



# Design Credits. Anurag Bhat (B20CS097)

**A Data-driven Approach towards Life Prediction of Aerospace Bearings**

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## Project Overview and Motivation

1. In this project we deal with different types of bearings and various methods to evaluate/predict their endurance limit data (or fatigue lives).
2. We go through all the models prepared starting from 1924 till date, study the parameters of each model .We also attempt to judge the correctness, reliability and modifications made to each model.
3. Another aspect of this project is the justification of the design approach because there are so many possible methods in material science to solve such research problems.
4. Later as the project progresses we study the re evaluation of parameters, it's need and the improvements it caused in fatigue life prediction.

This is a extremely widespread area of research for its modern requirements in terms of transportation and appliances. For that I have gone through 3-4 different research papers thoroughly and attempted to draw conclusions on how those old researches (done in 1947-1995) translate to today's industry where many factors have been modified / have improved.

As time has passed and people from various backgrounds have looked and researched into this field ,there was a need to change many parameters (and the constants related with them) which was targeted experimentally (changing some parameters lead to no improvement in life).

# Project Objectives

- Understand all load life parameters for bearings.
- Learn the models, their history, their assumptions and how they were further modified in response to the drawbacks they possessed.
- Realize shortcomings of earlier models and why re-evaluation of parameters was required to tackle it.
- To re-evaluate the parameters followed by analysis and results.

# Introduction - Mechanical Bearings

Bearings are used to enable rotational or linear movement, reduce friction between two objects. They are easier to move (both in a rotary or linear fashion) and when friction is reduced, speed and efficiency of an object will be enhanced in turn.

## How do bearings work?

Bearings are made up of a ball and an inside, outside smooth surface for rolling. The ball will carry the load weight, and the force of the load weight assists in rotation of the bearing



# Types Of Bearings -

## **1) Ball Bearings**

Ball bearings have a niche area in the field of bearings as they can handle both radial and thrust loads. Every bearing is given a naming based on its rolling element and the same applies to the ball bearings also.

## **2) Tapered Roller Bearings**

The tapered roller bearings are designed to provide huge load versatility and equally effective at handling the large thrust and radial loads. This bearing is extensively used in automotive industry as it can lift the load, the wheels are expected to tolerate.

## **3) Ball Thrust Bearings**

Ball Thrust Bearings can handle thrust loads in low-speed, low-weight applications

## **4) Roller Thrust Bearing**

# Specific Use Of Bearings In Aerospace Demonstration -



## Important Terminologies Followed in Deterministic and Probabilistic Approach For Life Prediction

### Equivalent Load

Palmgren realized in his research paper in 1924, that it was important to establish a relation for bearings under purely radial load and also conversion of axial loads into radial loads.

Palmgren stated that it is mostly impossible to find a simple yet accurate expression for radial and axial pressure. He assumed it to be linear initially and the equation is -

$$Q = P_{eq} = XFr + YFa$$

*P<sub>eq</sub> - Equivalent Load*

*Fr-Radial component*

*Fa-Axial component*

*X-Rotations*

*Y-Thrust factor of bearings*

## Important Terminologies Followed in Deterministic and Probabilistic Approach For Life Prediction

### Fatigue Limit

Palmgren knew that limited life was a consequence of fatigue phenomenon. He also thought about other factors on higher loads like permanent damages, internal fractures etc.

He proved the general misconception that bearing can undergo unlimited cycles below fatigue limit since every material has a fracture limit too.

He developed an exponential relationship between load and number of cycles .

$$k = C(a \cdot n + e)^{-x} + u$$

*k*-specific load

*C*-material constant

*a*-load cycles

*n*-revolutions in millions

*x*-1/3 power figured later.

*u*-fatigue limit

Hence it can also be written as-

$$\text{Life(millions of cycles)} = (C/(k-u))^{1/3-e}$$

After certain assumptions Palmgren also modified Load Capacity equation later :-

For ball bearings

$$C_D = f_c \frac{id^2 Z^{\frac{2}{3}} \cos \beta}{1 + 0.02d} \quad (9)$$

For roller bearings

$$C_D = f_c id^2 l_i Z^{\frac{2}{3}} \cos \beta \quad (10)$$

where

- $f_c$  material-geometry coefficient<sup>2</sup>
- $i$  number of rows of rolling elements (balls or rollers)
- $d$  ball or roller diameter
- $l_i$  roller length
- $Z$  number of rolling elements (balls or rollers) in a row  $i$
- $\beta$  bearing contact angle



