Introduction to Artificial Intelligence

Lecture 5: Adversarial Search

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- Driving in traffic.
- Bargain-hunting auctioning.
- Governmental elections.

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Game theory ‘works’ when agents behave **rationally**.

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> *Game theory is a sort of umbrella or 'unified field' theory for the rational side of social science, where 'social' is interpreted broadly, to include human as well as non-human players (computers, animals, plants).*
>
> *(Aumann 1987)*

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A **strategic game** is a game in which players involved make one decision each, and make it independently.

Toy Games: Matching Pennies

- Two players: Alice and Bob.
- Both show a coin.
- Bob wins if they show different faces.
- Alice wins if they show the same.
- they each have two strategies: *heads* and *tails*.
- All possible strategy combinations give payoffs for each player.

Toy Games: Matching Pennies (conflict)

Alice	heads	tails
heads	$+$	$-$
tails	$-$	$+$

Bob	**heads**	**tails**
heads	$-$	$+$
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Together:

	heads	tails
heads	$(+,-)$	$(-,+)$
tails	$(-,+)$	$(+,-)$

Toy Games: Driving Game (cooperation)

Alice	left	right
left	$+$	$-$
right	$-$	$+$

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	left	right
left	$(+,+)$	$(-,-)$
right	$(-,-)$	$(+,+)$

Numerical Payoffs

	heads	tails
heads	(1,-1)	(-1,1)
tails	(-1,1)	(1,-1)

	left	right
left	(1,1)	(-1,-1)
right	(-1,-1)	(1,1)

Table: Matching Pennies and Driving Game with payoffs -1 and 1

Any numbers as long as winning>losing.

Recall Matching Pennies game:

	heads	tails
heads	(1,-1)	(-1,1)
tails	(-1,1)	(1,-1)

Zero-sum Games

Recall Matching Pennies game:

	heads	tails
heads	(1,-1)	(-1,1)
tails	(-1,1)	(1,-1)

Payoffs in each cell sum up to 0.
This can be always done for games with pure conflict.

Games in Artificial Intelligence

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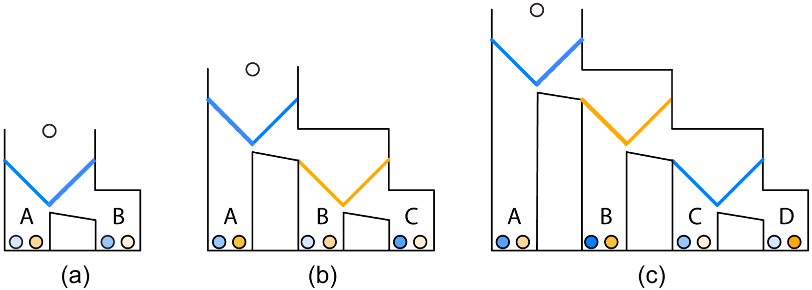
The games of interest are often also **perfect-information** games.

