# tf: The Transform Library

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Abstract—The tf library was designed to provide a standard way to keep track of coordinate frames and transform data within an entire system such that individual component users can be confident that the data is in the coordinate frame that they want without requiring knowledge of all the coordinate frames in the system. During early development of the Robot Operating System (ROS), keeping track of coordinate frames was identified as a common pain point for developers. The complexity of this task made it a common place for bugs when developers improperly applied transforms to data. The problem is also a challenge due to the often distributed sources of information about transformations between different sets of coordinate frames. This paper will explain the complexity of the problem and distill the requirements. Then it will discuss the design of the tf library in relation to the requirements. A few use cases will be presented to demonstrate successful deployment of the library. And powerful extensions to the core capabilities such as being able to transform data in time as well as in space.

#### I. INTRODUCTION

When doing tasks with a robot it is crucial that the robot be aware of where it is itself as well as where the rest of the world is in relation to itself. A simple example which demonstrates this well is a mobile robot finding a red ball and touching with it's gripper. The challenge is simply to move the gripper toward the ball. However, to do this simple task the relationship between the ball and the gripper must be known. If there is a sensor in the room which can find the ball in space the required computation is to compute the transform from the sensor, to the room, to the base of the robot to the torso, to the shoulder, to the elbow, to the wrist, to the gripper. Then to compare that to the position of the ball, as you can only compare the position of the two objects in the same coordinate frame. If the result of this is that the gripper must move 3 cm to the left in the coordinate frame of the sensor. To compute what motion the robot should make, the 3 cm to the left in the sensor frame must be transformed into the coordinate frame of the torso, by computing the transform from the sensor to the room, to the base, to the torso. And from that it can be found that 3 cm left in the sensor frame is actually 3cm down in the torso frame. So the robot simply has to move the arm down 3cm.

This is a relatively simple robotic system, but to do this calculation required knowledge of the entire system. Robotic systems, including sensors, motors, computation, and communication, quickly grow complex even when designed to be simple. As robotic systems grow in complexity the ability of any subsystem to have complete knowledge about the rest of the system diminishes, and the designer of a component must consider exactly what information is necessary for their module to complete its task. When in a single computer much of this is a challenge of designing interfaces to provide all

the information between modules. As robotic systems become more distributed across multiple computers there is a point where not all information can be made available due to limitations in bandwidth.

It would be very powerful if the programmer could simply request from a library, what is the vector that I need to move the gripper with respect to the torso such that it will touch the ball. This is the actual question which is relevant to the task, and the programmer does not need to know about the configuration of any of the intermediate links. If this was a more complicated system with a second robot observing the ball from the other side of the table it would not make a difference to the programmer, nor if the sensor was mounted on the head of the robot. The request is the same in every case, and the programmer only needs to know the coordinate frames in which they want to operate and those relevant to the task, as long as the system knows the intermediate frames and can compute the resultant transforms.

The *tf* library was designed to provide a standard way to keep track of coordinate frames and transform data within the entire system such that individual component users can be confident that the data is in the coordinate frame that they want without requiring knowledge of all the coordinate frames in the system. As robotic systems get more and more complicated, being able to focus on precisely the task frame and only the relevant coordinate frames becomes critical. Most robotic systems are fusing data from many different sensors with different coordinate frames.

The *tf* library was developed as ROS package to provide this capability. [1] The *tf* library has two standard modules, a *Broadcaster* and *Listener*. These two modules are designed to integrate with and the ROS ecosystem but are generally useful outside of ROS [2].

### II. RELATED WORK

The *tf* library is most closely related to the concept of a scene graph. A scene graph is a common type of data structure used to represent a 3D scene for rendering.

Scene graphs are used heavily in visualizers for rendering 3D scenes as well as in robotic simulators for basically the same purpose [3] [4] [5].

Scene graphs typically consist of a tree of objects to be rendered. Every object is attached to a parent object with a position and other information. Depending on the application the other information can range from visualization meshes for pure rendering to update rules and inertial properties for the simulators.