# L10 LIFE

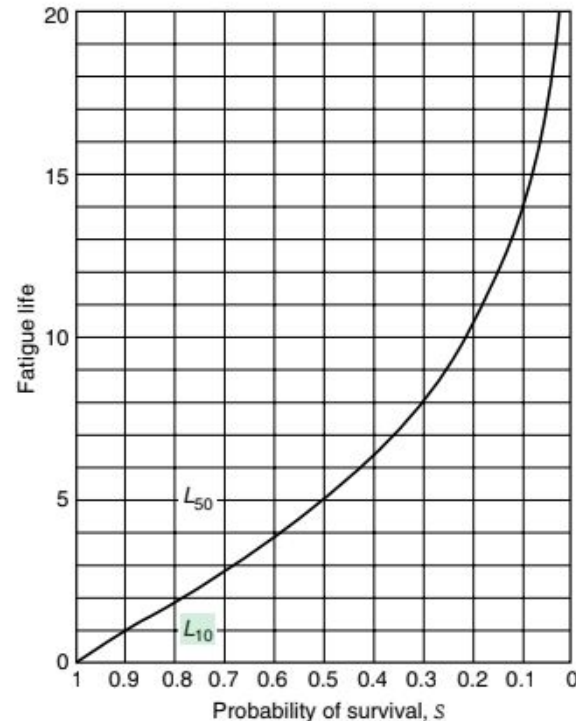
## Definition-

In the simplest terms, L10 life is calculating, with 90% reliability, how many hours a bearing will last under a given load and speed. There is a 10% probability that at the applied load and speed, 10% of a population of identical bearings would suffer a fatigue failure, typically pitting on the raceway of the bearing.

## **Why do these assumptions matter in real world ?**

In the ideal world, a bearing would be selected by testing it under actual load conditions, but in the real world, this is almost never practical. Although bearing life calculations are statistical predictions, and there are many factors that affect a bearing's actual service life, the dynamic load capacity ratings and life calculations provided by ISO 14728-1 are well-proven and accepted across the linear bearing industry.

Figure -



## Important Terminologies Followed in Deterministic and Probabilistic Approach For Life Prediction

### Linear Damage Rule

Palmgren recognized that the variation in both load and speed must be accounted for in order to predict bearing life .

#### Rule-

$$M_1/n_1 + M_2/n_2 + M_3/n_3 + M_4/n_4 + \dots = 1$$

$M_k$ - Million Revolutions on load  $k$

$n_k$ - Life of  $n$  million revolutions on load  $k$

## Important Terminologies Followed in Deterministic and Probabilistic Approach For Life Prediction

### Hertz Contact Stress Theory

The Lundberg-Palgren model paper was written in 1924 ,it missed 2 key considerations. The 1st missing element was the ability to calculate the subsurface principal stresses .The 2nd missing element was a comprehensive life theory that would fit the observations of Palmgren .

Palmgren discounted Hertz theory and derived his own from The load-life relation which is -

$$L_s = (C/F_e)^p$$

**C-Dynamic Capacity**  
**F<sub>e</sub>-Total Radial Load**

Later it was discovered that the life of a Bearing cannot be derived simply from maths but should also involve experimental tests on bearings .

Palmgren had to inculcate hertz theory later .

#### **Hertz theory assumptions-**

- 1.Contact area very small.
- 2.Friction negligible

Due to multiple assumptions which make the expression easy, Hertz theory indeed does not account for many things (like expansion/contraction) and in real life factors are used to account for them for better approximations.

Rearrange formula and parameters.



# Design Formulation

Different Approaches Of Life Prediction In Material Science



### DETERMINISTIC MODEL

Takes data and mathematical equations .Allows you to calculate the further event without the involvement of randomness .You must have the required data to prove the outcome with surety.

### PROBABILISTIC MODEL

Deals with the data in such a way that we have the capacity to deal with uncertainties .Hence also called the stochastic or slower model. They allow some inherent randomness meaning same input and same parameters can lead to different outputs .

### **Why probabilistic model is preferred in a real world scenario & used in this particular project ?**

1. They reflect range of possible outcomes taking into account scenarios with bearings like internal fractures ,permanent damages while mechanical work is going on ,the external surroundings etc.
2. These considerations allow us to deal with shortcomings and also prepare suitable backups to face them
3. They always provide us a range of values instead to jumping to a final conclusion.

# Different Bearing Life Models

1. WEIBULL LIFE MODEL
2. LUNDBERG-PALMGREN MODEL
3. IOANNIDES-HARRIS MODEL
4. ZARETSKY MODEL

## Weibull Analysis

An equation to statistically evaluate the fracture strength of materials based on a small sample space .It allows to predict the cumulative statistical distribution of fatigue failure . The Equation is -

$$\ln \ln(1/S) = e \cdot \ln((L - L_u)/(L_b - L_u)) \quad 0 < L < \infty ; 0 < S < 1$$

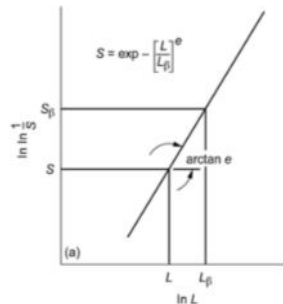
$e$ =slope of weibull plot

$S$ =probability of survival

$L_u$ =time below which no failure occurs. Usually assumed as 0 in manual solving

$L_b$ =time at which exponential(63.2%) population failure occurs

The slope - (Calculated for the available data set in further slides)



Weibull plot with slope  $e$  .

