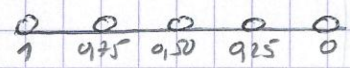
① 14.6

1) $l = 1\ m$

$m = 1\ kg$

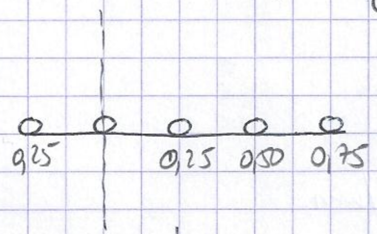


a) $I = \sum_{i=1}^{5} m_i x_i^2$

$= 1\times 0 + 1\times 0{,}25^2 + 1\times 0{,}50^2 + 1\times 0{,}75^2 + 1\times 1^2 = 1{,}88\ kg\cdot m^2$

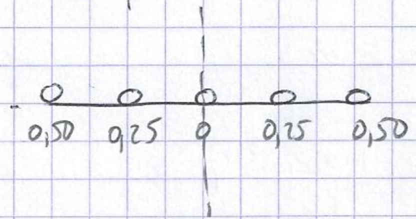
b) $I = 0{,}25^2 + 0{,}25^2 + 0{,}50^2 + 0{,}75^2$

$= 0{,}94\ kg\cdot m^2$



c) $I = 0{,}25^2 \times 2 + 0{,}50^2 \times 2$

$= 0{,}63\ kg\cdot m^2$



d) $I = mk^2$

a) $1{,}88 = 5 \times k^2 \Leftrightarrow k = 0{,}61\ m$

b) $0{,}94 = 5 \times k^2 \Leftrightarrow k = 0{,}43\ m$

c) $0{,}63 = 5 \times k^2 \Leftrightarrow k = 0{,}35\ m$

e) Teorema de Steiner: $I = I_{cm} + Md^2$

$I_c = \int r^2\,dm = \int x^2\,dm = \frac{M}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} x^2\,dx = \frac{M}{L}\left[2\times \frac{\frac{L^3}{8}}{3}\right] = \frac{ML^2}{12}$

a) $1{,}88 = 0{,}63 + 5\times 0{,}50^2$ ✓

b) $0{,}94 = 0{,}63 + 5\times 0{,}25^2$ ✓

2) $M_{haste} = 0{,}20\,kg$ $\quad I_{cm\ haste} = \frac{1}{12}ML^2 = 0{,}017\ kg\cdot m^2$

a) $I_{haste} = 0{,}017 + 0{,}20\times 0{,}50^2 = 0{,}067\ kg\,m^2$

$I = 1{,}88 + 0{,}067 = 1{,}95\ kg\,m^2$

b) $I_{haste} = 0{,}017 + 0{,}20\times 0{,}25^2 = 0{,}0295\ kg\,m^2$

$I = 0{,}94 + 0{,}0295 = 0{,}9695\ kg\cdot m^2$

c) $I_{haste} = 0{,}017$

$I = 0{,}63 + 0{,}017 = 0{,}647\ kg\cdot m^2$

3)

$m_C = 12\ u = 12 \times 1{,}66 \times 10^{-27}$.

$m_O = 16\ u = 16 \times 1{,}66 \times 10^{-27}$

$r = 1{,}13 \times 10^{-10}\ m$

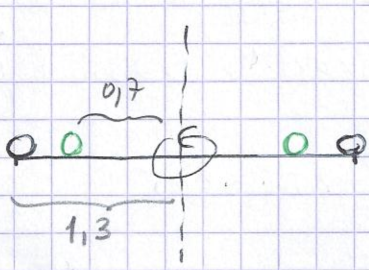
$$I = 16 \times 1{,}66 \times 10^{-27} \times (1{,}13 \times 10^{-10})^2 \times 2 = 6{,}78 \times 10^{-46}\ kg \cdot m^2$$