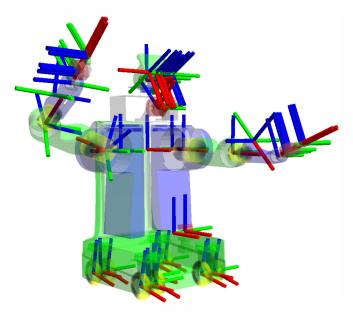


Fig. 1: A view of all the standard *tf* frames in Willow Garage's PR2 Robot with the robot meshes rendered transparently and the edges of the tree hidden. The RGB cylinders represent the X, Y, and Z axes of the coordinate frames. The names of each coordinate frame is suppressed for viewability as well.

One interesting extension of scene graphs is the OSGAR project which sought to leverage the Open Scene Graph project to help with Augmented Reality [6] [7].

The 3D engine OGRE uses scene graphs as well [8]. The *tf* tree has been integrated into the OGRE scene graph in the rviz rendering tool. [9] In Figure 1 you can see the *tf* tree both rendered as elements in the OGRE scene graph, as well as being used to inform the OGRE scene graph of the positions of the meshes of the PR2 body.

During the design phase the tree data structure has proved to be useful for the purposes of *tf*.

## III. REQUIREMENTS

The *tf* library can be separated into two different parts. The first part is disseminating transform information to the entire system. The second part of the library receives the transform information and stores it for later use. The second part is then able to respond to queries about the resultant transform between different coordinate frames.

There are often several different sources of information regarding the various coordinate frames in a system. Each of these sources of information is often connected to hardware and produce data(e.g sensor values, actuator feedback) at different frequencies, and could potentially be connected over a link with non-trivial latency or packet drops. As such the tf library must accept asynchronous inputs and be robust to delayed or lost information.

The *tf* library was designed to be a core library of the ROS ecosystem. To be able to support ROS applications it needed to be robust to distributed computing environments with unreliable networking and non negligible latency. The

design was also influenced by the need to communicate using anonymous publish subscribe message passing.

The library will need to be able to provide a transform between two coordinate frames at a requested time. If data is not available the library must provide the user with an appropriate error and not return invalid data. It also will be built on top of the data distribution and must be robust to the same types of noise.

It was not assumed that the system would have a constant structure, so it should also provide the ability to dynamically change the relationships between frames including adding, removing, and changing connections between coordinate frames.

#### IV. DESIGN

As the requirements were separated so to was the design. The distribution was developed in the *Broadcaster* module.

While the reception and queries for transforms were performed by the *Listener* module.

#### A. Data Structures

Transforms and coordinate frames can be expressed as a graph with the transforms as edges and the coordinate frames as nodes. In this representation the net transform is simply the product of the edges connecting any two nodes. The graph can exist with one or more disconnected subgraphs and the transform can be computed between nodes within the subgraphs, but not between disconnected subgraphs.

Transforms are inherently directed. To traverse up an edge the inverse of the transform can be used. However, with an arbitrary graph, two nodes may have multiple paths between them, resulting in two or more potential net transforms making the result of the query ambiguous. To avoid this the graph must be acyclic.

To provide quick look ups the tree must be quickly searchable. Limiting the graph to a tree enables fast searching for connectivity. This becomes important as graph complexity increases. As an example of a complex tree is the PR2 Robot whose graph can be seen in Figure 2.

The choice of a tree echoes those seen for scene graph development. A difference between the scene graph and the *tf* tree data structure is that the scene graph is designed to be iterated across periodically while the *tf* tree is designed to be queried for specific values asynchronously.

A tree structure also has the benefit of allowing for dynamic changes to the structure without using extra information except the directed graph edges. When an edge is published to a node referencing a different parent node, the tree will resolve to the new parent without extra information.

