GROUP ASSIGNMENT 05

Simultaneous Random Variables, Data Sampling, Transformation, Multivariate and CLT

I. Simultaneous Random Variables

Two discrete random variables X and Y have a joint pmf $f_{XY}(x,y)$ as given in the following table:

Y \ X	10	20	30	40
–100	0.04		0.08	0.14
0	0.06	0.06		0.08
100	0.18	0.00	0.08	

It is furthermore known, that Pr(Y=0) = 0.28 and Pr(Y=–100|X=20) = 0.50.

1. Fill in the empty places in the joint pmf-table.
2. Calculate and sketch the marginals $f_X(x)$ and $f_Y(y)$.
3. Calculate the means and variances EX, EY, Var(X), Var(Y) and E[XY].
4. Calculate the correlation corr(X,Y), covariance cov(X,Y) and correlation coefficient ρ_{XY}.
5. Are the random variables X and Y independent?

II. Data Sampling

You need testdata (virtuel measurements) for test or simulation of a flight simulator. The testdata should simulate the altitude of a plane and are supposed to be Rayleigh distributed:

$$f_X(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} \; ; \qquad x > 0$$

1. Find the cdf for the Rayleigh distribution $F_X(x)$.
2. Find the inverse of the Rayleigh cdf (ie. x as a function of $y = F_X(x)$).
3. Make a Matlab programme that makes 10000 random Rayleigh distributed testdata with σ=7.
4. Make a scatter-plot of the testdata.
5. Make a histogram of the data. Does it look like a Rayleigh distribution?
6. Extra: Repeat 1.–5. for exponential distributed data: $f_X(x) = \lambda e^{-\lambda x}; \;\; x \geq 0, \;$ with $\lambda = \frac{1}{8}$.

III. Transformation of Random Variable

A stochastic variable Y are given by: $Y = -3 \cdot X + 4$, where $X \sim \mathcal{U}(-1{,}2)$.

1. Determine the pdf $f_Y(y)$, and make a draw of it together with $f_X(x)$.
2. Calculate EX, EY, Var(X) and Var(Y).
3. Make a Matlab programme that:
    a. simulates 10 samples of one realizaton of X and Y by use of the *rand* function;
    b. plot the samples of the realizations of X and Y;
    c. plot the samples of Y against the samples of X.

IV. Multivariate Random Variables

A resistor R in an analog filter is made of two resistors in series: $R_1 = 2.4k\Omega$ and $R_2 = 100\Omega$. Both resistors are uniform distributed ±5%.

1. Determine and draw the pdf $f_R(r)$ for the sum of the two resistors $R = R_1 + R_2$.
2. Find the mean and standard deviation of $R = R_1 + R_2$.
3. What would the mean and standard deviation be if we instead used one uniform distributed 5%-resistor $R_0 = 2.5k\Omega$?
4. If you could chose any two uniform distributed 5%-resistors to build a 2.5 $k\Omega$ resistor, which resistor-values will you chose to get the most accurate total resistor? What will the standard deviation be? Determine and draw the pdf $f_R(r)$ for the sum of the two resistors.
5. Make a simulation of 1. – 4. in Matlab.

V. The Central Limit Theorem (CLT)

Make a Matlab simulation to illustrate the Central Limit Theorem (CLT):

Make 9 random variables $Y_1,\ldots,Y_9$ as the average of 1 – 9 i.i.d. random variables X_n (ie. $Y_1=X_1$, $Y_2=½(X_1+X_2)$ etc.) and make a plot of the 9 random variables Y_i.

Does the random variables approach a normal distribution when more terms are included?

Make the simulations for:

1. $X_n \sim \mathcal{U}(0{,}1)$
2. $X_n \sim Rayleigh\ distributed\ with\ \sigma = 1$