Producing Synthetic Seismograms through Convolution of Reflectivity Series with Source Wavelet

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Abstract

To understand the basic of reflection seismology, a code was developed in MATLAB to perform forward modelling of reflections assuming a plane wave in a 1D subsurface model by computing a reflectivity series, *r*, and convolving this with a source wavelet, *w*. Through this convolution, a synthetic seismogram or seismic trace, *s*, was produced. Using the knowledge of the reflection coefficient, the amplitudes for the primary reflection and first-order multiples reflection were determined corresponding to its two-way travel time. The combination of amplitudes between the primary and first-order reflection was found the be the same as the amplitudes of the primary reflection by itself. Considering the effect of attenuation that the seismic wave experienced as it propagates through the subsurface, the amplitudes of the reflected waves were observed to be smaller than the original amplitude without the attenuation.

Background / Theory

The concept of reflection coefficient or reflectivity is fundamental to reflection seismology as it is utilized to estimate the properties of the Earth's subsurface from reflected seismic waves. When a seismic wave encounters a boundary between two materials with different acoustic impedances, some of the energy in the wave will be reflected at the boundary, while some of the energy will be transmitted through the boundary (Lines, L. R. & Newrick, R. T., 2004). The acoustic impedance, *I*, is based on the density, *ρ*, and velocity of wave, *v*, through each layer (Equation 1). The amplitude of the reflected waves is governed by reflection coefficient - the amplitude ratio of the reflected wave to the incident wave. For a wave that hits a boundary normal incidence, the expression for the reflection coefficient, *R*, and transmission coefficient, *T*, for a downgoing and upgoing wave is shown in Equation 2 and 3 respectively.

$$I = \rho v \tag{1}$$

$$R_{\downarrow} = R = \frac{I_2 - I_1}{I_2 + I_1} \qquad R_{\uparrow} = -R \tag{2}$$

$$T_{\downarrow} = 1 - R \qquad T_{\uparrow} = 1 + R \tag{3}$$

A primary reflection event occurs as the seismic wave is reflected only once from an interface. When the seismic wave reflects off at multiple interfaces or boundaries, these can obstruct the desired primary reflections. Both primary and first order (three reflection) multiple reflection events are shown in Figure 1.

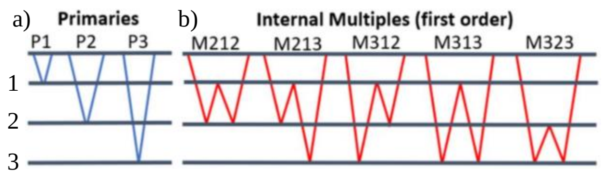

Figure 1 a) The primary events for a three-layer subsurface model, and b) the first order multiples for the three-layer model. (Iverson, A. S., 2020)

The primary reflection amplitudes at *n*-th interface, A_n, in the subsurface can be calculated using Equation 4. The two-way travel time at *n*-th interface (time for the wave to travel to the interface and back to the receiver), t_n, for the primary reflection can be calculated using Equation 5 where *z* is the depth of the interface.

$$A_n = R_n \sum_{k=1}^{n-1} (1 - R_k^2) \tag{4}$$

$$t_n = \sum_{n=1}^{n} (2\frac{z_n - z_{n-1}}{v_n}) \tag{5}$$

A seismic trace contains information that can be associated with changes in rock properties in the subsurface. The most common one-dimensional model for the seismic trace is referred to as the convolutional model, which states that the seismic trace, *s*, is simply the convolution of the earth's reflectivity, *r*, with a seismic source wavelet, *w*, (Russell, B. H., 1988) as shown mathematically in Equation 6 or illustratively in Figure 2 where * implies convolution. Convolution is a mathematical operation on two functions that produces a third function that expresses how the shape of one is modified by the other. This can be observed in Figure 2 where the spike in *r* (following its polarity) is replaced by the shape of the wavelet in *w* producing *s* after the convolution process.

$$s = r * w \tag{6}$$

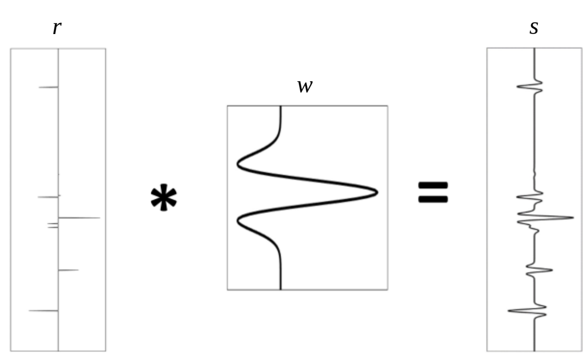

Figure 2 The convolution of the reflectivity with a source wavelet to produce a seismic trace or synthetic seismogram. (Keating, S., 2021)

