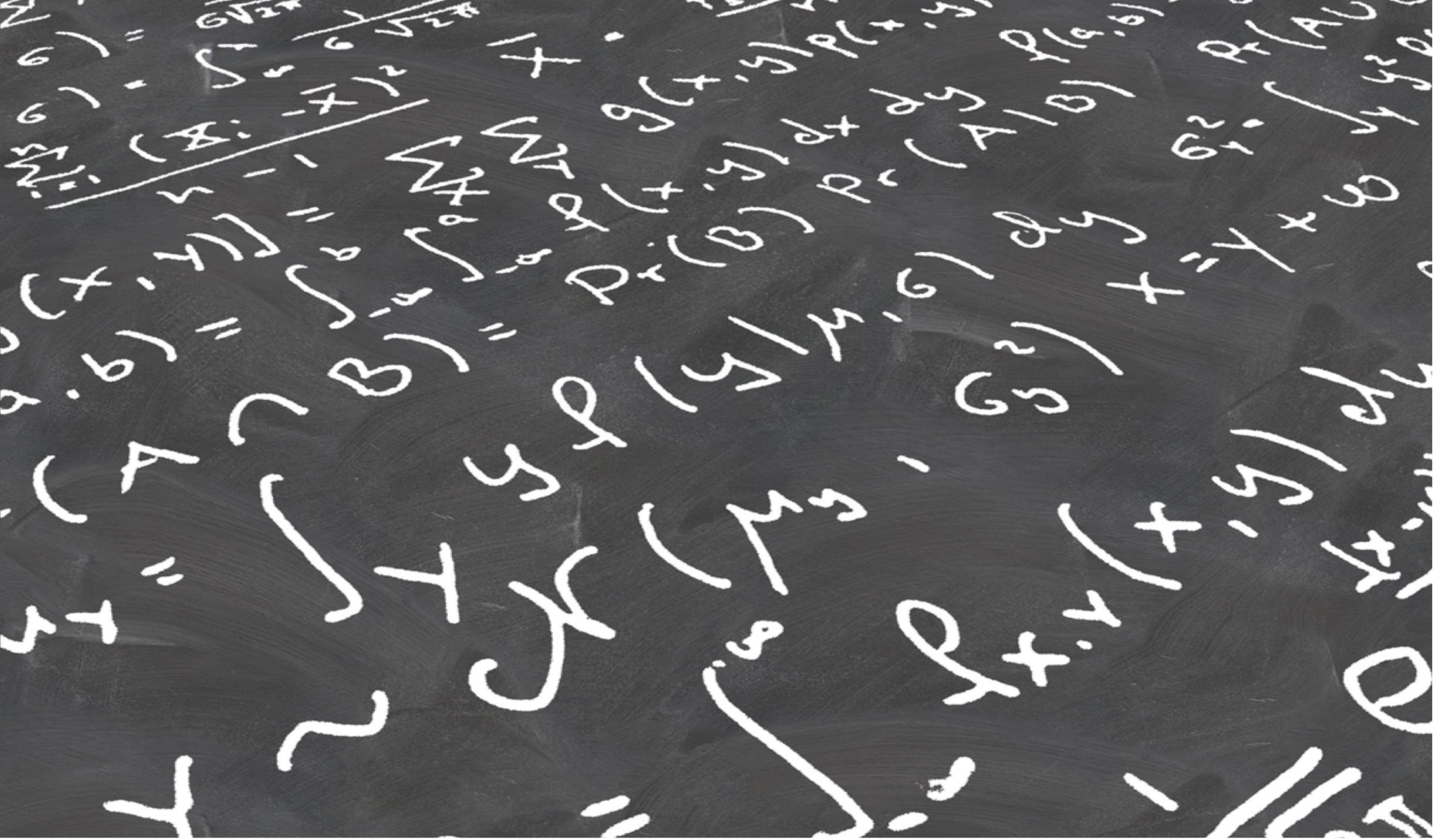


7. Correlation functions and Power Spectral Density

Gunvor Elisabeth Kirkelund
Lars Mandrup

Agenda for Today

- Stochastic Processes (repetition)
  - Mean and variance
  - Stationarity
  - Ergodic Processes
- Correlation functions
  - Autocorrelation functions
  - Cross-correlation functions
- Power spectrum density

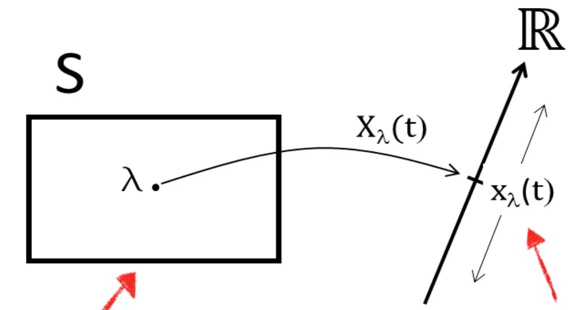
Stochastic Processes

Definitions:

- A stochastic process is a time dependent stochastic variable:

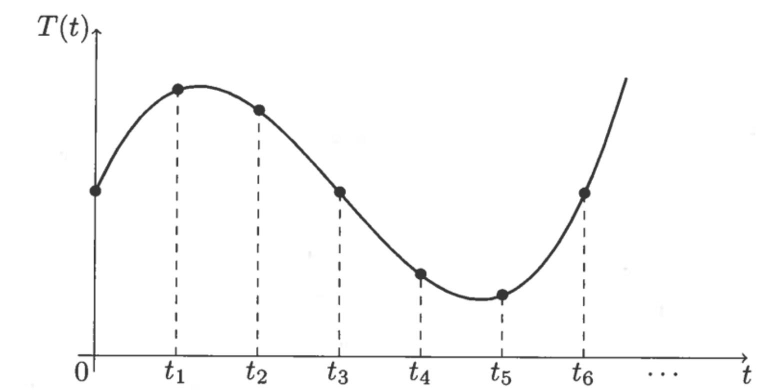
Continuous-time

$$X(t)$$



Sample space for stochastic experiment

Sample space for stochastic proces



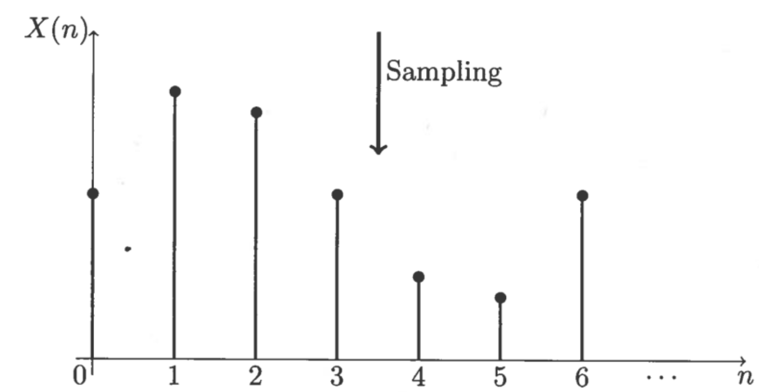
- A discrete stochastic process is given by:

time

$$X[n] = X(nT)$$

where n is an integer.

Discrete-time



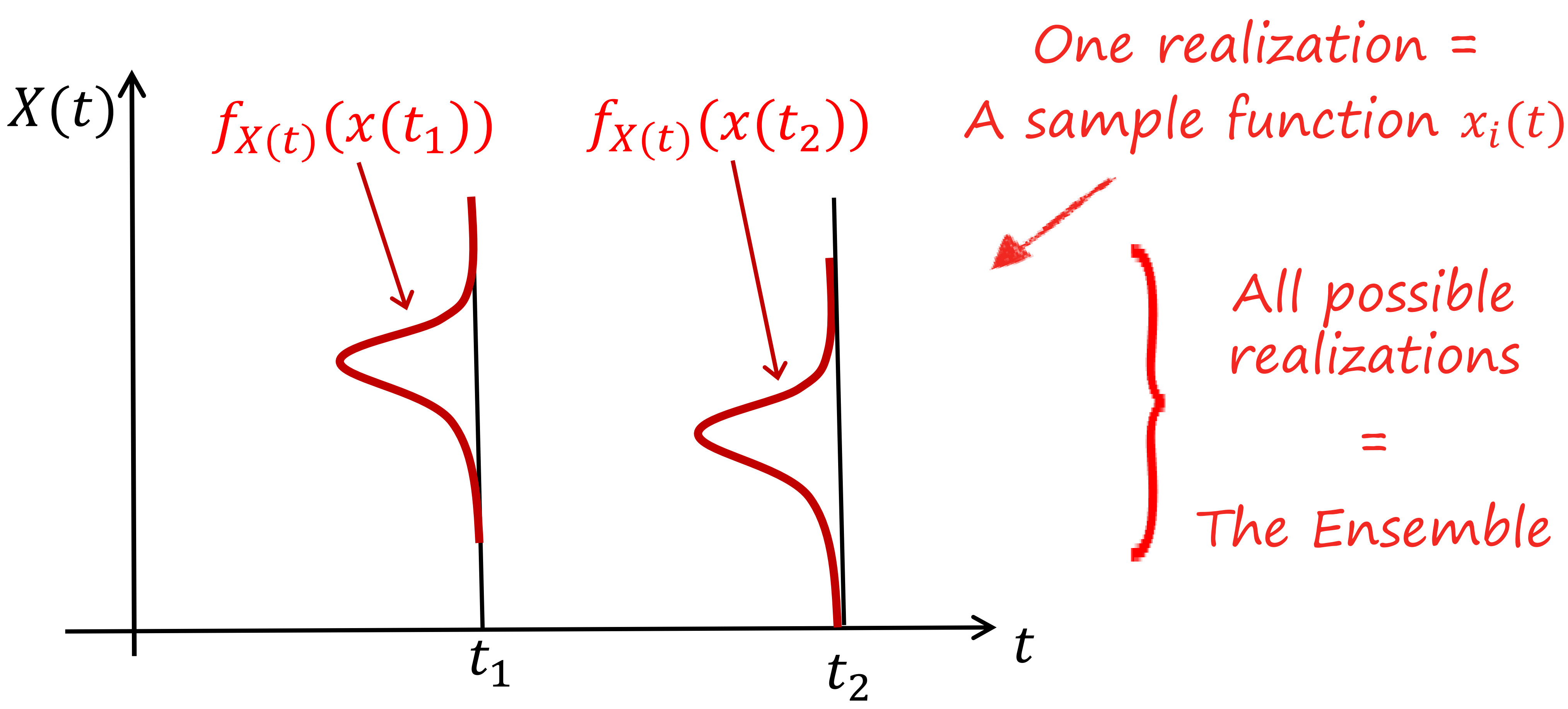
Notice:

- When we measure/sample a signal from a stochastic process, we observe only one realization of the process

Sample Functions – Realizations – Ensemble

Definition:

- A Sample Function $x(t)$ is a realization of a stochastic process X
- The Ensemble of the Stochastic Process is the collection of all possible realizations $x(t)$ of the Stochastic Process X



The Mean and Variance Functions

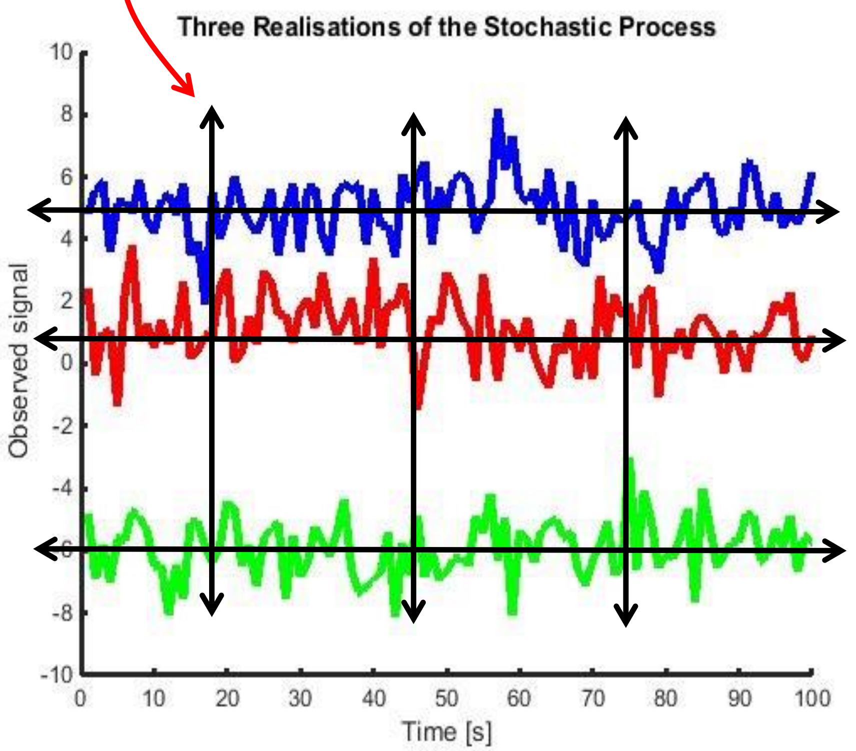
- Ensemple mean:

$$\mu_{X(t)}(t) = E[X(t)] = \int_{-\infty}^{\infty} x(t) f_{X(t)}(x(t))\, dx(t)$$

The mean/variance of all possible realizations to time t

- Ensemple variance:

$$Var\big(X(t)\big) = \sigma^2_{X(t)}(t) = E\Big[\big(X(t) - \mu_{X(t)}(t)\big)^2\Big]$$



The time average/variance for one realization of the stochastic process

- Temporal mean:

$$\hat{\mu}_{X_i} = \langle X_i \rangle_T = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x_i(t)\, dt$$

- Temporal variance:

$$\hat{\sigma}^2_{X_i} = \langle X_i^2 \rangle_T - {\langle X_i \rangle_T}^2 = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left(x_i(t)^2 - {\hat{\mu}_{X_i}}^2\right) dt = Var(X_i)$$

Stationarity in the Wide Sense (WSS)

- Ensemble mean is a constant

Can be tested.

$$\mu_X(t) = E[X(t)] = \mu_X$$ - independent of time

- Ensemble variance is a constant

$$\sigma_X^2(t) = E\left[X(t)^2\right] - E[X(t)]^2 = \sigma_X^2$$ - independent of time

Stationarity in the Strict Sense (SSS):

- The density function $f_{X(t)}\big(x(t)\big)$ do not change with time

Difficult to test in reality.

