Introduction to Artificial Intelligence
Lecture 9: Logical Inference

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Outline

- Propositional Theorem Proving
- Resolution
- Horn Clauses and Definite Clauses
- Effective Propositional Model Checking

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Truth-table Method for Inference

The most intuitive way to check validity of inference by brut-force truth-tables.

- *KB*: Robert does well in the exam if and only if he is prepared or lucky. Robert does not do well in the exam.
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0	1	0	1	0	1	0
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Semantic vs Syntactic Approach

- (Semantic) model checking:

  enumerating models and showing it holds in all models.

- (Syntactic) theorem proving:

  applying rules of inference directly the sentences in our knowledge base to construct a proof of the desired sentence without consulting models.

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  applying rules of inference directly the sentences in our knowledge base to construct a proof of the desired sentence without consulting models.

Reason: If the number of models is large but the length of the proof is short, then theorem proving can be more efficient than model checking.

Some Crucial Concepts

- Logical equivalence

  two formulas φ and ψ are logically equivalent:

  if they are true in the same set of models, or

  if each of them entails the other: $\varphi \equiv \psi$ if and only if $\varphi \models \psi$ and $\psi \models \varphi$.

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- Satisfiability

  φ is satisfiable if it is true in some model.

  The SAT problem, determining the satisfiability of sentences, was the first problem shown to be NP-complete.

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For any sentences φ and ψ of propositional logic

$$\varphi \models \psi \textit{ if and only if } \varphi \to \psi \textit{ is valid.}$$

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(reductio ad absurdum, proof by refutation, proof by contradiction)

Inference and Proofs

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Examples of inference rules

$$\frac{\varphi \rightarrow \psi, \ \varphi}{\psi} \qquad \text{(Modus Ponens)}$$

$$\frac{\varphi \wedge \psi}{\varphi} \qquad \text{(And-Elimination)}$$

Using Search Algorithms to Find Proofs

A friendly statement of the theorem proving problem as a search problem:

- Initial state: the initial knowledge base.
- Actions: the inference rules.
- Result: the result of an action is to add the conclusion to KB.
- Goal: the sentence we are trying to prove.

Monotonicity

In monotonic logics the set of entailed sentences can only increase as information is added to the knowledge base.

For any sentences φ and ψ, if $KB \models \varphi$ then $KB \cup \psi \models \varphi$.

Note: non-monotonic logics, which violate the monotonicity property, capture a common property of human reasoning: changing one's mind.

Outline

Resolution: one rule to rule them all

$$\frac{\ell_1 \vee \ldots \vee \ell_k, \; m}{\ell_1 \vee \ldots \vee \ell_{i-1} \vee \ell_{i+1} \vee \ldots \vee \ell_k} \qquad \text{(Unit Resolution)}$$

where literals ℓ_i and m are complementary (i.e., one is negation of the other).

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Practical comment: mind the Factoring (removing duplicates)!

E.g., if we resolve $(A \vee B)$ with $(A \vee \neg B)$, we obtain $(A \vee A)$.
Single A is enough!

Conjunctive normal form (CNF)

- Resolution applies to clauses (disjunctions of literals).
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4. Distribute $\wedge$ over $\vee$: $(\neg r \vee p \vee s) \wedge (\neg p \vee r) \wedge (\neg s \vee r)$.

A Resolution Algorithm

To show that $KB \models \varphi$, we show that $KB \wedge \neg\varphi$ is unsatisfiable.
We do this by proving a contradiction.

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3. Each pair that contains complementary literals is resolved to produce a new clause, which is added to the set if it is not already present.
4. The process continues until one of two things happens:
   - A there are no new clauses that can be added, in which case KB does not entail φ; or,
   - B two clauses resolve to yield the empty clause, in which case KB entails φ.

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5. $clauses$:=$clauses \cup \{\bot\}$ from resolving r with $\neg r$.
6. Result: empty clause, so $KB \models \varphi$.

Resolution algorithm is good

1. Always terminates.
2. Is complete, by the ground resolution theorem: If a set of clauses is unsatisfiable, then the resolution closure of those clauses contains the empty clause.

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Special kinds of clauses

Sometimes it's enough if we restrict the language to special types of clauses:

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Horn clauses are closed under resolution:
if you resolve two Horn clauses, you get back a Horn clause.

