SMP L7 - Efterår 2015 assignment 2

Assignment 2: Stochastic Processes

A discreet stochastic process is given by:

$X(n) = w(n),$

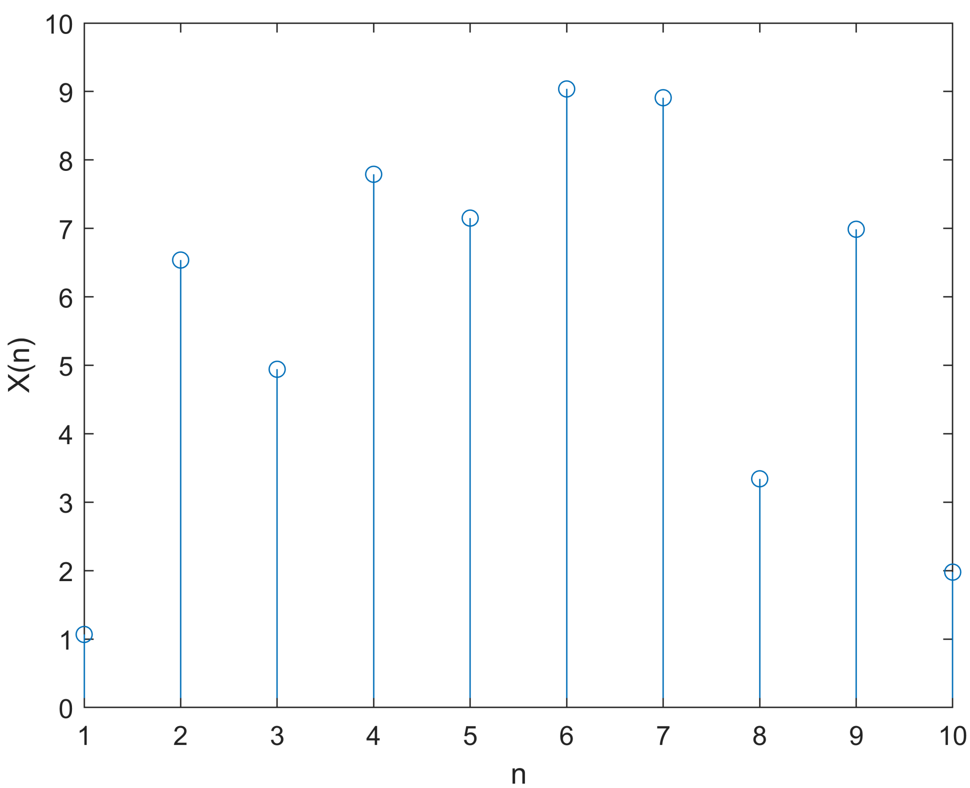
$w(n)$ is i.i.d. and distributed according to a uniform distribution according to $w(n) \sim \mathcal{U}(0,10)$.

1) Make a sketch with 10 samples of one realization of the process $X(n)$, i.e. for $n = 1, \cdots, 10$.

The matlab function unifrnd can be used to generate continuous uniform random numbers.

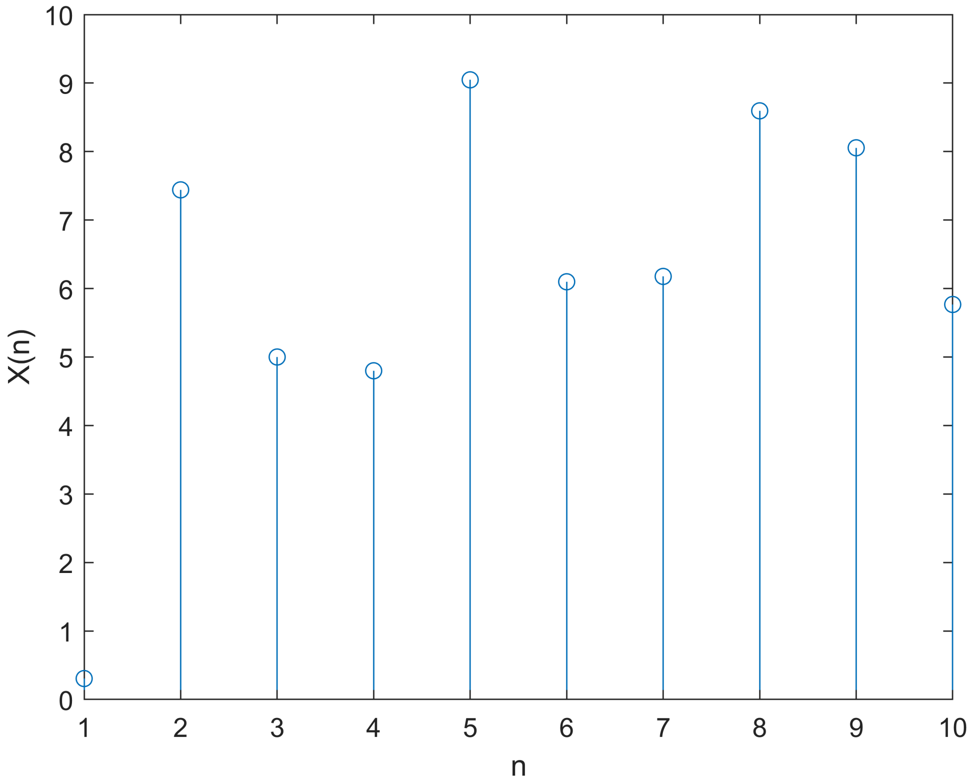
```
a = 0; % lower
b = 10; % upper
X = unifrnd(a,b,1,10);

stem(X)
xlabel('n')
ylabel('X(n)')
```