$S_\beta$  ( Prob. Of survival ) is 36.8% at  $L_\beta$

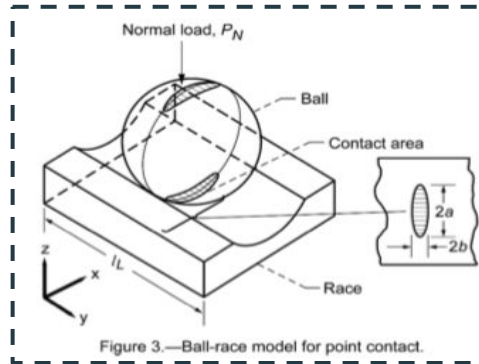
## Weibull Fatigue Life Model

Weibull had asked his contemporaries Lundberg and Palmgren to use his equation to predict bearing life -  $f(X) = (\tau^c) \cdot (\eta^e)$   
The model implies that relation of life vs stress is a function of Weibull plot data dispersion.

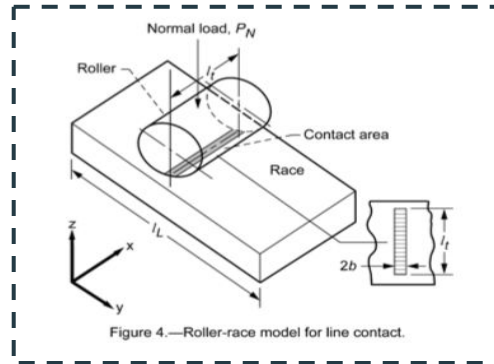
$\tau$ -shear stress  
 $\eta$ -stress cycles upto failure  
 $c/e$ - stress expo.

Model on Ball Race vs Roller Race -

### Ball Race (Point Contact)-



### Roller Race (Line Contact)-



Values of S and L for both contacts are derived from Hertz Theory using assumptions ( $\tau \sim S_{max}$  and  $V \sim S_{max}$  where V is stressed volume)

$S_{max}$  for point contact -

$$S_{max} \sim P^{1/3}$$

$S_{max}$  for line contact -

$$S_{max} \sim P^{1/2}$$



## Lundberg Palmgren Model

In 1947 they both applied Weibull's analysis to the prediction of rolling element bearing fatigue life. In order to better match the values of the Hertz stress-life exponent  $n$  and the load-life exponent  $p$  with experimentally determined values from pre-1940 tests on air-melt steel bearings, they introduced another variable, the depth to the critical shearing stress  $z$  to the  $h$  power

### New Equation-

$f(X) = (\tau^c \cdot \eta^e) / (z^h)$  :-  $z$  introduced because Lundberg and Palmgren assumed time for crack propagation was a func. of  $z^h$

### Solving for the stress life exponent $n$ and load life exponent $p$

They chose orthogonal shearing stress as the critical shearing stress and from Hertz theory  $Z \sim S_{max}$ .

Known  $L$  &  $P$  values for  $c, h, e$  are 10.33, 2.33, 1.125

So values of  $n$  and  $p$  come out to be about **8** and about **4** respectively as predicted earlier.

### Modifications which were done later and why were they required?

The values of  $n$  and  $p$  for point and line contacts correlated to the then-existing rolling-element bearing database. Lundberg and Palmgren modified their value of the load-life exponent  $p$  for roller bearings from 4 to 10/3. The rationale for doing so was that various roller bearing types had one contact that is line contact, as a rule the contacts between the rollers and the raceways transforms from a point to a line contact for some certain load so that the life exponent varies from 3 to 4 for differing loading intervals within the same bearing.

## Ioannides-Harris Model

Ioannides and Harris, using Weibull, introduced a fatigue-limiting shear stress  $\tau_u$

**New Equation-**  $f(X) = ((\tau - \tau_u)^c) \cdot (\eta^e) / (z^h)$  :- Same as LP model except fatigue limit stress

### Solving for the exponent stress life exponent $n$ and load life exponent $p$

Ioannides and Harris use same values of  $e, c, h$  as the LP model. For values of  $\tau_u > 0$ ,  $n$  becomes a function of  $\tau - \tau_u$ .