Ergodicity

- We can say something about the properties of the stochastic process in general based on one sample function, as long as we have observed it for long enough.

- If ensemble averaging is equivalent to temporal averaging:

Any realization

Ensemble (WSS)

$$\left.\begin{aligned} \langle X_i \rangle_T &= \mu_X \\ \hat{\sigma}^2_{X_i} &= \sigma^2_X \end{aligned}\right\} \rightarrow Ergodic$$

All information is achieved with one measurement (realization)

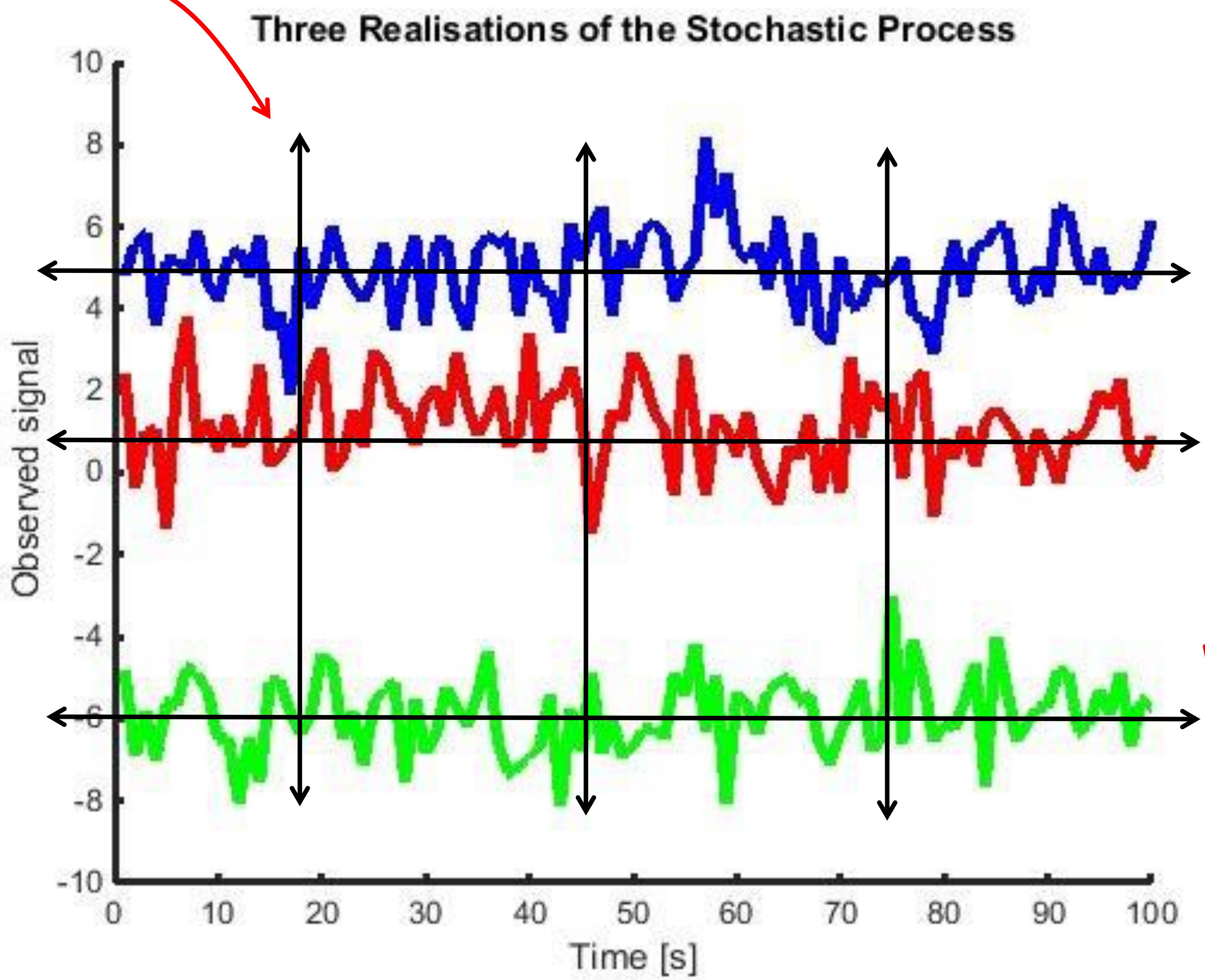
Stochastic Processes (signals)

Additive Noisemodel

$$observed\ signal = signal\ +\ noise$$

Ensemble mean and variance (to a specific time).

If indepentent of time: WSS



Time average and variance of each realization.

If equal (for all realizations): Ergodic

Tells of the coupling between variables

Correlation and Covariance – Stochastic Variables

Correlation tells of the (biased) coupling between variables

- Correlation: $corr(X,Y) = E[XY] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x \cdot y \cdot f_{X,Y}(x,y)dxdy$

Covariance is without bias from the mean

- Covariance: $cov(X,Y) = E\left[(X-\bar{X})(Y-\bar{Y})\right] = E[XY] - E[X] \cdot E[Y]$

Correlation Coefficient is the normalized Covariance

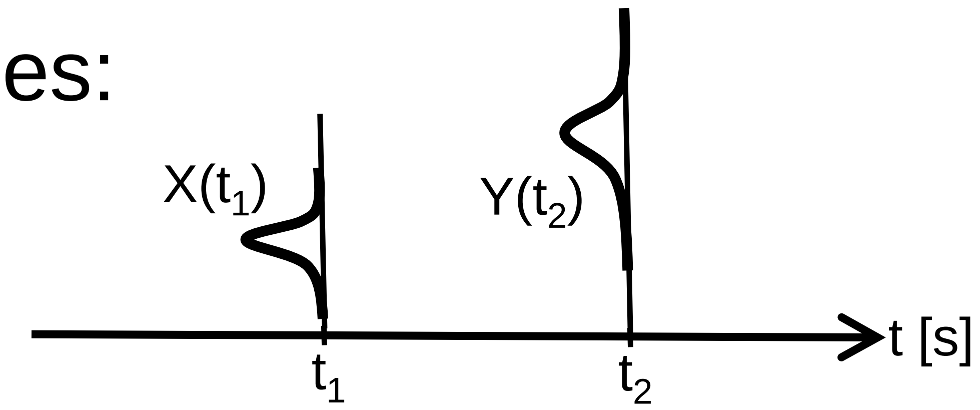
- Correlation coefficient: $\rho = E\left[\frac{X-\bar{X}}{\sigma_X} \cdot \frac{Y-\bar{Y}}{\sigma_Y}\right] = \frac{E[XY] - E[X]E[Y]}{\sigma_X \cdot \sigma_Y}$

$$-1 \le \rho \le 1$$

Comparing realizations

Correlations – Stochastic Processes

We compare processes at two different times:



Correlation of a process with itself

- Autocorrelation: $R_{XX}(t_1, t_2) = E[X(t_1)X(t_2)^*]$
  - Says something about how much the signal $X(t_1)$ resembles itself at time t_2
  - Depent on how rapidly the signal changes over time
  - Larger if $|t_1 - t_2|$ is small

Correlation of two different processes

- Cross-correlation: $R_{XY}(t_1, t_2) = E[X(t_1)Y(t_2)^*]$
  - Can be used to look for places where the signal $X(t)$ is simillar to the signal $Y(t)$

Correlations

Master of starry sky

Satellite navigation camera

How is the satellite oriented?

Ensemble means that it applied for the ensemble of the process

Ensemble Autocorrelation

Tells about the connection at two different times

- In general:

$$R_{XX}(t_1, t_2) = E[X(t_1)X(t_2)^*]$$

Complex conjugated

$$= \iint_{-\infty}^{\infty} x(t_1)\, x(t_2)^* f_{X(t_1),X(t_2)}\big(x(t_1), x(t_2)\big) dx(t_1) dx(t_2)$$

- For a stationary process (WSS):

$$R_{XX}(t_1, t_2) = R_{XX}(t_1 + T, t_2 + T) = E[X(t_1 + T)X(t_2 + T)^*]$$

Independent of time (t_1)
Depends only on $\tau = t_2 - t_1$

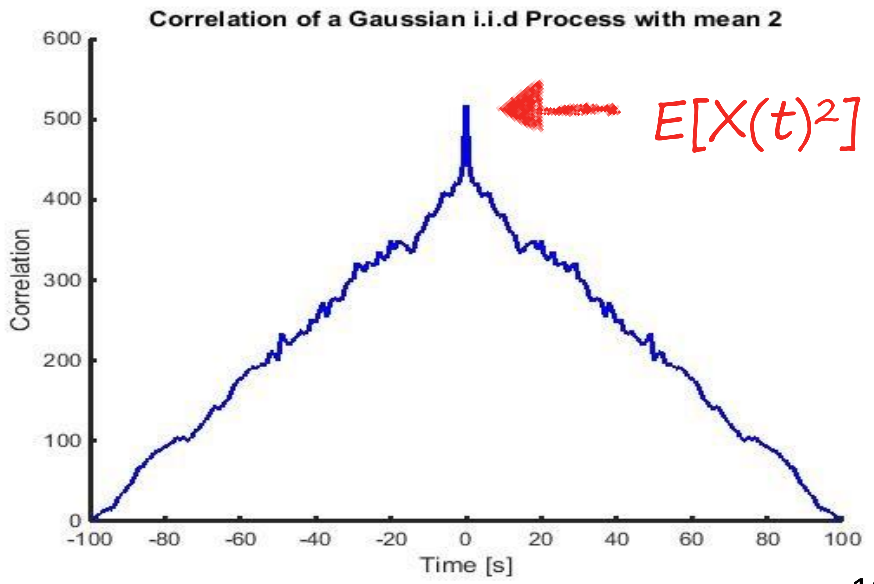
- We rewrite to: $R_{XX}(\tau) = E[X(t)X(t+\tau)^*]$

$\tau = t_2 - t_1$ is the lag!