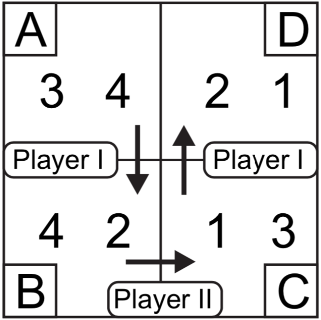
Turn-based Games: Marble Drop Game



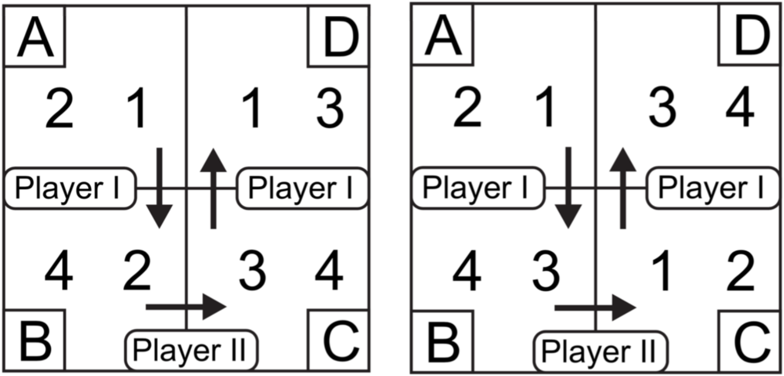
Turn-based Games: Matrix Game

There is one token in the game. It starts being at A. Player I can decide to *move* it to B or let it *stay* in A. If it is moved to B, player II can afterwards *move* it to C or let it *stay* in B. If it is moved to C, player I can *move* it to D or let it *stay* in C. The game ends when a player decides to *stay* or the token ends up in D.

The utility (payoff) of player I and II are the, respectively, left and right numbers in the cell that the token ends up in.



More Matrix Games: Exercise



Search Problems Formally

So far, search problems were defined by:

- **Initial state**.
- $\text{Actions}(s)$: **possible actions**.
- $\text{Result}(s, a)$: **transition model**.
- **Goal test**.

That induces a **state space** in which we search for a goal state.

GAME PROBLEMS FORMALLY

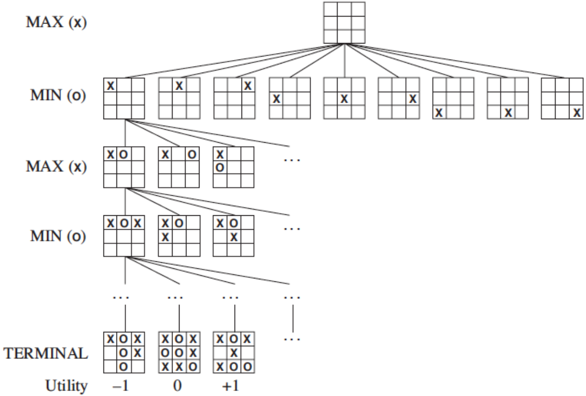
Similarly, games can be described by:

- s_0: **initial state**.
- PLAYER(s): who has the move in s.
- ACTIONS(s): Legal moves in s.
- RESULT(s, a): **transition model**.
- TERMINAL-TEST(S): **terminal test**. Is the game over?
- UTILITY(s, p): **utility function** (or **payoff function**).

  Numerical value for player p in terminal state s.

  Example: $+1$ for win and -1 for loose (zero-sum).

Tic-Tac-Toe: Game Tree Example



Minimax

- There are two players: MAX and MIN.
- The game is zero-sum.
- Nodes assigned values representing expected utility for MAX.

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In a state s with $\textsc{Player}(s) = \textsc{max}$, the following move is optimal for MAX:

$$\operatorname{argmax}_{a \in \textsc{Actions}(s)} \textsc{Minimax}(s).$$

That is, MAX chooses the move that maximises the minimax value.

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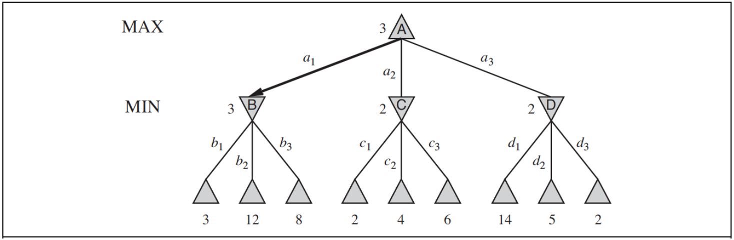
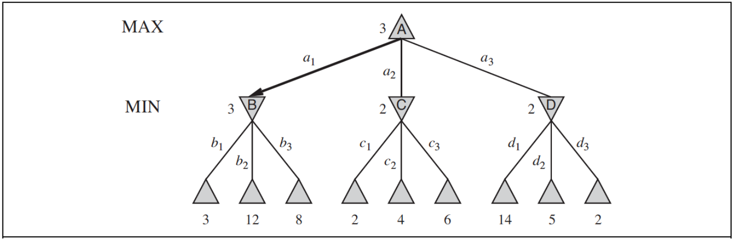
Is MINIMAX a breadth- or a depth-first search?

