Investigating Gravitational Potential and Gravity Effect Due to Mass Anomaly at Different Elevation of Investigation Point

Fazlie Latib, *Department of Geoscience, University of Calgary*

Abstract

To understand the basic of gravity surveying, a code was developed in MATLAB to perform forward modelling for gravity anomalies in the subsurface by measuring how the gravitational potential, U, and the gravity effect, g_z, change as the elevation of the point of investigation changes. Two functions are built to perform the computation of U and g_z. The gravitational potential and gravity effect due to a 10 million metric tonne spherical mass anomaly were computed at three different elevations of investigation point (0, 10 and 100 m). The results were plotted using three different grid spacings (5, 10 and 25 m). As the elevation of the investigation point increases, the magnitude of U and g_z decreases. Smaller grid spacing produced contour plots of U and g_z that are much more precise in terms of the resolution where it showed better gradual changes across the plots.

Background / Theory

Gravity method is a versatile geophysical technique used to detect and identify subsurface bodies and anomalies within the Earth. This method has been used extensively in the hydrocarbon and mineral exploration over the years. Gravity surveys exploit the very small changes in gravity from place to place that are caused by changes in subsurface rock density (Telford et al., 1990). Higher gravity values are found over rocks that are denser, and lower gravity values are found over rocks that are less dense. A buried body represents a subsurface zone of anomalous mass and causes a localized perturbation in the gravitational field known as a gravity anomaly. The basis of the gravity method is Newton's Law of Gravitation, which states that the force of attraction, F, between two masses (m_1 and m_2) is directly proportional to the product of the two masses and inversely proportional to the square of the distance between their centres of mass. The greater the distance separating the centres of mass, the smaller is the force of attraction between them.

The gravitational potential, U, due to a concentrated point mass, m, is given by Equation 1 where $G \approx 6.674 \times 10^{-11}$ N m^2 kg^{-2} is the universal constant of gravitation, and r is the distance from the mass. To calculate U at a point, $\mathrm{x} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ due to a mass at $\mathrm{x_m} = x_m\hat{\mathbf{i}} + y_m\hat{\mathbf{j}} + z_m\hat{\mathbf{k}}$, the distance between them first needs to be computed using Equation 2 where $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are the standard unit vectors in the direction of the x, y, and z axes. Generally, U represents the gravitational potential at a distance r from mass m.

$$U = \frac{Gm}{r} \tag{1}$$

$$r = \|\mathrm{x} - \mathrm{x_m}\| = \sqrt{(x - x_m)^2 + (y - y_m)^2 + (z - z_m)^2} \tag{2}$$

The principal component of gravity measured by gravimeters and used in most gravity analyses is in the vertical direction, hence, the gravity effect, g_z, can be derived from Equation 1 by finding the first derivative of U with respect to z. The first step in finding the equation for g_z is by using a chain rule (Equation 3). By solving the chain rule, g_z is finally given by Equation 4.

$$g_z = -\frac{\partial U}{\partial z} = -\frac{\partial U}{\partial r} \cdot \frac{\partial r}{\partial z} \tag{3}$$

$$g_z = -\left(-\frac{Gm}{r^2}\right)\left(\frac{1}{2}(r^{-0.5})\big(2(z - z_m)\big)\right) = \left(\frac{Gm}{r^2}\right)\left(\frac{z - z_m}{r}\right) = Gm\frac{z - z_m}{r^3} \tag{4}$$

Methods / Algorithm

The algorithm comprises of two main functions. The first function computes the gravitational potential due to a mass anomaly using Equation 1 while the second function computes the gravity effect of the mass anomaly using Equation 4. Both the functions take as inputs a vector containing the coordinates at which U and g_z is to be computed, a vector containing the coordinates of the location of the point mass, x, the mass of the anomaly, m, and the universal constant of gravitation, G. The functions gave outputs as U and g_z values respectively.

A 10 million metric tonne spherical mass anomaly ($m = 1.0 \times 10^7$ kg) with its centroid located at x = 0$\mathbf{\hat{i}}$ + 0$\mathbf{\hat{j}}$ - 10$\mathbf{\hat{k}}$ is used in the computation. On a grid with same grid spacings that runs from x_{min} = y_{min} = -100 m to x_{max} = y_{max} = +100 m, contour plots of U and g_z at three different elevations (z = 0, 10 and 100 m) were generated using the values of U and g_z calculated using the functions. These results were plotted using three different grid spacings (5, 10 and 25 m).

