SHP

Opg. 1 V20/21

$Pr(Sol|F) = 0.46$ $\quad$ $Pr(F) = Pr(S) = Pr(E) = Pr(V) = \frac{1}{4}$

$Pr(Sol|S) = 0.63$

$Pr(Sol|E) = 0.32$

$Pr(Sol|V) = 0.12$

a) $\underline{\underline{Pr(\overline{Sol}|V)}} = 1 - Pr(Sol|V) = 1 - 0.12 \underline{\underline{= 0.88}}$

b) $\underline{\underline{Pr(Sol)}} = Pr(Sol|F) \cdot Pr(F) + Pr(Sol|S) \cdot Pr(S) + Pr(Sol|E) \cdot Pr(E) + Pr(Sol|V) \cdot Pr(V)$

$= 0.46 \cdot \frac{1}{4} + 0.63 \cdot \frac{1}{4} + 0.32 \cdot \frac{1}{4} + 0.12 \cdot \frac{1}{4}$

$= 1.53 \cdot \frac{1}{4}$

$\underline{\underline{= 0.3825}}$

c) $\underline{\underline{Pr(F|Sol)}} = \frac{Pr(Sol|F) \cdot Pr(F)}{Pr(Sol)}$

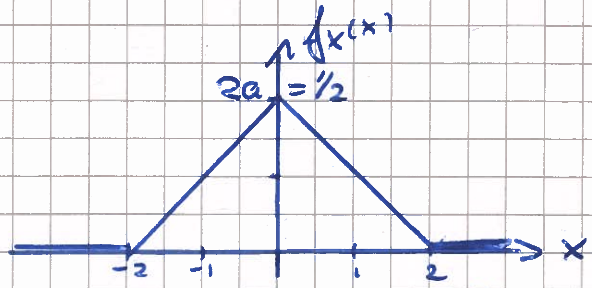
$= \frac{0.46 \cdot \frac{1}{4}}{0.3825}$

$\underline{\underline{= 0.301}}$

SMP

Opg. 2 V20/21

$$f_X(x) = \begin{cases} a\cdot(x+2) & ; -2 \le x \le 0 \\ -a(x-2) & 0 \le x \le 2 \\ 0 & \text{ellers} \end{cases}$$

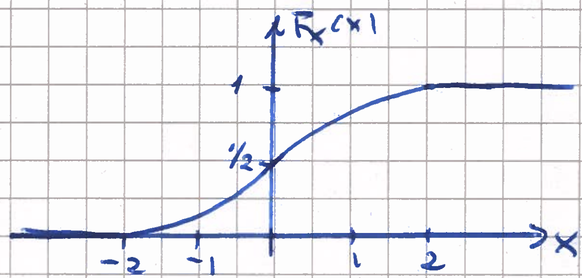


a) $\int_{-\infty}^{\infty} f_X(x)dx = \int_{-2}^{0} a\cdot(x+2)dx + \int_{0}^{2} -a\cdot(x-2)dx = a\cdot\left[\frac{1}{2}x^2+2x\right]_{-2}^{0} - a\cdot\left[\frac{1}{2}x^2-2x\right]_{0}^{2}$

$= 0 - a\left(\frac{1}{2}(-2)^2+2\cdot(-2)\right) - a\cdot\left(\frac{1}{2}\cdot 2^2 - 2\cdot 2\right) - 0 = 2a+2a = 4a = 1$

$\Downarrow$ $\underline{\underline{a = \frac{1}{4}}}$

b) $\underline{\underline{F_X(x)}} = \int_{-\infty}^{x} f_X(x)dx = \begin{cases} 0 & ; x < -2 \\ \frac{1}{4}\left(\frac{1}{2}x^2+2x-(2-4)\right) = \frac{1}{8}(x^2+4x+4) = \frac{1}{8}(x+2)^2 & , -2 \le x \le 0 \\ -\frac{1}{4}\left(\frac{1}{2}x^2-2x\right)+\frac{1}{2} = -\frac{1}{8}(x^2-4x+4)+1 = -\frac{1}{8}(x-2)^2+1 & , 0 \le x \le 2 \\ 1 & ; x > 2 \end{cases}$



c) $\underline{\underline{EX}} = \int_{-\infty}^{\infty} x\cdot f_X(x)dx$

$= \frac{1}{4}\int_{-2}^{0}(x^2+2x)dx - \frac{1}{4}\int_{0}^{2}(x^2-2x)dx$

$= \frac{1}{4}\left(\left[\frac{1}{3}x^3+x^2\right]_{-2}^{0} - \left[\frac{1}{3}x^3-x^2\right]_{0}^{2}\right)$

