

DTU



Lazaros Nalpantidis

Stereo Vision

some slides borrowed or adapted from:

- *Noah Snavely*
- *Aaron Bobick*
- *Antonio Torralba*



Outline

- What is Stereo Vision?
- Stereo/Epipolar Geometry
- Rectified Stereo Case
- Depth from Stereo Matches
- Correspondence Problem
 - **Dense** vs Sparse Correspondence
 - **Local** vs Global Correspondence
 - (Dis)-Similarity Measures
- Summary



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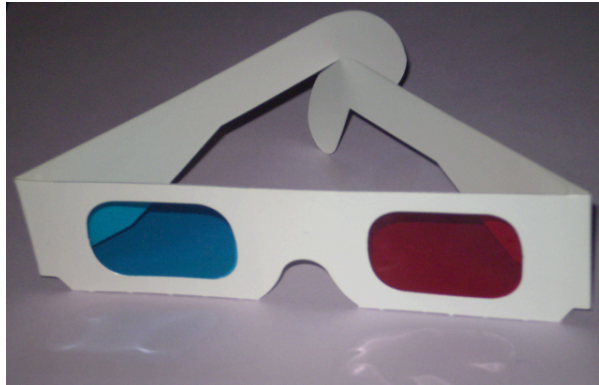


What is Stereo Vision?

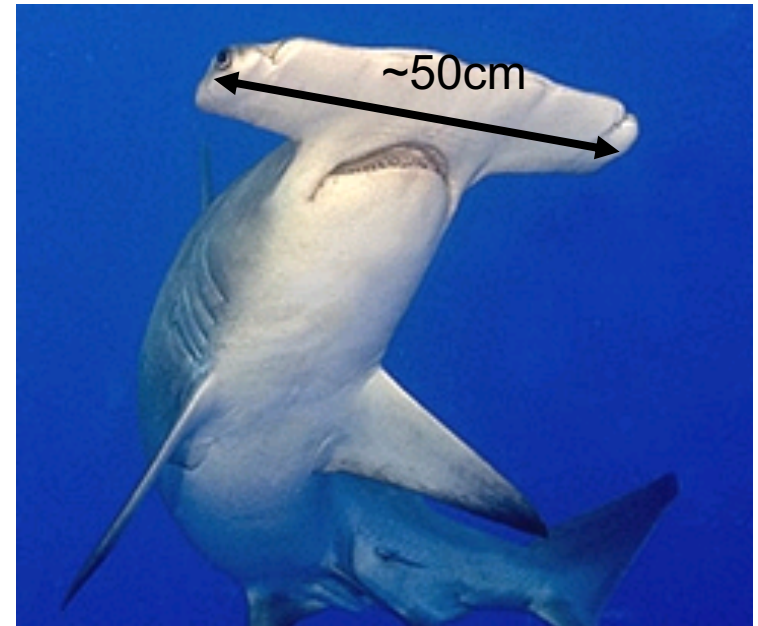
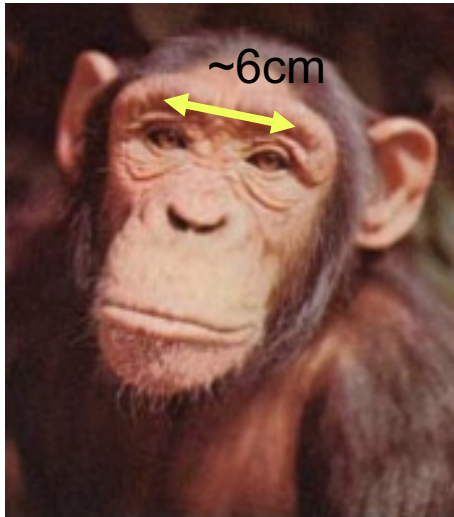


What is Stereo Vision?

- 3D cinema
- 3D television
- ...



What is Stereo Vision?

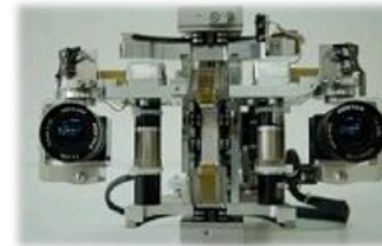
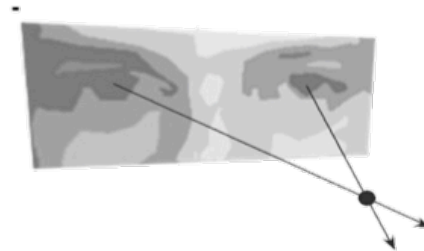


What is Stereo Vision?

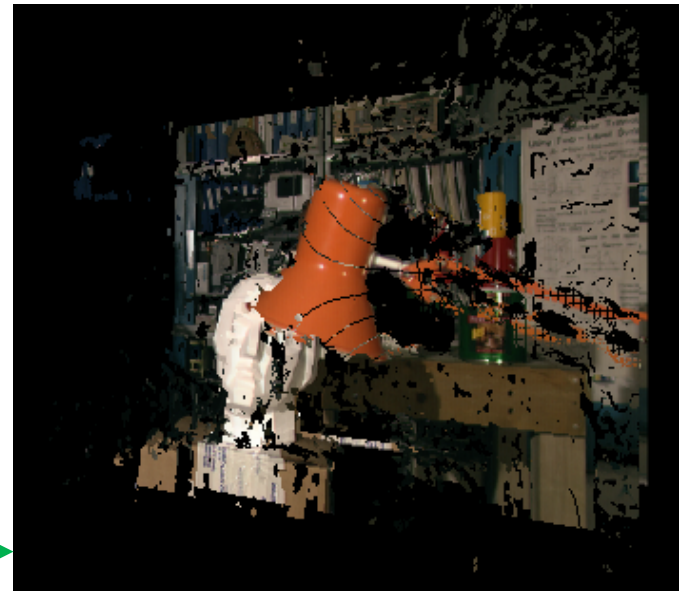


“... the mind perceives an object of three-dimensions by means of the two dissimilar pictures projected by it on the two retinae...”

Sir Charles Wheatstone, 1838



Stereo Vision Computation



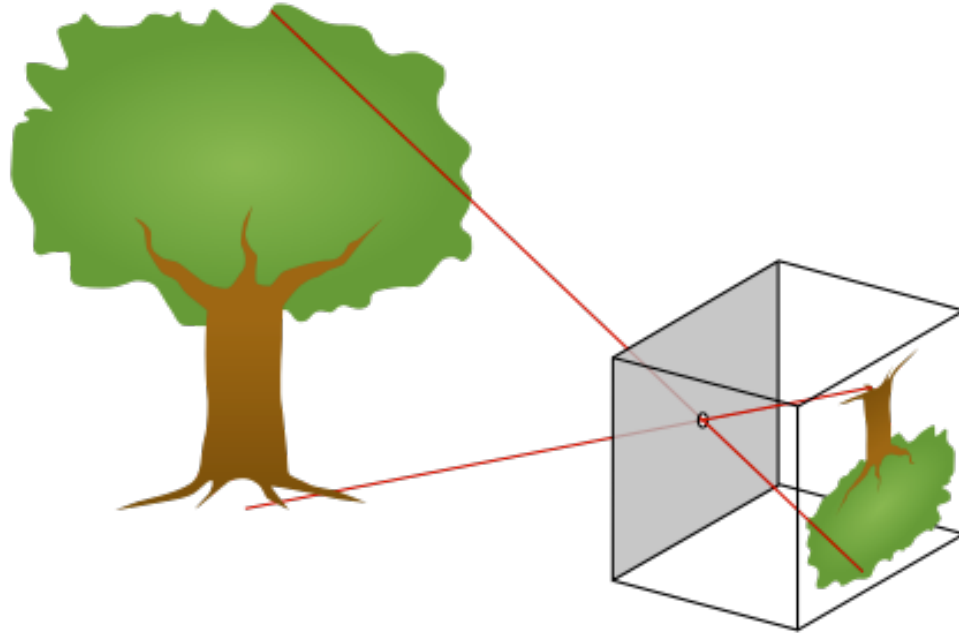


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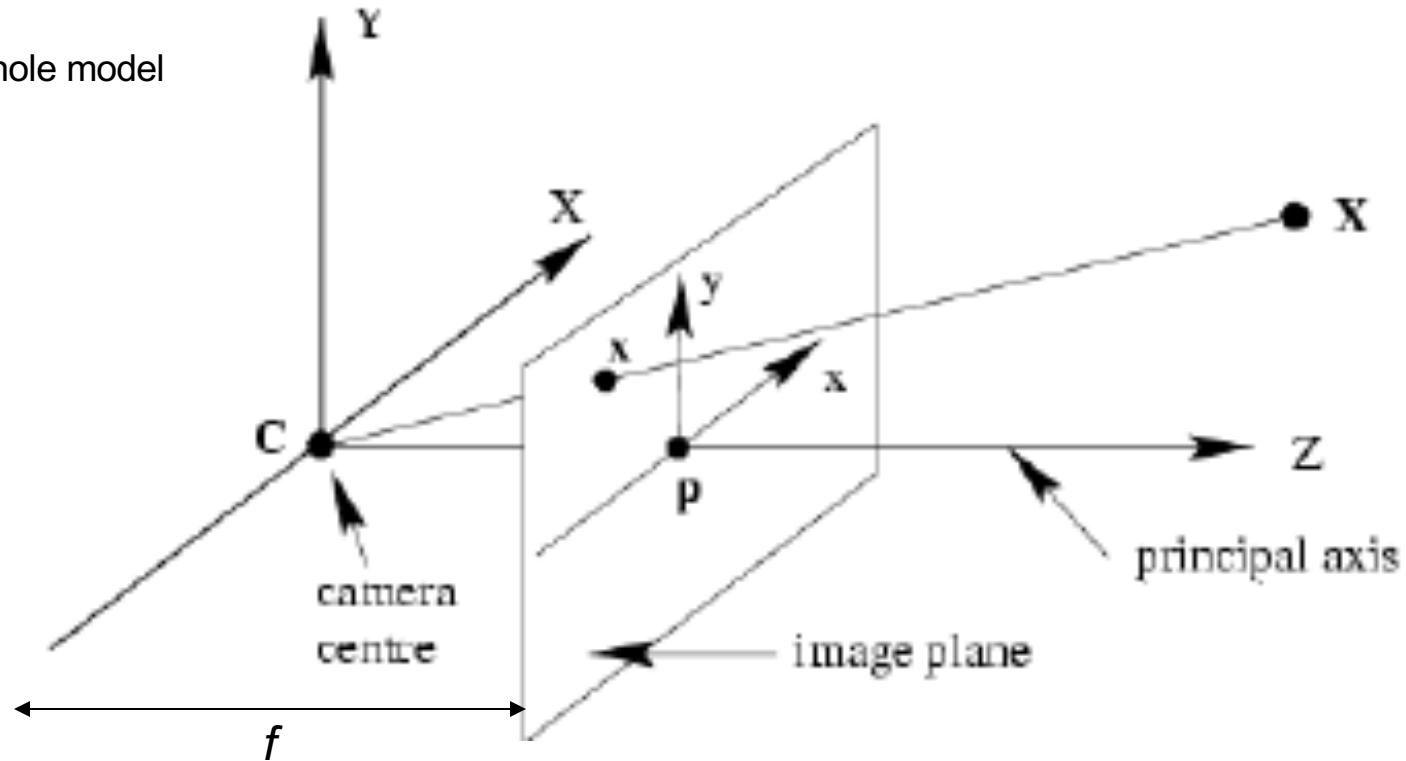
Camera Model