L10 life was rewritten to include fatigue limiting load  $P_u = (C_d) / (P_{eq} - P_u)^p$

When  $P_{eq} < P_u$ , bearing life is infinite and no failure would be expected. When  $P_u = 0$ , the life is the same as that for Lundberg and Palmgren. The reason Ioannides and Harris used the fatigue limit was to replace the material and processing life factors that are used as life modifiers in conjunction with the bearing lives calculated from the LP equations.

### Modern Day Issues With This Model -

There are two problems associated with the use of a fatigue limit for rolling-element bearing.

1. It does not reflect the presence of a fatigue limit but the presence of a compressive residual stress.
2. There are no data in the open literature that would justify the use of a fatigue limit for severely hardened bearing steels

## Zaretsky Model

Both the Weibull and Lundberg-Palmgren models relate the critical shear stress-life exponent  $c$  to the Weibull slope  $e$ . The parameter  $c/e$  thus becomes, in essence, the effective critical shear stress-life exponent, implying that the critical shear stress-life exponent depends on bearing life scatter or dispersion of the data. A search of the literature for a wide variety of materials and for non rolling element fatigue reveals that most stress-life exponents vary from 6 to 12. The exponent appears to be independent of scatter or dispersion in the data.

### New Equation-

$f(X)=(\tau^{ce}).(\eta^e)$  :- For critical shearing stress , Zaretsky chose the maximum shearing stress,  $\tau_{45}$ .

LP model initiated use of  $z$  for crack propagation and formation of fatigue spell .It means it is time dependent.Crack propagation time is an extremely small fraction of the total life or running time of the bearing. **This is opposite to the LP model** .Zaretsky dispensed with the Lundberg-Palmgren relation of  $L \sim z^{h/e}$ .

### Solving for the exponent stress life exponent $n$ and load life exponent $p$

If it is assumed that  $c = 9$  and  $e = 1.11$ ,  $n = 10.8$  for point contact and  $n = 9.9$  for line contact. If it is further assumed that  $c = 10$  and  $e = 1.0$ ,  $n = 12$  for point contact and  $n = 11$  for line contact acc. to the relation -

$$\begin{aligned} n &= c + 1/e \text{ (point)} \\ n &= c + 2/e \text{ (line)} \end{aligned}$$

What differentiates Equation, from those of Weibull ,Lundberg and Palmgren is that the relation between shearing stress and life is independent of the Weibull slope,  $e$ , or the distribution of the failure data. However, in all four models, there is a dependency of the Hertz stress-life exponent,  $n$ , on the Weibull slope. The magnitude of the variation is least with the Zaretsky model.



## Methodology adopted for the project

Reevaluation of Rolling Element Bearing Load-Life Equation Based on Fatigue Endurance Data



# Why is re-evaluation of parameters required ?

LP model was based on fracture mechanics according to which, the fatigue life is known to be inversely proportional to the square root of the size of the nonmetallic inclusions.

However, modern high-performance vacuum induction melt–vacuum arc remelt (VIMVAR) bearing steels are clean and nonmetallic inclusions are no longer the weak link. Fatigue life predictions (L10 life) for modern bearings using the modified load-life relations greatly underpredict observed life. The LP crack initiation-based life prediction worked reasonably well for older technology bearing steels that had nonmetallic inclusions, with expected fatigue life of about 600 million cycles.

However, these theories do not work well for modern “ultraclean” vacuum induction melt–vacuum arc remelt (VIMVAR) bearing steels, which have fewer and smaller non metallic inclusions.

Hence, there is a need to update parameters of these equations using more recent life data.

# Bearing Life Fatigue Dispersion Equation Approximation

Approximations to modify parameters -

1.  $\ln(1/S) = (\tau^c) \cdot (\eta^e) / (z^h) V$

$\tau$  - orthogonal shear stress  
 $z$  - depth of max. Stress  
 $n$  - number of stress cycles  
 $V$  - stressed volume

Substitute  $V$  as  $a(Z_0)L$

$a$  - semi-major axis of contact ellipse  
 $Z_0$  - depth of max. Stress  
 $L$  - circumference of raceway depth

2.  $\ln(1/S) = (\tau^c) \cdot (z^{1-h}) a \cdot (u^e) L L_s$

$L_s$  - no. of revs survived  
 $u$  - no. of stress cycles

Further simplified under particular load

3.  $\ln(1/S) = A \cdot L_s^e$  where  $e$  weibull's slope.

Simply taking logarithm on both sides

4.  $\ln \ln(1/S) = e \ln(L_s/L_b)$  where  $L_b$  is characteristic life when 63.2% of bearings fail.

*The final equation obtained is the latest equation present, used even in today's real world scenarios.*

# Load Life Equation Modifications

Initial Equation -  $L_s = (C/F_e)^p$ .