Each update to the edge of the tree is specific to the time at which it was measured. Likewise, queries against the tree are required to have a specific time at which to make the look up. To make this possible, a history of the values of an edge of the graph is stored in a chronologically sorted list to enable quick look up. In Figure 3 the debugging information shows the 5 seconds of received history for each edge. Data is stored for

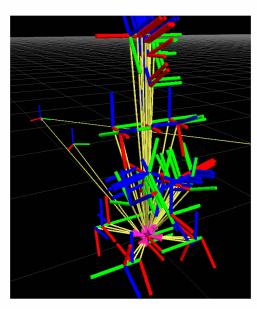


Fig. 2: A view of the standard *tf* tree of the PR2 in the *tf* rviz plugin. Two coordinate frames can be seen in the background showing the "odom" and "odom\_combined" frames from the active navigation task, and the origin of the map is out of the image.

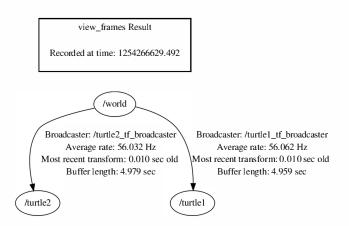


Fig. 3: A simple *tf* tree from two turtles (i.e. two simple virtual robots) in one of the ROS tutorials, with debugging information. The turtles are used as an example in section VI-B for more details.

a specified duration and within that period it can be expected to be able to query for a net transform within the *tf* tree.

To be able to operate, all data which is going to be transformed by the *tf* library must contain two pieces of information: the coordinate frame in which it is represented and the time at which it is valid. These two pieces of data are referred to as a *Stamp*. Data which contains the information in the *Stamp* can be transformed for known data types.

### B. Transform Broadcasting

The *Broadcaster* module was designed very simply. It broadcasts messages every time an update is heard about a specific transform with a minimum frequency.

#### C. Transform Listening

The *Broadcasters* sent updates periodically whether or not they have changed. The *Listener* collects the values into a sorted list and when queried can interpolate between the two nearest values. Because the *Broadcaster* send transforms regularly, the *Listener* does not ever assume the presence of a coordinate frame into the future. The frequency should be selected for the broadcaster which is high enough that spherical linear interpolation (SLERP) can approximate the motion of the joint between the two samples [10]. Although higher frequencies increase accuracy they also incur larger bandwidth requirements and its recommended to use as low a frequency as possible while still exceeding the system's accuracy requirements for cumulative errors.

The interpolation is a critical capability. It allows the publishers to be unsynchronized and to publish at different rates. As long as the update come in frequently enough, SLERP can allow a look up of an arbitrary position in time between two closely spaced samples. Being able to interpolate also helps make the system robust to lost packets. A few lost packets can be interpolated across without losing significant accuracy.

## D. Transform Computation

To compute the transforms between any two nodes the spanning set is computed and then the net transform is computed from that spanning set. To compute the spanning set between a source and target frame, the *Listener* module walks up the edges of the tree until a common parent node is found forming a spanning set. If no common parent is found the look up fails and will return an error. If the look up succeeds the *Listener* will compute the net transform of the edges from the source frame to the target frame along the spanning set. When travelling up an edge in the tree the inverse transform is used, and when travelling down an edge the value of the transform is used.

When computing transforms between two frames, a and c with b in between it is relatively straight forward to chain the transforms together such that:

$$T_a^c = T_a^b * T_b^c \tag{1}$$

#### E. Strengths

The simplicity of the core data structures carries through to the the overall system. It provides advantages in efficiency and flexibility and enables operating in distributed unsynchronized environments while still being debuggable.

1) Efficiency: The design of the tf library enables developers to only broadcast data once, which applies to both the sensor data as well as the transform data. The transform data is broadcast one time by the authority for that transform. For example, many transforms are simply published by the

motor controller. However there are usually other authorities on transform data such as the localization process. Each authority broadcasts the best estimate of the transform once, and all the *Listener* receive the data directly. The single broadcast with direct connections minimizes both bandwidth and latency.

For each message received, a *Listener* stores it in a chronologically sorted list. The design of the system only requires the *Listener* to insert the message into a sorted list on receipt of the data. The minimal number of operations when monitoring is important because when operating a *Listener* will be continuously receiving updates and using limited computational resources.