- Pinhole model

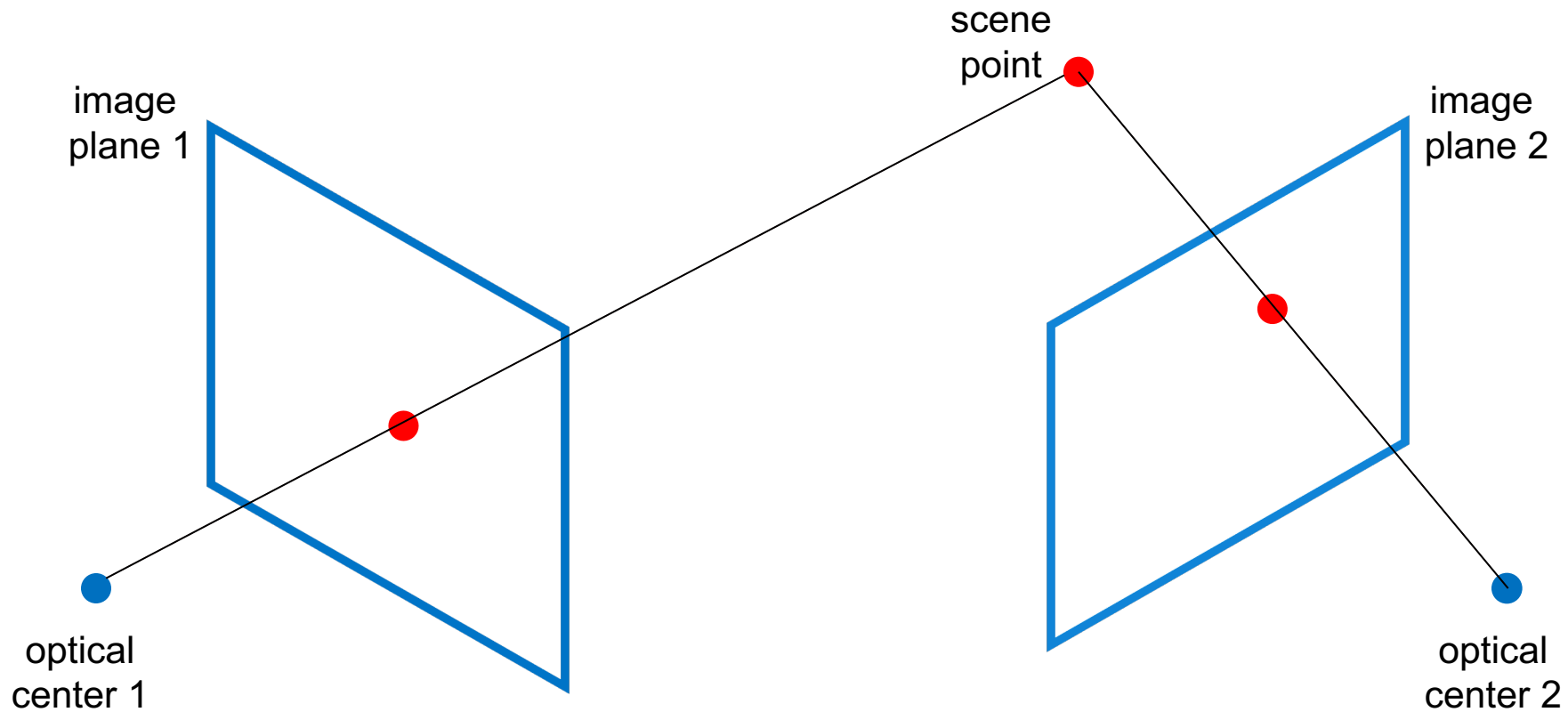


Camera Model

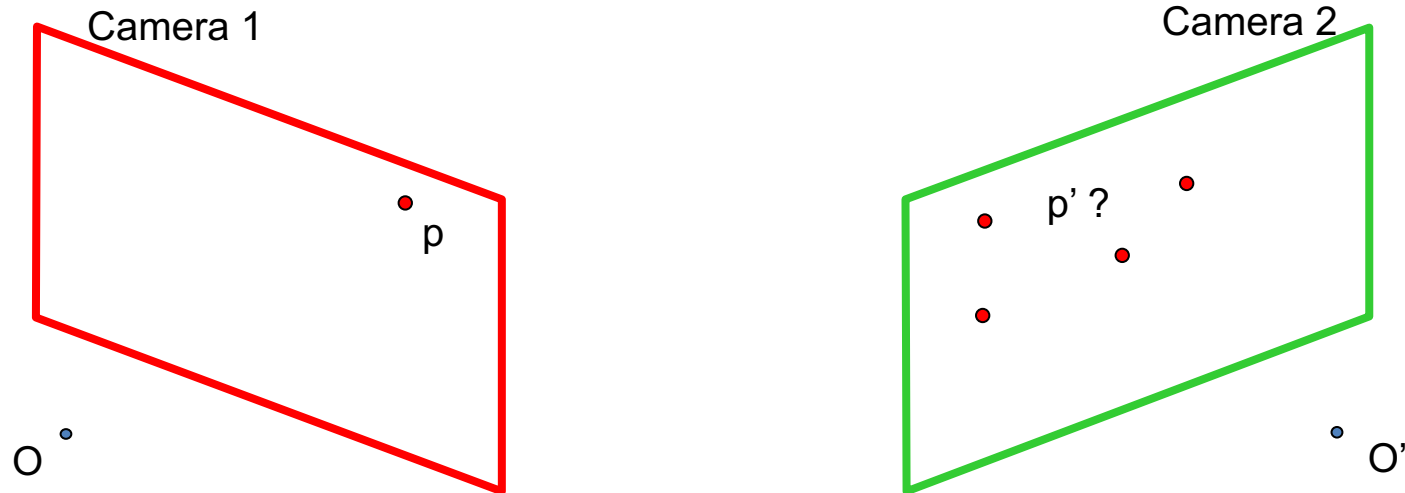
- Pinhole model



Epipolar Geometry

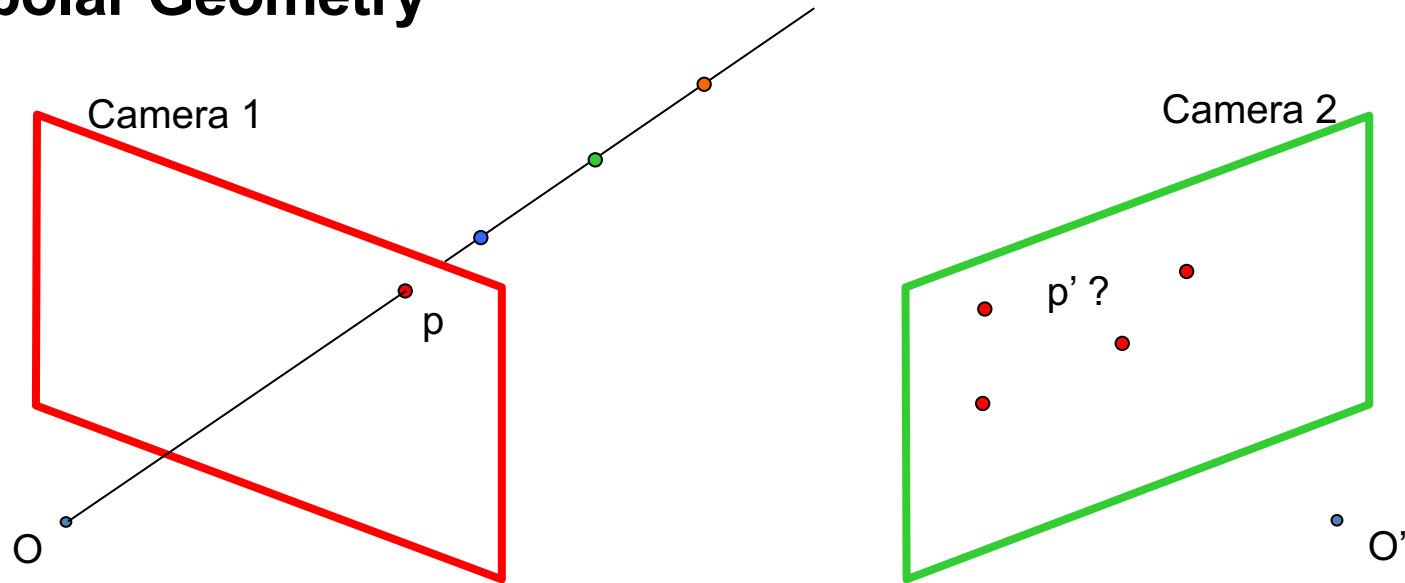


Epipolar Geometry



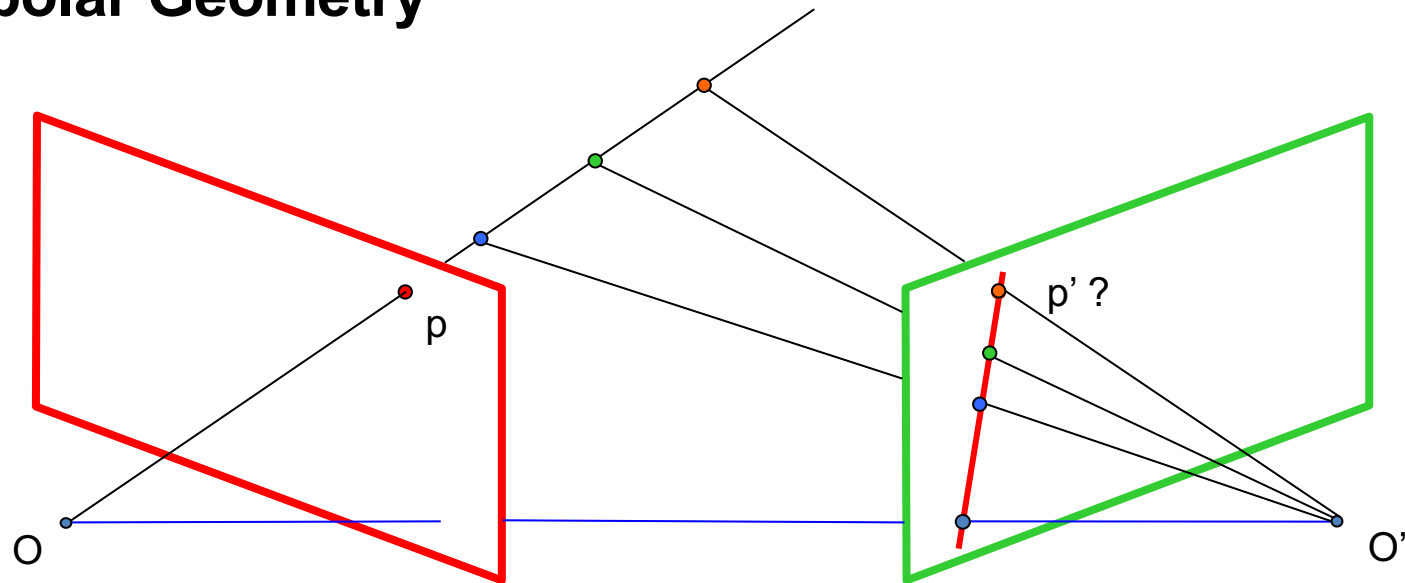
If we see a point in camera 1, are there any constraints on where we will find it on camera 2?

