Silicon materials and whose o/p is a V/g proportional to light intensity.

→ Using a filter in front of a sensor improves its selectivity.
Example:- an optical green transmission filter favors light in the green band of the color spectrum.
→ Sensor o/p would be stronger for green light than for other visible light components.

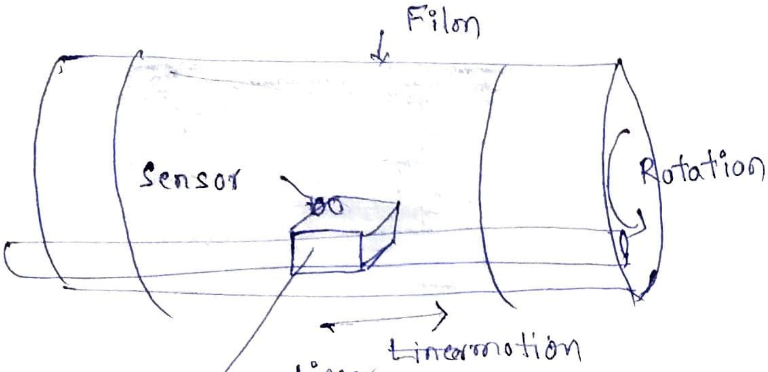
→ To generate a 2-D image using a single sensing element there has to be relative displacements in both the x and y directions b/w the sensor and the area to be imaged.



(b) Line Sensor



(c) Array sensor.



One image line out per increment of rotation and full linear displacement of sensor from left to right.

→ Arrangement used in high precision scanning, where a film negative is mounted onto a drum whose mechanical rotation provides displacement in one dimension.

→ Sensor is mounted on a lead screw that provides motion in the perpendicular direction.

→ Light source is contained inside the drum.

→ Light passes through the film, its intensity is modified by the film density before it is captured by the sensor

→ This "modulation" of the light intensity causes corresponding variations in the sensor voltage, which are ultimately converted to image intensity levels by digitization

→ This method is an ~~en~~expensive way to obtain high resolution images because mechanical motion can be controlled with high precision.

→ Disadvantages is slow and not readily portable.

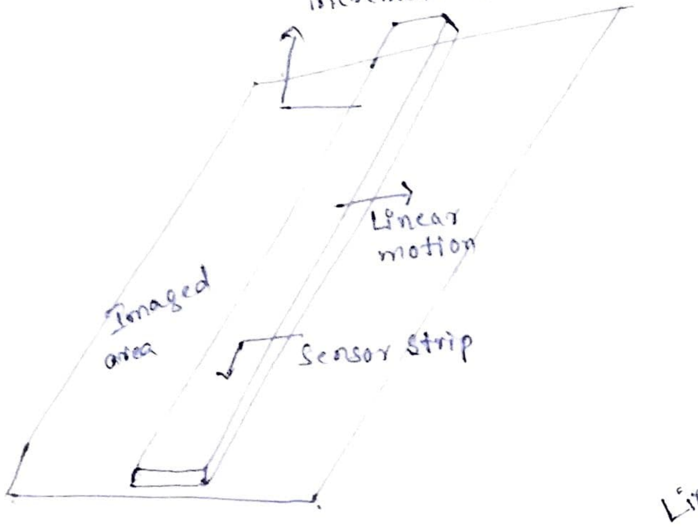
→ other Similar mechanical arrangements use a flat imaging bed with the ~~Sesor~~ Sensor moving in two linear directions. This type of mechanical digitizer called transmission microdensitometers.

→ Systems in which light is reflected from the medium, instead of passing through it, are called reflection microdensitometers.

→ Example Imaging with a single Sensing element places a laser Source Coincident with the Sensor.

→ Moving mirrors are used to control the outgoing beam in a Scanning pattern and to direct the reflected laser Signal onto the Sensor

Image Acquisition using sensor strips:-



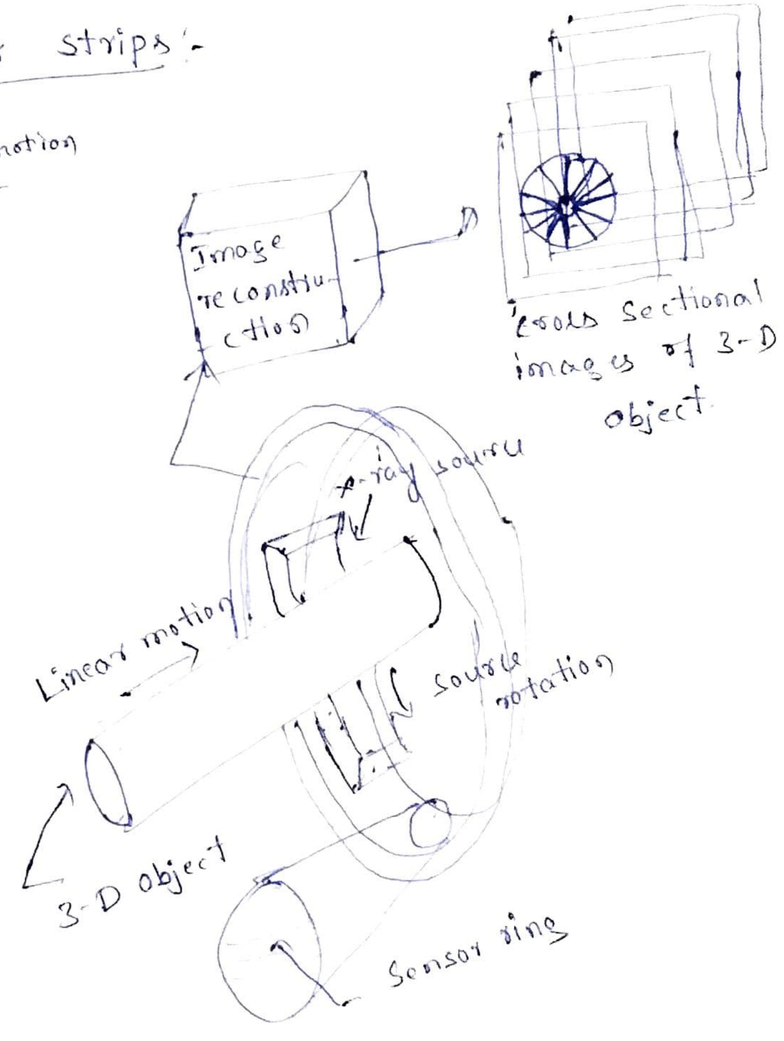


Fig. (a) Image acquisition using a linear sensor strip.

(b) Image acquisition using a circular sensor strip.

→ The strip provides imaging elements in one direction. Motion perpendicular to the strip provides imaging in the other direction.

