PART 2: Modeling

Exercise 2.1⋆

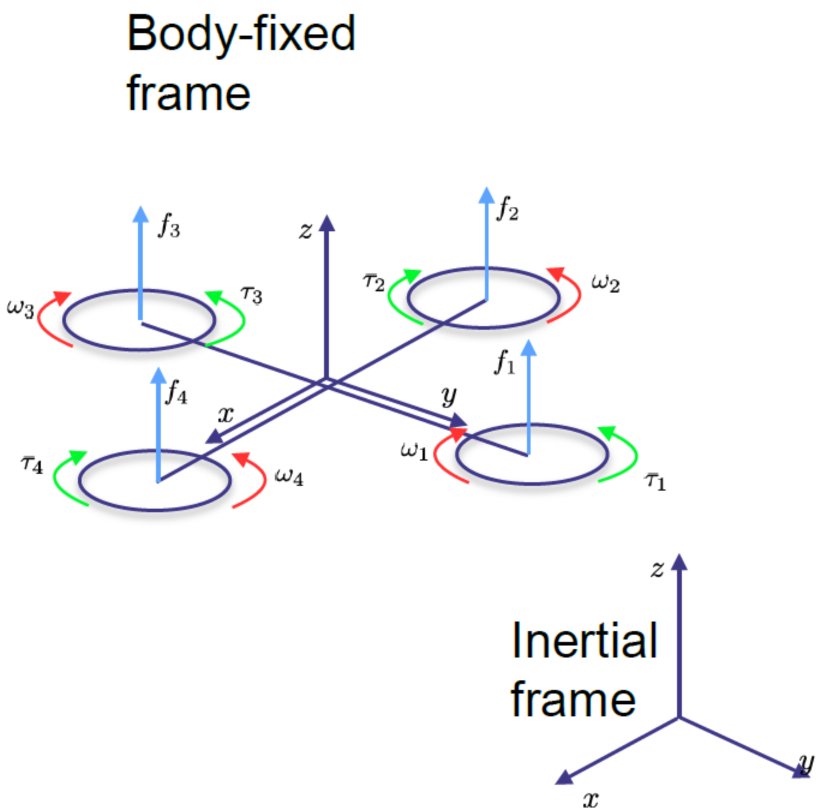

Figure 1: The free-body diagram of the quadrotor UAV.

Given the rigid body diagram as in Figure 1, derive the dynamic equation of the drone, given:

- m=0.5 Kg is the drone mass in the center of gravity of the drone
- L=0.225 m is the length of each arm of the quadrotor frame, i.e. the distance of the motors from the CoG.
- k=0.01 $N\frac{s^2}{rad^2}$ the overall aerodynamic coefficient necessary to compute the lift of the propellers as $f_i = b\omega_i$.
- b=0.001 $Nm\frac{s^2}{rad^2}$ is the overall aerodynamic coefficient necessary to compute the drag torque of the propellers as $\tau_i = b\omega_i$.
- $D = diag([D_x,\ D_y,\ D_z]^T)$ is matrix representing the the drag on the UAV moving in air with velocity $\dot{\mathbf{p}}$, being $D_x = D_y = D_z = 0.01\ Ns/m$
- $Ixx = Iyy = 3e^{-6}\ Nms^2/rad$ are the moment of inertia on the principal axis x and y

- $Izz = 1e^{-5}\ Nms^2/rad$ is the moment of inertia on the principal axis z
- The matrix of moment of inertial of the UAV is $I = diag([I_xx,\ I_yy,\ I_zz]^T)$
- gravity acts along the z-axis of the inertial frame of reference, and the gravitational acceleration $\mathbf{g}$ is given by $\mathbf{g} = [0,\ 0,\ -9.81]^T\ m/s^2$

Please consider the following notation:

- $\mathbf{p}$ [m] is the position of the CoG of the UAV w.r.t. a fixed inertial frame of reference. Let's denote $\mathbf{p} = [x,\ y,\ z]^T$
- Θ [rad] is the representation of the rotation of the body-fixed frame w.r.t. the fixed inertial frame of reference, according to the roll-pitch-yaw angular representation. Let's denote $\Theta = [\phi, \theta, \psi]^T$
- ω [rad/s] is the angular velocity of the body-fixed frame w.r.t. the inertial frame.
- Ω is the vector of the angular speed of the four propellers, $\Omega = [\Omega_1,\ \Omega_2,\ \Omega_3,\ \Omega_4]^T$

1. Define the rotation matrix representing the orientation of the body-fixed frame w.r.t. the inertial frame.

2. Define the relation between the angular velocity $\dot{\Theta}$ and the rotational velocity of the body-fixed frame ω.

3. Write the linear and angular dynamic equation of the drone in compact form, and clearly show each component of the equation explicitly

4. Make a MATLAB/Simulink model of the drone, given initial conditions $\mathbf{p}(0) = [0,\ 0,\ 0]^T$, $\dot{\mathbf{p}}(0) = [0,\ 0,\ 0]^T$, $\Theta(0) = [0, 0, 0]^T$ and $\dot{\Theta}(0) = [0, 0, 0]^T$, and report the following:

- Make a plot of $\mathbf{p}$ and Θ, given $\Omega = [0,\ 0,\ 0,\ 0]^T$ and explain the result
- Make a plot of $\mathbf{p}$ and Θ, given $\Omega = [10000,\ 0,\ 10000,\ 0]^T$ and explain the result
- Make a plot of $\mathbf{p}$ and Θ, given $\Omega = [0,\ 10000,\ 0,\ 10000]^T$ and explain the result

Exercise 2.2

Given the same UAV model as in Exercise 2.1, write the equations using the unit quaternions to represent the rotations and answer the following questions:
1. Define the rotation matrix representing the orientation of the body-fixed frame w.r.t. the inertial frame.

2. Define the relation between the angular velocity Θ and the rotational velocity of the body-fixed frame ω.

3. Write the linear and angular dynamic equation of the drone in compact form, and clearly

show each component of the equation explicitly

4. Make a MATLAB/Simulink model of the drone, given initial conditions $\mathbf{p}(0) = [0,\ 0,\ 0]^T$, $\dot{\mathbf{p}}(0) = [0,\ 0,\ 0]^T$, $\Theta(0) = [0, 0, 0]^T$ and $\dot{\Theta}(0) = [0, 0, 0]^T$, and report the following:

- Make a plot of $\mathbf{p}$ and Θ, given $\Omega = [0,\ 0,\ 0,\ 0]^T$ and explain the result
- Make a plot of $\mathbf{p}$ and Θ, given $\Omega = [10000,\ 0,\ 10000,\ 0]^T$ and explain the result
- Make a plot of $\mathbf{p}$ and Θ, given $\Omega = [0,\ 10000,\ 0,\ 10000]^T$ and explain the result

Exercise 2.3

Using the model in Exercise 2.1 (or 2.2), linearize the dynamic model of th eUAV in hovering conditions. Compare the linearized model with the non-linear one under the same input conditions as in previous exercises (2.1 and 2.2 if solved):

- Make a plot of $\mathbf{p}$ and Θ, given $\Omega = [0,\ 0,\ 0,\ 0]^T$ and explain the result
- Make a plot of $\mathbf{p}$ and Θ, given $\Omega = [10000,\ 0,\ 10000,\ 0]^T$ and explain the result
- Make a plot of $\mathbf{p}$ and Θ, given $\Omega = [0,\ 10000,\ 0,\ 10000]^T$ and explain the result