Perpetual Demand Lending Pools

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May 27, 2025

Abstract

Decentralized perpetuals protocols have collectively reached billions of dollars of daily trading volume, yet are still not serious competitors on the basis of trading volume with centralized venues such as Binance. One of the main reasons for this is the high cost of capital for market makers and sophisticated traders in decentralized settings. Recently, numerous decentralized finance protocols have been used to improve borrowing costs for perpetual futures traders. These protocols have grown to over \$2.5 billion dollars of assets while generating over \$890 million in fees in 2024. We formalize this class of mechanisms utilized by protocols such as Jupiter, Hyperliquid, and GMX, which we term *Perpetual Demand Lending Pools* (PDLPs). We then formalize a general target weight mechanism that generalizes what GMX and Jupiter are using in practice. We explicitly describe pool arbitrage and expected payoffs for arbitrageurs and liquidity providers within these mechanisms. Using this framework, we show that under general conditions, PDLPs are easy to delta hedge, partially explaining the proliferation of live hedged PDLP strategies. Our results suggest directions to improve capital efficiency in PDLPs via dynamic parametrization.

1 Introduction

Perpetual futures markets are the most liquid trading markets for cryptocurrencies. These markets facilitate daily volumes on the order of hundreds of billions of dollars of notional value and are the primary drivers of hedging and leverage. Although the majority of perpetual futures volume remains on centralized trading venues, such as Binance or Coinbase, a growing amount of perpetuals trading occurs on decentralized venues. Such venues, including Hyperliquid, Jupiter, GMX, and dYdX, have grown to capture almost 10% of the daily perpetual future volume [Blo24].

Although decentralized venues like dYdX have been live since 2019, the failure of FTX in 2023 has driven demand for decentralized venues. Exchanges that hold the state of all

balances and positions on a public blockchain can ensure that user funds are not misappropriated. However, these decentralized exchanges face a number of challenges that their centralized counterparts do not. The public nature of positions can lead to front-running and other forms of manipulation. In addition, the lack of identity verification (which permits traders to walk away from bad positions) leads to higher collateral requirements for margin trading.

Market maker loans. To promote liquidity on their platforms, centralized exchanges often offer market makers loans that can only be utilized on their exchange. In general, these loans are only partially-collateralized and, as a result, present risks for other users of the platform. For example, FTX used customer funds to offer a market maker loan to Alameda Research that did not require any collateral [DeK23, All22, Bre24]. Decentralized platforms, by construction, cannot lend out customer funds without their consent. However, these platforms have higher collateral requirements and must turn to other mechanisms to provide market maker loans.

Perpetual Demand Lending Pools. Decentralized perpetual future exchanges have created novel liquidity pools that lend to traders on their platform. GMX [Tea24] first pioneered these pools, which borrow aspects from both decentralized spot trading (e.g., Uniswap [AKC+21]) and decentralized lending (e.g., Aave and Compound [KCCM20]). A number of other protocols including Jupiter [Jup24] and Hyperliquid [Hyp24b] have since followed suit. These pools allow traders to borrow assets for only one purpose: to open positions on the associated perpetuals exchange. The protocol liquidates positions as soon as they become undercollateralized. Since these loans resemble demand loans [JM80], we call these mechanisms Perpetual Demand Lending Pools (PDLPs).

PDLP mechanics. Although perpetual demand lending pools have different implementation details with regard to how their lending functions operate, they share common traits:

- Users pool together funds into a liquidity pool. The protocol only permits certain assets in this pool and maintains a target composition.
- Traders, including market makers, borrow from the pool to open positions on the associated exchange.
- LPs earn fees from traders who borrow from the pool. These fees are proportional to the size of the position and are paid at regular intervals.
- Arbitrageurs ensure the pool stays at the protocol's desired target composition.
- LPs, rather than the protocol itself, realize losses when the pool cannot rebalance or when the protocol cannot liquidate underwater positions quickly enough.

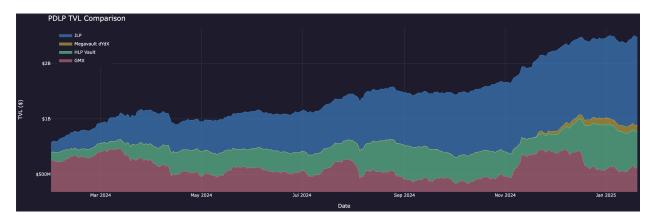


Figure 1: Total Value Locked for PDLPs (*i.e.* assets locked into PDLP pools) in 2024 (Link To Data).

Background on existing PDLPs. PDLPs have amassed over \$2.5 billion in assets and generated roughly \$897.73 million dollars in aggregate fees¹ since GMX launched in September 2021. Jupiter's JLP pool and Hyperliquid's HLP pool followed, each making changes to the original GMX PDLP mechanism. In the last year, the total value of assets in PDLPs has grown from under \$750 million to over \$2.5 billion, and these positions have yielded a return of approximately 40% in 2024. We note that the rising popularity of PDLPs has tracked the growth of perpetuals trading on decentralized exchanges.

Delta hedging. PDLP LPs have had success delta hedging in practice. These LPs, who provide assets to the pool, can take derivative positions to offset their exposure to risky asset price movement (the delta) while still capturing fees. This property allows LPs to enter and exit positions without realizing large convexity losses due to volatility [HW17] and, as a result, facilitates cheaper market making [HLM15, SS09]. We note that many previous attempts in decentralized finance to provide delta-hedged portfolios have been unsuccessful in practice (e.g., see [AEC23, KC22, LK22]). This observation begs the question: Why are PDLPs easier to hedge? We characterize PDLP mechanics and shed light on this question in this paper.

1.1 This paper.

We provide the first formalized (to the authors' knowledge) definitions and description of PDLPs. We aim to formalize the financial properties of these pools and explain their large amount of fee revenue.

In §2, we define PDLPs and describe how perpetuals exchanges use these pools. We then describe single-period arbitrage of the perpetual funding rate—the core mechanism that ensures the futures contract and the spot asset have consistent prices. We also describe arbitrage opportunities in the PDLP itself, which are similar to those in constant function

 $^{^{1}}$ As of December 27, 2024, the cumulative fees generated by each protocol are: Jupiter (\$445.48 million [ile24]), dYdX MegaVault (\$4.25 million [Gau24a]), HLP (\$58 million [Hyp24a]), GMX v1 + v2 (\$390 million [gmx24b])

market makers. We give necessary conditions on the PDLP fee so that both the funding rate arbitrageur and the PDLP liquidity providers are profitable. The arbitrage loss faced by LPs resembles loss-versus-rebalancing (LVR) from the CFMM literature [MMRZ22].

In §3, we formalize mechanisms used by PDLPs to stay near a target portfolio, which are usually specified in terms of desired relative weights, These mechanisms, called target weight mechanisms, provide economic incentives for liquidity providers to add or remove assets so that the PDLP maintains a target portfolio. Unlike CFMMs, PDLP liquidity providers may deposit (or withdraw) any subset of assets to (or from) the pool. This feature creates novel opportunities for arbitrageurs which resemble create-redeem arbitrage in exchange traded funds [PZ17]. We model these arbitrage opportunities as optimization problems, which can be approximately solved in practice. Using this model, we demonstrate that the target weight mechanism bounds an LP's delta exposure within in PDLPs. This fact may explain the proliferation of successful PDLP hedging strategies, especially for those utilizing Jupiter's JLP [Gau24c]. We note that hedging LVR in CFMMs is possible, but these protocols have far less usage in practice [Arj24, LLS24, LK22].

Finally, in §4, we describe dynamic delta hedging of risk assets in a PDLP using meanvariance optimization. We give simple sufficient conditions for when the delta hedge improves the Sharpe ratio of PDLP LP returns. We also describe when a single PDLP pool should be split into two pools to improve the delta-hedged Sharpe ratio.

Notation. We denote the unit simplex as $S^n = \{(w_1, \dots, w_n) \in \mathbf{R}_+^n : \sum_i w_i = 1\}$. We use \mathbf{R}_+^n for the nonnegative orthant and \mathbf{R}_{++}^n for the positive orthant in n dimensions. We denote set of natural numbers from 1 to k is denoted as $[k] = \{1, 2, \dots, k\}$. The L^p norm of a vector $x \in \mathbf{R}^n$ is denoted as $||x||_p = (\sum_i |x_i|^p)^{1/p}$.

