280 - THE1

Student Information

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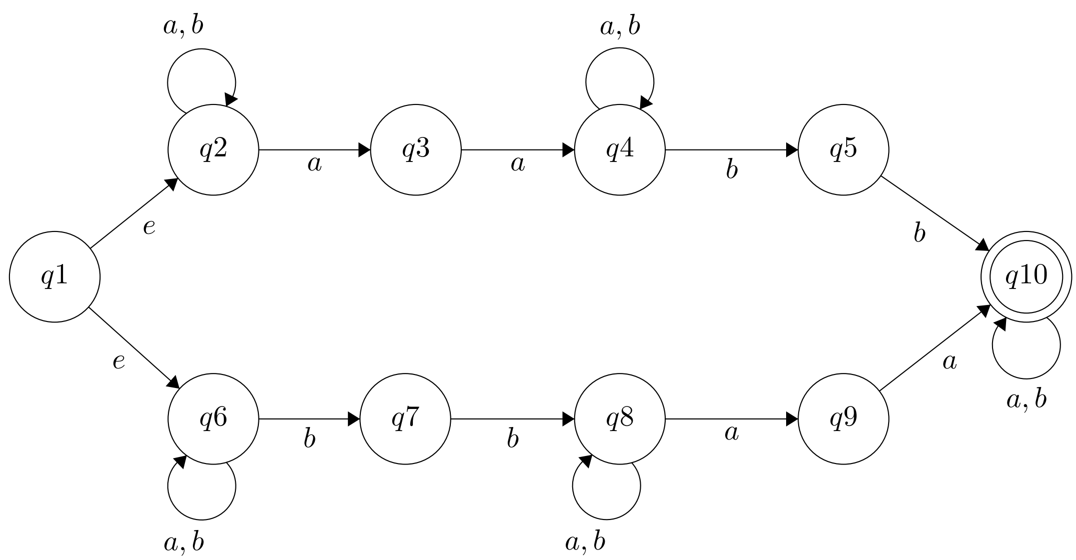
Q1

a)
The regular expression is;
$((a \cup b)^* aa(a \cup b)^* bb(a \cup b)^*) \cup ((a \cup b)^* bb(a \cup b)^* aa(a \cup b)^*)$

b)
$M = (K, \sum, \delta, s, F)$
where;
$K = \{q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}\}$
$\sum = \{a, b\}$
$\delta = \{(q_1, e, q_2), (q_1, e, q_6), (q_2, a, q_2), (q_2, b, q_2), (q_2, a, q_3), (q_3, a, q_4), (q_4, a, q_4), (q_4, b, q_4), (q_4, b, q_5), (q_5, b, q_{10}),$
$(q_6, a, q_6), (q_6, b, q_6), (q_6, b, q_7), (q_7, b, q_8), (q_8, a, q_8), (q_8, b, q_8), (q_8, a, q_9), (q_9, a, q_{10}), (q_{10}, a, q_{10}), (q_{10}, b, q_{10})\}$
$s = q_1$
$F = \{q_{10}\}$

Corresponding drawing for the NFA (Starting state being q_1):



c)

Initial state of the DFA is $s' = \{q_1, q_2, q_6\}$

Using the subset construction algorithm and finding the transitions for all the newly introduced states, we have the following.

$\delta'(\{q_1, q_2, q_6\}, a) = \{q_2, q_3, q_6\}$
$\delta'(\{q_1, q_2, q_6\}, b) = \{q_2, q_6, q_7\}$

$\delta'(\{q_2, q_3, q_6\}, a) = \{q_2, q_3, q_4, q_6\}$
$\delta'(\{q_2, q_3, q_6\}, b) = \{q_2, q_6, q_7\}$

$\delta'(\{q_2, q_6, q_7\}, a) = \{q_2, q_3, q_6\}$
$\delta'(\{q_2, q_6, q_7\}, b) = \{q_2, q_6, q_7, q_8\}$

$\delta'(\{q_2, q_3, q_4, q_6\}, a) = \{q_2, q_3, q_4, q_6\}$
$\delta'(\{q_2, q_3, q_4, q_6\}, b) = \{q_2, q_4, q_5, q_6, q_7\}$

$\delta'(\{q_2, q_6, q_7, q_8\}, a) = \{q_2, q_3, q_6, q_8, q_9\}$
$\delta'(\{q_2, q_6, q_7, q_8\}, b) = \{q_2, q_6, q_7, q_8\}$

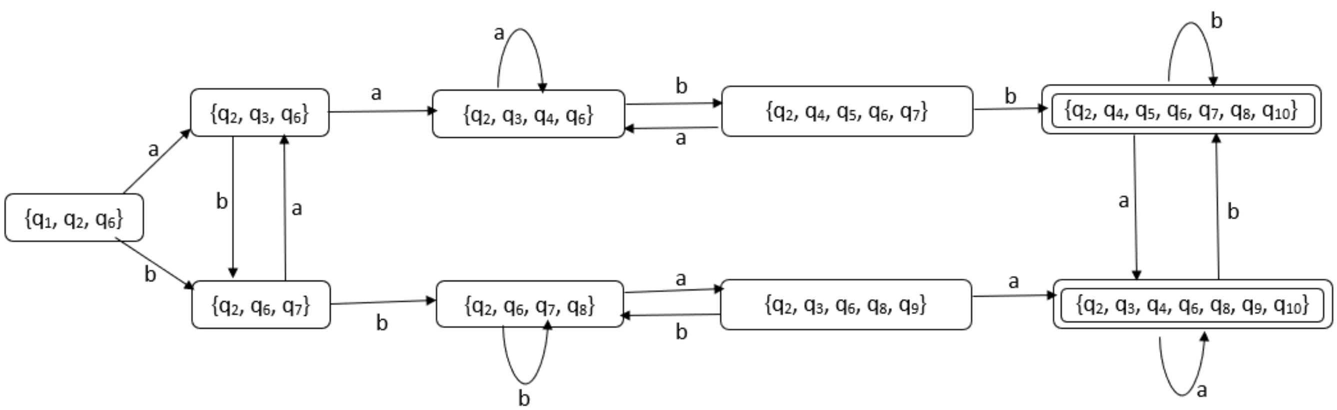
$\delta'(\{q_2, q_4, q_5, q_6, q_7\}, a) = \{q_2, q_3, q_4, q_6\}$
$\delta'(\{q_2, q_4, q_5, q_6, q_7\}, a) = \{q_2, q_4, q_5, q_6, q_7, q_8, q_{10}\}$