Minimax on an Example



Alpha-beta Pruning: An Example



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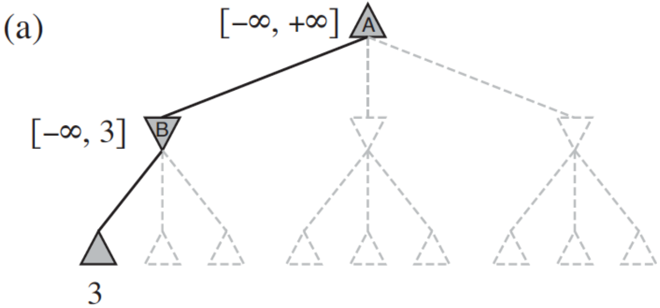

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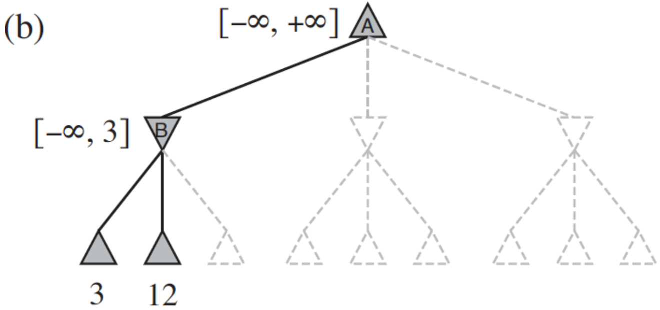

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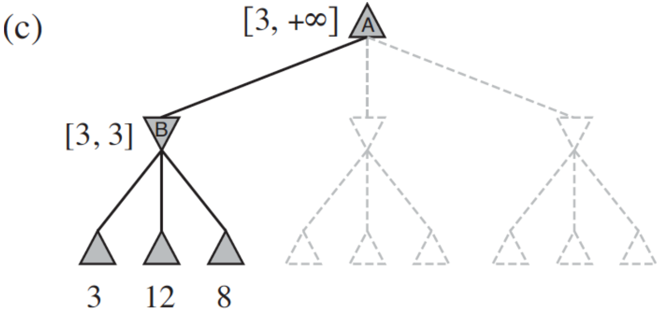

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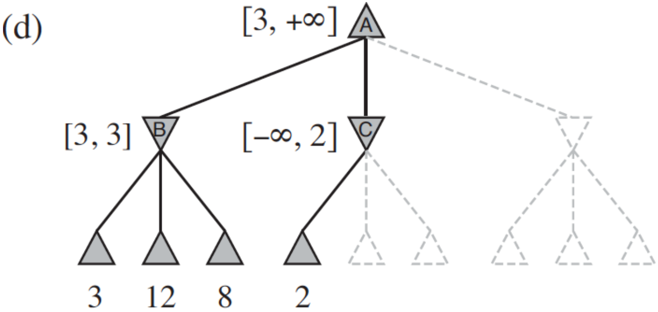

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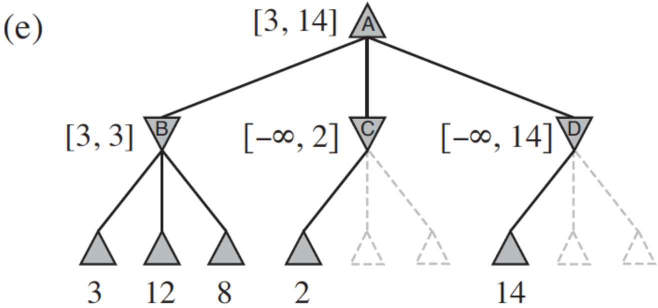