C-Dynamic Capacity

F<sub>e</sub>-Total Radial Load

However, it was observed that for modern bearing steels, the LP equation significantly underpredicts bearing fatigue lives. One of the reasons for this underprediction is that reliability, material, and lubrication conditions of the bearings were not considered in the original LP model. To account for these operating conditions, Zaretsky (4) modified the original LP equation with three life factors  $a_1$ ,  $a_2$ , and  $a_3$  as -

$$L_s = a_1 a_2 a_3 (C/F_e)^p$$

In the late 1980s better technology was developed by manufacturers in terms of surface finishes ,steel quality ,design ,production methods.By 1994 bearing life prediction was 14 times better than predicted by the LP model . Ionides and Harris proposed a new fatigue life model .Their work confirmed the existence of fatigue limit stress and interdependence of material and lubrication life factors. Based on finite element analysis, they replaced stress in the original LP with the difference between stress and fatigue limit stress at each location within the material.

$$L_s = a_1 (a_{ISO}) (C/F_e)^p$$
  $a_{ISO}$  is the integrated life adjustment factor

However US Standards institute didn't accept it and Zaretsky also raised objections .Due to existing conflict the modified LP model was still used.

## Drawbacks Of LP Model's Load Life Equations -

- 1.Assumes smooth surface and ignores irregularities that affect subsurface stress fields.
- 2.Does not consider surface shear stress or surface lubrication

## JUSTIFICATION OF THE DESIGN CHOICE - Recalibration Analysis And Need For It

For roller bearings with m6 interference fit, Oswald reported a stress life exponent  $n$  in the range of 7.4–7.7. Similarly, for ball bearings with m6 middle of the tolerance-band interference fit it was found to be 8.63 for deep-groove ball bearings and 8.74 for angular contact ball bearings. Work by Parker and Zaretsky suggests that the value of the stress life exponent for cleaner and more recent vacuum processed steels should be 12. In addition, Zaretsky established that the load-life exponent  $p$  will one third of the stress life exponent  $n$ . Therefore, his work suggests that the load-life exponent  $p$  should be 4 for ball bearings manufactured from cleaner vacuum-processed steels. To check the accuracy of existing and recommended load-life exponents validation analysis of the modified LP was performed.

**Recalibration Analysis-** In 1993, under a study sponsored by the United States Navy, ball and roller bearing fatigue endurance data were collected for 62 data sets of deep groove bearings, angular contact bearings and cylindrical bearings. Following ISO :1978 standards these are the best compilation of endurance data available in open literature.

### Significance Of Weibull's Slope

From the Weibull slope estimates for each data set, we can see that the quality of endurance data is mixed. Higher values of the Weibull slope indicate narrow dispersion and smaller values indicate wide dispersion of fatigue life values in that data set.

It is clear that majority of the  $L_{10}(LP)/L_{10}(act)$  ratios were close to 0 indicating that modified LP still significantly underpredicts bearing fatigue life .

Validation analysis was performed to statistically re evaluate the load-life exponent  $p$  used in the modified LP to correct for this underprediction.



## WEIBULL DISTRIBUTION PARAMETERS

The Weibull distribution is a continuous probability distribution function .Where k and lambda are shape and scale parameters,k=e and lambda =Lb . The distribution is plotted later depending on the scale and shape parameters.

$$f(x;\lambda,k) = \begin{cases} \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} \exp\left(-\left(\frac{x}{\lambda}\right)^k\right), & x \geq 0, \\ 0, & x < 0 \end{cases}$$

### Validation-

We can see from the table data that lives predicted by the modified LP model differ significantly from field data.If we calculate  $\log(L10(LP)/L10(act))$  and the predictions are accurate then we should get a straight line fit .The load life exponents 3 for ball and 3.33 for roller bearings are not reliable design tools for new generations of bearings.It can lead to oversized bearings and weight penalties.

### Uncertainty Quantification-

There is an epistemic uncertainty in L10 lives reported due to finite number of bearings.To quantify it virtual samples of fatigue lives were generated that follow weibull distribution with scale parameter Lambda i and shape parameter ki. Then this simulation is repeated 5000 times to generate 5000 virtual samples of L10 life.

# Statistical Reevaluation

If the observed fatigue life is known now load life exponent  $p$  can be written as  $p = 3 - (\log(L_{10}(LP)/L_{10}(act))) / (\log(Fe/c))$  for ball bearings and  $p = 10/3 - (\log(L_{10}(LP)/L_{10}(act))) / (\log(Fe/c))$  for cylindrical bearings

To improve the agreement between predicted fatigue lives and observed fatigue lives, it is necessary to reevaluate the load life exponent  $p$  values in the modified LP eq. We get estimated values of load-life exponent  $p$  from 5,000 virtual samples of  $L_{10}$  lives presented. These 5,000 probable estimates of load-life exponent represent uncertainty due to finite samples in the experiment, and for analysis purposes they should be represented with a probability distribution. We use parameters for this normal distribution -mean and standard deviation of 5000 samples.

