Introduction to Artificial Intelligence

Lecture 3: Informed Search

Nina Gierasimczuk

Problem solving is an essential part of intelligence.

Problem solving can be represented as a search strategy.

Uniformed searching is **blindly** following transitions from states to states.

Informed search uses extra information in making choices of transitions.

This information is given in the form of **heuristic function**.

Recap: Search problems formally

Formally, a **search problem** consists of the following elements:

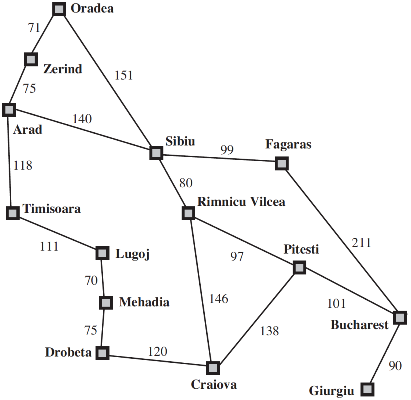
- s_0: Initial state
- $\textsc{Actions}(s)$: Returns the set of actions **applicable** in state s.
- $\textsc{Results}(s, a)$: Returns the state s' reached from s by executing action a.
- $\textsc{Goal-Test}(s)$: Returns `true` if s is goal state, otherwise `false`.
- $\textsc{Step-Cost}(s, a)$: The cost of executing action a in s. Most often we will assume $\textsc{Step-Cost}(s, a) = 1$ for all s and a.

A state g is called a **goal state** if $\textsc{Goal-Test}(g) = \texttt{true}$.

A **solution** to a search problem is a **sequence of actions** (a **path**) from s_0 to a goal state. It is **optimal** if it has minimum sum of step costs.

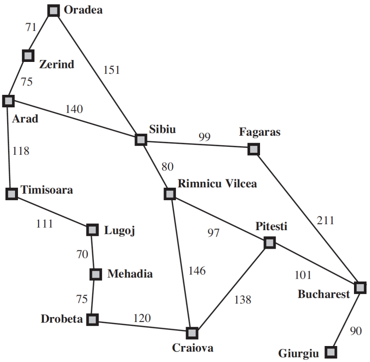
A Real World Problem

Route-finding



Route-finding Problem: From Arad to Bukarest

Problem Solving by BFS and DFS



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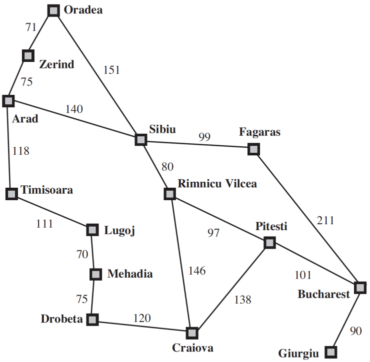

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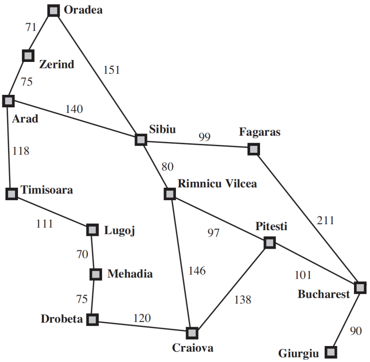

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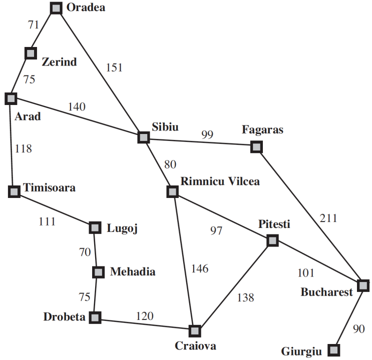