The Importance of Horn Clauses

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- Definite clause can be written as an implication whose premise is a conjunction of positive literals and whose conclusion is a single positive literal. E.g., $(\neg p \vee \neg s \vee r)$ is equivalent to $(p \wedge s) \rightarrow r$. The premise is called the body and the conclusion is called the head. A sentence consisting of a single positive literal, such as r, is called a fact $(\top \rightarrow r)$.

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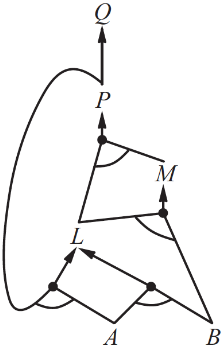
- Inference with Horn clauses can be done by forward- and backward-chaining.
- Deciding entailment with Horn clauses is linear in the size of the knowledge base.

Forward- and Backward-chaining on AND-OR Graphs

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$



Outline

General Model-Checking Algorithms for Propositional Inference

- the algorithms checking satisfiability: the SAT problem
- testing entailment $\varphi \models \psi$, is done by testing unsatisfiability of $\varphi \land \neg\psi$
- two types: backtracking and local search

Davis-Putnam Algorithm (DPLL Algorithm)

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Improvements over TT:

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2. Pure symbol heuristic, e.g., in $(A \vee \neg B)$, $(\neg B \vee \neg C)$, $(C \vee A)$, the symbol A is pure. If a sentence has a model, then it has a model with the pure symbols assigned so as to make their literals true, because doing so can never make a clause false.

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3. Unit clause heuristic, e.g., if the model contains $B = \top$, then $(\neg B \vee \neg C)$ simplifies to $\neg C$, which is a unit clause. Assigning one unit clause can create another unit clause, such 'cascade' of forced assignments is called unit propagation.

Davis-Putnam Algorithm (DPLL Algorithm)

function DPLL-SATISFIABLE?(s) **returns** *true* or *false*
  inputs: s, a sentence in propositional logic

  clauses ← the set of clauses in the CNF representation of s
  symbols ← a list of the proposition symbols in s
  return DPLL(*clauses*, *symbols*, { })

---

function DPLL(*clauses*, *symbols*, *model*) **returns** *true* or *false*

  if every clause in *clauses* is true in *model* **then return** *true*
  if some clause in *clauses* is false in *model* **then return** *false*
  P, *value* ← FIND-PURE-SYMBOL(*symbols*, *clauses*, *model*)
  if P is non-null **then return** DPLL(*clauses*, *symbols* – P, *model* ∪ {P=*value*})
  P, *value* ← FIND-UNIT-CLAUSE(*clauses*, *model*)
  if P is non-null **then return** DPLL(*clauses*, *symbols* – P, *model* ∪ {P=*value*})
  P ← FIRST(*symbols*); *rest* ← REST(*symbols*)
  return DPLL(*clauses*, *rest*, *model* ∪ {P=*true*}) **or**
      DPLL(*clauses*, *rest*, *model* ∪ {P=*false*}))

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- We flip the truth value of one symbol at a time and escape local minima with randomness.

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- The goal is to find an assignment that satisfies every clause, so
- evaluation function counts the number of unsatisfied clauses.
- We flip the truth value of one symbol at a time and escape local minima with randomness.

WalkSat: On every iteration, the algorithm picks an unsatisfied clause and picks a symbol in the clause to flip. It chooses randomly between two ways to pick which symbol to flip:

1. a min-conflicts step that minimises the number of unsatisfied clauses in the new state, and
2. a random walk step that picks the symbol randomly.

WalkSat Algorithm

function WALKSAT(*clauses*, p, *max_flips*) **returns** a satisfying model or *failure*
inputs: *clauses*, a set of clauses in propositional logic
p, the probability of choosing to do a "random walk" move, typically around 0.5
max_flips, number of flips allowed before giving up

model ← a random assignment of *true*/*false* to the symbols in *clauses*
for $i = 1$ **to** *max_flips* **do**
    if *model* satisfies *clauses* **then return** *model*
    clause ← a randomly selected clause from *clauses* that is false in *model*
    with probability p flip the value in *model* of a randomly selected symbol from *clause*
    else flip whichever symbol in *clause* maximizes the number of satisfied clauses
return *failure*

End of Lecture 9