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Reevaluation of Rolling Element Bearing Load-Life Equation Based on Fatigue Endurance Data

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The load-life exponents used in the modified life rating equation for rolling element bearings were determined by statistical analysis of the experimental data generated in the 1940s, following Lundberg and Palmgren's seminal work. Based on fracture mechanics arguments, the fatigue life is known to be inversely proportional to the square root of the size of the nonmetallic inclusions. However, modern high-performance vacuum induction melt–vacuum arc remelt (VIMVAR) bearing steels are clean and nonmetallic inclusions are no longer the weak link. Fatigue life predictions (L_{10} life) for modern bearings using the modified load-life relations greatly underpredict observed life. Hence, there is a need to update parameters of these equations using more recent life data. Based on the endurance data reported in Harris and McCool (1), validation analysis of the modified life rating equation was performed to reevaluate the values of load-life exponent for both ball and cylindrical roller bearings. The results from this study indicate that the load-life exponent for ball bearings should be 4.1, instead of 3, and for roller bearings it should be 5.5, instead of 3.33. Bearing L_{10} life calculated using the corrected load-life exponents values shows better agreement with observed life. Details of the sampling technique used for reducing epistemic uncertainty in experimental data and the process of statistical reevaluation using Bayesian updating are discussed in detail. The accuracy of reevaluated results is presented using logarithmic plots of the ratio of predicted to actual fatigue lives for all data samples.

KEY WORDS

Ball Bearing; Rolling Contact Fatigue; Bearing L_{10} Life; Load-Life Exponent; Validation; Statistical Reevaluation; Epistemic Uncertainty; Weibull Parameters; Bayesian Updating

INTRODUCTION

Subsurface material in high-performance aerospace bearings is subjected to a large number of rolling contact fatigue (RCF) stress cycles ($\sim 10^{10}$) with complex triaxial stress state and changing planes of maximum shear stress during a loading cycle. Bearing raceway surfaces subjected to RCF experience highly localized cyclic microplastic loading, leading to localized material degradation, nucleation and propagation of subsurface cracks, and the detachment of material (spall) from the main body of the bearing. Lundberg and Palmgren (LP) (2) were the first researchers to propose that the maximum orthogonal shear stress τ_o initiates subsurface cracks and this shear stress occur at a subsurface depth of z_o. They also applied Weibull statistical strength theory (Weibull (3)) to the stressed volume under Hertzian elastic contact to obtain the probability of survival of the volume susceptible to subsurface-initiated fatigue cracks. The LP crack initiation-based life prediction worked reasonably well for older technology bearing steels that had nonmetallic inclusions, with expected fatigue life of about 600 million cycles. However, these theories do not work well for modern "ultraclean" vacuum induction melt–vacuum arc remelt (VIMVAR) bearing steels, which have fewer and smaller nonmetallic inclusions. Because RCF effects are very localized, the spatial aspects of the microstructure (primary inclusions: oxides; and secondary inclusions: carbides and nitrides) and their attributes (volume fraction, size, morphology, and distribution) play an important role in bearing fatigue life dispersion.

Bearing Fatigue Life Dispersion

Even when nominally identical sets of bearings are tested under similar operating conditions for load, speed, lubrication, and environmental conditions, bearings fail according to a dispersion that varies over a wide range of values. Because of this life dispersion, bearing life is typically expressed by L_{10} life, defined as the number of cycles at which 90% of the identical bearings survive under the applied load.

In 1947, Lundberg and Palmgren *(2)* extended statistical work of Weibull *(3)* to predict probability of survival S of surface under rolling contact fatigue as a power function of orthogonal shear stress τ_0, number of stress cycles survived N, depth to maximum orthogonal shear stress z_0, and stressed volume V. In mathematical form, the LP equation is represented as

$$\ln\frac{1}{S} \sim \frac{\tau_0^c N^e}{z_0^h} V. \qquad [1]$$

Here c, e, and h are empirical constants. The exponent e is the Weibull slope and is a measure of bearing fatigue life dispersion. A lower value of e indicates greater dispersion of fatigue life.

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NOMENCLATURE

a = Semimajor axis of ellipsoidal contact area (m, in.)
a_1 = Reliability factor
a_2 = Material factor
a_3 = Lubrication factor
a_4 = Contamination factor
a_{iso} = Integrated life adjustment factor
b = Semiminor axis of ellipsoidal contact area (m, in.)
C = Dynamic capacity of the bearing (N, lbf)
c = Shear stress exponent
e = Weibull slope
F = Probability of failure
F_a = Axial/thrust load (N, lbf)
F_e = Equivalent radial load (N, lbf)
F_r = Radial load (N, lbf)
h = Maximum orthogonal shear stress depth exponent
i = Experiment number
k = Shape parameter
L_{10} = Bearing fatigue life with 90% reliability, millions of revolutions or millions of stress cycles
L_{10}(act) = Bearing fatigue life from actual experiments, millions of revolutions or millions of stress cycles
L_{10}(LP) = Bearing fatigue life from modified LP model, millions of revolutions or millions of stress cycles
L_{10}(reeval) = Bearing fatigue life from reevaluated load-life model, millions of revolutions or millions of stress cycles
L_s = Bearing fatigue life with 100 − s% reliability, millions of revolutions or millions of stress cycles
L_β = Characteristic parameter, millions of revolutions or millions of stress cycles
LP = Lundberg and Palmgren
$\mathcal{L}$ = Circumference of raceway (m, in.)
N = Number of stress cycles
n = Stress-life exponent
n_i = Number of bearings tested in experiment number i
p = Load-life exponent
r_i = Number of failures observed in experiment number i
S = Probability of survival of surface
u = Number of stress cycles per revolution
V = Stressed volume (m3, in3)
X = Radial load factor
Y = Axial load factor
z_0 = =Depth to maximum orthogonal shear stress (m, in.)
α = Normalization factor
λ = Scale parameter
$\left[\sigma_{(\text{total})}\right]_i$ = Normalized standard deviation for normal distribution corresponding to sample i
$\left[\sigma_{(\text{sample})}\right]_i$ = Weighted standard deviation of 5,000 probable estimates from sample i
τ_0 = Orthogonal shear stress (Pa, psi)

Equation [1] can further be simplified for a specific bearing under a particular load as (see Appendix A)

$$\ln \ln \frac{1}{S} = e \ln\left(\frac{L_s}{L_\beta}\right). \quad [2]$$

Equation [2] represents the Weibull distribution for bearing fatigue lives. Here L_s represents the number of revolutions for which the bearing will survive given load with reliability level of $100 - s$, and L_β is a characteristic parameter determined using bearing fatigue lives. It is clear from Eq. [2] that the $\ln \ln \frac{1}{S}$ vs. $\ln L_s$ plot will be a straight line fit.

Load-Life Equation

Lundberg and Palmgren *(2)* proposed that life rating for rolling element bearings can be obtained using the following relation:

$$L_s = \left(\frac{C}{F_e}\right)^p. \quad [3]$$

The basic dynamic capacity of a bearing, C, is the rated load that the bearing will successfully withstand for one million revolutions with reliability level of 90% (i.e., $s = 10\%$). For other reliability levels, the life modification factor a_1 is used. F_e is the equivalent radial load experienced by the bearing and in the standard method of life calculation is given by

$$F_e = XF_r + YF_a, \quad [4]$$

where X and Y are radial and axial load factors, and F_r and F_a are the applied radial and axial loads, respectively. Load factor values are dependent on the nominal contact angle of the bearings.

However, it was observed that for modern bearing steels, the LP Eq. [3] significantly underpredicts bearing fatigue lives. One of the reasons for this underprediction is that reliability, material, and lubrication conditions of the bearings were not considered in the original LP model. To account for these operating conditions, Zaretsky (4) modified the original LP equation with three life factors a_1, a_2, and a_3 as

$$L_s = a_1\, a_2\, a_3 \left(\frac{C}{F_e}\right)^p. \quad [5]$$

It was observed that these life adjustment factors are independent of load-life exponent p, except for the case when inner ring raceways experience tensile hoop stresses due to interference fits (Vlcek and Zaretsky *(5)*). This modified LP equation was part of ISO 281 standard launched in 1978 *(6)*. Harris *(7)* discussed the systematic procedure of obtaining modified life rating Eq. [5] from the basic LP Eq. [1].

