Notice: In the grading of the exercises special attention will be paid to check that the answers are substantiated and that the procedure of calculations is well documented. When results are achieved using a calculator or a pc it should be noted in the solution. All six problems are weighted equally in the grading. Your solution to the problem set can be written in Danish or English as you prefer.

Problem 1

Let the matrix A and vector $\boldsymbol{b}$ be given by:

$$A = \begin{bmatrix} -2 & -12 & 6 \\ 1 & 2 & 0 \\ 3 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix} \quad \text{and} \quad \boldsymbol{b} = \begin{bmatrix} 4 \\ 3 \\ 14 \\ 16 \end{bmatrix}$$

a) Determine all solutions to the homogene equation $A\boldsymbol{x} = \boldsymbol{0}$.

b) Determine all solutions to the inhomogene equation $A\boldsymbol{x} = \boldsymbol{b}$.

Problem 2

Let the matrix B be given by: $B = \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & \beta \\ -2 & 1 & 5 \end{bmatrix}$

a) Determine β so $\boldsymbol{v}_1 = \begin{bmatrix} 3 \\ 6 \\ 11 \end{bmatrix}$ is an eigenvector for the matrix B.

b) Find the corresponding eigenvalue λ_1.

Problem 3

Let A be a 6×4 matrix with $rank(A) = 3$.

For the statements given below, state whether they are true or false and justify your answer for each statement:

a) All the row vectors in the matrix A are linearly independent.

b) The solutions to the homogene equation $A\boldsymbol{x} = \boldsymbol{0}$ are given by one free parameter.

c) $A^T A$ is a 3×3 matrix.

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Problem 4

Let $C = \begin{bmatrix} 2 & -3 & 3 & -5 & 2 \\ 0 & 1 & 2 & 4 & 0 \\ -1 & 7 & -1 & 4 & 1 \end{bmatrix}$.

a) Find bases for $Col\ C$, $Row\ C$ and $Nul\ C$.

b) Is $\boldsymbol{v} = \begin{bmatrix} -8 \\ 14 \\ 2 \\ -21 \\ 9 \end{bmatrix}$ in $Row\ C$?

Problem 5

In an experiment a set of six data points of the variables x and y are measured:

x	0	1	2	3	4	5
y	6	4	7	10	15	35

The data are expected to fit into a linear model in parabolic form: $y(x) = \beta_0 + \beta_1 \cdot x + \beta_2 \cdot x^2$

a) Determine the design matrix, the parameter vector and the observation vector.

b) Determine the parameters β_0, β_1 and β_2 of the least-squares fit to the parabolic form.

c) Determine the residual vector and the sum of squares of the residuals for the parabolic fit.

Problem 6

A vector space V consist of all 2×2 matrices.

For two matrices $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ in the vector space V we define:

$$\langle A, B \rangle = a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22}$$

a) Show that $\langle A, B \rangle$ is an inner product on the vector space V.

Let $A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 5 & 1 \end{bmatrix}$.

b) Find the orthogonal projection of B on the subspace of V spanned by A.