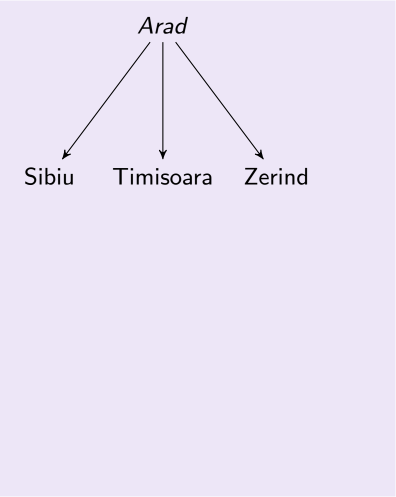

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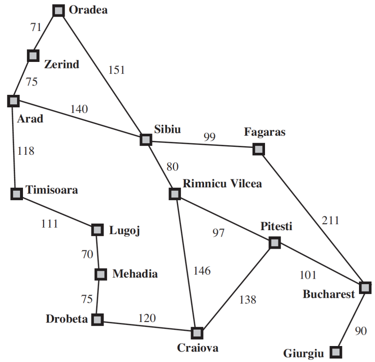

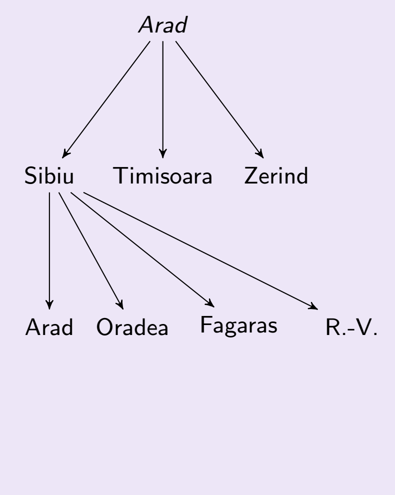

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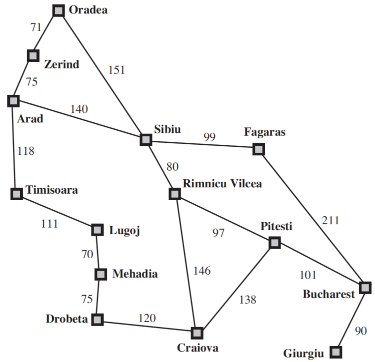

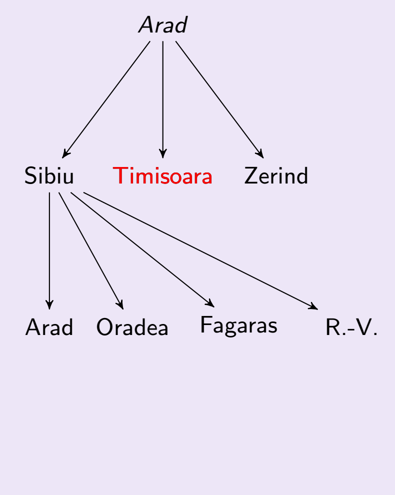

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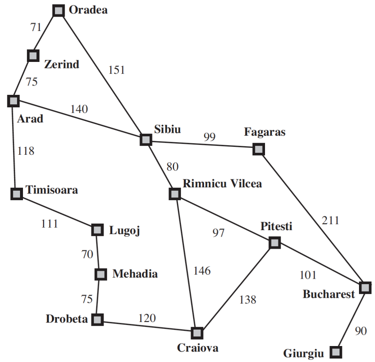

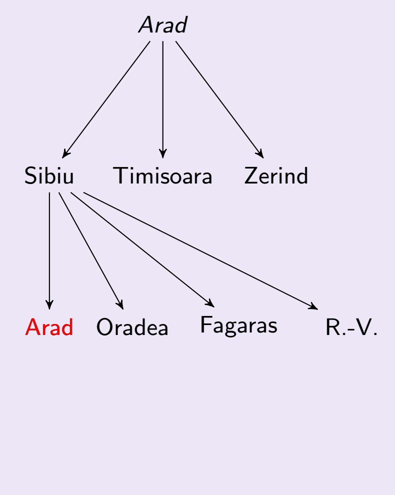

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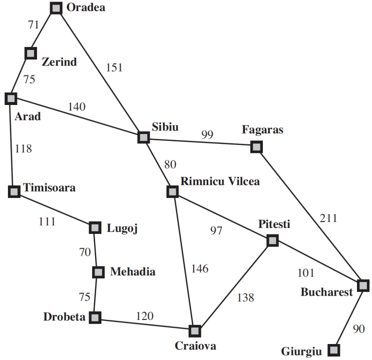

