Introduction to AI
Exercises at week 13

May 4th, 2021

Ex. 1 Translate the following natural language sentences into predicate logic formulas. Remember to indicate your domain of discourse and your vocabulary. (*Hint*: you can use "living bipeds of the Star Wars universe" as the domain for some of the following sentences.)

1. Some Jedi does not like all Wookies.
2. Every Jedi who likes a Wookie is also liked by some Wookie.
3. Every Wookie who likes all Jedies does not like every Wookie.
4. No girl who does not love a boy loves a girl who loves a boy.

Answers:

1. Some Jedi does not like all Wookies.

$$\begin{array}{c} \begin{array}{ccc} \exists x\,(Jx & \land & \text{not all Wookies are liked by } x) \\ & & \neg\forall y\,(Wy \to Lxy) \end{array} \\ \hline \exists x\,(Jx \land \neg\forall y\,(Wy \to Lxy)) \end{array}$$

2. Every Jedi who likes a Wookie is also liked by some Wookie.

$$\begin{array}{c} \begin{array}{cccc} \forall x & (x \text{ is a Jedi and } x \text{ likes a Wookie} & \to & \text{some Wookie likes } x) \\ & Jx \land \exists y\,(Wy \land Lxy) & & \exists z\,(Wz \land Lzx) \end{array} \\ \hline \forall x\,((Jx \land \exists y\,(Wy \land Lxy)) \to \exists z\,(Wz \land Lzx)) \end{array}$$

3. Every Wookie who likes all Jedis does not like every Wookie.

$$\begin{array}{c} \begin{array}{cccc} \forall x & (x \text{ is a Wookie and } x \text{ likes all Jedis} & \to & \text{not all Wookies are liked by } x) \\ & Wx \land \forall y\,(Jy \to Lxy) & & \neg\forall z\,(Wz \to Lxz) \end{array} \\ \hline \forall x\,((Wx \land \forall y\,(Jy \to Lxy)) \to \neg\forall z\,(Wz \to Lxz)) \end{array}$$

4. No girl who does not love a boy loves a girl who loves a boy.

$$\begin{array}{c} \begin{array}{cccccc} \neg\exists x & (x \text{ is a girl} & \land & \text{no boy is loved by } x & \land & x \text{ loves a girl who loves a boy}) \\ & Gx & & \neg\exists y\,(By \land Lxy) & & \exists z\,(Gz \land Lxz \land \exists w\,(Bw \land Lzw)) \end{array} \\ \hline \neg\exists x\,(Gx \land \neg\exists y\,(By \land Lxy) \land \exists z\,(Gz \land Lxz \land \exists w\,(Bw \land Lzw))) \end{array}$$

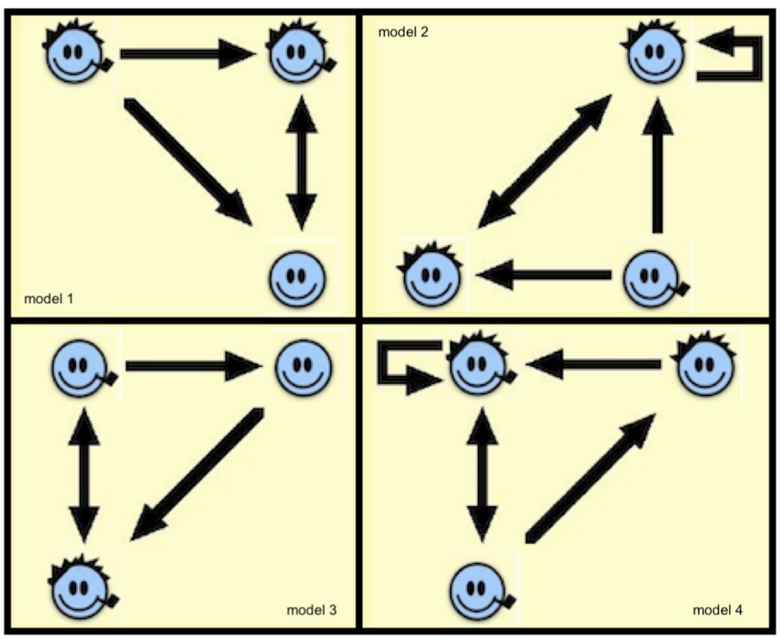
Ex. 2 For the formula you built for sentence (iii) of exercise 2, give a non-empty model in which the formula is **true**.
Answer:

Any model without Wookies. Less trivial is a model like $\bullet \longrightarrow \circ$, where $\bullet$ is a Wookie and $\circ$ a Jedi. The Wookie likes all Jedis, and this Wookie does not like all Wookies, because he does not like himself.