→ This arrangement is used in most ~~of the~~ flat bed Scanners.

→ Sensing devices with 4000 or more in-line sensors are possible.

→ In line Sensors are used routinely in airborne imaging applications.

→ Imaging System is mounted on the aircraft that flies at a constant altitude and speed over the geographical area to be imaged.

→ One dimensional imaging sensor strips that respond to various bands of the electromagnetic spectrum are mounted perpendicular to the direction of flight.

→ An imaging of strip gives one line of an image at a time, and the motion of the strip relative to the scene completes the other dimensi-on of a 2-D image.

→ Lenses or other focusing schemes are used to project the area to be scanned onto the sensors.

In fig (b) sensor strips in a ring configuration are used in medical and industrial imaging to obtain cross-sectional ("slice") image of 3-D objects.

→ A rotating X-ray source provides illumination and X-ray sensitive sensors opposite the source collect the energy that passes through the object.

→ This is the basis for medical and industrial computerized axial tomog-raphy (CAT) imaging.

→ The o/p of the sensor is processed by reconstruction algorithms whose objective is to transform the sensed data into meaningful cross sectional images.

→ In other words, images are not obtained directly from the sensors by motion alone. They also require extensive computer processing.

→ A 3-D digital volume consisting of stacked images is generated as the object is moved in a direction perpendicular to the sensor ring.

→ Other modalities of imaging based on the CAT principle include magnetic ~~rea~~ resonance imaging (MRI) and positron emission tomography (PET).

IMAGE ACQUISITION USING SENSOR ARRAYS

- Individual sensing elements arranged in the form of a 2-D array.
- Electromagnetic and Ultrasonic sensing devices frequently arranged in this manner.
- This is also the predominant arrangement found in digital cameras.
- A typical sensor for these cameras is a CCD (charge coupled device) array, which can be manufactured with a broad range of sensing properties and can be packaged in rugged arrays of 4000×4000 elements or more.
- CCD sensors are used widely in digital cameras and other light ~~ener~~ sensing instruments.
- The response of each sensor is proportional to the integral of the light energy projected onto the surface of the sensor, a property that is used in astronomical and other applications requiring low noise images.
- Noise reduction is achieved by letting the sensor integrate the i/p light signal over minutes or even hours.
- Advantage is that a complete image can be obtained by focusing the energy pattern onto the surface of the array.

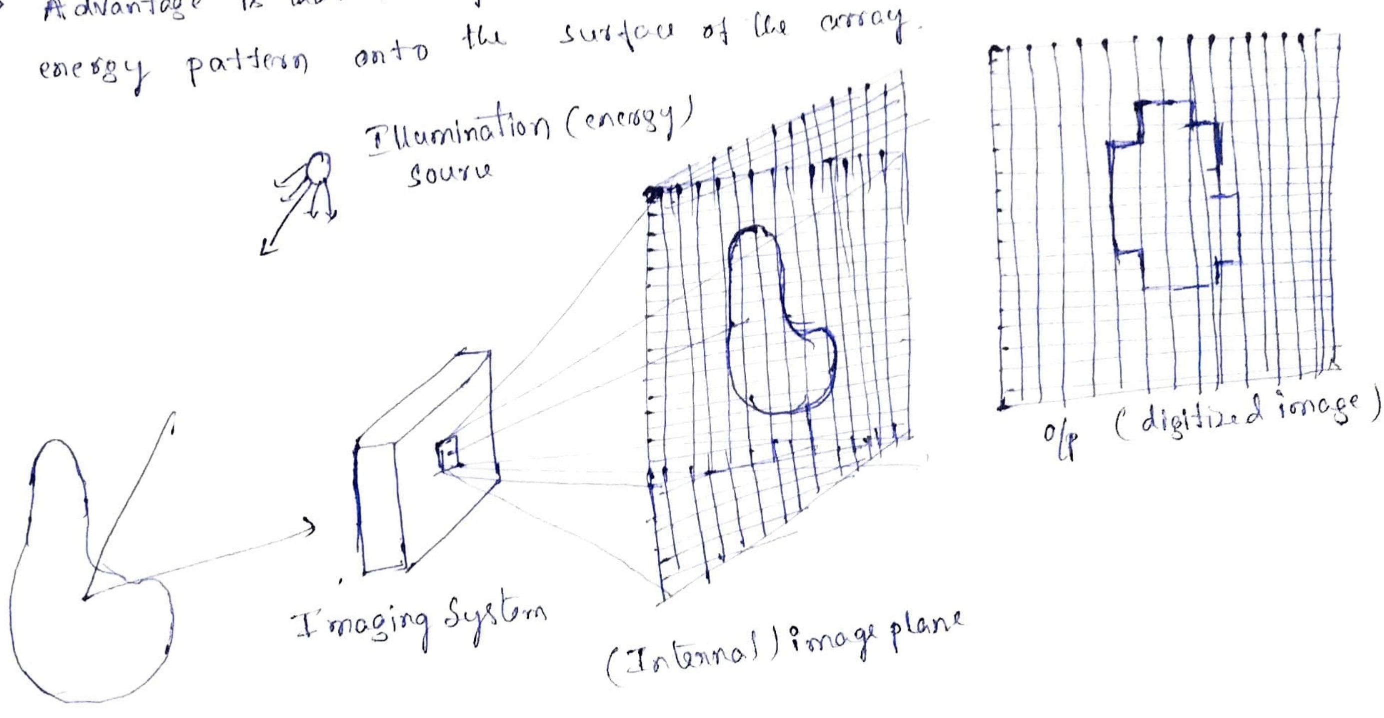


Fig. An example of digital image acquisition (a) Illumination (energy) source (b) Scene (c) Imaging system system (d) Projection of the scene onto the image plane (e) Digitized image.

Fig. shows the principal manner in which array sensors are used.
This figure shows the energy from an illumination source being reflected from a scene.

→ the first function performed by the imaging system in fig ⓐ is to collect the incoming energy and focus it onto an image plane.

→ If the illumination is light, the front end of the imaging system is an optical lens that projects the viewed scene onto the focal plane of the lens as fig ⓓ.

→ The sensor array, which is coincident with the focal plane, produces o/p's proportional to the integral of the light received at each sensor.

→ Digital and analog circuitry sweep these o/p's and convert them to an analog signal, which is then digitized by another section of the imaging system.