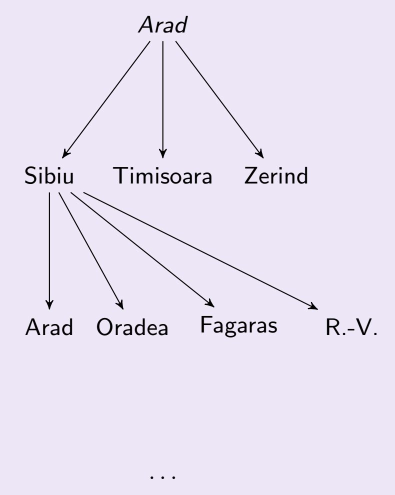

The Tree-Search algorithm

function TREE-SEARCH (problem) returns a solution, or failure
  frontier := $\{s_0\}$ (initial state)   // we initialise the frontier
  loop do
    if *frontier* $= \emptyset$ **then return** failure
    choose a node *n* from *frontier*
    remove *n* from *frontier*
    if *n* is a goal state **then return** solution
    for each child *m* of *n*   // we expand *n*
      add child m to frontier

Search strategies of Tree-Search and Graph-Search

Different search strategies can be achieved by simply changing how choose node from frontier and add child to frontier work.

Breadth-first search (BFS):

- Frontier is **queue** (FIFO).
- choose node from frontier: dequeue node from frontier.
- add child to frontier: enqueue node to frontier.

Evaluation of Search Algorithms

For Tree-Search

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional
Complete?	Yes[a]	Yes[a,b]	No	No	Yes[a]	Yes[a,d]
Time	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$
Optimal?	Yes[c]	Yes	No	No	Yes[c]	Yes[c,d]

Figure 3.21 Evaluation of tree-search strategies. b is the branching factor; d is the depth of the shallowest solution; m is the maximum depth of the search tree; l is the depth limit. Superscript caveats are as follows: [a] complete if b is finite; [b] complete if step costs $\geq \epsilon$ for positive ϵ; [c] optimal if step costs are all identical; [d] if both directions use breadth-first search.

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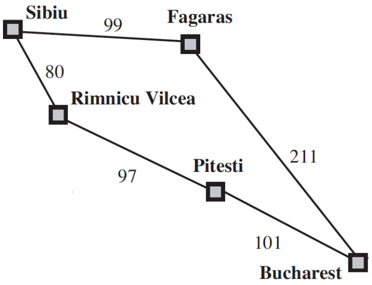
Differences with BFS:

- Goal-Test is applied when a node is selected for expansion
  (rather than when it is first generated)
  (the first goal node that is generated may be on a suboptimal path)
- a test is added for a better path to a node currently on the frontier

Back to route-finding: Sibiu to Bucharest

Uniform-cost Search



Back to route-finding: Sibiu to Bucharest

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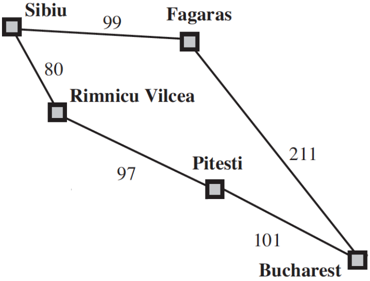

Back to route-finding: Sibiu to Bucharest

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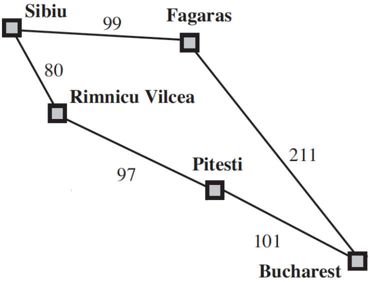

Sibiu

Back to route-finding: Sibiu to Bucharest

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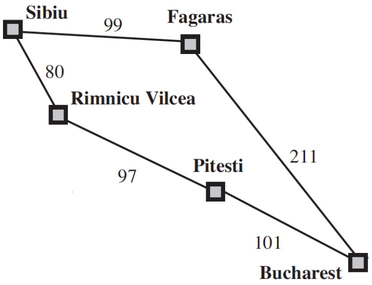

