Student Information

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Answer 1

a)
When finding the expectation of discrete random variables, we need to first multiply every possible value with its probability and then add them together. Below you can see the expected values of a single dice roll of all three colors.

$$E(Blue) = 2 * P(2) + 3 * P(3) + 4 * P(4) = 2 * \frac{4}{6} + 3 * \frac{1}{6} + 4 * \frac{1}{6} = \frac{5}{2}$$

$$E(Yellow) = 1 * P(1) + 2 * P(2) + 4 * P(4) = 1 * \frac{2}{6} + 2 * \frac{2}{6} + 4 * \frac{2}{6} = 2$$

$$E(Red) = 1 * P(1) + 2 * P(2) + 3 * P(3) + 5 * P(5) = 1 * \frac{1}{4} + 2 * \frac{1}{4} + 3 * \frac{3}{8} + 5 * \frac{1}{8} = \frac{5}{2}$$

b)
From the linearity of expectation:
First option: $E(2 * Red + Yellow) = 2 * E(Red) + E(Yellow) = 2 * \frac{5}{2} + 2 = 7$
Second option: $E(2 * Yellow + Blue) = 2 * E(Yellow) + E(Blue) = 2 * 2 + \frac{5}{2} = 6.5$
Hence, I would choose the first option.

c)If it is guaranteed that the blue dice' value will be 4; therefore, E(Blue) would be 4. In this case second option becomes 8. Hence, I would choose the second option.

d) Question asks us $P(Red|3)$
It is a conditional probability so we can use the conditional probability formula.

$$P(Red|3) = \frac{P(Red \cap 3)}{P(3)} = \frac{\frac{1}{3} * \frac{3}{8}}{\frac{1}{3} * \frac{1}{6} + \frac{1}{3} * \frac{2}{6} + \frac{1}{3} * \frac{3}{8}} = \frac{3}{7}$$

e) Total value will be 6 in two cases:
First Case: The value of red dice is 3 and the value of yellow dice is 3.
Second Case: The value of red dice is 5 and the value of yellow dice is 1.

The first case's probability is $\frac{3}{8} * \frac{2}{6}$ since events are independent.

The second case's probability is $\frac{1}{8} * \frac{2}{6}$

We can add them together since they are mutually exclusive. If we add them together we get $\frac{1}{6}$

Answer 2

a) From the Table 1, $P(A = 0, I = 2) = 0.17$

b) It is 0 since the table is exhaustive.

c) There are two electric outages in total only when:
1) In Ankara there is no electrical outage and in Istanbul there are 2 electrical outages
2)Both in Ankara and in Istanbul there is 1 electrical outage.
Since condition 1 and 2 is mutually exclusive, we can add them to find the answer.
$P(A = 0, I = 2) + P(A = 1, I = 1) = 0.17 + 0.11 = 0.28$

d) We can find it with using the addition rule:
We need to sum all the probabilities where random variable A has the value 1.

$$\begin{aligned} P(A=1) &= P(A=1,I=0)+P(A=1,I=1)+P(A=1,I=2) \\ &\quad +P(A=1,I=3) \\ &= 0.12+0.11+0.22+0.15 \\ &= 0.6 \end{aligned}$$

e)
Let $X = A + I$ be the total electrical outages in both cities.
We can see that X can be in the range 0 to 4.
Then,

$$\begin{aligned} P_X(0) &= P(A=0 \cap I=0) = P(0,0) \\ &= 0.08 \\ P_X(1) &= P(A=1 \cap I=0)+P(A=0 \cap I=1) = P(1,0)+P(0,1) = 0.13+0.12 \\ &= 0.25 \\ P_X(2) &= P(A=1 \cap I=1)+P(A=0 \cap I=2) = P(1,1)+P(0,2) = 0.11+0.17 \\ &= 0.28 \\ P_X(3) &= P(A=1 \cap I=2)+P(A=0 \cap I=3) = P(1,2)+P(0,3) = 0.22+0.02 \\ &= 0.24 \\ P_X(4) &= P(A=1 \cap I=3) = P(1,3) \\ &= 0.15 \end{aligned}$$

f)
To understand the independence of the random variables A and I, we need to find the marginal pmfs. Adding rowwise and columnwise, we get the marginal pmfs.
For random variables to be independent, product of all pairs of a and i should give us the joint probability. Therefore, if a pair violates this condition, we can say that they are dependent.
It is clear from the table that they are dependent since $P_{A,I}(0,1) = 0.13$ whereas $P_A(0)P_I(1) = 0.096$

		i				
		0	**1**	**2**	**3**	**P(A)**
a	**0**	**0.08**	**0.13**	**0.17**	**0.02**	**0.4**
	1	**0.12**	**0.11**	**0.22**	**0.15**	**0.6**
	P(I)	**0.2**	**0.24**	**0.39**	**0.17**	**1**