Autocorrelation

- For Real WSS: $R_{XX}(\tau) = E[X(t)X(t+\tau)]$

- Properties of the autocorrelation function $R_{XX}(\tau)$:
  - An even function of τ $(R_{XX}(\tau) = R_{XX}(-\tau))$
  - Bounded by: $|R_{XX}(\tau)| \leq R_{XX}(0) = E\left[X^2\right]$ (max. in $\tau = 0$)
  - If $X(t)$ changes fast, then $R_{XX}(\tau)$ decreases fast from $\tau = 0$
  - If $X(t)$ changes slowly, then $R_{XX}(\tau)$ decreases slowly from $\tau = 0$
  - If $X(t)$ is periodic, then $R_{XX}(\tau)$ is also periodic



Notice: If $X(t)$ is WSS, and $X(t)$ and $X(t+\tau)$ are independent for $\tau \neq 0$ then:
$R_{XX}(\tau) = E[X(t)X(t+\tau)] = E[X(t)] \cdot E[X(t+\tau)]$

Random Walk – Example

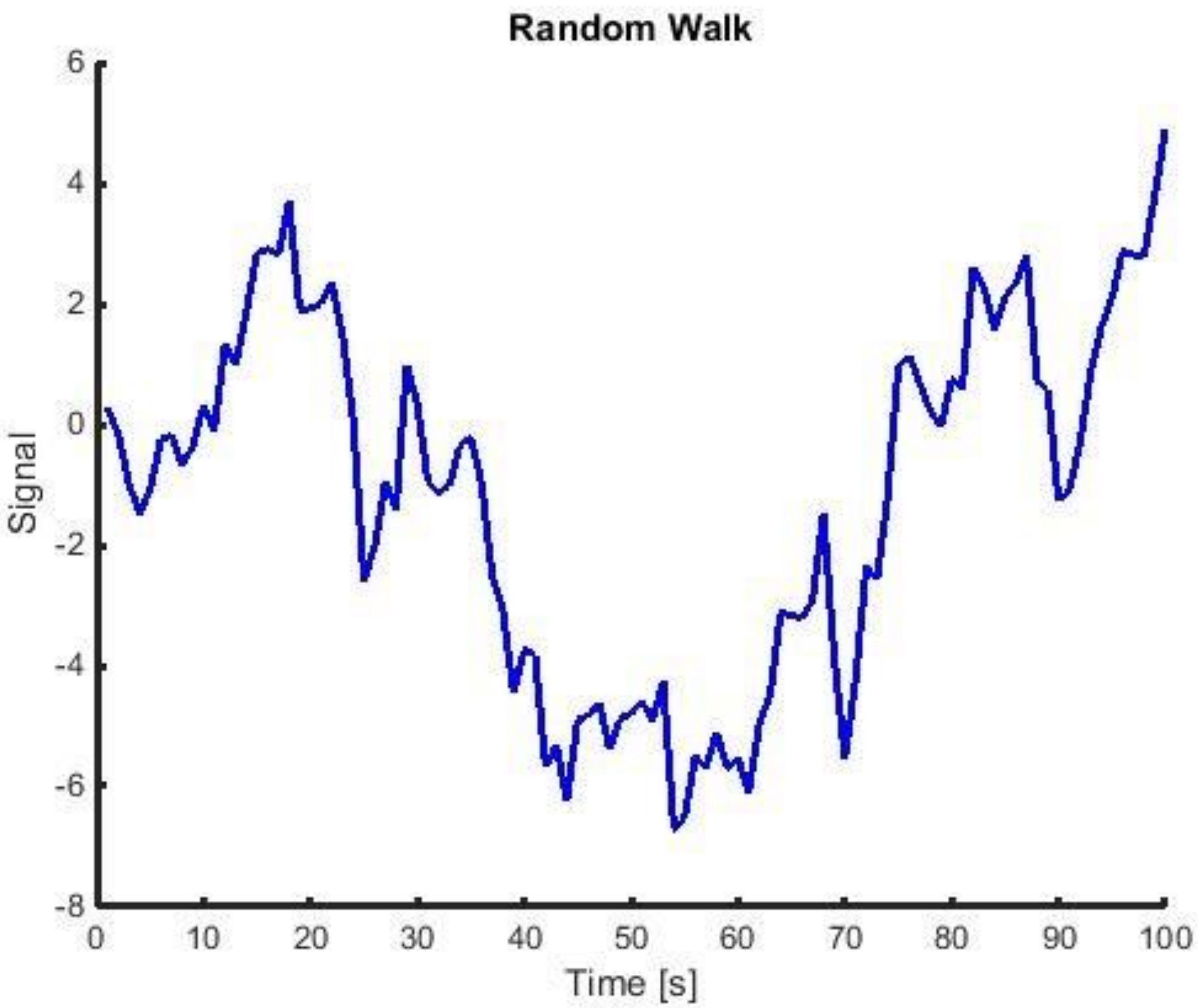
Brownish motions / Wiener Process

- We consider a random walk: $W[n] = W[n-1] + X[n]$

$$X[n] = \pm\sqrt{\delta}$$

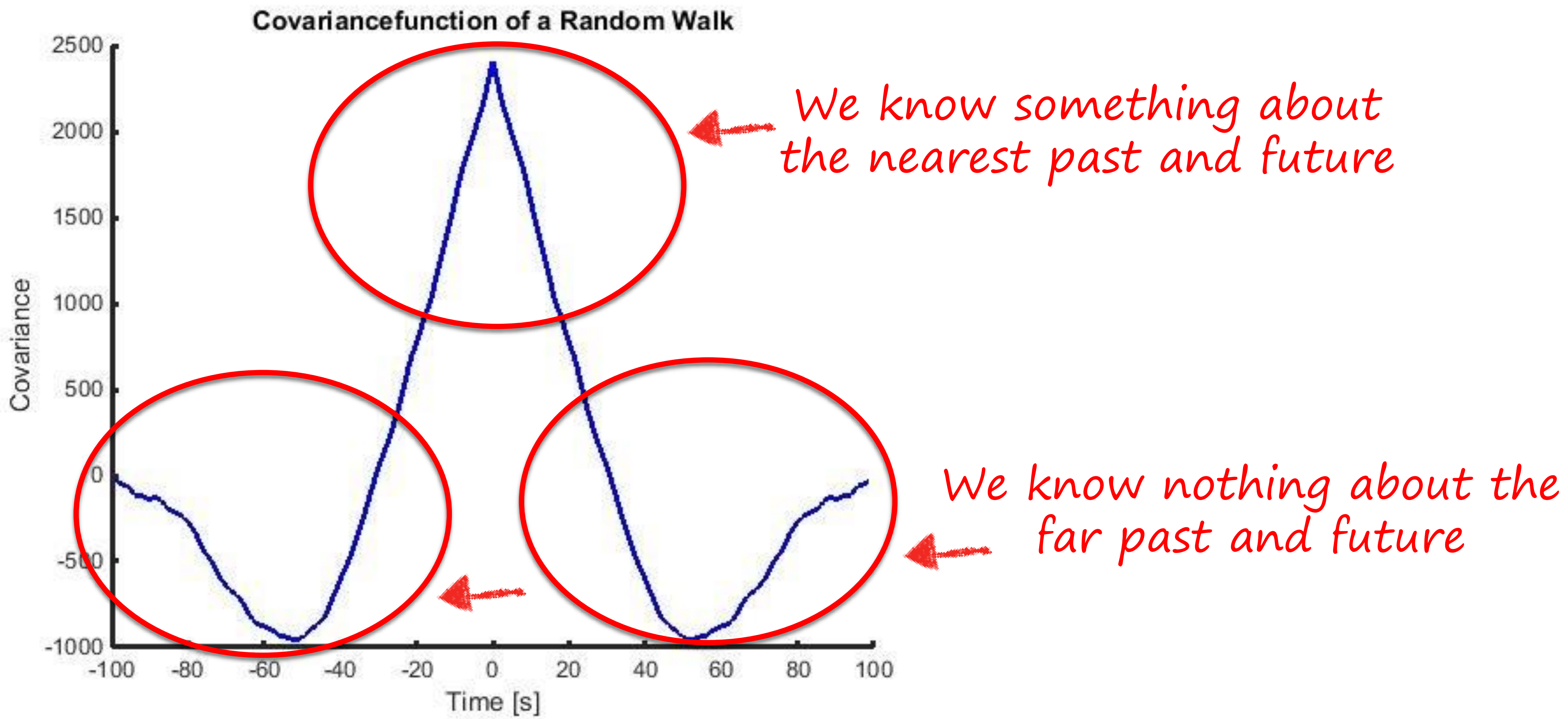
$$EX = 0$$

$$Var(X) = \delta$$



Random Walk – Example

- Sample of the autocovariance function:



Tells about how much we can predict the future

Autocovariances

- Autocovariance function:

Autocorrelation without DC

$$C_{XX}(t_1,t_2) = E\left[\left(X(t_1) - \mu_X(t_1)\right)\left(X(t_2) - \mu_X(t_2)\right)^*\right]$$
$$= R_{XX}(t_1,t_2) - \mu_X(t_1)\mu_X(t_2)$$

Especially: $C_{XX}(t,t) = E\left[(X(t) - \mu_X(t))^2\right] = E\left[X(t)^2\right] - E[X(t)]^2 = \sigma_X^2(t)$

- Autocorrelation coefficient:

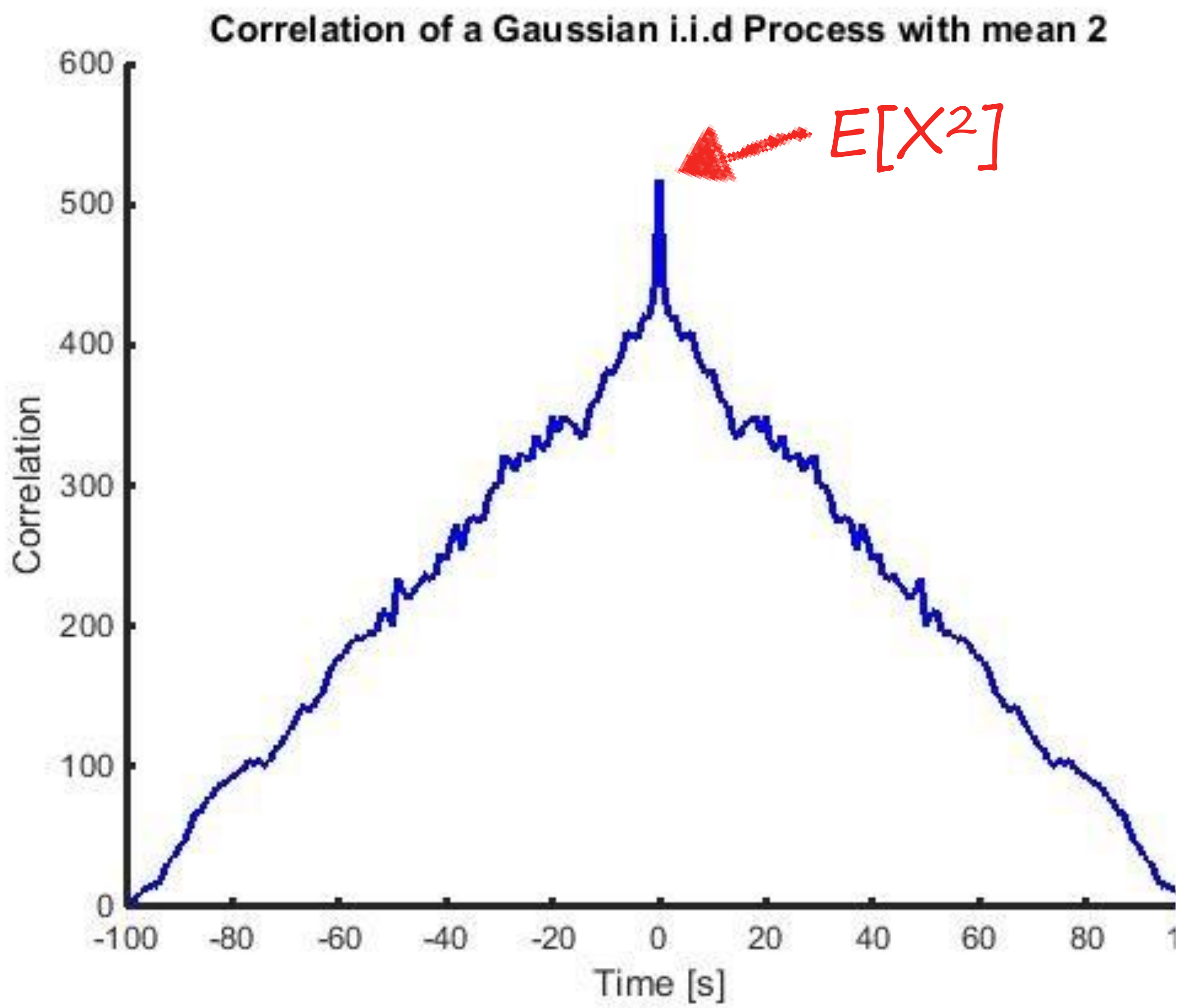
$$r_{XX}(t_1,t_2) = \frac{C_{XX}(t_1,t_2)}{\sqrt{C_{XX}(t_1,t_1)C_{XX}(t_2,t_2)}}; \qquad 0 \le r_{XX}(t_1,t_2) \le 1$$