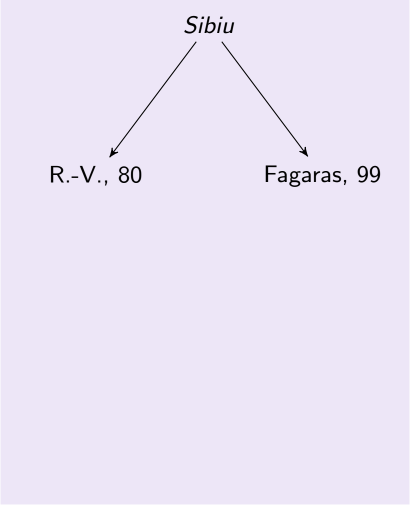

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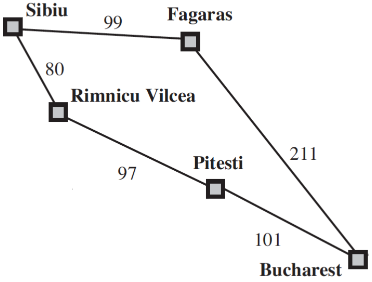

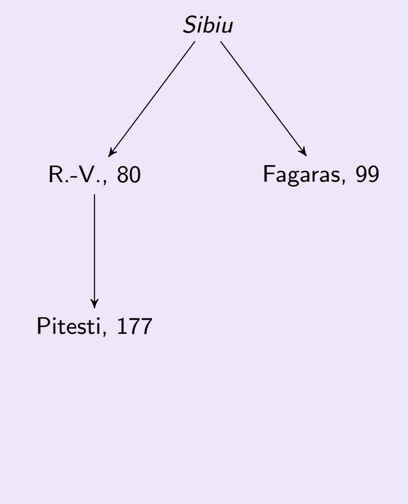

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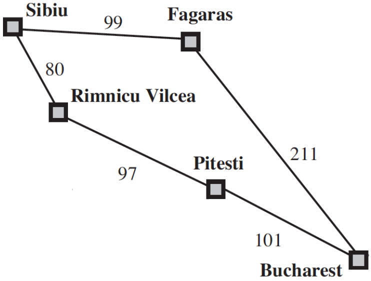

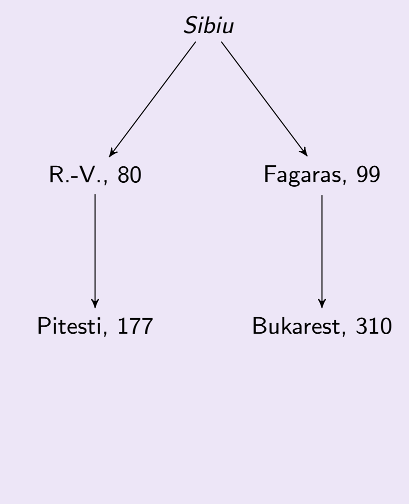

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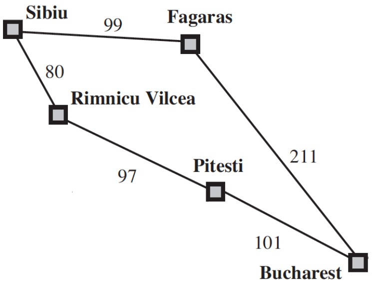

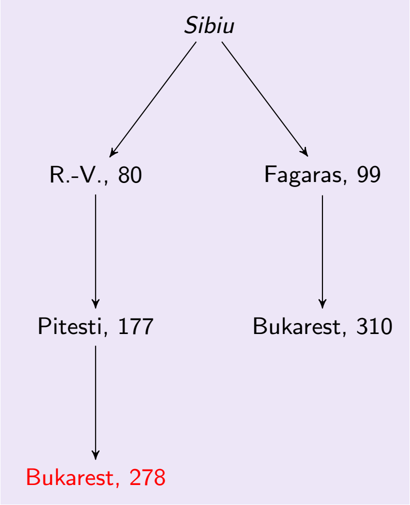

Best-first search

Best-first search is an instance of the general TREE- or GRAPH-SEARCH where node selection for expansion is based on an **evaluation function**, f.

The function f is a cost estimate: node n with lowest $f(n)$ is expanded first.