Epipolar Geometry

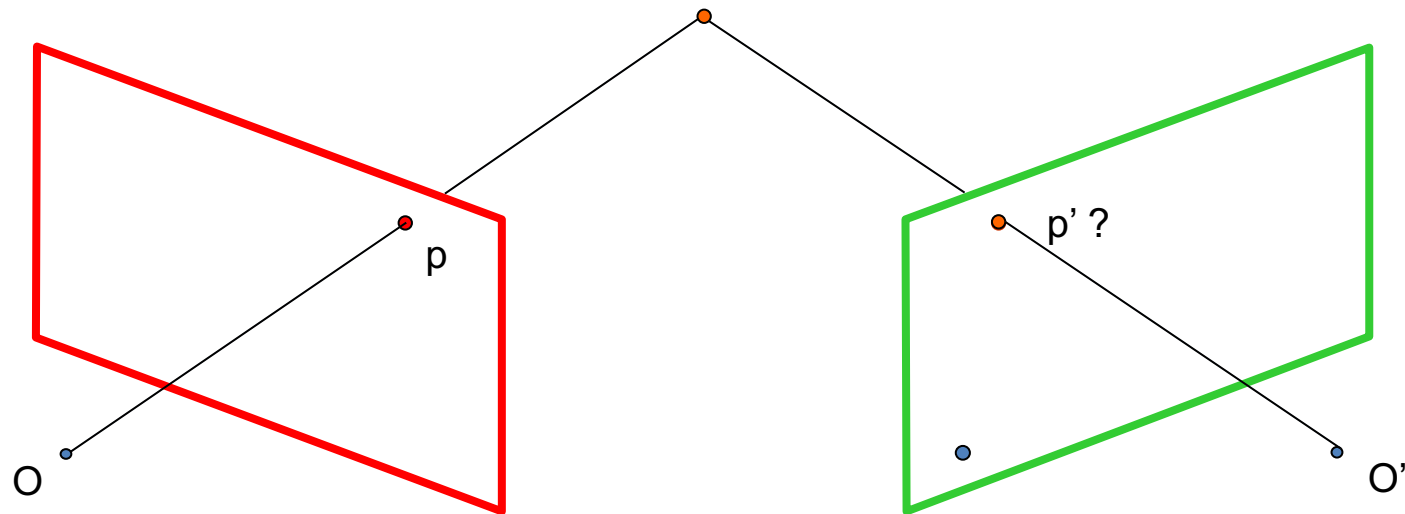


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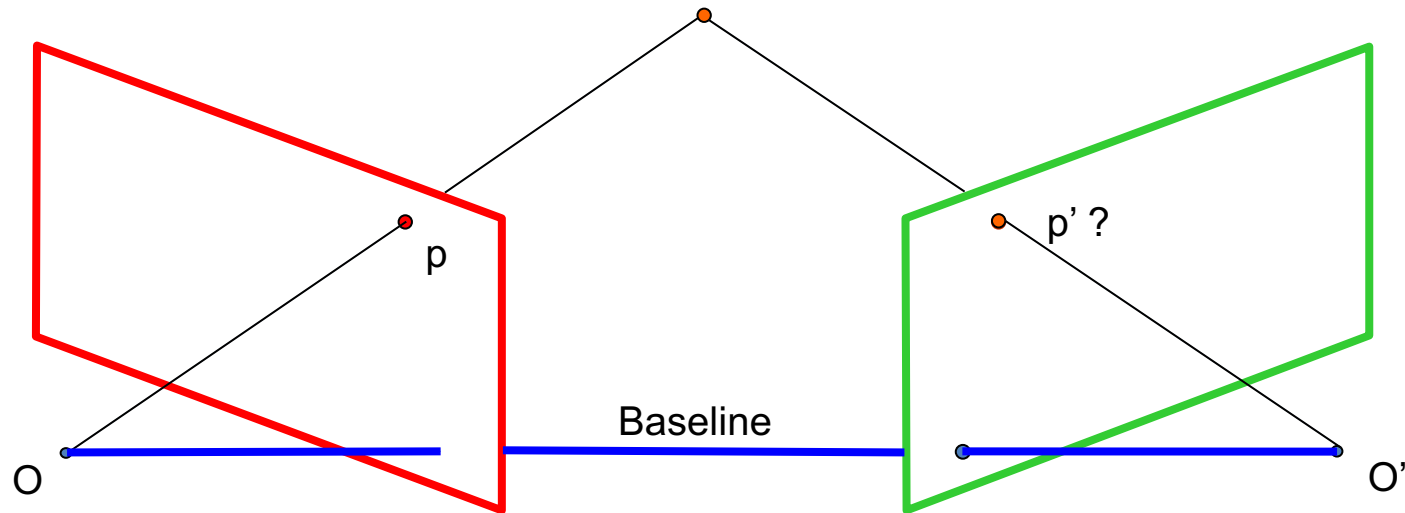
Epipolar Geometry



Epipolar Geometry

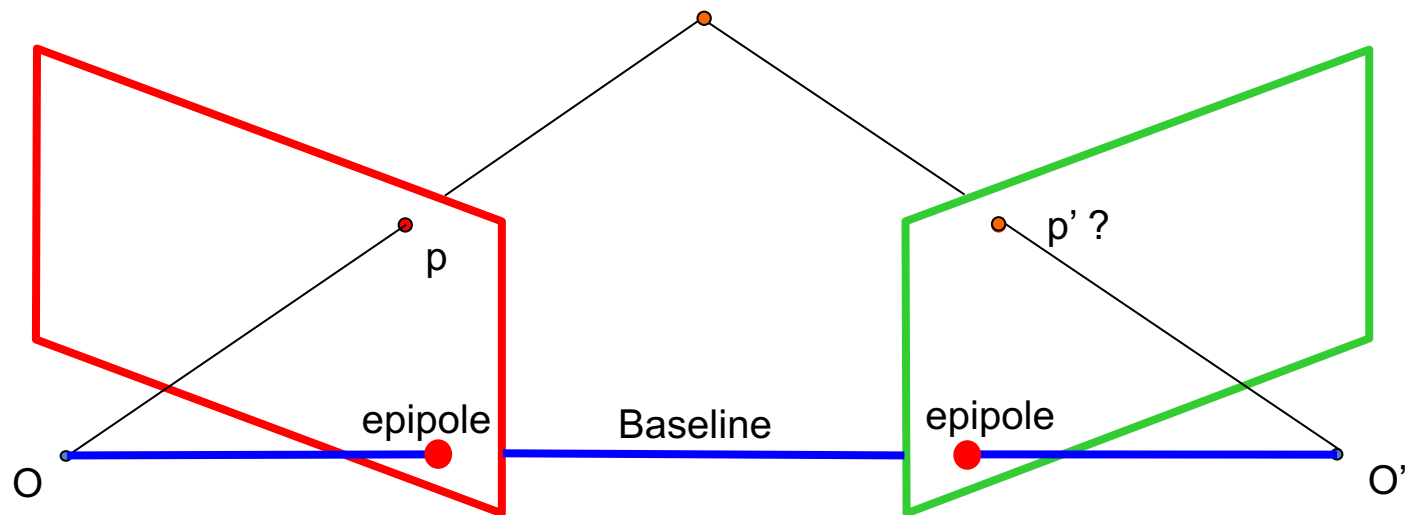


Epipolar Geometry



Baseline: the line connecting the two camera centers

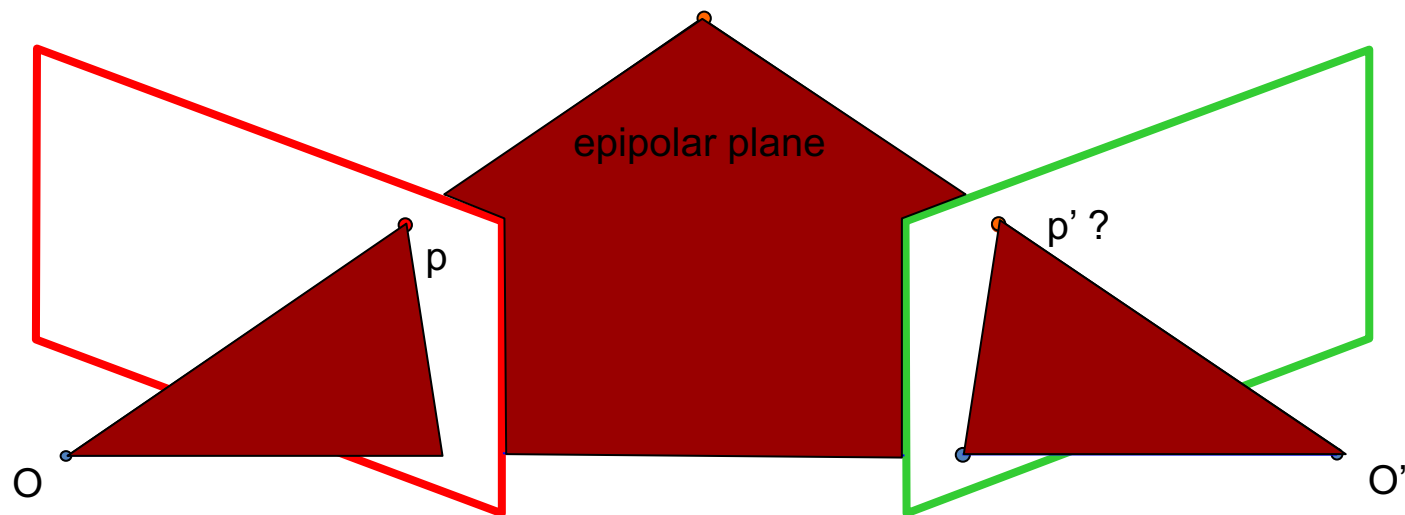
Epipolar Geometry



Baseline: the line connecting the two camera centers

Epipole: point of intersection of *baseline* with the image plane

Epipolar Geometry

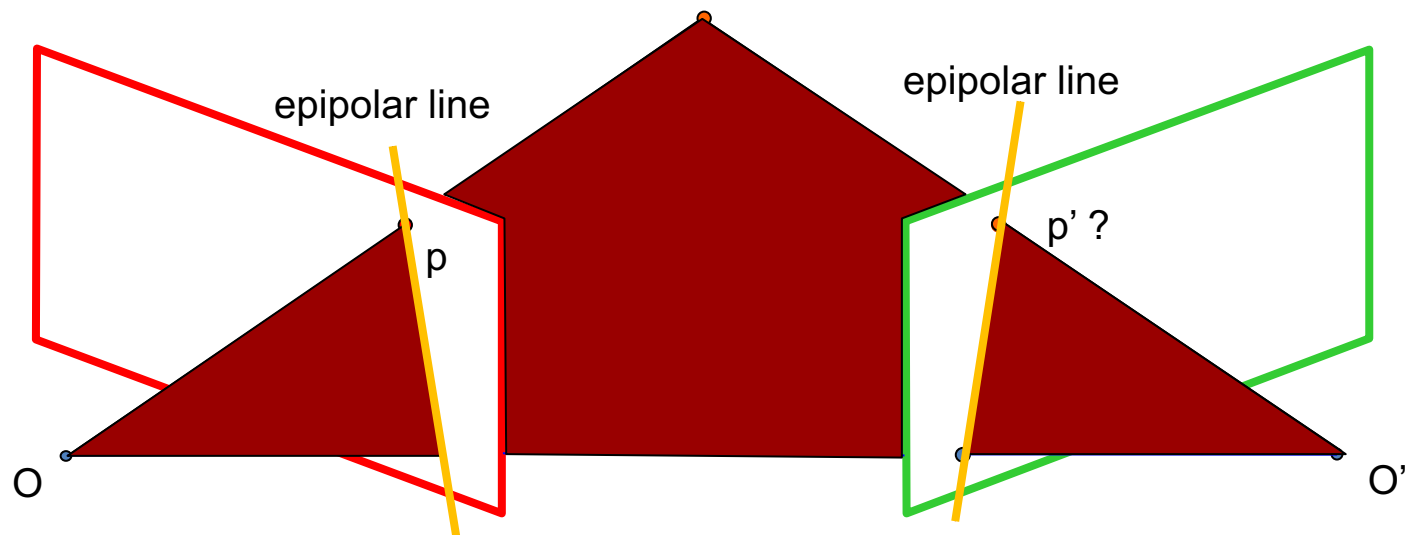


Baseline: the line connecting the two camera centers

Epipole: point of intersection of *baseline* with the image plane

Epipolar plane: the plane that contains the two camera centers and a 3D point in the world

Epipolar Geometry



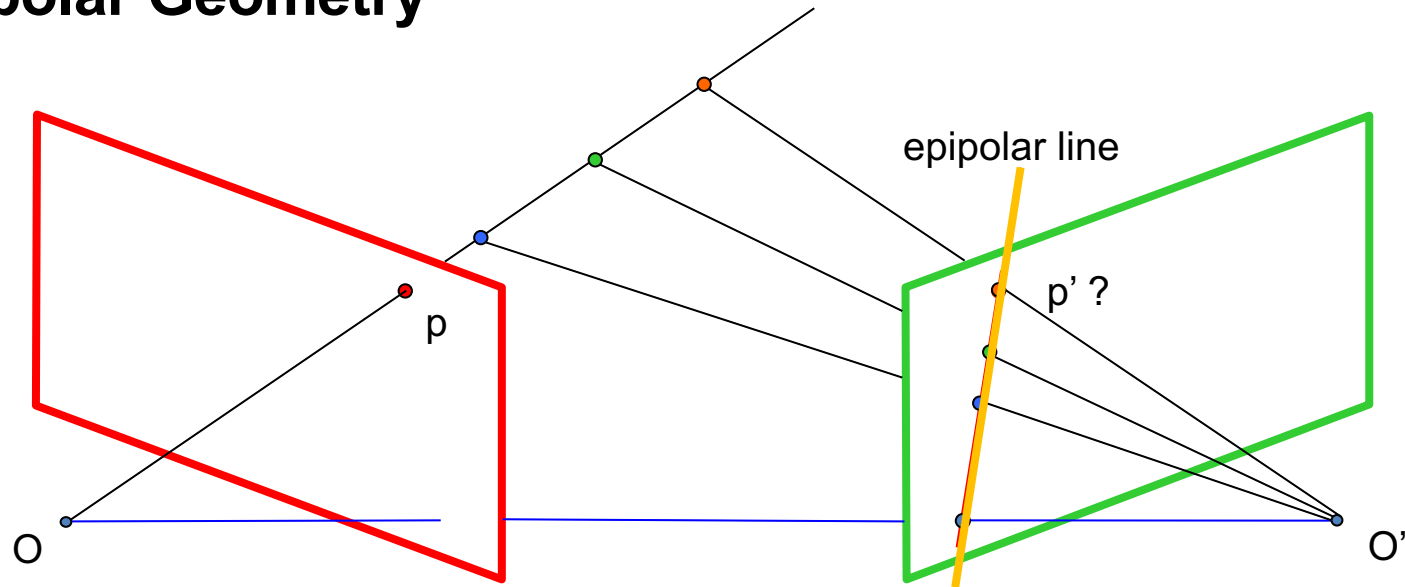
Baseline: the line connecting the two camera centers

