Opgave 1

Hændelser:

F: Chip-set har fejl nF: Chip-set har ingen fejl T: Test viser fejl nT: Test viser ingen fejl

Data:

$Pr\_T\_givet\_F := 0.34$ $Pr\_T\_givet\_nF := 0.08$ $Pr\_F := 0.025$

a) $Pr\_F\_og\_T := Pr\_T\_givet\_F \cdot Pr\_F = 0.0085$

b) $Pr\_nF := 1 - Pr\_F = 0.975$

c) $Pr\_nF\_og\_T := Pr\_T\_givet\_nF \cdot Pr\_nF = 0.078$

$Pr\_T := Pr\_F\_og\_T + Pr\_nF\_og\_T = 0.0865$

d) $Pr\_F\_givet\_T := \dfrac{Pr\_T\_givet\_F \cdot Pr\_F}{Pr\_T} = 0.09827$

$Pr\_F\_givet\_T := \dfrac{Pr\_F\_og\_T}{Pr\_T} = 0.09827$

Opgave 2

a) Simultan pmf:

$$f'_{XY}(x,y,K) := \begin{array}{l} \text{if } y=-1 \\ \quad \begin{array}{l} \text{if } x=2 \\ \quad \frac{K}{2} \\ \text{if } x=4 \\ \quad \frac{K}{4} \\ \text{if } x=6 \\ \quad \frac{K}{2} \\ \text{if } x=8 \\ \quad \frac{K}{2} \\ \text{if } x=10 \\ \quad \frac{K}{4} \end{array} \\ \text{if } y=1 \\ \quad \begin{array}{l} \text{if } x=2 \\ \quad \frac{K}{4} \\ \text{if } x=4 \\ \quad \frac{K}{2} \\ \text{if } x=6 \\ \quad \frac{K}{2} \\ \text{if } x=8 \\ \quad \frac{K}{4} \\ \text{if } x=10 \\ \quad \frac{K}{2} \end{array} \end{array}$$

$$\sum_{xx=1}^{5} \sum_{yy=0}^{1} f'_{XY}(2\ xx, 2\ yy-1, K) = 4\ K$$

--> $K := 4\ K = 1 \xrightarrow{solve, K} \frac{1}{4}$

$$f_{XY}(x,y) := f'_{XY}(x,y,K)$$

b) Marginal pmf:

$$f_X(x) := \sum_{yy=0}^{1} f_{XY}(x, 2\,yy - 1) \qquad x := 2, 4 .. 10$$

$f_X(2) = 0.188$

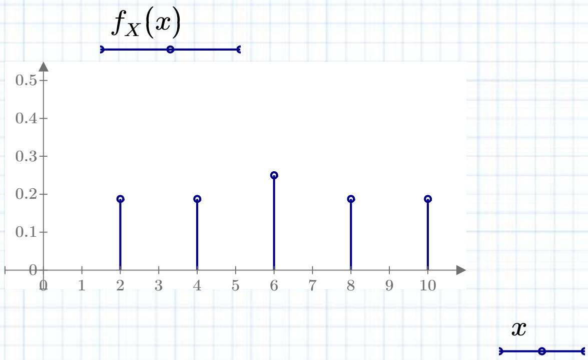
$f_X(4) = 0.188$

$f_X(6) = 0.25$

$f_X(8) = 0.188$

$f_X(10) = 0.188$

$$\sum_{xx=1}^{5} f_X(2\,xx) = 1$$ o.k.



c) Marginal cdf:

$$F_X(x) := \begin{cases} \text{if } x < 2 \\ \quad 0 \\ \text{else} \\ \quad \displaystyle\sum_{xx=1}^{\frac{x}{2}} f_X(2\,xx) \end{cases}$$

