Opgave 1

Hændelser: G=Godkendt, nG=Ikke-godkendt, A=Fejl A, B=Fejl B

Data: $Pr\_G := 0.86$ $Pr\_A\_given\_nG := 0.42$ $Pr\_B\_given\_nG := 0.72$

a) $Pr\_nG := 1 - Pr\_G = 0.14$ --> $Pr\_nGpct := Pr\_nG \cdot 100 = 14$

b) $Pr\_A := Pr\_A\_given\_nG \cdot Pr\_nG = 0.059$ --> $Pr\_Apct := Pr\_A \cdot 100 = 5.88$

c) $nG = A + B - A \cap B$ --> $Pr\_A{\cap}B\_given\_nG := Pr\_A\_given\_nG + Pr\_B\_given\_nG - 1 = 0.14$

--> $Pr\_A{\cap}B := Pr\_A{\cap}B\_given\_nG \cdot Pr\_nG = 0.0196$

--> $Pr\_A{\cap}Bpct := Pr\_A{\cap}B \cdot 100 = 1.96$

d) $Pr\_B\_given\_A := \dfrac{Pr\_A{\cap}B}{Pr\_A} = 0.333$ --> $Pr\_B\_given\_Apct := Pr\_B\_given\_A \cdot 100 = 33.333$

Opgave 2

a) Simultan pmf:

$$f'_{XY}(x,y) := \begin{cases} \text{if } x=-1 & \begin{cases} \text{if } y=1 & \frac{1}{20} \\ \text{if } y=2 & \frac{3}{20} \\ \text{if } y=3 & \frac{2}{20} \\ \text{if } y=4 & 0 \end{cases} \\ \text{if } x=0 & \begin{cases} \text{if } y=1 & 0 \\ \text{if } y=2 & \frac{2}{20} \\ \text{if } y=3 & \frac{0}{20} \\ \text{if } y=4 & \frac{1}{20} \end{cases} \\ \text{if } x=1 & \begin{cases} \text{if } y=1 & \frac{4}{20} \\ \text{if } y=2 & \frac{1}{20} \\ \text{if } y=3 & \frac{1}{20} \\ \text{if } y=4 & \frac{2}{20} \end{cases} \end{cases}$$

$$Pr(X=0, Y=3) = f_{XY}(0,3) = f_{XY\_0\_3}$$

$$f_{XY\_0\_3} := 1 - \sum_{x=-1}^{1} \sum_{y=1}^{4} f'_{XY}(x,y) = 0.15$$

-->

$$f_{XY}(x,y) := \begin{cases} \text{if } x=-1 & \begin{cases} \text{if } y=1 & \frac{1}{20} \\ \text{if } y=2 & \frac{3}{20} \\ \text{if } y=3 & \frac{2}{20} \\ \text{if } y=4 & 0 \end{cases} \\ \text{if } x=0 & \begin{cases} \text{if } y=1 & 0 \\ \text{if } y=2 & \frac{2}{20} \\ \text{if } y=3 & \frac{3}{20} \\ \text{if } y=4 & \frac{1}{20} \end{cases} \\ \text{if } x=1 & \begin{cases} \text{if } y=1 & \frac{4}{20} \\ \text{if } y=2 & \frac{1}{20} \\ \text{if } y=3 & \frac{1}{20} \\ \text{if } y=4 & \frac{2}{20} \end{cases} \end{cases}$$

b) Marginale pmf:

$$f_X(x) := \sum_{y=1}^{4} f_{XY}(x,y)$$

$f_X(-1) = 0.3$

$f_X(0) = 0.3$

$f_X(1) = 0.4$

$\sum_{x=-1}^{1} f_X(x) = 1$ o.k.

$$f_Y(y) := \sum_{x=-1}^{1} f_{XY}(x,y)$$

$f_Y(1) = 0.25$

$f_Y(2) = 0.3$

$f_Y(3) = 0.3$

$f_Y(4) = 0.15$

$\sum_{y=1}^{4} f_Y(y) = 1$ o.k.

