Opgave 1

P: Parasit T: Positiv test

$Pr\_P\_givet\_T := 0.51$ $Pr\_nP\_givet\_T := 1 - Pr\_P\_givet\_T = 0.49$

$Pr\_nP\_givet\_nT := 0.56$ $Pr\_P\_givet\_nT := 1 - Pr\_nP\_givet\_nT = 0.44$

$Pr\_T := 0.4$ $Pr\_nT := 1 - Pr\_T = 0.6$

a) $Pr\_P\_og\_T := Pr\_P\_givet\_T \cdot Pr\_T = 0.204$

b) $Pr\_P\_og\_nT := Pr\_P\_givet\_nT \cdot Pr\_nT = 0.264$

$Pr\_P := Pr\_P\_og\_T + Pr\_P\_og\_nT = 0.468$

c) $Pr\_T\_givet\_P := \frac{Pr\_P\_givet\_T \cdot Pr\_T}{Pr\_P} = 0.436$

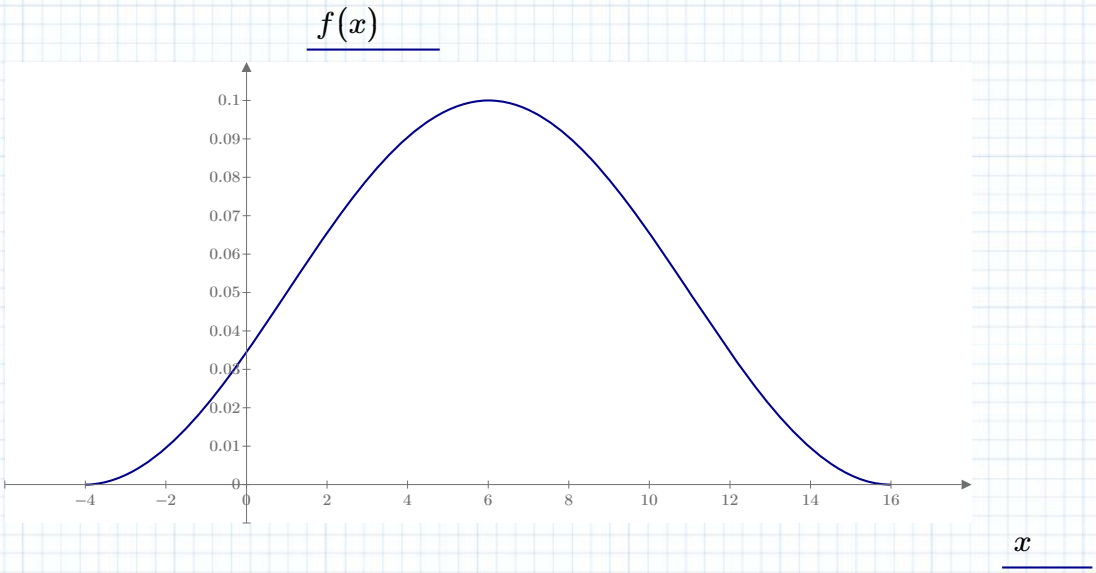
Opgave 2

$$F(x) := \frac{1}{2} \cdot \left(1 + \frac{x-6}{10} + \frac{1}{\pi} \cdot \sin\left(\pi \cdot \frac{x-6}{10}\right)\right)$$

a) $Pr(X>5) = 1 - Pr(X<5) = 1 - F(5)$ $\qquad 1 - F(5) = 0.599$

b) $Pr(X=5) = 0$

c) $f(x) := \frac{d}{dx} F(x) \rightarrow \frac{\cos\left(\pi \cdot \left(\frac{x}{10} - \frac{3}{5}\right)\right)}{20} + \frac{1}{20}$