2 Perpetual Demand Lending Pools

In this section, we formalize a model of Perpetual Demand Lending Pools (PDLPs). Note that we ignore implementation details of practical mechanisms that do not impact economic payoffs. We begin by describing a simplified linear model of a perpetuals exchange. We refer the interested reader to [AHJ24] for models that account for additional parameters found in practice.

2.1 Perpetuals exchange

A perpetuals exchange facilitates perpetual future contracts between two groups of users: those holding long positions and those holding short positions in some underlying asset. In these contracts, the group of users with the larger cumulative position pays a fee, called the funding rate, at regular intervals to the other group. This fee incentivizes the two positions to be roughly equal. Arbitrageurs who observe an imbalance between the long and short positions can equalize the positions and earn a profit if the price moves in the same direction as the imbalance.

Concretely, we define a perpetual future contract for some asset as a triple $(L, S, p_0) \in \mathbf{R}^3_+$, where L and S are respectively the cumulative long and short positions, and p_0 , is the mark

price of the underlying asset, which is set at the beginning of each time interval. The funding rate is a function of these three parameters, and of the price of the underlying asset at the end of this interval. We can interpret the mark price p_0 as the price at which two parties enter into the perpetual contract to bet on a price change during the time interval. (For example, large centralized exchanges such as Binance, OkEx, and Coinbase update this mark price every eight hours [AHJ24].) Arbitrageurs will ensure that, at the beginning of the next interval, the mark price is equal to the price of the underlying asset. A perpetuals exchange contains perpetual future contracts for some collection of $n \in \mathbb{N}$ assets.

Trades. A user trades on a perpetuals exchange by placing collateral on the exchange and then opening a long or short position in some asset. We will assume, for simplicity, that the collateral is a numéraire (such as US dollars) whose price at all times is 1. A *trade* is defined as a tuple $(c, \delta, \eta, p_0) \in \mathbf{R}_+ \times \mathbf{R}_+ \times (\mathbf{R} - \{0\}) \times \mathbf{R}_+$ where c is the collateral the user places, δ is the position size, η is the leverage level, and p_0 is the price when the trade is created. A position is a short position if $\eta < 0$ and has leverage if $|\eta| > 1$. At the time of creation, the position requires collateral c that satisfies the *collateral condition*:

$$p_0 \delta < |\eta| c. \tag{1}$$

This means that a user can create a notional position with size $p\delta$ that is up to $|\eta|$ times larger than the collateral they deposit.²

Liquidations. A trader must ensure that the trade has sufficient collateral as the mark price of the risky asset p fluctuates, otherwise the exchange will liquidate this position. Define the $liquidation\ condition$ as

$$\operatorname{sign}(\eta)\delta(p_0-p) \geq c.$$

In words, if the the loss in the perpetuals position due to price change is greater than the collateral value, the position will be liquidated. For example, a long position is liquidated once the price of the risky asset drops below $p_0 - c/\delta$. If this position was created with minimal collateral, saturating (1), then the position is liquidated when the price drops below $p_0(1-1/\eta)$.

Example. Consider a trade $T = (c, \delta, \eta, p_0)$ with collateral c = \$2000, position size $\delta = 1$ ETH, leverage $\eta = 4$ and initial ETH price $p_0 = \$2000$. The initial notional value of the position, δp_0 , is 8,000 USD. This trade satisfies the collateral condition (1). As the price of ETH p decreases, the position remains valid until the liquidation condition is met, which occurs when p = \$1,5000. Once this price is reached, the perpetuals exchange will liquidate this position.

²This condition is what is often meant when the claim that perpetual futures are more capital efficient for leverage than traditional lending protocols is made, see *e.g.*, [AHJ24].

Funding rate. We consider the linear funding rate

$$\gamma_L(L, S, p, p_0) = \kappa \left(\frac{L}{S} - \frac{p}{p_0}\right), \tag{2}$$

where p is the price of the underlying asset and $\kappa > 0$ is a constant. Note that on-chain perpetuals exchanges typically use a price oracle to determine the price p. This price oracle is often (but not always) the median of the prices on a set of centralized exchanges with sufficient liquidity. In practice, most exchanges use the linear funding rate model, although a number of other models exist in the literature (for examples, see [AHJ24, ACEL23, HMRvW22]). If the funding rate is positive ($\gamma_L > 0$), then the users with short positions pay those with long positions, and if the funding rate is negative ($\gamma_L < 0$), then the users with long positions pay those with short positions.

Example. Consider an ETH-USD perpetual futures contract (L, S, p_0) with cumulative long position L = 1000 ETH, short position S = 250 ETH, and mark price $p_0 = 2000$ USD, the funding rate is $\gamma_L = \kappa \left(\frac{L}{S} - \frac{p}{p_0}\right)$. When $p = p_0$, including at the start of the contract period, this equation simplifies to:

$$\gamma_L = \kappa \left(\frac{1000}{250} - 1 \right) = 3\kappa$$

Here, longs pay shorts, which incentivizes traders to take short positions until $\gamma_L = 0$.

2.2 Perpetual Demand Lending Pools

A perpetual demand lending pool allows traders to borrow assets to open positions on an associated perpetuals exchange. Yield-seeking depositors, called *liquidity providers* (LPs), deposit these assets into the pool. Traders can then use these assets to collateralize their positions on the perpetuals exchange, for which they pay fees to the liquidity providers. Since most of these positions are levered, liquidity providers effectively provide under-collateralized loans which only may be used for trading on the associated perpetuals exchange.

Definition. Formally, we specify a Perpetual Demand Lending Pool (PDLP) by its current portfolio $R \in \mathbf{R}_+^n$, target portfolio $\pi \in \mathbf{R}_+^b$, lending fee $f \in (0,1)$, and outstanding loans $\{c_i\}_{i\in[M]}$, where each loan c_i is the collateral used by position $i=1,\ldots,M$. Note that one can view the loans made by the PDLP as collateralizing an equal but opposite position to the one desired by the trader, much as a market maker takes the opposite size of a leveraged trade. The trader associated with position i must pay a fee fc_i for each time period that they hold a loan. We note that the target portfolio and fees may be dynamically updated based on external market conditions. The total value lent to traders must be less than the total value of the pool's assets, i.e.,

$$\sum_{i=1}^{M} c_i \le R.$$

If any position i becomes invalid, the protocol liquidates this position and returns the collateral c_i back to the PDLP.

Dynamics. Using the definitions above, we model how trades interact with the perpetuals exchange and how LPs utilize the PDLP via a fixed sequence of transaction execution. First, note that we assume that there exists an unmanipulable price oracle that submits a price update for the n assets at each block. The sequence of interactions executed by a solvent, collateralized PDLP is:

- 1. The price oracle is updated.
- 2. Liquidatable positions are removed.
- 3. Funding rates, based on the positions opened at the previous period and the oracle price, are paid out.
- 4. Fees for loans used in the previous period are paid out to LPs from trader positions.
- 5. LPs update their PDLP portfolios.
- 6. Traders submit new trades and valid trades are executed.

In Appendix A, we formalize these dynamics in terms of the state variables defined so far. We also note that the dynamics here do not include any constraints on updates to the target portfolio since different protocols use different mechanisms to update the target portfolio. We study the mechanism used by GMX and Jupiter to adjust and/or realize a target portfolio in §3.

Liquidity creation and redemption. When a liquidity provider (LP) deposits assets into a PDLP, they receive tokens which entitle them to a pro-rata claim on the pools' portfolio and fees. An LP may deposit or withdraw any subset of assets into the PDLP. Note that this mechanism differs from that of a constant function market maker (CFMM), where LPs generally must deposit or withdraw all assets and/or define a price range over which their liquidity can be used. (See [AAE+22, BOM23, FMCM+21] for details.) PDLPs also allow swaps between the assets in the pool, similarly to a CFMM. Unlike a CFMM, a PDLP typically sets swap prices using an external price oracle and charges a dynamic fee chosen to maintain the PDLP's target portfolio. LPs can create a share of a PDLP, entitling them to a pro-rata claim on the pool's assets and fees, by tendering any of the assets held by the PDLP. Similarly, they can redeem a PDLP share for any valid portfolio that has the same value as the share.

2.3 Arbitrage in a single period

We consider two arbitrage opportunities created by a price movement in the underlying asset: funding rate arbitrage and PDLP share arbitrage. We will show that both the pool liquidity providers, who deposit assets prior to the funding rate change, and funding rate arbitrageurs are profitable as long as the PDLP fees remain inside a particular interval.

