# 1 Corrections

This short note contains a correct proof of Theorem 1 in the Invertible Bloom Lookup Table paper.

**Theorem 1.** *If $m = e^2kt$ then ListEntries fails with probability $O(t^{-k+2})$ for $n \leq t$.*

*Proof.* We assume $k \geq 3$ as otherwise the guarantee $O(t^{-k+2})$ is trivial.

We describe the result in terms of the 2-core. Let $F$ be the event that there is a non-empty 2-core. Let $F_j$ denote the event that there is a non-empty 2-core consisting of $j$ hyperedges. Then by the union bound $\Pr[F] \leq \sum_{j=2}^{n} \Pr[F_j]$. To bound $F_j$, define a set $S$ for each subset of $j$ hyperedges out of the $n$ random hyperedges $E = \{e_1, \ldots, e_n\}$. We use the notation $\binom{E}{j}$ to denote the collection of all $j$-sized subsets of $E$. For each $S \in \binom{E}{j}$, let $F_{S,j}$ be the event that $S$ is the 2-core of size $j$. Then by the union bound, $\Pr[F_j] \leq \sum_{S \in \binom{E}{j}} \Pr[F_{S,j}]$. Observe that if a set $S$ with $|S| = j$ forms a 2-core, then the number of nodes $i$ with $i \in e$ for a hyperedge $e \in S$ is no more than $jk/2$. This is true since each hyperedge $e \in S$ is a subset of $k$ nodes, and any node must be contained in either zero or at least two hyperedges if $S$ is the 2-core. We therefore define yet another event $F_{S,j,T}$ for each subset $T$ of $jk/2$ nodes among all nodes $V = \{v_1, \ldots, v_m\}$. $F_{S,j,T}$ is the event that the 2-core consists of the $j$ hyperedges $S$ and that all the hyperedges $e \in S$ have $e \subseteq T$. A union bound gives $\Pr[F_{S,j}] \leq \sum_{T \in \binom{V}{jk/2}} \Pr[F_{S,j,T}]$.

To bound $\Pr[F_{S,j,T}]$, observe that for $F_{S,j,T}$ to happen, each of the $k$ endpoints of each hyperedge $e \in S$ must be in $T$. Thus $\Pr[F_{S,j,T}] \leq (|T|/m)^{kj} = (jk/(2m))^{kj}$. We therefore have $\Pr[F_{S,j}] \leq \sum_{T \in \binom{V}{jk/2}} \Pr[F_{S,j,T}] \leq \binom{m}{jk/2}(jk/(2m))^{kj}$ and then $\Pr[F_j] \leq \sum_{S \in \binom{E}{j}} \Pr[F_{S,j}] \leq \binom{n}{j}\binom{m}{jk/2}(jk/(2m))^{kj}$. Finally, we get $\Pr[F] \leq \sum_{j=2}^{n} \Pr[F_j] = \sum_{j=2}^{n} \binom{n}{j}\binom{m}{jk/2}(jk/(2m))^{kj}$. We finally do the calculations while using the inequality $\binom{n}{k} \leq (ne/k)^k$ and see:

$$\begin{aligned}
\sum_{j=2}^{n} \binom{n}{j}\binom{m}{jk/2}(jk/(2m))^{kj} &\leq \\
\sum_{j=2}^{n} (en/j)^j (2em/(jk))^{jk/2} (jk/(2m))^{kj} &= \\
\sum_{j=2}^{n} (en/j)^j (ejk/(2m))^{jk/2} &= \\
\sum_{j=2}^{n} (e^2kn/(2m))^j (ejk/(2m))^{j(k/2-1)} &\leq \\
\sum_{j=2}^{n} (1/2)^j (ejk/(2m))^{j(k/2-1)}.
\end{aligned}$$

We observe that the ratio between two consecutive terms in the sum is at most $(1/2)(ejk/(2m)) \leq (1/2)(1/(2e)) < 1/2$, hence the sum is asymptotically dominated by the first term. Thus

$$\Pr[F_j] = O((en/2)^2(2ek/(2m))^{2k/2}) = O(t^{2-k}).$$

□