Best-first graph search the same as UCS but with f for priority queue.

Heuristic functions

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Heuristic functions are the most common form in which additional knowledge of the problem is imparted to the search algorithm.

Heuristics are arbitrary, nonnegative, problem-specific functions, with one constraint: if n is a goal node, then $h(n) = 0$.

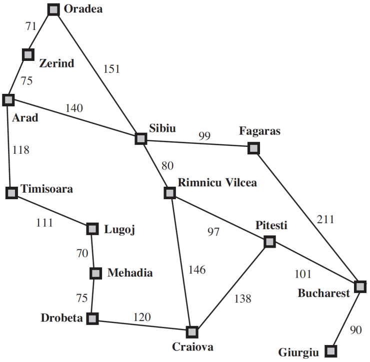
Greedy best-first search

Greedy best-first search tries to expand the node that is closest to the goal, on the grounds that this is likely to lead to a solution quickly.

Greedy best-first search evaluates nodes by using just the heuristic function:

$$f(n) = h(n).$$

Greedy best-first search: Arad to Bucharest



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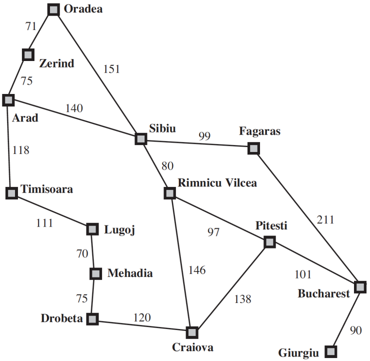
Greedy best-first search: Arad to Bucharest



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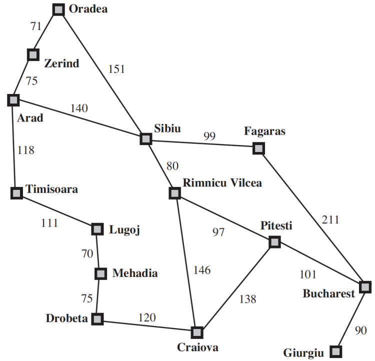

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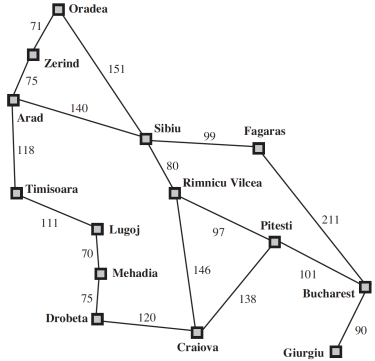
Arad

Greedy best-first search: Arad to Bucharest



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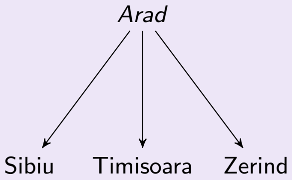
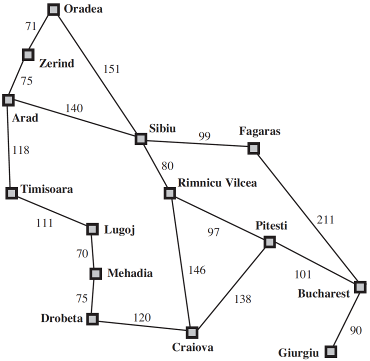


Greedy best-first search: Arad to Bucharest



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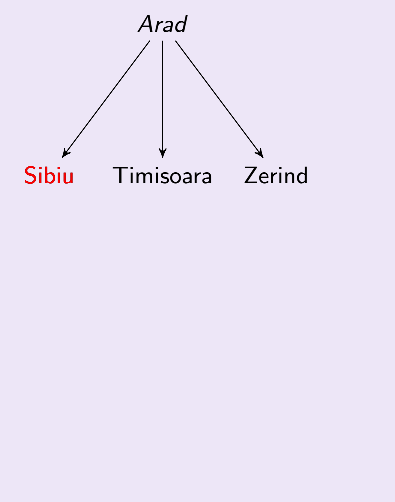