d) $E_X := \int_{-4}^{16} x \cdot f(x)\, dx = 6$

e) $E_{XX} := \int_{-4}^{16} x^2 \cdot f(x)\, dx = 49.069$

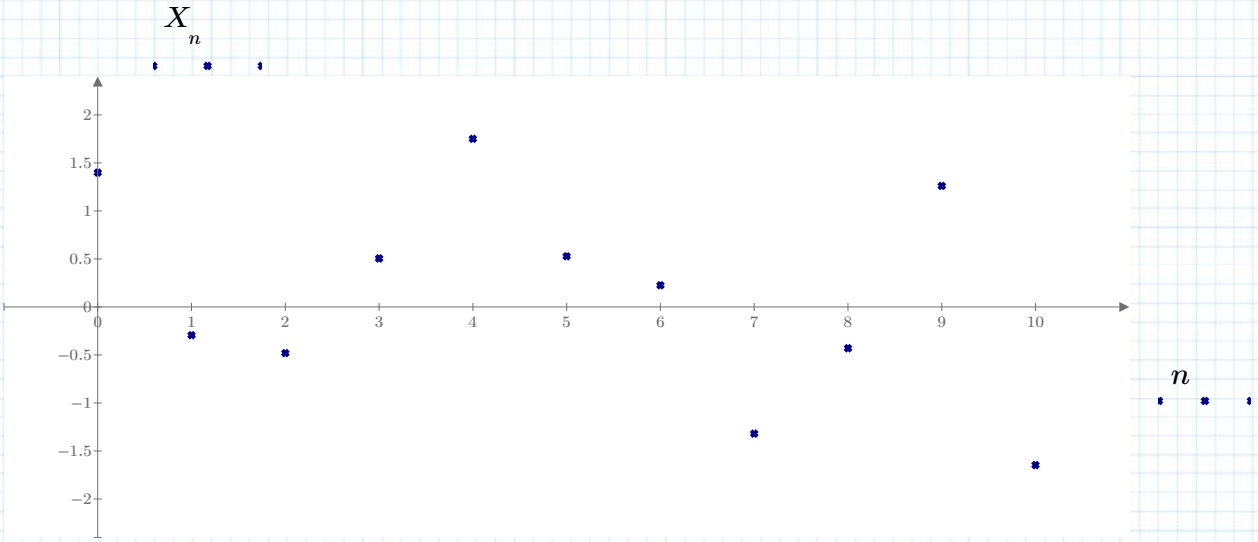
$Var_X := E_{XX} - E_X{}^2 = 13.069$

Opgave 3

a) Én realisation, 11 samples:

$$Y := \text{rbinom}(1, 2, 0.2) \cdot [1] = 0 \qquad W := \text{rnorm}\left(11, 0, \sqrt{2}\right) = \begin{bmatrix} 1.399 \\ -0.293 \\ -0.48 \\ 0.506 \\ 1.752 \\ 0.528 \\ 0.226 \\ -1.318 \\ -0.429 \\ 1.261 \\ -1.646 \end{bmatrix}$$

$$n := 0, 1..10 \qquad X_n := -Y + W_n \qquad X = \begin{bmatrix} 1.399 \\ -0.293 \\ -0.48 \\ 0.506 \\ 1.752 \\ 0.528 \\ 0.226 \\ -1.318 \\ -0.429 \\ 1.261 \\ -1.646 \end{bmatrix}$$



b) Ensemble middelværdi: $E(X) = E(-Y+W) = -E(Y) + E(W) = n \cdot p + \mu_W = 2 \cdot 0.2 + 0 = 0.4$

Ensemble varians: $Var(X) = Var(-Y+W) = (-1)^2 \cdot E(Y) + E(W)$

$Var(X) = n \cdot p \cdot (1-p) + {\sigma_W}^2 = 2 \cdot 0.2 \cdot 0.8 + 2 = 0.32 + 2 = 2.32$

c) Temporal middelværdi: $\mu_{X_n} = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N} X(n) = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N} (-Y + W(n)) = -Y + \mu_{W_n} = -Y + 0 = -Y$

d) $E(X)$ og $Var(X)$ uafhængig af n (tiden) --> X er WSS (Wide Sense Stationary)