Keeping the list sorted is important because transform data may be delayed in the transmission. This is most efficient if the data does arrive in order as the list insert will simplify to appending the new transform data, but is robust to out of order data.

To search an arbitrary graph the optimal search time will be O(|E| + |N| log|N|) with E being the number of edges and N being the number of coordinate frames [11]. However, limiting the graph to a tree makes searching for elements O(D) where D is the maximum depth of the tree. The tree also has the benefit that it does not need to do maintenance or analysis of the data structure before before querying it for a result, this enables the Listener to only do the list insert and not analyze the graph regularly.

The *tf* library enables sensor data, or any data with a *Stamp*, to be transmitted in its original frame across the network as well as to be stored in its original frame. When an algorithm wants to use data in the coordinate frame most relevant to the computation, it can query the *tf* library for the transform from the coordinate frame of the *Stamped* data to the desired coordinate frame. Using the resultant transform to transform the data at the time of need prevents unnecessary intermediate transforms, saving both computational time and degradation of data due to repeated processing with potential rounding issues.

For convenience, to get the latest data available, a request at time zero will return the latest common time across the queried values. If such a time does not exist the *Listener* will raise an exception, in the same way as if an unavailable time was queried outside of the cached history.

2) Flexibility: By enabling the user to efficiently store and transporting the data in its raw format tf allows users great flexibility. They are able to change the coordinate frame in which they operate dynamically and also to do things like reprocess logged data in a different coordinate frame than the ones used when collecting the data. This makes doing offline processing of logged data easy, raw sensor data and transform data is all that is needed to be recorded. At playback time, the transform data and raw sensor data is played back. While playing back the recorded data processing algorithms can be rerun with different parameters including operating in any arbitrary coordinate frame including one being generated by modules which were not running when the data was recorded. An example of a coordinate frame being produced during playback is a map frame being produced by a SLAM algorithm in which moving obstacles are being tracked. Using the raw data logged on the vehicle, both the SLAM algorithm as well as the obstacle tracking algorithms can be updated independently while using the same data set.

The *Broadcaster* and *Listener* modules pair well with the anonymous publish subscribe message passing system providing flexibility to developers to add or remove modules. As soon as a new *Broadcaster* module starts publishing transform data, all the *Listener* modules will be connected and their internal buffers will store the transform data.

Likewise any *Listener* module will connect on start-up to all *Broadcaster* modules, and immediately begin building up the buffer to be able to query for resultant transforms. This allows any *Listener* module to start-up without any configuration, but does not require any central coordination. Although the buffer is highly stateful when keeping track of transform data received until the buffer timeout, there is no required configuration for the user. There is one case which the user must be careful about, which is trying to query for transforms immediately after start-up, when the *Listener* module has not yet received full information about the system it will fail to look up. However there are many other reasons that the look up might fail. Robust user code will catch the error anyway and retry, after which the buffer likely will have been filled.

Robustness to network latency is often an issue in distributed systems. When transforming data, the transform data for the specific time at which the data was Stamped must be available. There are two ways in which latency can effect the system. The first case is if the Stamped data is delayed and the transform data is already available. In this case the buffered history of transform data will resolve the issue without further action. The second case is when the Stamped data arrives before the transform data. This case can happen if the Broadcaster is farther away on the network or if the transform needed is actually an intermediate result of another computation requiring processing of data to produce the estimate of the transform data, such as the result of a laser based localization algorithm. To resolve this second case, the Listener module provides both blocking and non-blocking methods to hold the Stamped data until the transform is available. The blocking method simply waits for updates, and the non blocking method will test after receiving each transform data message and provide a callback when the net transform can be computed at the desired time. In many cases a user will expect that extrapolating a small distance into the future will work as an approximation. There is an option to enable extrapolation in tf. However, from testing on the PR2 any extrapolation can lead to extremely unpredictable behavior because the tf library does not have information about the dynamics of the system to accurately extrapolate on the right time scale.