To generate randomly distributed values, we can use the rand() function.

```
b = 10; % upper
a = 0; % lower
n = 10; % number of samples
X = 10*rand([1,n]);
stem(X)
xlabel('n')
ylabel('X(n)')
```



2) Find the ensemble mean value and the variance for the process $X(n)$.

The ensemble mean can be calculated by looking at the formula book, p. 54 for continuous uniform distributions:

$$E[X] = \frac{a+b}{2} = \frac{0+10}{2} = 5$$

```
ensemble_mean = 5;
```

The ensemble variance can be calculated by using the formula in the book, p. 54:

$$\mathrm{Var}(X) = \frac{(b-a)^2}{12} = \frac{(10-0)^2}{12} = \frac{100}{12} = \frac{25}{3} = 8.33$$

```
ensemble_var = 25/3;
```

3) Find the autocorrelation $R_{XX}(\tau)$ for the process $X(n)$ for $\tau = 0, \cdots, 3$.

The autocorrelation is defined by: $R_{\mathrm{XX}}(\tau) = E[X(t)X(t+\tau)]$

$R_{\mathrm{XX}}(0) = E[w(n) \cdot w(n)] = \mathrm{Var}(w(n)) + E[w(n)]^2$

```
rxx0 = ensemble_var + ensemble_mean^2
```

```
rxx0 = 33.3333
```

$R_{\mathrm{XX}}(1) = E[w(n) \cdot w(n+1)] = E[w(n)] \cdot E[w(n+1)]$ We can do the last trick, since w(n) and w(n+1) are i.i.d.

```
rxx1 = ensemble_mean^2
```

```
rxx1 = 25
```

$R_{\mathrm{XX}}(2) = E[w(n) \cdot w(n+2)] = E[w(n)] \cdot E[w(n+2)]$

```
rxx2 = ensemble_mean^2
```

```
rxx2 = 25
```

$R_{\mathrm{XX}}(3) = E[w(n) \cdot w(n+3)] = E[w(n)] \cdot E[w(n+3)]$

```
rxx3 = ensemble_mean^2
```

```
rxx3 = 25
```

4) State whether the process is WSS (wide sense stationary) and whether it is ergodic, specify the reason behind your answers.

The process is WSS since the ensemble mean and variance are independent of time.

The process is also ergodic because the ensemble mean and variance are equal to the temporal mean and variance.

SMP L7 - Efterår 2015 Reeksamen assignment 2

Assignment 2: Stochastic Processes

A discreet stochastic process is given by:

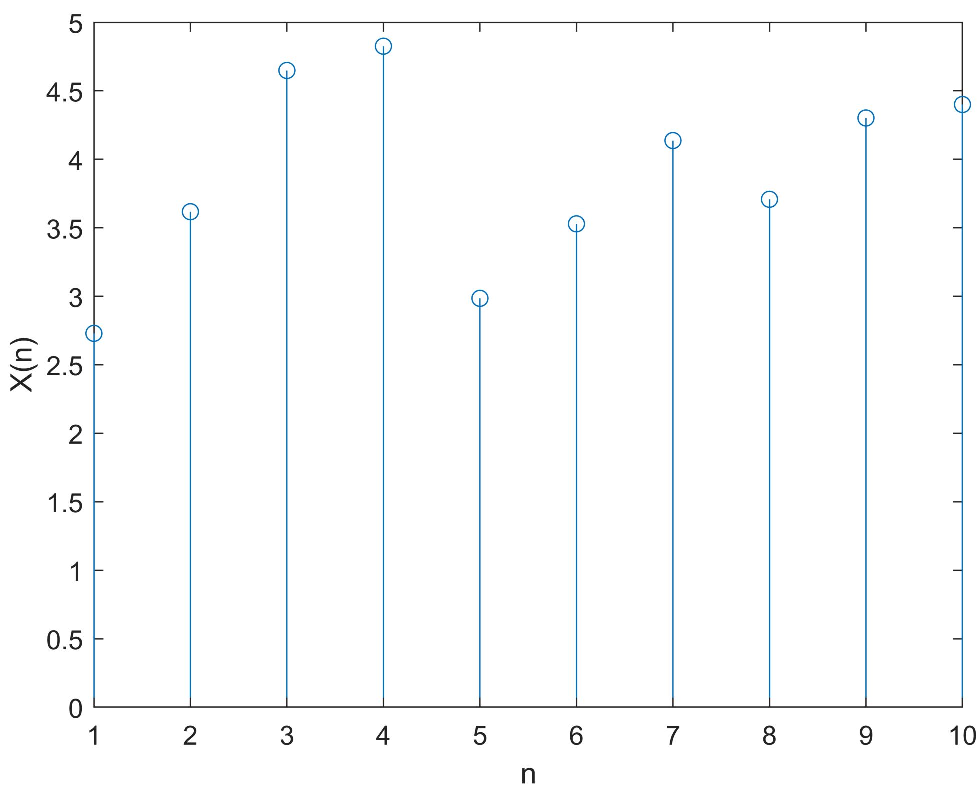
$$X(n) = w(n) + 4$$

Where a sample n of w is an i.i.d Gaussian distributed stochastic random variable $w(n) \sim N(0,1)$.

1) Sketch 10 samples ($n = 1, 2, \ldots, 10$) of a realization of the process $X(n)$.

A normal gaussian distribution can be created using matlabs function normrnd()

```
mu = 0;
sigma = 1;
n = 10;
wn = normrnd(mu,sigma,1,n);
X = wn + 4;
stem(X)
xlabel('n')
ylabel('X(n)')
```



2) Find the ensemble mean value and the ensemble variance for the process $X(n)$.

The ensemble mean can be calculated using the formula for a standard normal distribution on pg. 52: $E[w(n)] = 0$

$$E[X] = E[w(n) + 4] = E[w(n)] + E[4] = 0 + 4 = 4$$

```
ensemble_mean = 4;
```

The ensemble variance can be calculated using the formula for a standard normal distribution on pg. 52: $\mathrm{Var}(w(n)) = 1$

$$\mathrm{Var}(X) = E[X(n)^2] - E[X(n)]^2 = E[w(n)^2] + E[4^2] - 4^2 = E[w(n)^2] = 1$$

```
ensemble_var = 1;
```

3) State whether the process is WSS (wide sense stationary), and whether it is ergodic. State the reason behind your answers.

The process is WSS, since the ensemble mean and variance are not dependent of time.

The process is ergodic since the mean and variance can be calculated from just one realization.

SMP L7 - Forår 2016 assignment 2

Assignment 2: Stochastic Processes

A continuous stochastic process is given by:

$$X(t) = w + 4$$

Where w is Gaussian distributed after $w \sim N(5,1)$.