Results and Discussion

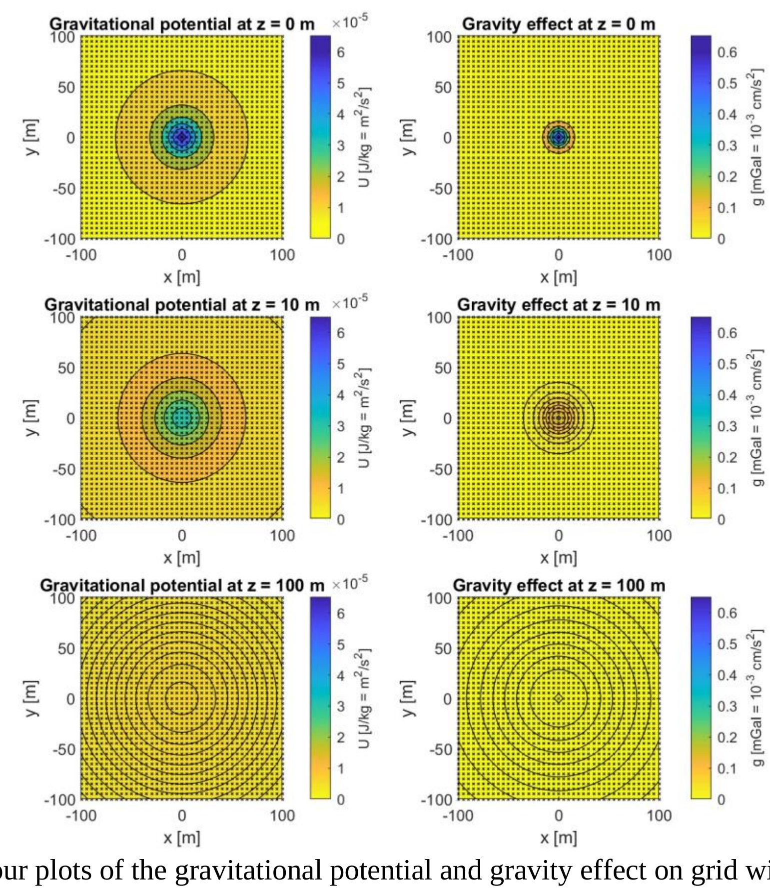

Figure 1 Contour plots of the gravitational potential and gravity effect on grid with 5 m spacings.

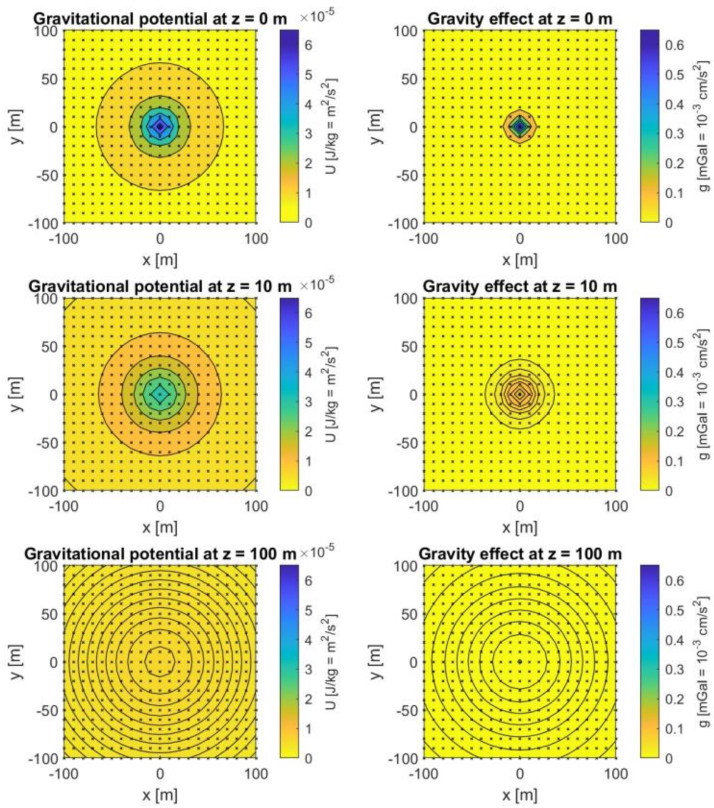

Figure 2 Contour plots of the gravitational potential and gravity effect on grid with 10 m spacings.

