Anamorphic Encryption, Revisited

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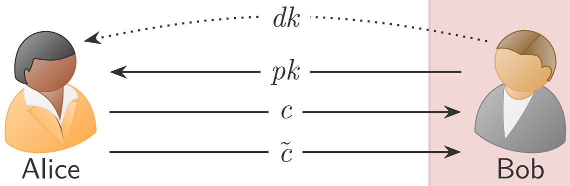
EUROCRYPT 2024
May 27, Zurich, Switzerland

(Receiver-)Anamorphic Encryption [Persiano et al., EUROCRYPT 2022]

$c \leftarrow \textbf{Enc}(pk, m)$

$\tilde{c} \leftarrow \textbf{aEnc}(dk, m', \widehat{m})$



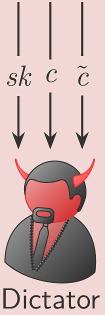
$(sk, pk) \leftarrow \textbf{Gen}()$

$m := \textbf{Dec}(sk, c)$

$dk \leftarrow \textbf{aGen}(sk, pk)$

$\widehat{m} := \textbf{aDec}(dk, \tilde{c})$

$m' := \textbf{Dec}(sk, \tilde{c})$



$m := \textbf{Dec}(sk, c)$

$m' := \textbf{Dec}(sk, \tilde{c})$

Bob uses a *well-established* PKE $\mathbf{\Pi} = (\textbf{Gen}, \textbf{Enc}, \textbf{Dec})$

Suddenly, Bob's country is led by a **dictator** D!

Bob can still use $\mathbf{\Pi}$, but must surrender sk to D

Use **anamorphic extension** $\mathbf{\Sigma} = (\textbf{aGen}, \textbf{aEnc}, \textbf{aDec})$

With **double key** dk, Alice embeds **covert message** $\widehat{m}$

Decoupling Keys & Security

In Persiano et al., double key dk was bound to key pair (sk, pk): $(sk, pk, dk) \leftarrow \textbf{aGen}()$

Limitation: impossible to associate a new double key to an *already deployed* key pair

We redefine **aGen** so that Bob can *later* associate $dk \leftarrow \textbf{aGen}(sk, pk)$ to his key pair

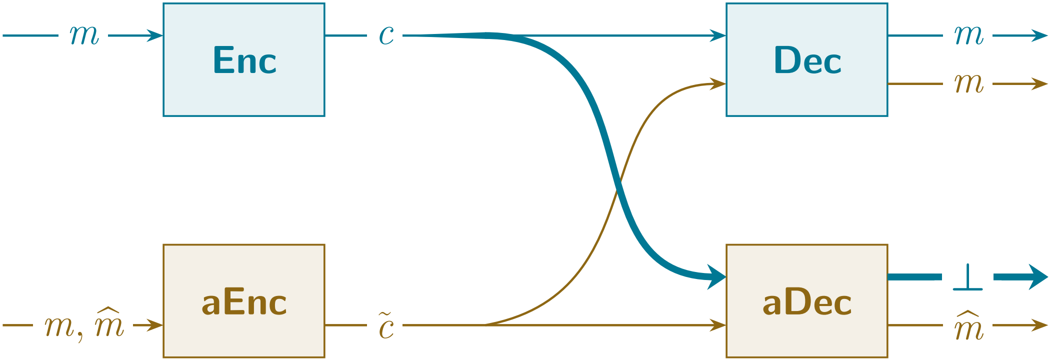
Advantages: can associate *multiple* double keys to a key pair and enables **deniability**

Recall the two modes Alice and Bob can use to communicate:

- **Normal:** $c \leftarrow \textbf{Enc}(pk, m)$; $m := \textbf{Dec}(sk, c)$
- **Anamorphic:** $\tilde{c} \leftarrow \textbf{aEnc}(dk, m, \widehat{m})$; $\widehat{m} := \textbf{aDec}(dk, \tilde{c})$, $m := \textbf{Dec}(sk, \tilde{c})$

Security: The two modes must be indistinguishable: $\tilde{c} \approx c$! **Is this all?**

Using Anamorphic Encryption



This case was not considered! Need to signal *"no covert message"* $\Longrightarrow$ **Robustness**

Why Robustness?

- **Functionality:** Bob might use $\mathbf{\Pi}$ *regularly* and $\mathbf{\Sigma}$ *sporadically*

  Therefore, more often than not: *ciphertexts carry no (intentional) covert message!*

  When Bob sees “garbage” covert messages, he could guess they were not meant ...