As seismic wave propagates through the Earth, the wave will experience attenuation which caused the amplitude of wave observed to be smaller than expected (Mickus, K., 2021). The attenuation factor is often expressed as seismic quality factor, *Q*. The amplitude loss to attenuation for each layer, A_{loss} is given by Equation 7 where f_{dom} is the dominant frequency of the wavelet and *t* is the times spent propagating through the layer.

$$A_{loss} = e^{-\frac{2\pi f_{dom} t}{2Q}} \quad (7)$$

Methods / Algorithm

The algorithm or code for this study is provided separately from the paper. For this study, the data file used to read by MATLAB contains density, ρ (in kg/m^3), p-wave velocity, v_p (in m/s) and quality factor, Q from a well log over a range of depths *z* up to a depth of 3000 m.

A source wavelet, *w*, is defined as a single period sine wave with a frequency of 32 Hz and unit amplitude at time step, *dt* = 0.001.

Using Equation 1 and 2, the impedance of each layer, *I*, and the reflection coefficient at every boundary, *R*, is calculated.

A function is written to perform the convolution of two vectors with the interface **[w] = conv_517(u,v)** where w is the output while u and v is the input of the convolution.

The two-way travel time to each boundary is calculated using Equation 5.

A vector of the reflection coefficients corresponding to each two-way travel time, r_R is formed and then is convolved with the source wavelet producing a synthetic seismogram, s_R.

The primary reflection amplitude at each boundary is calculated using Equation 4. A vector of the primary reflection amplitudes corresponding to each two-way travel time, r_P is formed and then is convolved with the source wavelet producing a synthetic seismogram, s_P.

The first-order multiples reflection amplitude at every possible scenario is calculated using Equation 2 and 3. Then, it is added together with the primary reflection amplitude to form a vector of the measured reflections relative to the down-going wave for both primaries and first-order (two-bounce) multiple corresponding to each two-way travel time, r_M. It is convolved with the source wavelet producing a synthetic seismogram, s_M.

Accounting for attenuation using Equation 7, a vector of the measured reflections relative to the down-going wave for both primaries and first-order (two-bounce) multiple corresponding to each two-way travel time, r_Q is formed and then is convolved with the source wavelet producing a synthetic seismogram, s_Q.

Results

The source wavelet, *w*, that is used for all the convolution process is shown in Figure 3.

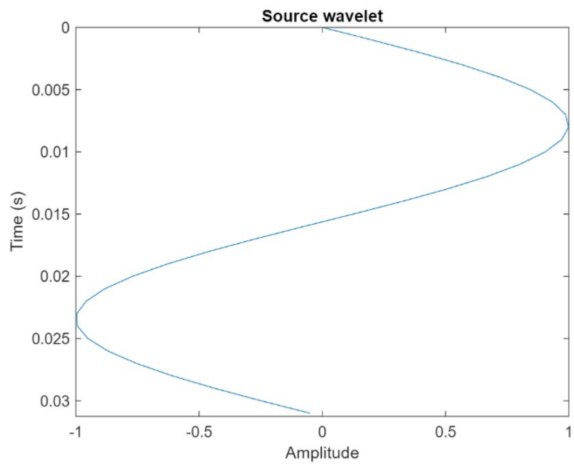

Figure 3 The source wavelet, *w*, of a single period 32 Hz sine wave.

In Figure 4, the four main physical properties of each layer or its boundary were plotted against depth.

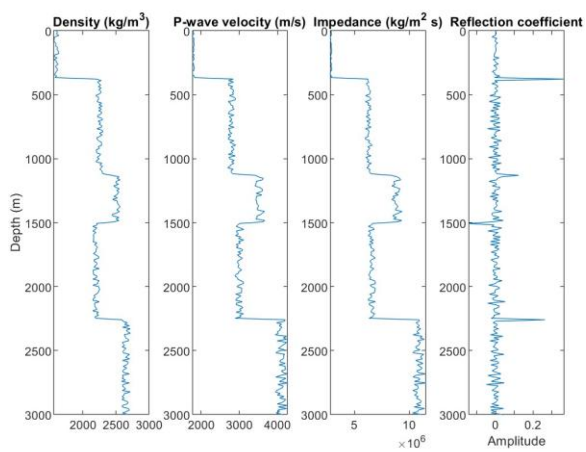

Figure 4 The density, p-wave velocity, impedance, and reflection coefficient plotted against depth.

