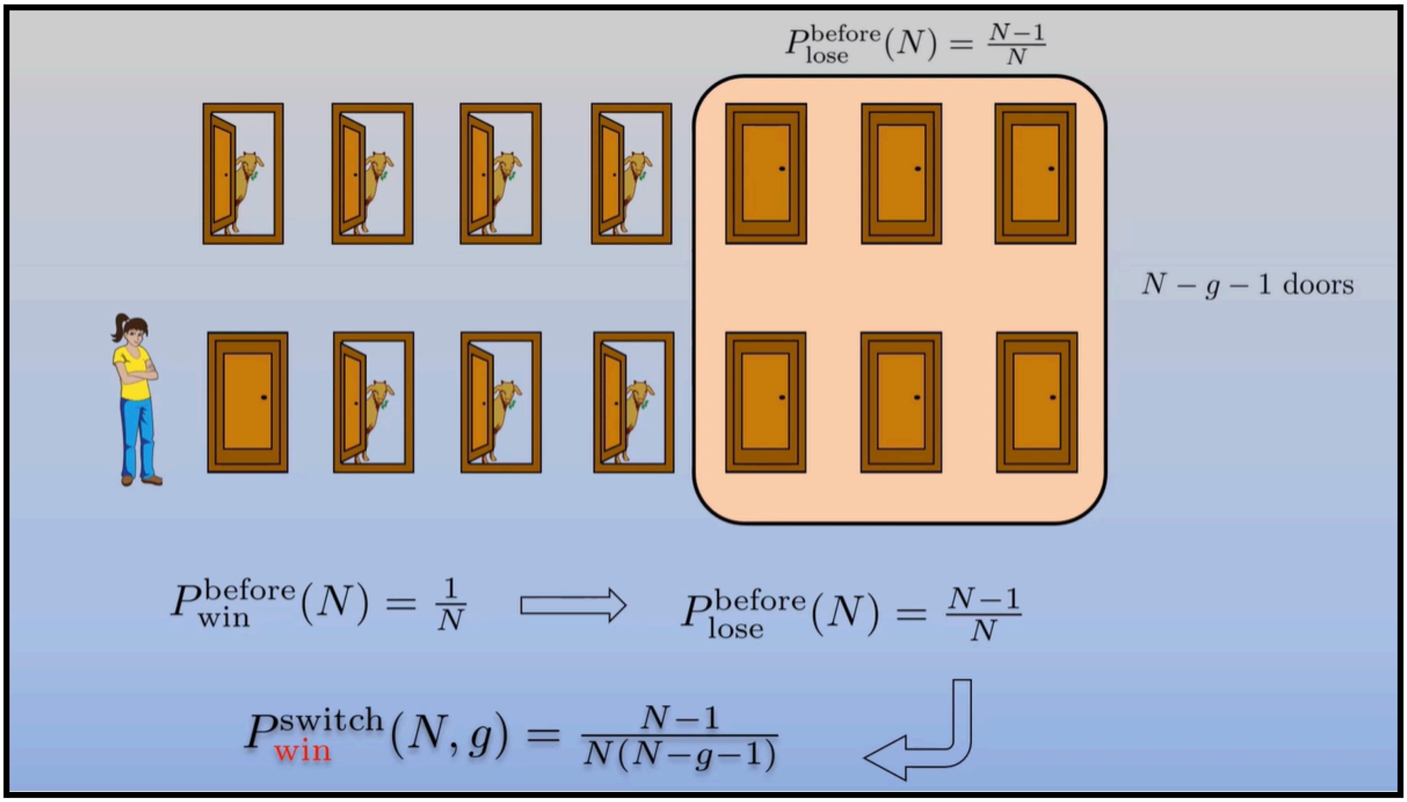
Monty Hall Simulator (Extended to n doors)

Probability Formula (Diagram + Derivation)



- Let the number of doors be N.
- Let the number of doors opened after choice 1 be g.
- Let $P(Ws)$ be the probability of winning if he stays.

$P(Ws)$ -

The initial probability of making the correct pick out of N doors should be $\frac{1}{N}$ .If the player keeps his initial pick, then he must have picked the car to win. Hence probability of winning the game stays the same.

$$\Rightarrow \qquad P(Ws) = \frac{1}{N}$$

- Let $P(Wc)$ be the probability of winning if he switches.

  $P(Wc)$ -
  To win after switch, the intersection of 2 events should happen.

  Event 1:
  The player must make an incorrect initial pick. The probability of that is $P(E1) = \frac{N-1}{N}$.

  Event 2:
  Following an incorrect pick in event 1, the number of doors to pick from are now reduced to ($N - g - 1$). Now, the player must be lucky enough to pick the correct door out of these doors if he wants to win on a switch.
  $P(E2) = \frac{1}{N-g-1}$.

  Taking the intersection of 2 events.

$$\Rightarrow \qquad P(E1 \cap E2) = P(E1).P(E2 \mid E1) = \frac{N-1}{N} \times \frac{1}{N-g-1} .$$

$$\Rightarrow \qquad P(Wc) = \frac{N-1}{N \times (N-g-1)}$$

Hence we have derived the probabilities of winning if the player stays with his initial pick and winning if the player switches to a new pick.
As asked in the question, the probabilities in terms of n and k would be.

Stays	Switches
$P(Ws) = \frac{1}{n}$	$P(Wc) = \frac{n-1}{n \times (n-k-1)}$