Solutions

1 Let μ_1 denote the mean body temperature of men, and let μ_2 denote the mean body temperature of women.

a. The null hypothesis is $H_0: \mu_1 = \mu_2$ or equivalently, $H_0: \mu_1 - \mu_2 = 0$.

b. The true variance is unknown, so we have to use the Test catalog for comparing two means (unknown variance).

Estimate of difference:

$$\hat{\delta} = \bar{x}_1 - \bar{x}_2 = 36.725 - 36.886 = -0.1610$$

Estimate of variance:

$$s^2 = \frac{1}{n_1 + n_2 - 2}\left((n_1 - 1)s_1^2 + (n_2 - 1)s_2^2\right)$$
$$= \frac{1}{65 + 65 - 2}\left((65 - 1)0.699^2 + (65 - 1)0.743^2\right) = 0.5203$$

Estimate of standard deviation:

$$s = \sqrt{0.5203} = 0.7213$$

Test size:

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{s\sqrt{1/n_1 + 1/n_2}} = \frac{-0.1610}{0.7213\sqrt{1/65 + 1/65}} = -1.2725 \sim t(65 + 65 - 2)$$

Approximate p-value:

$$2 \cdot \left(1 - t_{cdf}(|t|, n_1 + n_2 - 2)\right) = 2(1 - tcdf(1.2725, 65 + 65 - 2) = 2(1 - 0.8972)$$
$$= 0.2055$$

And we fail to reject the null hypothesis, meaning that the data do not suggest that there is a difference in mean body temperature between men and women.

c. The endpoints of the 95% confidence interval are

$$\delta_- = (\bar{x}_1 - \bar{x}_2) - t_0 \cdot s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = -0.1610 - 1.9787 \cdot 0.7213\sqrt{\frac{1}{65} + \frac{1}{65}} = -0.4114$$

$$\delta_+ = (\bar{x}_1 - \bar{x}_2) + t_0 \cdot s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = -0.1610 + 1.9787 \cdot 0.7213\sqrt{\frac{1}{65} + \frac{1}{65}} = 0.0894$$

where

```
t0 = tinv(1-0.05/2,n1+n2-2) = tinv(0.975,65+65-2) = 1.9787
```

Since δ=0 is included in the 95% confidence interval, we fail to reject the null hypothesis.

2 Let μ_1 denote the mean for the intensive tutoring group, and let μ_2 denote mean for the paced tutoring group. The true variance is unknown, so we have to use the Test catalog for comparing two means (unknown variance).

Estimate of difference:

$$\hat{\delta} = \bar{x}_1 - \bar{x}_2 = 46.31 - 42.79 = 3.52$$

Estimate of variance:

$$s^2 = \frac{1}{n_1 + n_2 - 2}\left((n_1 - 1)s_1^2 + (n_2 - 1)s_2^2\right) = \frac{1}{12 + 10 - 2}\left((12 - 1)6.44^2 + (10 - 1)7.52^2\right)$$

$$= 48.2582$$

Estimate of standard deviation:

$$s = \sqrt{48.2582} = 6.9468$$

Test size:

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{s\sqrt{1/n_1 + 1/n_2}} = \frac{3.52}{6.9468\sqrt{1/12 + 1/10}} = 1.1829 \sim t(12 + 10 - 2)$$

Approximate p-value:

$$2 \cdot \left(1 - t_{cdf}(|t|, n_1 + n_2 - 2)\right) = 2(1 - tcdf(1.1829{,}12 + 10 - 2) = 2(1 - 0.8746)$$

$$= 0.2507$$

And we fail to reject the null hypothesis, meaning that the data do not suggest that there is a difference between the mean for the paced group and the mean for the intensive group.

3 Let μ_1 denote the mean number of raisins per box in brand A, and let μ_2 denote mean number of raisins per box in brand B. The true variance is unknown, so we have to use the Test catalog for comparing two means (unknown variance).

