In the grading of the exercises special attention will be paid to check that answers are substantiated and that the procedure of calculations is well documented. When results are achieved using a calculator or a pc it should be noted in the paper. All 6 problems are weighted equally in the grading. Your solution to the problem set can be written in Danish or English as you prefer.

PROBLEM 1.
Let 3 invertible 2×2 matrices, A, B and C be given by

$$A = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & -2 \\ 1 & 0 \end{bmatrix}$$

1. Calculate the solutions, both algebraic and numerically, to the following five matrixequations.

$$AX = B, \qquad A^2X + B = \mathbf{0}, \qquad AXB = C, \qquad AX + BX = C, \qquad ACX = \mathbf{0}$$

PROBLEM 2.
This problem is built on case 5: Error-Detecting and Error-Correcting codes. Let a vector equation be given by

$$x_1 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

where all vectors are elements in $\mathbb{Z}_2^4$ and x_1, x_2 and x_3 are elements in $\mathbb{Z}_2$.

1. Determine all solutions to the vector equation.

PROBLEM 3.
The difference equation $\mathbf{x}(k+1) = A\mathbf{x}(k)$ has the generel solution $\mathbf{x}(k) = A^k\mathbf{x}(0)$. Consider a 3×3 matrix A with eigenvalues $\lambda_1 = 1.3$, $\lambda_2 = 1$ and $\lambda_3 = 0.7$ with corresponding eigenvectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$.

1. Discuss the behaviour of $\mathbf{x}(k)$ in the limit $k \to \infty$ for an arbitrary vector $\mathbf{x}(0)$.

PROBLEM 4.
Assume that corresponding values of time t and output y have been measured for a system as shown in this table

t	y
0	1
2	6
5	17
6	19

It is assumed that the system can be fitted to a linear model of the form $y_1(t) = \beta_0 + \beta_2 t^2$ or $y_2(t) = \gamma_1 t + \gamma_2 t^2$.

1. Determine the model parameters for the two models.
2. State, with justification, which of the two models best fit the data.

PROBLEM 5.
Let the inner product for matrices in $\mathbb{R}^{m\times n}$ be defined as

$$< A, B >= \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} b_{ij}$$

Two matrices in $\mathbb{R}^{2\times 2}$ are given by

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

1. Calculate $< A, B >$.

A and B span a 2-dimensional vector space, $\mathbb{H}$.

2. Determine an orthogonal basis for $\mathbb{H}$.

PROBLEM 6.
Let a matrix A and a vector $\mathbf{b}$ be given by

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 6 & -6 & 5 \\ 9 & -9 & 10 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

1. Show that $A\mathbf{x} = \mathbf{b}$ does not have a solution.
2. Calculate the least squares solution using the pseudoinverse matrix.