In the late 1980s, as manufacturers began designing for more demanding applications, steel quality, design, production methods, and surface finishes improved, which led to significantly higher test lives for bearings than predicted. By 1994, the bearing life was 14 times better than that predicted from the modified LP Eq. [5]. In addition, some manufacturers reported that in Eq. [5] factors a_2 and a_3 seemed to be dependent on each other *(6)*. In

1985, Ioannides and Harris *(8)* proposed a new fatigue life model for rolling bearings that addressed these issues. Their work confirmed the existence of fatigue limit stress and interdependence of material and lubrication life factors. Based on finite element analysis, they replaced "stress" in the original LP Eq. [1] with the difference between stress and fatigue limit stress at each location within the material. Using Ioannides and Harris's *(8)* model, the ISO launched its new standard ISO 281 in 2007 *(6)*:

$$L_s = a_1 \, a_{ISO} \left(\frac{C}{F_e} \right)^p, \qquad [6]$$

where, a_{ISO} is the integrated life adjustment factor, which is dependent on four interdependent factors: lubrication, contamination, load, and the fatigue stress limit of the bearing material. Gabelli, et al. *(9)* defined fatigue limit stress for rolling contacts as cyclic stress that leads to median fatigue failure corresponding to the very high cycle fatigue range of 10^9–10^{11} cycles. ISO 281: 2007 standard is been widely accepted by manufacturers and institutes for bearing designs; this standard is mandatory for certification of wind turbine gearboxes *(6)*. However, the American National Standards Institute has not adopted this standard and Zaretsky *(10)* objected to the use of fatigue stress limits and the interdependence of life adjustment factors. Therefore, due to existing conflict and insufficient information to determine life factor a_{iso} for each reported data set, the modified LP Eq. [5] is used for analysis in this work.

In the LP Eq. [1], exponents c and h are a function of exponents e and p, which are determined empirically. Evaluating the endurance test data of approximately 1,500 bearings, Lundberg and Palmgren *(2)* determined that for a bearing under point contact, $e = 10/9$, $c = 31/3$, and $h = 7/3$. Harris *(7)* showed that the load-life exponent p can be expressed as

$$p = \frac{c - h + 2}{3e}. \qquad [7]$$

For line contacts (roller bearings) Lundberg and Palmgren (11) reported that

$$p = \frac{c - h + 1}{2e} \qquad [8]$$

and $e = 9/8$. From Eqs. [7] and [8], Lundberg and Palmgren *(2)*, *(11)* estimated that load-life exponent $p = 3$ for point contacts and $p = 4$ for line contacts. However, Lundberg and Palmgren *(12)* conservatively chose to use $p = 10/3$ for roller bearings. The vast majority of data were based on endurance testing of bearings made from 52100 steel. Exponents c and h were found to be constant for both ball and roller bearings, indicating that they are material constants.

The load-life exponents 3 for ball bearings and 3.33 for roller bearings were determined empirically based on the endurance test data generated in 1947 on 52100 bearings. These values pertain to rolling bearings of specific design, properly manufactured from good quality air-melted steels and are based on work by Lundberg and Palmgren (2), (12) conducted during the 1930s and 1940s. The load rating and life calculation formulas developed in the middle of the 20th century are representative of the manufacturing practices, materials, and lubricating methods available at that time. Despite its popularity since 1950, the LP theory has several drawbacks. It did not consider the effect of surface shear stress and the presence of a surface lubricating film. It has been observed that some surface shear stress is always present and it shifts the location of maximum orthogonal shear stress closer to the surface (Sadeghi, et al. *(13)*). In addition, LP theory is developed based on the assumption of perfectly smooth contacting surfaces. However, in practice, surfaces contain irregularities that affect subsurface stress fields.

Recent endurance data for bearings manufactured from VIM-VAR steels indicated that bearings fail at much higher fatigue lives. The modified life rating Eq. [5] underpredicts bearing L_{10} life for the majority of applications. Hence, based on the latest endurance test data available, the current load-life exponent values for bearings must be updated to bridge the gap between theory and practice. For roller bearings with m6 interference fit, Oswald, et al. *(14)* reported a stress life exponent n in the range of 7.4–7.7. Similarly, for ball bearings with m6 middle-of-the-tolerance-band interference fit it was found to be 8.63 for deep-groove ball bearings and 8.74 for angular contact ball bearings (Oswald, et al. *(15)*). Work by Parker and Zaretsky *(16)* suggests that the value of the stress life exponent for cleaner and more recent vacuum processed steels should be 12. In addition, Zaretsky, et al. *(17)* established that the load-life exponent p will be one third of the stress life exponent n. Therefore, his work suggests that the load-life exponent p should be 4 for ball bearings manufactured from cleaner vacuum-processed steels. Sadeghi, et al. *(13)* proposed that bearing endurance data can be represented with the three-parameter Weibull distribution and the value of load-life exponent p should be 8/3 for both rotary and linear ball bearings undergoing point contact loading. To check the accuracy of existing and recommended load-life exponents, validation analysis of the modified LP Eq. [5] was performed. The endurance data presented in Harris and McCool *(1)* was used to reevaluate the load-life exponent p values so that observed bearing L_{10} lives can be best represented on the logarithmic plot of the ratio of predicted and actual fatigue lives.

RECALIBRATION ANALYSIS

In 1993, under a study sponsored by the United States Navy, ball and roller bearing fatigue endurance data were collected from four bearing manufacturers, two helicopter manufacturers, three aircraft engine manufacturers, and U.S. Government agency–sponsored technical reports. The collected endurance data contains fatigue lives for 62 data sets comprising deep-groove ball bearings (DGBBs), angular contact ball bearings (ACBBs), and cylindrical roller bearings (CRBs). The DGBBs and CRBs were operating under pure radial load. Some of the ACBBs were subjected to pure axial loading and the rest were subjected to combined radial and axial loads in which axial load is dominant. Of the 62 data sets, 11 data sets reported 0 or 1 failure; hence, their L_{10} life and Weibull slope estimates are not available. Therefore, these 11 data sets were rejected for the validation study. Harris and McCool *(1)* provide detailed discussion on the types of bearing tested and observed L_{10} lives for each data set. Sufficient technical information regarding bearing raceway curvatures, operating conditions, fit-ups, and material properties are not provided. These are the primary reasons for using ISO 281: 1978 standard over ISO 281: 2007 in this work. The individual data points and the Weibull plots used for determining L_{10} lives were also not provided. However, these test results are the best possible compilation of endurance data available in the open literature for the validation study. Results of these 51 data sets are summarized in Tables 1 and 2. Table 1 contains data for DGBBs and Table 2 contains data for ACBBs and CRBs. It should be noted that VIMVAR and VAR steels are denoted using the symbols VV and V, respectively. Tables 1 and 2 contain the dimensionless ratio F_e/C, which indicates the loads experienced by each bearing. F_e is the equivalent radial load for DGBBs and CRBs and equivalent thrust load for ACBBs. C represents the basic dynamic capacity rating for each bearing calculated based on carbon vacuum degassed 52100 steel. The mean shaft speed and r_i number of failures observed in sample of size n_i are provided as failure index r_i/n_i for each data set. Harris and McCool *(1)* determined unbiased maximum likelihood estimates of shape parameter and corresponding L_{10} lives for each data set. These characteristic parameters of each data set were used to generate virtual samples of bearing fatigue lives for respective experiments. From the Weibull slope estimates for each data set, we can see that the quality of endurance data is mixed. Higher values of the Weibull slope indicate narrow dispersion and smaller values indicate wide dispersion of fatigue life values in that data set. Harris *(7)* reported that for commonly used bearing steels, values of e are observed in the range 1.1 to 1.5 and for modern, ultraclean, vacuum remelted steels, they are in the range of 0.8 to 1.0. Harris and McCool *(1)* mentioned that though the majority of the data are from bearing testing in which operating conditions were carefully monitored, some data are from *(1)* poorly monitored tests of small samples of bearings with few failures, *(2)* large samples of bearings with few failures, and *(3)* tests with inadequate control of time-variant load and speed duty cycles, leading to large uncertainties in some of the observations. In some cases, failures of the bearings were not examined as per the laboratory standards, leading to substantial scatter in both L_{10} lives and Weibull slopes. Using the modified load-life Eq. [5], theoretical L_{10} lives were predicted for each operating bearing. The life modification factors used in this equation were obtained from STLE standards specified in Zaretsky *(4)* and are multiplicative. The material life factor a_2 used for bearing life calculations is given in Table 3. In addition, based on the work of Sayles and MacPherson *(18)*, life adjustment factor a_4 was used for level of contamination (Harris and McCool *(1)*). Tables 1 and 2 represent the ratio of theoretically predicted fatigue lives, from the modified LP model, to the experimental fatigue lives for DGBBs, ACBBs, and CRBs, respectively. It is clear that the majority of the $\frac{L_{10}(LP)}{L_{10}(act)}$ ratios are