Epipole: point of intersection of *baseline* with the image plane

Epipolar plane: the plane that contains the two camera centers and a 3D point in the world

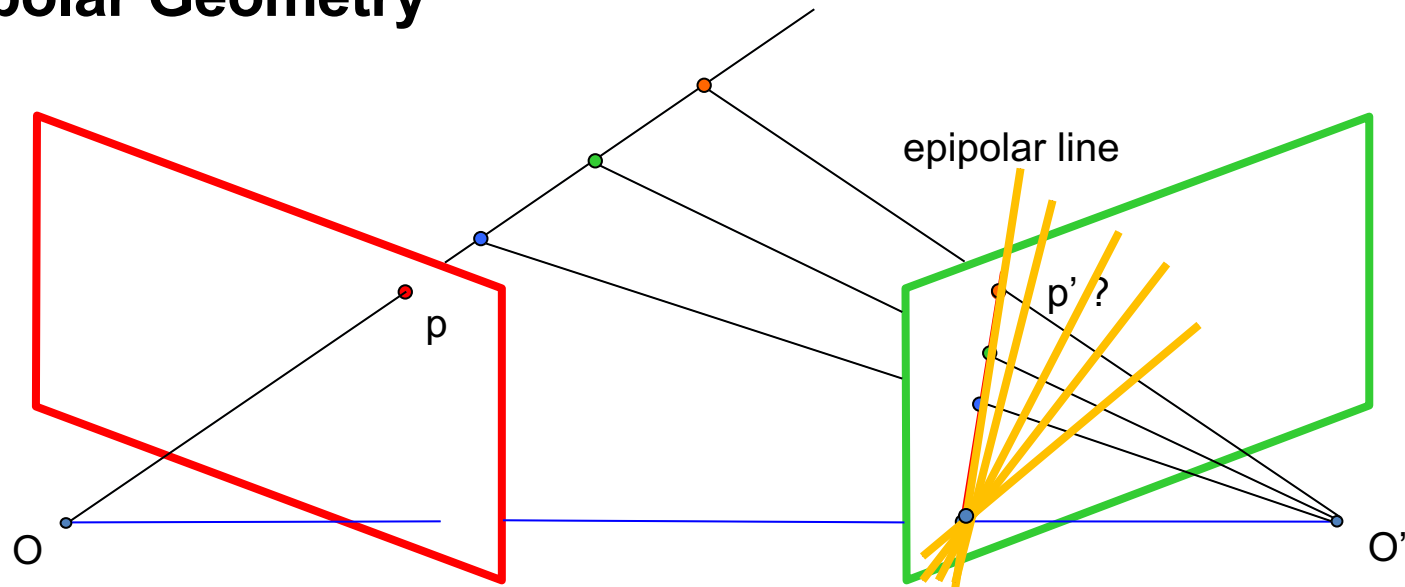
Epipolar line: intersection of the *epipolar plane* with each image plane

Epipolar Geometry



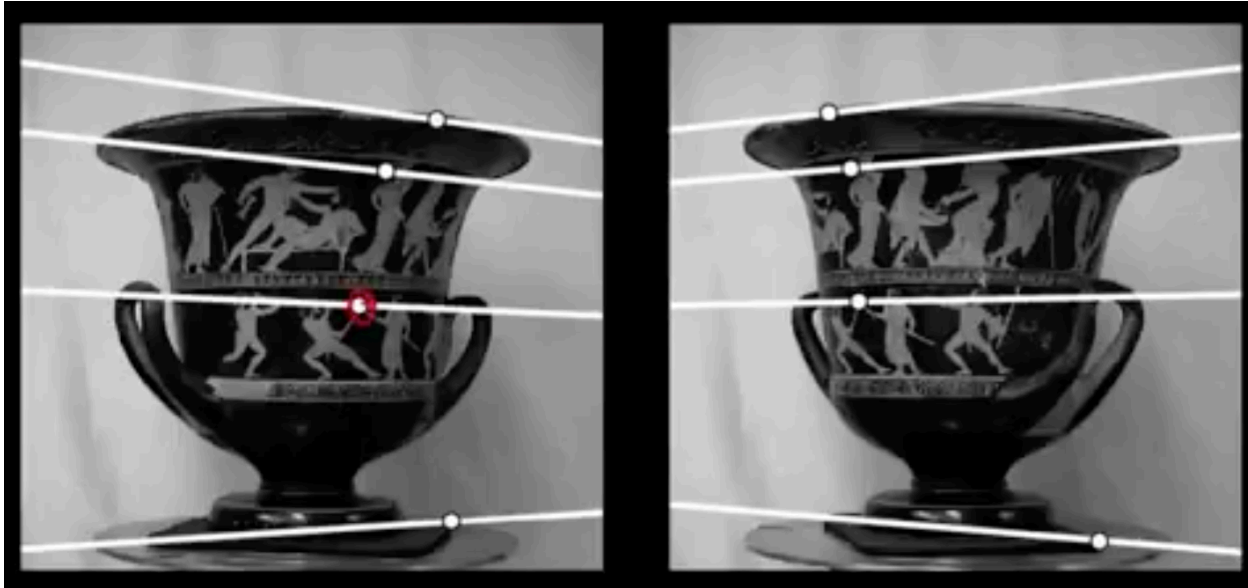
- We can search for matches across epipolar lines.
 - Search space for correspondences reduces to a 1D problem!

Epipolar Geometry



- We can search for matches across epipolar lines
 - Search space for correspondences reduces to a 1D problem!
- All epipolar lines intersect at the epipoles

Epipolar Geometry



- We can search for matches across epipolar lines
 - Search space for correspondences reduces to a 1D problem!
- All epipolar lines intersect at the epipoles
 - **Where are the epipoles in this case?**

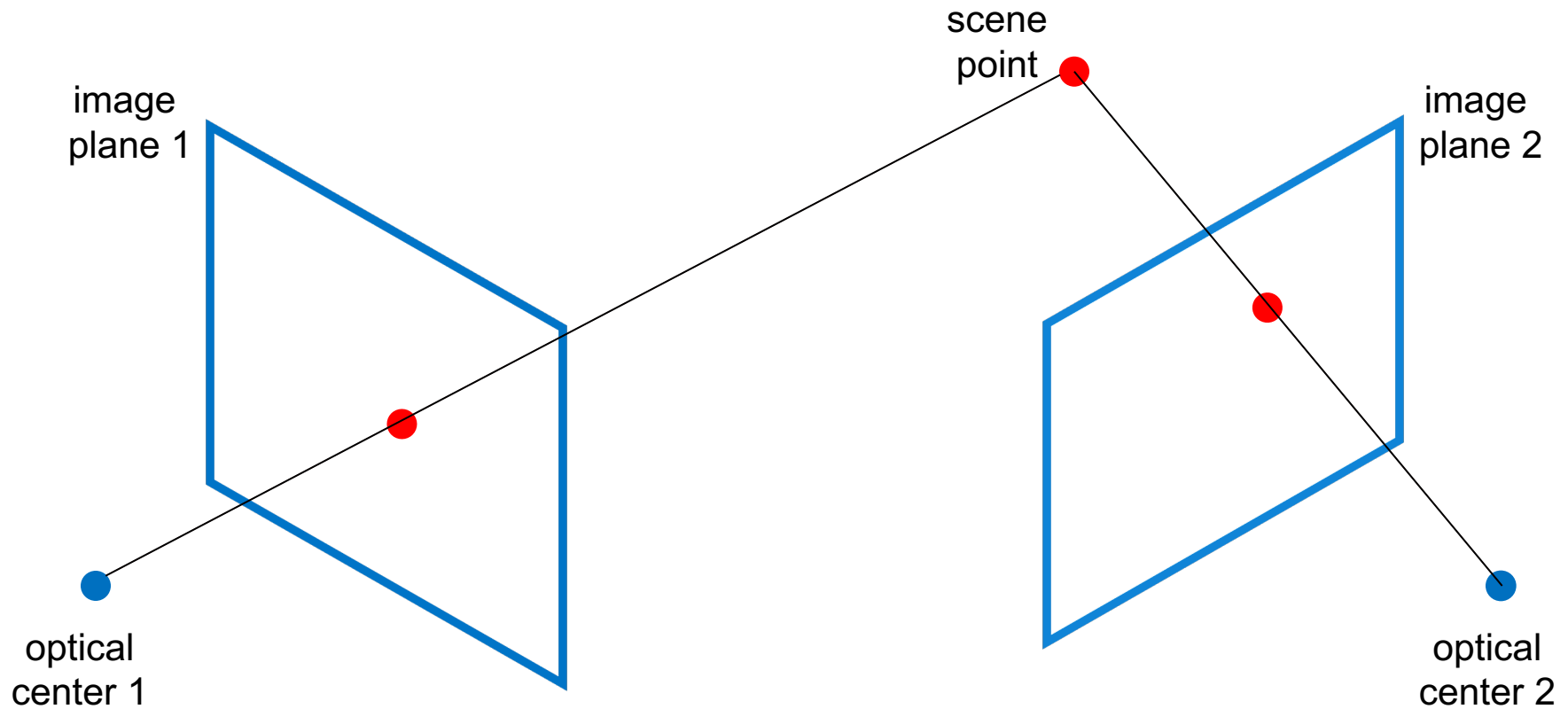


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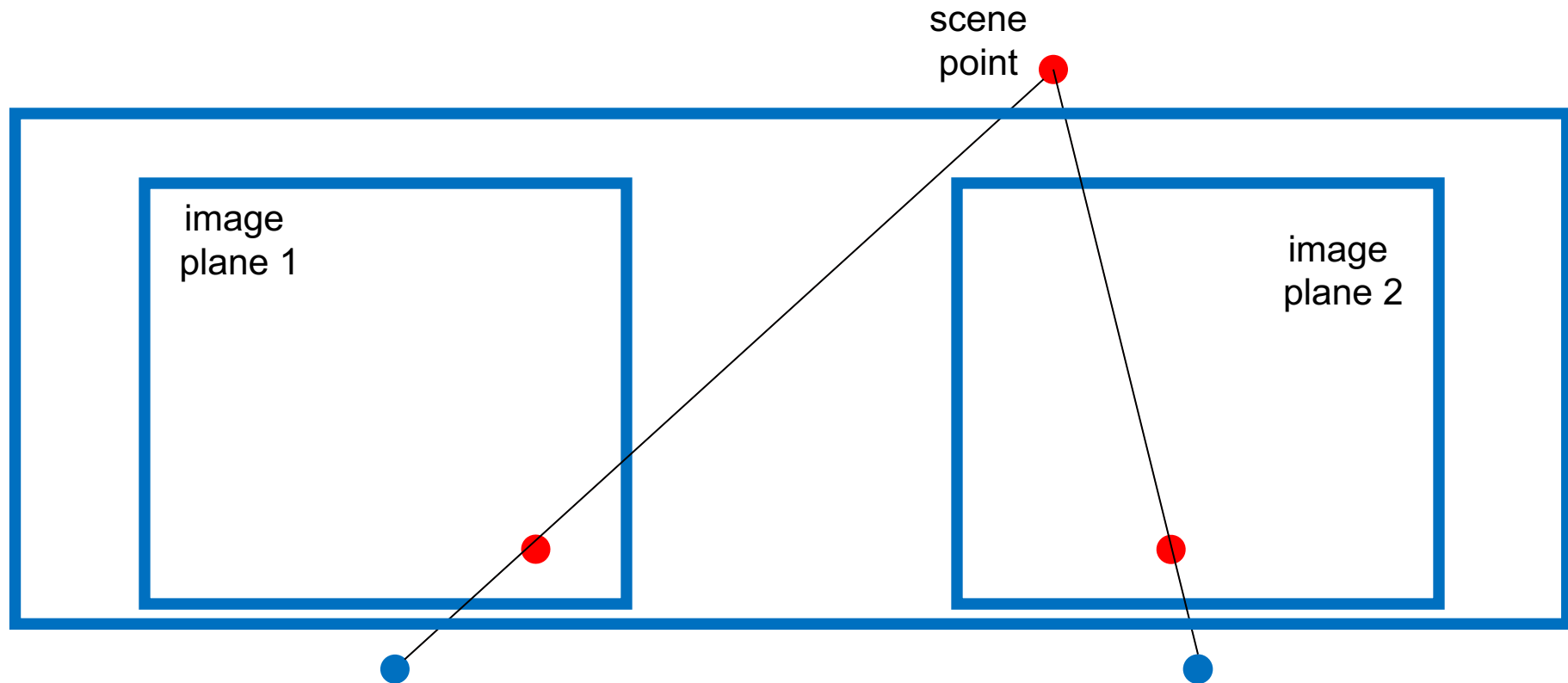
DTU