A Simple Image Formation Model

→ We denote images by two-dimensional functions of the form $f(x,y)$

→ The value of f at spatial coordinates (x,y) is a scalar quantity, whose physical meaning is determined by the source of the image, and whose values are propotional to energy radiated by a physical source (e.g. electromagnetic waves). As a consequence $f(x,y)$ must be nonnegative and finite.

$$0 \leq f(x,y) < \infty \quad -(1)$$

→ Function $f(x,y)$ is characterized by two components.
1) the amount of source illumination incident on the scene being viewed.
2) the amount of illumination reflected by the objects in the scene.
Appropriately, these are called the illumination and reflectance components and denoted by $i(x,y)$ and $r(x,y)$ respectively.
The two functions combine as a product to form $f(x,y)$

$$f(x,y) = i(x,y)\, r(x,y) \quad - (2)$$

where

$$0 \leq i(x,y) < \infty \quad - (3)$$

$$0 \leq r(x,y) \leq 1 \quad (4)$$

Thus reflectance is bounded by 0 (total absorption) and 1 (total reflectance).
The nature of $i(x,y)$ is determined by the illumination source, $r(x,y)$ is determined by the characteristics of the imaged objects.

→ These expressions are applicable also to images formed via transmission of the illumination through a medium, such as a chest X-ray.

→ We would deal with a transmissivity instead of a reflectivity function but the limits would be the same as in eqn (4) and the image function formed would be modeled as the product in eqn (2)

Let the intensity (gray level) of a monochrome image at any coordinates (x, y) be denoted by.

$$l = f(x, y) \quad \text{—} \quad (5)$$

From eqn (2) through (4) it is evident that l lies in the range

$$L_{min} \leq l \leq L_{max} \quad \text{—} \quad (6)$$

→ In theory, the requirement on L_{min} is that it be nonnegative and on L_{max} that it be finite.

→ In practice $L_{min} = i_{min} r_{min}$ and $L_{max} = i_{max} r_{max}$.

→ The interval $[L_{min}, L_{max}]$ is called the intensity (or gray) scale.

→ Common practice is to shift this interval numerically to the interval $[0, 1]$. 0 = Black 1 = white.

IMAGE SAMPLING AND QUANTIZATION

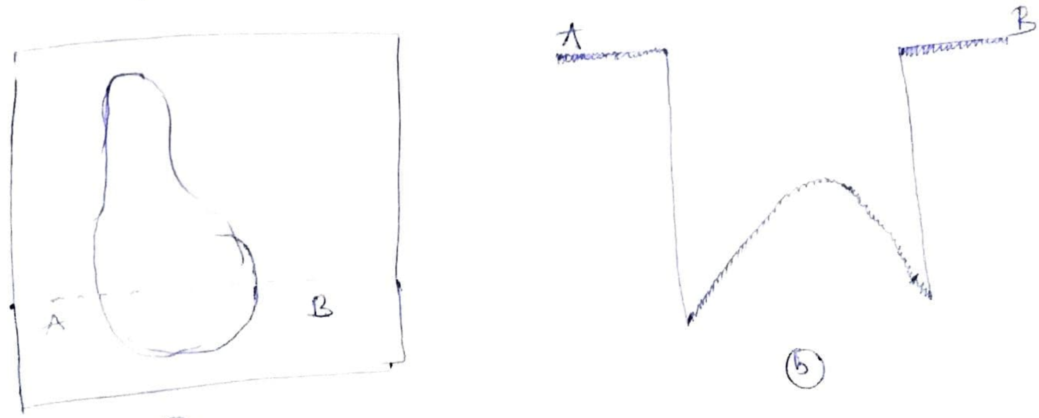
Main objective is

→ To generate digital images from sensed data.

→ The output of most sensors is a continuous v/g waveform whose amplitude and spatial behavior are related to the physical phenomenon being ~~viewed~~ sensed.

→ To create a digital image, we need to convert the continuous sensed data into a digital format.

→ This requires two processes: Sampling and quantization.



(a) (b)

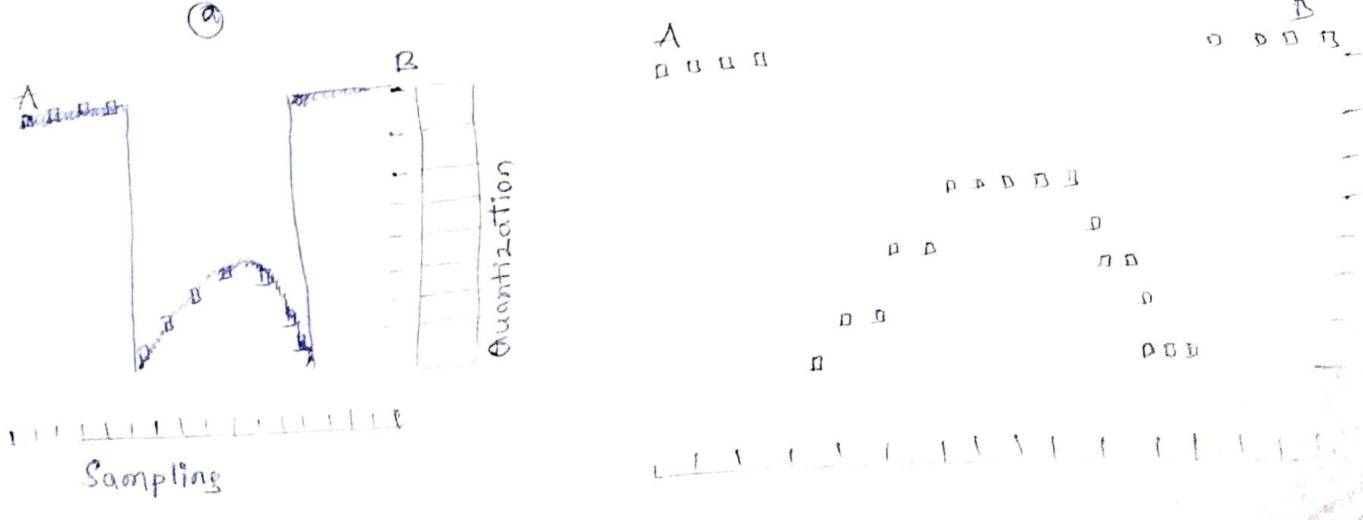


Fig ⓐ continuous image, ⓑ A scan line showing intensity variations along line AB in the Continuous image.
ⓒ Sampling and Quantization ⓓ Digital scan line.