TABLE 1—SUMMARY OF ENDURANCE DATA FOR DEEP-GROOVE BALL BEARINGS (DGBB)

Data Line.	Material	F_e/C	Failure Index	L_{10} life (h)	Weibull Slope (e)	L_{10} (LP)/L_{10} (act)	Speed (rpm)
			Deep-groove ball bearings; load-life exponent $p = 3$				
1	52100	0.357	22/40	527	2.22	0.335	1,500
2	52100	0.357	7/11	147	1.28	1.12	6,000
3	52100	0.379	3/6	21	2.22	0.114	8,000
4	52100	0.354	4/33	3,503	0.65	0.395	2,000
5	52100	0.919	9/67	1,644	0.65	0.635	263
6	52100	0.357	7/37	1,956	0.889	0.663	6,000
7	52100	0.212	3/28	654	1.33	0.316	8,000
8	52100	0.53	23/37	1,723	2.65	0.263	6,000
10	52100	0.357	2/41	8,856	0.7	0.146	1,500
12	52100	0.357	23/79	807	1.21	0.688	6,000
13	52100	0.53	11/40	513	0.513	1.12	6,000
14	52100	0.354	21/60	433	0.928	0.473	2,000
15	52100	0.379	12/30	115	0.695	0.43	8,000
16	52100	0.379	8/30	257	0.717	0.192	8,000
17	52100	0.379	43/103	1,813	1.36	0.306	8,000
18	52100	0.379	29/29	19	2.75	2.6	8,000
19	8620 car	0.379	8/29	240	0.686	0.183	8,000
20	8620 car	0.379	12/29	290	1.2	0.152	8,000
21	8620 car	0.379	57/57	296	0.947	0.192	8,000
22	VV M50	0.357	7/30	1,723	2.29	0.205	3,200
23	VV M50	0.357	3/28	2,678	1.06	0.102	3,500
24	VV M50	0.357	6/37	1,133	0.72	0.214	3,200
25	VV M50	0.357	33/40	203	3.48	1.92	3,200
26	M50NiL	0.357	6/40	1,202	0.681	0.667	3,500
27	M50NiL	0.357	5/40	1,744	1.23	0.201	3,500
29	V M50	0.366	11/18	58.3	2.29	0.292	21,200

TABLE 2—SUMMARY OF ENDURANCE DATA FOR ANGULAR CONTACT BALL BEARINGS (ACBBS) AND CYLINDRICAL ROLLER BEARINGS (CRBS)

Data Line.	Material	F_e/C	Failure Index	L_{10} life (h)	Weibull Slope (e)	L_{10} (LP)/L_{10} (act)	Speed (rpm)
Angular contact ball bearings; load-life exponent $p = 3$							
30	52100	0.993	5/8	15	1.91	0.297	263
31	VV M50	0.544	2/33	385,900	0.2	0.0778	3,130
33	VV M50	0.544	2/20	1,292	1.1	0.442	3,130
34	VV M50	0.544	7/362	103,900	0.953	0.289	3,130
36	VV M50	1.393	2/634	3,264,000	0.671	0.00984	12,400
37	VV M50	0.349	2/64	20,260	0.796	0.48	9,800
38	VV M50	1.344	3/1199	17,200	1.15	0.0104	12,400
39	VV M50	0.224	3/20	504	0.81	0.902	12,000
40	VV M50	0.159	3/30	3,010	1.08	0.158	19,000
41	M50NiL	0.224	2/12	41	0.136	0.131	25,000
43	V M50	0.359	5/17	344.2	0.836	0.307	5,500
45	V M50	0.244	2/10	1,503	1.14	0.224	5,500
46	V M50	0.592	5/10	67	0.843	0.903	9,700
47	V M50	0.592	4/10	70	0.689	0.964	9,700
Cylindrical roller bearings; load-life exponent $p = 3.33$							
49	52100	0.091	4/6	14	0.752	0.0951	1,050
50	52100	0.35	6/6	46	3.63	0.0625	19,400
51	52100	0.38	4/6	63	1.23	0.0389	7,540
52	52100	0.322	4/7	4	0.736	0.635	19,400
53	52100	0.387	3/6	8	1.9	0.209	7,540
54	52100	0.441	3/6	41	1.08	0.152	2,550
55	VV M50	0.073	13/5321	14,290	1	0.0276	27,600
56	V M50	0.325	6/6	83	2.43	0.0462	9,700
58	V M50	0.51	2/6	22	0.48	0.154	1,050
61	V M50	0.35	4/6	25	0.674	0.131	19,400
62	V M50	0.38	2/19	895	0.21	0.011	7,540

close to 0, indicating that the modified LP Eq. [5] still significantly underpredicts bearing fatigue lives. In this study, validation analysis was performed to statistically reevaluate the load-life exponent p used in the modified LP Eq. [5] to correct for this underprediction.

TABLE 3—STLE MATERIAL-LIFE FACTORS (HARRIS AND MCCOOL (1))

Material Parameter	a_2
Composition	
AISI 52100	3
M50	2
M50NiL	4
Heat processing	
Air melt	1
Carbon vacuum degassed (CVD)	1.5
Vacuum arc remelted (VAR)	3
Vacuum induction melted–vacuum arc remelted (VIMVAR)	6
Surface working	
Radial ball bearing raceways	1.2
Angular contact ball bearing raceways	1
Forged angular contact ball bearing raceways	1.2
Cylindrical roller bearing raceways	1

Weibull Distribution Parameters

The Weibull distribution is a continuous probability distribution and its probability density function is represented as (Papoulis and Pillai (19))

$$f(x;\lambda,k) = \begin{cases} \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}\exp\left(-\left(\frac{x}{\lambda}\right)^{k}\right), & x \geq 0, \\ 0, \quad x < 0 \end{cases} \qquad [9]$$

where k and λ are the positive shape and scale parameters of the distribution, respectively. If quantity x is failure time, then Eq. [9] gives the distribution for which the failure rate is proportional to a power of time. From Eqs. [B5] and [B6] in Appendix B we can conclude that the shape parameter of the Weibull distribution for each data set in Tables 1 and 2 can be obtained as

$$k = e \qquad [10]$$

and the scale parameter as

$$\lambda = (A)^{\frac{-1}{e}} = L_{\beta}. \qquad [11]$$

In Eq. [11], A is material factor that is constant for rolling bearings (see Appendix A). Equation [2] can be solved for each bearing data set. Values of Weibull slope e and characteristic parameter L_β were determined using the data reported for each experiment. Then using Eqs. [10] and [11] the shape and scale parameters of Weibull distribution of each data set were determined for the validation study. These Weibull distribution parameters were used to represent corresponding data sets in the statistical reevaluation process.

Validation

Issues surrounding validation include validity of the theoretical model and uncertainty quantification. As we can see from Tables 1 and 2, fatigue lives predicted by the modified LP model differ significantly from field data. Figures 1 and 2 present plots of the logarithmic ratio of predicted fatigue life to the experimental fatigue lives for the reported bearing data sets. Figure 1 shows $\log\frac{L_{10}(LP)}{L_{10}(act)}$ values for ball bearings (DGBBs and ACBBs) and Fig. 2 shows the $\log\frac{L_{10}(LP)}{L_{10}(act)}$ values for CRBs. $L_{10}(LP)$ is the theoretically predicted fatigue life from the modified LP model; that is, load-life Eq. [5]. As shown in the figures, if the life rating Eq. [5] accurately predicts fatigue life, we should get a straight line fit at $\log\frac{L_{10}(LP)}{L_{10}(act)} = 0$ for all data sets. Confidence intervals of 90% around the target are also shown in these two figures. From Figs. 1 and 2 we can see that there is significant underprediction of fatigue lives from this equation. The LP-based life models with load-life exponents of 3 for ball bearings and 3.33 for cylindrical roller bearings generally result in wide variation in predicted vs. observed life and thus are not a reliable design tool for the new generations of bearings and bearing steels. This can result in the use of oversized bearings and weight penalty for the required load conditions, a serious concern in aerospace applications. To address this issue and update the load-life exponents using the available and more recent experimental data, validation and reevaluation analysis were performed for the modified load-life Eq. [5].

