

# P-values

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We consider a non-parametric test with an unknown distribution of values (in our case, the distribution of  $\eta$  by Gross et al., 2013), under the assumption that time series are not cross-correlated. We don't want to assume that the distribution is symmetric around its mean, nor that it is centered on 0. We don't know if there is compensation or synchrony, so we want a two-tailed test.

There are two methodological questions associated with this analysis (even though they are intricate):

- How to write the p-value exactly?
- How to compare them to the Type I error rate?

## P-value definition

Based on [1]:

$$p_1 = \frac{\#(|\eta^*| \geq |\eta_{emp}|) + 1}{\#(\eta^*) + 1}$$

where  $\eta^*$  are the values of the Gross index under  $H_0$  (uncorrelated surrogates) and  $\eta_{emp}$  is the observed value/  
Based on [2]:

$$p_2 = 2 \min (Pr(\eta^* \leq \eta_{emp}), Pr(\eta^* \geq \eta_{emp}))$$

where we can assume that  $Pr(\eta^* \leq \eta_{emp}) = \frac{\#(\eta^* \leq \eta_{emp}) + 1}{\#(\eta^*) + 1}$  (and conversely).

## Test

We use a Beta(2,5) distribution which we shift to the left by 0.6 (Fig 1). It seems that  $p_2$  is more accurate (?).

## References

Gross, K., B. J. Cardinale, J. W. Fox, A. Gonzalez, M. Loreau, H. Wayne Polley, P. B. Reich, and J. van Ruijven, 2013. Species richness and the temporal stability of biomass production: a new analysis of recent biodiversity experiments. *The American Naturalist* **183**:1–12.