$= \frac{1}{4}\left(0 - \frac{-8}{3} - 4 - \frac{8}{3} + 4 - 0\right) = \underline{\underline{0}}$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f_X(x)dx = \frac{1}{4}\int_{-2}^{0}(x^3+2x^2)dx - \frac{1}{4}\int_{0}^{2}(x^3-2x^2)dx$

$= \frac{1}{4}\left(\left[\frac{1}{4}x^4+\frac{2}{3}x^3\right]_{-2}^{0} - \left[\frac{1}{4}x^4-\frac{2}{3}x^3\right]_{0}^{2}\right) = \frac{1}{4}\left(0 - \frac{16}{4} - \frac{-16}{3} - \frac{16}{4} + \frac{16}{3} - 0\right)$

$= \frac{1}{4}\left(-8+\frac{32}{3}\right) = \frac{1}{4}\cdot\frac{8}{3} = \frac{2}{3}$

$\Downarrow$ $\underline{\underline{Var(x)}} = E(x^2) - EX^2 = \frac{2}{3} - 0 = \underline{\underline{\frac{2}{3}}}$

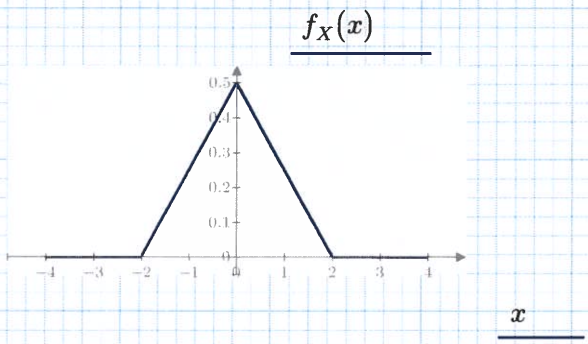
d) $\underline{\underline{Pr(X=0)}} = \int_0^0 f_X(x)dx = \underline{\underline{0}}$ (kontinuert stokastisk variabel)

e) $\underline{\underline{Pr(-1 \le X \le 1)}} = F_X(1) - F_X(-1) = -\frac{1}{8}(-1)^2 + 1 - \frac{1}{8}\cdot 1^2 = 1 - \frac{2}{8} = \underline{\underline{\frac{3}{4}}}$

Opgave 2

a) $$a := \int_{-2}^{0} a\cdot(x+2)\,\mathrm{d}x + \int_{0}^{2} -a\cdot(x-2)\,\mathrm{d}x = 1 \xrightarrow{solve,\,a} \frac{1}{4}$$

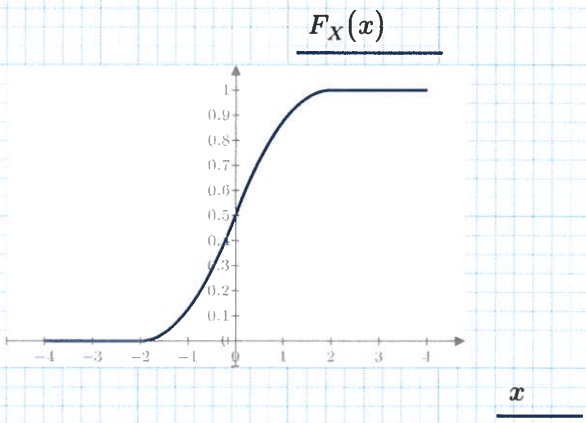
$$f_X(x) := \begin{array}{l} \text{if } x<-2 \\ \quad 0 \\ \text{else if } -2\le x\le 0 \\ \quad a\cdot(x+2) \\ \text{else if } 0<x\le 2 \\ \quad -a\cdot(x-2) \\ \text{else if } x>2 \\ \quad 0 \end{array}$$



b) $$F_{X\_1}(x) := \int_{-2}^{x} a\cdot(x+2)\,\mathrm{d}x \rightarrow \frac{(x+2)^2}{8}$$

$$F_{X\_2}(x) := F_{X\_1}(0) + \int_{0}^{x} -a\cdot(x-2)\,\mathrm{d}x \rightarrow \frac{1}{2} - \frac{x\cdot(x-4)}{8}$$

$$F_X(x) := \begin{array}{l} \text{if } x<-2 \\ \quad 0 \\ \text{else if } -2\le x\le 0 \\ \quad \dfrac{(x+2)^2}{8} \\ \text{else if } 0<x\le 2 \\ \quad \dfrac{1}{2} - \dfrac{x\cdot(x-4)}{8} \\ \text{else if } x>2 \\ \quad 1 \end{array}$$



c) $$EX := \int_{-2}^{0} x\cdot a\cdot(x+2)\,\mathrm{d}x + \int_{0}^{2} -x\cdot a\cdot(x-2)\,\mathrm{d}x = 0$$