Uncertainty Quantification

Because the number of bearings tested in each experiment were finite, there exists epistemic uncertainty in the reported L_{10} lives. To quantify this epistemic uncertainty in each experiment/data set, virtual samples of fatigue lives were generated using MATLAB simulation software. Let i be the subscript that denotes experiments from 1 to 51. For simulation, *wblrnd* (λ_i, k_i, n_i) function was used to generate a virtual sample of n_i fatigue lives that follow a Weibull distribution with scale parameter λ_i and shape parameter k_i. Here it should be noted that n_i represents the number of bearings tested in the ith experiment, whose Weibull distribution parameters are as follows: shape parameter k_i and scale parameter λ_i, were determined using Eqs. [10] and [11], respectively. From Table 1 we can see that for $i = 1$, $n_i = 40$; Fig. 3 presents a Weibull plot of 40 probable fatigue lives corresponding to data set 1. For this sample of 40 virtual lives, L_{10} life—that is, the 10th percentile value of the sample—was estimated using the '*prctile*' function available in MATLAB. By this method we get one estimate of virtual L_{10} life corresponding to data set 1. Then this simulation was repeated 5,000 times to generate 5,000 virtual samples of L_{10} lives corresponding to data set 1. The number 5,000 was based on the fact that results were found to converge beyond 5,000 and considerations of computational time. Figure 4 presents a histogram of these 5,000 virtual L_{10} lives for data set 1. We can assume that the distribution in Fig. 4 represents uncertainty in conducting the experiment corresponding to data set 1. This process of estimation of 5,000 virtual L_{10} lives was then repeated for all 51 data sets of rolling element bearings to quantify uncertainties in corresponding experiments. These $5,000 \times 51$ probable fatigue life estimates were then used to reevaluate the parameter p in the modified LP Eq. [5].

Statistical Reevaluation

Because bearing preloading conditions are not specified in the reported data, life modification factors cannot be reevaluated. However, based on the observed fatigue lives and operating

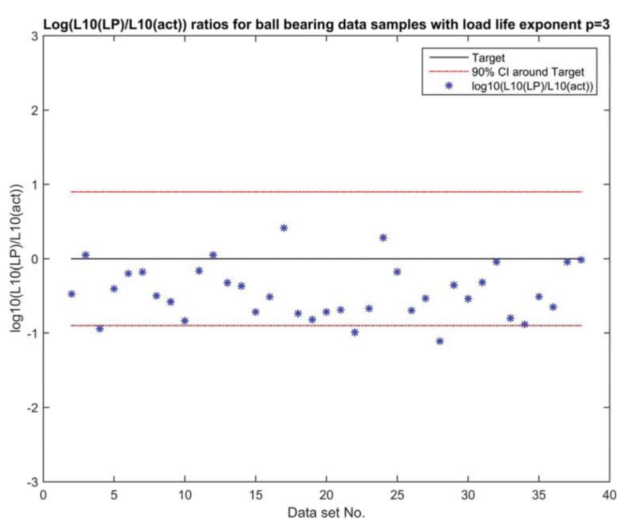

Fig. 1—$\log\frac{L_{10}(\mathbf{LP})}{L_{10}(\mathbf{act})}$ plot of data points for DGBBs and ACBBs; load-life exponent $p = 3$.

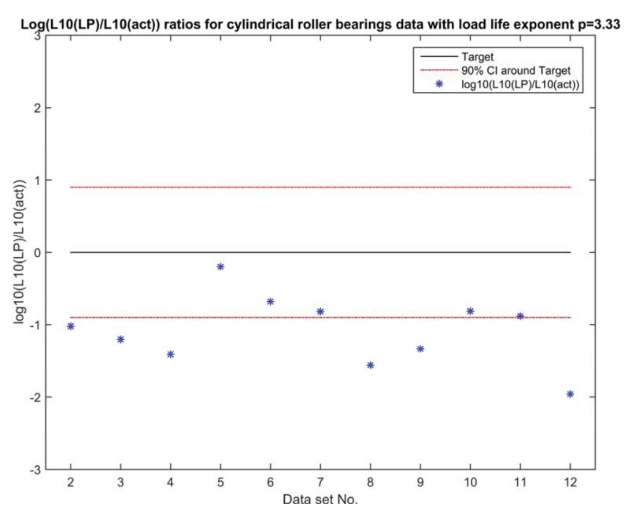

Fig. 2—$\log\frac{L_{10}(\mathbf{LP})}{L_{10}(\mathbf{act})}$ plot of data points for the CRBs; load-life exponent $p = 3.33$.

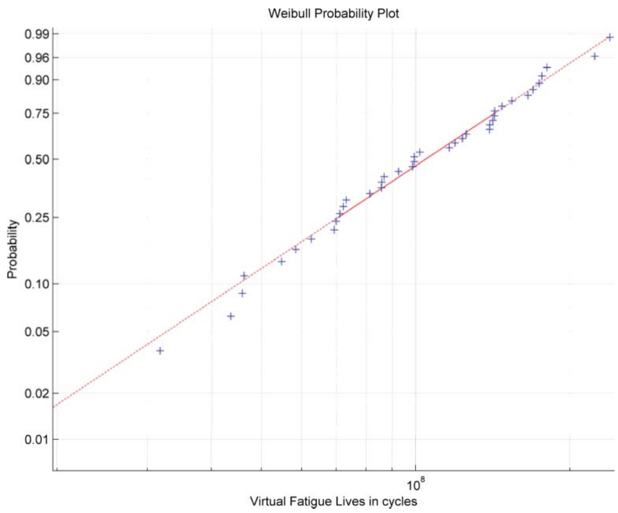

Fig. 3—Weibull plot of 40 virtual fatigue lives for data set 1.

loading conditions we can reevaluate the values of load-life exponent p for both ball and cylindrical roller element bearings. If the observed fatigue life is known, then Eq. [C4] from Appendix C can be used to determine load-life exponent p for ball bearings as

$$p = 3 - \frac{\log \frac{L_{10}(act)}{L_{10}(LP)}}{\log \frac{F_e}{C}}. \quad [12]$$

Similarly, for cylindrical roller bearings, Eq. [C5] can be used to determine load-life exponent p as

$$p = \frac{10}{3} - \frac{\log \frac{L_{10}(act)}{L_{10}(LP)}}{\log \frac{F_e}{C}}. \quad [13]$$

To improve the agreement between predicted fatigue lives and observed fatigue lives, it is necessary to reevaluate the load-life exponent p values in the modified LP Eq. [5]. For every 5,000 virtual samples of L_{10} lives obtained in the previous step, Eq. [12] can be solved to estimate the corresponding 5,000 values of load-life exponent p for ball bearings. Figure 5 shows the histogram of the estimated values of load-life exponent p from 5,000 virtual samples of L_{10} lives presented in Fig. 4. Similarly, Eq. [13] can be solved to estimate values of load-life exponent p from probable fatigue life samples of cylindrical roller bearings. These 5,000 probable estimates of load-life exponent represent uncertainty due to finite samples in the experiment, and for analysis purposes they should be represented with a probability distribution function. Hence, the 5,000 probable estimates of load-life exponent from each data set, as shown in Fig. 5, were approximated using a normal distribution function, Weibull distribution function, and kernel smoothing function estimate; that is, the ksdensity fitting tool available in MATLAB. It was observed that there was no significant difference in the reevaluated value of load-life exponent between these three approximations. Hence, we can assume that the uncertainty due to finite samples can be approximately represented with a normal distribution and the parameters for this normal distribution—that is, mean and standard deviation—are the mean and standard deviations of the 5,000 probable samples. Furthermore, consideration should be given to the fact that some of the samples contain very few failures. Thus, estimates for samples with a large number of failures are more accurate than those for samples with very few failures. Therefore, the standard deviations of each likelihood function were weighted with square root number of failures of the corresponding sample. This approach is similar to method used by Harris and McCool *(1)* for statistical analyses. Likelihood function properties—that is, mean and weighted standard deviation corresponding to each experimental data set—are presented in Table 4. It was observed that for all of the data samples the maximum error in symmetric approximation of sampling uncertainty is less than 1%. It attains a maximum value of 6% only for data set 41 at a confidence interval of 20%. This uncertainty is expected due to the low Weibull slope corresponding to this data set. But even for this sample, symmetric approximation is valid beyond 20% confidence intervals with maximum error less than 1%. Moreover, this data set corresponds to only two failures. Therefore, it was observed that through the weighted