Rectified Stereo – a simpler case



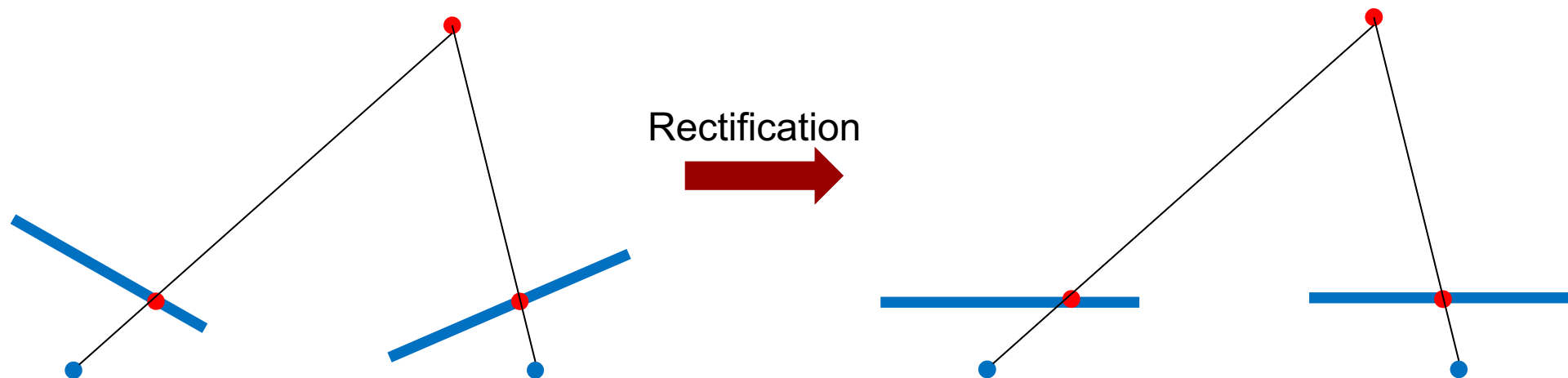
DTU

Rectified Stereo – a simpler case

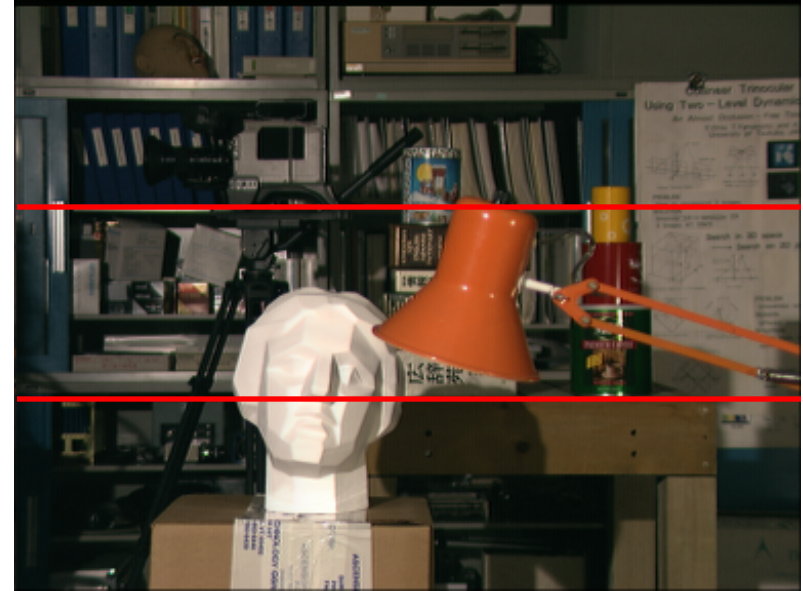
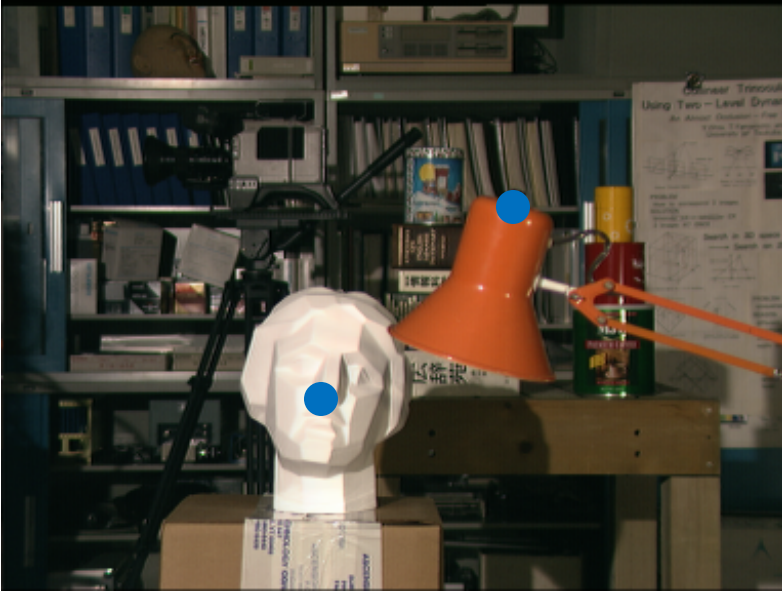


Rectified Stereo – a simpler case

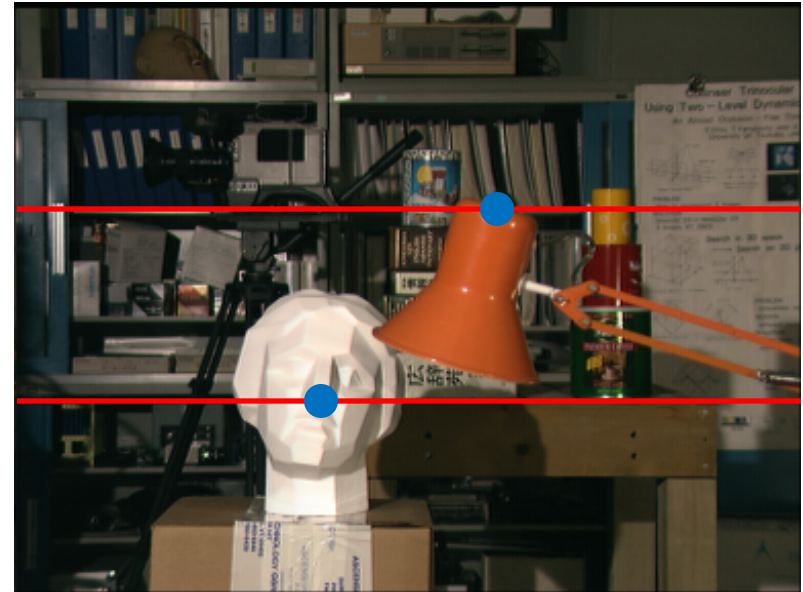
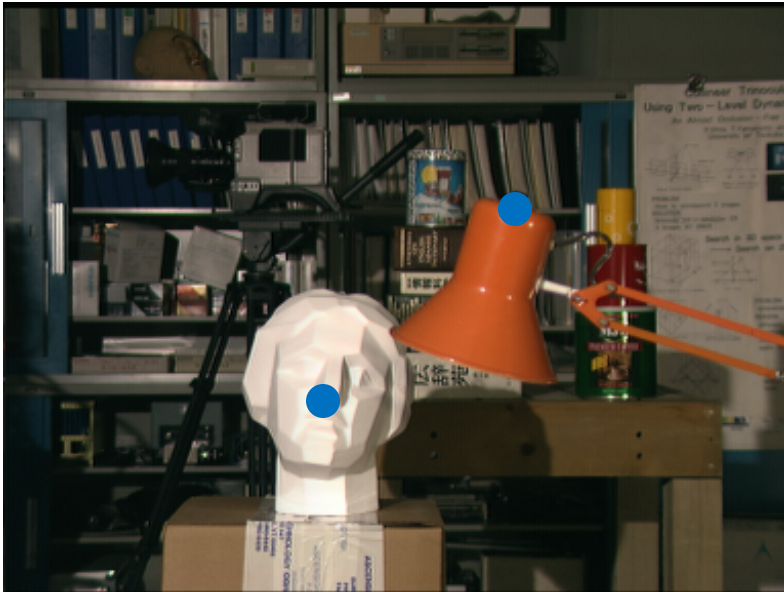
- Rectification:
 - The initial images are reprojected on a common plane that is parallel to the baseline B joining the optical centers of the initial images.
 - Epipolar lines become parallel (and under certain conditions they become also horizontal)



Rectified Stereo



Rectified Stereo



- “All epipolar lines intersect at the epipoles”
 - **Where are the epipoles in this case?**



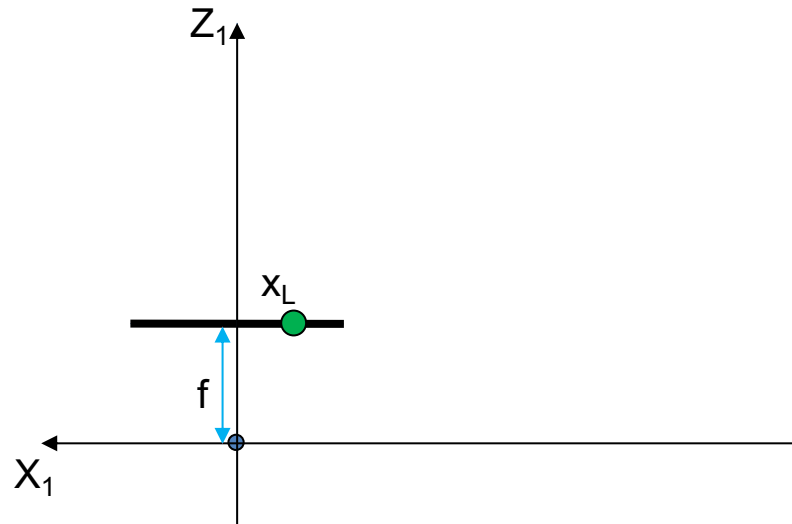
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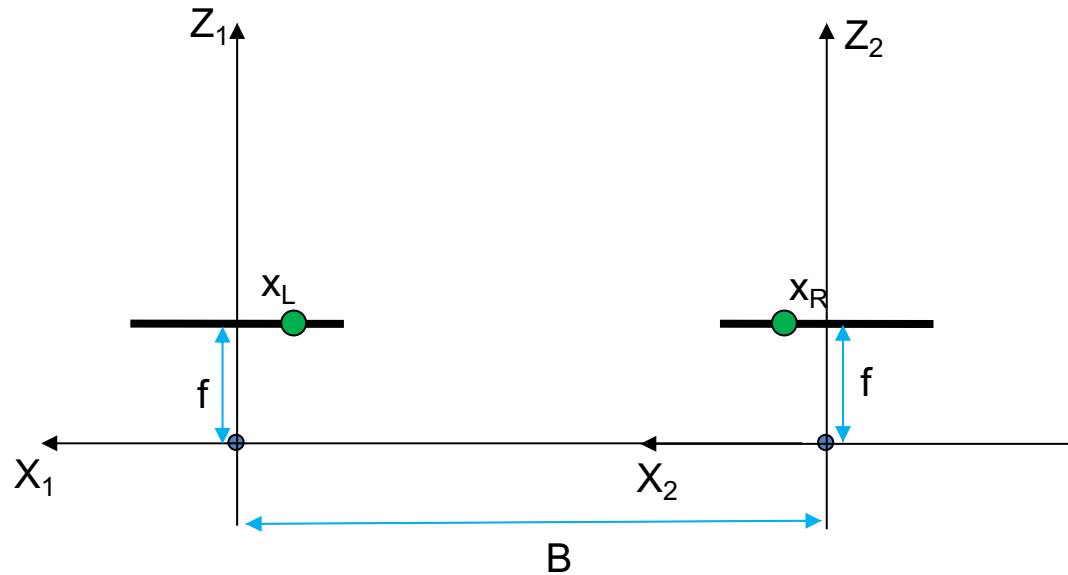
Depth from Stereo Matches

- Let us assume (for now!!) that:
 - we can check the points along the epipolar line and
 - we can find the point (on the right image) that is most similar to our reference point (in the left image), i.e. we can solve the correspondence problem!

