SMP

GA4 : Resistor Production

Gauss fordeling: $f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, $\mu = 100\,\Omega$

$F_x(x) = \phi\left(\frac{x-\mu}{\sigma}\right)$ $(\mathcal{N}(0,1))$

Sortering: A $(|R-100\,\Omega| \leq 5\%)$ $\rightarrow$ $95 \leq R \leq 105\,\Omega$

B $(5\% < |R-100\,\Omega| \leq 10\%)$ $\rightarrow$ $90 \leq R < 95 \vee 105 < R < 110\,\Omega$

C $(|R-100\,\Omega| > 10\%)$ $\rightarrow$ $R < 90 \vee R > 110\,\Omega$

1) $\sigma = 5\,\Omega$:

$$\underline{\underline{P(A)}} = \int_{95}^{105} f_x(x)dx = F_x(105) - F_x(95) = \phi\left(\frac{105-100}{5}\right) - \phi\left(\frac{95-100}{5}\right)$$

$$= \phi(1) - \phi(-1) = \phi(1) - (1-\phi(1)) = 2\phi(1) - 1$$

$$= 2 \cdot 0.8413 - 1 = 1.6826 - 1 = \underline{\underline{68.3\%}}$$

$$\underline{\underline{P(B)}} = P(90 < R < 95) + P(105 < R < 110)$$

$$= 2 \cdot P(105 < R < 110)$$

$$= 2 \cdot (F_x(110) - F_x(105)) = 2 \cdot \left(\phi\left(\frac{110-100}{5}\right) - \phi\left(\frac{105-100}{5}\right)\right)$$

$$= 2 \cdot (\phi(2) - \phi(1)) = 2 \cdot (0.9772 - 0.8413) = 2 \cdot 0.1359 = \underline{\underline{27.2\%}}$$

$$\underline{\underline{P(C)}} = 1 - P(A) - P(B) = 1 - (0.6826 + 0.2718) = 1 - 0.9544$$

$$= \underline{\underline{4.6\%}}$$

$$= P(R < 90) + P(R > 110) = F_x(90) + 1 - F_x(110)$$

$$= 1 + \phi\left(\frac{90-100}{5}\right) - \phi\left(\frac{110-100}{5}\right) = 1 + \phi(-2) - \phi(2)$$

$$= 1 + 1 - \phi(2) - \phi(2) = 2 \cdot (1 - \phi(2)) = 2 \cdot (1 - 0.9772)$$

$$= 2 \cdot 0.228 = \underline{\underline{4.6\%}}$$

3) $P(95 < R < 105) = \frac{1}{2}$, $\sigma = ?$

$\Downarrow$

$$F_x(105) - F_x(95) = \phi\left(\frac{105-100}{\sigma}\right) - \phi\left(\frac{95-100}{\sigma}\right) = \phi\left(\frac{5}{\sigma}\right) - \phi\left(\frac{-5}{\sigma}\right)$$
$$= \phi\left(\frac{5}{\sigma}\right) - \left(1 - \phi\left(\frac{5}{\sigma}\right)\right) = 2\phi\left(\frac{5}{\sigma}\right) - 1$$
$$= \frac{1}{2}$$

$\Downarrow$

$$\phi\left(\frac{5}{\sigma}\right) = \frac{3/2}{2} = \frac{3}{4} = 0.75$$

$\Downarrow$

$$\frac{5}{\sigma} = \phi^{-1}(0.75) = 0.6745 \Rightarrow \underline{\underline{\sigma}} = \frac{5}{0.6745} = \underline{\underline{7.41\,\Omega}}$$

4) $$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi \cdot 7.41^2}} e^{-\frac{(x-100)^2}{2 \cdot 7.41^2}} = 0.0538\, e^{-\frac{(x-100)^2}{109.8}}$$

$$F_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{x} f_X(x)\,dx$$

5) Middelværdi: $\overline{R} = \frac{1}{N} \sum_{i=1}^{N} R_i$

Varians: $Var(R) = \frac{1}{N} \sum_{i=1}^{N} R_i^2 - \overline{R}^2$

6) $\overline{R}$ og $Var(R)$ afviger fra μ og σ^2, da sample-prøven ikke er uendelig stor (kun indeholde N samples).