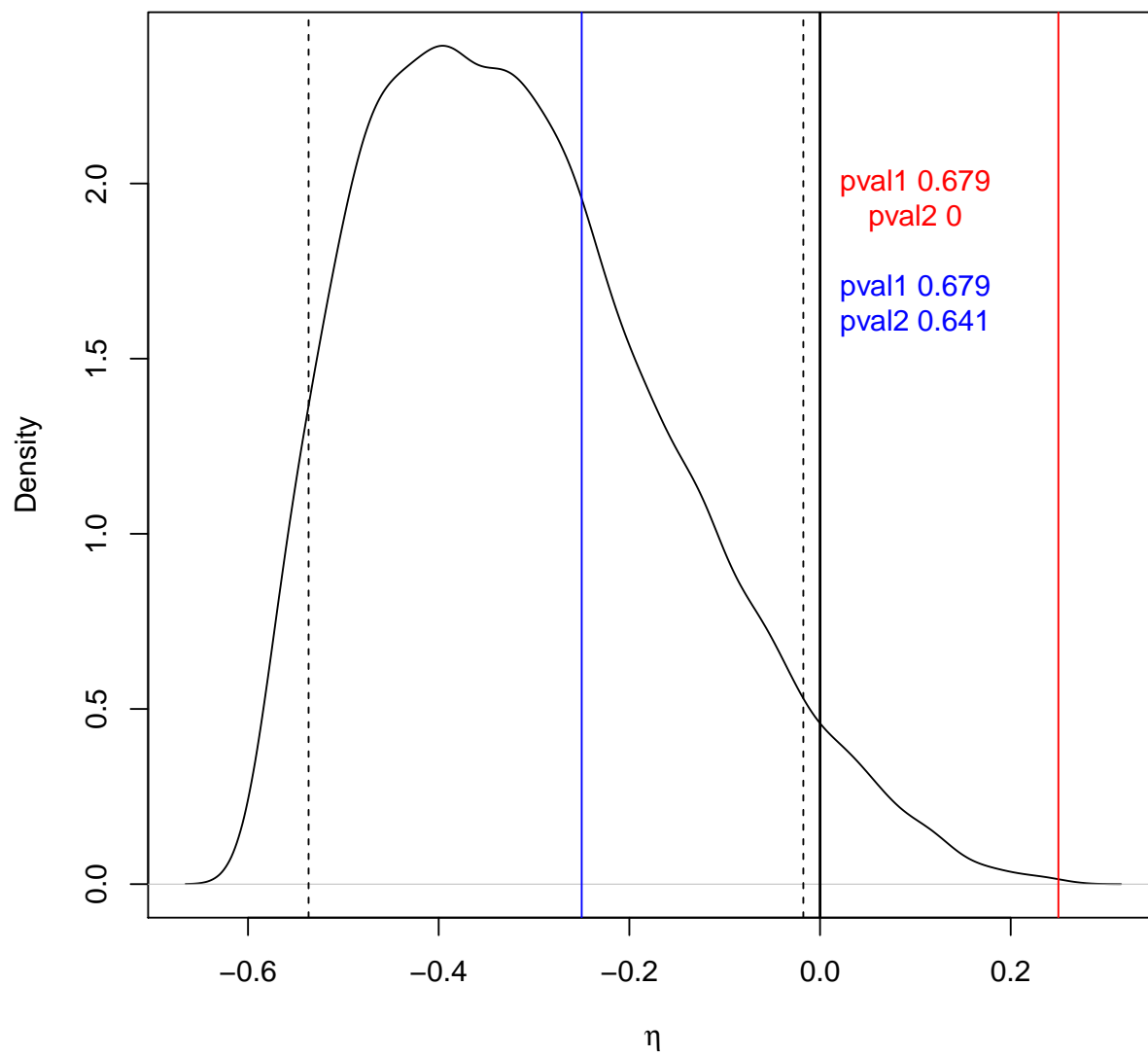


Figure 1: Distribution of the beta law and examples of pvalues associated with potential values of  $\eta$

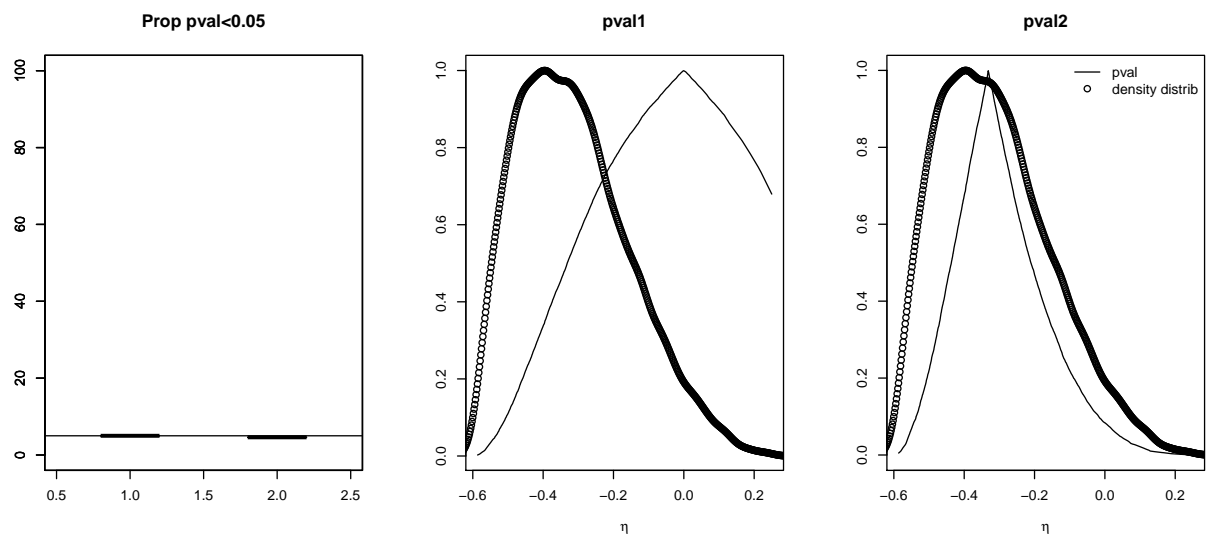


Figure 2: Comparison between pvalues and a 5% threshold