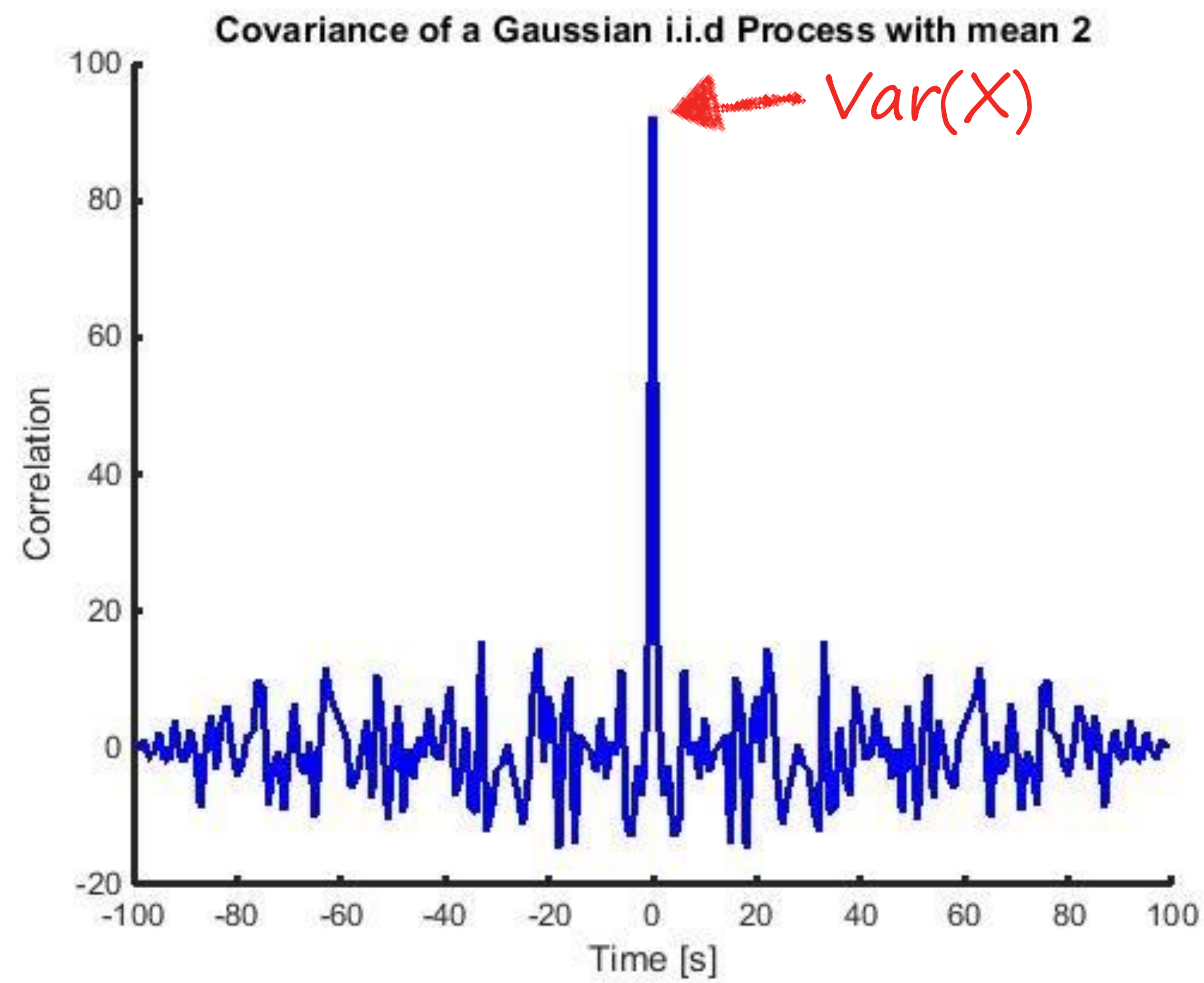
Especially: $r_{XX}(t,t) = 1$ ($X(\mathrm{t})$ is totally correlated to itself!)

Autocovariances

For i.i.d. Gaussian (stationary) noise

- Autocorrelation and autocovariance





Temporal Autocorrelation

Temporal only looks at one realization of the stochastic process.

Convolution

- Temporal autocorrelation:

$$\mathcal{R}_{XX}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x\,(t) \cdot x\,(t+\tau) dt$$

- If the process is ergodic the temporal autocorrelation is equal to the ensemble autocorrelation:

$$R_{XX}(\tau) = \mathcal{R}_{XX}(\tau)$$

Ensemble

Temporal

Estimate Autocorrelation

We only have measurements of one realization of X(t)

Autocorrelation function:

- In practise, with respect to the lag:

temporal $$\mathcal{R}_{XX}(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x\,(t)\cdot x\,(t+\tau)dt$$

N+1 measurements/samples $$x(0), x(\Delta t), x(2\Delta t), \dots, x(N\Delta t)$$

- The estimated autocorrelation function:

^ = estimation

$$\hat{R}_{XX}(n) = \frac{1}{N-n+1}\sum_{k=0}^{N-n} \underbrace{x(k)}_{t}\cdot \underbrace{x(k+n)}_{t+\tau}$$

Number of terms (T/ Δt)

Random Binary Noise – Example

- One measurement (5 samples) of a WSS and independent binary stochastic process $X[n]$:

n	0	1	2	3	4 = N
$X[n]$	1	0	1	1	0

- The estimated autocorrelation function: $\hat{R}_{XX}(\tau) = \frac{1}{N-\tau+1}\sum_{k=0}^{N-\tau} x(k)\cdot x(k+\tau)$

$\tau = 0$:

$X[n]$	1	0	1	1	0
$X[n+0]$	1	0	1	1	0

$$\hat{R}_{XX}(0) = \frac{1}{5}\sum_{k=0}^{4} x(k)\cdot x(k) = \frac{1}{5}\cdot(1+0+1+1+0) = \frac{3}{5}$$

$\tau = 1$:

$X[n]$	1	0	1	1	0
$X[n+1]$	0	1	1	0	

$$\hat{R}_{XX}(1) = \frac{1}{4}\sum_{k=0}^{3} x(k)\cdot x(k+1) = \frac{1}{4}\cdot(0+0+1+0) = \frac{1}{4}$$

$\tau = 2$:

$X[n]$	1	0	1	1	0
$X[n+2]$	1	1	0		

$$\hat{R}_{XX}(2) = \frac{1}{3}\sum_{k=0}^{2} x(k)\cdot x(k+2) = \frac{1}{3}\cdot(1+0+0) = \frac{1}{3}$$

$\tau = 3$:

$X[n]$	1	0	1	1	0
$X[n+3]$	1	0			

$$\hat{R}_{XX}(3) = \frac{1}{2}\sum_{k=0}^{1} x(k)\cdot x(k+3) = \frac{1}{2}\cdot(1+0) = \frac{1}{2}$$

$\tau = 4$:

$X[n]$	1	0	1	1	0
$X[n+4]$	0				

$$\hat{R}_{XX}(4) = \frac{1}{1}\sum_{k=0}^{0} x(k)\cdot x(k+4) = \frac{1}{1}\cdot(0) = 0$$

Random Binary Noise – Example

- Let a stochastic process be defined as:

$$X[n] = \begin{cases} 0 & p = ½ \\ 1 & p = ½ \end{cases} \sim \mathcal{U}[0,1] \text{ i.i.d.}$$

- $E\big[X[n]\big] = ½; \quad E\big[X^2[n]\big] = ½; \quad Var(X[n]) = \frac{1}{4}$
- WSS
- Ergodic

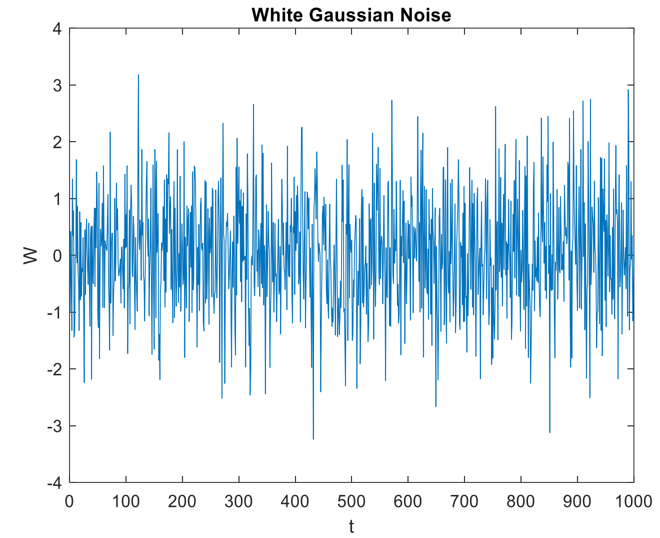
- Autocorrelation:

$$R_{XX}(\tau) = E\big[X[n]X[n+\tau]\big] = \begin{cases} E\big[X[n]^2\big] & ;\tau = 0 \\ E\big[X[n]\big] \cdot E\big[X[n+\tau]\big] & ;\tau \neq 0 \end{cases} = \begin{cases} \frac{1}{2} & ;\tau = 0 \\ \frac{1}{4} & ;\tau \neq 0 \end{cases}$$

White Gaussian Noise – Example

- Let a stochastic process be defined as:

$$W(t) \sim \mathcal{N}(0,1) \text{ i.i.d.}$$



- $E[W(t)] = 0; \quad E\left[W^2(t)\right] = 1; \quad Var(W(t)) = 1$
- WSS
- Ergodic

- Autocorrelation:

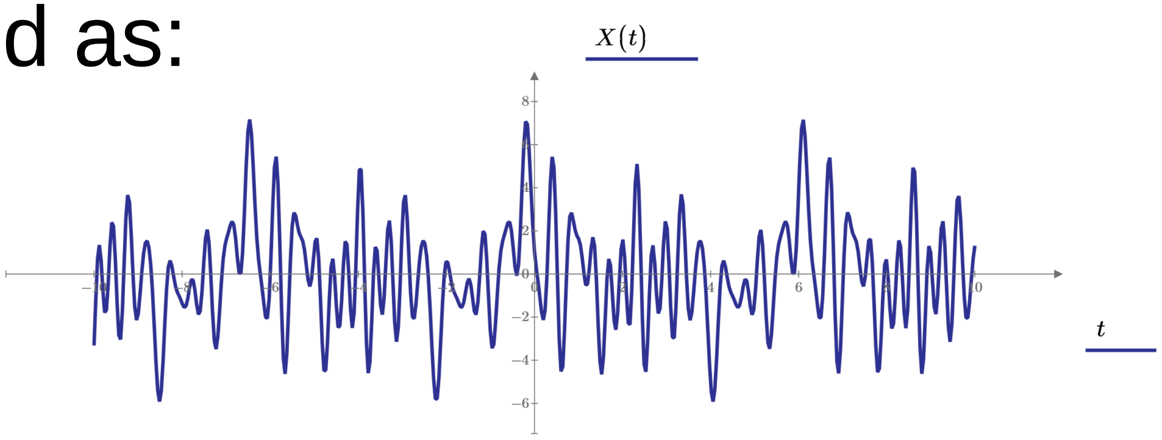
$$R_{WW}(t_1, t_2) = E[W(t_1) \cdot W(t_2)] = \begin{cases} E\left[W^2(t_1)\right] & t_1 = t_2 \\ E[W(t_1)] \cdot E[W(t_2)] & t_1 \neq t_2 \end{cases}$$

$$= \begin{cases} 1 & \tau = t_1 - t_2 = 0 \\ 0 & \tau = t_1 - t_2 \neq 0 \end{cases} = R_{XX}(\tau)$$

Random Sinusoid – Example

- Let a stochastic process be defined as:

$$X(t) = \sum_{i=1}^{n} A_i \cos(\omega_i t + \varphi_i)$$



where $A_i \sim \mathcal{N}(0, \sigma^2)$ i.i.d., $\varphi_i \sim \mathcal{U}(0,2\pi)$ i.i.d. and $\omega_i = i \cdot \omega_0$

- ➢ $E[X(t)] = 0; \quad E\left[X^2(t)\right] = ½\sigma^2; \quad Var(X(t)) = ½\sigma^2$
- ➢ WSS
- ➢ ÷ Ergodic

Random Sinusoid – Example (cont'd)

- Autocorrelation:

$$R_{XX}(\tau) = E[X(t)X(t+\tau)]$$

$$= E[\sum_{i=1}^{n} \sum_{j=1}^{n} A_i \cos(\omega_i t + \varphi_i) \cdot A_j \cos(\omega_j (t+\tau) + \varphi_j)]$$

➢ Since A_i i.i.d. with $E[A_i] = 0$: $i \neq j \implies E\left[A_i A_j\right] = 0$

➢ $R_{XX}(\tau) = \sum_{i=1}^{n} E\left[A_i^2\right] \cdot E[cos(\omega_i t + \varphi_i) \cdot cos(\omega_i (t+\tau) + \varphi_i)]$

