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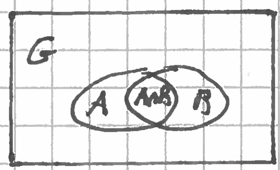
Opg. 1

Hændelser: G = Godkendt $\quad \overline{G}$ = Ikke-godkendt = Fejl

A = Fejltype A $\quad B$ = Fejltype B

Data: $Pr(G) = 0.86 = 86\%$; $Pr(A|\overline{G}) = 0.42 = 42\%$; $Pr(B|\overline{G}) = 0.72 = 72\%$

Venn-diagram:



a) $\underline{\underline{Pr(\text{Fejl})}} = Pr(\overline{G}) = 1 - Pr(G) = 1 - 0.86 = 0.14 = \underline{\underline{14\%}}$

b) $\underline{\underline{Pr(A)}} = Pr(A|\overline{G}) \cdot Pr(\overline{G}) = 0.42 \cdot 0.14 = 0.059 = \underline{\underline{5.9\%}}$

c) Fejl $= A + B - A \cap B$

$\Downarrow$

$$\underline{\underline{Pr(A \cap B)}} = Pr(A) + Pr(B) - Pr(\overline{G}) = (Pr(A|\overline{G}) + Pr(B|\overline{G}) - 1) \cdot Pr(\overline{G})$$

$$= (0.42 + 0.72 - 1) \cdot 0.14$$

$$= 0.14 \cdot 0.14 = 0.0196 = \underline{\underline{1.96\%}}$$

d) $$\underline{\underline{Pr(B|A)}} = \frac{Pr(A \cap B)}{Pr(A)} = \frac{0.0196}{0.059} = \underline{\underline{0.332 = 33.2\%}}$$

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Opg. 2

Stokastiske variable X og Y med simultan pmf $f_{XY}(x,y)$:

X\Y	1	2	3	4	f_X
−1	1/20	3/20	2/20	0	6/20
0	0	2/20	p	1/20	
1	4/20	1/20	1/20	2/20	8/20
f_Y	5/20	6/20		3/20	1

a) $\sum_{x=-1}^{1}\sum_{y=1}^{4} f_{XY}(x,y) = \sum\sum_{(x,y)\neq(0,3)} f_{XY}(x,y) + f_{XY}(0,3) = 1$

$\Downarrow$

$$\underline{\underline{\Pr(X=0 \wedge Y=3)}} = f_{XY}(0,3) = p = 1 - \sum\sum_{(x,y)\neq(0,3)} f_{XY}(x,y)$$

$$= 1 - \frac{1+3+2+2+1+4+1+1+2}{20} = 1 - \frac{17}{20} = \underline{\underline{\frac{3}{20} = 0.15}}$$

b) Marginale pmf:

$$\left.\begin{aligned} \underline{\underline{f_X(-1)}} &= \sum_{y=1}^{4} f_{XY}(-1,y) = \frac{1+3+2+0}{20} = \underline{\underline{\frac{6}{20} = 0.30}} \\ \underline{\underline{f_X(0)}} &= \sum_y f_{XY}(0,y) = \frac{0+2+3+1}{20} = \underline{\underline{\frac{6}{20} = 0.30}} \\ \underline{\underline{f_X(1)}} &= \sum_y f_{XY}(1,y) = \frac{4+1+1+2}{20} = \underline{\underline{\frac{8}{20} = 0.40}} \end{aligned}\right\} \sum_x f_X(x) = \frac{6+6+8}{20} = 1 \text{ O.K.}$$

$$\left.\begin{aligned} \underline{\underline{f_Y(1)}} &= \sum_{x=-1}^{1} f_{XY}(x,1) = \frac{1+0+4}{20} = \frac{5}{20} = \underline{\underline{0.25}} \\ \underline{\underline{f_Y(2)}} &= \sum_x f_{XY}(x,2) = \frac{3+2+1}{20} = \frac{6}{20} = \underline{\underline{0.30}} \\ \underline{\underline{f_Y(3)}} &= \sum_x f_{XY}(x,3) = \frac{2+3+1}{20} = \frac{6}{20} = \underline{\underline{0.30}} \\ \underline{\underline{f_Y(4)}} &= \sum_x f_{XY}(x,4) = \frac{0+1+2}{20} = \frac{3}{20} = \underline{\underline{0.15}} \end{aligned}\right\} \sum_y f_Y(y) = \frac{5+6+6+3}{20} = 1 \text{ O.K.}$$

