# Introduction to Artificial Intelligence

## Lecture 10: Belief Revision

Nina Gierasimczuk

# The problem of Belief Revision

Belief revision is a topic of much interest in theoretical computer science and logic, and it forms a central problem in research into artificial intelligence. In simple terms: how do you update a database of knowledge in the light of new information? What if the new information is in conflict with something that was previously held to be true?

Gärdenfors, Belief Revision

# History: Data Bases, Theories and Beliefs

- Computer science: updating databases (Doyle 1979 and Fagin et al. 1983)
- Philosophy (epistemology):
  - scientific theory change and revisions of probability assignments;
  - belief change (Levi 1977, 1980, Harper 1977) and its rationality.

# AGM Belief Revision Model

- Names: Carlos **A**lchourrón, Peter **G**ärdenfors, and David **M**akinson.
- 1985 paper in the Journal of Symbolic Logic.
- Starting point of belief revision theory.

# Belief Representation in AGM

We are talking about **beliefs** rather than **knowledge**.

In this context, the difference is that beliefs are changeable and can be false.

# Belief Representation in AGM

We are talking about **beliefs** rather than **knowledge**.

In this context, the difference is that beliefs are changeable and can be false.

- belief := sentence

# Belief Representation in AGM

We are talking about **beliefs** rather than **knowledge**.

In this context, the difference is that beliefs are changeable and can be false.

- belief := sentence
- belief := sentence in some formal language

# Belief Representation in AGM

We are talking about **beliefs** rather than **knowledge**.

In this context, the difference is that beliefs are changeable and can be false.

- belief := sentence
- belief := sentence in some formal language
- beliefs of an agent := a set of such sentences

# Belief Representation in AGM

We are talking about **beliefs** rather than **knowledge**.

In this context, the difference is that beliefs are changeable and can be false.

- belief := sentence
- belief := sentence in some formal language
- beliefs of an agent := a set of such sentences

## Language of Beliefs in AGM

Beliefs are expressed in propositional logic:

- propositions $p, q, r, \ldots$
- connectives: negation ($\neg$), conjunction ($\wedge$), disjunction ($\vee$), implication ($\rightarrow$), and biconditional ($\leftrightarrow$).

**Example (What are the consequences of my beliefs?)**

# Belief Sets

Example (What are the consequences of my beliefs?)

1. John is a bachelor. John is handsome.

# Belief Sets

Example (What are the consequences of my beliefs?)

1. John is a bachelor. John is handsome. **John is a handsome bachelor.**

# Belief Sets

Example (What are the consequences of my beliefs?)

1. John is a bachelor. John is handsome. **John is a handsome bachelor.**
2. If we charge high fees for university, only the rich enroll. We charge high fees for university.

# Belief Sets

Example (What are the consequences of my beliefs?)

1. John is a bachelor. John is handsome. **John is a handsome bachelor.**
2. If we charge high fees for university, only the rich enroll. We charge high fees for university. **Only the rich enroll.**

**Example (What are the consequences of my beliefs?)**

1. John is a bachelor. John is handsome. **John is a handsome bachelor.**
2. If we charge high fees for university, only the rich enroll. We charge high fees for university. **Only the rich enroll.**
3. The barber is male. The barber shaves only those men in town who do not shave themselves.

Example (What are the consequences of my beliefs?)

1. John is a bachelor. John is handsome. **John is a handsome bachelor.**
2. If we charge high fees for university, only the rich enroll. We charge high fees for university. **Only the rich enroll.**
3. The barber is male. The barber shaves only those men in town who do not shave themselves. **The barber is female.**

# Belief Sets

### Example (What are the consequences of my beliefs?)

1. John is a bachelor. John is handsome. **John is a handsome bachelor.**
2. If we charge high fees for university, only the rich enroll. We charge high fees for university. **Only the rich enroll.**
3. The barber is male. The barber shaves only those men in town who do not shave themselves. **The barber is female.**

**Belief set** is a set of formulas that is **deductively closed**.
As such it is an abstract object.

# Why are Belief Sets important?

**Example**

Assume Bob tells you that their beliefs include:

$$p, q.$$

Upon further inquiry it turns out that Bob also believes:

$$p \rightarrow \neg q.$$

# Why are Belief Sets important?

### Example

Assume Bob tells you that their beliefs include:

$$p, q.$$

Upon further inquiry it turns out that Bob also believes:

$$p \to \neg q.$$

Bob's beliefs are inconsistent!