7

→ Fig ⓐ shows a continuous image f that we want to convert to digital form.
→ An image may be continuous with respect to the x and y coordinates, and amplitude.
→ To digitize it, we have to sample the function in both coordinates and also in amplitude.
→ Digitizing the coordinate values is called sampling.
→ Digitizing the amplitude values is called quantization.
→ the Fig ⓑ is a plot of amplitude (intensity levels) values of the continuous image along the line segment. AB in fig ⓐ.
→ The random variations are due to image noise.
→ To sample this function, we take equally spaced samples along line AB as shown in fig ⓒ.
→ the samples are shown as small dark squares superimposed on the function and their (discrete) spatial locations are indicated by corresponding tick marks in the bottom of the figure.
→ the set of dark squares constitute the sampled function.
→ the values of the sample still vertically a continuous range of intensity values.
→ The intensity values also must be converted (quantized) into discrete quantities.
→ The vertical gray bar in fig ⓒ depicts the intensity scale divided into eight discrete intervals ranging from black to white.
→ the vertical tick marks indicate the specific values assigned to each of the eight intensity intervals.
→ The continuous intensity levels are quantized by assigning one of the eight value to each sample, depending on the vertical proximity of a sample to a vertical tick mark.
→ The digital samples resulting from both sampling and quantization are shown as white squares in fig ⓓ

REPRESENTING DIGITAL IMAGES

→ Let $f(s,t)$ represent a continuous image function of two continuous variables s and t.

→ We convert this function into a digital image by sampling and quantization.

→ The continuous image into a digital image $f(x,y)$. Containing M rows and N columns, where (x,y) are discrete coordinates.

→ Integer values for ~~di~~ these discrete coordinates $x = 0, 1, 2 \ldots M-1$ and $y = 0, 1, 2 \ldots N-1$.

→ The value of the digital image at the origin is $f(0,0)$ and its value at the next coordinates along the first row is $f(0,1)$

The representation of an M×N numerical array

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & & & \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1) \end{bmatrix}$$

Represent a digital image in a traditional matrix form

$$A = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \vdots & \vdots & & \vdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}$$

$L = 2^K$

L = no of intensity levels. K is an Integer. Integer range $[0, L-1]$

The number b of bits required to store a digital image is

$b = M \times N \times K$

when $M = N$

$\therefore b = N^2 K$

SPATIAL AND INTENSITY RESOLUTION

→ Spatial resolution is a measure of the smallest discernible detail in an image.

→ Spatial resolution can be stated in several ways, with line pairs per unit distance, and dots (pixels) per unit distance being common measures.

→ Dots per unit distance is a measure of image resolution used in the printing and publishing industry.

→ In USA this measure usually is expressed as dots per inch (dpi)

→ Newspapers are printed with a resolution of 75 dpi, magazines at 133 dpi, glossy brochures at 175 dpi, and book page at which you are presently looking was printed at 2400dpi

→ Example: A digital camera with a 20- megapixel CCD imaging chip can be expected to have a higher capability to resolve detail than a 8 - megapixel camera, assuming that both cameras are equipped with comparable lenses and the comparision images are taken at the same distance.

→ Intensity resolution refers to the smallest discernible change in intensity level.

→ Discernible changes in intensity are influenced also by noise and saturation values.

Image Interpolation

→ Interpolation is used in tasks such as zooming, shrinking, rotating and geometrically correcting digital images.

→ Interpolation is the process of using known data to estimate values at unknown locations.

→ Suppose that an image of size 500 x 500 pixels has to be enlarged 1.5 times to 750 x 750 pixels.

→ A simple way to visualize zooming is to create an imaginary 750 x 750 grid with the same pixel spacing as the original image.

Intensity Transformations and Spatial Filtering

Spatial domain :- The image plane itself and image processing methods in this category are based on direct manipulation of pixels in an image.

Two principal categories of Spatial processing :-

1. Intensity transformations
2. Spatial filtering

Intensity transformations operate on single pixels of an image for tasks such as contrast manipulation and image thresholding.

Spatial filtering performs operations on the neighborhood of every pixel in an image.
Ex: Spatial filtering include image smoothing and sharpening.

→ Spatial domain techniques operate directly on the pixels of an image.

→ Frequency domain in which operations are performed on the Fourier transform of an image.

THE BASICS OF INTENSITY TRANSFORMATIONS AND SPATIAL FILTERING

The spatial domain process discuss based on the expression

$$g(x,y) = T[f(x,y)] \quad \text{--- (1)}$$

where $f(x,y)$ → Input image $g(x,y)$ - output image.

T is an operator on f defined over a neighborhood of point (x,y)

→ The operator can be applied to the pixels of a single image or to the pixels of a set of images, such as performing the elementwise sum of a sequence of images for noise reduction.

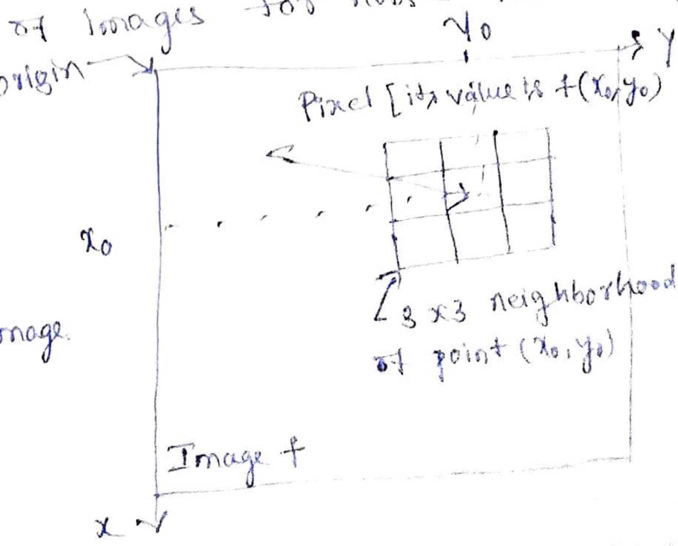


Fig. A 3x3 neighborhood about a point (x_0, y_0) in an image. The neighborhood is moved from pixel to pixel in the image to generate an o/p image.

Fig. 1. shows the basic implementation of image and the small region shown an neighborhood of (x_0, y_0)

→ Typically the neighborhood is rectangular, centered on (x_0, y_0) so and much smaller in size than the image.

→ The fig 1. illustrate consists of moving the center of the neighborhood from pixel to pixel and applying the operator T to the pixels in the neighborhood to yield an o/p value at that location.

→ Thus, for any specific location (x_0, y_0) the value of the o/p image g at those coordinates is equal to the result applying T to the neighborhood with origin at (x_0, y_0) in f.

Ex' Suppose that the neighborhood is a square of size 3x3 and that operate T is defined as "compute the average intensity of the pixels in the neighborhood".

→ consider orbitory location in an image, say (100, 150), the result at that location in the o/p image g(100, 150) is the

→ sum of f(100, 150) and its 8 neighbors divided by 9.

→ The center of the neighborhood is then moved to the next adjacent location and the procedure is repeated to generate the next value of the o/p image g.

→ The process start at the top left of the input image and proceeds pixel by pixel in a horizontal (vertical) scan one row (column) at a time.