c) $EX := \sum_{x=-1}^{1} x \cdot f_X(x) = 0.1$

$EY := \sum_{y=1}^{4} y \cdot f_Y(y) = 2.35$

$EX2 := \sum_{x=-1}^{1} x^2 \cdot f_X(x) = 0.7$

$EY2 := \sum_{y=1}^{4} y^2 \cdot f_Y(y) = 6.55$

d) Hændelser: A = $\{X \le 0\}$; B = $\{Y \ge 3\}$; C = $\{X \le 0 \,|\, Y \ge 3\}$ = A|B;

$Pr\_C = Pr(X \le 0 \,|\, Y \ge 3) = Pr(X \le 0 \cap Y \ge 3) / Pr(Y \ge 3) = Pr\_A \cap B / Pr\_B$ <-- Bayes rule

$Pr\_A\cap B := f_{XY}(-1,3) + f_{XY}(-1,4) + f_{XY}(0,3) + f_{XY}(0,4) = 0.3$

$Pr\_B := f_Y(3) + f_Y(4) = 0.45$

$Pr\_C := \frac{Pr\_A\cap B}{Pr\_B} = 0.667$

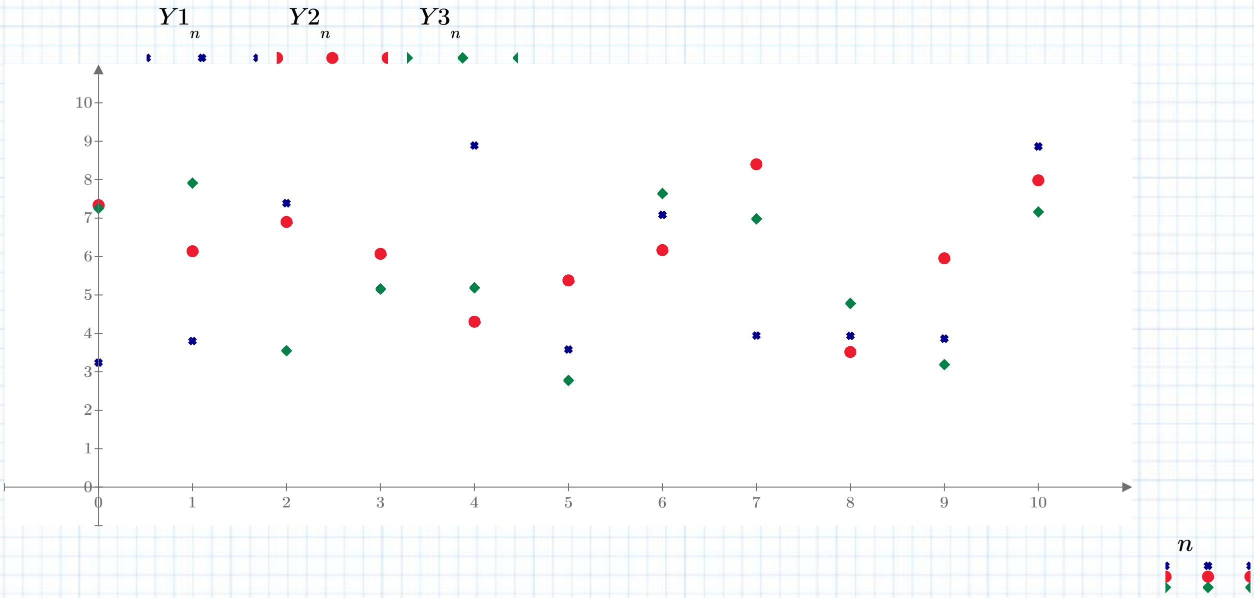
Opgave 3

a) Stokastisk proces : $Y(n) = 3 \cdot X(n) + W(n)$ $X(n)$ --> U(1,3); $W(n)$ --> N(0,0.5)

Tre realisationer: $W1 := \mathrm{rnorm}\left(11, 0, \sqrt{0.5}\right)$ $X1 := \mathrm{runif}(11, 1, 3)$ $Y1 := 3 \cdot X1 + W1$

$n := 0, 1..10$ $W2 := \mathrm{rnorm}\left(11, 0, \sqrt{0.5}\right)$ $X2 := \mathrm{runif}(11, 1, 3)$ $Y2 := 3 \cdot X2 + W2$