It was observed that for all of the data samples the maximum error in symmetric approximation of sampling uncertainty is less than 1%. It attains a maximum value of 6% only for data set 41 at a confidence interval of 20%. This uncertainty is expected due to the low Weibull slope corresponding to this data set. But even for this sample, symmetric approximation is valid beyond 20% confidence intervals with maximum error less than 1%. In addition to uncertainty due to the finite number of samples, there is uncertainty due to experimental conditions and the adequacy of the model.

## ERRORS-

1. **Uncertainty due to finite samples and Weibull distribution.**
2. **Experimental errors.**

## Relation - Normalized vs Weighted Std Deviation

$$[\sigma_{(total)}]_i = \sqrt{[\sigma_{(sample)}]_i^2 + 0.25 \times \left[ \text{Standard deviation of means of 5,000 probable estimates for all samples} \right]^2}$$

## Reevaluated Value Of P After Bayesian Inference and Updating

Earlier values of  $p$  for ball and roller bearings were 3 and 3.33 respectively. The relevant results and plots in comparison to Nikhil's research paper will be shown after seeing his reevaluated value .

Bearing Type	Reevaluated Value of $p$ (Mode)	Standard Deviation	95% Confidence Bounds
Ball bearings	4.1	0.11	[3.87, 4.33]
Cylindrical roller bearings	5.5	0.26	[4.98, 6.02]

### Material Dependent Values of $p$

Material Type	Reevaluated Value of $p$ (Mode)	Standard Deviation	95% Confidence Bounds
Through-hardened steels	4.0	0.13	[3.74, 4.26]
Case-hardened steels	4.5	0.17	[4.16, 4.84]
AISI 52100 Steels	3.9	0.20	[3.50, 4.30]
M50 steels (VAR, VIMVAR)	4.1	0.18	[3.74, 4.46]



# Results and Analysis

By exporting the data on Excel we take 2 approaches -

1 . Formula based approach .

2. Graphical approach .

The dataset used is 1995 US Navy data.( 1 out of available 62 in total, taken in consideration )



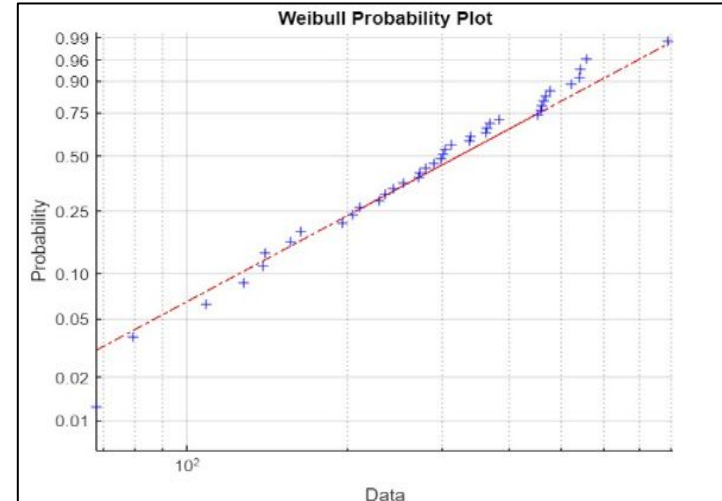
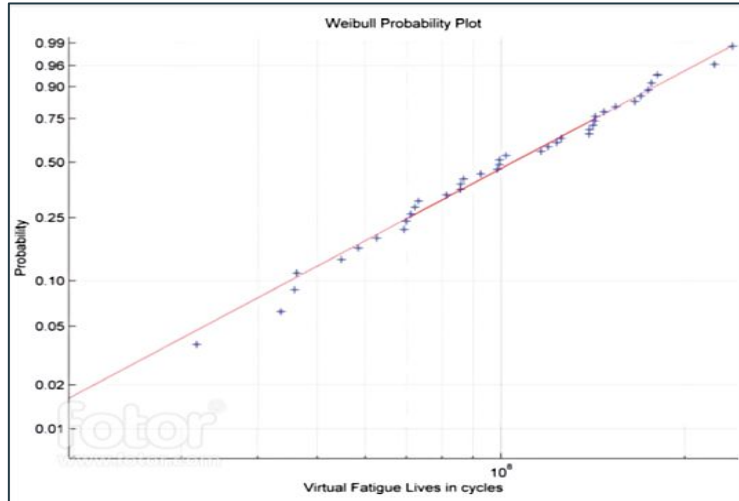
# Graphical Analysis

Taking dataset and plotting graphs of mathematical relations mentioned in reference research paper -

I have used MATLAB ,online probability distribution plotters and excel to plot these graphs.