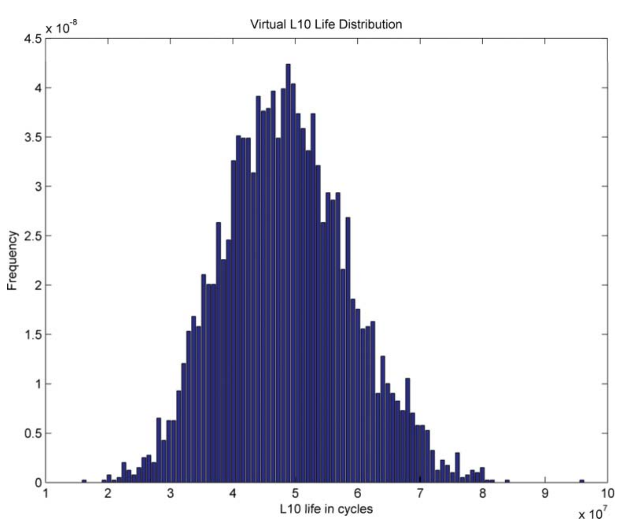

Fig. 4—Histogram of 5,000 probable L_{10} lives from data set 1.

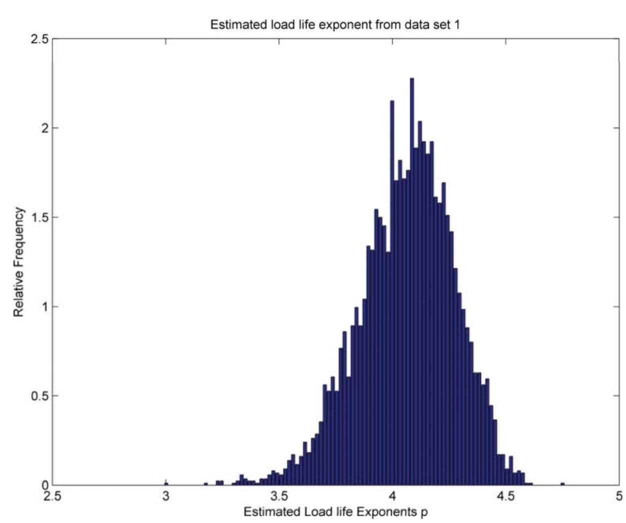

Fig. 5—Histogram of 5,000 estimated load-life exponents p from data set 1.

TABLE 4—NORMAL DISTRIBUTION/LIKELIHOOD FUNCTION PARAMETERS DURING EACH STEP OF BAYESIAN UPDATING PROCESS

	Corresponding Parameters of Likelihood Function/Normal Distribution for Load-Life Exponent p				Corresponding Parameters of Likelihood Function/Normal Distribution for Load-Life Exponent p		
Data Line	Mean	Weighted Standard Deviation $\sigma_{(sample)}$	Normalized Standard Deviation $\sigma_{(total)}$	Data Line	Mean	Weighted Standard Deviation $\sigma_{(sample)}$	Normalized Standard Deviation $\sigma_{(total)}$
	Ball bearings (DGBBs and ACBBs)						
1	4.06	0.05	0.61	31	7.49	3.08	3.14
2	2.94	0.23	0.65	33	4.37	0.67	0.90
3	5.27	0.29	0.67	34	5.03	0.11	0.61
4	3.89	0.40	0.73	36	−10.94	0.40	0.40
5	8.43	2.25	2.33	37	3.71	0.33	0.69
6	3.42	0.20	0.64	38	−12.45	0.15	0.15
7	3.74	0.16	0.62	39	3.10	0.31	0.68
8	5.10	0.06	0.61	40	4.01	0.16	0.62
10	4.88	0.48	0.77	41	5.28	2.81	2.88
12	3.35	0.06	0.61	43	4.20	0.35	0.70
13	2.89	0.43	0.74	45	4.12	0.36	0.70
14	3.72	0.09	0.61	46	3.48	0.84	1.03
15	3.92	0.22	0.64	47	3.48	1.15	1.30
16	4.75	0.28	0.66		Cylindrical roller bearings (CRBs)		
17	4.22	0.04	0.61				
18	2.02	0.04	0.61				
19	4.79	0.29	0.67	49	4.45	0.26	0.73
20	4.95	0.14	0.62	50	5.99	0.12	0.69
21	4.70	0.06	0.61	51	6.80	0.43	0.80
22	4.54	0.09	0.61	52	4.02	0.50	0.84
23	5.23	0.30	0.67	53	5.01	0.35	0.77
24	4.52	0.27	0.66	54	5.81	0.64	0.93
25	2.37	0.02	0.60	55	4.70	0.0046	0.68
26	3.42	0.28	0.66	56	6.09	0.17	0.70
27	4.57	0.17	0.63	58	7.19	1.87	1.99
29	4.22	0.09	0.61	61	5.65	0.63	0.93
30	180.96	30.76	30.76	62	8.74	2.21	2.31

standard deviation approach, this data set has very little influence on reevaluated values of load-life exponent p for ball bearings. In addition, data sets 31, 46, and 47, which attain a maximum error of 1% in symmetric approximation, have large weighted standard deviations. Therefore, their effect on the final results is also negligible.

In addition to uncertainty due to the finite number of samples, there is uncertainty due to experimental conditions and the adequacy of the model. That is, even if each experiment had an infinite number of samples, we would expect to see differences between the values of p calculated from Eq. [12] or Eq. [13]. Part of the differences is due to unavoidable experimental errors, and the model of the Eq. [5] is not perfect. Therefore, errors that led to different means of each likelihood function can be classified into two categories: (1) sampling uncertainty containing errors due to finite samples and Weibull distribution function and (2) experimental uncertainty that contains inherent experimental errors in each data set along with the imperfect nature of the modified LP model. It is reasonable to assume that both types of errors are independent and they contribute equally. Therefore, half of the standard deviation of p values obtained from different sets of tests is due to unaccounted experimental errors and the inadequate LP model. Hence, the total standard deviation associated with the ith sample is given as

$$[\sigma_{(\text{total})}]_i = \sqrt{[\sigma_{(\text{sample})}]_i^2 + 0.25 \times \begin{bmatrix} \text{Standard devitation} \\ \text{of means of } 5,000 \\ \text{probable estimates} \\ \text{for all samples} \end{bmatrix}^2}. \quad [14]$$

This total standard deviation is used for defining the normal distribution that provides the likelihood of any value of p based on the ith set of data. The total/normalized standard deviations are given in Table 4. For the majority of data sets presented in Table 4, we can see that the normalized standard deviations of likelihood functions are significantly higher than their

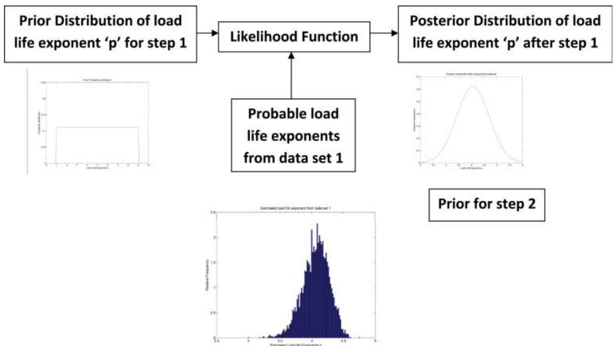

Fig. 6—Schematic of Bayesian updating process for determining posterior distribution of load-life exponent *p* for ball bearings.

corresponding weighted standard deviations. This shows the low precision of the reported endurance data, which leads to large uncertainties in each likelihood function. It was also observed that confidence intervals in the reevaluated values of load-life exponents are significantly dependent on the normalization factor 0.25 used in Eq. [14]. However, reevaluated values of load-life exponent p are not significantly affected due to changes in this factor. Detailed discussion regarding effect of this normalization factor is provided in Appendix D.