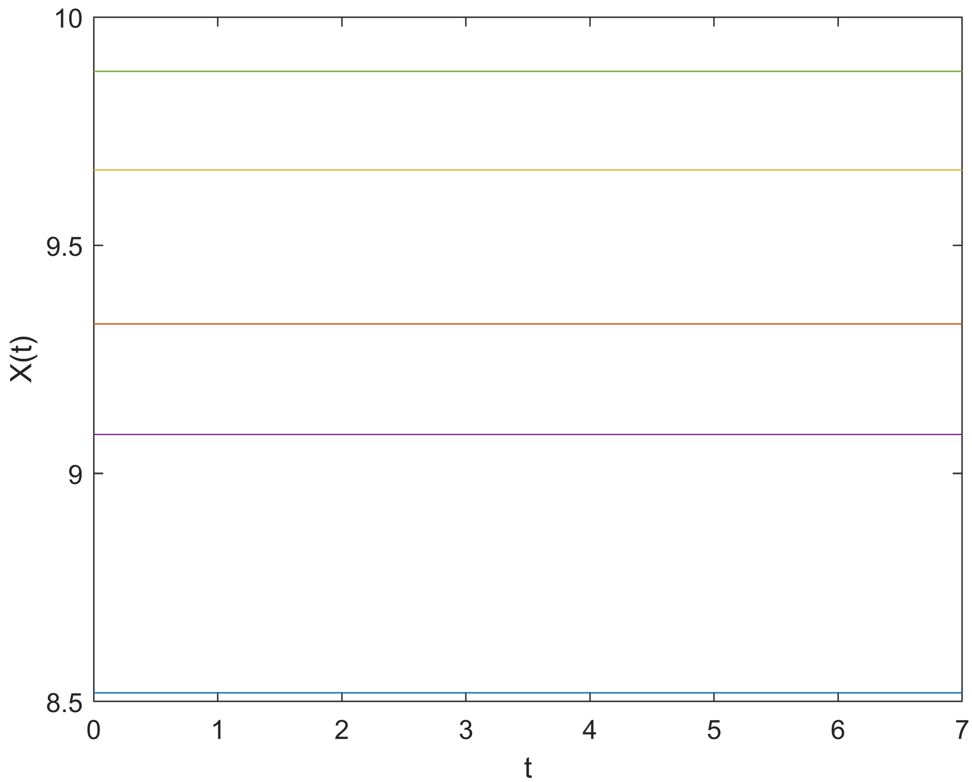
1) Sketch five realizations of the process $X(t)$ between $t \in [0; 7]$. Use a Gaussian random number generator, it can e.g. be the build in generator in matlab, *randn()*. State how the five realizations are generated.

The random number generator randn() from matlab can be used.

```
mu = 5;
sigma = 1;
n_realizations = 5;
wn = sigma*randn(1,n_realizations)+5;

X = wn + 4;

t = 0:7;
for i=1:n_realizations
    plot(t,X(i)*ones(1,length(t)))
    hold on
end
hold off
xlabel('t')
ylabel('X(t)')
```



2) Find the ensemble mean value and the ensemble variance for the process $X(t)$.

The ensemble mean can be calculated using the formula for a normal distribution on pg. 53: $E[w(n)] = \mu$

$E[X] = E[w(n)] + E[4] = 5 + 4 = 9$

```
ensemble_mean = 9;
```

The ensemble variance can be calculated using the formula for a normal distribution on pg. 53: $\mathrm{Var}(w(n)) = \sigma^2$

$\mathrm{Var}(X) = E[X(n)^2] - E[X(n)]^2 = E[w(n)^2] + E[4^2] - 4^2 = E[w(n)^2] = 1^2 = 1$

```
ensemble_var = 1;
```

3) Select one of the five realizations, and decide the mean value and the variance for that realization.

The temporal mean can be calculated using the equation from pg. 39 in the book: $\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} X_i$

The temporal variance can be calculated using the equation: $\widehat{\sigma^2} = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \widehat{\mu}\right)^2$

Or using the matlab functions *mean* and *var*. Since each realization is a constant, the mean will be that value, and the variance will be 0.

```
% realization 1
temporal_mean = mean(X(1))
```

```
temporal_mean = 8.5191
```

```
temporal_var = var(X(1))
```

```
temporal_var = 0
```

4) State whether the process $X(t)$ is WSS (wide sense stationary), and whether it is ergodic. State the reason behind your answers.

The process is WSS, since the ensemble mean and variance are independt of time.

However it is not ergodic because the temporal and ensemble mean are not identical. This can also be thought of as 1 realization does not contain all information about the process

SMP L7 - Forår 2016 Reeksamen assignment 2

Assignment 2: Stochastic Processes

A continuous stochastic process is given by:

$$X(t) = w(t)$$

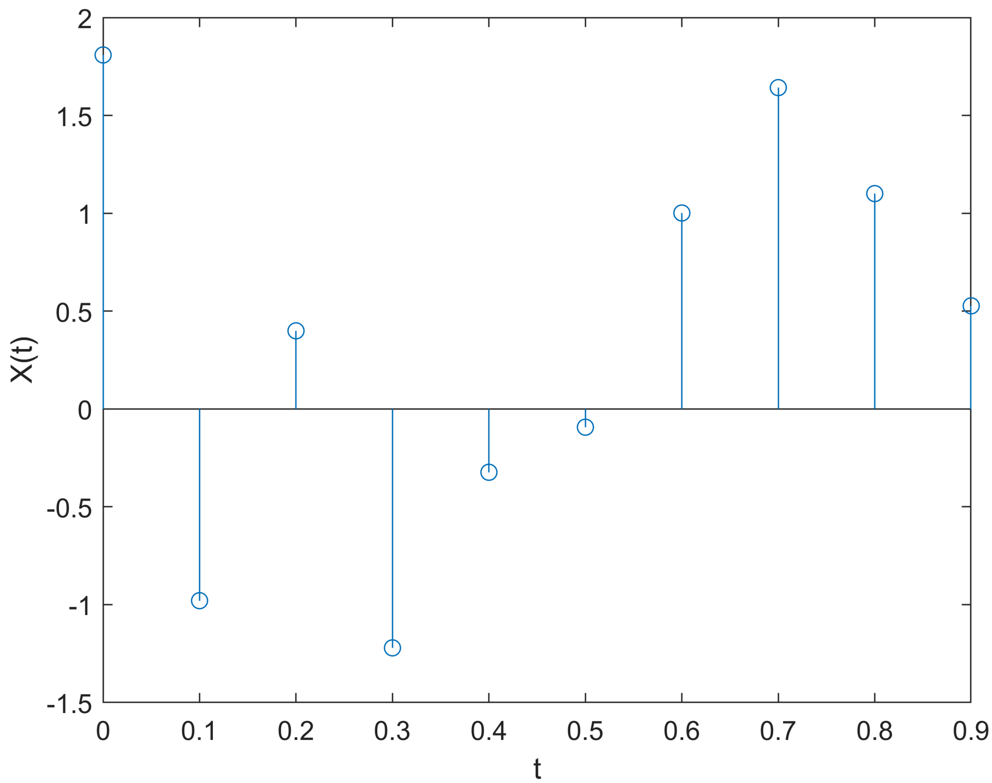
Where $w(t)$ is i.i.d. (independent and identically distributed) and Gaussian distributed according to $w(t) \sim N(t, 1)$.

1) Sketch one realization of the process $X(t)$, where it is sampled at the times: $t = [0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9]$. Use a Gaussian random number generator, it can e.g. be the build in generator in matlab, *randn()*. State how the realization is generated.

```
t = 0:0.1:0.9;
sigma = 1;
wt = sigma*randn(1,length(t))+t;
Xt = wt;

stem(t,Xt)
xlabel('t')
```

```
ylabel('X(t)')
```



2) Find the ensemble mean value and the ensemble variance for the process $X(t)$.

$$E[X(t)] = E[w(t)] = t$$

$$\mathrm{Var}(X(t)) = \mathrm{Var}(w(t)) = 1$$

3) What do you expect the timely mean value of a random realization of $X(t)$ in the time inverval $t = [0; 100]$ to be? State the reason for your answer.

I expect it to be 50 since it is the mean value of the time interval.

$$\hat{\mu} = \frac{1}{N}\int_{i=1}^{N} E[X(t)]\,\mathrm{dt} = \frac{1}{100}\sum_{i=1}^{100} t\,\mathrm{dt} = 50$$

```
t1 = 0:100;
wt1 = sigma*randn(1,length(t1))+t1;
Xt1 = wt1;
mean(Xt1)
```

```
ans = 50.0937
```

4) State whether the process $X(t)$ is WSS (wide sense stationary), and whether it is ergodic. State the reason behind your answers.

The process is not WSS since the ensemble mean is dependent of time.

Since the process is not WSS it can not be ergodic either.

5) Construct the equation to determine the autocorrelation $R_{X(t_1)X(t_2)}(t_1 = 1, t_2 = 2)$ and calculate the value.

Look at slide 14.

$$R_{\mathrm{XX}}(t_1, t_2) = E\left[X(t_1)X(t_2)^*\right] = \int\int_{-\infty}^{\infty} X(t_1)X(t_2) \cdot f_{X(t_1),X(t_2)}(x(t_1), x(t_2))\, \mathrm{d}x(t_1)\mathrm{d}x(t_2)$$

Since the process is i.i.d we can calculate the autorcorrelation as (x1 and x2 are independent):

$$R_{X(t_1)X(t_2)}(t_1 = 1, t_2 = 2) = E[X(1)] \cdot E[X(2)] = 1 \cdot 2 = 2$$