# Why are Belief Sets important?

### Example

Assume Bob tells you that their beliefs include:

$$p, q.$$

Upon further inquiry it turns out that Bob also believes:

$$p \to \neg q.$$

Bob's beliefs are inconsistent!

Even though Bob's expressed beliefs do not include two complementary literals, $\neg q$ can be deduced from $p$ and $p \to \neg q$.

He is committed to both $q$ and $\neg q$, and so both are in Bob's belief set.

# Logical Consequence

## Definition

For any set $B$ of sentences, $Cn(B)$ is the set of **logical consequences** of $B$.

If $\varphi$ can be derived from $B$ by classical propositional logic, then $\varphi \in Cn(B)$.

## Example (cntd)

Bob's expressed beliefs form the set: $B = \{p, q, p \rightarrow \neg q\}$, then $B \subset Cn(B)$, $p \wedge q \in Cn(B)$, but also $\neg q \in Cn(B)$.

# Logical Consequence

### Definition

For any set $B$ of sentences, $Cn(B)$ is the set of **logical consequences** of $B$.

If $\varphi$ can be derived from $B$ by classical propositional logic, then $\varphi \in Cn(B)$.

### Example (cntd)

Bob's expressed beliefs form the set: $B = \{p, q, p \to \neg q\}$, then $B \subset Cn(B)$, $p \wedge q \in Cn(B)$, but also $\neg q \in Cn(B)$.

Why is it problematic to have inconsistent beliefs?

DEFINITION

For any set $B$ of sentences, $Cn(B)$ is the set of **logical consequences** of $B$.

If $\varphi$ can be derived from $B$ by classical propositional logic, then $\varphi \in Cn(B)$.

EXAMPLE (CNTD)

Bob's expressed beliefs form the set: $B = \{p, q, p \rightarrow \neg q\}$, then $B \subset Cn(B)$, $p \wedge q \in Cn(B)$, but also $\neg q \in Cn(B)$.

Why is it problematic to have inconsistent beliefs?

If $B$ is an inconsistent set of formulas, then for any formula $\varphi$, $\varphi \in Cn(B)$.

In other words, we can deduce anything from inconsistent beliefs, as such they are uninteresting and uninformative!

# Let's help Bob revise his beliefs

**Example**

Assume Bob believes: $Cn(\{p, q\})$

# Let's help Bob revise his beliefs

**Example**

Assume Bob believes: $Cn(\{p, q\})$

He learns, from a reliable source: $\neg q$

# Let's help Bob revise his beliefs

**Example**

Assume Bob believes: $Cn(\{p, q\})$

He learns, from a reliable source: $\neg q$

What should his new belief set be?

# Let's help Bob revise his beliefs

### Example

Assume Bob believes: $Cn(\{p, q\})$

He learns, from a reliable source: $\neg q$

What should his new belief set be?

---

Bob's new belief set should be:

$$Cn(\{p, \neg q\})$$

## Let's help Bob revise his beliefs

**Example**

Assume Bob believes: $Cn(\{p, q, r\})$

He learns, from a reliable source:

What should his new belief set be?

---

Bob's new belief set should be:

# Let's help Bob revise his beliefs

### Example

Assume Bob believes: $Cn(\{p, q, r\})$

He learns, from a reliable source: $\neg(q \vee r)$

What should his new belief set be?

---

Bob's new belief set should be:

## Let's help Bob revise his beliefs

**Example**

Assume Bob believes: $Cn(\{p, q, r\})$

He learns, from a reliable source: $\neg(q \vee r)$

What should his new belief set be?

---

Bob's new belief set should be:

$$Cn(\{p, \neg(q \vee r)\})$$

# Let's help Bob revise his beliefs

**Example**

Assume Bob believes: $Cn(\{p, q, p \rightarrow q\})$

He learns, from a reliable source:

What should his new belief set be?

---

Bob's new belief set should be:

# Let's help Bob revise his beliefs

**Example**

Assume Bob believes: $Cn(\{p, q, p \to q\})$

He learns, from a reliable source: $\neg q$

What should his new belief set be?

