

Terningespil: 3 kast med en terning ( $X_1, X_2, X_3$ ) -  $X_1, X_2, X_3 = 1, \dots, 6$   
 $X = X_1 + X_2 + X_3 = \text{summen af \u00f8jne} = 3, \dots, 18$

1) Kombinationsmuligheder:

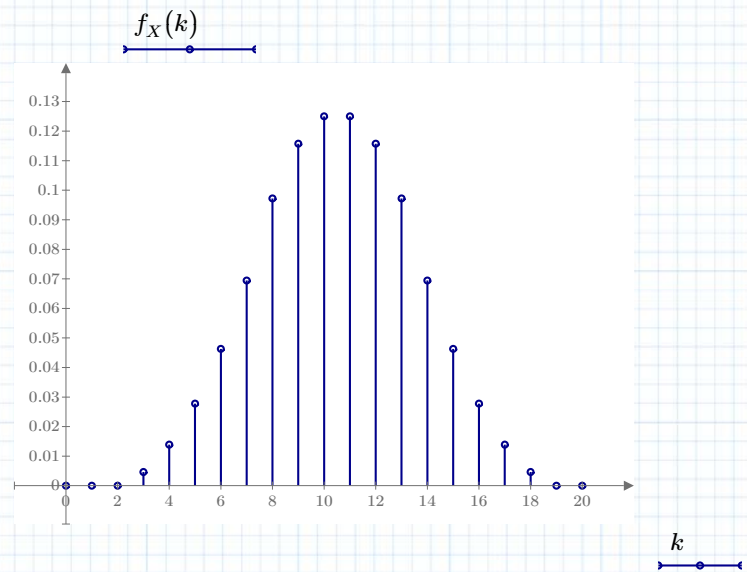
$X=3$ (18):	(1,1,1) $1 = 1$	$X=8$ (13):	(1,1,6) $6 + 5 + 4 + 3 + 2 + 1 = 21$ (1,2,5)	
$X=4$ (17):	(1,1,2) $2 + 1 = 3$ (1,2,1) (2,1,1)	(1,3,4) (1,4,3) (1,5,2) (1,6,1)	$X=9$ (12):	(1,2,6) $5 + 6 + 5 + 4 + 3 + 2 = 25$ (1,3,5) (1,4,4) (1,5,3) (1,6,2) (2,1,6) (2,2,5) (2,3,4) (2,4,3) (2,5,2) (2,6,1) (3,1,5) (3,2,4) (3,3,3) (3,4,2) (3,5,1) (4,1,4) (4,2,3) (4,3,2) (4,4,1) (5,1,3) (5,2,2) (5,3,1) (6,1,2) (6,2,1)
$X=5$ (16):	(1,1,3) $3 + 2 + 1 = 6$ (1,2,2) (1,3,1) (2,1,2) (2,2,1) (3,1,1)	(2,1,5) (2,2,4) (2,3,3) (2,4,2) (2,5,1) (3,1,4) (3,2,3) (3,3,2) (3,4,1) (4,1,3) (4,2,2) (4,3,1) (5,1,2) (5,2,1) (6,1,1)		$4 + 5 + 6 + 5 + 4 + 3 = 27$ $X=10$ (11): (1,3,6) (1,4,5) (1,5,4) (1,6,3) (2,2,6) (2,3,5) (2,4,4) (2,5,3) (2,6,2) (3,1,6) (3,2,5) (3,3,4) (3,4,3) (3,5,2) (3,6,1) (4,1,5) (4,2,4) (4,3,3) (4,4,2) (4,5,1) (5,1,4) (5,2,3) (5,3,2) (5,4,1) (6,1,3) (6,2,2) (6,3,1)
$X=6$ (15):	(1,1,4) $4 + 3 + 2 + 1 = 10$ (1,2,3) (1,3,2) (1,4,1) (2,1,3) (2,2,2) (2,3,1) (3,1,2) (3,2,1) (4,1,1)			
$X=7$ (14):	(1,1,5) $5 + 4 + 3 + 2 + 1 = 15$ (1,2,4) (1,3,3) (1,4,2) (1,5,1) (2,1,4) (2,2,3) (2,3,2) (2,4,1) (3,1,3) (3,2,2) (3,3,1) (4,1,2) (4,2,1) (5,1,1)			

Kombinationsmuligheder i alt:  $N := 2 \cdot (1 + 3 + 6 + 10 + 15 + 21 + 25 + 27) = 216$

## 2) Probability Mass Function pmf:

 $k := 0, 1 \dots 20$ 

$$f_X(x) := \begin{cases} \frac{1}{216} & \text{if } x = 3 \vee x = 18 \\ \frac{3}{216} & \text{else if } x = 4 \vee x = 17 \\ \frac{6}{216} & \text{else if } x = 5 \vee x = 16 \\ \frac{10}{216} & \text{else if } x = 6 \vee x = 15 \\ \frac{15}{216} & \text{else if } x = 7 \vee x = 14 \\ \frac{21}{216} & \text{else if } x = 8 \vee x = 13 \\ \frac{25}{216} & \text{else if } x = 9 \vee x = 12 \\ \frac{27}{216} & \text{else if } x = 10 \vee x = 11 \\ 0 & \text{else} \end{cases}$$