Assume that, at the beginning of the period, the price of the underlying asset is p_0 and the funding rate is zero, so long and short positions have the same size: $L = S = L_0$ (see (2)). When the price of the underlying asset moves from p_0 to p, this price movement opens two arbitrage opportunities:

- 1. A funding rate arbitrageur borrows from the PDLP pool and opens a long or short position to capture the funding rate until the mark price and true price converge.
- 2. A PDLP arbitrageur creates or redeems shares of the PDLP to capture the spread between the value of the PDLP's assets and the LP token's spot price.

We describe each of these opportunities and bound the loss of the PDLP. We assume that $p > p_0$, but an entirely symmetric derivation gives the case when $p < p_0$.

2.3.1 Funding rate arbitrage.

The funding rate (2) after price movement is

$$\gamma_L(L_0, S_0, p, p_0) = \kappa \left(\frac{L_0}{S_0} - \frac{p}{p_0}\right) = \kappa \left(1 - \frac{p}{p_0}\right) < 0.$$

Thus, the users with short positions must pay those with long positions. A funding rate arbitrageur opens a long position of size ℓ to capture this funding rate prior to the next funding payment made by the PDLP. This arbitrage exists as long as the funding rate continues to be non-positive; thus, the largest long position that can capture this funding rate (i.e., the ℓ that makes $\gamma_L(L_0 + \ell, S_0, p, p_0) = 0$) is given by

$$\ell = L_0 \left(\frac{p}{p_0} - 1 \right). \tag{3}$$

After opening this position, the arbitrageur receives a pro-rata share of the funding rate: $\ell/(L_0 + \ell) \cdot |\gamma_L(L_0, S_0, p, p_0)|$. To open this position, the arbitrageur must pay a fee $f \cdot \ell$. Thus, the arbitrageur makes a profit when the fee is less than the revenue, *i.e.*

$$f\ell \le \frac{\kappa\ell}{L_0 + \ell} \left(\frac{p}{p_0} - 1\right) = \frac{\kappa\ell}{L_0} \left(1 - \frac{p_0}{p}\right)$$

where we utilized (3) in the equality. If the relative price increment is bounded, $1 < \frac{p}{p_0} \le B$, then we have that

$$f \le \frac{\kappa}{L_0} \left(1 - \frac{p_0}{p} \right) \le \frac{\kappa (1 - B^{-1})}{L_0}. \tag{4}$$

This means that a fee that is inversely proportional to the size of the cumulative long position (the open interest) ensures that the funding rate arbitrage is profitable. As the open interest increases, this fee must decrease (or the funding rate must increase) to offset the increase in arbitrage size.

Discussion. We note that the assumption $1 \leq \frac{p}{p_0} \leq B$ is not restrictive when compared to other forms of optimal fees (e.g., the assumptions of LVR with fees [MMR23]). If one has a model for the price process (such as a geometric brownian motion, which is used in LVR), one can use Chebyschev's inequality to get a bound of the form $\mathbf{Prob}[\frac{p}{p_0} \leq B] \leq f(\sigma)$ for an increasing function f of the price process volatility σ . In short, one can turn a price process into a pointwise bound $\frac{p}{p_0} \leq B$ that holds with high probability.

2.3.2 PDLP arbitrage.

The same price movement also introduces a discrepancy between the price at which the PDLP is willing to exchange assets and the external market price. An arbitrageur may buy the asset at the old price p_0 (which need not be the mark price) on the PDLP and sell it on an external market for the new, higher price p_0 . The price impact of this trade on both markets and the associated fees bound the size of this trade. For simplicity, we focus only on the price impact of buying on the PDLP. (We will return to the PDLP's fees in §3.) We model the PDLP's swap functionality using the forward exchange function $G: \mathbf{R}_+ \to \mathbf{R}_+$, which is nonnegative, concave, and nondecreasing [AAE+22]. This function denotes, for a quantity x of the numéraire, the amount of the asset, G(x) that the PDLP is willing to sell. We assume that G(0) = 0 (no free lunch) and that the PDLP initially quotes at the old price: $G'(0) = 1/p_0$. If the arbitraguer uses x units of the numéraire to purchase the asset on the PDLP, their profit is

$$pG(x) - x$$
.

When G is differentiable, the most profitable trade easily follows from the first order conditions:

$$G'(x^{\star}) = 1/p$$

The net change in the PDLP's value is then

$$\underbrace{x^{\star}}_{\text{numéraire in}} - p \cdot \underbrace{G(x^{\star})}_{\text{asset out}}.$$

This arbitrage is equivalent to the standard price arbitrage in a CFMM [AC20], and the LP loss is analogous to loss-versus-rebalancing in CFMMs [MMRZ22].

2.3.3 Choosing fees

After arbitrage, PDLP LPs have earned a fee from the PDLP borrow but also suffered a rebalancing loss from the pool. Their profit is

$$\mathsf{Profit} = \underbrace{f\ell}_{\text{fee revenue}} + \underbrace{x^{\star} - pG(x^{\star})}_{\text{rebalancing cost}}.$$

Note that the second term is negative since

$$x - pG(x) \le x - pG'(0)(x - 0) = x(1 - p/p_0) < 0,$$

for any x, where the first inequality follows from the definition of concavity and the second from the fact that $p > p_0$. Using (3) and the definitions above, this profit is nonnegative whenever

$$f \ge \frac{1}{L_0} \cdot \frac{G(x^*) - x^*/p}{G'(0) - 1/p}$$
 (5)

Using the definition of concavity, we have the following sufficient condition for the fee lower bound (5) to hold:

$$f \ge \frac{x^*}{L_0}$$
.

In other words, LPs are profitable as long as there is sufficient liquidity and the PDLP swap functionality, described by the price impact function, ensures the arbitrage size is sufficiently small. A stronger condition follows from additional assumptions on the forward exchange function G (for example, strong concavity).

Combining the upper bound (4) and lower bound (5), we have conditions for fees that ensure both LPs make a profit and arbitrageurs arbitrage the funding rate, which balances the long and short positions. Specifically, if price changes are bounded then the PDLP can set parameters so that fees inversely proportional to the open interest (i.e., $f = \Theta(1/L_0)$) ensure that both LPs and funding rate arbitrageurs are profitable. This result suggests that PDLPs can have a sustainable equilibrium between traders and LPs under mild conditions. Moreover, these results suggest that PDLPs with dynamic fees (i.e., where the fee f depends on the cumulative long and short positions) are more likely to realize this equilibrium.

Example. Consider a PDLP with a forward exchange function that resembles Uniswap v2:

$$G(x) = \frac{R_2 x}{R_1 + x}.$$

The forward exchange rate is $G'(x) = R_1R_2/(R_1 + x)^2$, and the pool initially quotes the original price: $R_2/R_1 = 1/p_0$. The LP loss can be calculated as

$$R_1 + pR_2 - 2\sqrt{pR_1R_2}$$

Using the bounds on the price change, we can show that the LP is profitable as long as

$$f \ge \frac{(B-1)R_1}{L_0}.$$

The fee must increase as the size of the PDLP increases, but it may decrease as the amount of open interest increases.

3 Weight-based arbitrage

Perpetual demand lending pools typically have a target portfolio that they aim to maintain. For example, GMX's GLP and Jupiter's JLP pool aim to maintain a constant relative composition of assets. These target portfolios ensure the lending pools stay diversified and maintain inventory across the different assets used for collateral. To maintain these target portfolios, PDLPs provide economic incentives to liquidity providers and traders to rebalance the pool. We describe the mechanism for distributing these incentives, which resemble PID controls, and the associated arbitrage problems in this section.

3.1 Target Weight Mechanisms

A target weight mechanism (TWM) for a PDLP takes a target portfolio and attempts to minimize the deviation between the PDLP's current portfolio and this target portfolio. The PDLP constructs this target portfolio to limit its exposure to a single asset or a small set

of correlated positions,³ and this target portfolio also ensures that liquidity providers add diversified assets to the pool to meet demand. Typically, PDLPs specify this portfolio in terms of a target price-weighted relative asset composition. We consider this case here, but our analysis easily extends to considering the target portfolio directly.