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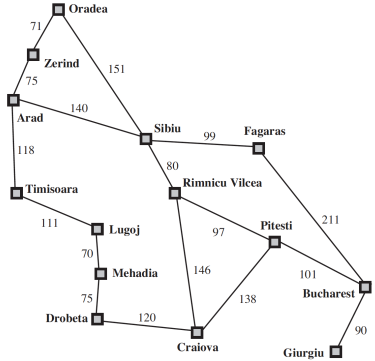

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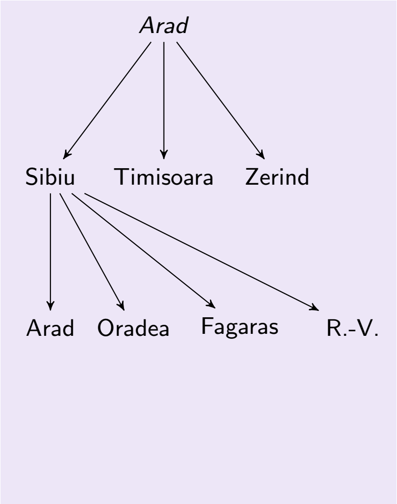

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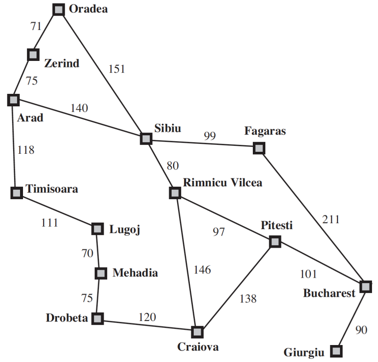

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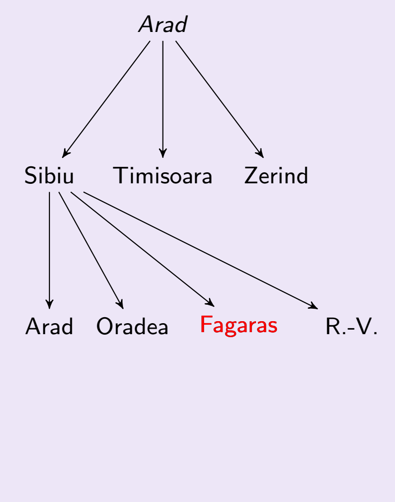

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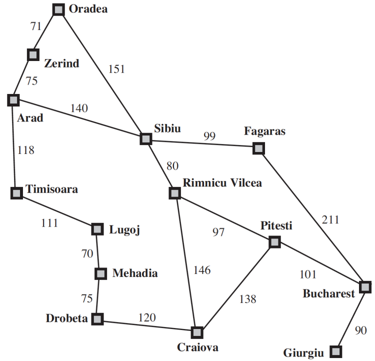

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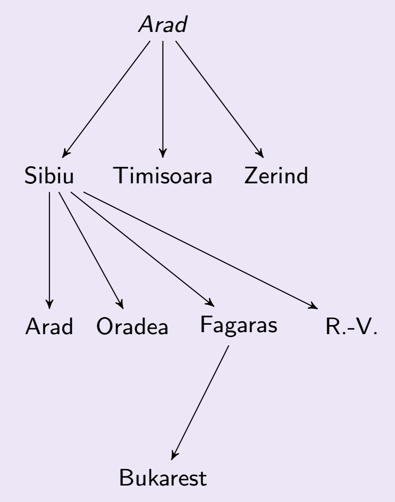

Properties of Greedy Best-First Search

- It's greedy.
- It's incomplete.
- The complexity (for tree version) is as DFS.

$A^\star$ SEARCH

$A^\star$ evaluates a node n by combining:

- $g(n)$, the cost to reach the node, and
- $h(n)$, the estimated cost to get from the node to the goal:
- $f(n) = g(n) + h(n)$.

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The $A^\star$ algorithm is UCS except that it uses $g + h$ instead of g.

$A^\star$ for Super Mario

Video of $A^\star$ playing Super Mario

Designing heuristics

Designing good heuristic function $h(n)$ is in many cases a great art form.

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Q: What properties should a good heuristic function have?

A: Estimate cost to goal as precisely as possible; be as cheap as possible to compute (cheaper than computing the actual cost).

ADMISSIBILITY AND OPTIMALITY

Recall: STEP-COST(s, a) is the **cost** of executing a in s.