$W3 := \mathrm{rnorm}\left(11, 0, \sqrt{0.5}\right)$ $X3 := \mathrm{runif}(11, 1, 3)$ $Y3 := 3 \cdot X3 + W3$



b) Ensemble middelværdi: $EY(n) = E(3 \cdot X(n) + W(n)) = 3 \cdot EX(n) + EW(n) = 3 \cdot \frac{3+1}{2} + 0 = 6$ --> $EY(n) := 6$

Ensemble varians: $VarY(n) = Var(3 \cdot X(n) + W(n)) = 3^2 \cdot Var(X(n)) + Var(W(n)) = 9 \cdot \frac{(3-1)^2}{12} + 0.5$

--> $VarY(n) := 9 \cdot \frac{(3-1)^2}{12} + 0.5 = 3.5$

c) $R_{YY}(\tau) = E(Y(n) \cdot Y(n+\tau)) = EY(n) \cdot EY(n+\tau) = 6 \cdot 6 = 36$ for $\tau \neq 0$, da Y(n) er i.i.d.

--> $R_{YY}(1) = R_{YY}(2) = R_{YY}(3) = 36$

$\tau = 0$: $R_{YY}(0) = E\left(Y(n)^2\right) = Var(Y(n)) + EY(n)^2$ --> $R_{YY}(\tau) := VarY(n) + EY(n)^2 = 39.5$ for $\tau = 0$

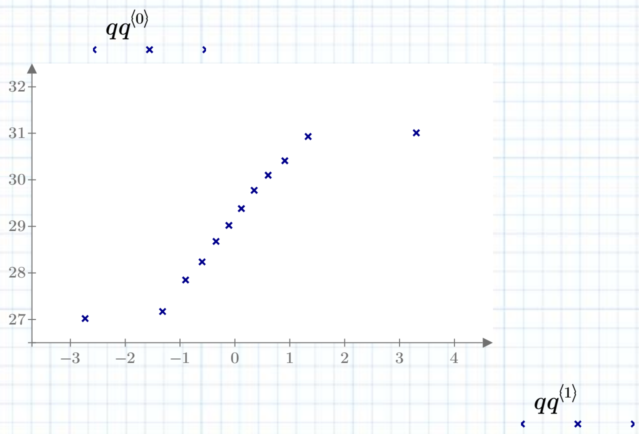
Opgave 4

$\text{clear}(x)$

Data: $x := [30.09\ \ 28.78\ \ 31.01\ \ 27.02\ \ 30.11\ \ 29.35\ \ 28.37\ \ 29.65\ \ 27.71\ \ 30.58\ \ 28.06\ \ 29.04]$ $\qquad xt := x^{T}$

$N := 12$

a) $qq := \text{qqplot}(xt)$



--> Ret linje --> Data normalfordelt

b) Data normalfordelt, ukendt varians --> Student t-test

c) Nul-hypotese H0: $\mu_0 := 30$ $\qquad$ Alternativ hypotese H1: $\mu_0 \neq 30$

d) Sample mean: $\mu_x := \frac{1}{N} \cdot \sum_{i=0}^{11} xt_i = 29.148$

Sample varians: $s2_x := \frac{1}{N-1} \cdot \sum_{i=0}^{11} \left(xt_i - \mu_x\right)^2 = 1.469$

e) Test-værdi: $t := \frac{\mu_x - \mu_0}{\sqrt{\frac{s2_x}{N}}} = -2.437$

p-værdi: $p\_val := 2 \cdot (1 - \text{pt}(\text{abs}(t), N-1)) = 0.033$ $<$ 0.05 = α --> H0 afvises! --> $\mu_0 \neq 30$

f) 95% konfidensinterval: $t_0 := \text{qt}(0.975, N-1) = 2.201$

$\mu_{min} := \mu_x - t_0 \cdot \sqrt{\frac{s2_x}{N}} = 28.377$ $\qquad$ $\mu_{max} := \mu_x + t_0 \cdot \sqrt{\frac{s2_x}{N}} = 29.918$

Konfidensinterval: $[\mu_{min}; \mu_{max}] = [28.377; 29.918]$

Resultatet for konfidensintervallet betyder, at flødebollemaskinen er indstillet til at lave for små flødeboller!