→ The smallest possible neighborhood of is of size 1x1.

→ In this case g depends only on the value of f at a single point (x, y) and T in eqn ① becomes an intensity transformation function. (Gray-level, or mapping). of the form — ②

$$s = T(r)$$

→ Use s and r denote the intensity of g and f at any point (x, y)

→ Example: If T(r) has the form in fig 2ⓐ

→ The result of applying the transformation to every pixel in f to generate the corresponding pixel in g would be to produce an image of higher contrast than the original by darkening the intensity levels below K and brightening the levels above K.

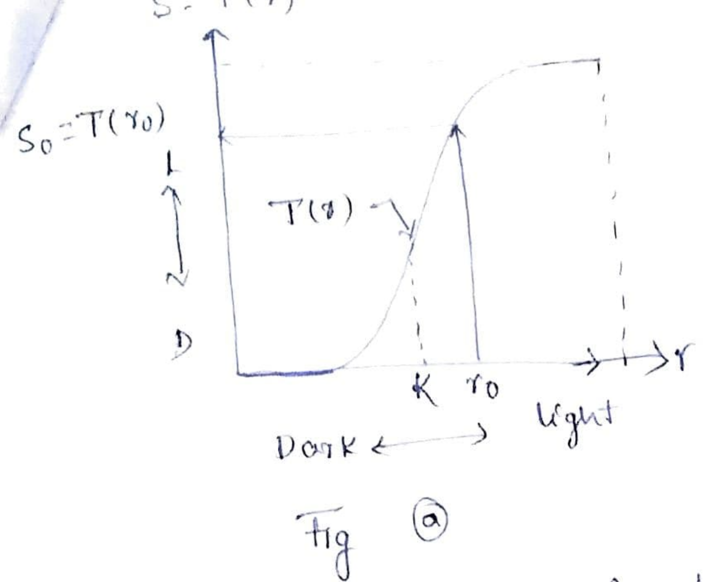


Fig (a)

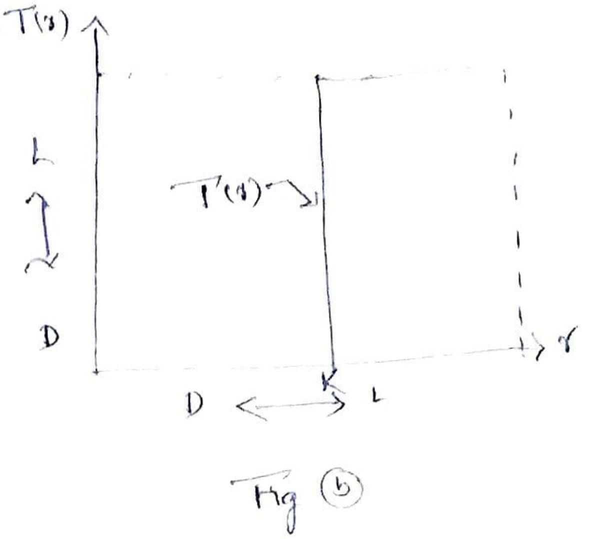


Fig (b)

Intensity Transformation functions
(a) Contrast stretching function (b) Thresholding function.

→ In this technique sometimes called contrast stretching. the values of s toward values of r lower than K reduce (darken) the values of s toward black.

→ The opposite is true for values of r higher than k.

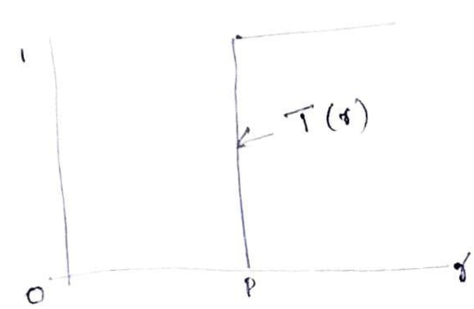
→ observe how an intensity value r_0 is mapped to obtain the corresponding value s_0.

→ Fig (b) T(r) produces a two level (binary) image.

→ A mapping of this form is called a thresholding function.

→ Approaches whose results depend only on the intensity at a point sometimes are called point processing techniques.

Example



r = 256
P = 127

→ Here we have only two levels of intensities that are 0 and 1.

→ The pixel intensity values that are below 127 are 0 means black.

→ The pixel intensity values that are greater than 127 are 1 that means white.

→ The exact point of 127 there is a sudden change in transformation so we cannot tell that at that exact point the value would be (0) or (1).

Mathematically this transformation function can be denoted as

$$g(x,y) = \begin{cases} 0 & f(x,y) < 127 \\ 1 & f(x,y) \geq 127. \end{cases}$$

Histogram Processing :-

let r_k, for $K = 0, 1, 2 \ldots L-1$ denote the intensities of an L-level digital image, $f(x, y)$.

The unnormalized histogram of f is defined as

$$h(r_k) = n_k \quad \text{for } K = 0, 1, 2 \ldots L-1$$

where n_k is the no of pixels in f with intensity r_K. and Subdivisions of the intensity scale are called histogram bins.

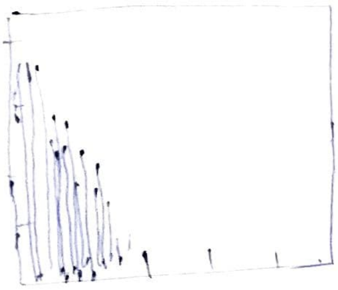
$||^{ly}$ the normalized histogram of f is defined as

$$P(r_K) = \frac{h(r_K)}{MN} = \frac{n_K}{MN}$$

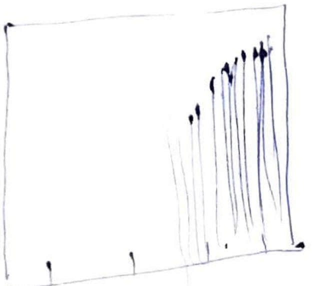
where M and N are the no of image rows and columns.

The sum of $p(r_k)$ for all values of K is always 1.

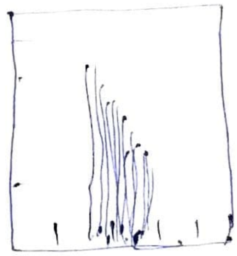
Four basic intensity characteristics: dark, light, low contrast, and high contrast.