4) $m = 25\ kg$

$l = 2{,}6\ m$

$M = 10\ kg$

$w = 5\ rev/min$



$I_{barra} = \frac{ML^2}{12}$

a) Momento de inércia mantém-se: $I_i w_i = I_f w_f$

$$I_i = \frac{10 \times 2{,}6^2}{12} + 25 \times 1{,}3^2 \times 2 = 90{,}13\ kg \cdot m^2$$

$$I_f = \frac{10 \times 2{,}6^2}{12} + 25 \times 0{,}7^2 \times 2 = 30{,}13\ kg \cdot m^2$$

$$w_i = \frac{5 \times 2\pi}{60} = 0{,}52\ rad/s$$

logo, $90{,}13 \times 0{,}52 = 30{,}13 \times w_f$

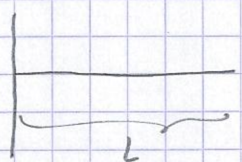
$w_f = 1{,}56\ rad/s$

b) $\Delta_{EC} = E_{cf} - E_{ci}$

$$= \frac{1}{2} I w_f^2 - \frac{1}{2} I w_i^2 = \frac{1}{2} \times 30{,}13 \times 1{,}56^2 - \frac{1}{2} \times 90{,}13 \times 0{,}52^2$$

$$= 24{,}48\ J$$

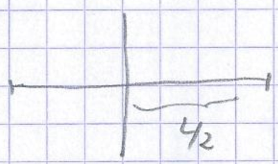
5) a)



$$I = \int r^2\, dm = \frac{M}{L} \int_0^L x^2\, dx = \frac{M}{L} \left[\frac{x^3}{3} \right]_0^L =$$

$$= \frac{M}{L} \frac{L^3}{3} = \frac{ML^2}{3}$$

b)



$$I = \frac{M}{L} \int_{-L/2}^{L/2} x^2\, dx = \frac{M}{L} \left[\frac{x^3}{3} \right]_{-\frac{L}{2}}^{L/2}$$

$$= \frac{M}{L} \left[\frac{\frac{L^3}{8}}{3} + \frac{\frac{L^3}{8}}{3} \right] = M \left[\frac{L^2}{24} + \frac{L^2}{24} \right] = \frac{ML^2}{12}$$

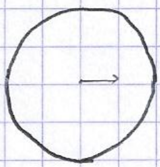
6)

$$I = \int r^2\, dm = \frac{2M}{R^2}\int_0^R x^2 \times x\, dx = \frac{2M}{R^2}\left[\frac{x^4}{4}\right]_0^R$$

$$dm = \frac{M}{\cancel{\pi} R^2} \times 2\cancel{\pi} x\, dx$$

$$= \frac{2M}{R^2}\,\frac{R^4}{4} = \frac{MR^2}{2}$$

7)



$\tau = 10\,N.m$ $\qquad R = 0{,}6\,m$

$M = 100\,kg$

$w = 175\,rad/s$

$\tau = I\alpha$

$$I = \frac{MR^2}{2} = \frac{100 \times 0{,}6^2}{2} = 18\ kg\cdot m^2$$

$$\tau = 10 \iff 18 \,.\, \alpha = 10 \iff \alpha = 0{,}56\ rad/s^2$$

mas $w_f = 0\ rad/s$

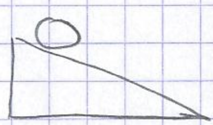
$$\alpha = \frac{w_f - w_i}{t} \iff 0{,}56 = \frac{|0 - 175|}{t}$$

$$\iff t = 312{,}5\ s \rightarrow \frac{312{,}5}{60} = 5{,}21\ min$$

8) $r = 1\,cm = 0{,}01\,m$

$m = 5\,g = 0{,}005\,kg$

$w = 6\,rev/s = 6 \times 2\pi = 12\pi\ rad/s$



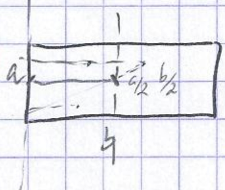
a) $E_{c_r} = \frac{1}{2} I w^2 = \frac{1}{2} \times \frac{MR^2}{2} \times (12\pi)^2 = 1{,}78 \times 10^{-4}\ J$

b) $E_{c_T} = \frac{1}{2} m v^2 = \frac{1}{2} \times 0{,}005 \times (0{,}12\pi)^2 = 3{,}55 \times 10^{-4}\ J$

$v = wR = 12\pi \times 0{,}01 = 0{,}12\pi\ m/s$

c) $E_c = E_{c_r} + E_{c_T} = 5{,}33 \times 10^{-4}\ J$

9)



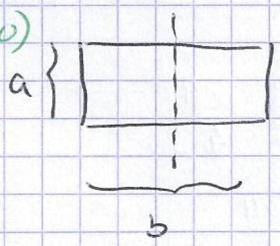
$\cancel{I_{CM} = \frac{1}{12} M(a^2+b^2)}$