  Is this satisfactory? **No!**

- **Security:** it could get even worse!

  Without robustness, D might find out that Bob has established a covert channel!

  1. Send encryption of random message to Bob
  2. If D is lucky, the covert message is not “garbage” and Bob detectably reacts!

Construction Σ_1: A Naive Robust Scheme

Keep $\widehat{\mathcal{M}}$ small (poly. size), share key K of PRF F as part of double key dk, and then:

- **Alice:** map $\widehat{m} \in \widehat{\mathcal{M}}$ to $r \in \mathcal{R}$ via F_K and counter `ctr`, use r to encrypt m into $\tilde{c}$:

  $$\mathbf{aEnc}(dk, m, \widehat{m}) := \mathbf{Enc}(pk, m; F_K(\texttt{ctr} \| \widehat{m}))$$

- **Bob:** decrypt $\tilde{c}$ into m, and check which $\widehat{m} \in \widehat{\mathcal{M}}$ yields $\tilde{c}$:

  $$\mathbf{aDec}(dk, \tilde{c}) := \{ \text{let } m := \mathbf{Dec}(sk, \tilde{c});$$
  $$\text{find } \widehat{m} \text{ s.t. } \mathbf{Enc}(pk, m; F_K(\texttt{ctr} \| \widehat{m})) = \tilde{c} \text{ or return } \perp; \}$$

Problem: Alice and Bob need to keep **synchronized** counters and **aDec** uses **Dec**!

Solution: use PKEs with a special property: **Selective Randomness Recoverability**

Selective Randomness Recoverability (SRR)

PKE scheme $\mathbf{\Pi} = (\textbf{Gen}, \textbf{Enc}, \textbf{Dec})$ is SRR if the following conditions are met:

(i) Randomness space $\mathcal{R}$ must form a group with some operation $\star$

(ii) Ciphertexts *"have two parts"*: for $c := \textbf{Enc}(pk, m; r)$ we want $c = (A, B)$ where:

- Part A depends on pk, m, and r: $A = \alpha(pk, m, r)$
- Part B depends **only** on r: $B = \beta(r)$

(iii) Can compute $\beta(a)$ from $\beta(a \star b)$ and b:

- There exists an efficiently computable function γ s.t. $\gamma(\beta(a \star b), b) = \beta(a)$

Both **ElGamal** and **Cramer-Shoup** are SRR

Construction Σ_2: Using an SRR Scheme

Keep $\widehat{\mathcal{M}}$ small (poly. size), share key K of PRF F as part of double key dk, and then:

- **Bob:** precompute β^{-1} in *table* $\mathbf{T}$: set $\mathbf{T}[\beta(\widehat{m})] := \widehat{m}$ for each $\widehat{m} \in \widehat{\mathcal{M}}$
- **Alice:** use $F_K(\texttt{ctr})$ as otp for $\widehat{m}$ and use result as r to enc. m into $\tilde{c} = (A, B)$:

$$\textbf{aEnc}(dk, m, \widehat{m}; \texttt{ctr}) := \textbf{Enc}(pk, m; \widehat{m} \star F_K(\texttt{ctr}))$$

- **Bob:** use F_K and γ to extract $\widehat{m}$ from B:

$$\textbf{aDec}(dk, (A, B); \texttt{ctr}) := \mathbf{T}[\gamma(B, F_K(\texttt{ctr}))] \qquad [\textbf{Dec} \text{ not needed!}]$$

Still need to keep **synchronized** counters!

Construction Σ_3: Getting Rid of Synchronization

Idea: pick random $\mathtt{ctr}$, until can *partially* extract $\mathtt{ctr}$ from B via some function δ

aEnc$(dk, m, \widehat{m})$:

1. Pick u.a.r. $(\mathtt{x}, \mathtt{y}) \in [\sigma] \times [\tau]$, set $\mathtt{ctr} := \mathtt{x} \| \mathtt{y}$, $r := \widehat{m} \star F_K(\mathtt{ctr})$, and $B := \beta(r)$
2. Repeat until $\delta(B) = \mathtt{x}$, let r^* be the such first r
3. Return $(A, B) :=$ **Enc**$(pk, m; r^*)$

aDec$(dk, (A, B))$:

1. Set $\mathtt{x} := \delta(B)$
2. For each possible value $\mathtt{y}$: if $\widehat{m} := \mathbf{T}[\gamma(B, F_K(\mathtt{x} \| \mathtt{y}))] \neq \perp$, return $\widehat{m}$
3. If no such $\mathtt{y}$ found, return $\perp$