## V. Example Use Cases

The ability to have multiple *Listener* modules is useful in that they can be instantiated both in scripts as well as in libraries without worrying about them colliding when combined. A common use case is for libraries to integrate a *Listener* module inside of them to do their core work, such as the PR2 navigation libraries which among other things transforms sensor data to build a collision map. While at the same time *Listener* modules are being used inside the navigation libraries

another instance can be used in scripting the robot to monitor progress toward a waypoint. When the robot achieves its goal the script monitoring progress can simply destruct the *Listener* module without affecting the instance internal to the library which continues to provide transforms for computing the obstacle maps. This was used heavily during the PR2 office marathon. [12] [13]

Regardless of the number of *Listener* modules the ability to transform data into arbitrary coordinate frames is a powerful tool to simplify tasks. An example of using task specific coordinate frames can be seen in both the door detection and self-plugging demonstrations of the PR2 [13]. In both of these demonstrations, perception algorithms are used to detect fixtures in the world which can then be transformed from the observed frame into the most convenient task frame for the particular application.

Task specific frames can be used even if they do not have physical representation. A use case which has shown this to be the case is simple navigation when localization information is poor. A simple example is to consider the robot driving forward through obstacles and recording them into the map. If the localization of the robot abruptly jumps, such as happens when GPS is reacquired, the obstacles recently recorded as next to the robot may now be represented as intersecting the robot, despite the fact that the robot has not moved significantly since the last observation. This issue can be resolved by having a locally continuous frame which represents the piston of the robot in relationship to the original position based solely on relative updates such as odometry. If obstacles are recorded in this coordinate frame they will be unaffected by updates to the localization. This coordinate frame however will drift over time, requiring the data to expire before the drift accumulates too much. When using this technique the localization process can be changed to publish just the corrections to the drift of the odometry, instead of publishing the position of the robot in the world. This technique was critical to enabling the PR2 to pass through standard doors because it allowed the robot to pass through doorways with less tolerance than the accuracy of its localization [12] [13]. Objects such as goal waypoints were Stamped in the global map frame, however by transforming them into the locally continuous frame the robot can do its path planning in the coordinate frame with the stable obstacles.

The power of being able to dynamically change the structure of the tf tree can be seen in basic table top manipulation tasks [14] [15]. As an example consider the case where a robot approaches a table, picks up an object, places it on its base and drives away. As the robot approaches the object, it may make multiple observations of the object from one or more sensors. From these readings the size and position of the object can be estimated in the world. However when the robot reaches out and grasps the object, it transitions from being attached to the world to being attached to the gripper. By attaching the object to the gripper, it can be added to the collision model as the arm moves collisions with the object can be avoided at the same time as collisions with the arm, even if the object is not observable due to occlusions. When placed down on the base again the attachment should change to be attached to the base of the robot. This allows the robot to accurately model the fact that the object will move with the robot when it starts driving again.

#### VI. EXTENSIONS

There are even more things which can be done with the same infrastructure used to compute net transforms between two coordinate frames at a specified time. Two extensions which have been implemented are support for basic velocity transformations and the ability to compute the current position of a object observed at some point in the past.

#### A. Support for Velocity

To extend support for transforming velocities, the *tf* library takes advantage of the time history of each transform and does discrete differentiation.

$$Velocity_{t,\Delta t} = \frac{Position_t - Position_{t-\Delta t}}{\Delta t}$$

The API provides a parameter to allow the user to choose the time-scale over which to do the differentiation. The choice of this parameter is important for obtaining accurate information. If the time-scale is too small, the noise in the position measurements will dominate. Extending the timescale is effectively applying a low pass filter, so if the velocities of interest are changing faster than the time-scale they will not be measured.

## B. Transforming Data in Time

The other extension of the *tf* library is to be able to transform data in time as well as in space.