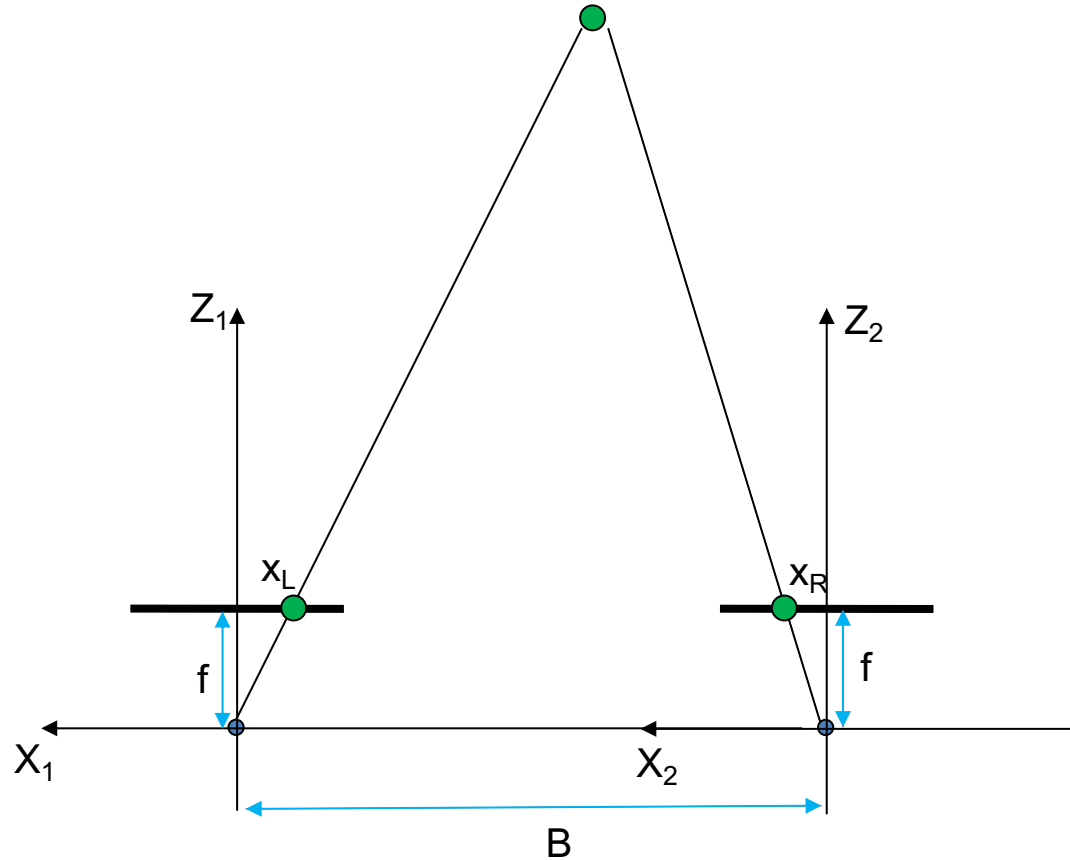
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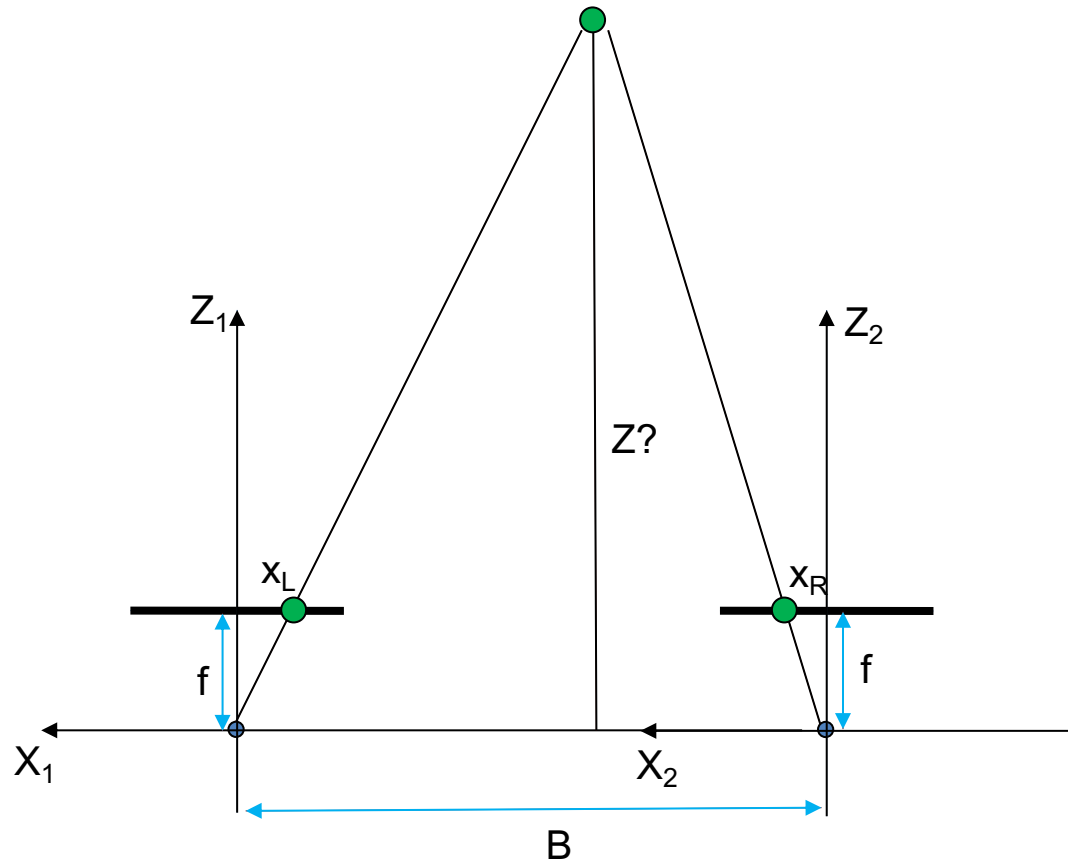
Depth from Stereo Matches



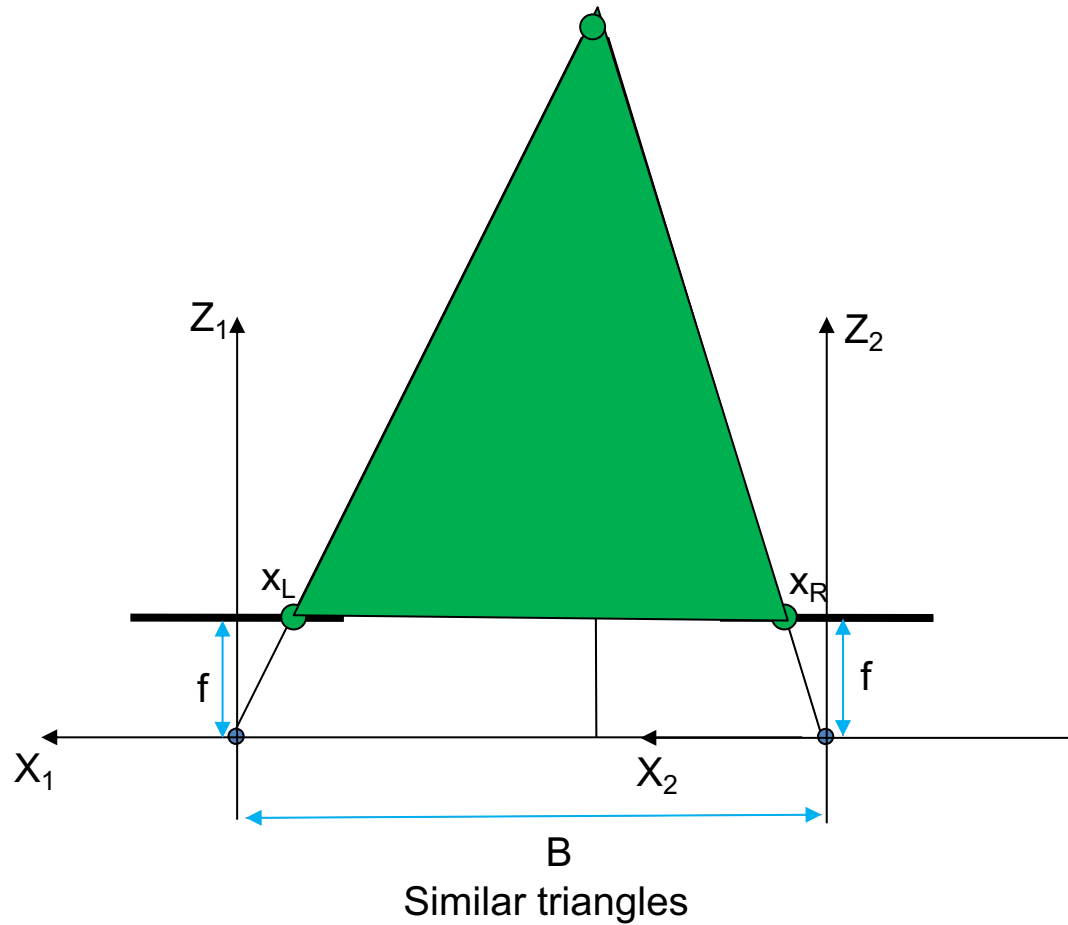
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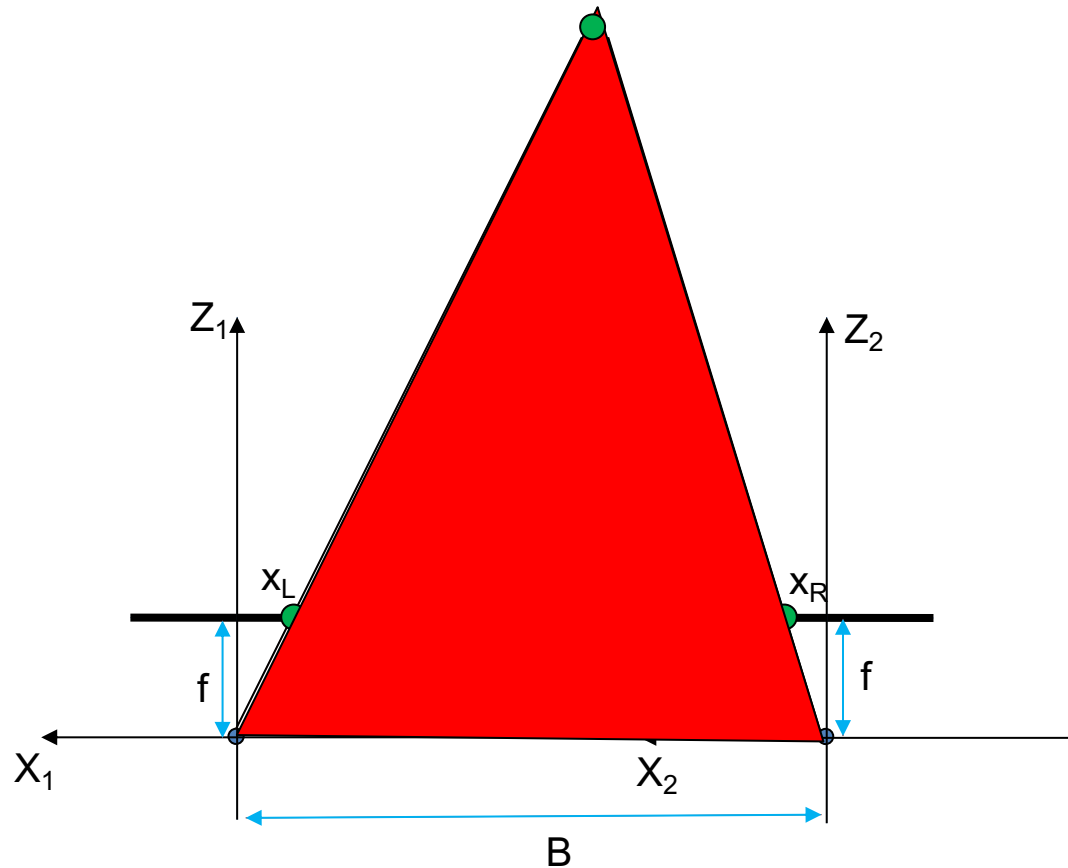
Depth from Stereo Matches