$E(X) \neq \mu_{X_n}$ --> X er ikke ergodisk

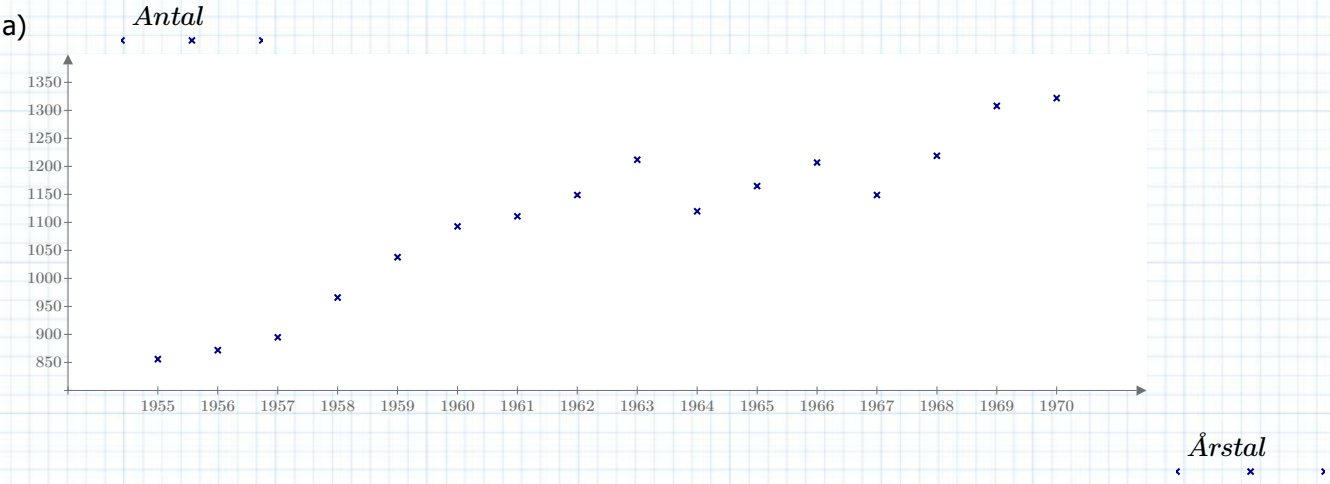
Opgave 4

Data:

$$Årstal := [1955 \;\; 1956 \;\; 1957 \;\; 1958 \;\; 1959 \;\; 1960 \;\; 1961 \;\; 1962 \;\; 1963 \;\; 1964 \;\; 1965 \;\; 1966 \;\; \ldots]$$

$$Antal := [856 \;\; 872 \;\; 895 \;\; 966 \;\; 1038 \;\; 1093 \;\; 1111 \;\; 1149 \;\; 1212 \;\; 1120 \;\; 1165 \;\; 1207 \;\; \ldots]$$

a)



b) Middelværdier:

$$\mu_{År} := \frac{1}{16} \cdot \sum_{i=0}^{15} Årstal_{0,i} = 1962.5$$

$$\mu_{Antal} := \frac{1}{16} \cdot \sum_{i=0}^{15} Antal_{0,i} = 1105.1$$

c) Lineær regression:

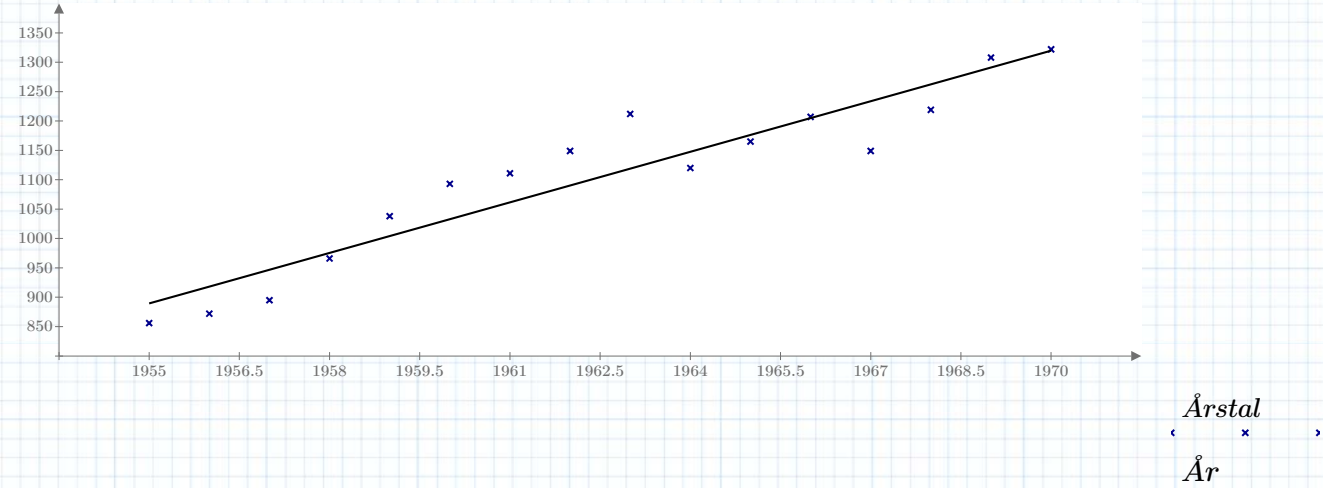
Hældning:

$$\beta := \frac{\sum_{i=0}^{15} \left(\left(Årstal_{0,i} - \mu_{År} \right) \cdot \left(Antal_{0,i} - \mu_{Antal} \right) \right)}{\sum_{i=0}^{15} \left(Årstal_{0,i} - \mu_{År} \right)^2} = 28.679$$