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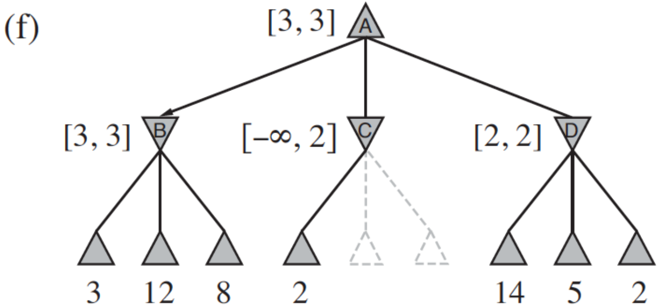

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The recursive MINIMAX algorithm **passes down** an α and β value to each node:

- α: **lower bound** on what MAX can achieve when playing through the choice points leading to the current node.
- β: **upper bound** on what MIN can achieve when playing through the choice points leading to the current node.

The root node has $\alpha = -\infty$ and $\beta = \infty$.

Additionally, the algorithm iteratively updates the value v of each node. Initially $v = -\infty$ for MAX nodes and $v = \infty$ for MIN nodes.

- **Alpha-cut**: If $v \leq \alpha$ in a MIN node, we can prune further search below that node: MAX has a better choice at a previous choice point.
- **Beta-cut**: If $v \geq \beta$ in a MAX node, we can prune further search below that node: MIN has a better choice at a previous choice point.

α-values concern the choice of MAX, β-values concern the choice of MIN.

ALPHA-BETA-SEARCH algorithm (see pseudocode in R&N):

- MAX nodes passes α values down:

  the max of the α- and v-values of the parent.

- MIN nodes passes β values down:

  the min of the β and v-values of the parent.

When searching to the end is not possible

- Use $\textsc{Cutoff-Test}(\textsc{s})$ instead of $\textsc{Terminal-Test}(\textsc{s})$. E.g. limit by depth (search only to depth 5).
- Use $\textsc{Eval}(s, p)$ instead of $\textsc{Utility}(s, p)$. This is called an **evaluation function**.

$\textsc{Eval}(s, p)$: How desirable is state s for player p?
E.g. an estimate of the chance of winning or the expected utility.

Expected utility. If 20% chance of getting utility 5 and 80% chance of getting utility 2, expected utility is $0.2 * 5 + 0.8 * 2 = 2.6$.

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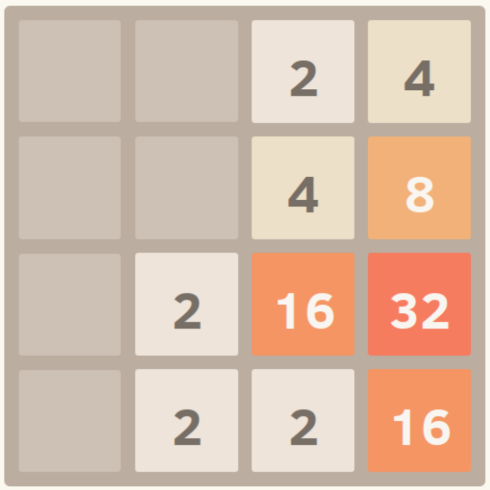
Evaluation functions in adversarial search play approximately the same role as heuristic functions in classical search: an estimate of "how good" the state is that can be used to guide the search/decisions.