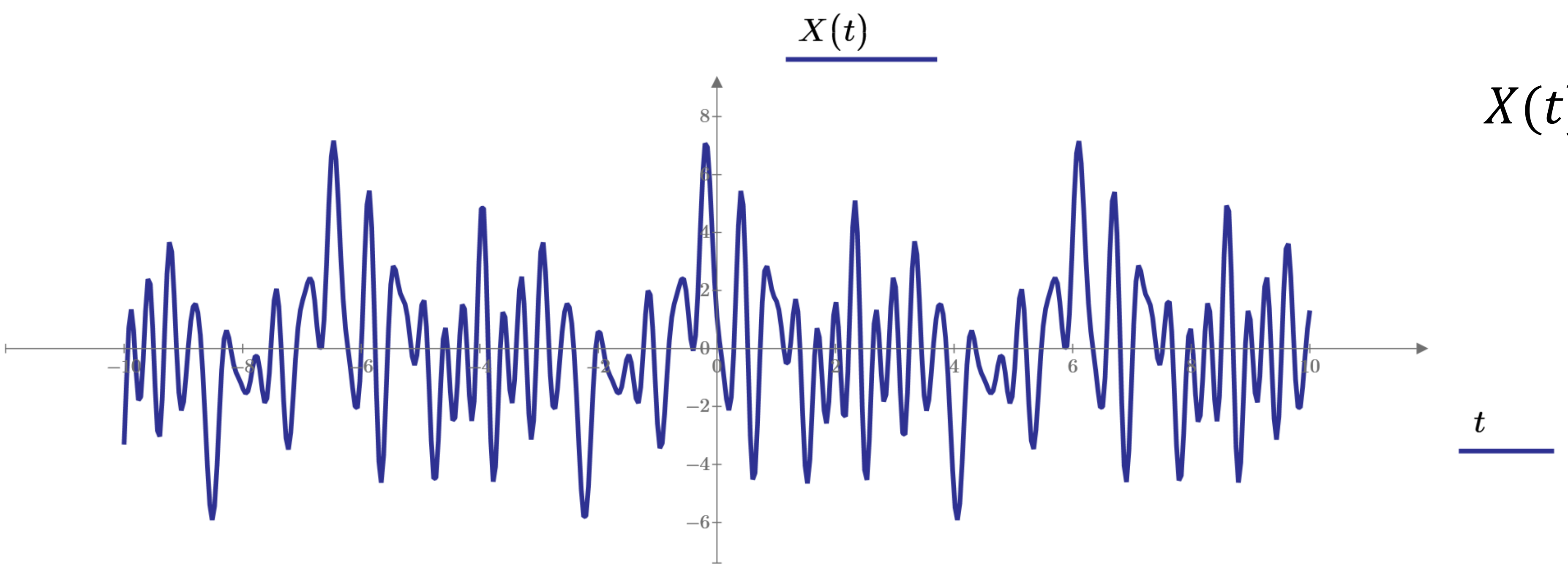
$$= E\left[A_i^2\right] \cdot \sum_{i=1}^{n} E[½ \cdot (cos(\omega_i \tau) + cos(\omega_i (2t+\tau) + 2\varphi_i))]$$

$$= \frac{\sigma^2}{2} \cdot \sum_{i=1}^{n} cos(\omega_i \tau)$$

[Using: $cos\theta_1 \cdot cos\theta_2$
$= ½(\cos(\theta_1 - \theta_2) + \cos(\theta_1 + \theta_2))$]

➢ Especially we have: $R_{XX}(0) = n\frac{\sigma^2}{2}$

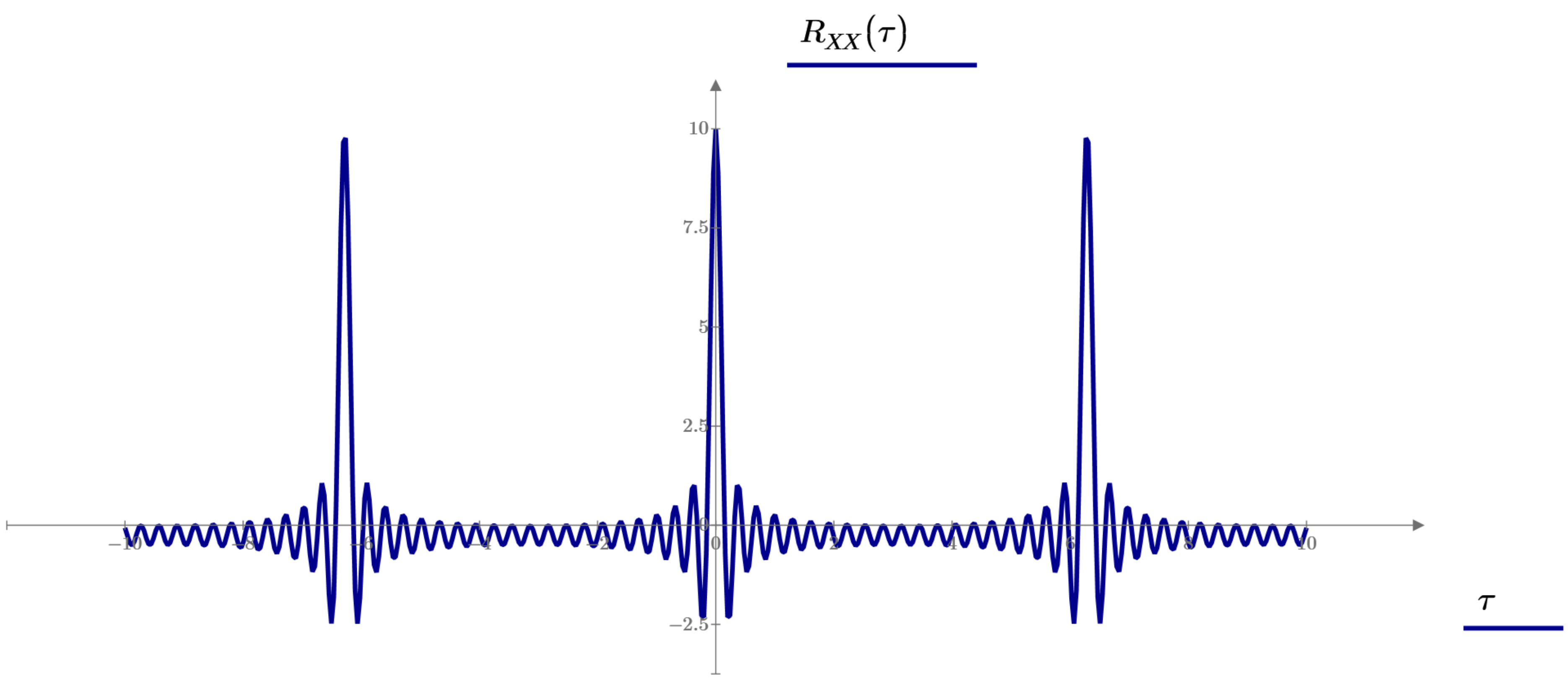
Random Sinusoid – Example (cont'd)



$$X(t) = \sum_{i=1}^{n} A_i \cos(\omega_i t + \varphi_i)$$

$$A_i \sim \mathcal{N}(0, \sigma^2)$$
$$\varphi_i \sim \mathcal{U}(0, 2\pi)$$
$$\omega_i = i \cdot \omega_0$$

$$\omega_0 = 1$$
$$\sigma^2 = 1$$
$$n = 20$$



$$R_{XX}(0) = n\frac{\sigma^2}{2} = 10$$

Two Stochastic Processes

- If we have two stochastic processes $X(t)$ and $Y(t)$
  - We can compare them by looking at the cross-correlation and cross-covariance:

Cross-correlation $R_{XY}(t_1, t_2) = E[X(t_1)Y(t_2)^*]$

Cross-covariance $C_{XY}(t_1, t_2) = E[X(t_1)Y(t_2)^*] - E[X(t_1)]E[Y(t_2)]$

Ensemble means that it applied for the ensemble of the two processes

Ensemble Cross-correlation

Tells about the connection between two different processes

- In general:

$$R_{XY}(t_1, t_2) = E[X(t_1)Y(t_2)^*]$$

$$= \iint_{-\infty}^{\infty} x(t_1)\, y(t_2)^* f_{X(t_1),Y(t_2)}\big(x(t_1), y(t_2)\big) dx(t_1) dy(t_2)$$

- For two WSS stationary processes:

$$R_{XY}(t_1, t_2) = R_{XY}(t_1 + T, t_2 + T) = E[X(t_1 + T)Y(t_2 + T)^*]$$

- We rewrite to: $R_{XY}(\tau) = E[X(t) \cdot Y(t + \tau)^*]$

Cross-Correlation Functions

- For Real WSS processes $X(t)$ and $Y(t)$:

$$R_{XY}(\tau) = E[X(t)Y(t+\tau)]$$

- Properties of the cross-correlation function $R_{XY}(\tau)$:
  - $R_{XY}(\tau) = R_{YX}(-\tau))$
  - $|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0)R_{YY}(0)} = \sqrt{E[X^2]E[Y^2]}$
  - $|R_{XY}(\tau)| \leq \frac{1}{2}(R_{XX}(0) + R_{YY}(0))$
  - If $X(t)$ and $Y(t)$ are orthogonal, then $R_{XY}(\tau) = 0$
  - If $X(t)$ and $Y(t)$ are independent, then $R_{XY}(\tau) = \mu_X \cdot \mu_Y$

Notice: If $X(t)$ and $Y(t)$ are WSS, and $X(t)$ are independent of $\mathrm{Y}(t)$ then:
$R_{XY}(\tau) = E[X(t)Y(t+\tau)] = E[X(t)] \cdot E[Y(t+\tau)]$

Temporal only looks at one realization of the two stochastic processes.

Temporal Cross-correlation

- The temporal cross-correlation between X and Y:

Convolution

$$\mathcal{R}_{XY}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x\,(t)\cdot y\,(t+\tau)dt$$

- If the two processes are ergodic the temporal cross-correlation is equal to the ensemble cross-correlation:

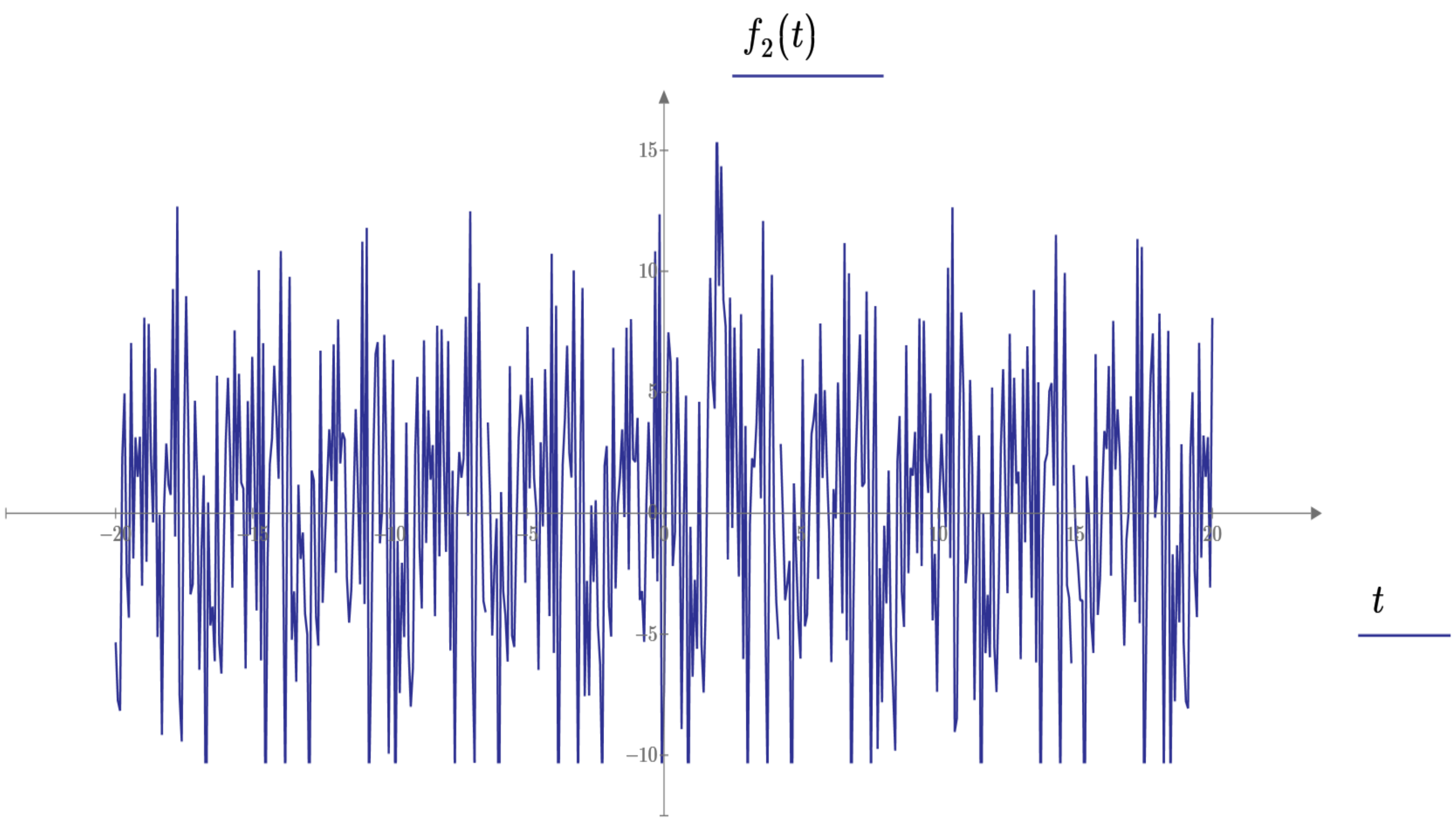
$$R_{XY}(\tau) = \mathcal{R}_{XY}(\tau)$$
$$R_{YX}(\tau) = \mathcal{R}_{YX}(\tau)$$