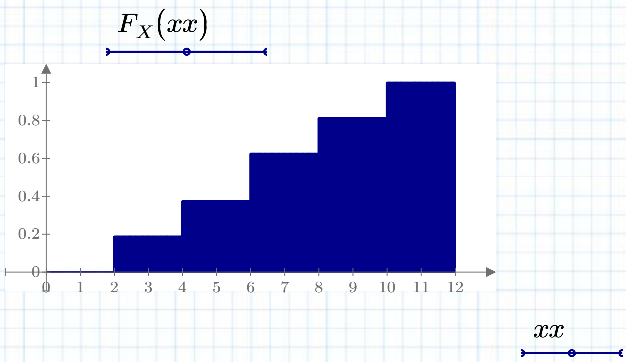
$F_X(2) = 0.188$

$F_X(4) = 0.375$

$F_X(6) = 0.625$

$F_X(8) = 0.813$

$F_X(10) = 1$



d) $$EX := \sum_{xx=1}^{5} 2\,xx \cdot f_X(2\,xx) = 6$$

$$EX2 := \sum_{xx=1}^{5} (2\,xx)^2 \cdot f_X(2\,xx) = 43.5$$ --> $$VarX := EX2 - EX^2 = 7.5$$

e) $$EXY := \sum_{xx=1}^{5} \sum_{yy=0}^{1} (2 \cdot xx) \cdot (2 \cdot yy - 1) \cdot f_{XY}(2 \cdot xx, 2 \cdot yy - 1) = 0.25$$

f) $$Pr(Y = 1 \mid X = 6) = Pr(X = 6 \cap Y = 1) / Pr(X = 6) = \frac{f_{XY}(6, 1)}{f_X(6)} = 0.5$$

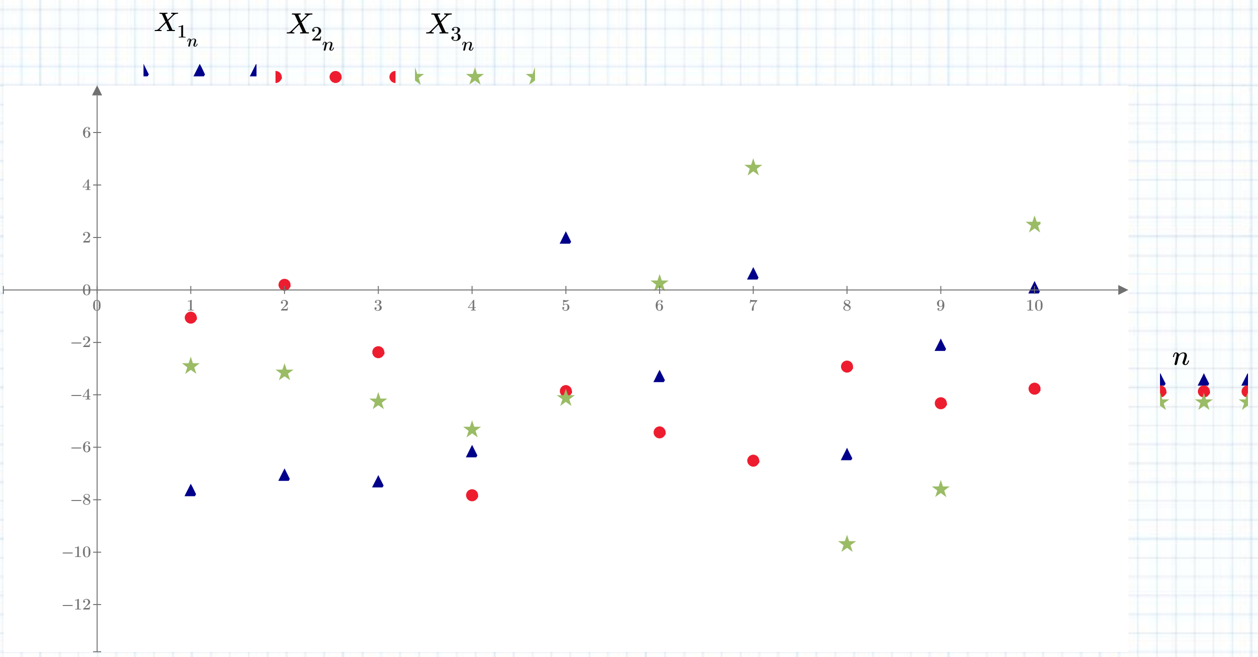
Opgave 3

a) Tre realisationer, 10 samples:

$$Z_1 := \text{rnorm}\left(10, 1, \sqrt{10}\right) = \begin{bmatrix} 4.115 \\ 3.727 \\ 3.895 \\ 3.128 \\ -2.302 \\ 1.218 \\ -1.389 \\ 3.203 \\ 0.425 \\ -1.038 \end{bmatrix} \quad \text{-->} \quad n := 1, 2..10 \quad X_{1_n} := -1.5 \cdot \left(Z_{1_{n-1}} + 1\right) = \begin{bmatrix} -7.673 \\ -7.09 \\ -7.343 \\ -6.192 \\ 1.954 \\ -3.328 \\ 0.584 \\ -6.305 \\ -2.137 \\ 0.057 \end{bmatrix}$$

$$Z_2 := \text{rnorm}\left(10, 1, \sqrt{10}\right) \quad \text{-->} \quad X_{2_n} := -1.5 \cdot \left(Z_{2_{n-1}} + 1\right)$$

$$Z_3 := \text{rnorm}\left(10, 1, \sqrt{10}\right) \quad \text{-->} \quad X_{3_n} := -1.5 \cdot \left(Z_{3_{n-1}} + 1\right)$$



b) Tidslig middelværdi af realisation X_1: $\mu_T := \frac{1}{10} \cdot \sum_{i=1}^{10} X_{1_i} = -3.747$

c) Ensemble middelværdi: $E(X) = E(-1.5\ (Z+1)) = -1.5\ (E(Z)+1) = -1.5\ (1+1) = -3$

Ensemble varians: $Var(X) = Var(-1.5\ (Z+1)) = (-1.5)^2 \cdot (Var(Z) + Var(1)) = 2.25\ (10+0) = 22.5$

d) $E(X)$ og $Var(X)$ uafhængig af n (tiden) --> X er WSS (Wide Sense Stationary)

Opgave 4

Data: $N := 12$

Højde [cm]: $H := [55\ \ 60\ \ 70\ \ 75\ \ 79\ \ 90\ \ 101\ \ 112\ \ 121\ \ 129\ \ 134\ \ 143]$

Alder [mdr]: $A := [1\ \ 3\ \ 6\ \ 9\ \ 12\ \ 24\ \ 36\ \ 48\ \ 60\ \ 72\ \ 84\ \ 96]$

a) Signal-model: $H = \alpha + \beta \cdot A + \varepsilon$ Residual (støj): ε --> $N(0, \sigma^2)$

b) Middelværdier:

$$\mu_A := \frac{1}{N} \cdot \sum_{i=0}^{N-1} A_{0,i} = 37.583$$

$$\mu_H := \frac{1}{N} \cdot \sum_{i=0}^{N-1} H_{0,i} = 97.4$$

Lineær regression:

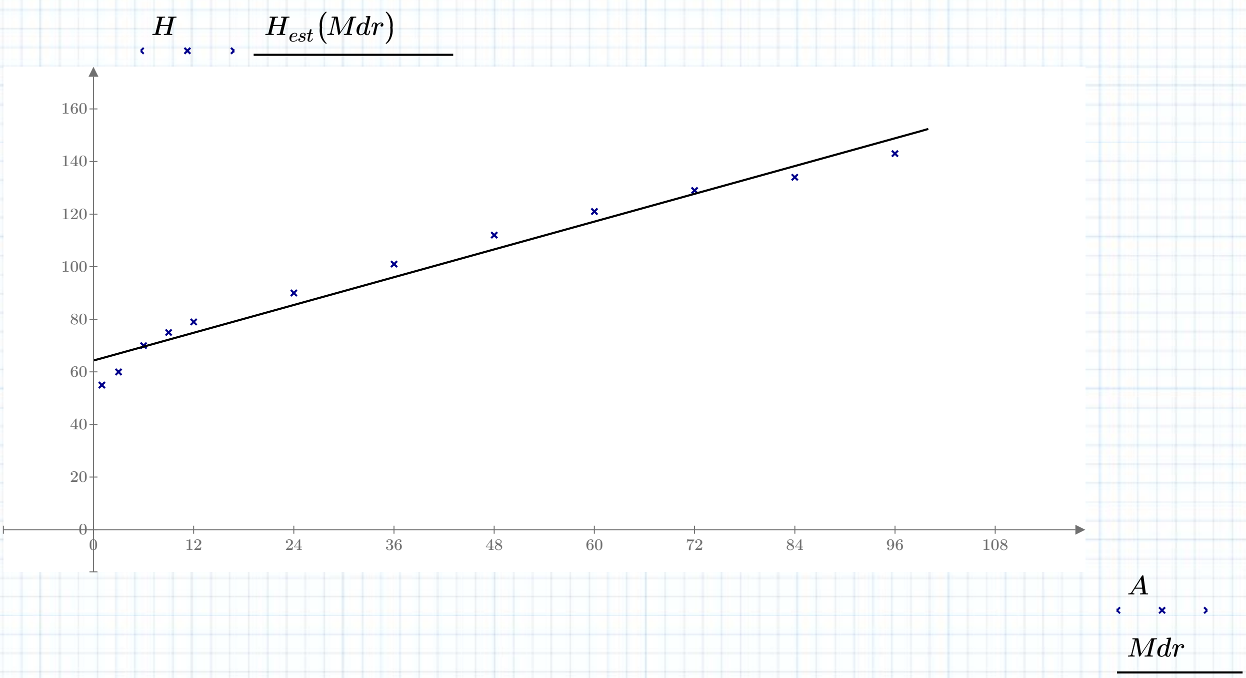
Hældning:

$$\beta := \frac{\sum_{i=0}^{N-1} \left(\left(A_{0,i} - \mu_A \right) \cdot \left(H_{0,i} - \mu_H \right) \right)}{\sum_{i=0}^{N-1} \left(A_{0,i} - \mu_A \right)^2} = 0.88$$

Skæring: $\alpha := \mu_H - \beta \cdot \mu_A = 64.342$

$$H_{est}(A) := \alpha + \beta \cdot A \xrightarrow{float, 5} 0.88004 \cdot A + 64.342$$

$Mdr := 0, 1..100$



c) Hypotesetest på hældning β=0: H0: $\beta=0$ H1: $\beta\neq 0$

d) $s2_x := \sum_{i=0}^{N-1} \left(A_{0,i} - \mu_A\right)^2 = 12552.917$ $s_x := \sqrt{s2_x} = 112.04$

$s2_r := \frac{1}{N-2} \cdot \sum_{i=0}^{N-1} \left(H_{0,i} - H_{est}\left(A_{0,i}\right)\right)^2 = 32.103$ $s_r := \sqrt{s2_r} = 5.666$

$t := \frac{\beta - 0}{\frac{s_r}{s_x}} = 17.4$ $p_{value} := 2 \cdot \left(1 - \mathrm{pt}\left(t, N-2\right)\right) = 8.325 \cdot 10^{-9}$

$p_{value} = 8.325 \cdot 10^{-9}$ < 0,05 --> Hypotesen afvises --> Hældningen $\beta\neq 0$

e) 95% konfidensinterval:

$t_0 := \mathrm{qt}\left(0.975, N-2\right) = 2.228$

$\Delta\beta := t_0 \cdot \frac{s_r}{s_x} = 0.113$

$\beta_{min} := \beta - \Delta\beta = 0.767$ $\beta_{max} := \beta + \Delta\beta = 0.993$

$\beta = 0 < \beta_{min}$ --> Hypotesen afvises --> Hældningen $\beta\neq 0$

f) Residualer: $\varepsilon := H - H_{est}(A) = [-10.2 \;\; -7 \;\; 0.4 \;\; 2.7 \;\; 4.1 \;\; 4.5 \;\; 5 \;\; 5.4 \;\; 3.9 \;\; 1.3 \;\; -4.3 \;\; -5.8]$