Opg. 1 S19 re

P : Parasit $\qquad$ T : Positiv test

$Pr(P|T) = 0.51 \Rightarrow Pr(\overline{P}|T) = 1 - Pr(P|T) = 0.49$

$Pr(\overline{P}|\overline{T}) = 0.56 \Rightarrow Pr(P|\overline{T}) = 1 - Pr(\overline{P}|\overline{T}) = 0.44$

$Pr(T) = 0.4 \Rightarrow Pr(\overline{T}) = 1 - Pr(T) = 0.6$

a) $Pr(P \cap T) = Pr(P|T) \cdot Pr(T) = 0.51 \cdot 0.4 = \underline{\underline{0.204}}$

b) $Pr(P) = Pr(P \cap T) + Pr(P \cap \overline{T})$

$= Pr(P \cap T) + Pr(P|\overline{T}) \cdot Pr(\overline{T})$

$= 0.204 + 0.44 \cdot 0.6$

$= 0.204 + 0.264$

$= \underline{0.468}$

c) $Pr(T|P) = \dfrac{Pr(P|T) \cdot Pr(T)}{Pr(P)} = \dfrac{Pr(T \cap P)}{Pr(P)}$

$= \dfrac{0.204}{0.468}$

$= \underline{\underline{0.436}}$

Opg. 2 S19 re

$$F_x(x) = \begin{cases} 0 & ; \quad x \le -4 \\ \frac{1}{2} \cdot \left(1 + \frac{x-6}{10} + \frac{1}{\pi} \cdot \sin\left(\frac{\pi}{10}(x-6)\right)\right) & ; \quad -4 < x < 16 \\ 1 & ; \quad x \ge 16 \end{cases}$$

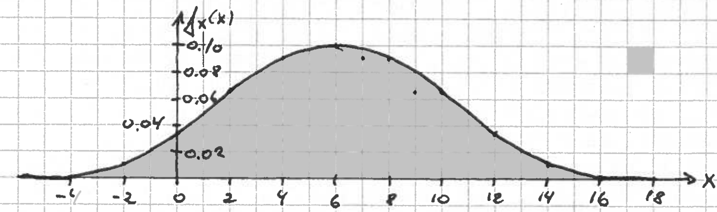
a) $Pr(X > 5) = 1 - Pr(X \le 5) = 1 - F_x(5)$

$= 1 - \frac{1}{2} \cdot \left(1 - \frac{1}{10} + \frac{1}{\pi} \cdot \sin\left(-\frac{\pi}{10}\right)\right) = 1 - \frac{1}{2} \cdot \left(\frac{9}{10} + \frac{-0.3090}{\pi}\right) = 1 - 0.4008 = \underline{\underline{0.5992}}$

b) $Pr(X = 5) = 0$, da X er en kontinuert stokastiske variabel

c) $$f_x(x) = \frac{dF_x(x)}{dx} = \begin{cases} \frac{1}{20} \cdot \left(1 + \cos\left(\frac{\pi}{10}(x-6)\right)\right) & ; \quad -4 < x < 16 \\ 0 & ; \quad \text{ellers } (x \le -4 \vee x \ge 16) \end{cases}$$

x:	-4	-2	0	2	4	6	8	10	12	14	16
f_x:	0	0.010	0.035	0.065	0.090	0.10	0.090	0.065	0.035	0.010	0



d) $E[X] = \int_{-\infty}^{\infty} x \cdot f_x(x)\,dx = \int_{-4}^{16} \frac{1}{20} \cdot \left(x + x \cdot \cos\left(\frac{\pi}{10}(x-6)\right)\right) dx = \frac{1}{20}\left[\frac{1}{2}x^2 + \frac{100}{\pi^2} \cdot \cos\left(\frac{\pi}{10}(x-6)\right) + \frac{10}{\pi} \cdot x \cdot \sin\left(\frac{\pi}{10}(x-6)\right)\right]_{-4}^{16}$