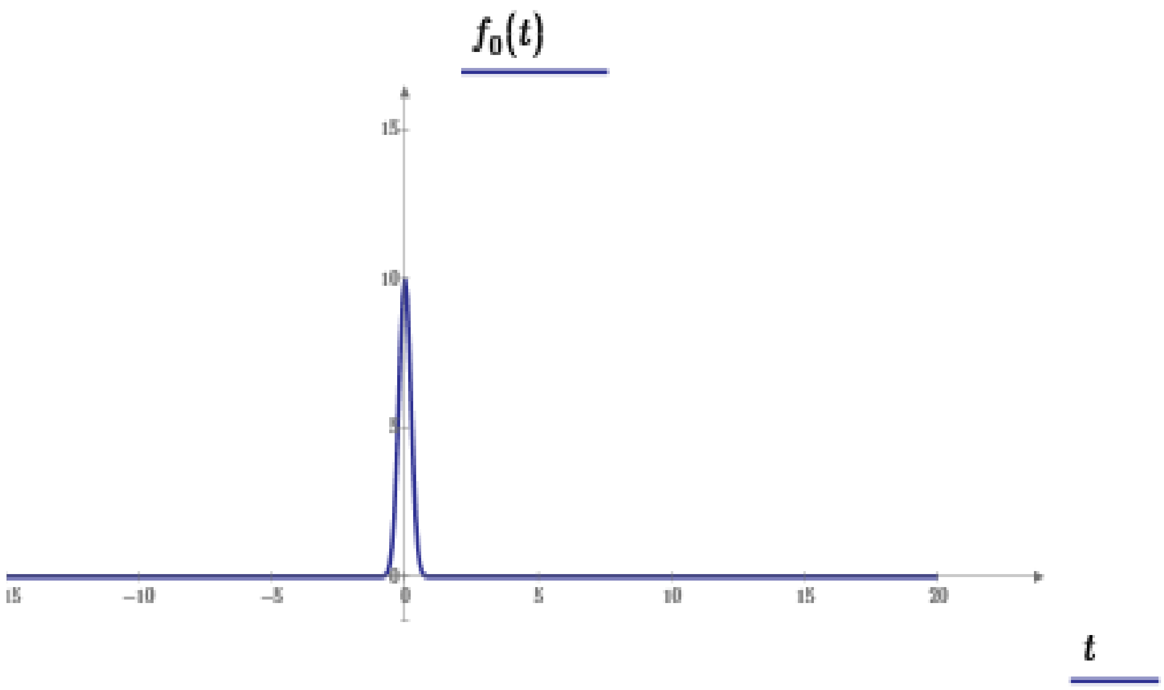
Ensemble

Temporal

Cross-correlation – Uncalibrated noisy signal

- Comparing two signals:
  - An uncalibrated and noisy signal: $f_2(t)$
  - Reference signal: $f_0(t) = 10 \cdot e^{-10t^2}$

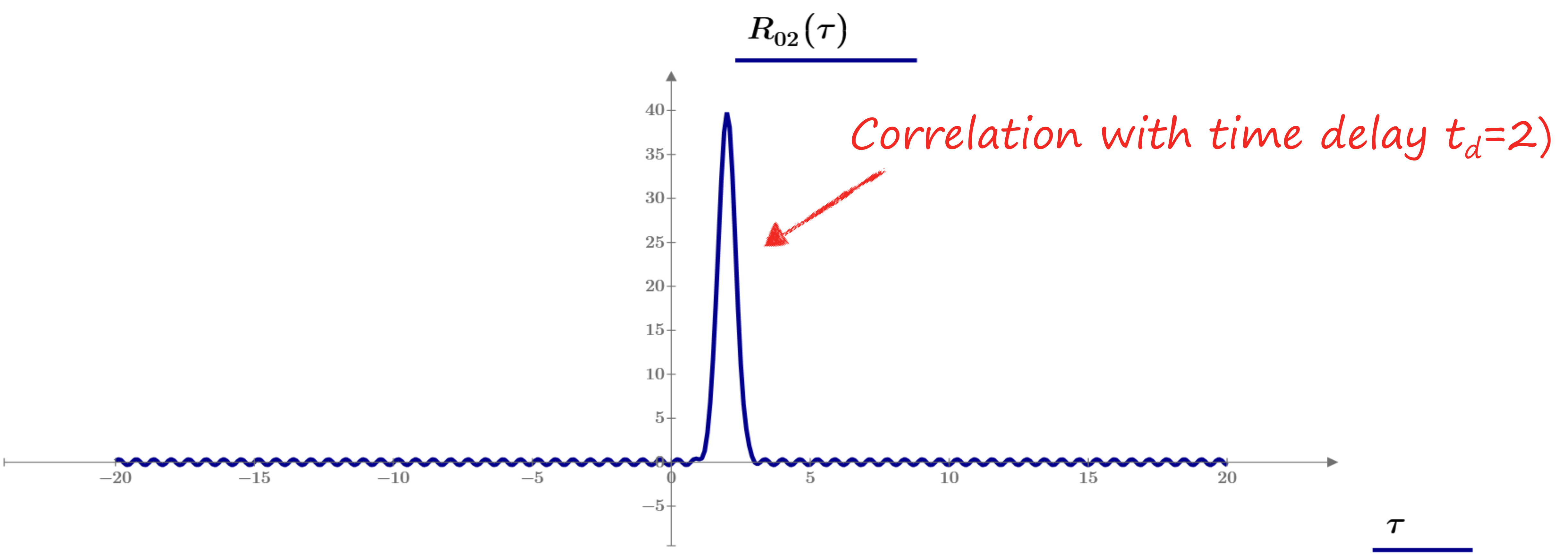




Cross-correlation – Uncalibrated noisy signal

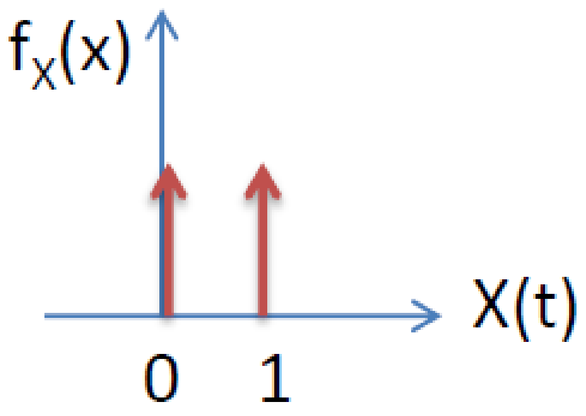
- Comparing two signals:
  - An uncalibrated and noisy signal: $f_2(t)$
  - Reference signal: $f_0(t) = 10 \cdot e^{-10t^2}$

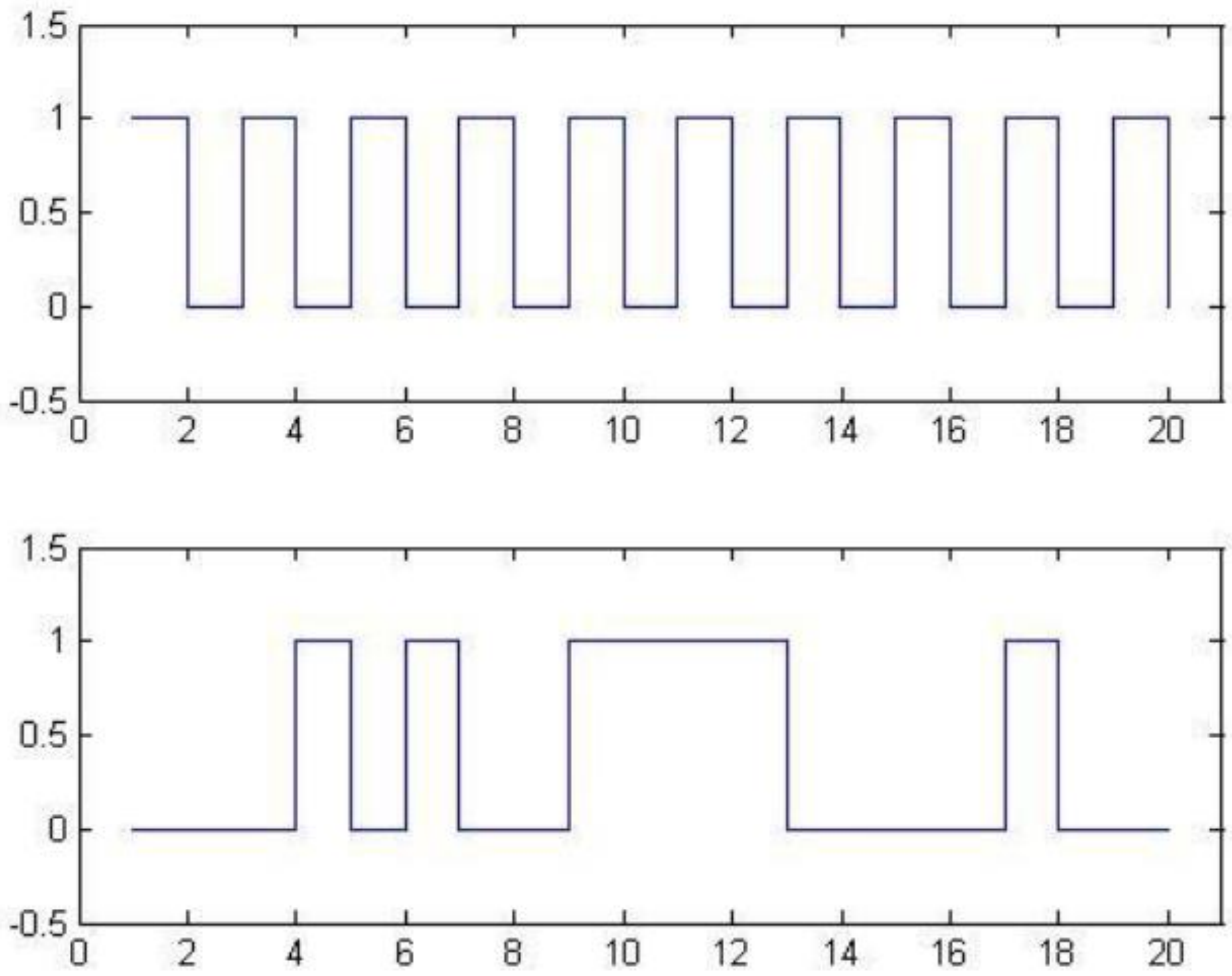
- Cross-correlation: $R_{02}(\tau) = \int_{-\infty}^{\infty} f_0(t) \cdot f_2(t+\tau) dt$



Deterministic vs. Stochastic signal

The probability mass function:





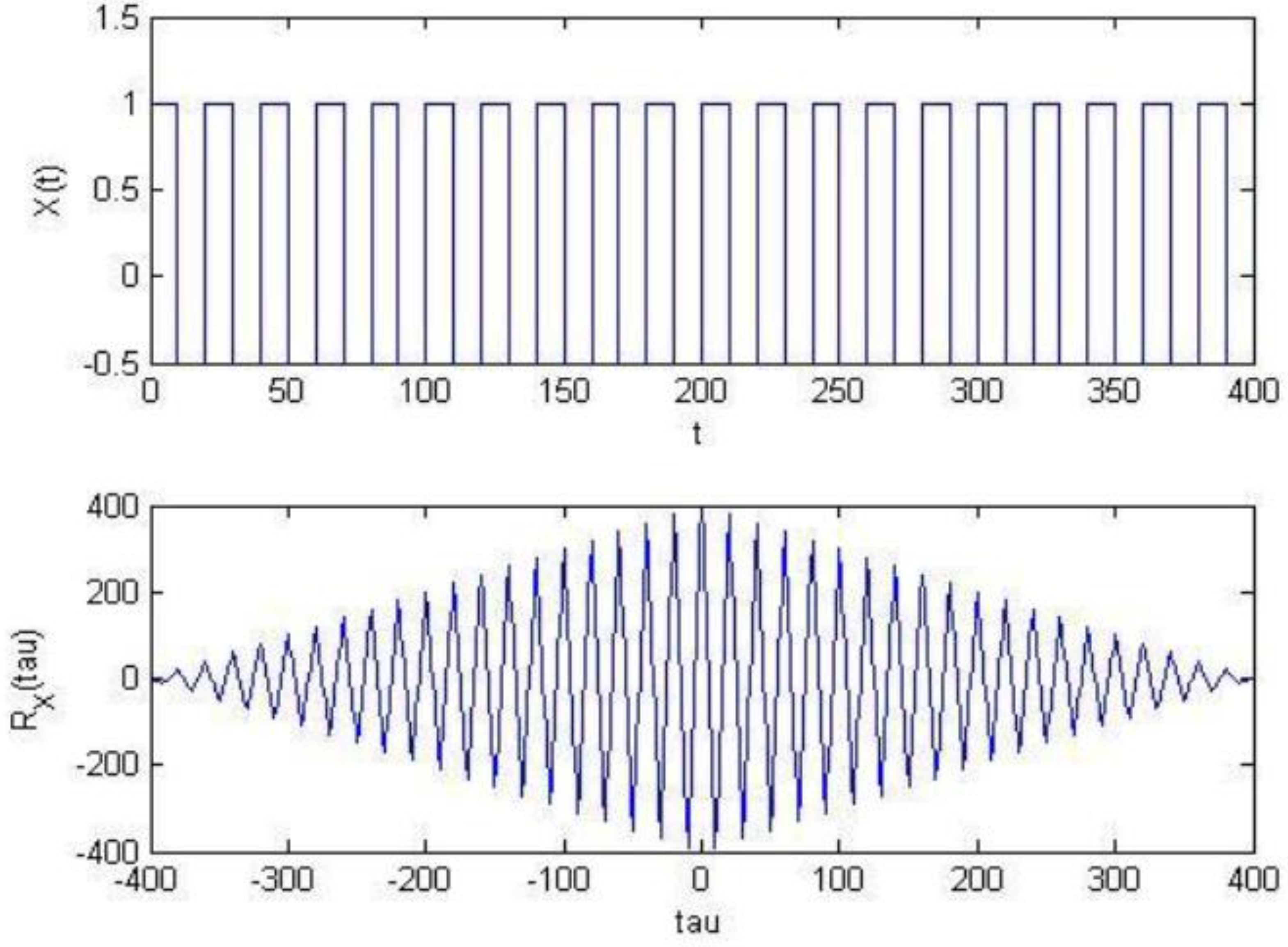
The two random processes have the same pmf.

Deterministic signal

Periodic signal

↓

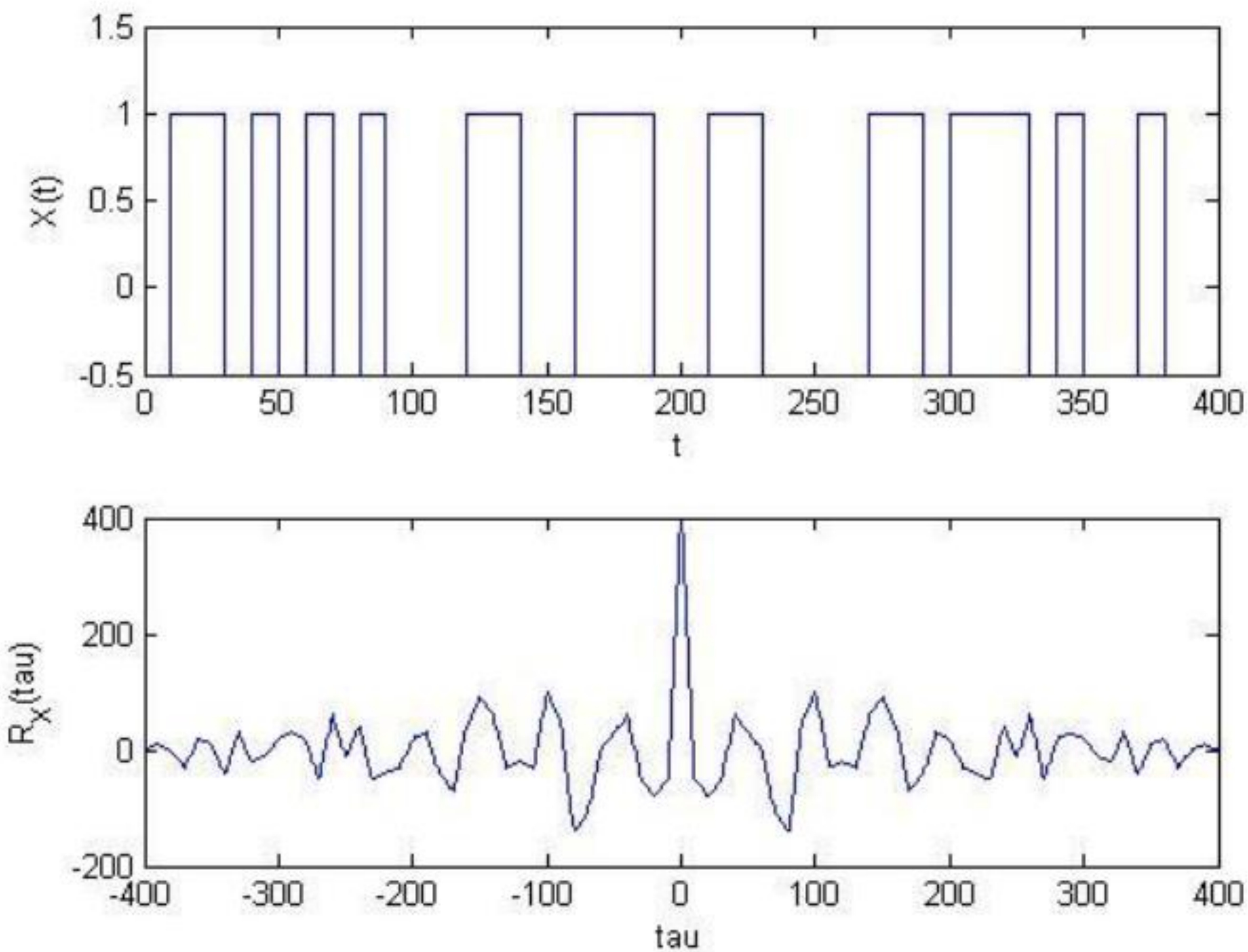
R_{XX} periodic




```
Rx = conv(x,fliplr(x));
```

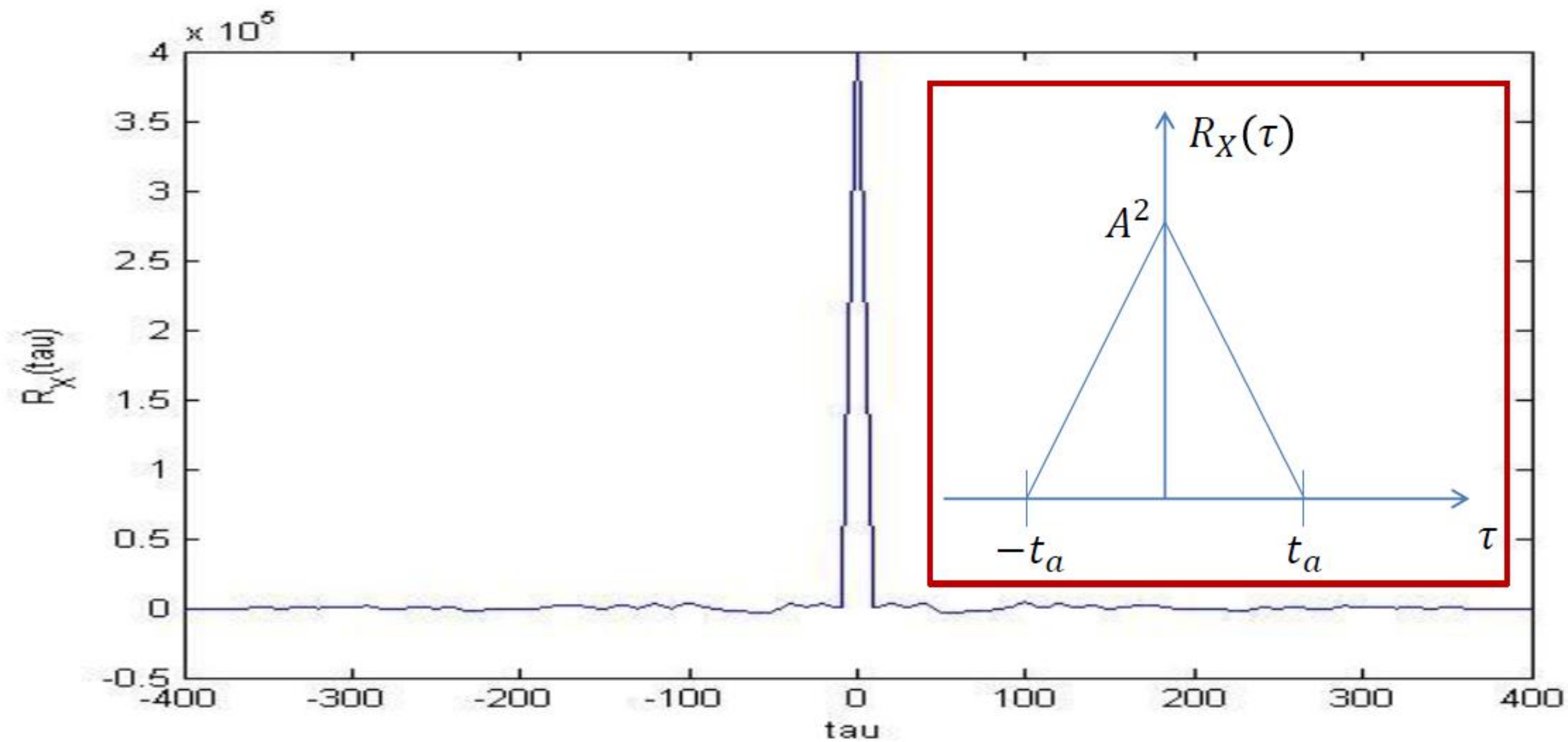
Stochastic signal

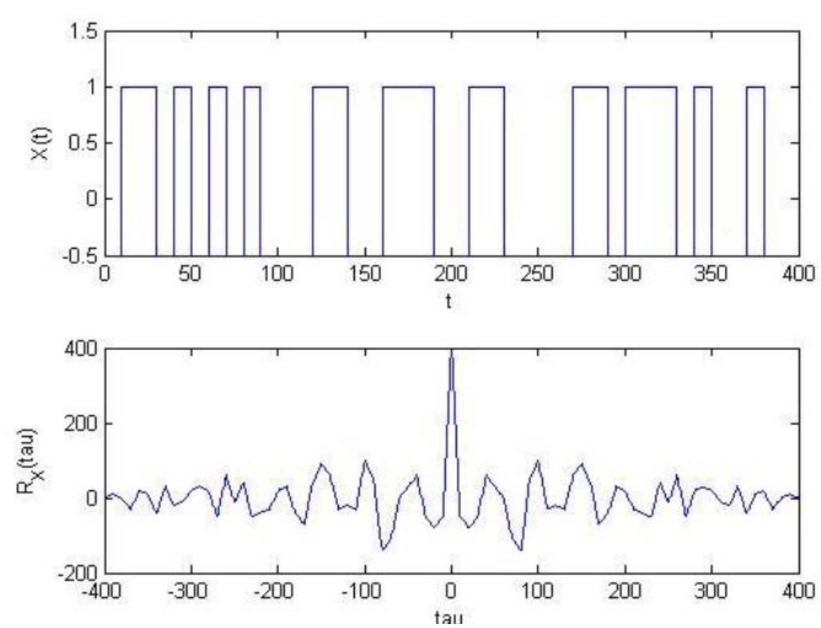
Also called Non-deterministic




```
Rx = conv(x,fliplr(x));
```

Autocorrelation for Stochastic signal

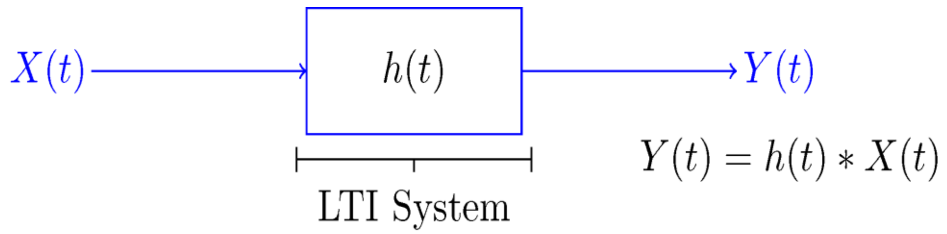




Autocorrelation function averaged over 1000 simulations.