---

Bob's new belief set should be:

# Let's help Bob revise his beliefs

### Example

Assume Bob believes: $Cn(\{p, q, p \to q\})$

He learns, from a reliable source: $\neg q$

What should his new belief set be?

---

Bob's new belief set should be:
go to https://www.menti.com/ and enter code: 9730 8621

Option A: $Cn(\{p\})$
Option B: $Cn(\{p \to q, \neg q\})$
Option C: $Cn(\{p, q, p \to q\})$
Option D: $Cn(\{p, \neg q\})$
Option E: $Cn(\{p, p \to q\})$
Option F: $Cn(\{p, \neg q, p \to q\})$

# Let's help Bob revise his beliefs

### Example

Assume Bob believes:

He learns, from a reliable source:

What should his new belief set be?

---

Bob's new belief set should be:

Option B: $Cn(\{p \to q, \neg q\})$

Option D: $Cn(\{p, \neg q\})$

# Three Parts of Taking in New Information

What can I do to my belief set?

1. **Revision**: $B * \varphi$; $\varphi$ is added and other things are removed, so that the resulting new belief set $B'$ is consistent.
2. **Contraction**: $B \div \varphi$; $\varphi$ is removed from $B$ giving a new belief set $B'$.
3. **Expansion**: $B + \varphi$; $\varphi$ is added to $B$ giving a new belief set $B'$.

# Levi Identitiy

One formal way to combine those two is to use:

Levi identity

$$B * \varphi := (B \div \neg\varphi) + \varphi.$$

Belief revision can be defined as first removing any inconsistency with the incoming information and then adding the information itself.

# Thinking in Terms of Plausibility Orders: Prior

Bob believes: $Cn(\{p, q, p \rightarrow q\})$, i.e., the state $x$ is the most plausible.

But there are different ways in which the remaining options can be ordered.

| $p, q$ | $p, \bar{q}$ | $\bar{p}, q$ | $\bar{p}, \bar{q}$ |
|---|---|---|---|
| | | | $w$ |
| | $y$ | | |
| | | $z$ | |
| $x$ | | | |

| $p, q$ | $p, \bar{q}$ | $\bar{p}, q$ | $\bar{p}, \bar{q}$ |
|---|---|---|---|
| | $y$ | | |
| | | $z$ | |
| | | | $w$ |
| $x$ | | | |

In the above pictures, the lower the state the more plausible it is.

# Thinking in Terms of Plausibility Orders

## Revision Posterior

Bob believes: $Cn(\{p, q, p \rightarrow q\})$, i.e., the state $x$ is the most plausible.

After revising with $\neg q$ his **posterior plausibility** changes differently depending on the **prior plausibility**.

We are looking for prior-minimal states that do not satisfy $q$.

| $p, q$ | $p, \bar{q}$ | $\bar{p}, q$ | $\bar{p}, \bar{q}$ |
|---|---|---|---|
| | | | $w$ |
| | $y$ | | |
| | | $z$ | |
| $x$ | | | |

Table: Option B: $Cn(\{p, \neg q\})$

| $p, q$ | $p, \bar{q}$ | $\bar{p}, q$ | $\bar{p}, \bar{q}$ |
|---|---|---|---|
| | $y$ | | |
| | | $z$ | |
| | | | $w$ |
| $x$ | | | |

Table: Option E: $Cn(\{p \rightarrow q, \neg q\})$

# Thinking in Terms of Plausibility Orders

## Contraction

Bob believes: $Cn(\{p, q, p \rightarrow q\})$, i.e., the state $x$ is the most plausible.

After contracting with $q$, Bob has to expand his view.

We are looking for prior-minimal states that do not satisfy $q$.

| $p, q$ | $p, \bar{q}$ | $\bar{p}, q$ | $\bar{p}, \bar{q}$ |
|---|---|---|---|
| | | | $w$ |
| | $y$ | | |
| | | $z$ | |
| $x$ | | | |

Table: $Cn(\{p\})$

| $p, q$ | $p, \bar{q}$ | $\bar{p}, q$ | $\bar{p}, \bar{q}$ |
|---|---|---|---|
| | $y$ | | |
| | | $z$ | |
| | | | $w$ |
| $x$ | | | |

Table: $Cn(\{p \leftrightarrow q\})$

After contraction Bob's beliefs are specified by the union of his prior most plausible world and the prior most plausible word not-entailing $q$.

THE END OF LECTURE 10