$$EX\_2 := \int_{-2}^{0} x^2\cdot a\cdot(x+2)\,\mathrm{d}x + \int_{0}^{2} -x^2\cdot a\cdot(x-2)\,\mathrm{d}x = 0.667$$

$$VarX := EX\_2 - EX^2 = 0.667$$

d) Pr_0 =Pr(X=0): $$Pr\_0 := \int_{0}^{0} a\cdot(x+2)\,\mathrm{d}x = 0$$

e) Pr_1 =Pr(-1<X<1): $$Pr\_1 := F_X(1) - F_X(-1) = 0.75$$

SMP

Opg. 3 V20/21

$$P[n] = 3 \cdot Y[n] + 2 \cdot W \quad ; \quad Y[n] \sim \mathcal{N}(-2; 0.01),\ W \sim \mathcal{U}[3, 5]$$

a) $W = (b-a) \cdot \text{rand} + a = 2 \cdot \text{rand} + 3$;

$Y = \sigma \cdot \text{randn}(1, 11) + \mu = 0.1 \cdot \text{randn}(1, 11) - 2$;

Realisationer: Se bilag.

b) Ensemble middelværdi:

$$\underline{\underline{EP}} = E[3Y + 2W] = 3EY + 2EW = 3 \cdot (-2) + 2 \cdot \frac{5+3}{2} = -6 + 8 = \underline{\underline{2}}$$

Ensemble varians:

$$\underline{\underline{Var(P)}} = Var(3Y + 2W) = 9\, Var\, Y + 4\, Var(W) \quad (Y \text{ og } W \text{ iid})$$
$$= 9 \cdot 0.01 + 4 \cdot \frac{(5-3)^2}{12} = 0.09 + \frac{4}{3} = \underline{\underline{1.423}}$$

c) Da EP og $Var(P)$ er uafh. af n (tiden), $\underline{\underline{\text{er } P \text{ WSS}}}$.

Da W bliver valgt én gang for en realisation, vil både den temporale middelværdi og varians for en realisation givet noget andet end ensemble-værdierne:

$$\hat{\mu}_P = 3 \cdot \hat{\mu}_Y + 2 \cdot W = 3 \cdot (-2) + 2W = -6 + 2W \in [0; 4] \text{ afh. af } W.$$

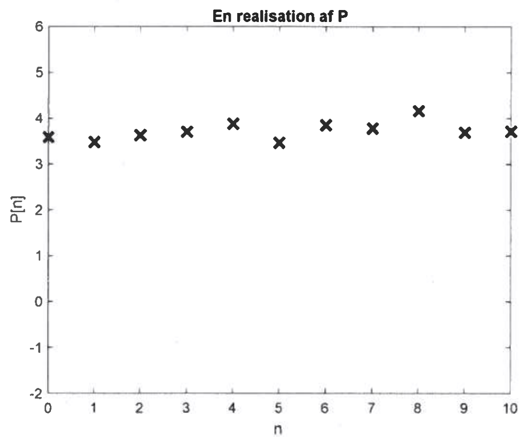
$$\hat{\sigma}_P^2 = 9\hat{\sigma}_P^2 + 0 = 9 \cdot 0.01 = 0.09 \quad (\text{da } \hat{\sigma}_W^2 = 0 \text{ (W konst.) for én realisation})$$

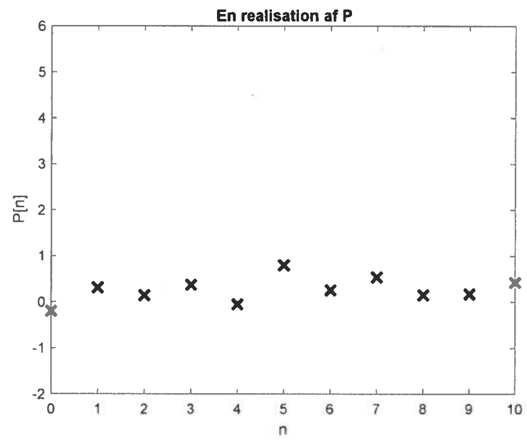
Dvs. $\underline{\underline{\text{P er ikke ergodisk.}}}$