Utilization. Unlike a CFMM, the PDLP has some set of assets that cannot be redeemed: those pledged as collateral for a perpertual future position. We say that these assets are utilized. We define the available portion of the pool, R^A , as

$$R^A = R - \sum_{i=1}^M c_i.$$

By construction, this vector is nonnegative; the pool itself does not use leverage.

Target weight. Given a PDLP with reserves $R \in \mathbf{R}_{+}^{n}$ and asset prices $p \in \mathbf{R}_{++}^{n}$, the weight of a PDLP is the price-weighted relative composition of the assets in the pool:

$$w(p,R) = \frac{p \odot R}{p^T R},$$

where \odot is the element-wise (Hadamard) product. We normalize by the portfolio value, p^TR . The PDLP's TWM attempts to minimize the deviation between this weight and a target weight w^* by providing economic incentives to arbitrageurs to add or remove assets⁴. By providing these incentives, the TWM essentially aims to (indirectly) solve the optimization problem

minimize
$$\|w(p, R + \Delta) - w^*\|$$
,
s.t. $\Delta \ge -R^A$, (6)

with variable $\Delta \in \mathbb{R}^n$ denoting the assets to be added or removed from the PDLP. The constraint indicates that the amount removed cannot be in excess of the unutilized assets \mathbb{R}^A . The act of providing incentives to approximately solve (6) resembles both the Hedge Algorithm [FS97] and other online learning algorithms on the simplex [HK16]. We suspect that weight update rules could be designed to be no-regret, as they are in resource markets in [ADM24, DECA23]. We leave formalizing this connection to future work.

Incentives and the discount rate. PDLPs give liquitity providers of underweight assets a discounted pro-rata ownership of the PDLP. (Equivalently, these depositors receive a subsidy, denominated in the pool's portfolio, for depositing these underweight assets.) Let $\Delta \in \mathbb{R}^n$ denote an LP's proposed change to the PDLP reserves. If this update is valid, *i.e.*, $-\Delta \geq R^A$, then the PDLP's reserves are updated to $R + \Delta$ and this LP recieves the pro-rata ownership in the pool

$$(1 + F(p, w^*, R, \Delta)) \cdot \frac{p^T \Delta}{p^T (R + \Delta)},$$

³We suspect that the PDLP can choose this target portfolio with approximation algorithms akin to those proposed for decentralized lending [BNW⁺24, NKV24, BBGD⁺24].

⁴The target weight is typically set to reflect expected loan quantities. See, for example, [Gau24d].

with discount rate $F: \mathbf{R}_{+}^{n} \times S^{n} \times \mathbf{R}_{+}^{n} \times \mathbf{R}^{n} \to \mathbf{R}$. LPs who move the weights closer to the target receive a discount on the pool share price, i.e., F > 0. In particular, we assume the following conditions on F hold:

- There is no discount for a zero trade: $F(p, w^*, R, 0) = 0$ for all p, w^*, R .
- The discount rate F is concave in its last argument Δ .
- The discount rate is maximized by trades that achieve the target weights: when $\Delta \in \operatorname{argmax}_{\delta} F(p, w^{\star}, R, \delta)$, we have $w(p, R + \Delta) = w^{\star}$ and $w(p, R) \neq w^{\star}$.

The first condition states that there is a constant discount at all prices, weights, and reserves for an empty trade. The second condition states that the discount rate has constant or diminishing growth. And the final condition states that the discount is maxmized for trades that achieve the target weight. Existing PDLP LPs implicitly pay the new pool LP via dilution when F > 0, and this new LP dilutes the existing LPs by a factor of 1/(1+F). (Alternatively, existing LPs receive a subsidy when F < 0.)

Target weight arbitrage. TWMs rely on discount rate arbitrage to ensure that PDLPs remain near their target weights. The discount rate F must not only incentivize arbitrauers to solve (6) but also not excessively dilute existing LPs. This arbitrage mechanism resembles that of exchange-traded funds with target portfolios within traditional finance (see [Pet17] and citations within). However, the TWM introduces a variable create-redeem fee that can introduce arbitrage opportunities even when the price of the PDLP share does not differ from the value of the pool's assets. We construct the relevant optimization problem in the case of share creation and of share redemption.

Example. Here, we walk through a simple example to demonstrate the benefits of the target weight mechanism. Consider a PDLP with two assets, the numéraire and a risky asset, that are equally weighetd. Assume that all of the risky asset is being used as collateral for perpetual positions. What happens when the price of the risky asset changes? If the risky asset decreases in price, the PDLP will be underweight the risky asset. Arbitrageurs will be incentivitized to add more of this asset to the pool, which will allow for more positions that use the risky asset as collateral to be opened. On the other hand, if the risky asset increases in price, the PDLP will be overweight the risky asset. Arbitrageurs can either close positions in the money, redeeming the risky asset, or add numéraire to the pool.

3.2 TWMs in practice

Here, we review several of the most popular target weight mechanisms used in practice. We note that PDLPs often have differing requirements for which colleteral traders may use for which positions.

GMX GLP pool and Jupiter JLP pool. GMX introduced the first PDLP, launched on the Arbitrum blockchain, in September 2021 [Tea24]. This PDLP provided incentives for users to maintain a target weight: changes to the pool that brought the asset composition closer to the target received a rebate, whereas changes that pushed the asset composition further from the target incurred a fee. As implemented, the mechanism resembles a simple proportional-integral-derivative (PID) controller mechanism that requires arbitrageurs to implement the control policy [xvi23]. Jupiter's JLP pool introduced a similar mechanism but with changes to reduce the funding rate risk borne by LPs using delta-hedge strategies. As of December 2024, GMX's PDLP has approximately \$650 million in assets [DeF24a] and supports \$10 million in open interest (dollar notional value of open positions). Jupiter's PDLP is significantly larger, with roughly \$1.5 billion of assets and \$870 million of open interest [Gau24b].

Hyperliquid HLP pool. Hyperliquid, a large exchange with over \$3 billion of open interest in December 2024, similarly uses a PDLP but with a much more opaque mechanism. The Hyperliquid PDLP uses a closed-source strategy, managed by a single whitelisted entity, to adjust its target portfolio and lend to traders [Hyp24b]. This PDLP still allows for permissionless liquidity provision and distributes its rewards amongst the LPs. As of December 2024, the Hyperliquid PDLP holds approximately \$350 million of assets [DeF24b].

dYdX MegaVault. The dYdX MegaVault [dYd24] PDLP uses a fixed, open-source market making strategy (see [tqi24] for details). This strategy depends on certain parameters, which are periodically adjusted by a single whitelisted actor. These public strategies are similar to classical market-making strategies [AS08] and generally perform well in lower-frequency and liquidity markets. As of December 2024, the dYdX MegaVault PDLP holds approximately \$65 million of assets [Gau24a].

3.3 Share creation

When there is a positive discount, F > 0, an arbitraguer can create a PDLP share worth more than the assets she provides to the PDLP. Recall from §2.2 that, an LP who deposits a portfolio $\Delta \in \mathbb{R}^n_+$ receives a pro-rata ownership in the PDLP worth

$$(1 + F(p, w^*, R, \Delta)) \cdot \frac{p^T \Delta}{p^T (R + \Delta)} \cdot V(p, R + \Delta) = (1 + F(p, w^*, R, \Delta)) \cdot p^T \Delta.$$

This costs the LP $p^T\Delta$ in assets. Thus, the maximum value arbitrage is given by the solution to the optimization problem

maximize
$$F(p, w^*, R, \Delta)(p^T \Delta)$$
, s.t. $\Delta \ge -R^A$. (7)

It is clear that share creation is only rational for the arbitrageur when there is a positive discount. We claim that if F is an admissible discount rate and is concave in Δ and decreasing, then this optimization problem's maxima can be approximated when F is strongly concave.

Approximate optimum. Often, PDLPs do not disclose the exact form of the discount function F. Instead, arbitrageurs have black-box access to the function F and can query the discount for a given trade size δ . We demonstrate that, under certain assumptions on F, arbitrageurs can approximately solve the creation arbitrage problem (7) with only this black-box discount access. This fact suggests that arbitrageurs can still successfully trade on closed-source PDLPs, such as Jupiter's JLP.

