Chapter 4.1

2. a. If $\mathbf{u} = \begin{bmatrix} x \\ y \end{bmatrix}$ is in W, then the vector $c\mathbf{u} = c\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} cx \\ cy \end{bmatrix}$ is in W because $(cx)(cy) = c^2(xy) \geq 0$ since $xy \geq 0$.

b. *Example:* If $\mathbf{u} = \begin{bmatrix} -1 \\ -7 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, then $\mathbf{u}$ and $\mathbf{v}$ are in W but $\mathbf{u} + \mathbf{v}$ is not in W.

8. Yes. The zero vector is in the set H. If $\mathbf{p}$ and $\mathbf{q}$ are in H, then $(\mathbf{p} + \mathbf{q})(0) = \mathbf{p}(0) + \mathbf{q}(0) = 0 + 0 = 0$, so $\mathbf{p} + \mathbf{q}$ is in H. For any scalar c, $(c\mathbf{p})(0) = c \cdot \mathbf{p}(0) = c \cdot 0 = 0$, so $c\mathbf{p}$ is in H. Thus H is a subspace by Theorem 1.

9. The set $H = \text{Span}\,\{\mathbf{v}\}$, where $\mathbf{v} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$. Thus H is a subspace of $\mathbb{R}^3$ by Theorem 1.

13. a. The vector $\mathbf{w}$ is not in the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. There are 3 vectors in the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

b. The set $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ contains infinitely many vectors.

c. The vector $\mathbf{w}$ is in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ if and only if the equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{w}$ has a solution. Row reducing the augmented matrix for this system of linear equations gives

$$\begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ -1 & 3 & 6 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

so the equation has a solution and $\mathbf{w}$ is in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

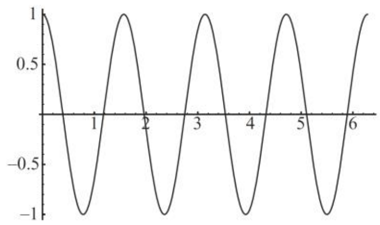
23. a. False. The zero vector in V is the function $\mathbf{f}$ whose values $\mathbf{f}(t)$ are zero *for all* t in $\mathbb{R}$.

b. False. An arrow in three-dimensional space is an example of a vector, but not every arrow is a vector.

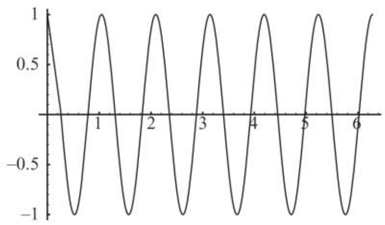
c. False. See Exercises 1, 2, and 3 for examples of subsets which contain the zero vector but are not subspaces.

d. True. See the paragraph before Example 6.

37. **[M]** The graph of $\mathbf{f}(t)$ is given below. A conjecture is that $\mathbf{f}(t) = \cos 4t$.



The graph of $\mathbf{g}(t)$ is given below. A conjecture is that $\mathbf{g}(t) = \cos 6t$.



Chapter 4.2

1. One calculates that $A\mathbf{w} = \begin{bmatrix} 3 & -5 & -3 \\ 6 & -2 & 0 \\ -8 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, so $\mathbf{w}$ is in Nul A.

5. First find the general solution of $A\mathbf{x} = \mathbf{0}$ in terms of the free variables. Since $[A \quad \mathbf{0}] \sim \begin{bmatrix} 1 & -2 & 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & -9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$, the general solution is $x_1 = 2x_2 - 4x_4$, $x_3 = 9x_4$, $x_5 = 0$, with x_2 and x_4 free. So $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_2\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} -4 \\ 0 \\ 9 \\ 1 \\ 0 \end{bmatrix}$, and a spanning set for Nul A is $\left\{\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 9 \\ 1 \\ 0 \end{bmatrix}\right\}$.

7. The set W is a subset of $\mathbb{R}^3$. If W were a vector space (under the standard operations in $\mathbb{R}^3$), then it would be a subspace of $\mathbb{R}^3$. But W is not a subspace of $\mathbb{R}^3$ since the zero vector is not in W. Thus W is not a vector space.

17. The matrix A is a 4×2 matrix. Thus

(a) Nul A is a subspace of $\mathbb{R}^2$, and

(b) Col A is a subspace of $\mathbb{R}^4$.

19. The matrix A is a 2×5 matrix. Thus

(a) Nul A is a subspace of $\mathbb{R}^5$, and

(b) Col A is a subspace of $\mathbb{R}^2$.

21. Either column of A is a nonzero vector in Col A. To find a nonzero vector in Nul A, find the general solution of $A\mathbf{x} = \mathbf{0}$ in terms of the free variables. Since $\begin{bmatrix} A & \mathbf{0} \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, the general solution is $x_1 = 3x_2$, with x_2 free. Letting x_2 be a nonzero value (say $x_2 = 1$) gives the nonzero vector $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, which is in Nul A.

25. **a**. True. See the definition before Example 1.

b. False. See Theorem 2.

c. True. See the remark just before Example 4.

d. False. The equation $A\mathbf{x} = \mathbf{b}$ must be consistent *for every* **b**. See #7 in the table on page 206.

e. True. See Figure 2.

f. True. See the remark after Theorem 3.

27. Let A be the coefficient matrix of the given homogeneous system of equations. Since $A\mathbf{x} = \mathbf{0}$ for $\mathbf{x} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$, **x** is in Nul$A$. Since Nul$A$ is a subspace of $\mathbb{R}^3$, it is closed under scalar multiplication.

Thus $10\mathbf{x} = \begin{bmatrix} 30 \\ 20 \\ -10 \end{bmatrix}$ is also in Nul$A$, and $x_1 = 30$, $x_2 = 20$, $x_3 = -10$ is also a solution to the system of equations.

31. **a**. Let **p** and **q** be arbitary polynomials in $\mathbb{P}_2$, and let c be any scalar. Then

$$T(\mathbf{p}+\mathbf{q}) = \begin{bmatrix} (\mathbf{p}+\mathbf{q})(0) \\ (\mathbf{p}+\mathbf{q})(1) \end{bmatrix} = \begin{bmatrix} \mathbf{p}(0)+\mathbf{q}(0) \\ \mathbf{p}(1)+\mathbf{q}(1) \end{bmatrix} = \begin{bmatrix} \mathbf{p}(0) \\ \mathbf{p}(1) \end{bmatrix} + \begin{bmatrix} \mathbf{q}(0) \\ \mathbf{q}(1) \end{bmatrix} = T(\mathbf{p}) + T(\mathbf{q})$$

and $T(c\mathbf{p}) = \begin{bmatrix} (c\mathbf{p})(0) \\ (c\mathbf{p})(1) \end{bmatrix} = c\begin{bmatrix} \mathbf{p}(0) \\ \mathbf{p}(1) \end{bmatrix} = cT(\mathbf{p})$, so T is a linear transformation.

b. Any quadratic polynomial **q** for which $\mathbf{q}(0) = 0$ and $\mathbf{q}(1) = 0$ will be in the kernel of T. The polynomial **q** must then be a multiple of $\mathbf{p}(t) = t(t-1)$. Given any vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ in $\mathbb{R}^2$, the polynomial $\mathbf{p} = x_1 + (x_2 - x_1)t$ has $\mathbf{p}(0) = x_1$ and $\mathbf{p}(1) = x_2$. Thus the range of T is all of $\mathbb{R}^2$.