The approach used for this analysis is bolstered by the work of Parker and Zaretsky *(16)* based on the data obtained from a five-ball fatigue tester. Their work aims to solve for a unique value of the stress life exponent based on a straight-line fit approach coupled with the method of least squares, whereas the approach presented in this work aims to estimate the load-life exponent p based on actual bearing fatigue endurance data. In addition, instead of using a classical approach, a Bayesian probability approach was used to quantify uncertainty in the estimated load-life exponents.

Bayesian Inference

Bayes' rule states that a posterior probability distribution can be obtained by combining prior knowledge with observed data.

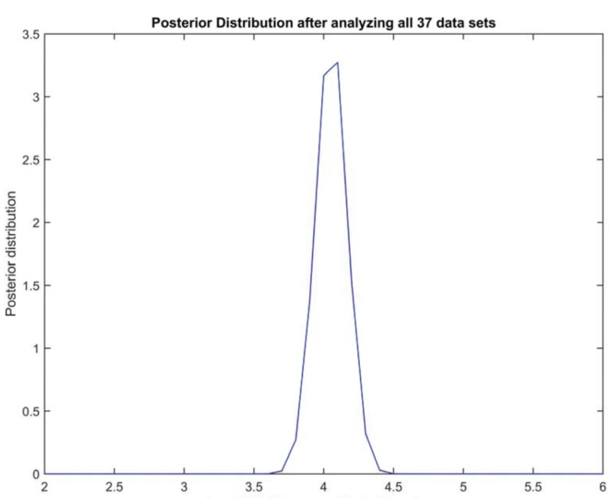

Fig. 7—Resultant posterior probability distribution of load-life exponent *p* for ball bearings.

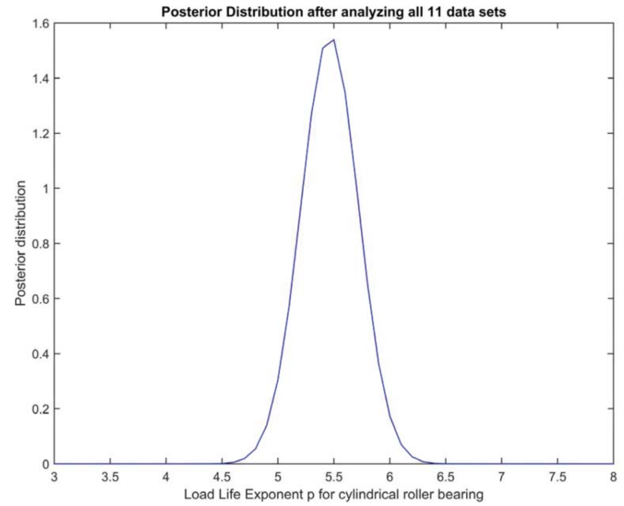

Fig. 8—Resultant posterior probability distribution of load-life exponent *p* for cylindrical roller bearings.

Bayes' rule can be used to update the probability estimate for a hypothesis as additional data is collected. This method of inference is popularly known as Bayesian inference. The standard procedure of calibration aims at inferring a hypothesis based on observed experimental data. This is an inductive process that can be solved using Bayes' rule in the Bayesian probability approach as follows:

$$P\,(\text{hypothesis}\,|\,\text{data}) \propto L(\text{data}\,|\,\text{hypothesis}) \times P(\text{hypothesis}). \qquad [15]$$

In expression [15], $P(\text{hypothesis}\,|\,\text{data})$ represents the posterior probability of a hypothesis based on observed data; $L(\text{data}\,|\,\text{hypothesis})$ is the likelihood function that represents the probability of observing data based on the hypothesis; and $P(\text{hypothesis})$ represents the prior probability of the hypothesis before any observation was made. Therefore, from expression [15], we say that the posterior probability is proportional to the product of likelihood times prior probability.

Bayesian Updating

A Bayesian inference technique can be used to combine estimates from different data sets that have different accuracies. Therefore, using a Bayesian approach, the posterior probability distribution for load-life exponent p can be determined based on the probable fatigue life data for rolling element bearings. The

TABLE 5—PROPERTIES OF RESULTANT POSTERIOR PROBABILITY DISTRIBUTION FOR DIFFERENT BEARING TYPES

Bearing Type	Reevaluated Value of p (Mode)	Standard Deviation	95% Confidence Bounds
Ball bearings	4.1	0.11	[3.87, 4.33]
Cylindrical roller bearings	5.5	0.26	[4.98, 6.02]

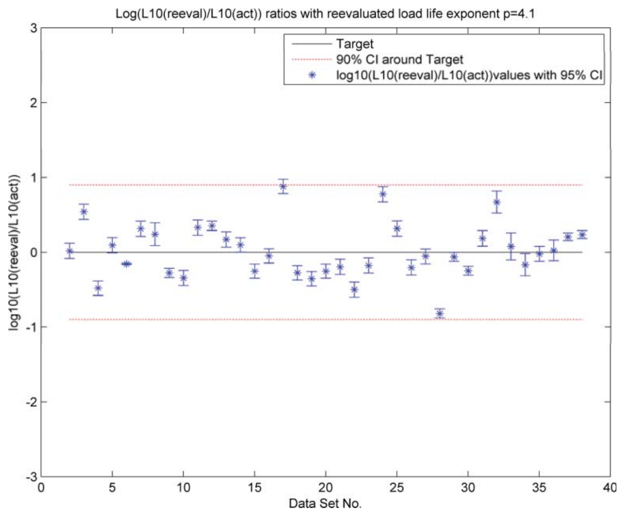

Fig. 9—$\log \frac{L_{10}(\mathbf{reeval})}{L_{10}(\mathbf{act})}$ plot of data points for DGBBs and ACBBs; that is, ball bearings with 95% confidence bounds.

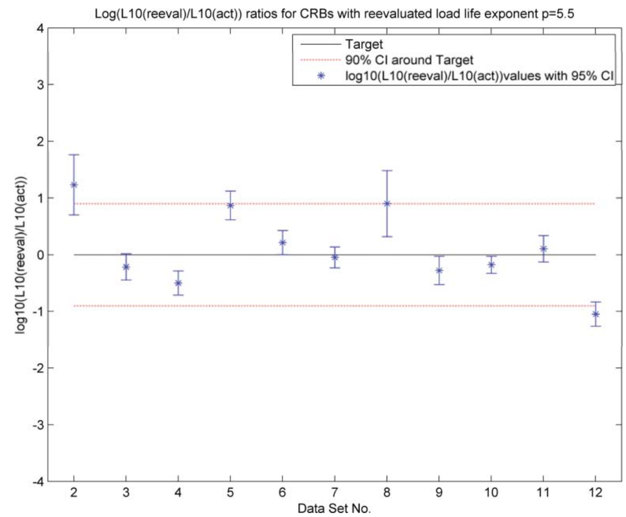

Fig. 10—$\log \frac{L_{10}(reeval)}{L_{10}(act)}$ plot of data points for cylindrical roller bearings with 95% confidence bounds.

process of modifying the prior distribution based on observed data is generally termed Bayesian updating. A schematic of the Bayesian updating process is shown in Fig. 6. The prior function can be thought of as information regarding existing knowledge. This means that the prior in this case is $p = 3$ for ball bearings and 10/3 for cylindrical roller bearings. Likelihood functions can be thought of as measurements of some physical experiments, which are bearing endurance data in this case. Multiplication of the prior with the likelihood gives the posterior. The posterior can be thought of as updated information on existing knowledge based on observed experimental data. As a general case, instead of assuming p as a constant initially, it was assumed that the load-life exponent varies uniformly over the range from 1 to 9. This prior distribution of p is shown in Fig. 6. Then the likelihood function with a standard deviation given by Eq. [14] and the calculated p as the mean is used in the Bayesian inference process. The likelihood function of each data set was added sequentially and the posterior distribution of the current step was used as the prior distribution for the next step.

After adding all data sets in the Bayesian updating process, the resultant posterior probability distribution of load-life exponents p for ball bearings and cylindrical roller bearings are presented in Figs. 7 and 8, respectively. It should be noted that data sets 30, 36, and 38 were not used in this analysis because the means of their corresponding likelihood functions are beyond the possible range of load-life exponent p and they do not satisfy the basic definition of the LP model. For the posterior distribution presented in Fig. 7, the mode and mean are found to be 4.1 and 4.05, respectively, and for posterior distribution presented in Fig. 8 the mode and mean are found to be 5.5 and 5.46, respectively. These distributions clearly show that current industrial standards of $p = 3$ for ball bearings and $p = 3.33$ for cylindrical roller bearings are far from reality. Hence, this underlines the need for updating the modified load-life Eq. [5] for more realistic fatigue life predictions of rolling element bearings.