1. **MATLAB inbuilt function 'prctile()'** used to calculate L-10 life
2. **MATLAB inbuilt function 'wblrnd()'** used to calculate Weibull distribution of samples. Mention reference

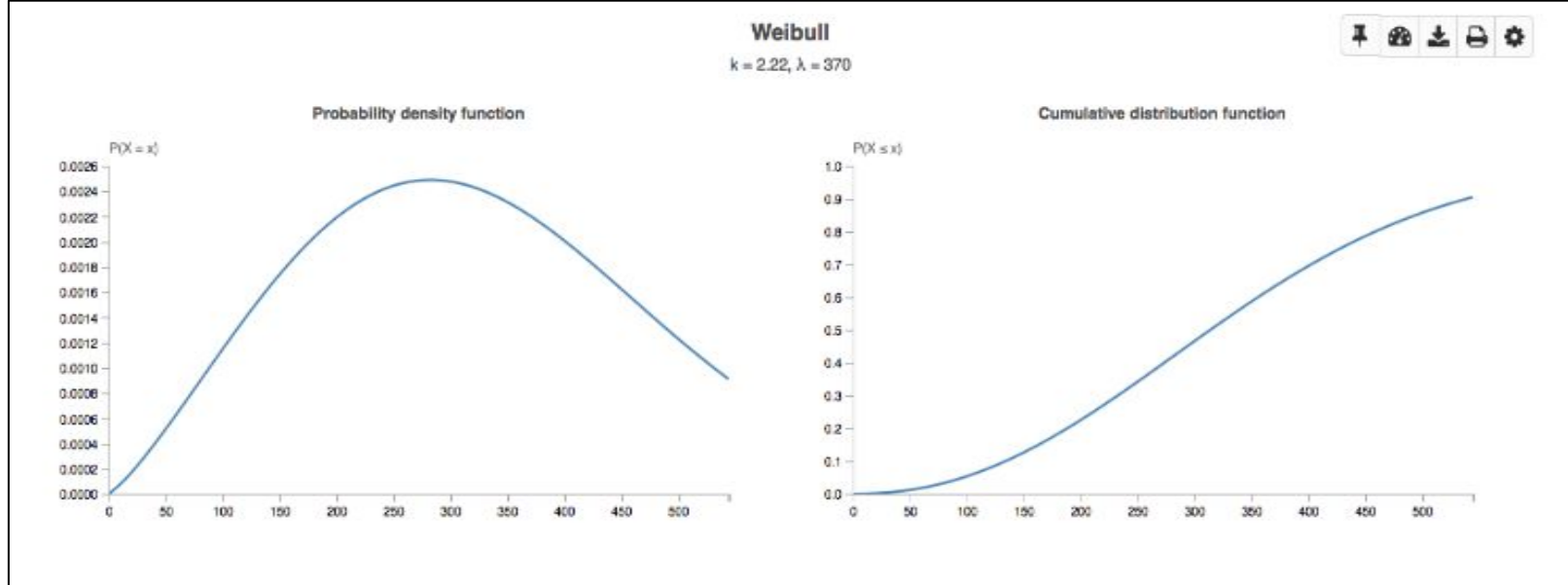
**Weibull Probability Plot Expected (paper taking 62 datasets) vs Observed ( for less samples on MATLAB)**



# Graphical Analysis

Weibull distribution using online probability distribution plotter( values of k shape and lambda scale parameters calculated manually from the dataset) .

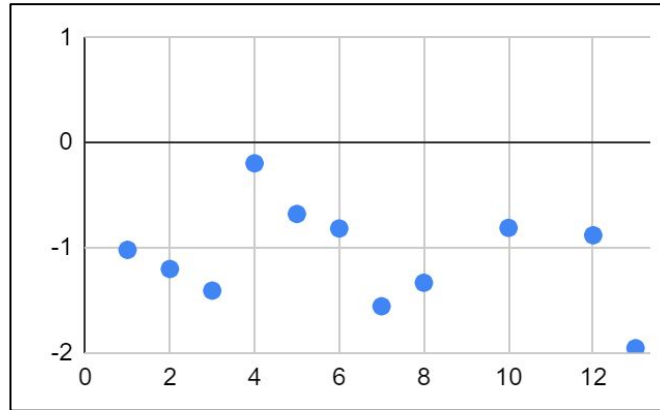
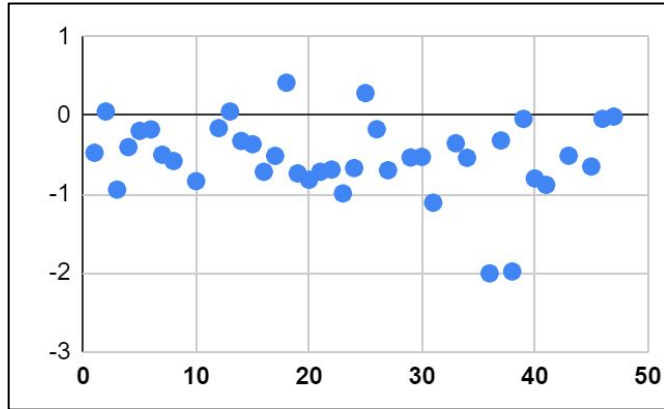
Probability of survival vs Frequency (High because of large number of samples)