The reflection coefficient corresponding to its two-way travel time, r_R, and its seismic trace, s_R, after convolution with the source wavelet is shown in Figure 5. The reflection coefficient can be seen as spikes in the figure.

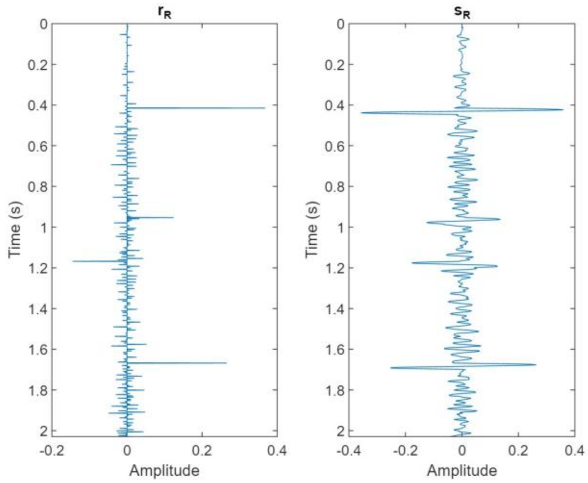

Figure 5 The reflection coefficient corresponding to each two-way travel time, r_R and the synthetic seismogram, s_R produced after its convolution with the source wavelet.

The primary reflection amplitudes corresponding to its two-way travel time, r_P, and its seismic trace, s_P, after convolution with the source wavelet is shown in Figure 6.

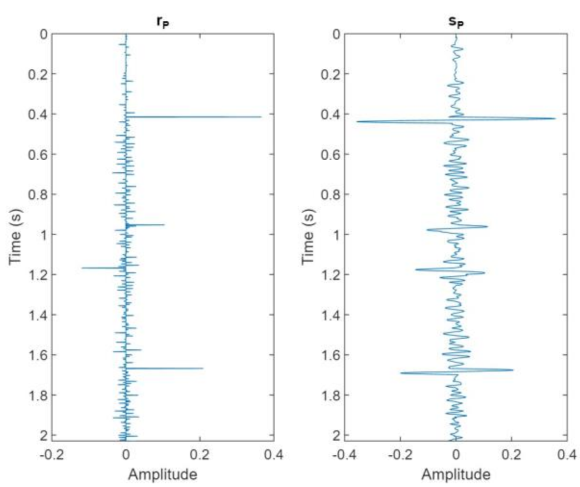

Figure 6 The primary reflection amplitudes corresponding to each two-way travel time, r_P and the synthetic seismogram, s_P produced after its convolution with the source wavelet.

The primary and first order multiple reflection amplitudes corresponding to its two-way travel time, r_M, and its seismic trace, s_M, after convolution with the source wavelet is shown in Figure 7.

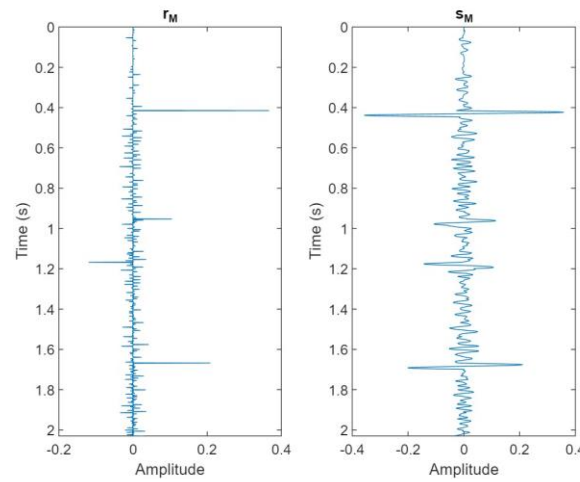

Figure 7 The primary and first order multiples reflection amplitudes corresponding to each two-way travel time, r_M and the synthetic seismogram, s_M produced after its convolution with the source wavelet.

The quality factor, *Q*, a value characterizing attenuation were plotted against depth and two-way travel time in Figure 8. In the same figure, the attenuation or amplitude loss was also shown corresponding to the two-way travel time.