$\delta'(\{q_2, q_3, q_6, q_8, q_9\}, a) = \{q_2, q_3, q_4, q_6, q_8, q_9, q_{10}\}$
$\delta'(\{q_2, q_3, q_6, q_8, q_9\}, b) = \{q_2, q_6, q_7, q_8\}$

$\delta'(\{q_2, q_4, q_5, q_6, q_7, q_8, q_{10}\}, a) = \{q_2, q_3, q_4, q_6, q_8, q_9, q_{10}\}$
$\delta'(\{q_2, q_4, q_5, q_6, q_7, q_8, q_{10}\}, b) = \{q_2, q_4, q_5, q_6, q_7, q_8, q_{10}\}$

$\delta'(\{q_2, q_3, q_4, q_6, q_8, q_9, q_{10}\}, a) = \{q_2, q_3, q_4, q_6, q_8, q_9, q_{10}\}$
$\delta'(\{q_2, q_3, q_4, q_6, q_8, q_9, q_{10}\}, b) = \{q_2, q_4, q_5, q_6, q_7, q_8, q_{10}\}$

The corresponding DFA is (Starting state being $\{q_1, q_2, q_6\}$):



d)

For NFA;

For NFA, there are more than one trace.

$(q_1, bbabb) \vdash (q_2, bbabb)$
$\vdash (q_2, babb)$
$\vdash (q_2, abb)$
$\vdash (q_3, bb)$
It is stuck.

$(q_1, bbabb) \vdash (q_2, bbabb)$
$\vdash (q_2, babb)$
$\vdash (q_2, abb)$
$\vdash (q_2, bb)$
$\vdash (q_2, b)$
$\vdash (q_2, e)$
q_2 is not a final state

$(q_1, bbabb) \vdash (q_6, bbabb)$
$\vdash (q_6, babb)$
$\vdash (q_6, abb)$
$\vdash (q_6, bb)$
$\vdash (q_6, b)$
$\vdash (q_6, e)$
q_6 is not a final state

$(q_1, bbabb) \vdash (q_6, bbabb)$
$\vdash (q_7, babb)$
$\vdash (q_8, abb)$
$\vdash (q_8, bb)$
$\vdash (q_8, b)$
$\vdash (q_8, e)$
q_8 is not a final state

$(q_1, bbabb) \vdash (q_6, bbabb)$
$\vdash (q_7, babb)$
$\vdash (q_8, abb)$
$\vdash (q_9, bb)$
It is stuck.

There are couple of more traces; however, none of them results in final state. Hence, NFA does not accept this string.

For DFA;

$(\{q_1, q_2, q_3\}, bbabb) \vdash (\{q_2, q_6, q_7\}, babb)$
$\vdash (\{q_2, q_6, q_7, q_8\}, abb)$
$\vdash (\{q_2, q_3, q_6, q_8, q_9\}, bb)$
$\vdash (\{q_2, q_6, q_7, q_8\}, b)$
$\vdash (\{q_2, q_6, q_7, q_8\}, e)$

Since $\{q_2, q_6, q_7, q_8\}$ is not a final state, automaton does not accept this string.

Q2

a)
Suppose it is regular, and let us choose the string $w = a^{p+1}b^p$ which is in the language.

The string can be divided only one way where the first and second dividend contains only a, such that;

$x = a^q$ such that $q \geq 0$
$y = a^b$ such that $b \geq 1$
$z = a^{p+1-q-b}b^p$

For the language to be regular xy^iz should be in the language for all $i \geq 0$. If we choose i to be 0, the new string will be xz;
$xz = a^q a^{p+1-q-b}b^p = a^{p+1-b}b^p$
xz is in the language iff $p + 1 - b > p$
Subtracting p from both sides and add b to both sides, we get;
$1 > b$
Since, above we said that $b \geq 1$. We reached a contradiction. Hence, L_1 is not regular.

Since, a complement of a non-regular language is also non-regular, L_2 is also not regular.

b)
The language L_5 can be written with a regular expression since m and n is said to be natural numbers. It states that $m \geq 0$ and $n \geq 0$, which is simply the definition of Kleene Star. So, it can be rewritten as $L_5 = a^*b^*$. Since, there is a regular expression to show the language L_5 it is a regular language.

Since the language L_6 is already written as a regular expression, it is a regular language.

For the language L_4, there does not seem to exist a regular expression; however, not being able to find a regular expression does not give us the right to say that it is not regular. In the case of this question, we don't even need to know that it is regular or not. It is because question asks the union of the three languages.
It is clear that L_4 is a subset of the language L_5 since L_5 contains the strings in which m is equal to n. So, we can say that $L_4 \cup L_5 = L_5$

Both L_5 and L_6 is regular as stated above. Hence, $L_4 \cup L_5 \cup L_6 = L_5 \cup L_6$ is regular since regular languages are closed under union operation.