Ex. 3 For the formula you built for sentence (iv) of exercise 2, give a non-empty model in which the formula is **false**.
Answer:

A model which makes the formula false is $\circ \longrightarrow \circ \longrightarrow \bullet$ where the $\circ$'s are the girls and the $\bullet$ a boy. The left most girl does not love a boy but, at the same time, she loves a girl who loves a boy.

Ex. 4 Consider the following models:



Let Hx represent that x is 'hairy', Px that x is a 'pipe-smoker' and Kxy that x 'knows' y represented by the arrows (arrows point from the first argument of the predicate, the knower, to the second, the one who is known). Decide for each of the following formulas in which of the four models that formula is true.

1. $\exists x\, Kxx$ is true in model 2 and 4
2. $\forall x\, (Hx \to Px)$ is true in model 1 and 3
3. $\forall x\, (Hx \to \exists y\, (Kxy \wedge Py))$ is true in model 3 and 4
4. $\exists y \forall x\, Kxy$ is true in model 2 and 4
5. $\exists y \forall x\, Kyx$ is false in all models
6. $\forall x \forall y\, (Kxy \to \exists z\, Kzx)$ is true in model 3 and 4

Ex. 5 (9.6 from the textbook) Formalise the following sentences in FOL, so that they are suitable for applying Generalised Modus Ponens:

1. Horses, cows, and pigs are mammals.

2. An offspring of a horse is a horse.
3. Bluebeard is a horse.
4. Bluebeard is Charlie's parent.
5. Offspring and parent are inverse relations.
6. Every mammal has a parent.

Answer:

1. $Horse(x) \rightarrow Mammal(x)$
   $Cow(x) \rightarrow Mammal(x)$
   $Pig(x) \rightarrow Mammal(x)$
2. $Offspring(x, y) \wedge Horse(y) \rightarrow Horse(x)$
3. $Horse(Bluebeard)$
4. $Parent(Bluebeard, Charlie)$
5. $Offspring(x, y) \rightarrow Parent(y, x)$
   $Parent(x, y) \rightarrow Offspring(y, x)$
6. $Mammal(x) \rightarrow Parent(G(x), x)$

Ex. 6 (9.4 from the textbook) For each of the following pairs give the most general unifier (if it exists):

1. $P(A, A, B)$, $P(x, y, z)$
2. $Q(y, G(A, B))$, $Q(G(x, x), y)$
3. $Older(Father(y), y)$, $Older(Father(x), Jerry)$
4. $Knows(Father(y), y)$, $Knows(x, x)$

Answer:

1. {x/A,y/A,z/B}
2. No unifier
3. {x/Jerry,y/Jerry}
4. No unifier (occurs check)

Ex. 7 (* 9.20 from the textbook) Let $\mathcal{L}$ be the first-order language with a single predicate $S(p, q)$, meaning "p shaves q". Take a domain of people.

(a) Consider the sentence "There exists a person P who shaves everyone who does not shave themselves, and only people that do not shave themselves." Express this in $\mathcal{L}$.

(b) Convert the sentence in (a) into a CNF.

(c) Construct a resolution proof to show that the clauses in (b) are inherently inconsistent.

Answer:

(a) $\exists p \forall q \; S(p, q) \leftrightarrow \neg S(q, q)$

(b) $\neg S(C_1, q) \vee \neg S(q, q)$
$S(C_1, q) \vee S(q, q)$

(c) Using the substitution $\{q/C_1\}$ we get the following clauses:
$\neg S(C_1, C_1)$
$S(C_1, C_1)$
which resolves to the null clause.