In the grading of the exercises special attention will be paid to check that answers are substantiated and that the procedure of calculations is well documented. When results are achieved using a calculator or a pc it should be noted in the paper. All 6 problems are weighted equally in the grading. Your solution to the problem set can be written in Danish or English as you prefer.

PROBLEM 1.

Consider the following matrix and vector

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 1 \\ -2 & 3 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 7 \\ -3 \\ c \end{bmatrix}$$

where c is a scalar.

1. Solve $A\mathbf{x} = \mathbf{b}$ for $c = 1$.
2. Can $A\mathbf{x} = \mathbf{b}$ be solved for any value of c?

PROBLEM 2.

Let the following 2×2 matrix be given

$$A = \begin{bmatrix} 1 & 4 \\ 1 & 2 \end{bmatrix}.$$

1. Find the inverse of A using the $[A|I] \sim [I|A^{-1}]$ algorithm from the textbook and show the steps during the row reduction.

Consider another matrix B with the property

$$B^2 - 3B + I = 0.$$

2. Show that $B^{-1} = 3I - B$.

PROBLEM 3.

The vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

are a very simple example of a *wavelet* basis. Wavelets are used in e.g. signal and image processing.

1. Show that the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_4$ form a basis for R^4.

Consider the vector $\mathbf{x} = \begin{bmatrix} 4 & -2 & 1 & 5 \end{bmatrix}^T$.

2. Find the coordinates of $\mathbf{x}$ in the wavelet basis.

PROBLEM 4.

For the statements given below, state whether they are true or false and justify your answer for each statement.

1. The equation $A\mathbf{x} = \mathbf{b}$ with A an $n \times n$ matrix is inconsistent if rank $A < n$.
2. The distance between the vectors $\begin{bmatrix} 4 & 3 & 2 \end{bmatrix}^T$ and $\begin{bmatrix} 3 & 2 & 1 \end{bmatrix}^T$ is $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$.
3. An $n \times n$ matrix A can *not* be diagonalized if two or more of the eigenvalues are identical.

PROBLEM 5.

In case 3, Computer Graphics in Automotive Design, homogeneous coordinates were introduced. In this problem, homogeneous coordinates in $\mathbb{R}^2$ are used.

An object $\mathcal{O}$ has the nodes $n_1, n_2, \ldots, n_5$ with coordinates

$$\mathcal{C}_n = \{(1,1), (1,3), (3,3), (3,1), (2,2)\}$$

and adjacency matrix

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

1. Sketch the shape of the object, $\mathcal{O}$.
2. Find the matrix T, which translates $\mathcal{O}$, such that it is centered on $(0,0)$.

Consider the matrix

$$T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$

3. What is the effect of T_2 applied to $\mathcal{O}$?

PROBLEM 6.

Consider the polynomial space $\mathbb{P}_2$ and let an inner product be defined as

$$< f, g >= \int_0^1 f(x)g(x)dx.$$

Let the following three vectors be given

$$p_1(x) = x + 1, \quad p_2(x) = x - 1, \quad p_3(x) = x^2 + x.$$

1. Show that the three vectors form a basis for $\mathbb{P}_2$.
2. Show that the three vectors are not orthogonal.
3. Use the Gram-Schmidt procedure to construct an orthogonal basis for $\mathbb{P}_2$.