$= \frac{1}{20}\left(\frac{256}{2} + \frac{100}{\pi^2} \cdot \cos(\pi) + \frac{10}{\pi} \cdot 16 \cdot \sin(\pi) - \frac{16}{2} - \frac{100}{\pi^2} \cdot \cos(-\pi) - \frac{10}{\pi}(-4) \cdot \sin(-\pi)\right)$

$= \frac{1}{20} \cdot \left(128 - \frac{100}{\pi^2} + 0 - 8 + \frac{100}{\pi^2} - 0\right) = \frac{120}{20} = \underline{\underline{6}}$

e) $E[X^2] = \int_{-\infty}^{\infty} x^2 \cdot f_x(x)\,dx = \int_{-4}^{16} \frac{1}{20}\left(x^2 + x^2 \cdot \cos\left(\frac{\pi}{10}(x-6)\right)\right) dx$

$= \frac{1}{20}\left[\frac{1}{3}x^3 + \left(\frac{10}{\pi}\right)^3 \cdot \left(2x \cdot \frac{\pi}{10} \cdot \cos\left(\frac{\pi}{10}(x-6)\right) + \left(x^2 \cdot \left(\frac{\pi}{10}\right)^2 - 2\right) \cdot \sin\left(\frac{\pi}{10}(x-6)\right)\right)\right]_{-4}^{16}$

$= \frac{1}{20}\left(\frac{4096}{3} + \left(\frac{10}{\pi}\right)^3 \cdot \left(2 \cdot 16 \cdot \frac{\pi}{10} \cdot \cos(\pi) + \left(256 \cdot \left(\frac{\pi}{10}\right)^2 - 2\right) \cdot \sin(\pi)\right) + \frac{64}{3} - \left(\frac{10}{\pi}\right)^3 \cdot \left(2 \cdot (-4) \cdot \frac{\pi}{10} \cos(-\pi) + \left(16 \cdot \left(\frac{\pi}{10}\right)^2 - 2\right) \cdot \sin(-\pi)\right)\right)$

$= \frac{1}{20} \cdot \left(\frac{4160}{3} + \left(\frac{10}{\pi}\right)^3 \cdot \left(-32 \cdot \frac{\pi}{10} - 8 \cdot \frac{\pi}{10}\right)\right) = \frac{208}{3} - \frac{200}{\pi^2} = 49.07$

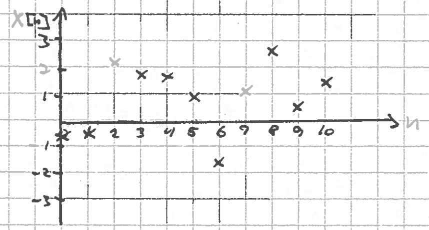
$Var[X] = E[X^2] - E[X]^2 = 49.07 - 6^2 = 49.07 - 36 = \underline{13.07}$

Opg. 3 S19 re

$X[n] = -Y + W[n]$; $Y \sim B(n=2, p=0.2)$, $W[n] \sim \mathcal{N}(0,2)$

Y og $W[n]$ er uafhængige

a) $X[n] = -\text{binornd}(2, 0.2) + \sqrt{2}\cdot \text{randn}(1,n)$; $n = 0, 1, \cdots, 10$



Se bilag (Matlab)

b) Ensemble: $\underline{\underline{E[X]}} = E[-Y] + E[W] = -n\cdot p + \mu_W = -2\cdot 0.2 + 0 = \underline{\underline{-0.4}}$

$\underline{\underline{Var[X]}} = Var[-Y] + Var[W]$ (Y og W uafh.)