Because there is no significant difference between the mean and mode for both posterior distributions, we can use the maximum likelihood estimate; that is, the mode as the reevaluated value of load-life exponent p for the two types of bearings. Table 5 shows the reevaluated value of load-life exponents for both point contact and line contact bearings. From Table 5 we can see that for ball bearings, the reevaluated value of load-life exponent p is 4.1 with 95% confidence bounds as [3.87, 4.33]. This value is much higher than the currently used industrial standard of 3. A standard deviation of 0.11 confirms that the current industrial standard is far from reality. Similarly, for cylindrical roller bearings the reevaluated value of load-life exponent was found to be 5.5 with 95% confidence bounds as [4.98, 6.02]. This value is also much higher compared to the current industrial standard of 3.33. Because the number of data sets reported for cylindrical roller bearings was small, we can see that the resultant posterior distribution with a standard deviation of 0.26 is much wider. Using the reevaluated value of load-life exponents, the ratios of predicted to actual fatigue lives were recalculated and plotted on logarithmic plots as shown in Figs. 9 and 10. Figure 9 presents a $\log \frac{L_{10}(\text{reeval})}{L_{10}(\text{act})}$ plot for ball

TABLE 6—PROPERTIES OF RESULTANT POSTERIOR PROBABILITY DISTRIBUTION FOR DIFFERENT BALL BEARING STEELS

Material Type	Reevaluated Value of p (Mode)	Standard Deviation	95% Confidence Bounds
Through-hardened steels	4.0	0.13	[3.74, 4.26]
Case-hardened steels	4.5	0.17	[4.16, 4.84]
AISI 52100 Steels	3.9	0.20	[3.50, 4.30]
M50 steels (VAR, VIMVAR)	4.1	0.18	[3.74, 4.46]

TABLE 7—PROPERTIES OF RESULTANT POSTERIOR PROBABILITY DISTRIBUTION FOR DIFFERENT CYLINDRICAL ROLLER BEARING STEELS

Material Type	Reevaluated Value of p (Mode)	Standard Deviation	95% Confidence Bounds
AISI 52100 steel	5.4	0.27	[4.86,5.94]
M50 steel	5.7	0.48	[4.74, 6.66]

bearings—that is, DGBBs and ACBBs—and Fig. 10 presents the corresponding plot for cylindrical roller bearings. L_{10}(reeval) represents the predicted fatigue life from the modified LP Eq. [5] with reevaluated values of load-life exponent p. It should be noted that original life modification factors a_1, a_2, and a_3 are still applicable to fatigue lives derived using reevaluated load-life exponent values. From Fig. 9 we can see that there are 19 data points on the underprediction side and 18 data points are on the overprediction side. Similarly, in Fig. 10, five data points are on the overprediction side and six data points are on the underprediction side. Generally after calibration it is expected that data points should be symmetric about our target line. However, this is difficult to achieve in this analysis because standard deviations of each likelihood function differ significantly. These figures indicate that even after normalization, the resultant posterior distribution is slightly biased toward data sets with low standard deviations. Thus, the modified life rating Eq. [5] with reevaluated values of load-life exponent will approximately overpredict ball bearing fatigue lives 49% of the time and underpredict bearing fatigue lives 51% of the time. Figures 9 and 10 also indicate 90% confidence bounds around our target. From Fig. 9 we can see that all 37 data points for the ball bearings are within 90% confidence bounds, whereas from Fig. 10 we can see that only two data points for cylindrical roller bearings fall outside the 90% confidence bounds around the target. In addition, in both figures , 95% confidence bounds in the estimated ratio are also presented at each point. From Fig. 9 we can see that for ball bearings all of the points with their 95% confidence bounds lie within the 90% interval around the target. Figure 10 indicates that for cylindrical roller bearings, seven points along with their 95% confidence bounds lie within 90% confidence intervals around the target. This confirms the suitability of a Bayesian statistics approach for reevaluation of load-life exponent p.

Dependence of Load-Life Exponent on Material Type

It has been reported that for case-hardened bearing steels there exists a gradient in the carbide volume fraction at subsurface depths. This results in an approximately linear variation of hardness and elastic modulus as a function of depth (Klecka, et al. *(20)*) for case-hardened steels. They also have a strong residual compressive stress through the case layer and as a result generally tend to have considerably higher fatigue lives than the corresponding through-hardened bearing steels. Of the 37 data sets reported on ball bearings, 31 are from experiments on through-hardened bearing steels (52100, VIMVAR M50, and VAR M50) and six data sets are from experiments on case-hardened steels (SAE 8620 and M50NiL). For experiments on cylindrical roller bearings, no case-hardened steels were used. To check the dependency of the load-life exponent on the type of bearing material, separate validation and reevaluation analyses were performed for through-hardened steels and case-hardened steels. Table 6 shows the reevaluated values of load-life exponent for ball bearings manufactured from different types of steel. We can see that for ball bearings manufactured from through-hardened steels the maximum likelihood estimate of load-life exponent p is 4 with 95% confidence bounds as [3.74, 4.26] and for case-hardened steels it is about 4.5 with 95% confidence bounds as [4.16, 4.84]. These reevaluated values clearly show improved endurance capability of case-hardened steels compared to through-hardened steels. In addition, Table 6 indicates reevaluated load-life exponents for ball bearings manufactured from AISI 52100 and M50 steels. For accurate fatigue life predictions of bearings manufactured from AISI 52100 steels, the load-life exponent p of 3.9 with 95% confidence bounds as [3.5, 4.3] can be used and for ball bearings with M50 steel raceways the load-life exponent of 4.1 with 95% confidence bounds as [3.74, 4.46] can be used. In addition, as shown in Table 7, for cylindrical roller bearings manufactured from AISI 52100 steel, the reevaluated value of load-life exponent is 5.4 with 95% confidence bounds as [4.86, 5.94] and for M50 steels it is 5.7 with 95% confidence bounds as [4.74, 6.66]. These test data indicate that M50 steels have better endurance capability than AISI 52100 steel for both ball bearings and cylindrical roller bearings.

CONCLUSIONS

After analyzing extensive rolling bearing fatigue data gathered by the United States Navy in 1993 (Harris and McCool *(1)*), it was found that the widely used LP method and the derivative modified load-life Eq. [5] tend to significantly underestimate bearing fatigue lives. Therefore, continuing the use of standard parameters based on this method tends to result in larger size bearings and mechanisms than are necessary. This is a critical issue for aerospace engine components where the weight penalty is high. This underlines the need for a more realistic rolling bearing life prediction method. Currently used values of the load-life exponent for ball and cylindrical roller bearings were determined based on the statistical study done in 1955 and there is a practical need to reevaluate these values based on more recent experimental data.

To address this issue, validation analysis of the modified load-life Eq. [5] used for rolling bearing design was performed in this study. The epistemic uncertainties in the reported experimental data were reduced by generating 5,000 virtual samples of fatigue lives corresponding to each data set. Using statistical calibration techniques along with a Bayesian probability approach, it was found that load-life exponent p values should be corrected for ball bearings from 3 to 4.1 with 95% confidence bounds as [3.87, 4.33] and for cylindrical roller bearings it should be corrected from 3.33 to 5.5 with 95% confidence bounds as [4.98, 6.02]. Life predictions using reevaluated load-life exponents show better agreement with observed fatigue life data. In addition, as part of the reevaluation analysis, the dependence of the load-life exponent on bearing material type was studied for both ball and cylindrical roller bearings. It was observed that the reevaluated value of load-life exponent is much higher for case-hardened steels compared to through-hardened steels due to their better fatigue endurance capabilities.