Estimate of difference:

$$\hat{\delta} = \bar{x}_1 - \bar{x}_2 = 102.1 - 93.6 = 8.5$$

Estimate of variance:

$$s^2 = \frac{1}{n_1 + n_2 - 2}\left((n_1 - 1)s_1^2 + (n_2 - 1)s_2^2\right) = \frac{1}{6 + 9 - 2}\left((6 - 1)12.3^2 + (9 - 1)7.52^2\right)$$

$$= 104.6264$$

Estimate of standard deviation:

$$s = \sqrt{104.6264} = 10.2287$$

The endpoints of the 95% confidence interval are

$$\delta_- = (\bar{x}_1 - \bar{x}_2) - t_0 \cdot s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 8.5 - 2.1604 \cdot 10.2287\sqrt{\frac{1}{6} + \frac{1}{9}} = -3.1467$$

$$\delta_+ = (\bar{x}_1 - \bar{x}_2) + t_0 \cdot s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 8.5 + 2.1604 \cdot 10.2287\sqrt{\frac{1}{6} + \frac{1}{9}} = 20.1467$$

where

```
t0 = tinv(1-0.05/2,n1+n2-2) = tinv(0.975,6+9-2) = 2.1604
```

Since δ=0 is included in the 95% confidence interval, we fail to reject the null hypothesis.

4 The data are paired, so we must use the Test catalog for paired data.

a. Hypothesis test:

Null hypothesis:

$$H_0: \delta = 0$$

Data in Matlab notation:

```
x1 = [88 92 85 80 83 84 86 78 81 95 ]
x2 = [89 90 87 84 84 86 86 84 83 92 ]
d = x1-x2
```

Average difference ('before' minus 'after'):

$$\hat{\delta} = \bar{d} = \frac{1}{n}\sum_{i=1}^{n} X_{1i} - X_{2i} = \text{mean(d)} = \text{-}1.3$$

Estimated variance (use `var` in Matlab)

$$s_d^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(d_i - \bar{d}\right)^2 = \text{var(d)} = 6.9$$

Estimated standard deviation

$$s_d = \sqrt{6.9} = 2.6268$$

Test size

$$t = \frac{\bar{d} - \delta_0}{s_d/\sqrt{\mathrm{n}}} = \frac{-1.3 - 0}{2.6268/\sqrt{10}} = -1.5650 \sim t(\mathrm{n} - 1)$$

p-value

$$2 \cdot \left(1 - t_{cdf}(|t|, \mathrm{n} - 1)\right) = 2 \cdot \left(1 - t_{cdf}(-1.5650, 10 - 1)\right) = 2 \cdot (1 - 0.9240) = 0.1520$$

Since p>0.05, we fail to reject the null hypothesis.

b. 95% confidence interval

$$\delta_- = \bar{d} - t_0 \cdot \frac{s_d}{\sqrt{n}} = -1.3 - 2.2622 \cdot \frac{2.6268}{\sqrt{10}} = -3.1791$$

$$\delta_+ = \bar{d} + t_0 \cdot \frac{s_d}{\sqrt{n}} = -1.3 + 2.2622 \cdot \frac{2.6268}{\sqrt{10}} = 0.5791$$

where `t0 = tinv(1-0.05/2,n-1) = tinv(0.975,9-1) = 2.2622`

We fail to reject the null hypothesis because $\delta = 0$ is included in the 95% confidence interval.

5 The data are paired, so we must use the Test catalog for paired data.
The null hypothesis is:

$$H_0: \delta = 0$$

Average difference:

$$\hat{\delta} = \bar{d} = \frac{1}{n}\sum_{i=1}^{n} X_{1i} - X_{2i} = 2.6$$

Estimated variance (use `var` in Matlab)

$$s_d^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(d_i - \bar{d}\right)^2 = \frac{(6-2.6)^2 + (1-2.6)^2 + \cdots + (6-12.6)^2}{9-1} = 8.9333$$

Estimated standard deviation

$$s_d = \sqrt{8.9333} = 2.9889$$

Test size

$$t = \frac{\bar{d} - \delta_0}{s_d/\sqrt{\mathrm{n}}} = \frac{2.6 - 0}{2.9889/\sqrt{10}} = 2.7508 \sim t(\mathrm{n}-1)$$

p-value

$$2 \cdot \left(1 - t_{cdf}(|t|, \mathrm{n}-1)\right) = 2 \cdot \left(1 - t_{cdf}(2.7508, 10-1)\right) = 2 \cdot (1 - 0.9888) = 0.0224$$

Since $p<0.05$, we reject the null hypothesis and conclude that there is indeed a difference in the recall of words when participants looked at a simple text and a complex text.