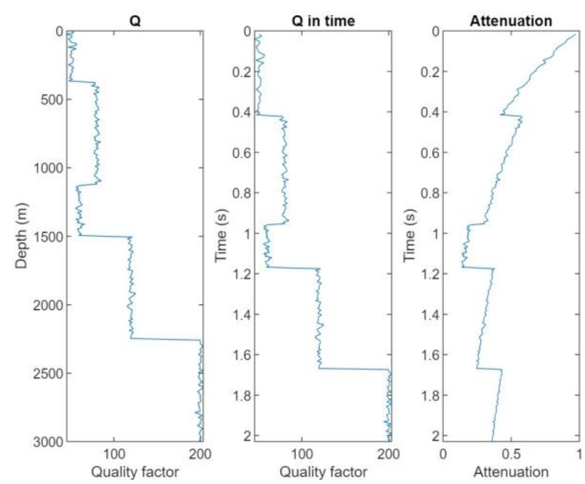

Figure 8 The quality factor, *Q*, plotted against depth and two-way travel time and the corresponding attenuation.

Applying the attenuation effect, the primary and first order multiple reflection amplitudes corresponding to its two-way travel time, r_Q, and its seismic trace, s_Q, after convolution with the source wavelet is shown in Figure 9.

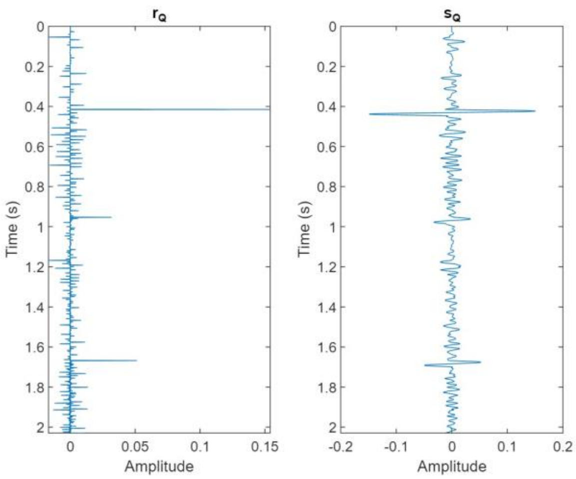

Figure 9 The primary and first order multiples reflection amplitudes corresponding to each two-way travel time and attenuation, r_Q and the synthetic seismogram, s_M produced after its convolution with the source wavelet.

Discussion

In Figure 4, the impedance and reflection coefficient were plotted against depth. As discussed earlier, the reflection coefficient value depends on the change in acoustic impedance. The greater the change in impedance, the greater the reflection coefficient or the 'spike' in the plot.

After the process of convolution between the reflectivity, *r*, and the source wavelet, *w*, the spikes of reflection coefficient in r_R were replaced by the shape of the source wavelet as shown in Figure 5, 6, 7 and 9.

The amplitude of the primary reflection in Figure 6 is a little bit smaller than the amplitude of the reflection coefficient in Figure 5. This is because the primary reflection amplitudes calculation considers the transmission coefficient between layers before reaching the specified interface.

Comparing Figure 6 and 7, they both exhibit the same amplitude against two-way travel time even though in Figure 7 it added the amplitudes of the first order multiples reflection with the amplitudes of the primary reflection. This shows that the first order multiple reflection amplitudes were much smaller in magnitude than the primary reflection amplitudes.

In Figure 8, as the seismic waves propagates through a deeper depth (longer two-way travel time), the attenuation experienced by the wave increases. This can be due to the geometric spreading of wave or energy losses throughout the propagation.

After applying the effect of attenuation towards the amplitudes of the primary and first order multiples reflection, the final amplitudes decrease accordingly. This can be seen by comparing Figure 7 and 9. These aligned with our theory which implies that attenuation would cause amplitude losses for the reflected waves.

Conclusion

The process of convolution of reflectivity, *r*, with the source wavelet, *w*, gave us the seismic trace, *s*, or synthetic seismogram successfully. The "spikes" or amplitudes of the reflections were replaced by the shape of the chosen wavelet as seen in Figure 5, 6, 7 and 9. The first order multiple reflection amplitude is calculated to be very much smaller than the primary reflection amplitude. As depth travelled by seismic waves increases, the attenuation increases. Finally, the attenuation effect did decrease the amplitude of the reflected waves due to the amplitude loss.

References

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