Student Information

Name : Fazlı Balkan

ID : 2380178

Answer 1

a) $We\ test\ the\ null\ hypothesis\ H_0 : \mu = 7.8\ against\ a\ one\ sided\ right-tail\ alternative\ H_A :\ \mu > 7.$
$We\ are\ given\ \sigma = 1.4,\ n = 17\ and\ \overline{X} = 7.8.\ And,\ we\ want\ a\ confidence\ level\ of\ 95\%,\ which\ means\ that\ \alpha = 0.05.$
$Step\ 1 :\ Test\ Statistic :$

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{7.8 - 7}{1.4/\sqrt{17}} = \frac{0.8 * \sqrt{17}}{1.4} = 2.356$$

$Step\ 2 :\ Acceptance\ and\ rejection\ regions.\ The\ critical\ value\ is :$
$z_\alpha = z_{0.05} = 1.645$

$Step\ 3 :\ Result :$
$Our\ test\ statistic\ Z\ is\ bigger\ than\ the\ z_\alpha.\ Therefore,\ we\ reject\ the\ null\ hypothesis,\ which\ means\ that\ it\ can\ be\ regarded\ as\ successful.$

b) $If\ a\ customer\ gives\ 1\ instead\ of\ 10,\ the\ mean\ would\ change.\ New\ mean\ will\ become :$

$$\overline{X} = \frac{7.8 * 17 - 10 + 1}{17} = 7.27.$$

$We\ know\ that\ the\ standard\ deviation\ does\ not\ change.\ We\ can\ apply\ the\ same\ steps\ from\ part\ a :$

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{7.27 - 7}{1.4/\sqrt{17}} = \frac{0.27 * \sqrt{17}}{1.4} = 0.795$$

$It\ is\ clear\ that\ the\ new\ Z\ is\ in\ the\ favor\ of\ the\ null\ hypothesis,\ which\ means\ that\ it\ can\ NOT\ be\ regarded\ as\ successful.$

c) z_α *is still the same. This time mean does not change as much as it changed in the part b. The new sample mean is :*

$$\mu = \frac{7.8 * 45 - 10 + 1}{45} = 7.6.$$

Now, we should calculate the new Z.

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} = Z = \frac{7.6 - 7}{1.4/\sqrt{45}} = 2.875.$$

We see that $Z > z_\alpha$.
So, Z is in the favor of alternate hypothesis which means that it can be regarded as successful.

d) *If the threshold for success is set to* 8 *for* 95% *confidence, we don't need any calculations to say that it will always be regarded as unsuccessful. It is because in this case we used the one sided right tail hypothesis, and we want the standardized Z to be greater than* $z_\alpha(1.645)$. *But with the threshold* 8, *it will always be negative; thus, unsuccessful.*

Answer 2

In this question we should test $H_0 : \mu_{new} - \mu_{old} = D = 0$. *Andthe alternate hypothesis should be* $H_A : \mu_{new} > \mu_{old}$ *because we are asked if the new vaccine protects longer than the old vaccine.*
Test statistics :

We are given :
$n_{new} = 55,\ n_{old} = 55,$
$\overline{X}_{new} = 6.2,\ \overline{X}_{old} = 5.8,$
$\sigma_{new} = 1.5,\ \sigma_{old} = 1.1$
Our α *is* 0.05 *since we want to have a* 5% *level of significance. Therefore,*
$z_\alpha = 1.645.$

$$Z = \frac{\overline{\mu_{new}} - \overline{\mu_{new}} - D}{\sqrt{\frac{\sigma^2 y}{m} + \frac{\sigma^2 x}{n}}} = \frac{\overline{6.2} - \overline{5.8} - 0}{\sqrt{\frac{1.5^2 y}{55} + \frac{1.1^2 x}{55}}} = 1.594$$

It is clear that Z is bigger than z_α. *Since Z is in the favor of null hypothesis, we can NOT conclude that the new vaccine protects longer than the old vaccine.*

Answer 3

a) *To calculate the margin error we need to use the formula* $\Delta = Z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$
Using this formula we can calculate the margin error for the both parties :

$$\Delta_{red} = 1.96\sqrt{\frac{0.48*(1-0.48)}{400}} = 0.049$$

$$\Delta_{blue} = 1.96\sqrt{\frac{0.37*(1-0.37)}{400}} = 0.0473$$

b) *In this part we can use the formula* $\Delta = Z_{\alpha/2}\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$
Using this formula we can calculate the margin of error for the estimated lead :

$$\Delta = 1.96\sqrt{\frac{0.48(1-0.48)}{400} + \frac{0.37(1-0.37)}{400}} = 1.96\sqrt{0.5012/400} = 0.0693$$

c) *Red party's margin of error is larger than the blue party's. It is because that the only difference in the formula is the variable p. Although the variable p is not directly proportional it is multiply by the unit difference of itself. More precisely* $p(1-p)$*.And we know that if the sum of two numbers are equal then the two which are closer to each other will produce a bigger number if they are multiplied*

d) *It will be decreased because in the formula margin is inversely proportional to the squared root of n. Margin of errors will drop to* 0.471 *times since n increases* $\frac{1800}{400}$ *times.*