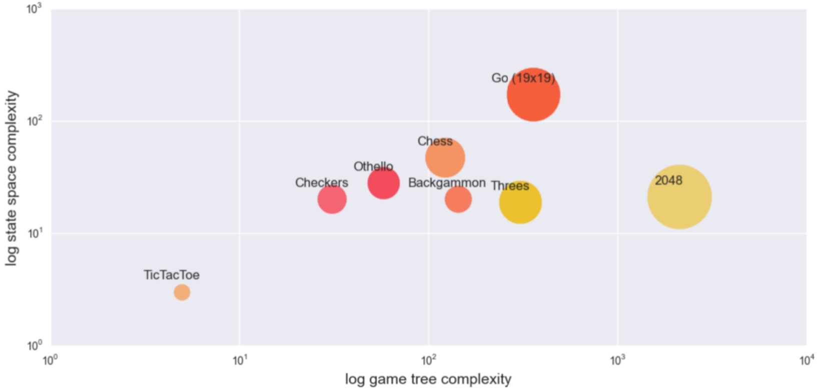
$$\textsc{H-Minimax}(s, d) = \begin{cases} \textsc{Eval}(s) & \text{if } \textsc{Cutoff-Test}(s, d) \\ \max_{a \in \textsc{Actions}(s)} \textsc{Minimax}(\textsc{Result}(s, a), d+1) & \text{if } \textsc{Player}(s) = \textsc{max} \\ \min_{a \in \textsc{Actions}(s)} \textsc{Minimax}(\textsc{Result}(s, a), d+1) & \text{if } \textsc{Player}(s) = \textsc{min} \end{cases}$$

Example: AI for 2048



Games and Their Combinatorial Complexity

- State space complexity: number of possible states of the game.
- Game tree complexity: number of possible games.



10^n on these axes mean that the complexity/size is $10^{(10^n)}$.

Stochastic games: Expectimax

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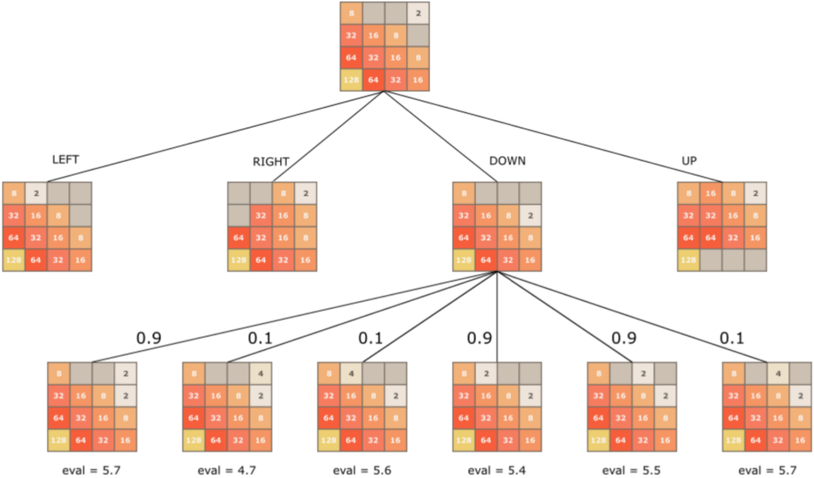
$$\textsc{Expectimax}(s) =$$

$$\begin{cases} \textsc{Utility}(s, \textsc{max}) & \text{if } \textsc{Terminal-Test}(s) \\ \max_{a \in \textsc{Actions}(s)} \textsc{Expectimax}(\textsc{Result}(s, a)) & \text{if } \textsc{Player}(s) = \textsc{max} \\ \Sigma_{a \in \textsc{Actions}(s)} P(a, s) \cdot \textsc{Expectimax}(\textsc{Result}(s, a)) & \text{if } \textsc{Player}(s) = \textsc{chance} \end{cases}$$

Here $P(a, s)$ is the probability of CHANCE chosing action a in s.



Expectimax value of going down from root state:

$$\frac{0.9 \cdot (5.7 + 5.4 + 5.5) + 0.1 \cdot (4.7 + 5.6 + 5.7)}{3} = 5.512.$$

Demo of Expectimax on 2048

Expectimax for 2048 (by a previous student Kristine Strandby):

http://kstrandby.github.io/2048-Helper/

THE END OF LECTURE 5