Equation 1 is accurate for instantaneous values. However if an observation was made at  $t_0$  and it's now  $t_1$  equation 1 is now missing a term. There is a need to transform between  $t_0$  and  $t_1$ . A transform can be added into the middle of 1 which will transform in time.

$$T_{a@t_0}^{c@t_1} = T_{a@t_0}^{b@t_0} * T_{b@t_1}^{b@t_0} * T_{b@t_1}^{c@t_1}$$
 (2)

With the standard interface to the *Listener* module the first and last elements of equation can be readily evaluated. However,  $T_{b@t_1}^{b@t_0}$  is generally unknown. There is one case where  $T_{b@t_1}^{b@t_0}$  is known and that is the case that the object is stationary in the coordinate frame between  $t_0$  and  $t_1$  then this transform is simply the identity.

Thus by choosing a coordinate frame in which the data is expected to be static. The transform can be accurately computed between two different frames at different times using the following arguments: source frame, source time, fixed frame, target frame, target time.

A concrete example of doing this is if two turtles were walking around, and it was decided that turtle1 should follow 5 seconds behind turtle2. To compute turtle1's goal point in his own coordinate frame you would need to compute the value of Equation 3. An illustration of the transform can be see in Figure 4 and Figure 3 shows an example of the state of the tree when doing this search.

$$T_{turtle1@t}^{turtle2@t-5} = T_{world@t-5}^{turtle2@t-5} * T_{world@t}^{world@t-5} * T_{turtle1@t}^{world@t} \quad (3)$$

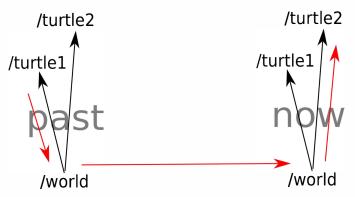


Fig. 4: A simple tf tree from a core ROS tutorial, with debugging information.

Long Term Data Storage: The above example assumes that you have the ability to transform between frames a and b at time 0 and b and c at time 1. This will only work if the length of the buffer in your Listener module is long enough to encompass both time 0 and time 1. To be able to store data for a long time, it should be transformed into the fixed frame, b, when saved. Then the source frame is the same as frame b and consequently the transform between the two is the identity, leaving only  $T_{b@t_1}^{c@t_1}$  needing to be computed to find the current location of any previously observed object. This simple case is common in many robotic systems because they do not have the tools to compute the more complicated cases. This will only work when the assumptions about where the identity transform can be used is maintained.

## VII. FUTURE WORK

Although the *tf* library is an actively used library and the core functionality has remained relatively stable, there is still active development seeking to improve and extend the library. Some of the upcoming challenges will be to optimize *tf* to work better over limited bandwidth links, potentially looking at how to extend *tf* to support partially partitioned or bridged networks and consequently trees. For example, the default setting for the PR2 publishing transforms at 1kHz for 60 frames can take up 3Mbps of bandwidth. This is both linear in the frequency and number of coordinate frames, which uses a large fraction of a standard wifi link under common operating conditions. Decreasing the bandwidth will also help make *tf* easier to use on lower power CPUs, as subscribing to the full stream can take non-trivial CPU load.

A highly requested feature, which is an open problem, is how to integrate uncertainty into a *tf* tree.

There are also lots of software engineering improvements to be made. Fewer software dependencies would be preferable, as well as support for down-sampling data to reduce memory storage and enable longer buffers. Also potentially useful on low bandwidth links is the ability to subscribe to persistent queries about specific transforms remotely.

There also exist a lot of tools for debugging running systems. These tools are useful but also leave room for extension and improvement to make the system more accessible to less experienced developers.

#### VIII. CONCLUSION

The core of how the *tf* library works internally has been presented above. As well as the novel extension to transforming data both in time as well as in space. The library has been adopted by the greater ROS community as the primary way to keep track of positional information. This includes deployment to thousands of robots worldwide. Also presented in the paper are several use cases where the *tf* library has made development much simpler for expert roboticists. The simplicity of using transforms with the library also has made processing sensor data more accessible to many more people.

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