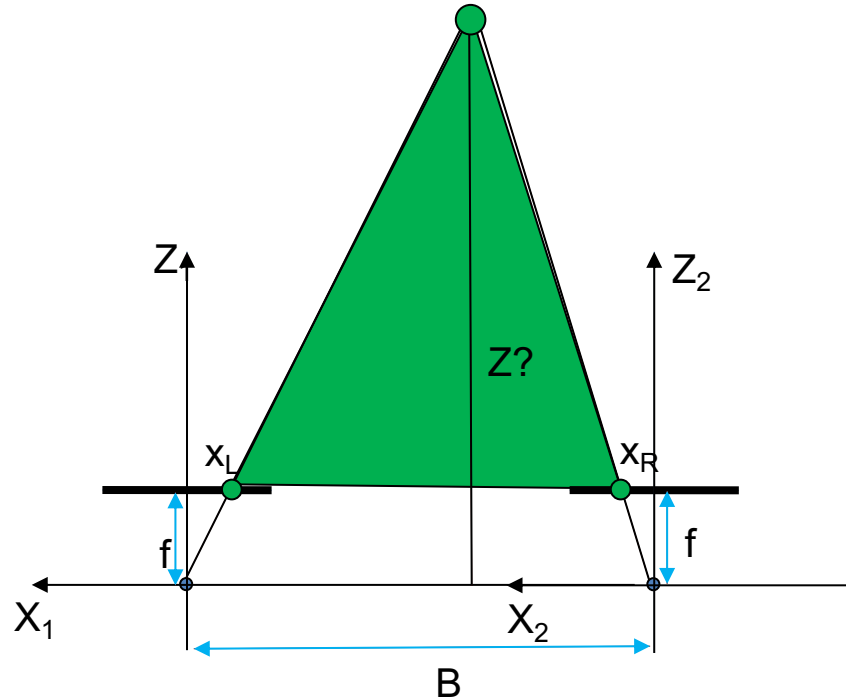
Depth from Stereo Matches



Depth from Stereo Matches



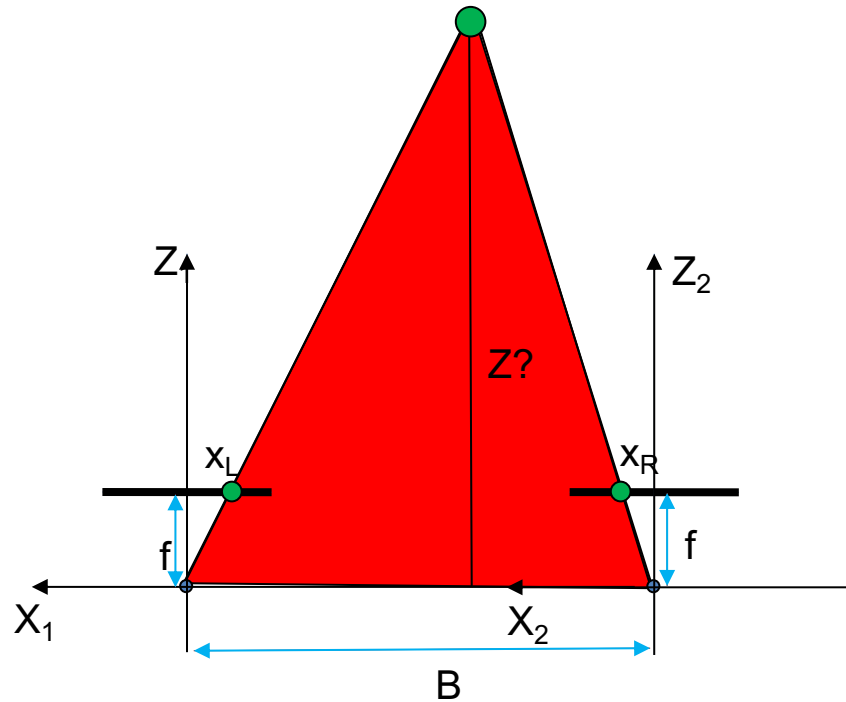
Depth from Stereo Matches



Similar triangles:

$$\frac{B + X_L - X_R}{Z - f} =$$

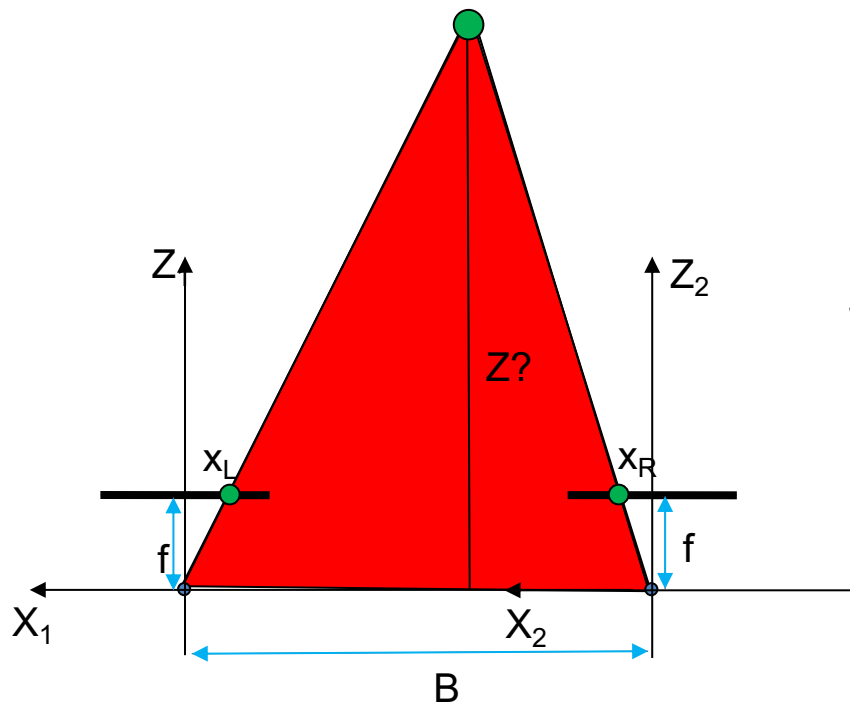
Depth from Stereo Matches



Similar triangles:

$$\frac{B + X_L - X_R}{Z - f} = \frac{B}{Z}$$

Depth from Stereo Matches



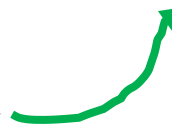
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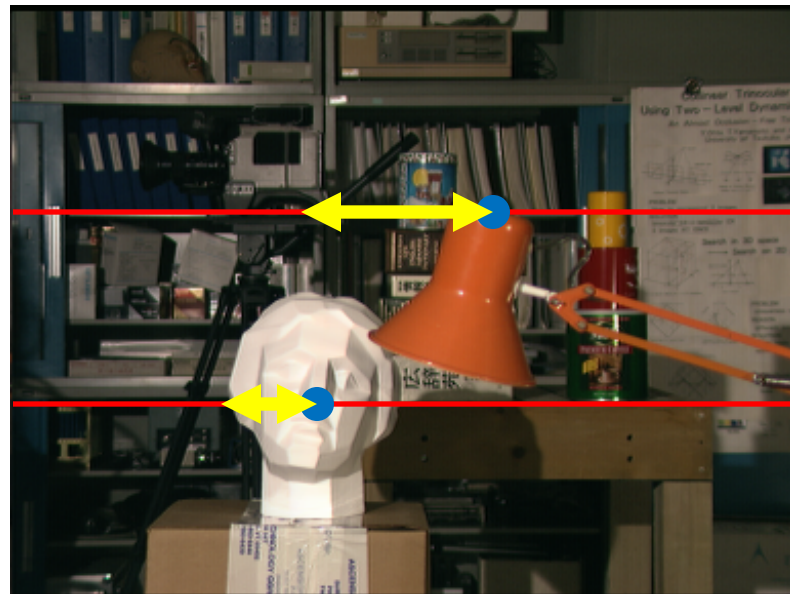
Solving for Z :

$$Z = f \frac{B}{X_L - X_R}$$

Disparity

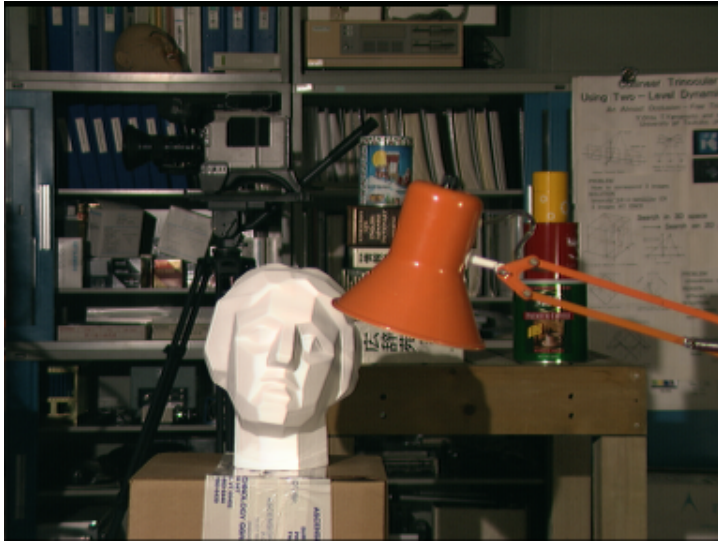


Depth from Stereo Matches



Depth from Stereo Matches

Reference Image



Disparity Map



$$Z = f \frac{B}{X_L - X_R}$$

Disparity



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Correspondence Problem

- We have assumed (up to now!!) that:
 - we can check the points along the epipolar line and
 - we can find the point (on the right image) that is **most similar** to our reference point (in the left image), i.e. we can **solve the correspondence problem!**
- *How can we indeed match corresponding pixels between the two stereo images?*

Correspondence Problem

- Beyond the hard constraint of epipolar geometry, there are “soft” constraints to help identify corresponding points
 - Similarity
 - Uniqueness
 - Ordering
 - Disparity gradient is limited



Correspondence Problem

- To find matches in the image pair, we will assume
 - Most scene points visible from both views
 - Image regions for the matches are similar in appearance

Correspondence Problem

- It depends!
 - Do we need **dense** or **sparse** stereo matching?

Stereo Vision

```
graph TD; A[Stereo Vision] --> B[Sparse output]; A --> C[Dense output]; C --> D[Local Methods (Area-based)]; C --> E[Global Methods (Energy-based)]; C --> F[Other Methods];
```

Sparse output

Dense output

- Local Methods (Area-based)
- Global Methods (Energy-based)
- Other Methods

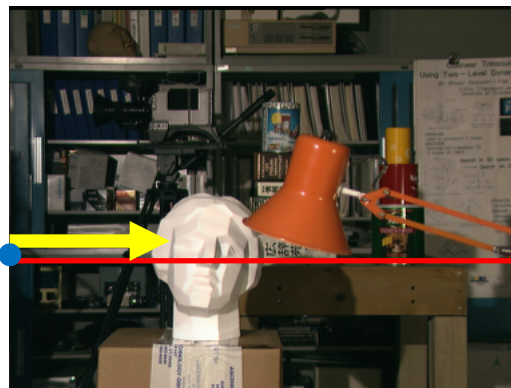


Sparse Stereo Correspondence

- Extract features (e.g. SIFT, SURF, Harris,...) and match them!
 - Pros?
 - Cons?

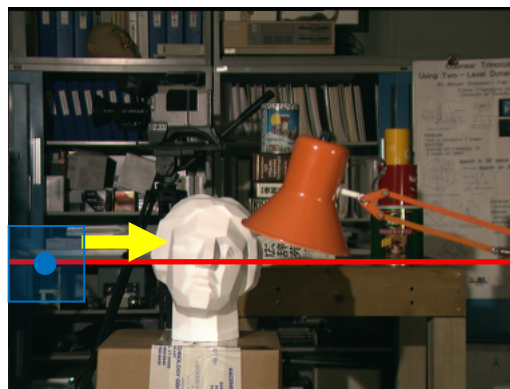
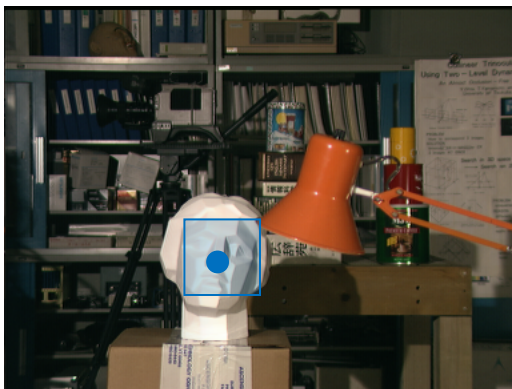
Dense Stereo Correspondence : Local Methods

- Try to find correspondences for all the pixels of the reference image.
- For each epipolar line
 - For each pixel in the left image
 - Compare with every pixel on same epipolar line in right image
 - Choose the pixel that maximizes a similarity metric (or minimizes a dissimilarity metric!).



Dense Stereo Correspondence

- Try to find correspondences for all the pixels of the reference image.
- For each epipolar line
 - For each pixel in the left image
 - Compare with every pixel on same epipolar line in right image
 - Choose the pixel that maximizes a similarity metric (or minimizes a dissimilarity metric!).
- Improvement: don't match individual pixels, but rather match windows!



