In the grading of the exercises special attention will be paid to check that answers are substantiated and that the procedure of calculations is well documented. When results are achieved using a calculator or a pc it should be noted in the paper. All 6 problems are weighted equally in the grading. Your solution to the problem set can be written in Danish or English as you prefer.

PROBLEM 1.
Let A be a 2×2 matrix with the property that

$$A\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \end{bmatrix} \quad \text{and } A\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

1. Determine the values of the four elements, a_{11}, a_{12}, a_{21} and a_{22} of A.

PROBLEM 2.
A matrix A is given by

$$A = \begin{bmatrix} 1 & -1 & 3 & 5 \\ -1 & -3 & 1 & -1 \\ 2 & 6 & -2 & 2 \end{bmatrix}$$

1. Determine bases for the null space, column space and row space of A.
2. How many solutions are there to the homogenous equation $A\mathbf{x} = \mathbf{0}$?
3. Find a vector $\mathbf{b}$ such that $A\mathbf{x} = \mathbf{b}$ can be solved.

PROBLEM 3.
Consider the matrix equation $A\mathbf{x} = \mathbf{b}$ where A and $\mathbf{b}$ are given by

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 2 & -1 & 4 \\ 0 & 1 & -1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

1. The equation $A\mathbf{x} = \mathbf{b}$ can not be solved. Show why.

Two guesses of approximate solutions to $A\mathbf{x} = \mathbf{b}$ are

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \quad \text{and} \quad \mathbf{x}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

2. Determine which of the two proposed solutions is the best one in the least squares sense.

PROBLEM 4.

Inner products can be defined with weight functions. One example of this is continuous functions defined on the interval $[a, b]$. Here, the weigthed inner product is given by

$$< f, g >= \int_a^b f(x)g(x)w(x)dx$$

Where $w(x)$ is the weight function. Here we consider the space of all continuous functions $C[0, 1]$ with $w(x) = x$.

Let two vectors be given by

$$f(x) = e^{-x} \qquad \text{and} \qquad g(x) = e^{-2x}$$

1. Show that the two vectors are linearly independent.
2. Determine if the two vectors are orthogonal with respect to the above defined inner product.

Part of the following table of integrals might be useful in the above problem

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$
$$\int x e^{ax} dx = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right)$$
$$\int x^2 e^{ax} dx = \frac{e^{ax}}{a}\left(x^2 - \frac{2x}{a} - \frac{2}{a^2}\right)$$

PROBLEM 5.
A set of three coupled differential equations are given by

$$\begin{aligned} x_1'(t) &= 4x_1(t) - x_2(t) + x_3(t) \\ x_2'(t) &= -2x_1(t) + 5x_2(t) + x_3(t) \\ x_3'(t) &= 2x_1(t) - x_2(t) + 3x_3(t) \end{aligned}$$

1. Rewrite as three decoupled differential equations.

PROBLEM 6.
In case 3, Computer Graphics in Automotive Design, homogeneous coordinates were introduced. With homogenous coordinates a point (x, y, z) in $\mathbb{R}^3$ is written as a 4-dimensional vector $(x, y, z, 1)$.

1. Explain the purpose of the 1 in $(x, y, z, 1)$.