Surrogate function. Suppose that F is μ -strongly concave [BV04, §9.1] in δ . That is, if we fix p, w^*, R and let $f(\delta) = F(p, w^*, R, \delta)$, then $f(\delta)$ satisfies

$$f(\delta) \le f(\delta') + \nabla f(\delta')^T (\delta - \delta') - \frac{\mu}{2} \|\delta - \delta'\|_2^2$$
(8)

for all δ' in the domain of f. Moreover, since $F(p, w^*, R, 0) = 0$ for all p, w^*, R , we have that

$$f(\delta) \le g^T \delta - \frac{\mu}{2} \delta^T \delta = h(\delta)$$

for any subgradient $g \in \partial F(p, w^*, R, 0)$. This also implies that $f(\delta) \leq g^T \delta$ for any $g \in \partial f(0)$. We define $\delta_H^* = \min_{\delta} h(\delta)$, which can be easily computed as

$$\delta_H^{\star} = \frac{1}{\mu}g$$

Using the strong convexity of $f(\delta)$, we can show that δ_H^* is close to $\delta_F^* \in \operatorname{argmax}_{\delta} f(\delta)$, as illustrated in the following claim:

Claim 3.1. Suppose $f(\delta) = 0$, $f(\delta)$ is μ -strongly concave, and $\max_{\delta} \|\nabla f(\delta)\|_2^2 \leq G$. Then

$$\|\delta_F^{\star} - \delta_H^{\star}\|_2 \le 2G/\mu.$$

If the constraint is not active, then we have a tighter bound: $\|\delta_F^{\star} - \delta_H^{\star}\|_2 \leq \sqrt{G}/\mu$.

Proof. Recall the first-order condition for strong concavity [BV04, §9.1]:

$$f(\delta_F^\star) \le f(\delta_H^\star) + \nabla f(\delta_H^\star)^T (\delta_F^\star - \delta_H^\star) - \frac{\mu}{2} \|\delta_F^\star - \delta_F^\star\|_2^2$$

The first bound follows from using the fact that $0 \leq f(\delta_F^*) - f(\delta_H^*)$ and Cauchy-Schwarz. For the second bound, recall that a μ -strongly convex function $g: \mathbf{R} \to \mathbf{R}$ also satisfies the Polyak-Lojasiewicz condition:

$$\frac{1}{2} \|\nabla g(x)\|_2^2 \ge \mu(g(x) - g(x^*))$$

where $x^* \in \operatorname{argmax}_x g(x)$. Since -f is μ -strongly convex, this implies that

$$f(\delta_F^{\star}) - f(\delta_H^{\star}) \le \frac{1}{2\mu} \|\nabla f(\delta)\|_2 \le \frac{G}{2\mu}$$

From the first-order condition for strong concavity, we have that

$$f(\delta_H^\star) \leq f(\delta_F^\star) + \nabla f(\delta_F^\star)^T (\delta_H^\star - \delta_F^\star) - \frac{\mu}{2} \|\delta_H^\star - \delta_F^\star\|_2^2$$

Using the fact that $\nabla f(\delta_F^*) = 0$, we get the second bound:

$$\|\delta_F^* - \delta_H^*\|_2^2 \le \frac{2}{\mu} \left(f(\delta_F^*) - f(\delta_H^*) \right) \le \frac{G}{\mu^2}.$$

For any μ -strongly concave function f with optimum x^* , note that for any $y \in \mathbf{Dom} f$

$$f(y) \le f(x^*) + \nabla f(x^*)^T (y - x^*) - \mu \|y - x^*\|_2^2 = f(x^*) - \mu \|y - x^*\|_2^2$$

which implies that $\mu \|y - x^*\|_2^2 \le f(x^*) - f(y)$. Applying this to f gives the bound on $\|\delta_F^* - \delta_H^*\|_2$.

This bound implies that we can approximate the optimum of δ_F^\star using δ_H^\star , which is easily computable. Next, define $S(\delta) = f(\delta)(p^T\delta)$, where we assume that p is fixed. Let $\delta_S^\star \in \operatorname{argmax}_\delta S(\delta)$. We show that the approximate optimum of f, δ_H^\star and δ_S^\star are close:

Claim 3.2. Suppose the hypotheses of Claim 3.1 hold. Then we have that

$$\|\delta_S^{\star} - \delta_H^{\star}\| \le 4G/\mu.$$

Proof. Since Claim 3.1 bounds the distance between δ_F^* and any δ , apply this claim to both δ_H^* and δ_S^* , then use the triangle inequality.

Finally, we can bound the objective difference between the approximate and true optimal values, which bounds the profit lost by only approximately solving (7).

Claim 3.3. Suppose the hypotheses of Claim 3.1 hold and that $\min_i p \geq C$. Then we have

$$S(\delta_S^*) - S(\delta_F^*) \le \left(40 + \frac{8}{C}\right) \|p\|_2 \frac{G^3}{\mu^2}$$

Proof. Firstly note that the gradient condition implies f is Lipschitz. Since f(0)=0, this implies $|f(\delta)| \leq G\|\delta\|_2$ for all δ . Using Cauchy-Schwarz, we have $p^T\delta \leq \|p\|_2\|\delta\|_2$. From strong concavity, we have $f(\delta) \leq f(\delta_F^\star) - \frac{\mu}{2}\|\delta_F^\star - \delta\|_2^2$. This implies that $\|\delta_F^\star - \delta\| \leq \sqrt{\frac{2}{\mu}f(\delta_F^\star) - f(\delta)} \leq \sqrt{\frac{2}{\mu}f(\delta_F^\star)}$. If $\delta = 0$, this is equivalent to $\frac{\mu}{2}\|\delta_F^\star\|^2 \leq f(\delta_F^\star) \leq G\|\delta_F^\star\|$, which implies $\|\delta_F^\star\| \leq \frac{2G}{\mu}$, $\|\delta_F^\star - \delta\|_2 \leq \frac{4G}{\mu}$, and $f(\delta_F^\star) \leq \frac{2G^2}{\mu}$. Therefore we have

$$\begin{split} S(\delta_{S}^{\star}) - S(\delta_{F}^{\star}) &\leq |S(\delta_{S}^{\star}) - f(\delta_{S}^{\star})| + |S(\delta_{F}^{\star}) - f(\delta_{F}^{\star})| \\ &\leq f(\delta_{S}^{\star})|p^{T}\delta_{S}^{\star} + 1| + f(\delta_{F}^{\star})|p^{T}\delta_{F}^{\star} + 1| \\ &\leq G\|\delta_{S}^{\star}\|_{2} \left(1 + \|p\|_{2}\|\delta_{S}^{\star}\|_{2}\right) + G\|\delta_{F}^{\star}\|_{2} \left(1 + \|p\|_{2}\|\delta_{F}^{\star}\|_{2}\right) \\ &\leq G(\|\delta_{S}^{\star} - \delta_{F}^{\star}\|_{2} + \|\delta_{F}^{\star}\|_{2}) \left(1 + \|p\|_{2}(\|\delta_{S}^{\star} - \delta_{F}^{\star}\|_{2} + \|\delta_{F}^{\star}\|_{2})\right) + G\|\delta_{F}^{\star}\|_{2} \left(1 + \|p\|_{2}\|\delta_{F}^{\star}\|_{2}\right) \\ &= G\left(\frac{6G}{\mu}\right) \left(1 + \|p\|_{2}\frac{6G}{\mu}\right) + G\left(\frac{2G}{\mu}\right) \left(1 + \|p\|_{2}\frac{2G}{\mu}\right) \\ &\leq \left(40 + \frac{8}{\|p\|_{2}}\right) \|p\|_{2}\frac{G^{3}}{\mu^{2}} \end{split}$$

These claim show that if we find a trade that only approximately optimizes F, (possibly via the quadratic proxy, H), we are close to the optimal trade (the optimum of (7)) and the difference in profit between these two trades is bounded. From the approximate optimum, one can utilize a local method such as (sub)gradient descent to do further refinement.

3.4 Share redemption

An arbitrageur may alternatively buy PDLP shares on the market and redeem them for a pro-rata share of the portfolio value. This direction of arbitrage can be profitable even when the discount is negative. If the arbitrageur buys σ PDLP shares, which has S total shares outstanding, they may redeem these shares for a portfolio $\lambda \in \mathbb{R}^n_+$ satisfying

$$p^{T}\lambda = (1 + f(\lambda)) \cdot \frac{\sigma}{S} V(p, R) = (1 + f(\lambda)) \cdot \frac{\sigma}{S} \cdot p^{T} R$$
(9)

where we use $f(\lambda)$ as shorthand to denote $F(p, w^*, R, -\lambda)$. In other words, the PDLP requires the value of the shares redeemed, measured by the price p, to equal the LP's prorata share of the PDLP reserves, modified by the discount rate.