# Graphical Analysis

$\text{Log}(L_{10}(L_p)/L_{10}(\text{act}))$  ratios for ball bearings data samples with load life exp.  $p = 3$

**Observed**

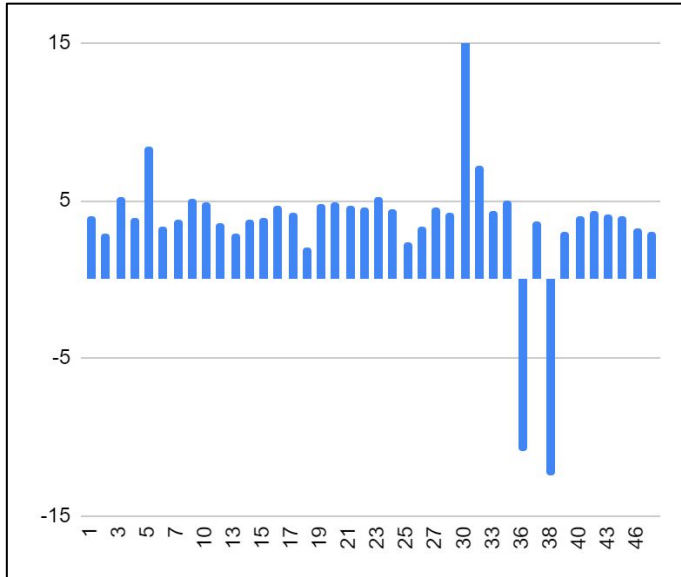


$\text{Log}(L_{10}(L_p)/L_{10}(\text{act}))$  vs Sample Number

# Graphical Analysis

**L-10 Life Distribution of dataset values . Graph between frequency and L10 lives in cycles**

**Observed**





# Conclusion based on the analysis -

- LP model which predicted 3 as load life exponent ( of ball bearings ) and 3.33 as load life ( of cylindrical bearings ) underpredicts the life to a great extent.
- There are some evident noises in the data which if not ignored will lead to obvious wrong results .These samples give high negative values in the L-10 life distribution graph and some be removed.
- Take average of values obtained with the new parameters the load life exponent of this data is **4.069179486** for ball bearings and **4.726283297** for cylindrical roller bearings.
- Applying 95% confidence bounds on load life exponent of ball bearings provides a range of [3.86,4.31] and applying 95% confidence bounds on load life exponent of cylindrical roller bearings gives a range of [4.5,5.06]

# Conclusion based on research papers -

Case Hardened steels have a strong residual compressive stress through the case layer and as a result generally tend to have considerably higher fatigue lives than the corresponding through-hardened bearing steels.

To check the dependency of the load-life exponent on the type of bearing material, separate validation and re evaluation analyses were performed for through-hardened steels and case-hardened steels. Ball bearings manufactured from through-hardened steels the maximum likelihood estimate of load-life exponent  $p$  is 4 with 95% confidence bounds as [3.74, 4.26] and for case-hardened steels it is about 4.5 with 95% confidence bounds as [4.16, 4.84]. These reevaluated values clearly show improved endurance capability of case-hardened steels compared to through-hardened steels. The test data also indicates that M50 steels have better endurance capability than AISI 52100 steel for both ball bearings and cylindrical roller bearings.

**Conclusion-** It is now clear that the widely used LP model significantly underestimates the fatigue bearing life. Therefore, continuing the use of standard parameters based on this method tends to result in larger size bearings and mechanisms than are necessary.

Using statistical calibration techniques along with a Bayesian probability approach, it was found that load-life exponent  $p$  values should be corrected for ball bearings from 3 to 4.1 with 95% confidence bounds as [3.87, 4.33] and for cylindrical roller bearings it should be corrected from 3.33 to 5.5 with 95% confidence bounds as [4.98, 6.02].

# Future Work

I believe work done in this project is satisfactory, but these are some of the stretch goals planned by me. These can be carried forward by students who participate in this project in future.

1. Add sampling through online samplers or implement a data set sampler to mimic the procedure followed by US Navy which is using 62 different data sets . A hybrid dataset can be created by using metrics like mean/median/mode and they re evaluation can be done for better results.
2. To evaluate more recent experimental datasets. Industries in India can be contacted to obtain more recent data . Machine learning can be used to predict life . Some models like Decision Tree Regressor , KNN Regressor model , Random Forest Regressor can be implemented as scope of this project.