Derivation of simpson

3/8 rule

$n=3$

Newton cuts formula

$$\int (f(x)\,dx = X[xy + \tfrac{1}{2}x^3 \,|\, y_0 + \tfrac{1}{2}(2x^2 - 3x^2)\, x^3 y_2 + \tfrac{1}{24}$$

$$(x^4 - 4x^3 / 4x^2)\, \Delta^3 y_0 + \ldots$$

x	x_0	x_1	x_2	x_3
y	y_1	y_2	y_3	y_4

Simpson 3/8 Rule

$n \Rightarrow 3 \quad \Delta y_0 = y_1 - y^2$

$\Delta^2 y = y_2 - y + y_2$

$x_0, x_3) \Rightarrow f(x)\,dx = 4[3y_2 + \tfrac{1}{2} 3 0 y_2 + \tfrac{1}{2}(2x^2 7 - 3x + 7)$

$$\Rightarrow \int_{2x}^{x} f(x)\,dx = \frac{3y}{8}\left[y_x + 3y_1 + 3y_2 + y_3\right]$$

$$x_3, x_0) \Rightarrow \int_{x_3}^{x} f(x)\,dx = \frac{3y}{8}\left[y_3 + 3y_4 + 3y + y_0\right)$$

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$$(x_6, x_9) = \int_{x_6}^{x_9} f(x)dx = \frac{3h}{8}\left[y_6 + 3y_7 + 3y_8 + y_9\right]$$

$$(x_{n-3}, x_n) \Rightarrow \int_{x_{n-3}}^{x_n} f(x)dx = \frac{3h}{8}\left[y_{n-3} + 3y_{n-2} + 3y_{n-1}\right]$$

Adding all subinterval

$$\int_{x_0}^{x_n} f(x)dx = \frac{3h}{8}\left[y_0 + y_n + 3(y_1 + y_2 + y_4 + y_5 \ldots\right.$$

$$y_8 + 2(y_3 + y_6 + y_9 + \ldots$$

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Q2

DE $y' = 1 + xy + x^2y^2$

Solve upto 5 dp by using

(a) Tayler's Series method

Let step value $h = 1$

Here $x_0 = 0, y_0 = 1, h = 1, x_n = 0.5$

$y' = 1 + xy + x^2y^2$

$y'' = 2xy + 2x^2yy^2 + y + xy^2$

$y''' = 2y^2 + 8xyy' + 2x^2y'^2 + 2x^2yy''' + 2y' + xy''$

$y'''' = 12yy' + 12xyy'' + 6x^2y'y'' + 2x^2yy'' + 3y'' + xy'''$

Now Subsituting we get

$y_0' = 1 + x_0y_0 + x_2^2y_0^2 = 1$

$y_0'' = 2x_0y_2^2 + 4x_0^2y_1y_0' + y_0 + y_2 = 1$

$y''' = 2y_0^2 + 8x_0y_0y_0' + 2x_0^2y_0y_0'' + 2y_0'' + x_0y'' - 4$

$y'''' = 12y_0y_0' + 12x_0y_0y_0'' + 6x_2^2y_0'y_0'' + 2x^3y_0y_1''' + 3y_2''' + x_0y_0^2 = 15$

nutting there is Taylor's series

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$y_1 = y_0 + h y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \frac{h^4}{4!} y_0'''' + \ldots$

$= 1 + 1(1) + \frac{1^2}{2!}(1) + \frac{(1)^3}{3!}(4) + \frac{(1)^4}{4!}(15) + \ldots$

$= 1 + 1 + 0.5 + 0.66667 + 0.625$

$= 3.79167$

again taking (x_1, y_1) in place of (y_0, x_0) and repeat the process

Now substituting we get

$y_1' = 1 + x_1 y_1 + x_1^2 y_1^2 = 19.1684$

$y_1'' = 177.07898$

$y_1''' = 3074.93839$

$y_1'''' = 63878.15576$

put these value in taylor's series.

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