Arbitrage problem. Assume that the external market price equals the implied price (sometimes called the virtual price) of the PDLP shares, *i.e.*,

$$p^{\text{mkt}} = \frac{p^T R}{S}.$$

This means that an arbitrage only exists when the discount $f(\lambda)$ is positive. The arbitrageur aims to redeem PDLP shares for the most valuable basket of assets λ . They may not withdraw more than the available assets (those not used for loans) R^A from the PDLP. This arbitrage problem can be written as

$$\begin{aligned} \text{maximize}_{\sigma,\lambda} & & p^T \lambda - p^{\text{mkt}} \sigma \\ \text{subject to} & & p^T \lambda = (1 + f(\lambda)) \cdot \sigma \cdot \frac{p^T R}{S} \\ & & 0 \leq \lambda \leq R^A \\ & & 0 \leq \sigma \leq S. \end{aligned}$$

Substituting in equation (9) for σ , and using our market price assumption, this problem becomes

maximize_{$$\lambda$$} $p^T \lambda \cdot \frac{f(\lambda)}{1 + f(\lambda)}$ (10)
subject to $0 \le \lambda \le R^A$.

Approximate optimization. Similar to the creation problem, we can find a surrogate function that approximates the objective. Using strong concavity, we have that

$$p^T \lambda \cdot \frac{f(\lambda)}{1 + f(\lambda)} \le p^T \lambda \cdot f(\lambda) \le p^T \lambda \left(g^T \lambda - \mu \|\lambda\|^2 \right),$$

where $g = \nabla f(\lambda)$. We can now approximately optimize the redemption problem (10) similarly to the creation problem (7).

Price impact. Some PDLPs enforce their discounting mechanism via a protocol-controlled secondary market where users can trade PDLP shares for the numéraire. These exchanges, such as Jupiter's JupiterSwap [Jup24], give users discounts on the fees that they pay to purchase a PDLP share from the secondary market. This mechanism is similar to having a discount rate and a price impact for the share purchase.

3.5 TWMs reduce volatility for LPs

In addition to ensuring asset diversity, TWMs reduce the volatility of LP positions. This benefit permits the associated tokens to be used as higher-quality collateral elsewhere in the DeFi ecosystem.

Portfolio dilution. The portfolio value of the PDLP share in one time period is the sum of several components: the assets in the PDLP, the fees earned from loans, and the loss from subsidizing the discount rate. Given a starting portfolio of assets R with lent assets ℓ , the portfolio value after a single TWM update of size δ is

$$V^{\text{new}} = \frac{1}{1 + F(p, w^*, R, \delta)} \cdot p^T R + f p^T \ell.$$

The first term captures the diluted value of the initial portfolio for existing LPs, and the second term captures the fees earned from lending.

PDLPs improve portfolio delta. The *delta* of a portfolio measures the sensitivity of the portfolio value to price changes in each of the underlying assets. Here, we measure this via the gradient of the portfolio value with respect to the asset prices. Passive LPs typically aim to have low delta portfolios. We show that the difference between the delta of the original portfolio, $\nabla_p V^{\text{old}} = R$, and that of the new portfolio, $\nabla_p V^{\text{new}}$, is bounded in the following claim:

Claim 3.4. Suppose that $\nabla_p F \geq \frac{8fR}{p^T R}$ and $F \leq 1$. Then we have

$$\nabla_p V^{\text{new}} \le \left(\frac{1}{2} - f\right) \cdot \nabla_p V^{\text{old}}.$$

Proof. We have that

$$\nabla_p V^{\text{new}} = \frac{1}{1 + F(p, w^*, R, \delta)} \cdot R - \frac{p^T R}{(1 + F(p, w^*, R, \delta))^2} \cdot \nabla_p F + f \ell$$

$$\leq \left(\frac{1}{2} + f\right) \cdot R - \frac{p^T R}{(1 + F(p, w^*, R, \delta))^2} \cdot \nabla_p F$$

$$\leq \left(\frac{1}{2} + f\right) \cdot R - \frac{1}{4} \cdot 8f R$$

$$= \left(\frac{1}{2} - f\right) R$$

In the first inequality, we used that $F \leq 1$ and $\ell \leq R$, and in the second we used the assumption on the gradient of F.

Discussion. In words, this claim shows that as long as the discount decays fast enough at higher prices and there is a minimal amount of lending demand, the excess delta added from participating in a PDLP is at most $(\frac{1}{2} - f)$ times greater than that of holding the portfolio R. Note the upper bound on PDLP delta decreases as fees increase, which suggests that the worst-case risky exposure the protocol takes decreases at higher fees. This is often argued to be a reason the JLP style assets are good collateral for lending protocols, where there exist nearly \$500 million dollars of outstanding JLP loans on Kamino. [y2k25, Kam25].⁵

3.6 Example

Recall that the weight of a PDLP with portfolio R and asset prices p is $w(p, R) = (p \odot R)/p^T R$. GMX's GLP pool [xvi23] has the discount function

$$F(p, w^*, R, \delta) = \max(0, \gamma_b + \max_{i \in [n]} G_i(w(p, R), w(p, R + \delta), w^*))$$
(11)

where $G(w^b, w^a, w^*)$ is the piecewise function

$$G_i(w^b, w^a, w^*) = \begin{cases} 0 & \text{if } w_i^a = w_i^b \\ \gamma_t \left| \frac{w_i^b - w_i^*}{w_i^*} \right| & \text{if } |w_i^a - w_i^*| < |w_i^b - w_i^*| \\ -\frac{\gamma_t}{2} \left(\left| \frac{w_i^b - w_i^*}{w_i^*} \right| + \left| \frac{w_i^a - w_i^*}{w_i^*} \right| \right) & \text{else} \end{cases}$$

and $\gamma_b, \gamma_t \in (0, 1)$ are called the base and tax scalars. We plot this function in Figure 2. We note that while this function is not μ -strongly concave, one can apply the proximal gradient method [PB⁺14] to construct a μ -strongly concave approximation of this function for which our results apply.

4 Hedged PDLPs

Empirical observations suggest that liquidity providers can more easily hedge their positions on PDLPs than on CFMMs. In this section, we study delta hedging of PDLP LP positions: removing volatility in an LP position by shorting the volatile assets in the PDLP portfolio. A delta-hedged LP position aims to only earn fees in the numéraire. In contrast to CFMM positions, LPs can cheaply approximately delta hedge PDLP positions, which likely contributed to the rapid rise of PDLP LP vaults (e.g., [Gau24c]).

Delta hedging. The delta of an n-asset portfolio, denoted by $\Delta \in \mathbf{R}^n$, measures the sensitivity of the portfolio value to price changes in each of the underlying assets. (For a practical example, see [Gau24c, §4.4].) Given a portfolio of n risky assets $R \in \mathbf{R}^n_+$, a delta

⁵As a concrete example, suppose we have a lending protocol where where are two assets A_1, A_2 that the protocol offers loan-to-values (LTVs) of $L_1, L_2 \in (0,1)$. This means that a user can borrow up to $L_i\%$ percent of the value of their A_i collateral in numéraire. A portfolio with weights (w, 1-w) for A_1, A_2 can borrow up to $L(w) = wL_1 + (1-w)L_2 = w(L_1-L_2) + L_2 \in (0,1)$ percent of the portfolio value. Claim 3.4 implies that a lending protocol (under no arbitrage) can offer an LTV of $\frac{1}{\frac{3}{2}-f}L(w)$ and have at most the same price risk as the portfolio (w, 1-w).