Security-Efficiency Trade-Off for $\mathbf{\Sigma}_3$

Security of $\mathbf{\Sigma}_3$: can safely transmit *at most* $\sigma \cdot \tau$ covert messages

Efficiency of $\mathbf{\Sigma}_3$:

- **aEnc** takes σ tries *in expectation*
- **aDec** takes *at most* τ tries

Trade-off:

- For **aEnc** and **aDec** to be *efficient*, σ and τ must be small (poly.)
- This means, the limit on transmitted covert messages $\sigma \cdot \tau$ will also be small

Mitigation: in our new model, we can simply *update the double key!*

Conclusions

- Our abstract scheme can be made concrete for **ElGamal** and **Cramer-Shoup**
- We also show how to make (fully) rand. recoverable schemes robustly anamorphic
  - Use small subset of randomness as covert message space (concrete for **RSA-OAEP**)
- **Open questions:**
  - Is the trade-off between security and efficiency for Σ_3 optimal?
  - Are there more robust anamorphic schemes? (see next talk 😉)

Thank You For Your Attention!

Appendix: The Evolution of Anamorphic Encryption

- Persiano et al. [EUROCRYPT 2022]: first receiver- and sender-anam. schemes
- Kutyłowski et al. [CRYPTO 2023]: sender-anamorphic signatures
- Kutyłowski et al. [PoPETs 2023(4)]: more receiver-anamorphic PKE schemes
- Wang et al. [ASIACRYPT 2023]: sender-anam. **robustness** (inspired by our work)
- Our work [EUROCRYPT 2024]: receiver-anamorphic **robustness**
- Catalano et al. [EUROCRYPT 2024]: receiver-anam. homomorphic encryption

  \+ new receiver-anamorphic **robust** schemes
- More to come ...

Appendix: Deniability

Why does decoupling key-pair (sk, pk) and double key dk enable **deniability**?

Assume $dk \leftarrow$ **aGen**(pk) instead of $dk \leftarrow$ **aGen**(sk, pk) (true for all our constructions)

Then, a malicious sender holding dk *cannot* convince D that Bob also holds dk:

- The double key dk can be generated either by the sender or the receiver
- The sender can simulate dk and some ciphertexts, without the help of the receiver

This is **not true** for Persiano et al.'s anamorphic Naor-Yung transform:

- The malicious sender hands dk to the dictator
- The dictator can then detect whether key-pair was deployed in *anamorphic mode*

Appendix: An SRR Scheme

ElGamal on cyclic group $\mathbb{G} = \langle g \rangle$ of order q is SRR:

(i) $\mathcal{R} = \mathbb{Z}_q$, and $\langle \mathbb{Z}_q; \oplus \rangle$ is a group with $\oplus$ addition modulo q

(ii) With $A = \alpha(pk, m, r) = m \cdot pk^r$ and $B = \beta(r) = g^r$: **Enc**$(pk, m; r) = (A, B)$

(iii) With $\gamma(a, b) := a \cdot g^{-b}$: $\gamma(\beta(a \oplus b), b) = \gamma(g^{a \oplus b}, b) = g^{a \oplus b} \cdot g^{-b} = g^a = \beta(a)$

Analogously for **Cramer-Shoup**

Appendix: Correctness and Robustness of Σ_2

Correctness: with $(A, \beta(\widehat{m} \star F_K(\texttt{ctr}))) := \textbf{aEnc}(dk, m, \widehat{m}; \texttt{ctr})$:

$$\textbf{aDec}(dk, (A, \beta(\widehat{m} \star F_K(\texttt{ctr}))); \texttt{ctr}) = \mathbf{T}[\gamma(\beta(\widehat{m} \star F_K(\texttt{ctr})), F_K(\texttt{ctr}))]$$

$$= \mathbf{T}[\beta(\widehat{m})] = \widehat{m}$$

Robustness: with $(A, \beta(r)) := \textbf{Enc}(pk, m; r)$, for $r \xleftarrow{\$} \mathcal{R}$:

$$\textbf{aDec}(dk, (A, \beta(r)); \texttt{ctr}) = \mathbf{T}[\gamma(\beta(r), F_K(\texttt{ctr}))] = \mathbf{T}[\beta(r \star F_K(\texttt{ctr})^{-1})] \overset{(*)}{\approx} \bot$$

$(*)$: w.o.p., since $r \star F_K(\texttt{ctr})^{-1} \notin \widehat{\mathcal{M}}$ with probability $1 - |\widehat{\mathcal{M}}|/|\mathcal{R}|$