Skæring:

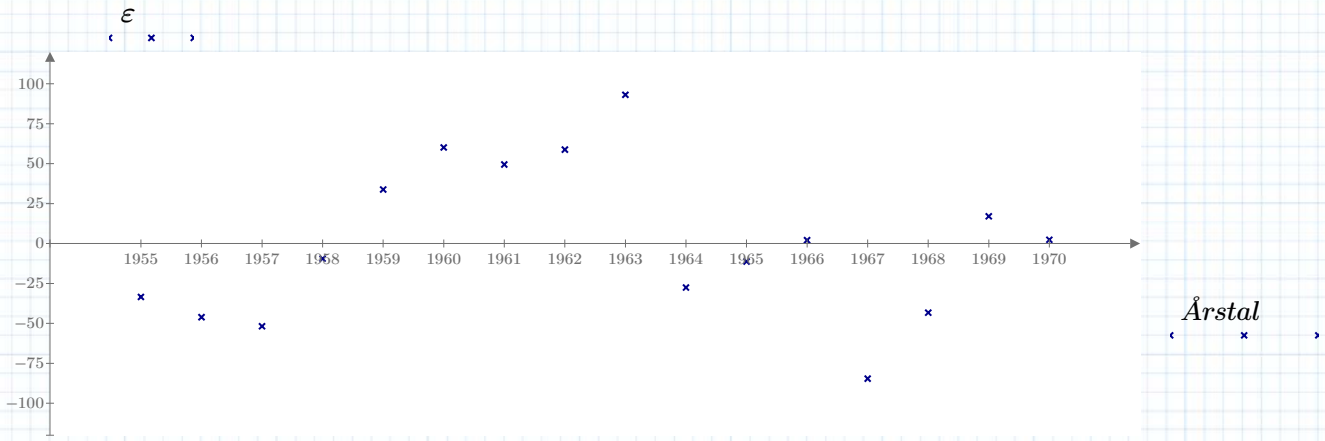
$$\alpha := \mu_{Antal} - \beta \cdot \mu_{År} = -55178.221$$

$$Antal_{est}(År) := \alpha + \beta \cdot År \xrightarrow{float, 5} 28.679 \cdot År - 55178.0 \qquad År := 1955, 1956 .. 1970$$



d) Residualer: $\varepsilon := Antal - Antal_{est}(Årstal)$

$$\varepsilon = \begin{bmatrix} -33.4 & -46.1 & -51.8 & -9.5 & 33.8 & 60.2 & 49.5 & 58.8 & 93.1 & -27.6 & -11.2 & 2.1 & -84.6 & -43.3 & 17 & 2.4 \end{bmatrix}$$



e) Hypotesetest på hældning β=0: H0: $\beta = 0$ H1: $\beta \neq 0$

f) $s2_x := \sum_{i=0}^{15} \left(Årstal_{0,i} - \mu_{År}\right)^2 = 340$ $s_x := \sqrt{s2_x} = 18.439$

$s2_r := \frac{1}{14} \cdot \sum_{i=0}^{15} \left(Antal_{0,i} - Antal_{est}\left(Årstal_{0,i}\right)\right)^2 = 2541.166$ $s_r := \sqrt{s2_r} = 50.41$

$t := \frac{\beta - 0}{\frac{s_r}{s_x}} = 10.5$ $p := 2 \cdot (1 - \mathrm{pt}(t, 14)) = 5.146 \cdot 10^{-8}$

g) $p = 5.146 \cdot 10^{-8}$ < 0,05 --> Hypotesen afvises --> Hældningen $\beta \neq 0$

95% konfidensinterval:

$t_0 := \mathrm{qt}(0.975, 14) = 2.145$

$\Delta\beta := t_0 \cdot \frac{s_r}{s_x} = 5.864$

$\beta_{min} := \beta - \Delta\beta = 22.816$ $\beta_{max} := \beta + \Delta\beta = 34.543$

$\beta = 0 < \beta_{min}$ --> Hypotesen afvises --> Hældningen $\beta \neq 0$