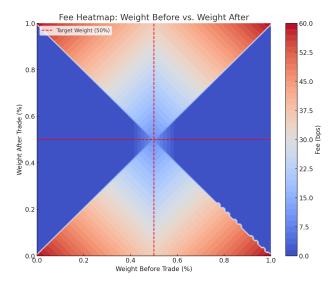


Figure 2: Heatmap of the GMX GLP discount function with two assets for a target weight $w^* = 0.5$

hedge is an offsetting portfolio $\pi \in \mathbf{R}^n$ so that the combined portfolio is less sensitive to price fluctuations (the delta) than the original portfolio. If the PDLP has $\ell \in \mathbf{R}^n_+$ lent out of the n assets, the the delta hedge portfolio under the mean-variance framework [Mad16, HW17] is updated by solving the following optimization problem:

$$\pi^{\text{new}} = \underset{x}{\operatorname{argmax}} \underbrace{f \cdot \ell^{T}(x+R)}_{\text{fees}} - \underbrace{\frac{1}{2}(x-\pi)^{T} \operatorname{Diag}(c)(x-\pi)}_{\text{rebalancing cost}} - \underbrace{\frac{\gamma}{2}(x+\Delta)^{T} \Sigma(x+\Delta)}_{\text{risk}}, \quad (12)$$

where $\Sigma \succ 0$ is the covariance matrix for the risky assets, $c \in \mathbf{R}^n_+$ is the vector of rebalancing costs, and $\gamma \in \mathbf{R}_+$ is a risk aversion parameter. This strategy aims to maximize the fees earned from loans while minimizing the risk of the delta-hedged portfolio, measured by the variance. The parameter γ controls the tradeoff between these two objectives. The exact solution to this problem is

$$\pi^{\text{new}} = (\gamma \Sigma + \mathsf{Diag}(c))^{-1} \left[f \cdot \ell + \mathsf{Diag}(c) \pi - \gamma \Sigma \Delta \right].$$

Without transaction costs, this expression simplifies to

$$\pi^{\text{new}} = (f/\gamma)\Sigma^{-1}\ell - \Delta. \tag{13}$$

For simplicity, we will prove our main result without transaction costs.

4.1 When is a hedged PDLP's Sharpe ratio improved?

We assess the performance of this portfolio using it's Sharpe ratio: the expected return per unit of standard deviation (i.e., risk). In this subsection, we give sufficient conditions such that the delta-hedged portfolio's Sharpe ratio is improved over the unhedged portfolio. We prove this result in two parts. First we show that the expectation of the delta-hedged

portfolio is non-decreasing. Second, we show that the variance of the delta-hedged portfolio is non-increasing. These two facts together imply that the Sharpe ratio of the delta-hedged portfolio is no worse than the unhedged portfolio.

Expectation is non-decreasing. Here, we derive conditions under which the expectation of the delta-hedged portfolio is at least as large as the unhedged portfolio. Since the delta-hedged portfolio is an additive offsetting portfolio, we aim to see when

$$\mathbf{E}\left[p^{T}\pi\right] = \mathbf{E}\left[(f/\gamma)p^{T}\Sigma^{-1}\ell - p^{T}\Delta\right] \ge 0,$$

where π is the solution to (13). If λ_{\min} and λ_{\max} are the minimum and maximum eigenvalues of the covariance matrix Σ , then we have the elementary inequality

$$\lambda_{\max}^{-1} \cdot p^T \ell \le p^T \Sigma^{-1} \ell.$$

This implies that the sufficient condition for non-decreasing expectation is

$$f \cdot \mathbf{E} \left[p^T \ell \right] \ge \gamma \lambda_{\text{max}} \cdot \mathbf{E} \left[p^T \Delta \right] \tag{14}$$

This condition states that as long as the fee revenue, driven by loan demand, is $\gamma \lambda_{\text{max}}$ times the portfolio delta, the expectation is non-decreasing. This implies that the correlation between the assets in the pool drives how profitable one needs to be for delta hedging to work. We discuss when a PDLP should be split into multiple pools in Appendix B.

Variance is non-increasing. Next, we derive conditions under which the variance of the delta-hedged portfolio is at most as large as that of the unhedged portfolio. The variance of the delta-hedged portfolio can be written as

$$\mathbf{Var}\left[p^T\pi + p^TR\right] = \mathbf{Var}\left[p^T\pi\right] + 2\mathbf{Cov}\left(p^T\pi, p^TR\right) + \mathbf{Var}\left[p^TR\right].$$

Thus, we must have that

$$\operatorname{Var}\left[p^T\pi\right] \leq -2\operatorname{Cov}\left(p^T\pi, p^TR\right).$$

Using the fact that the delta hedge in negatively correlated to the unhedged portfolio value and that covariance is bounded by the product of the standard deviations, we have the sufficient condition

$$\operatorname{Var}\left[p^T\pi\right] \leq 4 \cdot \operatorname{Var}\left[p^TR\right].$$

In other words, as long as the hedge is anticorrelated to the portfolio and that its variance is less than 4 times the variance of the portfolio, variance is non-increasing under the delta hedge.

Main result. Putting the two conditions together, we have conditions under which the Sharpe ratio of the delta-hedged portfolio is at least as good as the unhedged portfolio. We summarize this result in the following claim:

Claim 4.1. Suppose we have a PDLP with assets $R \in \mathbf{R}^n_+$ which have covariance $\Sigma \prec \lambda_{\max} I$, loan fees f, and loan demand ℓ . Denote the asset prices by $p \in \mathbf{R}^n_{++}$ and set the risk parameter $\gamma \in \mathbf{R}_+$. Then the Sharpe ratio of the delta-hedged portfolio, which includes the original portfolio plus an offseting portfolio $\pi \in \mathbf{R}^n$, is at least as high as the unhedged portfolio if the following conditions hold:

1. The loan fees are at least $\gamma \lambda_{max}$ times the portfolio delta:

$$f \cdot \mathbf{E}[p^T \ell] \ge \gamma \lambda_{\max} \cdot \mathbf{E}[p^T \Delta].$$

2. The variance of the offsetting portfolio is at most 4 times the variance of the unhedged portfolio:

$$\mathbf{Var}[p^T \pi] \le 4 \cdot \mathbf{Var}[p^T R].$$

5 Conclusion and future work

We presented the first formalization of Perpetual Demand Lending Pools (PDLPs), which hold \$2.5 billion in assets as of December 2024 and have generated over \$700 million in fees by lending to traders on decentralized perpetuals exchanges. Our formulation of the arbitrage problems for various counterparties suggests a wide, largely-unexplored design space for such protocols. The scale and rapid growth of PDLPs begs a question: Can novel PDLPs service the same notional volume with less capital?

Our framework provides a number of directions to explore. First, formalizing multi-period PDLP arbitrage could inform the design of dynamic fees and discounting mechanisms. Such multi-period models exist for CFMMs (e.g. via the quantification of loss-versus-rebalancing). Second, dynamic target portfolios may improve target weight mechanisms. Such adjustments have been proposed for CFMMs and lending protocols, but these mechanisms will need different properties for perpetuals protocols. Finally, the plethora of hedged and levered PDLP strategies (especially for Jupiter [Dri24]) suggest that levered PDLPs present an interesting avenue of future exploration. We hope that this work helps researchers improve these mechanisms and make decentralized exchanges competitive with their centralized counterparts.

6 Acknowledgments

We want to thank Tim Copeland, Alex Evans, Kshitij Kulkarni, Horace Pan, Krane, Victor Xu, JD Maturen, Diogenes Casares, Marius Ciubotariu, and Madison Piercy for helpful comments and suggestions.

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A Trading Dynamics

We more formally describe the update dynamics for a perpetuals exchange using a PDLP in this section. For each time $t \in \mathbb{N}$, let $\tau_t = \{(c_t^i, \delta_t^i, \eta_t^i, p_t)\}$ be the set of trades created by users at time t and let $\mathcal{A}_t = \bigcup_{s \leq t} \tau_s$ be the set of all trades created at or before time t. We define the set of liquidatable long and short trades at time t as

$$\ell_L(t, p_1, \dots, p_t) = \left\{ (c_s^i, \delta_s^i, \eta_s^i, p_s) \in \mathcal{A}_t : \eta_s^i > 0, \land \forall s' \in [s, t) \ p_{s'} > L(\eta, p_s) \land p_t \le L(\eta, p_s) \right\}$$

$$\ell_S(t, p_1, \dots, p_t) = \left\{ (c_s^i, \delta_s^i, \eta_s^i, p_s) \in \mathcal{A}_t : \eta_s^i < 0, \land \forall s' \in [s, t) \ p_{s'} < L(\eta, p_s) \land p_t \ge L(\eta, p_s) \right\}$$

In words, this is the set of trades at times s < t such that the trade was not liquidatable at any time s' with $s \le s' < t$ but was liquidatable at time t. That is, the trades in these sets are first liquidatable at time t. We define the set \mathcal{T}_t of valid trades at time t to be

$$\mathcal{T}_t = \mathcal{A}_t - \bigcup_{s < t} (\ell_L(t, p_1, \dots, p_t) \cup \ell_S(t, p_1, \dots, p_t))$$