$= (-1)^2 \cdot Var[Y] + Var[W]$

$= 1 \cdot n \cdot p \cdot (1-p) \cdot \sigma_W^2 = 1\cdot 2 \cdot 0.2 \cdot 0.8 + 2 = 0.32 + 2 = \underline{\underline{2.32}}$

c) Temporal: $\underline{\underline{\hat{\mu}_X = \lim_{N\to\infty} \frac{1}{N}\sum_{n=0}^{N}(-Y + W[n])}} = -Y + \hat{\mu}_W = -Y + 0 = -Y$

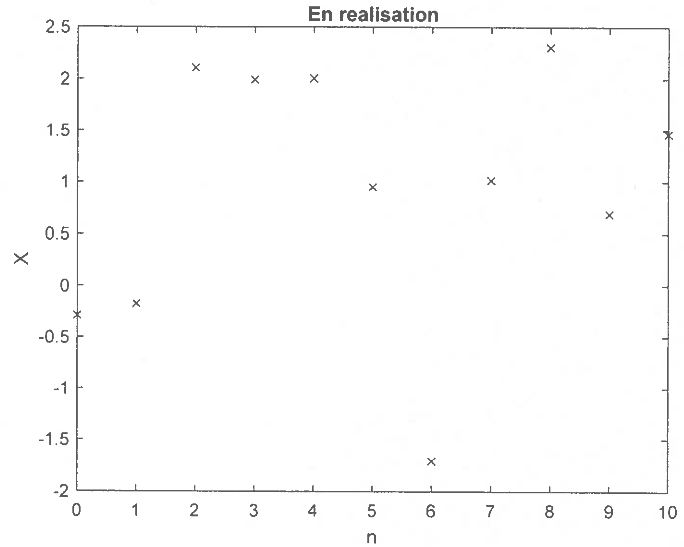
d) $E[X]$ og $Var[X]$ er uafhængige af n (tiden) $\rightarrow$ $\underline{\underline{X \text{ er WSS}}}$

$E[X] \neq \hat{\mu}_X \rightarrow$ X er ikke ergodisk

Afh. af den enkelte realisation (Y)

```
%%Opg3_S19 re

X=-binornd(2,0.2)+sqrt(2)*randn(1,11);
plot(0:10,X,'x')
title('En realisation')
xlabel('n')
ylabel('X')
```



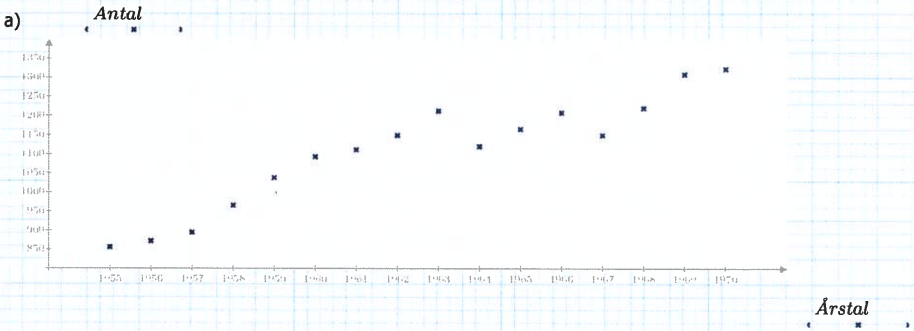
Opgave 4

Data:

$\mathit{Årstal} := [1955\ \ 1956\ \ 1957\ \ 1958\ \ 1959\ \ 1960\ \ 1961\ \ 1962\ \ 1963\ \ 1964\ \ 1965\ \ 1966\ \ \ldots]$

$\mathit{Antal} := [856\ \ 872\ \ 895\ \ 966\ \ 1038\ \ 1093\ \ 1111\ \ 1149\ \ 1212\ \ 1120\ \ 1165\ \ 1207\ \ \ldots]$

a)



b) Middelværdier: $\mu_{År} := \frac{1}{16} \cdot \sum_{i=0}^{15} \mathit{Årstal}_{0,i} = 1962.5$

