Introduction to Artificial Intelligence
Lecture 10b: Belief Revision on Plausibility Orders

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Three Parts of Taking in New Information

What can I do to my belief?

1. **Revision**: $B * \varphi$ is a new belief that includes φ.
2. **Contraction**: $B \div \varphi$ is a new belief that does not include φ.

Thinking in Terms of Plausibility Orders: Prior

Bob believes: $Cn(\{p, q, p \to q\})$, i.e., the state x is the most plausible.

But there are different ways in which the remaining options can be ordered.

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
			w
	y		
		z	
x			

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
	y		
		z	
			w
x			

In the above pictures, the lower the state the more plausible it is.

Thinking in Terms of Plausibility Orders

Revision Posterior

Bob believes: $Cn(\{p, q, p \to q\})$, i.e., the state x is the most plausible.

After revising with $\neg q$ his **posterior plausibility** changes differently depending on the **prior plausibility**.

We are looking for prior-minimal states that do not satisfy q.

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
			w
	y		
		z	
x			

Table: Option B: $Cn(\{p, \neg q\})$

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
	y		
		z	
			w
x			

Table: Option E: $Cn(\{p \to q, \neg q\})$

Bob believes: $Cn(\{p, q, p \to q\})$, i.e., the state x is the most plausible.

After contracting with q, Bob has to expand his view.

We are looking for prior-minimal states that do not satisfy q.

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
			w
	y		
		z	
x			

Table: $Cn(\{p\})$

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
	y		
		z	
			w
x			

Table: $Cn(\{p \leftrightarrow q\})$

After contraction Bob's beliefs are specified by the union of his prior most plausible world and the prior most plausible word not-entailing q.

Recall: Revision and Contraction on Plausibility Orders

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
		z	
			w
	y		
x			

more plausible

Table: Plausibility order over valuations

Recall: Revision and Contraction on Plausibility Orders

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
		z	
			w
	y		
x			

more plausible

Table: Plausibility order over valuations

Definition

Let P be a set of propositions (e.g. above, $P = \{p, q\}$). A **plausibility order** is a total preorder $\leq$ over the possible truth assignments W on P. A total preorder on X is a binary relation that is:

- transitive: for all $x, y, z \in X$, if $x \leq y$ and $y \leq z$, then $x \leq z$;
- complete: for all $x, y \in X$, $x \leq y$ or $y \leq x$.

Recall: Revision and Contraction on Plausibility Orders

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
		z	
			w
	y		
x			

more plausible

Table: Plausibility order over valuations

Let B be a belief set, φ a formula, and let $|\varphi| := \{x \in W \mid \varphi \text{ is true in } x\}$.

Recall: Revision and Contraction on Plausibility Orders

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
		z	
			w
	y		
x			

more plausible

Table: B is determined by the most plausible world(s)

Let B be a belief set, φ a formula, and let $|\varphi| := \{x \in W \mid \varphi \text{ is true in } x\}$.

- $\varphi \in B$ iff $min_{\leq}(W) \subseteq |\varphi|$;

Recall: Revision and Contraction on Plausibility Orders

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
		z	
			w
	y		
x			

more plausible

Table: $B * \neg p$ is determined by min world(s) with $\neg p$

Let B be a belief set, φ a formula, and let $|\varphi| := \{x \in W \mid \varphi \text{ is true in } x\}$.

- $\varphi \in B$ iff $min_{\leq}(W) \subseteq |\varphi|$;
- $\varphi \in B * \psi$ iff $min_{\leq}(|\psi|) \subseteq |\varphi|$;

Recall: Revision and Contraction on Plausibility Orders

p, q	$p, \bar{q}$	$\bar{p}, q$	$\bar{p}, \bar{q}$
		z	
			w
	y		
x			

more plausible

Table: $B \div \neg p$ is the union of the previous two

Let B be a belief set, φ a formula, and let $|\varphi| := \{x \in W \mid \varphi \text{ is true in } x\}$.

- $\varphi \in B$ iff $min_{\leq}(W) \subseteq |\varphi|$;
- $\varphi \in B * \psi$ iff $min_{\leq}(|\psi|) \subseteq |\varphi|$;
- $\varphi \in B \div \psi$ iff $min_{\leq}(|\neg\psi|) \cup min_{\leq}(W) \subseteq |\varphi|$

Exercise 1 in Week 11

Assume Bob's belief set $B = Cn(\{p, p \leftrightarrow q, \neg r\})$. Come up with an appropriate prior plausibility order on W (the set of all possible truth assignments over p, q, r), which will satisfy both of the requirements below:

1. after revision with r Bob would believe that $\neg q$;
2. after contraction with $p \rightarrow q$ Bob would believe that p.

Solution

p, q, r	$p, q, \bar{r}$	$p, \bar{q}, r$	$p, \bar{q}, \bar{r}$	$\bar{p}, q, r$	$\bar{p}, q, \bar{r}$	$\bar{p}, \bar{q}, r$	$\bar{p}, \bar{q}, \bar{r}$
a			d	e	f	g	h
		c					
	b						

Table: An example of a prior plausibility

1. After revision with r Bob would believe that $\neg q$: after revision with r the minimal world becomes c, and $\neg q$ is true in c.
2. After contraction with $p \to q$ Bob would believe that p: after contraction with $p \to q$, c is the minimal world which does not satisfy $p \to q$, since p is true in c and q is false in c. Note that, $p \in B \div (p \to q)$ because p is true in both b and c.

THE END OF LECTURE 10b