Optimal cost of a node n:
The minimal cost to achieve a goal from n, denoted $h^*(n)$.

Admissible heuristics: cost of reaching the goal is never overestimated:
the heuristics is **always optimistic**. Formally: $h(n) \leq h^*(n)$ for all nodes n.

Optimal search algorithm: The algorithm always returns an **optimal solution**:
a solution of minimal cost.

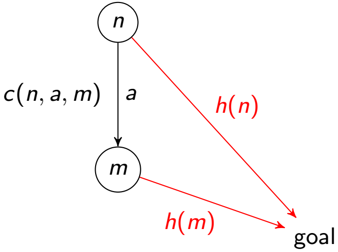
Theorem.
$A^\star$ TREE-SEARCH is optimal when h is admissible.

CONSISTENCY

Let $c(n, a, m)$ denote the STEP-COST of executing action a to go from n to m.

Consistent heuristics: If m is reached by executing a in n then

$$h(n) \leq c(n, a, m) + h(m)$$

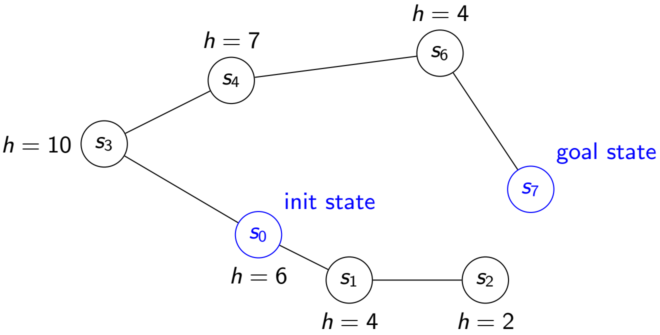


Theorem. Consistency $\Rightarrow$ admissibility.

Theorem. $A^\star$ GRAPH-SEARCH is optimal when h is consistent.
(See R&N, Chapter 3, for the discussion and proof outlines.)

Best-first tree-search vs graph-search

Consider the state space below. All step costs are 1.



Q1 How does greedy best-first tree search behave on the problem?

Q2 How about greedy best-first graph-search?

Q3 And A* tree-search?

Dominating heuristics

A heuristics h_2 is said to **dominate** a heuristics h_1 if

$$h_2(s) \geq h_1(s) \text{ for all } s.$$

If h_2 dominates h_1 and both are admissible, $A^\star$ will never expand more nodes using h_2 than using h_1. So h_2 is better in that sense.

If $h_1, \ldots, h_n$ are admissible heuristics, then so is $h(s) = \max\{h_1(s), \ldots, h_n(s)\}$.

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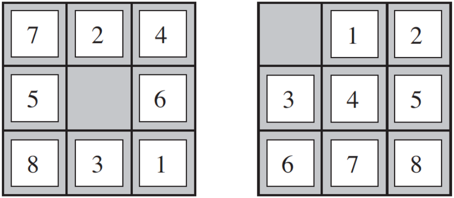
It is always better to use h as heuristics than either of the h_i,
unless the penalty in computation time of h is too high.

Often heuristics are generated via **relaxed problems**.

Relaxed problem:
A simplified version of a problem with fewer restrictions.
A solution to the original problem is also a solution to the relaxed problem.

Example



Start State Goal State

Relaxing the 8-puzzle:

1. A tile can move to any adjacent square.
2. A tile can move to any square.

Generating heuristics via relaxed problems

Take a problem P and let $h_P^*(n)$ be the optimal cost to get to goal from n in P.

Given any problem P and relaxation P', a heuristics h for P can be defined by:

$$h(n) = h_{P'}^*(n).$$

In words: The **estimated cost** of a solution to the **real problem** is taken to be the **actual cost** of a solution to the **relaxed problem**.

- Q1 Why is h defined above admissible?
- Q2 Why is h defined above consistent?
- Q3 Which heuristics do we get from the sliding puzzle relaxations of the previous slide (1. Move to any adjacent square; 2. Move to any square)?
- Q4 Does one of the heuristics from the previous question dominate the other?
- Q5 Given P', how do we calculate $h_{P'}^*$?

THE END OF LECTURE 3