Power Spectral Density (psd)

- Linear Time-Invariant (LTI) Systems



- Signal analysis → Frequency domain:
  - ➢ Deterministic signals $\mathrm{f}(t)$ → Fourier-transformation $\mathcal{F}(\mathrm{f}(t))$
  - ➢ Random signals $X(t)$ → ÷Fourier-transformation

- For Real WSS:
- Properties of the autocorrelation function $R_{XX}(\tau)$:
  - ➢ If $X(t)$ changes fast, then $R_{XX}(\tau)$ decreases fast from $\tau = 0$
  - ➢ If $X(t)$ changes slowly, then $R_{XX}(\tau)$ decreases slowly from $\tau = 0$
  - ➢ If $X(t)$ is periodic, then $R_{XX}(\tau)$ is also periodic

→ $R_{XX}(\tau)$ contain information about the frequency content in $X(t)$

Power Spectral Density (psd)

- WSS random signals $X(t)$:
- Power Spectral Density Function (psd):

Fourier-transform

- ➢ $S_{XX}(f) = \mathcal{F}(R_{XX}(\tau)) = \int_{-\infty}^{\infty} R_{XX}(\tau) e^{-j \cdot 2\pi f \cdot \tau} d\tau$

Invers Fourier-transform

- ➢ $R_{XX}(\tau) = \mathcal{F}^{-1}(S_{XX}(f)) = \int_{-\infty}^{\infty} S_{XX}(f) e^{j \cdot 2\pi f \cdot \tau} df$

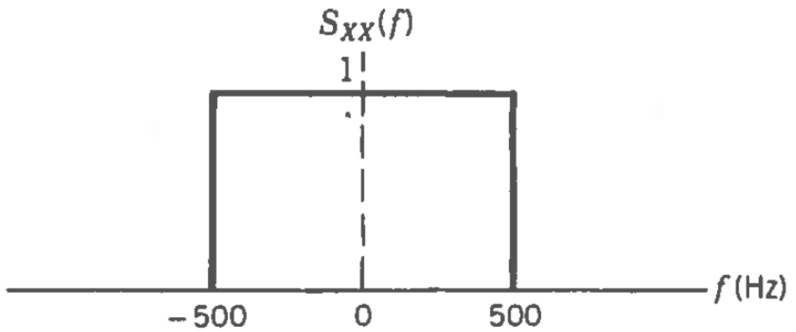


Figure 3.19a Psd of a lowpass random process $X(t)$.

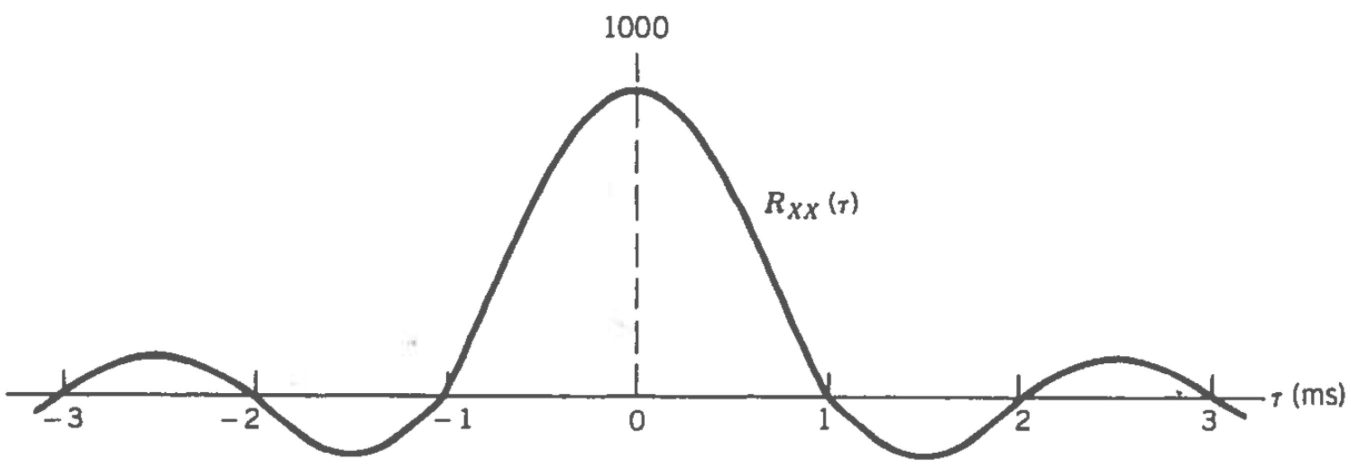


Figure 3.19b Autocorrelation function of $X(t)$.

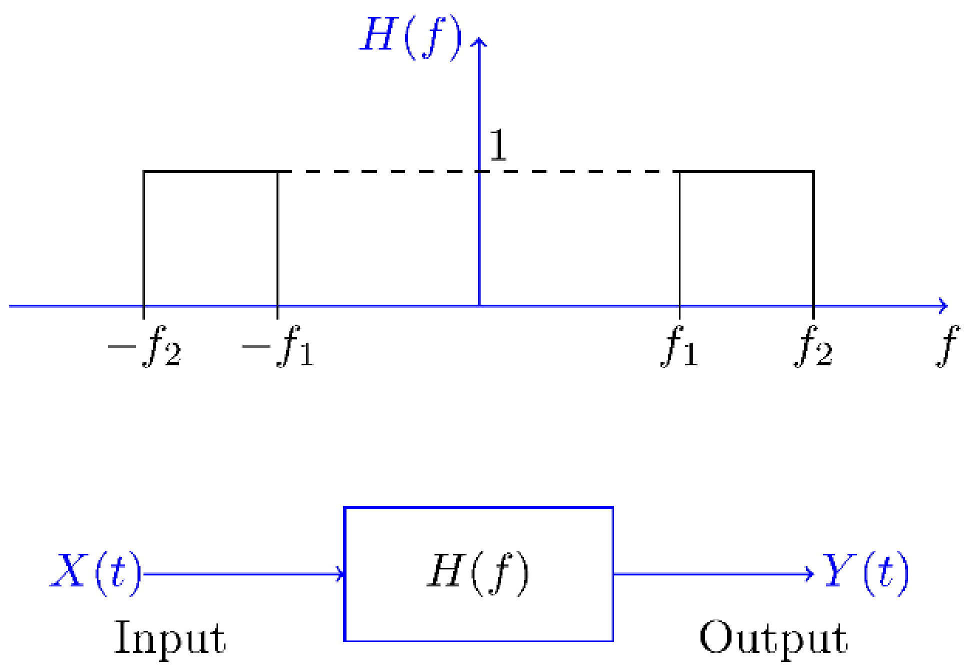
Figures from ”Random Signals”

Power Spectral Density (psd)

- Properties of psd $S_{XX}(f) = \mathcal{F}(R_{XX}(\tau))$ (spectrum of $X(t)$):
  - $S_{XX}(f) \in \mathbb{R}$
  - $S_{XX}(f) \geq 0$
  - If $X(t) \in \mathbb{R}$: $R_{XX}(-\tau) = R_{XX}(\tau)$ and $S_{XX}(-f) = S_{XX}(f) \rightarrow$ even functions
  - If $X(t)$ periodic components: $S_{XX}(f)$ will have impulses (δ−functions)

Impulse response

- Properties of psd $S_{XX}(f) = \mathcal{F}(R_{XX}(\tau))$ (spectrum of $X(t)$):
  - $S_{XX}(f) \in \mathbb{R}$
  - $S_{XX}(f) \geq 0$
  - If $X(t) \in \mathbb{R}$: $R_{XX}(-\tau) = R_{XX}(\tau)$ and $S_{XX}(-f) = S_{XX}(f) \rightarrow$ even functions
  - If $X(t)$ periodic components: $S_{XX}(f)$ will have impulses (δ−functions)



$S_{XX}(f)=S_{XX}(f) \cdot |H(f)|^2$

Power Spectral Density (psd)

Time-average

- Deterministic signals $x(t)$:
  - Average power: $P_X = \langle x(t)^2 \rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)^2 dt$

    i.e. if $x(t) = V(t)$ (voltage signal) $\rightarrow P_X =$ power in 1Ω-resistor

Average power in $X(t)$

- Stochastic WSS signals $X(t)$:
  - Average power: $P_X = E\left[X(t)^2\right] = R_{XX}(0) = \int_{-\infty}^{\infty} S_{XX}(f)df$
  - $[S_{XX}(f)] = \frac{W}{Hz} \rightarrow$ Distribution of power with frequency (power spectral density of the stationary random process $X(t)$)
  - $P_X[f_1, f_2] = 2\int_{f_1}^{f_2} S_{XX}(f)df$

    $\rightarrow$ Power in the frequency-interval $[f_1, f_2]$

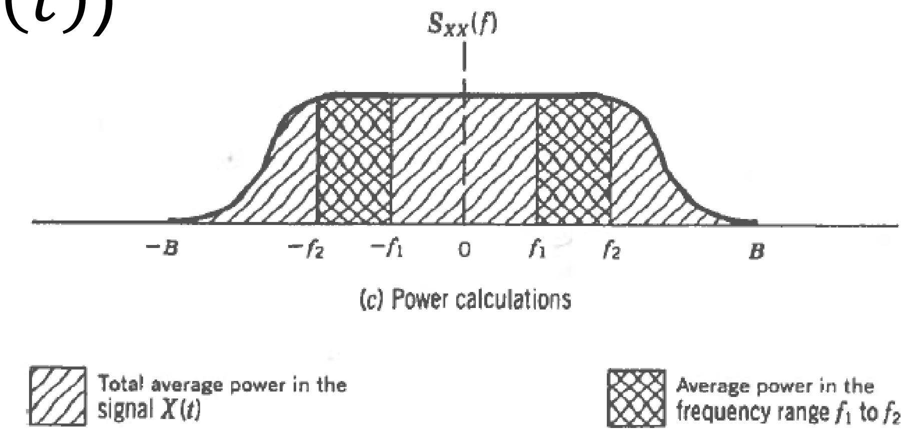
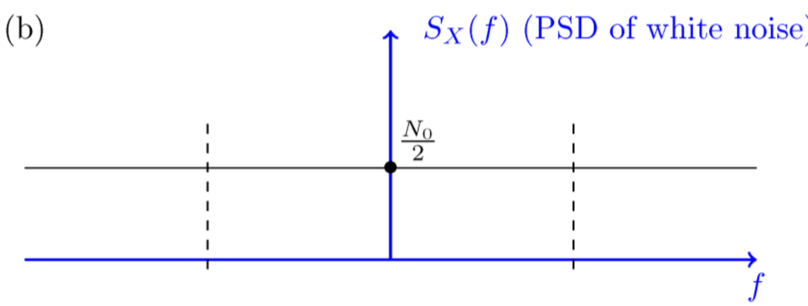


Figure from "Random Signals"

Power Spectral Density – White Gaussian Noise

- White Gaussian Noise ($\mu = 0$):



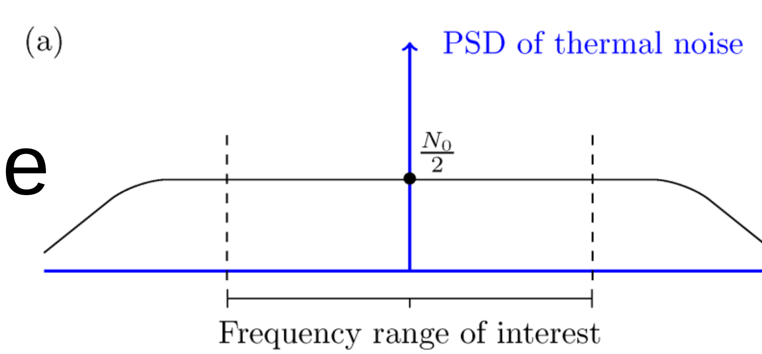
  - $S_{XX}(f) = \frac{N_0}{2}$ for all f
  - $R_{XX}(\tau) = \mathcal{F}^{-1}\big(S_{XX}(f)\big) = \frac{N_0}{2} \cdot \delta(\tau) = \begin{cases} \infty & for\ \tau = 0 \\ 0 & for\ \tau \neq 0 \end{cases}$

    $\rightarrow X(t_1)$ and $X(t_2)$ uncorrelated/independent for $t_1 \neq t_2$

  - $P_X = E\big[X(t)^2\big] = R_{XX}(0) = \int_{-\infty}^{\infty} S_{XX}(f)df = \infty$

- Thermal Noise

  $\sim$ White Gaussian Noise in frequency range



Words and Concepts to Know

Cross-correlation

psd

Power Spectral Density

Random walk

Deterministic

Cross-covariance

Autocorrelation

Temporal autocorrelation

Temporal Autocovariance

Autocorrelation Coefficient

Temporal cross-correlation

White Gaussian Noise

Auto-covariance

Non-deterministic