Finally, we let $\mathcal{L}_t, \mathcal{S}_t \subset \mathcal{T}$ be the set of long and/or short positions alive at time t. Given these definitions, we can describe the dynamics formally as follows:

- 1. The price is updated by the price oracle to p_{t+1}
- 2. The liquidatable positions are removed:

$$\mathcal{L}_t \leftarrow \mathcal{L}_t - \ell_L(t, p_1, \dots, p_{t+1})$$
 $\mathcal{S}_t \leftarrow \mathcal{S}_t - \ell_S(p_1, \dots, p_{t+1})$

3. For each market, the funding rate γ_t^i for the previous period is paid out, where

$$\gamma_t^i = \gamma_L(L_t^i, S_t^i, p_{t+1}, p_t) = \kappa \left(\frac{L_t^i}{S_t^i} - \frac{p_{t+1}}{p_t}\right)$$

and the long and short positions are updated as

$$L_t^i \leftarrow L_t^i + \gamma_t^i S_t^i \mathbf{1}_{\gamma_t^i \ge 0}$$

$$S_t^i \leftarrow S_t^i + \gamma_t^i L_t^i \mathbf{1}_{\gamma_t^i \le 0}$$

Each trader $j \in [M]$ receives a pro-rata percentage of the funding rate based on the relative size of their position. Note that the payout for the previous period occurs before the new trades for the following period are added.

- 4. LPs submit updates to their portfolio, $q_t^i \in \mathbf{R}^n$, which modifies the current portfolio R_t to $R_{t+1} = R_t + \sum_{i=1}^K q_t^i$ subject to the constraint that $R_t + \sum_{i=1}^\ell q_t^i \ge 0$ for all $\ell \in [k]$. This constraint ensures that the LPs cannot drain the pool.
- 5. For each trader $j \in [M]$ that submits a trade $(c_t^i, \delta_t^i, \eta_t^i, p_t) \in \tau_t$:
 - Collateral c_t^i is chosen such that $(1+\gamma)|\eta_t^i|p_{t+1}^Tc_i^t=p_{t+1}^T\delta_t^i$. If this is not possible due to the user's previous positions, the trade is rejected
 - If $c_{L,t} + c_{S,t} + \sum_{i \leq j} c_t^i > R_{t+1}$, the trade from trader j is rejected as it violates the solvency requirement
 - Otherwise update $\mathcal{L}_{j,t}$, $\mathcal{S}_{j,t}$ via

$$\mathcal{L}_{j,t+1} = \mathcal{L}_{j,t} + \mathbf{1}_{\eta_t^j > 0}(c_t^j, \delta_t^j, \eta_t^j) \qquad \qquad \mathcal{S}_{j,t+1} = \mathcal{S}_{j,t} + \mathbf{1}_{\eta_t^j < 0}(c_t^j, \delta_t^j, \eta_t^j)$$

6. Fees from the last period are paid out to liquidity providers from the traders

$$R_{t+1} \leftarrow R_{t+1} + f \cdot (c_{L,t+1} + c_{S,t+1})$$

B When is it better to have multiple pools?

Recall that GMX moved from a single-pool PDLP model to a multiple pool PDLP system. In this system, a PDLP portfolio R is partitioned into sub-portfolios $R = \sum_{i=1}^{k} R_i$ where R_i represents the assets in pool $i \in [k]$. Each pool services a different set of perpetuals with a unique target weight, among other parameters. A natural question to ask is when is it more efficient to aggregate the k pools into a single pool.

We will consider the simplest model to analyze this question with k=2 and $\operatorname{supp}(R_1) \cap \operatorname{supp}(R_2) = \emptyset$, *i.e.*, the two PDLP pools have no assets in common. We will assume that $|\operatorname{supp}(R_1)| = m_1, |\operatorname{supp}(R_2)| = m_2$ and that $m_1 + m_2 = n$. We will abuse notation slightly as also refer to the set of assets in R_i as R_i . We consider the full symmetric stochastic covariance matrix $\Sigma \in \mathbf{R}^{n \times n}$ and write

$$\Sigma = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$$

where $A \in \mathbf{R}^{m_1 \times m_1}$, $B \in \mathbf{R}^{m_1 \times m_2}$, $C \in \mathbf{R}^{m_2 \times m_2}$ are matrices with A, C, non-singular and positive definite.

For mean-variance optimization, one computes the conditional covariance⁶ using Schur Complements [Oue81, Zha06]. We denote the Schur complements as $\Sigma/A = A - BC^{-1}B^T$ and $\Sigma/C = C - B^TA^{-1}B$. Σ/A represents the covariance matrix after marginalizing the variables in R_2 , and similarly for Σ/C [Zha06, §6.2.3]. For a covariance matrix S, we define the portfolio values V_t^S as the delta hedged portfolio values when the covariance matrix is S.

Our goal is to find conditions under which the delta hedged portfolio for a single pool is better than an isolated pool. We can view the isolated pool as having either the covariance A or C without the impact of the other, whereas the single pool needs to incorporate information from both pools. We can view the conditional covariance matrices Σ/A and Σ/C as incorportating information from both pools and hence solving the mean-variance problem for these matrices represents the single pool solution.

If the single pool provides better returns than multiple pools, this corresponds to the following conditions: $\mathbf{E}[V_t^{\Sigma/A} - V(\mathcal{P}, p)] \geq \mathbf{E}[V_t^A - V(\mathcal{P}, p)]$ and $\mathbf{E}[V_t^{\Sigma/C} - V(\mathcal{P}, p)] \geq \mathbf{E}[V_t^C - V(\mathcal{P}, p)]$ which reduces to

$$\mathbf{E}[V_t^{\Sigma/A}] \ge \mathbf{E}[V_t^A] \qquad \qquad \mathbf{E}[V_t^{\Sigma/C}] \ge \mathbf{E}[V_t^C] \qquad (15)$$

We prove the following sufficent condition for when this occurs:

Claim B.1. Suppose that $\sigma_{\min}(\Sigma_X) > \sigma_{\max}(A)$ and $\sigma_{\min}(\Sigma_Y) > \sigma_{\max}(B)$. Then (15) holds and a single pool provides better delta hedged returns than multiple pools.

Proof. Conditions (15) are equivalent to

$$\mathbf{E}[p^T(1, \pi_t^{\Sigma/A})] \ge \mathbf{E}[p^T(1, \pi_t^A)] \tag{16}$$

⁶Note that this is technically only the conditional covariance for multivariate normal distributions; for generic distributions it corresponds to the partial correlation [Zha06, §6.2.3].

where π_t^{Σ} is the optimal portfolio with covariance Σ . We can proceed analogously to 4.1, which makes (16) equivalent to the condition

$$\frac{f}{\gamma}\tilde{p}^T(\Sigma/A)\tilde{\ell} - \tilde{p}^T\tilde{\Delta}(\mathcal{P}) \ge \frac{f}{\gamma}\tilde{p}^TA\tilde{\ell} - \tilde{p}^T\tilde{\Delta}(\mathcal{P})$$

which is equivalent to

$$\frac{f}{\gamma}\tilde{p}^T(\Sigma/A)\tilde{\ell} \ge \frac{f}{\gamma}\tilde{p}^TA\tilde{\ell}$$

A sufficient condition for this to hold for all $\tilde{p}, \tilde{\ell}$ is for $\sigma_{\max}(A) < \sigma_{\min}(\Sigma/A)$. We note that this doesn't violate Cauchy's interlacing theorem style results which show that $\sigma_{n-m_1}(\Sigma) < \sigma_{\max}(\Sigma/A) \leq \sigma_{\max}(\Sigma)$ where $\sigma_k(X)$ is the kth eigenvalue of a matrix X since those are between the spectral of the full matrix Σ and the Schur complement as opposed to the subcomponent [Zha06, Ch. 2]. The same proof applies for the other condition.

We note these conditions are similar to hierarchical risk-parity methods [Cot24, LdP16]. We also note that these conditions suggest that GMX V2's recent dynamic pricing upgrade [GMX24a] is not sufficient alone for ensuring that V2 vaults are easily delta hedgeable as it doesn't take into account pool asset covariance as is done here. We leave it for future work to connect the dynamic pricing model with the cost of delta hedging.