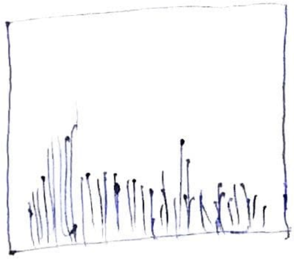
Histogram of dark image



Histogram of light image



Histogram of low-contrast image



Histogram of high contrast image

Histogram Equalization:-

let the varible r denote the intensities of an image to be processed r is in the range $[0, L-1]$ with $r=0$ representing black and $L-1$ representing white.

$$s = T(r) \qquad 0 \leq r \leq L-1 \quad \text{—①}$$

→ For r satisfying these conditions, we focus attention on transformations of the form that produce an o/p intensity values, s for a given intensity value r in the o/p image.

We assume that.

(a) $T(r)$ is a monotonic increasing function in the interval

$0 \le r \le L-1$; and

(b) $0 \le T(r) \le L-1$ for $0 \le r \le L-1$

In some formulations to be discussed shortly. We use the inverse transformation

$$r = T^{-1}(s) \qquad 0 \le s \le L-1 \quad - (2)$$

in which case we change condition (a) to;

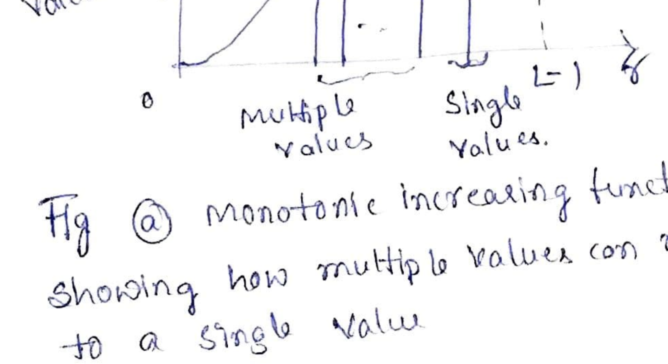
(a') $T(r)$ is a strictly monotonic increasing function in the interval

$0 \le r \le L-1$.

→ The condition in (a) that $T(r)$ be monotonically increasing guarantees that output intensity values will never be less than corresponding i/p values,

→ Condition (b) guarantees that the range of o/p intensities is the same as the i/p.

→ Finally, condition (a') guarantees that the mappings from s back to r will be one to one thus preventing ambiguities.



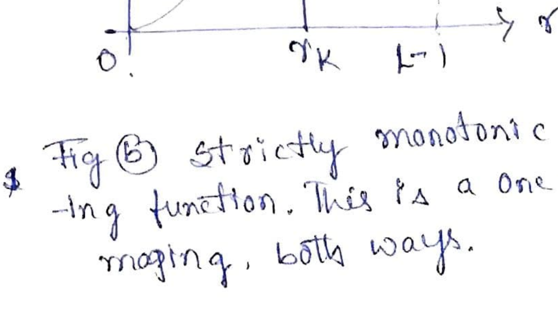


Fig (a) Monotonic increasing function showing how multiple values can map to a single value

Fig (b) strictly monotonic increasing function. This is a one to one maping, both ways.

Fig (a) shows the satisfies conditions (a) and (b). Here we see that it is possible for multiple i/p values to map to a single o/p value. and still satisfy these two conditions.

→ Monotonic transformation function performs a one to one or many to one mapping.

→ This is perfectly fine when mapping from r to s.

g. (b) shows, requiring that $T(r)$ be strictly monotonic guarantees that the inverse mapping will be single valued (i.e. the mapping is one to one in both directions).

→ The intensity of an image may be viewed as a random variables in the interval $[0, L-1]$.

→ let $P_r(r)$ and $P_s(s)$ denote the PDFs of intensity values r and s in two different images.

→ The subscripts on P indicate that P_r and P_s are different functions.

→ A fundamental result from probability theory is that If $P_r(r)$ and $T(r)$ are known and $T(r)$ is continuous and differentiable. over the range of values of intrest, then the PDF of the transformed (mapped) variable s can be obtained as

$$P_s(s) = P_r(r) \left| \frac{dr}{ds} \right| \quad - (3)$$

A transformation function of particular importance in image processing is

$$s = T(r) = (L-1) \int_0^r P_r(w)\, dw \quad - (4)$$

where w = dummy variable of integration.

→ The integral on the right side is the cumulative distribution function (CDF) of random variable r.

→ Because PDF always are positive and the integral of a function is the area under the function. Eqn (4) satisfies condition (a)

→ when the upper limit in this eqn is $r = (L-1)$ the integral evaluates to 1, as it must for a PDF,

→ Thus, the maximum value of s is $L-1$ and condition (b) is satisfied also.

→ Eqn (3) to find the $P_s(s)$

$$\frac{ds}{dr} = \frac{dT(r)}{dr}$$

$$= (L-1)\frac{d}{dr}\left[\int_0^r P_r(w)\, dw\right] \quad - (5)$$

$$\frac{ds}{dr} = (L-1)\, P_r(r)$$

Substituting this result for $\frac{dr}{ds}$ in eqn (3)

$$P_s(s) = P_r(r)\left|\frac{dr}{ds}\right|$$

$$= P_r(r)\left|\frac{1}{(L-1)\,P_r(r)}\right| \quad - (6)$$

$$P_s(s) = \frac{1}{L-1} \qquad 0 \le s \le L-1$$

$P_s(s)$ - uniform probability density function.

→ For discrete values, we work with probabilities and summations instead of probability density functions and integrals.

→ The probability of occurance of intensity level r_k in a digital image is approximated by.

$$P_r(r_k) = \frac{n_k}{MN} \quad - (7)$$

where MN = Total number of pixels in the image

n_k = the number of pixels that have intensity r_k.

$r_k \in [0, L-1]$ - normalized image histogram.

The discrete form of the transformation in eqn (4) is

$$S_k = T(r_k) = (L-1)\sum_{j=0}^{k} P_r(r_j) \qquad k = 0, 1, 2 \ldots L-1 \quad - (8)$$

where L = Number of possible intensity levels in the image

Thus, a processed (output) image is obtained by using eqn (8) to map each pixel in the input image with intensity r_k into a corresponding pixel with level S_k in the o/p image

This is called a histogram equalization or histogram linearization transformation. satisfies condition (a) and (b)

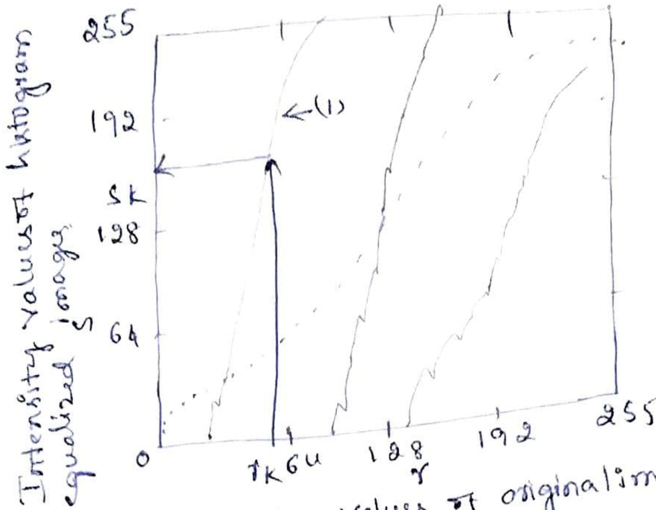
→ The inverse transformation from s back to r is denoted by

$$r_K = T^{-1}(S_K) \quad - (8)$$

Inverse transformation satisfies condition (a') and (b)

Histogram Matching (Specification)

Histogram equalization produces a transformation function that seeks to generate an output image with a uniform histogram



→ The method used to generate images that have a specified histogram is called histogram matching or histogram specification

→ Consider continuous intensities r and z. PDFs $P_r(r)$ and $P_z(z)$.

