CENG223 - THE 1

Student Information

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Q. 1

$\neg(p \wedge q) \longleftrightarrow (\neg q \rightarrow p)$
$\equiv (\neg(p \wedge q) \rightarrow (\neg q \rightarrow p)) \wedge ((\neg q \rightarrow p) \rightarrow \neg(p \wedge q))$ *By logical equivalences involving biconditional statements*
$\equiv (\neg(p \wedge q) \rightarrow (q \vee p)) \wedge ((q \vee p) \rightarrow \neg(p \wedge q))$ *By logical equivalences involving conditional statements*
$\equiv ((p \wedge q) \vee (q \vee p)) \wedge (\neg(q \vee p) \vee \neg(p \wedge q))$ *By logical equivalences involving conditional statements*
$\equiv ((p \wedge q) \vee (q \vee p)) \wedge ((\neg q \wedge \neg p) \vee (\neg p \vee \neg q))$ *By De Morgans Laws*
$\equiv (((p \wedge q) \vee q) \vee p) \wedge (((\neg q \wedge \neg p) \vee \neg p) \vee \neg q)$ *By Associative Laws*
$\equiv (q \vee p) \wedge (\neg q \vee \neg p)$ *By Absorption Laws*
$\equiv (p \vee q) \wedge (\neg p \vee \neg q)$ *By Commutative Laws*

Q. 2

a) $\forall x \forall y \exists (I(x, z) \wedge I(y, z) \wedge x \neq y) \rightarrow (\neg \exists n(E(x, n) \wedge E(y, n)))$
b) $\exists x \forall y (I(x, y) \wedge S(x, x))$
c) *bul*

Q. 3

a)
First I will be proving $p \vee \neg q, q \vdash p$, so that I can derive it in my main proof.

$$p \vee \neg q, q \vdash p \tag{1}$$

1. $p \vee \neg q$	premise
2. q	premise
3. p	assumption
4. $\neg q$	assumption
5. $\neg p$	assumption
6. q	*identical* 2
7. $\perp$	$\neg e$ 4,6
8. $\neg\neg p$	$\neg i$ 5-7
9. p	$\neg\neg e$ 8
10. p	$\vee e$ 1,3,4-9

Here is the main proof.

$$p \vee q, p \vee r \vdash (r \to q) \to p \tag{2}$$

1.	$p \vee \neg q$	premise
2.	$p \vee r$	premise
3.	$r \to q$	assumption
4.	p	assumption
5.	r	assumption
6.	q	$\to e$ 3,5
7.	p	(1)*derived* 1,6
8.	p	$\vee e$ 2,4,5-7
9.	$(r \to q) \to p$	$\to i$ 3-8

b)

$$\vdash ((q \to p) \to q) \to q \tag{3}$$

1.	$(q \to p) \to q$	*assumption*
2.	$\neg q$	*assumption*
3.	q	*assumption*
4.	$\neg p$	*assumption*
5.	$-q$	*copy* 2
6.	q	*copy* 3
7.	$\perp$	$\neg e$ 5,6
8.	$\neg\neg p$	$\neg i$ 4-7
9.	p	$\neg\neg e$ 8
10.	$q \to p$	$\to i$ 3-9
11.	q	$\to e$ 1,10
12.	$\perp$	$\neg e$ 2,11
13.	$\neg\neg q$	$\neg i$ 2-12
14.	q	$\neg\neg e$ 13
15.	$((q \to p) \to q) \to q$	$\to i$ 1-14

Q. 4

a)

$$\neg\forall(P(x) \to Q(x)) \vdash \exists x(P(x) \land \neg Q(x)) \tag{4}$$

1. $\neg\forall(P(x) \to Q(x))$ — *premise*
2. $\neg\exists x(P(x) \land \neg Q(x))$ — *assumption*
3. $x_0 \quad P(x_0) \land \neg Q(x_0)$ — *assumption*
4. $P(x_0)$ — *assumption*
5. $\neg Q(x_0)$ — *assumption*
6. $P(x_0) \land \neg Q(x_0)$ — $\land i$ 4,5
7. $\bot$ — $\neg e$ 3,6
8. $\neg\neg Q(x_0)$ — $\neg i$ 5-7
9. $Q(x_0)$ — $\neg\neg e$ 8
10. $P(x_0) \to Q(x_0)$ — $\to i$ 4-9
11. $\forall x(P(x) \to Q(x))$ — $\forall x i$ 3-10
12. $\bot$ — $\neg e$ 1,11
13. $\neg\neg\exists x(P(x) \land \neg Q(x))$ — $\neg i$2-12
14. $\exists x(P(x) \land \neg Q(x))$ — $\neg\neg e$13

b) I could not complete this one.

$$\forall x\forall y(P(x,y) \to \neg P(y,x)), \forall x\exists y P(x,y) \vdash \neg\exists v\forall z P(z,v) \tag{5}$$

1. $\forall x\forall y(P(x,y) \to \neg P(y,x))$ — *premise*
2. $\forall x\exists y P(x,y)$ — *premise*