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<u>Opg. 2 fortsat</u>

c) Middelværdier:

$$\underline{\underline{E[X]}} = \sum_{x=-1}^{1} x \cdot f_X(x) = -1 \cdot \frac{6}{20} + 0 \cdot \frac{6}{20} + 1 \cdot \frac{8}{20} = \frac{2}{20} = \underline{\underline{\frac{1}{10} = 0.10}}$$

$$\underline{\underline{E[Y]}} = \sum_{y=1}^{4} y \cdot f_Y(y) = 1 \cdot \frac{5}{20} + 2 \cdot \frac{6}{20} + 3 \cdot \frac{6}{20} + 4 \cdot \frac{3}{20} = \underline{\underline{\frac{47}{20} = 2.35}}$$

d) Hændelser: $A = \{X \leq 0\}$; $B = \{Y \geq 3\}$

$$\Pr(A \cap B) = \Pr(X \leq 0 \wedge Y \geq 3) = f_{XY}(-1,3) + f_{XY}(-1,4) + f_{XY}(0,3) + f_{XY}(0,4)$$
$$= \frac{2+0+3+1}{20} = \frac{6}{20}$$

$$\Pr(B) = \Pr(Y \geq 3) = f_Y(3) + f_Y(4) = \frac{6+3}{20} = \frac{9}{20}$$

$$\underline{\underline{\Pr(X \leq 0 \mid Y \geq 3)}} = \Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{6/20}{9/20} = \underline{\underline{\frac{2}{3} = 0.667}}$$

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Opg. 3

Stokastisk proces: $Y[n] = 3 \cdot X[n] + W[n]$, WSS

$X[n] \sim \mathcal{U}(1,3)$ iid

$W[n] \sim \mathcal{N}(0, 0.5)$ iid

a) Tre realisationer (10 samples):

Matlab:
```
Wn = sqrt(0.5)·randn(1,10);
Xn = 2·rand(1,10) + 1;
Yn = 3·Xn + Wn;
```
} Gentages 3 gange → Se bilag

b) Ensemble middelværdi:

$$\underline{\underline{EY[n]}} = E[3 \cdot X[n] + W[n]] = 3 \cdot EX[n] + EW[n] = 3 \cdot \frac{3+1}{2} + 0 = \underline{\underline{6}}$$

Ensemble varians:

$$\underline{\underline{\mathrm{Var}Y[n]}} = \mathrm{Var}(3X[n] + W[n]) = 9\,\mathrm{Var}X[n] + \mathrm{Var}W[n]$$
$$= 9 \cdot \frac{(3-1)^2}{12} + 0.5 = 3 + 0.5 = \underline{\underline{3.5}}$$

Både $EY[n]$ og $\mathrm{Var}Y[n]$ er uafh. af n (tiden) → o.k. (WSS)

c) Auto-korrelation: $R_{YY}(t, t+\tau) = R_{YY}(\tau)$ da Y er WSS.

$$R_{YY}(\tau) = E(Y[n] \cdot Y[n+\tau]) = EY[n] \cdot EY[n+\tau] = 6 \cdot 6 = 36 \quad \text{for } \tau \neq 0$$