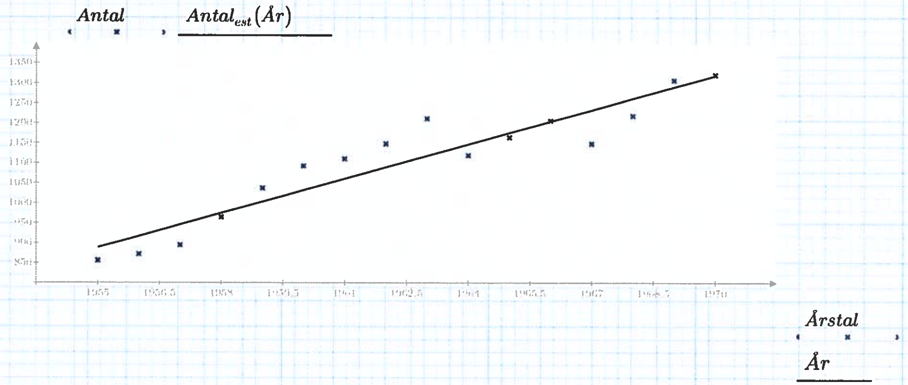
$\mu_{Antal} := \frac{1}{16} \cdot \sum_{i=0}^{15} \mathit{Antal}_{0,i} = 1105.1$

c) Lineær regression:

Hældning: $\beta := \frac{\sum_{i=0}^{15} \left(\left(\mathit{Årstal}_{0,i} - \mu_{År} \right) \cdot \left(\mathit{Antal}_{0,i} - \mu_{Antal} \right) \right)}{\sum_{i=0}^{15} \left(\mathit{Årstal}_{0,i} - \mu_{År} \right)^2} = 28.679$

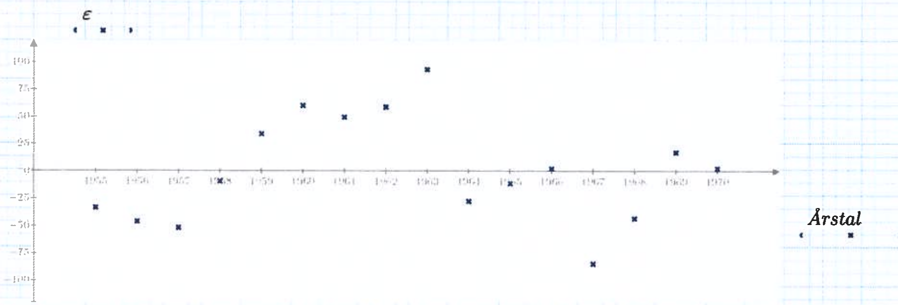
Skæring: $\alpha := \mu_{Antal} - \beta \cdot \mu_{År} = -55178.221$

$\mathit{Antal}_{est}(\mathit{År}) := \alpha + \beta \cdot \mathit{År} \xrightarrow{float,\,5} 28.679 \cdot \mathit{År} - 55178.0$ $\qquad \mathit{År} := 1955, 1956 .. 1970$



d) Residualer: $\varepsilon := \mathit{Antal} - \mathit{Antal}_{est}(\mathit{Årstal})$

$\varepsilon = [-33.4\ \ -46.1\ \ -51.8\ \ -9.5\ \ 33.8\ \ 60.2\ \ 49.5\ \ 58.8\ \ 93.1\ \ -27.6\ \ -11.2\ \ 2.1\ \ -84.6\ \ -43.3\ \ 17\ \ 2.4]$



e) Hypotesetest på hældning β=0: H0: $\beta = 0$ H1: $\beta \neq 0$

f) $s2_x := \sum_{i=0}^{15} \left(\mathit{Årstal}_{0,i} - \mu_{År} \right)^2 = 340$ $\qquad s_x := \sqrt{s2_x} = 18.439$

$s2_r := \frac{1}{14} \cdot \sum_{i=0}^{15} \left(\mathit{Antal}_{0,i} - \mathit{Antal}_{est}\left(\mathit{Årstal}_{0,i} \right) \right)^2 = 2541.166$ $\qquad s_r := \sqrt{s2_r} = 50.41$

$t := \frac{\beta - 0}{\frac{s_r}{s_x}} = 10.5$ $\qquad p := 2 \cdot (1 - \mathrm{pt}(t, 14)) = 5.146 \cdot 10^{-8}$

g) $p = 5.146 \cdot 10^{-8}$ < 0,05 --> Hypotesen afvises --> Hældningen $\beta \neq 0$