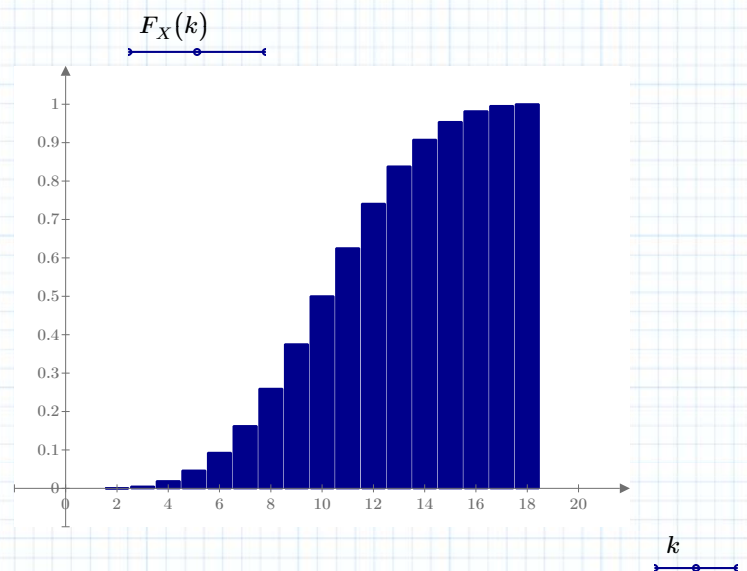


## 3) Fordelingsfunktion cdf:

$$F_X(x) := \sum_{k=0}^x f_X(k)$$

$$F_X(k) =$$

0  
0.005  
0.019  
0.046  
0.093  
0.162  
0.259  
0.375  
0.5  
0.625  
0.741  
0.838  
0.907  
0.954  
0.981  
0.995  
1

 $k := 2, 3 \dots 18$ 



$$Pr_X(k) := f_X(k)$$

$$4) \quad Pr_X(8) = 0.097$$

$$5) \quad Pr_X(X \geq 12) = 1 - Pr_X(X < 12) = 1 - F_X(11) \quad 1 - F_X(11) = 0.375$$

$$6) \quad Pr_X(9 \leq X \leq 15) = Pr_X(X \leq 15) - Pr_X(X < 9) = F_X(15) - F_X(8) \quad F_X(15) - F_X(8) = 0.694$$

$$7) \quad Pr_X(X < 4 \vee X > 16) = Pr_X(X < 4) + Pr_X(X > 16) = Pr_X(X < 4) + (1 - Pr_X(X \leq 16)) = F_X(3) + (1 - F_X(16))$$

$$F_X(3) + (1 - F_X(16)) = 0.023$$

$$8) \quad EX := \sum_{k=3}^{18} k \cdot f_X(k) = 10.5$$

$$9) \quad EX^2 := \sum_{k=3}^{18} k^2 \cdot f_X(k) = 119$$

$$VarX := EX^2 - EX^2 = 8.75$$

$$SDX := \sqrt{VarX} = 2.958$$

$$10) \quad Pr_X(EX - SDX \leq X \leq EX + SDX) = F_X(EX + SDX) - F_X(EX - SDX - 1)$$

$$F_X(EX + SDX) - F_X(EX - SDX - 1) = 0.745$$

$$Pr_X(EX - 2 SDX \leq X \leq EX + 2 SDX) = F_X(EX + 2 SDX) - F_X(EX - 2 SDX - 1)$$

$$F_X(EX + 2 SDX) - F_X(EX - 2 SDX - 1) = 0.977$$

$$Pr_X(EX - 3 SDX \leq X \leq EX + 3 SDX) = F_X(EX + 3 SDX) - F_X(EX - 3 SDX - 1)$$

$$F_X(EX + 3 SDX) - F_X(EX - 3 SDX - 1) = 1$$

$$11) \quad X_1 := 6$$

$$Pr_X(X \geq 12) = Pr_X(X_2 + X_3 \geq 6) = \sum_{i=1}^6 Pr(X_2 = i) \cdot Pr(X_2 \geq 6 - i) = \blacksquare$$

$$Pr(X_2 = 1) Pr(X_2 \geq 5) + Pr(X_2 = 2) Pr(X_2 \geq 4) + Pr(X_2 = 3) Pr(X_2 \geq 3) + Pr(X_2 = 4) Pr(X_2 \geq 2) + Pr(X_2 = 5) Pr(X_2 \geq 1) + Pr(X_2 = 6) Pr(X_2 \geq 0)$$

$$\blacksquare = \frac{1}{6} \cdot \left( \frac{2+3+4+5+6+6}{6} \right)$$

$$\frac{1}{6} \cdot \left( \frac{2+3+4+5+6+6}{6} \right) = 0.722$$