→ r and z ~~are the~~ denote the intensity level of the i/p and o/p images.

→ $P_r(r)$ - given input image & $P_z(z)$ specified PDF - o/p image

→ Let s be a random variable with the property.

$$s = T(r) = (L-1)\int_0^r P_r(w)\,dw \quad - (9)$$

where w - dummy variable.

→ Define a function G on variable z with the property.

$$G(z) = (L-1)\int_0^z P_z(v)\,dv = s \quad - (10)$$

where v - dummy variable.

→ $G(z) = s = T(r)$

$$z = G^{-1}(s) = G^{-1}[T(r)] \quad - (11)$$

→ Equation (9) through (11) imply that an image whose intensity levels have a specified PDF can be obtained using the following procedure.

1. obtain $P_r(r)$ from the i/p image to use in eqn (9)

2. Use the Specified PDF $P_z(z)$ in eqn (10) to obtain the function $G(z)$

3. Compute the inverse transformation $z = G^{-1}(s)$: this is a mapping from s to z, the latter being the values that have the specified PDF.

4. obtain the o/p image by first equalizing the i/p image using eqn (9) the pixel values in this image are the s values. For each pixel with value s in the equalized image perform the inverse mapping $z = G^{-1}(s)$ to obtain the corresponding pixel in the o/p image.

When all pixels have been processed with this transformation the PDF of the o/p image, $P_z(z)$ will be equal to the specified PDF.

→ The discrete formulation of eqn (9) in the histogram equalization transformation in eqn (7) which we repeat here for convenience.

$$S_K = T(r_K) = (L-1)\sum_{j=0}^{K} P_r(r_j) \quad K = 0, 1, 2 \ldots L-1 \quad - (12)$$

→ lly the discrete formulation of eqn (10) involves computing the transformation function

$$G(z_q) = (L-1)\sum_{i=0}^{q} P_z(z_i) \quad - (13)$$

for a value of q so that

$$G(z_q) = S_K \quad - (14)$$

where $P_z(z_i)$ - i^{th} value of the specified histogram.

→ Finally, we obtain the desired value z_q from the inverse transformation.

$$z_q = G^{-1}(S_K) \quad - (15)$$

- This is a mapping from the s values in the histogram equalized image to the corresponding z values in the output image.

Fundamentals of Spatial filtering.

→ Filtering refers to passing, modifying or rejecting specified frequency components of an image.

→ For example, a filter that passes low frequencies is called a lowpass filter.

→ The net effect produced by a lowpass filter is to smooth an image by blurring it.

→ Spatial filtering modifies an image by replacing the value of each pixel by a function of the values of the pixel and its neighbors.

→ If the operation performed, on the image pixels is linear, then the filter is called linear spatial filter.

→ otherwise the filter is a nonlinear spatial filter.

The Mechanics of Linear Spatial filtering

→ A linear spatial filter perform a sum of products operation between an image f and a filter kernel w.

→ The kernel is an array whose size defines the neighborhood of operation and who coefficients determine the nature of the filter.

→ other terms used to refer to a spatial filter kernel are mask, template and window.

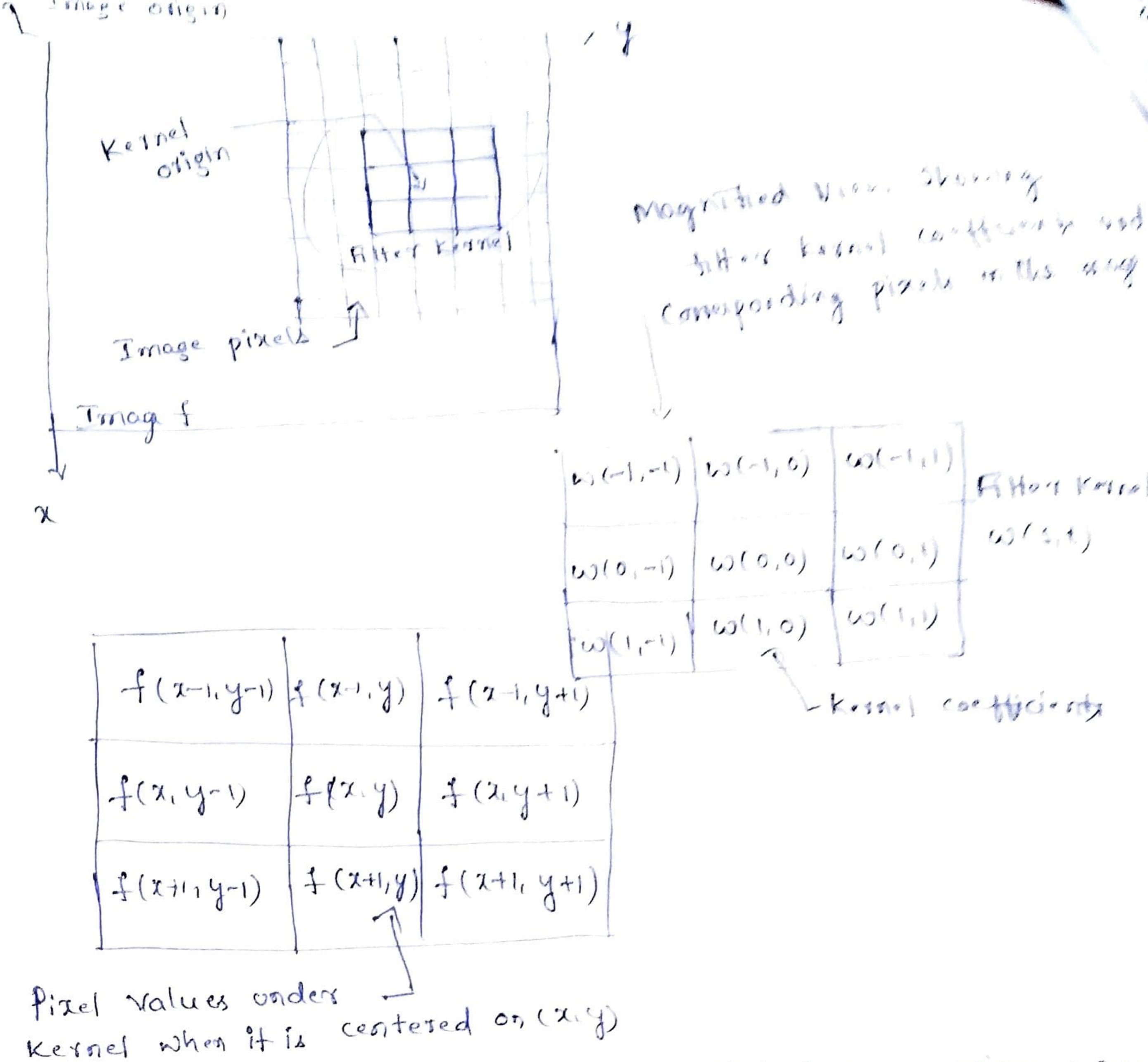
→ we use the term filter kernel or simply kernel.

