SMP S20

Opg. 1

Handelser: F = Chipset har en fejl $\quad$ $\bar{F}$: Chipset har ingen fejl

T: Test viser fejl $\quad$ $\bar{T}$: Test viser ingen fejl

Sandsynligheds data: $Pr(T|F) = 0.34$, $Pr(T|\bar{F}) = 0.08$

$Pr(F) = 0.025$

a) $\underline{\underline{Pr(F \cap T)}} = Pr(T|F) \cdot Pr(F) = 0.34 \cdot 0.025 = \underline{\underline{0.0085 = 0.85\%}}$

b) $\underline{\underline{Pr(\bar{F})}} = 1 - Pr(F) = 1 - 0.025 = \underline{\underline{0.975 = 97.5\%}}$

c) $\underline{\underline{Pr(T)}} = Pr(T \cap F) + Pr(T \cap \bar{F}) = Pr(T \cap F) + Pr(T|\bar{F}) Pr(\bar{F})$

$= 0.0085 + 0.08 \cdot 0.975 = \underline{\underline{0.0865 = 8.65\%}}$

d) $\underline{\underline{Pr(F|T)}} = \dfrac{Pr(T|F)\, Pr(F)}{Pr(T)} = \dfrac{0.34 \cdot 0.025}{0.0865} = \dfrac{Pr(F \cap T)}{Pr(T)} = \underline{\underline{0.098 = 9.8\%}}$

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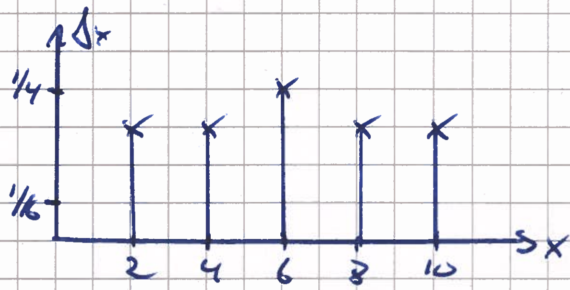
Opg. 2

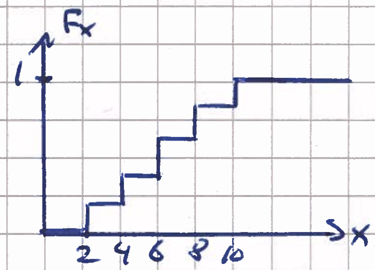
$f_{X,Y}$	2	4	6	8	10	f_Y
-1	$K/2$	$K/4$	$K/2$	$K/2$	$K/4$	$2K$
1	$K/4$	$K/2$	$K/2$	$K/4$	$K/2$	$2K$
f_X	$\frac{3}{4}K$	$\frac{3}{4}K$	K	$\frac{3}{4}K$	$\frac{3}{4}K$	$4K=1$

a) $\sum_x \sum_y f_{X,Y}(x,y) = K\cdot\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{2}+\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\frac{1}{2}+\frac{1}{2}+\frac{1}{4}+\frac{1}{2}\right) = 4K = 1 \Rightarrow \underline{\underline{K=\frac{1}{4}}}$

b) $f_X(x) = \sum_y f_{X,Y}(x,y) = \begin{cases} 3/16 & x=2 \\ 3/16 & x=4 \\ 1/4 & x=6 \\ 3/16 & x=8 \\ 3/16 & x=10 \end{cases}$



c) $F_X(x) = \Pr(X\le x) = \sum_{X\le x} f_X(x) = \begin{cases} 0 & x<2 \\ 3/16 & 2\le x<4 \\ 3/8 & 4\le x<6 \\ 5/8 & 6\le x<8 \\ 13/16 & 8\le x<10 \\ 1 & x\ge 10 \end{cases}$



d) $\underline{\underline{E[X] = \sum_x x f_X(x)}} = 2\cdot\frac{3}{16}+4\cdot\frac{3}{16}+6\cdot\frac{1}{4}+8\cdot\frac{3}{16}+10\cdot\frac{3}{16} = \frac{6+12+24+24+30}{16} = \frac{96}{6} = \underline{\underline{6}}$

$E[X^2] = \sum_x x^2 f_X(x) = 4\cdot\frac{3}{16}+16\cdot\frac{3}{16}+36\cdot\frac{1}{4}+64\cdot\frac{3}{16}+100\cdot\frac{3}{16} = \frac{12+48+144+192+300}{16}$

