Student Information

Full Name : Fazlı Balkan
Id Number : 2380178

Answer 1

Let A(x) be the generating function corresponding $a_n = \sum_{n=0}^{\infty} a_n x^n$
$\sum_{n=1}^{\infty} a_n x^n = \sum_{n=1}^{\infty} (a_{n-1} + 2^n) x^n$

0^{th} term is missing on the left hand side. Separating the terms on the right hand side;
$A(x) - a_0 x^0 = \sum_{n=1}^{\infty} a_{n-1} x^n + \sum_{n=1}^{\infty} 2^n x^n$

On the left hand side 0^{th} term is 1. On the right hand side getting one of the x's from the first term out and rearranging the second term, we get;
$A(x) - 1 = x \sum_{n=1}^{\infty} a_{n-1} x^{n-1} + \sum_{n=1}^{\infty} (2x)^n$

First term on the left hand side is equal to $xA(x)$ and the second term is equal to $\frac{1}{1-2x} - 1$;
$A(x) - 1 = xA(x) + \frac{1}{1-2x} - 1$

After rearranging,
$A(x) = \frac{1}{(1-2x)(1-x)}$

Now, we need to do partial fraction decomposition;
$A(x) = \frac{1}{(1-2x)(1-x)} = \frac{B}{1-2x} - \frac{C}{1-x} = \frac{B+C-(B+2C)}{(1-2x)(1-x)}$

We see that B = 2 and C = -1. Hence;
$A(x) = \frac{2}{1-2x} - \frac{1}{1-x}$

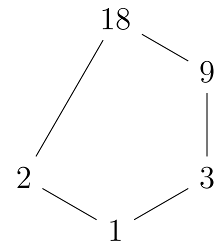
From generating functions, we know what each term corresponds to. Hence a_n is equal to;
$a_n = 2.2^n - 1$

$a_n = 2^{n+1} - 1$

Answer 2

$R = \{(1,1),(1,2),(1,3),(1,9),(1,18),(2,2),(2,18),(3,3),(3,9),(3,18),(9,9),(9,18),(18,18)\}$

a)



b)

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

c)

Yes, it is a lattice. There is no pair of elements that does not have a least upper bound or a greatest lower bound. Each pair of elements has both a least upper bound and a greatest lower bound.

d)

To find the symmetric closure of R, we need to add the elements (b,a) to the relation for each of the elements (a,b) in the relation. So, basically we need to include the inverse relation in the symmetric closure. This can be found by taking the union of the original relation and inverse relation. If we use matrix representation for both relations, we can or the matrices.

$$M_{R_s} = M_R \cup M_{R^{-1}} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

e)

For two element a and b to be comparable, the relation must include either (a, b) or (b, a). So,

2 and 9 are not comparable since $2 \nmid 9$ and $9 \nmid 2$. So, the relation does not have either (2, 9) or (9, 2).

3 and 18 are comparable since $3 \mid 18$. So, the relation has the element (3, 18).

Answer 3

a)

First let's consider the upper triangle of the matrix. Let an element in the upper triangle be (x_i, x_j). If that element exist in the relation then the element (x_j, x_i) should not exist in the relation for the relation to be anti-symmetric.
There are 3 cases:
1) (x_i, x_j) exists in the relation and (x_j, x_i) does not exist in the relation.
2) (x_j, x_i) exists in the relation and (x_i, x_j) does not exist in the relation.
3) Both (x_j, x_i) and (x_i, x_j) do not exist in the relation

In the upper triangle of the relation matrix, there exists $(n^2 - n)/2$ elements ((All the elements minus the diagonal) over 2).
Hence there are $3^{(n^2-n)/2}$ different possible upper triangles that are able to form an anti-symmetric binary relation.

Now, let's consider the diagonal elements. They can be anything and the result would still be an anti-symmetric binary relation. There are 2^n different possibilities for the diagonal.

Hence, the answer is $2^n.3^{(n^2-n)/2}$

b)

For finding the anti-symmetric part we use the same approach that we used on part a.

First let's consider the upper triangle of the matrix. Let an element in the upper triangle be (x_i, x_j). If that element exist in the relation then the element (x_j, x_i) should not exist in the relation for the relation to be anti-symmetric.
There are 3 cases:
1) (x_i, x_j) exists in the relation and (x_j, x_i) does not exist in the relation.
2) (x_j, x_i) exists in the relation and (x_i, x_j) does not exist in the relation.
3) Both (x_j, x_i) and (x_i, x_j) do not exist in the relation

In the upper triangle of the relation matrix, there exists $(n^2 - n)/2$ elements ((All the elements minus the diagonal) over 2).
Hence there are $3^{(n^2-n)/2}$ different possible upper triangles that are able to form an anti-symmetric binary relation.

For a binary relation to be reflexive, all diagonal elements should exist in the relation. Hence, in the matrix they should all be 1. This corresponds to 1 possibility.

Hence, the answer is $3^{(n^2-n)/2}$