→ Fig. illustrates the mechanics of linear spatial filtering using a 3 x 3 kernel.

→ At any point (x, y) in the image, the response g(x, y) of the filter is the sum of products of the kernel coefficients and the image pixels en compassed by the kernel.

$$g(x,y) = w(-1,1)f(x-1,y-1) + w(-1,0)f(x-1,y) + \ldots + w(0,0)f(x,y) + \ldots + w(1,1)f(x+1,y+1) \quad - (1)$$

→ x and y are varied, the center of the kernel moves from pixel to pixel, generating the filtered image g.



→ Fig illustrates the mechanics of linear spatial filtering using a 3×3 Kernel.

→ At any point (x,y) in the image, the response $g(x,y)$ of the filter is the sum of products of the kernel coefficients and the image pixels encompassed by the kernel

$$g(x,y) = w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,y) + \dots + w(0,0)f(x,y) + \dots + w(1,1)f(x+1,y+1) \quad -①$$

→ As coordinates x and y are varied, the center of the kernel moves from pixel to pixel, generating the filtered image g in the process.

→ Observe that the center coefficient of the kernel $w(0,0)$ aligns with the pixel at location (x,y).

In general, linear spatial filtering of an image of size M×N with a kernel of size m×n is given by the expression

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)\, f(x+s, y+t) \quad (2)$$

Where x and y are varied so that the center (origin) of the kernel visit every pixel in f once.

Spatial correlation and convolution:-

→ Spatial correlation consists of moving the center of a kernel over an image and computing the sum of products at each location.

→ The mechanics of spatial convolution are the same, except that the correlation kernel is rotated by 180°.

1-D illustration of eqn (2) becomes

$$g(x) = \sum_{s=-a}^{a} w(s) f(x+s) \quad - (3)$$

Convolution

Correlation

(a) origin f: 0 0 0 1 0 0 0 0 w: 1 2 4 2 8

Correlation result: 0 8 2 4 2 1 0 0

(b) 0 0 0 1 0 0 0 0
1 2 4 2 8
starting position alignment

Extended (full) correlation result: 0 0 0 8 2 4 2 1 0 0 0 0

zero padding

(c) 0 0 0 0 0 1 0 0 0 0 0 0
1 2 4 2 8
starting position

$$g(0) = \sum_{s=-2}^{2} w(s) f(s+0) = 0$$

(d) 0 0 0 0 0 1 0 0 0 0 0 0
1 2 4 2 8
position after 1 shift

$g(1) =$

(e) 0 0 0 0 0 1 0 0 0 0 0 0
1 2 4 2 8
Position after 3 shifts

(f) 0 0 0 0 0 1 0 0 0 0 0 0
1 2 4 2 8
Final position

Smoothing (Lowpass) Spatial filters:-

Smoothing (averaging) Spatial filters are used to reduce sharp transitions in intensity.

→ Smoothing is used to reduce irrelevant details in an image, where irrelevent refers to pixel regions that are small with respect to the size of the filter kernel.

→ Another application is for smoothing the false contours that result from using an insufficient no of intensity levels in an image.

→ Smoothing kernel with an image blurs the image with the kernel degree of blurring being determined by the size of the kernel and the values of its coefficients.

Low pass filters
- Box Filter kernels
- Gaussian kernels

Box Filter Kernels:-

→ The simplest, separable lowpass filter kernel is the box kernel, whose coefficients have the same value (typically 1).

→ the name "box kernel" comes from a constant kernel resembling a box when viewed in 3-D.

→ An m×n box filter is an m×n array of 1s with a normalizing constant in front, whose value is 1 divided by the sum of the values of the coefficients.

→ Apply low pass kernels has two purposes
1. the average value of an area of constant intensity would equal that intensity in the filtered image.
2. Normalizing the kernel in this way prevents introducing a bias during filtering. i.e. the sum of the pixels in the original and filtered images will be the same.

bcz Box kernel all rows and columns are identical. the rank of these kernels is 1.

→ Box filters are suitable for quick experimentation and they often yield smoothing results that are visually acceptable.

Gaussian kernels of the form

$$w(s,t) = G(s,t) = K e^{-\frac{s^2+t^2}{2\sigma^2}} \quad -(1)$$

Variables s and t in eqn (1) are real numbers.

By letting $r = [s^2+t^2]^{1/2}$

We can write Eqn (1) as. $G(r) = K e^{-\frac{r^2}{2\sigma^2}}$ - (2)

$\sqrt{2}$	1	$\sqrt{2}$
1	0	1
$\sqrt{2}$	1	$\sqrt{2}$

3x3

Sharpening (Highpass) Spatial filters:-

- Sharpening highlights transitions in intensity.
- Uses of image sharpening range from electronic printing and medical imaging to industrial inspection and autonomous guidance in military systems.
- Sharpening is often referred to as highpass filtering.
- In this case, high frequencies are passed, while low frequencies are attenuated or rejected.

Sharpening the filter based on first and second order derivates.

First derivative:-

1. Must be zero in areas of constant intensity.
2. Must be nonzero at the onset of an intensity step or ramp.
3. Must be nonzero along intensity ramps.

My any defn of a second derivative.

1. Must be zero in areas of constant intensity.
2. Must be nonzero at the onset and end of an intensity step or ramp.
3. Must be zero along intensity ramps.

A basic defn of the first order derivate of a one-dimensional function f(x) is the difference $\frac{\delta f}{\delta x} = f(x+1) - f(x)$