$I_{CM} = \frac{b^2}{12} M$ (vi num livro)

$$I = I_{CM} + Md^2$$

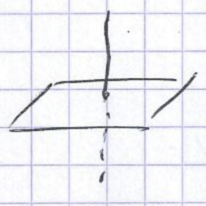
$$= \frac{b^2}{12}M + M\left(\frac{b}{2}\right)^2 = \frac{b^2}{12}M + \frac{3b^2}{12}M = \frac{Mb^2}{3}$$

10)



$$I = \frac{1}{12} M b^2$$ (livro)

11)



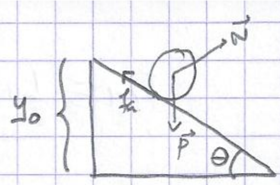
$$I = \frac{M(a^2+b^2)}{12}$$

12)

13)

3

14) esfera : $I = \frac{2}{5} MR^2$



soluções erradas! Está bem com os slides

$$E_{M_i} = E_{M_f}$$

$$E_{pg_i} + E_{c_{R_i}} + E_{c_{T_i}} = E_{pg_f} + E_{c_{R_f}} + E_{c_{T_f}}$$

$$Mgy_0 + \frac{1}{2} I w_i^2 + \frac{1}{2} m v_i^2 = Mg \times 0 + \frac{1}{2} I w_f^2 + \frac{1}{2} m v_f^2$$

$v_f = v$

$$\cancel{M} g y_0 = \frac{1}{\cancel{2}} \times \frac{2}{5} \cancel{M} \cancel{R^2} \times \frac{v^2}{\cancel{R^2}} + \frac{1}{2} \cancel{M} v^2$$

$$g y_0 = \frac{v^2}{5_{(2)}} + \frac{v^2}{2_{(5)}} \Leftrightarrow g y_0 = \frac{7v^2}{10}$$

$$\Leftrightarrow v = \sqrt{\frac{10}{7} g y_0}$$

cilindro : $I = \frac{MR^2}{2}$

$$Mgy_0 = \frac{1}{2} \times \frac{M\cancel{R^2}}{2} \times \frac{v^2}{\cancel{R^2}} + \frac{Mv^2}{2}$$

$$g y_0 = \frac{v^2}{4} + \frac{v^2}{2_{(2)}}$$

$$4 g y_0 = 3v^2 \Leftrightarrow v = \sqrt{\frac{4}{3} g y_0}$$

Anel : $I = MR^2$

$$M g y_0 = \frac{1}{2} M\cancel{R^2} \times \frac{v^2}{\cancel{R^2}} + M \frac{v^2}{2}$$

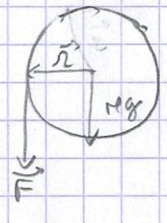
$$g y = \frac{v^2}{2} + \frac{v^2}{2}$$

$$v = \sqrt{gh}$$

15) $R = 0{,}5\ m$ $\quad I_{CM} = \frac{1}{2} M R^2$ $\quad F = 9{,}8\ N$

$M = 20\ Kg$

$t = 2\ s$ $\quad \alpha = ?$ $\quad w = ?$



F tangente

$\vec{\tau} = \vec{r} \times \vec{F}$

$\tau = r \times F \times \operatorname{sen} \theta$

$\tau = 0{,}5 \times 9{,}8 \times \operatorname{sen} 90°$

$\tau = 4{,}9\ N.m$

Mas por outro lado $\quad \tau = I \alpha$

$4{,}9 = \frac{1}{2} \times 20 \times 0{,}5^2 \times \alpha$

$\alpha = 1{,}96\ rad/s^2$

Considerando $wi = 0$

$\alpha = \frac{wf - wi}{t} \iff 1{,}96 \times 2 = wf \iff wf = 3{,}92\ rad/s$

16) $M_1 = 1\ Kg$ $\quad Mg - T = F$ $\quad a_t = r\alpha$

o enunciado sugere que F aplicada na corda é igual a T

$M_1 g - T = M_1 r \alpha$

$T = M_1 g - M_1 r \alpha$ N.m.