$= \frac{696}{16} = \frac{87}{2}$

$\underline{\underline{Var(X)}} = E[X^2]-E[X]^2 = \frac{87}{2}-36 = \frac{87-72}{2} = \frac{15}{2} = \underline{\underline{7.5}}$

e) $\underline{\underline{E[XY] = \sum_x\sum_y xy f_{XY}(x,y)}} = -2\cdot\frac{1}{8}-4\cdot\frac{1}{16}-6\cdot\frac{1}{8}-8\cdot\frac{1}{8}-10\cdot\frac{1}{16}+2\cdot\frac{1}{16}+4\cdot\frac{1}{8}+6\cdot\frac{1}{8}+8\cdot\frac{1}{16}+10\cdot\frac{1}{8}$

$= \frac{1}{16}\cdot(-4-4-12-16-10+2+8+12+8+20)$

$\underline{\underline{= \frac{4}{16} = \frac{1}{4}}}$

f) $\underline{\underline{\Pr(Y=1|X=6)}} = \frac{\Pr(Y=1 \wedge X=6)}{\Pr(X=6)} = \frac{f_{X,Y}(6,1)}{f_X(6)} = \frac{1/8}{1/4} = \underline{\underline{\frac{1}{2}}}$

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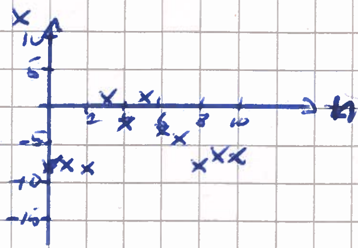
Opg. 3

$$X[n] = -1.5\cdot(Z[n]+1) = -1.5\,Z[n] - 1.5 \quad , \; Z[n] \sim \mathcal{N}(1, 10)$$

a) 3 realisationer, 10 samples:

Matlab: $X_n = -1.5\cdot(\underbrace{(\mathrm{sqrt}(10)\cdot \mathrm{randn}(1,10)+1)}_{Z[n] = \sigma\cdot\mathcal{N}(0,1)+\mu}+1);$



Se bilag!

b) Realisation 1: −6.5219, −8.4246, 1.3389, −3.8629, 1.6644, −4.8254, −4.5449, −9.1489, −8.2141, −6.0985

Tidslig middelværdi: $\underline{\underline{\langle X\rangle_T}} = \frac{1}{N}\sum_{n=1}^{N} X[n] = \frac{1}{10}\sum_{n=1}^{10} X[n] = \underline{\underline{-4.8628}}$

c) Ensemble middelværdi:

$$\underline{\underline{\mu_X[n]}} = E[-1.5(Z[n]+1)] = -1.5(E[Z[n]]+1) = -1.5\cdot(1+1) = \underline{\underline{-3}}$$

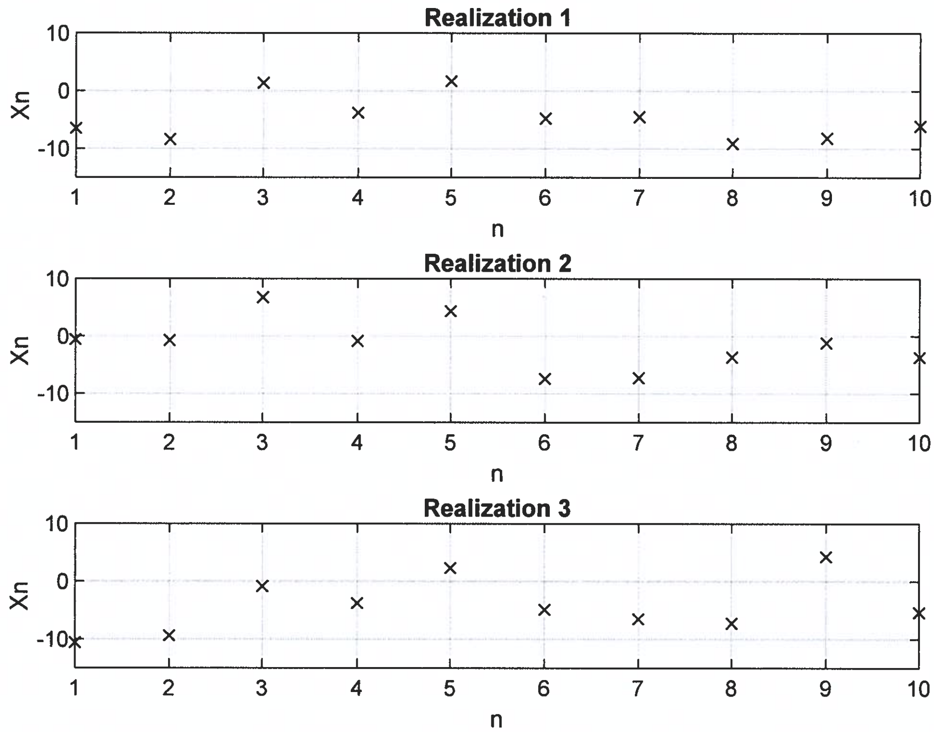
Ensemble varians:

$$\underline{\underline{\sigma_X^2[n]}} = Var(-1.5(Z[n]+1)) = (-1.5)^2(Var(Z[n]) + Var(1))$$
$$= \frac{9}{4}\cdot(10+0) = \frac{45}{2} = \underline{\underline{22.5}}$$

d) Da μ_X og σ_X^2 er uafhængig af n (tiden), $\underline{\underline{\text{er } X \text{ WSS}}}$

```
%%Opgave 3a S20
for i=1:3
    Xn=-1.5*((sqrt(10)*randn(1,10)+1)+1) %10 samples of stochastic proces
    Temporal_mean=sum(Xn)/10 %Mean of the 10 samples

    figure(1)
    ax=subplot(3,1,i); %Plot of three realizations
    n=1:10; %10 samples between 1 and 10
    plot(ax,n,Xn,'kx')
    grid
    axis([1,10,-15,10])
    title(ax,['Realization ',num2str(i)])
    xlabel(ax,'n')
    ylabel(ax,'Xn')
end
```



Realization 1: Xn = -6.5219 -8.4246 1.3389 -3.8529 1.6644 -4.8254 -4.5449 -9.1489 -8.2141 -6.0985

Temporal_mean = -4.8628

Realization 2: Xn = -0.6042 -0.7422 6.7314 -0.8735 4.3580 -7.4106 -7.2783 -3.6560 -1.2051 -3.6786

Temporal_mean = -1.4359

Realization 3: Xn = -10.6134 -9.3994 -0.8729 -3.7988 2.3150 -4.9008 -6.5057 -7.2693 4.2632 -5.3936

Temporal_mean = -4.2176

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Opg. 4

H [cm]	55	60	70	75	79	90	101	112	121	129	134	143
A [mdr]	1	3	6	9	12	24	36	48	60	72	84	96

, $N = 12$

a) $H = \alpha + \beta \cdot A + \varepsilon \quad , \quad \varepsilon \sim \mathcal{N}(0, \sigma^2)$

b) $\mu_H = \frac{1}{12}\sum_{i=1}^{12} H_i = 97.4 \qquad \mu_A = \frac{1}{12}\sum_{i=1}^{12} A_i = 37.583$

$$\left.\begin{aligned} \beta &= \frac{\sum (A_i - \mu_A)(H_i - \mu_H)}{\sum (A_i - \mu_A)^2} = 0.88 \\ \alpha &= \mu_H - \beta \cdot \mu_A = 64.3 \end{aligned}\right\} \Rightarrow \underline{\underline{H = 64.3 + 0.88 \cdot A \quad [cm]}}$$

c) $H_0 : \beta = 0 \quad , \quad H_1 = \beta \neq 0$

d) $s_x^2 = \sum (A_i - \mu_A)^2 = 12553 \Rightarrow s_x = \sqrt{12553} = 112.0$

$s_r^2 = \frac{1}{10}\sum (H_i - (\alpha + \beta A_i))^2 = 32.10 \Rightarrow s_r = \sqrt{32.1} = 5.67$

$$t = \frac{\beta - 0}{s_r / s_x} = \frac{0.88}{5.67/112} = 17.4 \sim t(10)$$

p-val $= 2 \cdot (1 - \text{tcdf}(17.4, 10)) \simeq 8 \cdot 10^{-9} << 0.05 \Rightarrow$ $\underline{\underline{H_0 \text{ afvises} \Rightarrow \beta \neq 0}}$

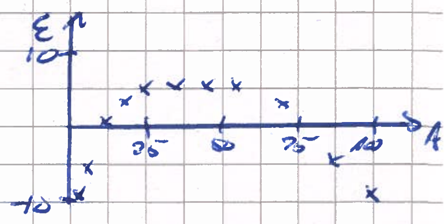
e) $t_0 = \text{tinvcdf}(0.975, 10) = 2.228$

$$\Delta\beta = t_0 \cdot \frac{s_r}{s_x} = 2.228 \cdot \frac{5.67}{112.0} = 0.113$$

$\beta_{min} = \beta - \Delta\beta = 0.88 - 0.113 = 0.767$

$\beta_{max} = \beta + \Delta\beta = 0.88 + 0.113 = 0.993$

95% konfidensinterval : $\underline{\underline{[0.767 ; 0.993]}}$



f) Residualer : $\varepsilon_i = H_i - (\alpha + \beta A_i)$

Residualerne danner en ∩-form. Det betyder, at der formodentligt <u>ikke</u> <u>er en lineær sammenhæng</u> mellem alder og højde.