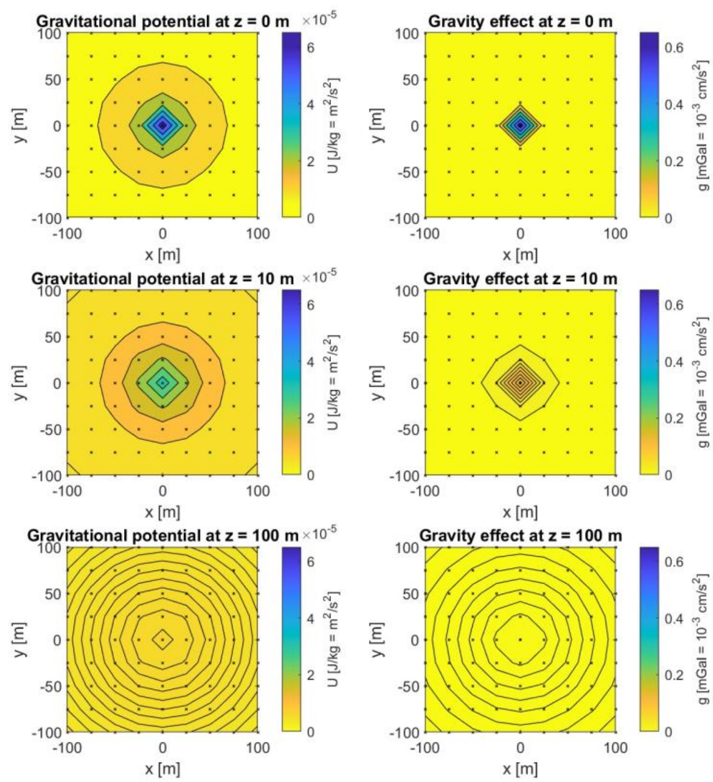

Figure 3 Contour plots of the gravitational potential and gravity effect on grid with 25 m spacings.

From Figure 1, 2 and 3, the maximum gravitational potential and gravity effect (represented by dark blue area in the contour plots) can be found when $z = 0$ m where the values are around 6.5×10^{-5} m^2/s^2 and 0.65 mGal respectively. As the elevation of investigation is increased to 10 m, the maximum gravitational potential and gravity effect decreased to around 3.0×10^{-5} m^2/s^2 and 0.1 mGal respectively. As we increased the elevation again, this time to a 100 m, both the gravitational potential and gravity effect decreased much more where the values are around 0.6×10^{-5} m^2/s^2 and 0-0.03 mGal respectively. Hence, U and g_z decrease as z increases. Ground-based gravity survey is done at $z = 0$ m whereas airborne gravity survey is done at $z > 0$ m. We can say that for the mass anomaly at the same depth in the subsurface, the ground-based gravity survey can detect a greater gravitational potential and gravity effect than the airborne survey. If only a ground-based gravity survey is done for the same mass anomaly but at a much deeper depth, the gravitational potential and gravity effect would decrease.

In Figure 1, 2, and 3, 5 m, 10 m and 25 m grid spacings is used respectively. The contour plots of U and gz between 5 m and 10 m grid spacings did not change much. However, as the grid spacings are increased to 25 m, the resolution of the contour plots became less precise where the gradual changes across the contour plot becomes less smooth. In other words, as the grid spacings increase, the quality of the contour plot output decreases.

Conclusion

The gravitational potential and gravity effect due to a mass anomaly changes as we change the elevation of the investigation point. The higher the elevation point, the more the gravitational potential and gravity effect decreases. As we increase the grid spacings used in the contour plot, the resolution or quality of the plot decreases.

References

Telford, W.M., Geldart, L.P., Sheriff, R.E. and Keys, D.A. (1990) Applied Geophysics (2nd edn), Cambridge