CENG223 - THE 2

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Q. 1

$(A \cup B) \setminus (A \cap B)$
$= \{x | x \in (A \cup B) \wedge x \notin (A \cap B)\}$ Definition of set difference
$= \{x | (x \in A \vee x \in B) \wedge x \notin (A \cap B)\}$ Definition of set union
$= \{x | (x \in A \vee x \in B) \wedge \neg(x \in (A \cap B))\}$ Definition of $\notin$
$= \{x | (x \in A \vee x \in B) \wedge \neg(x \in A \wedge x \in B))\}$ Definition of intersection
$= \{x | (x \in A \vee x \in B) \wedge (\neg(x \in A) \vee \neg(x \in B))\}$ De Morgan's rule for logical equivalences.
$= \{x | (x \in A \vee x \in B) \wedge (x \notin A \vee x \notin B)\}$ Definition of $\notin$
$= \{x | ((x \in A \vee x \in B) \wedge x \notin A) \vee ((x \in A \vee x \in B) \wedge x \notin B)\}$ Distribution laws for logical equivalences
$= \{x | ((x \in A \wedge x \notin A) \vee (x \in B \wedge x \notin A)) \vee ((x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin B))\}$ Distribution laws for logical equivalences
$= \{x | (\emptyset \vee (x \in B \wedge x \notin A)) \vee ((x \in A \wedge x \notin B) \vee \emptyset)\}$ Complement law
$= \{x | (x \in B \wedge x \notin A) \vee (x \in A \wedge x \notin B)\}$ Identity law
$= \{x | (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)\}$ Commutative law
$= \{x | x \in (A \setminus B) \vee x \in (B \setminus A)\}$ Definition of set difference
$= (A \setminus B) \cup (B \setminus A)$ Definition of set union

Q. 2

In this question I have 3 steps(a, b, c).

a) Let's see the if the first part of the set difference is uncountable.
Assume that the $\{ f : f \subseteq \mathbb{N}x\{0,1\}\}$ countable.
Then there exists a 1-to-1 correspondence between $\mathbb{N}$ and $\{ f : f \subseteq \mathbb{N}x\{0,1\}\}$
Suppose we have it:
$1 \to f_a = \{(a_{11}, a_{12}), (a_{21}, a_{22}), (a_{31}, a_{32})...\}$
$2 \to f_b = \{(b_{11}, b_{12}), (b_{21}, b_{22}), (b_{31}, b_{32})...\}$
$3 \to f_c = \{(c_{11}, c_{12}), (c_{21}, c_{22}), (c_{31}, c_{32})...\}$
Now let's see if we can construct a new subset that is missed by the enumeration.
$f_x \to \{(x_{11}, x_{12}), (x_{21}, x_{22}), (x_{31}, x_{32})...\}$
Where
$x_{11} \neq a_{11} \vee x_{12} \neq a_{12}$
$x_{21} \neq b_{21} \vee x_{22} \neq b_{22}$
$x_{31} \neq c_{31} \vee x_{32} \neq c_{32}$
By design $f_x \subseteq \mathbb{N}x\{0,1\}$ and it is missed by the enumeration. Hence, the set is uncountable.

b) Now let's see the if the second part of the set difference is uncountable.
We can enumerate all functions as such:
Let $f(x) = y$
$f(0) = 0, \quad f(0) = 1, f(1) = 0, \quad f(0) = 2, f(1) = 1...$
$(x + y = 0) \quad (x + y = 1) \quad (x + y = 2)...$
Since we found an enumeration, this set is countable.

c) This is the last part for this question.
What we have is a set difference:
$A \setminus B$ where $A = \{f : f \subseteq \mathbb{N}x\{0, 1\}\}$ and $B = \{f|f : 0, 1 \rightarrow \mathbb{N}$, f is a function $\}$

Suppose that $A \setminus B$ is countable.
Since B is countable, $A \cap B$ is also countable.
We know that $A = (A \setminus B) \cup (A \cap B)$
So, from here the set A should be also countable. However, we proved that A is uncountable. So we reached a contradiction.
Hence, our assumption is not correct.
So, $\{f : f \subseteq \mathbb{N}x\{0, 1\}\} \setminus \{f|f : 0, 1 \rightarrow \mathbb{N}$, f is a function $\}$ is uncountable.

Q. 3

Let's assume that $f(n)$ is $O(2^n)$. Then there exists constants C and k such that:
$4^n + 5n^2 logn \leq C2^n$ for all $n > k$

We can organize the expression as:
$(2^n)(2^n) + 5n^2 logn \leq C2^n$ for all $n > k$
Dividing both sides by 2^n, we get:
$2^n + \frac{5n^2 logn}{2^n} < C$ for all $n > k$

This cannot hold for all $n > k$, whatever the k and C is;
Hence, $f(n)$ is not $O(2^n)$

Q. 4

In this question we can organize the terms according to congruence rules:
$(2x - 1)^n - x^2 \equiv -x - 1(mod(x - 1))$
$(2x - 1)^n \equiv x^2 - x - 1(mod(x - 1))$
$(2x - 1)^n \equiv x(x - 1) - 1(mod(x - 1))$
$(2x - 1)^n \equiv -1(mod(x - 1))$
$(2(x - 1) + 1)^n \equiv -1(mod(x - 1))$
$(2(x - 1) + 1)(2(x - 1) + 1)^{n-1} \equiv -1(mod(x - 1))$

((2(x-1)+1) mod(x-1) is 1 and (multiplication of several of them) mod(x-1) is always 1. So left hand side can be written as 1.

$1 \equiv -1(mod(x - 1))$ Right hand side is the max value that is smaller than 0. So if we add (x-1) then it will be equal to 1 since 1 mod(any number) is 1.
$1 \equiv x - 2(mod(x - 1))$ So from here we can see that
$x - 2 = 1$
$x = 3$