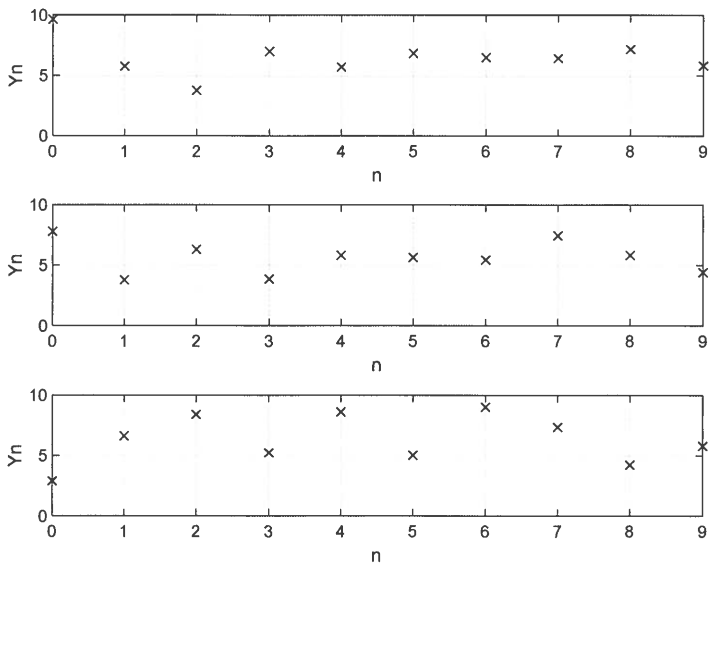
da Y er iid.

$$\Downarrow$$
$$\underline{\underline{R_{YY}(1) = R_{YY}(2) = R_{YY}(3) = 36}}$$

$$\underline{\underline{R_{YY}(0)}} = E[Y[n] \cdot Y[n]] = E[Y[n]^2] = \mathrm{Var}Y[n] + (EY[n])^2$$
$$= 3.5 + 6^2 = \underline{\underline{39.5}}$$

```
%% Opgave_3a_V19/20 Realization of a discrete stochastic process
for i=1:3
    %% 10 samples of one realization of a discrete stocastic process
    Wn=sqrt(0.5)*randn(1,10) %10 samples of gaussian distributed random proces with
                              mean=0 and variance=0.5
    Xn=2*rand(1,10)+1 %10 samples of a uniformly distributed random proces between -1
                       and +1
    Yn=3*Xn+Wn %10 samples (n=0,..,9) of the stochastic process: Y(n)=3*X(n)+W(n)

    %% Plot of 10 samples of one realization
    figure(1)
    ax=subplot(3,1,i); %3 plots on one figure
    n=0:9; %10 samples between 0 and 9
    plot(ax,n,Yn,'kx')
    grid
    axis([0,9,0,10])
    xlabel(ax,'n')
    ylabel(ax,'Yn')
 end
```

```
>> Opgave_3a_V19_20

Wn = 1.2621 -0.1416  0.6650  0.2469  1.3147  0.6555 -0.8676 -0.2314  0.6305  0.2038

Xn =  2.7928  1.9645  1.0282  2.2458  1.4622  2.0549  2.4500  2.2148  2.1767  1.8669

Yn =  9.6405  5.7518  3.7495  6.9842  5.7013  6.8201  6.4824  6.4131  7.1607  5.8044


Wn =-0.1035  -0.0729 -1.9792  0.2781  0.7002 -0.9176  -1.0762  0.4391 -1.0660  -1.1875

Xn =  2.6298  1.2810  2.7597  1.1908  1.7051  2.1868  2.1704  2.3354  2.2961  1.8667

Yn =  7.7860  3.7701  6.3000  3.8503  5.8156  5.6430  5.4349  7.4452  5.8222  4.4127


Wn =-0.4096  0.3398  -0.2735  0.2981  0.7691 -1.5905  1.2760  -0.4469  0.9309  1.0971

Xn =  1.0997  2.0918  2.8863  1.6429  2.6129  2.2028  2.5792  2.5984  1.0991  1.5664

Yn =  2.8896  6.6151  8.3855  5.2270  8.6079  5.0179  9.0137  7.3482  4.2283  5.7963
```



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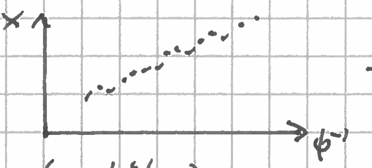
Opg. 4

Flødeboller: $\mu_0 = 30.0\,g$ Test: $n = 12$

a) Q-Q plot (Matlab: qqplot(x))