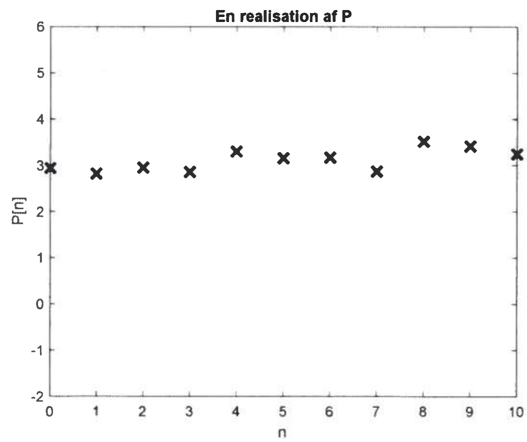
%%Opgave_3a: Plot af tre realisationer

```
clear all
for i=1:3
    W=2*rand+3; %Uniformt fordelt tal mellem 3 og 5
    Y=0.1*randn(1,11)-2; %Normalfordelt tal med my=-2 og var=0.01
    P=3*Y+2*W; %Stokastiske proces

    figure(i)
    plot(0:10,P,'kx','Linewidth',2,'Markersize',10)
    axis([0 10 -2 6])
    title('En realisation af P')
    xlabel('n')
    ylabel('P[n]')
 end
```







SMP

Opg. 4 V20/21

a) Da testene er udført parvis for produkt A og B - og varianserne er ukendte - benyttes en parret sammenligningstest med student-t fordelingen som statistisk model for forskellen i energiforbrug med $\hat{\delta} = A - B$ som teststørrelse.

b) Se qq-plot (bilag): $\hat{\delta} = A-B$ ligger på en ret linje
$\rightarrow$ Data ($\hat{\delta}$) er normalfordelte.

c) Hypoteser: $H_0: \delta = A - B = -10 \text{ kWh} = \delta_0$

$H_1: \delta = A - B \neq -10 \text{ kWh}$

d) $$\left.\begin{aligned} \bar{\delta} &= \frac{1}{14}\sum_{i=1}^{14}(A_i - B_i) = -6.7143 \\ s_\delta^2 &= \frac{1}{13}\sum_{i=1}^{14}(A_i - B_i - \bar{\delta})^2 = 31.0552 \end{aligned}\right\} \Rightarrow t = \frac{\hat{\delta} - \delta_0}{\sqrt{s_\delta^2/14}} = \frac{-6.7143-(-10)}{\sqrt{31.0552/14}} = 2.2061 \sim t(13)$$

$\Downarrow$

$$\underline{\underline{p\text{-val}}} = 2\cdot(1 - t_{cdf}(2.2061, 13)) = 2\cdot(1-0.9770) = \underline{\underline{0.0460}} < 0.05 = \alpha$$

e) $t_0 = t_{inv}(0.975, 13) = 2.1604$

$\Downarrow$

$$\underline{\underline{\delta_-}} = \bar{\delta} - t_0 \cdot \sqrt{\frac{s_\delta^2}{n}} = -6.7143 - 2.1602 \cdot \sqrt{\frac{31.0552}{14}} = \underline{\underline{-9.9319}}$$

$$\underline{\underline{\delta_+}} = \bar{\delta} + t_0 \cdot \sqrt{\frac{s_\delta^2}{n}} = -6.7143 + 2.1602 \cdot \sqrt{\frac{31.0552}{14}} = \underline{\underline{-3.4967}}$$

f) Da p-val $< \alpha = 0.05$ kan <u>firmaets påstand (H_0) afvises</u>.

Det ses også, at $\delta_0 = -10$ kWh ikke ligger i konfidensintervallet.

```
%%ETSMP V20_21_Opgave_4
%Data:
A=[31.2 55.3 74.0 15.1 68.6 45.6 103.1  92.3 20.1 12.8 57.2 78.0 23.2 136.5];
B=[36.6 71.4 85.9 12.0 75.1 49.9 113.7 102.2 20.6 12.2 65.5 85.2 26.3 150.4];

%Parret sammenligningstest
delta=A-B
n=length(delta);

%Undersøgelse af normalfordeling
qqplot(delta)

%Null-hypotese
delta_0=-10

%Statistiske estimater
mean_delta=mean(delta)
var_delta=var(delta)

%p-værdi
t=(mean_delta-delta_0)/(sqrt(var_delta/n))
t_cdf= tcdf(abs(t),n-1)
p_val=2*(1-t_cdf)

%Konfidens interval
t0=tinv(0.975,n-1)
delta_minus=mean_delta-t0*sqrt(var_delta/n)
delta_minus=mean_delta+t0*sqrt(var_delta/n)
```

>> Opgave_4

delta = -5.40 -16.10 -11.90 3.10 -6.50 -4.30 -10.60 -9.90 -0.50 0.60 -8.30 -7.20 -3.10 -13.90

delta_0 = -10

mean_delta = -6.7143

var_delta = 31.0552

t = 2.2061

t_cdf = 0.9770

p_val = 0.0460

t0 = 2.1604

delta_minus = -9.9319

delta_minus = -3.4967