Stereo Correspondence Metrics

- Sum of Absolute Differences (SAD)

$$SAD(x, y, d) = \sum_{x, y \in W} |I_l(x, y) - I_r(x, y - d)|$$

- Sum of Squared Differences (SSD)

$$SSD(x, y, d) = \sum_{x, y \in W} (I_l(x, y) - I_r(x, y - d))^2$$

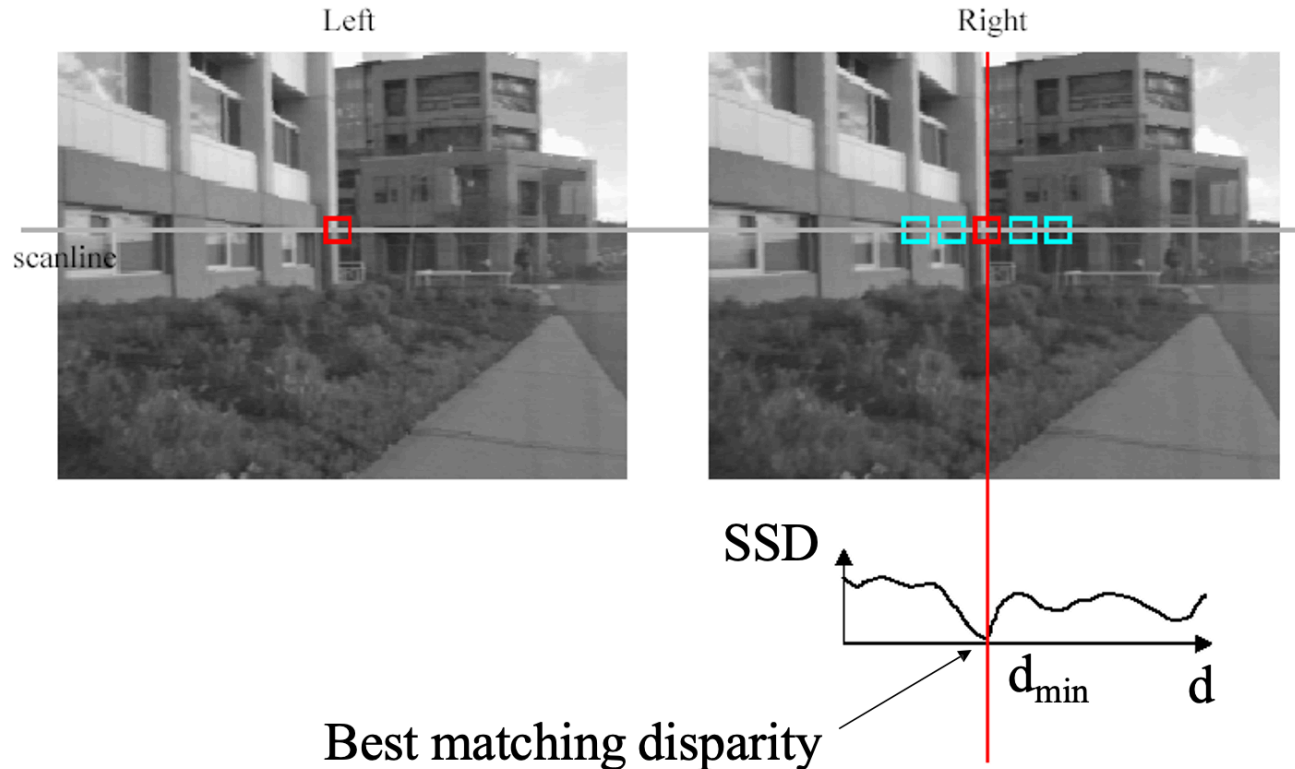
- Normalized Cross-Correlation

$$NCC(x, y, d) = \frac{\sum_{x, y \in W} I_l(x, y) \cdot I_r(x, y - d)}{\sqrt{\sum_{x, y \in W} I_l^2(x, y) \cdot \sum_{x, y \in W} I_r^2(x, y - d)}}$$

- ...many many more!!!



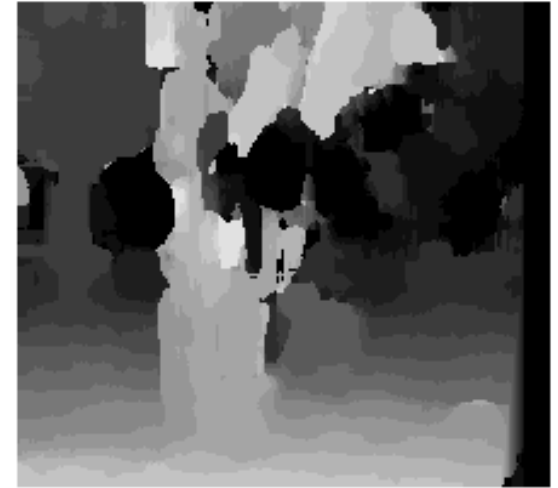
Stereo Correspondence Metrics: SSD



Stereo Correspondence Metrics: SSD on various windows



$W = 3$



$W = 20$

- Small vs Big windows
- What are their Pros and Cons?

Stereo Correspondence Metrics: Good/Bad areas



- In this stereo image pair:
 - what would be good areas to match?
 - where would you expect to face problems and why?

Global Stereo Correspondence

- Up to this point, the disparity of each pixel was determined only by the information of the pixel itself and its neighborhood.
 - Thus, those methods are called "local" or "area-based" methods.
- Example: Result of a **local** SSD algorithm with $W=21$:



Global Stereo Correspondence

- Up to this point, the disparity of each pixel was determined only by the information of the pixel itself and its neighborhood.
 - Thus, those methods are called "local" or "area-based" methods.
- Global methods find better solutions in expense of more computations
 - Optimize jointly the disparity values of all the pixels of each scanline (e.g. Dynamic Programming)



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 - Optimize jointly the disparity values of all the pixels of each scanline (e.g. Dynamic Programming)
 - Optimize jointly the disparity values of all the pixels of the image (e.g. graph cuts)



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 - Thus, those methods are called "local" or "area-based" methods.
- Global methods find better solutions in expense of more computations
 - Optimize jointly the disparity values of all the pixels of each scanline (e.g. Dynamic Programming)
 - Optimize jointly the disparity values of all the pixels of the image (e.g. graph cuts)
- *In global algorithms, stereo correspondence is formulated as an energy function minimization problem, consisting of data and smoothness terms.*

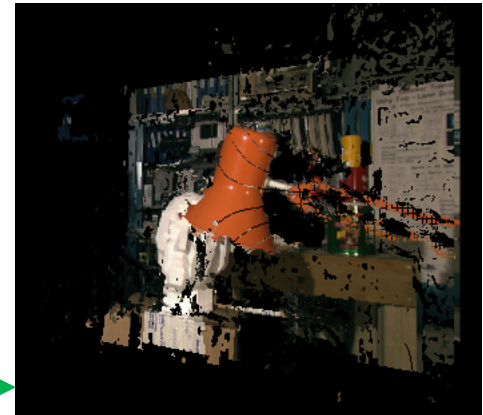
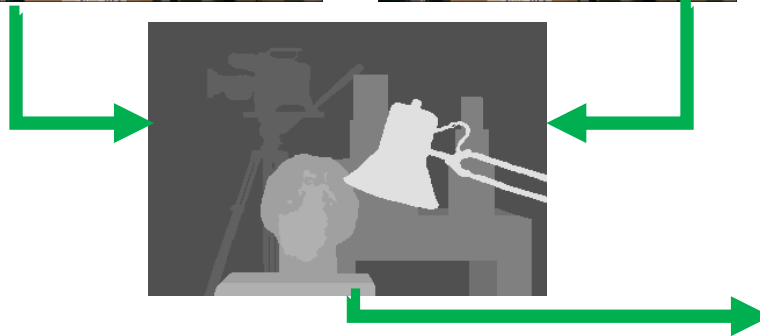
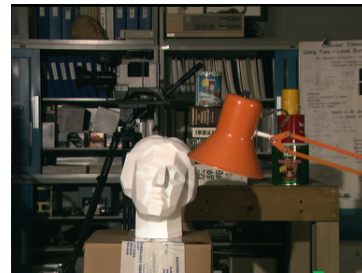


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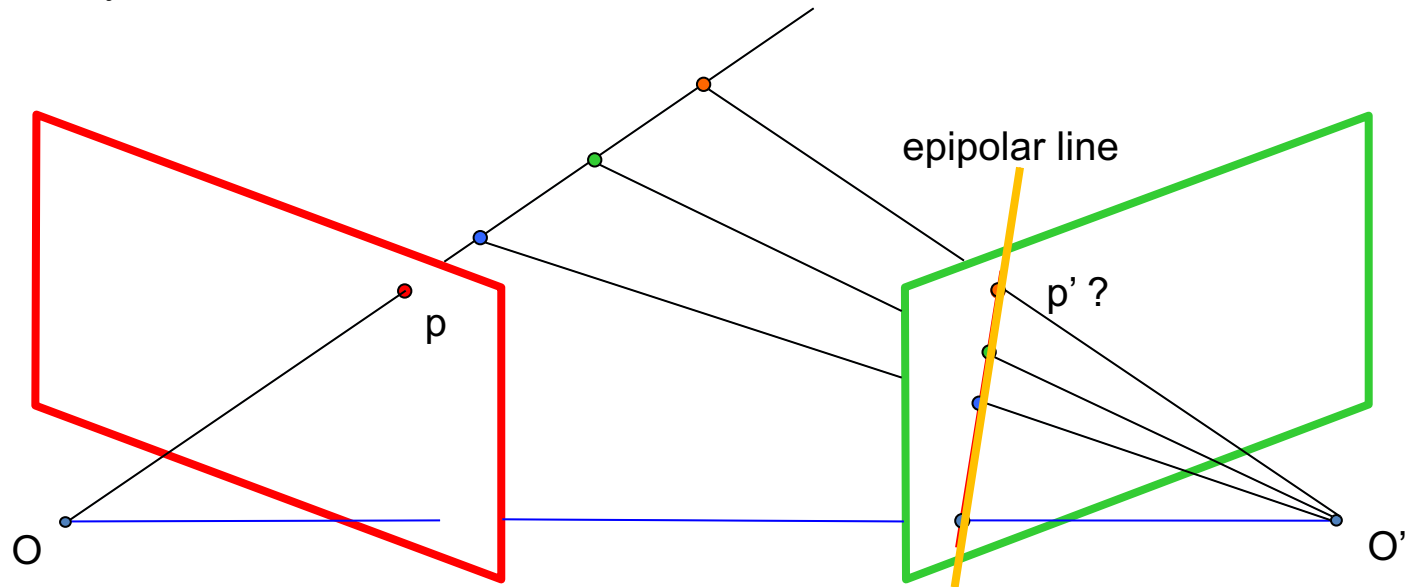
Summary

- We discussed about what Stereo Vision is.



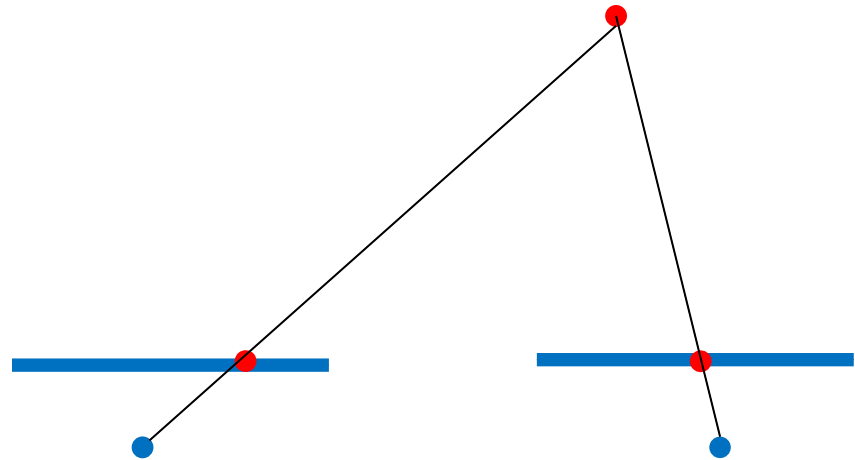
Summary

- We discussed about what Stereo Vision is.
- We learned about :
 - Stereo/Epipolar Geometry



Summary

- We discussed about what Stereo Vision is.
- We learned about :
 - Stereo/Epipolar Geometry
 - Rectified Stereo Case



Summary

- We discussed about what Stereo Vision is.
- We learned about :
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 - Depth from Stereo Matches

Reference Image



Disparity Map



$$Z = f \frac{B}{X_L - X_R}$$

Disparity

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- We learned about :
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Stereo Vision

Sparse output

Dense output

- Local Methods (Area-based)
- Global Methods (Energy-based)
- Other Methods

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