$\tau = r . T . \cos 90°$

$\tau = 0{,}5\,(Mg - Mr\alpha) . \operatorname{sen} 90°$

$\tau = 4{,}9 - 0{,}25\,\alpha$ N.m

Mas $\quad \tau = I \alpha$

$4{,}9 - 0{,}25 . \alpha = \frac{1}{2} \times 20 \times 0{,}5^2\, \alpha$

$4{,}9 - 0{,}25\,\alpha = 2{,}5\,\alpha$

$2{,}75\,\alpha = 4{,}9$

$\alpha = 1{,}8\ rad/s^2$

(4)

17) $M_{disco} = 20\ kg$ $\qquad F = Mg$

$R = 0,5\ m$ $\qquad F = 20 \times 9,8 = 196\ N$

$\tau = 0,5 \times 196 \times \text{sen}\,90°$

$\tau = 98\ N.m$

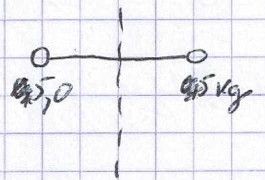
Mas $\quad \tau = I\alpha \qquad\qquad I = I_{CM} + Md^2$

$98 = 7,5\alpha \qquad\qquad I = \frac{1}{2}MR^2 + M \times R^2 = \frac{3}{2}MR^2$

$\alpha = 13,07\ rad/s^2 \qquad\qquad I = \frac{3}{2} \times 20 \times 0,5^2 = 7,5\ kgm^2$

$a = R\alpha \iff a = 0,5 \times 13,07$

$\iff a = 6,54\ rad/s^2$

18) 

~~w=~~ $\quad 1\ rot \rightarrow 2,0\ s$

OMG
a), b) e
c)
ld

$I = 5,0\ kg/m^2$

$l_i = 0,90\ m \quad l_f = 0,15\ cm$

$w = ?$

$w_i = \frac{1}{2}\ rot/s = \frac{1}{2} \times 2\pi\ rad/s = \pi\ rad/s$

O momento angular mantém-se: $I_i w_i = I_f w_f$

$I_i = 5 + 5 \times 0,9^2 \times 2 = 13,1\ kg.m^2$

$I_f = 5 + 5 \times 0,15^2 \times 2 = 5,225\ kg.m^2$

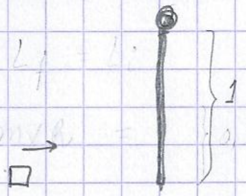
subst.: $\quad 13,1 \times \pi = 5,225\ w_f$

$w_f = 7,88\ rad/s$

$m = m_1 = m_2$

O ponto de rotação mantém-se

24)



$$L_f = L_i$$

$$I w_f = m v r$$

$$\left(\frac{mL^2}{12}+\frac{mL^2}{4_{(3)}}+\frac{mL^2}{(12)}\right)\times w_f = m v L$$

$$\frac{16}{12}L^2 w_f = vL$$

$$w_f = v\times\frac{12}{16L}$$

$$w_f = \frac{3}{4l}v$$

25) a) $J = I\alpha$ $\qquad I = \frac{4}{3}ml^2$

$$\alpha = \frac{J}{I}$$

$$\alpha = \frac{J}{\frac{4}{3}ml^2} = \frac{3J}{4ml^2}$$

b) $a_t = \alpha R = \frac{3J}{4ml}$

$$v_f^2 = v_i^2 - 2aS$$

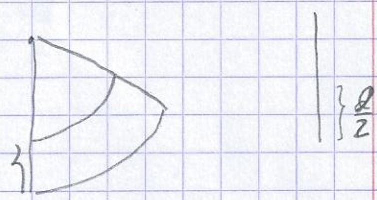
$$S = \frac{v^2}{2a}$$

$$S = \frac{v^2}{\frac{6J}{4ml}}$$

$$S = \frac{4mlv^2}{6J}$$

$$S = \frac{2mlv^2}{3J}$$

c)



$$y_{CM} = \frac{m_1 \frac{l}{2}}{m_1+m_2} = \frac{l}{2}\times\frac{1}{2} = \frac{l}{4}$$

$$l = l - \frac{l}{4} = \frac{3}{4}l$$

logo

$$S_1 = \frac{2m\left(\frac{3}{4}l\right)v^2}{3J} = \frac{3}{4}S$$

(6) 27)

a) $L_i = L_f$

$m v R = I w_f$ Supondo que o homem se encontra no centro da roda

$-m_H v_H R = \frac{m_m R^2}{2} w_f$

$-\frac{2 m_H v_H}{m_m R} = w_f$

b) $W_H = \Delta E_c = \frac{1}{2} m_H v_H^2 + \frac{1}{2} I w_m^2$

$= \frac{1}{2} m_H v_H^2 + \frac{1}{2} \times \frac{m_m}{2} \times R^2 w^2$

$= \frac{1}{2} m_H v_H^2 - \frac{1}{4} m_m R^2 v^2$