$\rightarrow$ ~Ret linje $\rightarrow$ Data normalfordelte

(se bilag)

b) Data normalfordelte, ukendt varians $\rightarrow$ Student t-test

c) Nul-hypotese: $H_0: \mu = \mu_0 = 30.0\,g$

Alternativ hypotese: $H_1: \mu \neq \mu_0 = 30.0\,g$

d) Sample middelværdi: $\hat{\mu} = \frac{1}{n}\cdot\sum_{i=1}^{n} x_i = \frac{1}{12}\sum_{i=1}^{12} x_i = 29.15$

Sample varians: $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \hat{\mu})^2 = \frac{1}{11}\sum_{i=1}^{12}(x_i - 29.15)^2 = 1.47$

e) Test-værdi: $t = \frac{\hat{\mu} - \mu_0}{\sqrt{s^2/n}} = \frac{29.15 - 30.0}{\sqrt{1.47/12}} = -2.44$

p-værdi: $p = 2\cdot(1 - t_{cdf}(|t|, n-1)) = 2\cdot(1 - t_{cdf}(2.44, 11))$

$= 2\cdot(1 - 0.9885)$

$= 0.033 < 0.05 \rightarrow H_0$ afvises

f) 95% konfidens interval:

$t_0 = t_{inv}(0.975, n-1) = t_{inv}(0.975, 11) = 2.201$

$\mu_- = \hat{\mu} - t_0\cdot\sqrt{s^2/n} = 29.15 - 2.201\cdot\sqrt{1.47/12} = 28.38$

$\mu_+ = \hat{\mu} + t_0\cdot\sqrt{s^2/n} = 29.15 + 2.201\cdot\sqrt{1.47/12} = 29.92$

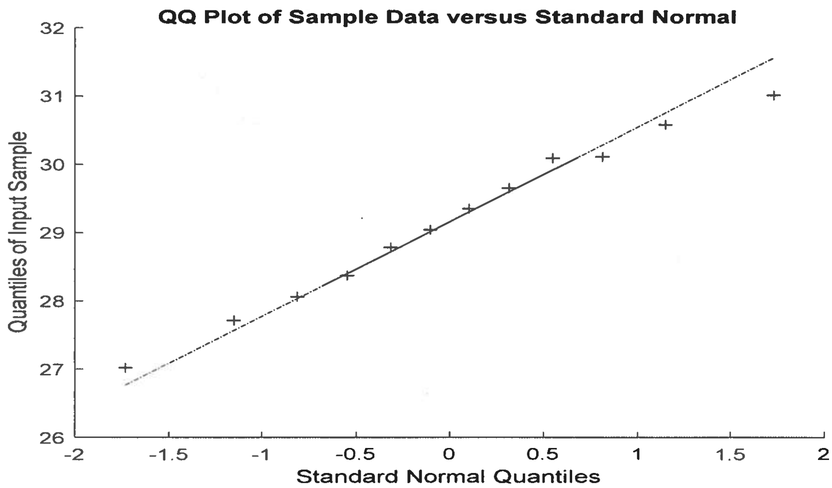
$[\mu_-;\mu_+] = [28.38; 29.92] \rightarrow$ Den sande μ vil med 95% sandsynlig hed ligge i dette interval.

$\mu_0 \notin [\mu_-;\mu_+] \rightarrow H_0$ afvises.

%% Opgave_4_V19_20 Flødebolle-test

```
N=12;
mu0=30
x=[30.09 28.78 31.01 27.02 30.11 29.35 28.37 29.65 27.71 30.58 28.06 29.04]
qqplot(x)
Meanx=mean(x)
Varx=var(x)
t=(Meanx-mu0)/sqrt(Varx/N)
p_val=2*(1-tcdf(abs(t),N-1))
t0=tinv(0.975,N-1)
x_min=Meanx-t0*sqrt(Varx/N)
x_max=Meanx+t0*sqrt(Varx/N)
```

>> Floedebollemaalinger



mu0 = 30

x = 30.09 28.78 31.01 27.02 30.11 29.35 28.37 29.65 27.71 30.58 28.06 29.04

Meanx = 29.1475

Varx = 1.4687

t = -2.4368

p_val = 0.0330

t0 = 2.2010

x_min = 28.3775

x_max = 29.9175