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APPENDIX A

Bearing Fatigue Life Dispersion

In 1947, Lundberg and Palmgren (2) showed that the probability of survival S of a surface under rolling contact fatigue can be expressed as a power function of orthogonal shear stress τ_0, number of stress cycles survived N, depth to maximum orthogonal shear stress z_0, and stressed volume V. In mathematical form the LP equation is represented as

$$\ln\frac{1}{S} \sim \frac{\tau_0^c N^e}{z_0^h} V. \qquad \text{[A1]}$$

Here c, e, and h are empirical constants and the exponent e is the Weibull slope. Lundberg and Palmgren (2) also proposed that the volume of RCF-affected material V should be

$$V = a z_o \mathcal{L}, \qquad \text{[A2]}$$

where a is the semimajor axis of the contact ellipse, z_o is the depth where the orthogonal shear stress is maximum, and $\mathcal{L}$ is the circumference of the raceway at depth z_o. Therefore, Eq. [A1] can be simplified using Eq. [A2] as

$$\ln\frac{1}{S} = \tau_o^c\, z_o^{1-h}\, a\mathcal{L}u^e L_s^e. \qquad \text{[A3]}$$

Here u is number of stress cycles per revolution and L_s is the number of revolutions survived with probability S.

Equation [A3] can be further simplified for a specific bearing under a particular load as

$$\ln\frac{1}{S} = A L_s^e, \qquad \text{[A4]}$$

where A is a material factor for rolling bearings and its value is considered constant (Harris *(7)*). Vlcek, et al. *(21)* reported that Eq. [A4] can be rearranged as

$$\ln\ln\frac{1}{S} = e\ln\left(\frac{L_s}{L_\beta}\right). \qquad \text{[A5]}$$

Equation [A5] represents the Weibull distribution for bearing fatigue lives. It is clear from Eq. [A5] that $\ln\ln\frac{1}{S}$ vs. $\ln L_s$ plot will be a straight line fit. L_β is the characteristic life defined as the life at which 63.2% of the bearings fail.

APPENDIX B

Weibull Distribution

The Weibull distribution is a continuous probability distribution and its probability density function is represented as

(Papoulis and Pillai *(19)*)

$$f(x;\lambda,k)=\begin{cases}\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}exp\left(-\left(\frac{x}{\lambda}\right)^{k}\right), & x\geq 0,\\ 0, \quad x<0\end{cases} \qquad [B1]$$

where k and λ are the positive shape and scale parameters of the distribution, respectively. If quantity x is failure time, then Eq. [B1] gives the distribution for which the failure rate is proportional to a power of time. The cumulative distribution function for Eq. [B1] is

$$F(x;k,\lambda)=\begin{cases}1-exp\left(-\left(\frac{x}{\lambda}\right)^{k}\right), & x\geq 0,\\ 0, \quad x<0\end{cases} \qquad [B2]$$

where F represents the cumulative probability of x. For $x \geq 0$, Eq. [B2] can be simplified as

$$\ln\ln\frac{1}{1-F}=k\ \ln\ x-k\ \ln\ \lambda. \qquad [B3]$$

If S is the probability of survival, then substituting$S=1-F$, in Eq. [B3] we get

$$\ln\ln\frac{1}{S}=k\ \ln\ x-k\ \ln\ \lambda. \qquad [B4]$$

Comparing Eq. [B4] and Eq. [A5], we can conclude that the shape parameter of the Weibull distribution for each data set reported can be obtained as

$$k=e \qquad [B5]$$

and the scale parameter as

$$\lambda=(A)^{\frac{-1}{e}}=L_{\beta}. \qquad [B6]$$

This indicates that the characteristic life parameter is the same as the scale parameter of the Weibull distribution.

APPENDIX C

For each individual data set, the modified life rating Eq. [5] can be rearranged as

$$L_s \propto \left(\frac{C}{Fe}\right)^{p}. \qquad [C1]$$

For 90% reliability and load-life exponent of $p = 3$ for ball bearings, Eq. [C1] can be simplified as

$$L_{10_{p=3}} \propto \left(\frac{C}{F_e}\right)^{3}. \qquad [C2]$$

Equations [C1] and [C2] can be combined to rewrite the generalized relation between L_{10} life and load exponent p for ball bearings. Replacing $L_{10_{p=3}}$ with $L_{10}(LP)$ we get

$$L_{10_p}=L_{10}(LP)\times\left(\frac{C}{F_e}\right)^{p-3}. \qquad [C3]$$

Based on the given loading condition and actual fatigue life, Eq. [C3] can be rearranged to solve for load-life exponent p as

$$p\ =\ 3-\frac{\log\frac{L_{10}(act)}{L_{10}(LP)}}{\log\frac{F_e}{C}}. \qquad [C4]$$

Similarly, for roller bearings, the load-life exponent p can be derived from observed fatigue life as

$$p\ =\ \frac{10}{3}-\frac{\log\frac{L_{10}(act)}{L_{10}(LP)}}{\log\frac{F_e}{C}}. \qquad [C5]$$

APPENDIX D

As discussed in previous sections, the quality of endurance data used in this analysis is mixed. Some of the tests were poorly monitored and in some cases very few failures were observed. This indicates that uncertainties in the reported data can be classified into following two categories:

1. Experimental uncertainty: Errors due to unavoidable differences between the experimental conditions and inadequacy of the LP model.
2. Sampling uncertainty: It is representation of errors due to finite samples in each data set and the Weibull distribution followed by bearing fatigue lives.

Experimental and sampling uncertainties are the primary errors that lead to different estimates of load-life exponent p from different data sets. Therefore, standard deviation of means of 5,000 probable estimates of p from all data sets approximately represent the total error in the reported endurance data. Let α be the normalization factor, which represents the contribution of experimental uncertainty in the total error of reported endurance data. Therefore, Eq. [14] can be written as

$$[\sigma_{(\text{total})}]_i=\sqrt{[\sigma_{(\text{sample})}]_i^2+(\alpha)^2\times\begin{bmatrix}\text{Standard devitation}\\ \text{of means of } 5,000\\ \text{probable estimates}\\ \text{for all samples}\end{bmatrix}^2}. \qquad [D1]$$

Tables D1 and D2 present the influence of normalization factor α on the properties of the resultant posterior distribution of load-life exponent p for ball bearings and cylindrical roller bearings, respectively. From the results presented in these tables we can see that the effect of the normalization factor is significant on the standard deviation of the resultant posterior distribution.

TABLE D1—EFFECT OF NORMALIZATION FACTOR ON RESULTANT POSTERIOR DISTRIBUTION OF p FOR BALL BEARINGS

Normalization Factor	Resultant Posterior Properties		No. of Data Points	
α	Mode p	Standard Deviation	Underprediction Side	Overprediction Side
0.1	3.9	0.05	22	15
0.2	4	0.05	22	15
0.3	4	0.07	22	15
0.4	4	0.09	22	15
0.5	4.1	0.11	19	18
0.6	4.1	0.13	19	18
0.7	4.1	0.15	19	18
0.8	4.1	0.17	19	18
0.9	4.1	0.19	19	18
1	4.1	0.21	19	18

TABLE D2—EFFECT OF NORMALIZATION FACTOR ON RESULTANT POSTERIOR DISTRIBUTION OF p FOR CYLINDRICAL ROLLER BEARINGS

Normalization Factor	Resultant Posterior Properties		No. of Data Points	
α	Mode p	Standard Deviation	Underprediction Side	Overprediction Side
0.1	5.3	0.09	6	5
0.2	5.4	0.13	6	5
0.3	5.4	0.17	6	5
0.4	5.4	0.21	6	5
0.5	5.5	0.26	6	5
0.6	5.5	0.30	6	5
0.7	5.5	0.34	6	5
0.8	5.5	0.38	6	5
0.9	5.5	0.42	6	5
1	5.6	0.45	6	5

This confirms the large experimental uncertainty and poor precision of reported endurance data. Equation [D1] indicates that smaller values of the normalization factor α should be neglected because they will reduce the influence of experimental uncertainty on the resultant posterior distribution of p. This can also be confirmed from the data presented in Tables D1 and D2. In addition, it was observed that higher values of α make the second term more dominant in Eq. [D1] for all data samples. This is undesirable because the effect of data samples with small weighted standard deviations (i.e., first term in Eq. [D1])—meaning samples with a large number of failures—will be poorly represented in final posterior distribution of load-life exponents. Another characteristic of reported data is that some of the samples in Tables 1 and 2 contain very few numbers of failures, which means that the contribution of sampling uncertainty in total error is not negligible and hence due to these reasons higher values of α are not justified in the analysis. Therefore, it is reasonable to assume half contribution of experimental uncertainty in the total error of the reported data sets. From Tables D1 and D2 we can see that for $\alpha = 0.5$, the number data of points on the underprediction side is nearly the same as the number of data points on the overprediction side; this validates this approach for estimation of load-life exponent p for both ball and cylindrical roller bearings.