Stable Flocking of Mobile Agents
Part II: Dynamic Topology

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***Abstract*—This is the second of a two-part paper, investigating the stability properties of a system of multiple mobile agents with double integrator dynamics. In this second part, we allow the topology of the control interconnections between the agents in the group to vary with time. Specifically, the control law of an agent depends on the state of a set of agents that are within a certain neighborhood around it. As the agents move around this set changes, giving rise to a dynamic control interconnection topology and a switching control law. This control law consists of a a combination of attractive/repulsive and alignment forces. The former ensure collision avoidance and cohesion of the group and the latter result to all agents attaining a common heading angle, exhibiting flocking motion. Despite the use of only local information and the time varying nature of agent interaction which affects the local controllers, flocking motion is established, as long as connectivity in the neighboring graph is maintained.**

I. Introduction

Over the past decade a considerable amount of attention has been focused on the problem of coordinated motion of multiple autonomous agents. Related problems have been studied in ecology and theoretical biology, in the context of animal aggregation and social cohesion in animal groups, statistical physics and complexity theory, non-equilibrium phenomena in many-degree-of-freedom dynamical systems, as well as in distributed control of multiple vehicles and formation control (see Part I of this paper [10] and the references within). Researchers from many different communities have been trying to develop an understanding of how a group of moving agents can move in a formation *only* using local interactions and without a global supervisor.

In 1986 Craig Reynolds [8] developed a computer animation model for coordinated motion of groups of animals such as bird flocks and fish schools. A similar model was proposed in 1995 by Vicsek *et al.* [11]. In Vicsek model, each agent heading is updated as the average of the headings of agent itself with its nearest neighbors plus some additive noise. Numerical simulations in [11] indicate the spontaneous development of coherent collective motion, resulting in the headings of all agents to converge to a common value. The first rigorous proof of convergence for Vicsek's model (in the noise-free case) was given in [6]. Reynolds' model suggests that flocking is the combined result of three simple steering rules, which each agent independently follows:

- **Separation:** steer to avoid crowding local flockmates.
- **Alignment:** steer towards the average heading of local flockmates.
- **Cohesion:** steer to move toward the average position of local flockmates.

In Reynolds' model, each agent can access the whole scene's geometric description, but flocking requires that it reacts only to flockmates within a certain small neighborhood around itself. The superposition of these three rules results in all agents moving in a formation, while avoiding collision.

In the first part of the paper, we demonstrated how flocking occurs when each agent is steered using state information from a fixed set of interconnected neighbors. The topology of the control interconnections was fixed and time invariant. In this paper we show that this can also be achieved in the case where the topology is dynamic. Distance-based dynamic agent interactions can now guarantee collision avoidance, regardless of the structure of the interconnection graph. Another distinguishing characteristic of range-dependent agent interactions is that the control laws may be switching. Control discontinuities require a stability analysis within the framework of Filippov solutions and nonsmooth stability. Our stability analysis and control design combines results from classical control theory, mechanics, algebraic graph theory, nonsmooth analysis and Lyapunov stability for nonsmooth systems. We show that whenever the the graph representing the nearest neighbor relations is connected, all agent velocities converge to the same vector and pairwise distances are stabilized.

This paper is organized as follows: in Section II we define the problem addressed in this paper and sketch the solution approach. Some basic facts from algebraic graph theory are presented in Section III. Section IV introduces the control scheme. A brief introduction to nonsmooth stability is given in section IV, to pave the way for the stability analysis of Section VI. The results of Section VI are verified in Section VII via numerical simulations. Section VIII summarizes the results and highlights our key points.

II. Problem Description

Consider N agents, moving on the plane with the following dynamics:

$$\dot{r}_i = v_i \tag{1a}$$
$$\dot{v}_i = u_i \quad i = 1, \dots, N , \tag{1b}$$

where $r_i = (x_i, y_i)^T$ is the position of agent i, $v_i = (\dot{x}_i, \dot{y}_i)^T$ is its velocity and $u_i = (u_{x_i}, u_{y_i})^T$ its control inputs. The heading angle of agent i, θ_i, is defined as:

$$\theta_i = \arctan(\dot{y}_i, \dot{x}_i) . \tag{2}$$

Relative position vectors are denoted $r_{ij} = r_i - r_j$.

The control objective is to generate coordinated motion in the same direction with constant pairwise distances using local, decentralized control action. The control input consists of two components (Figure 1):

$$u_i = a_i + \alpha_i . \tag{3}$$

The first component, a_i, is attributed to an artificial potential field generated by a function V_i, which encodes relative position information between agent i and its neighbors. This term ensures collision avoidance and cohesion in the group. The second component, α_i regulates the velocity vectors agent i to the average of that of its neighbors.

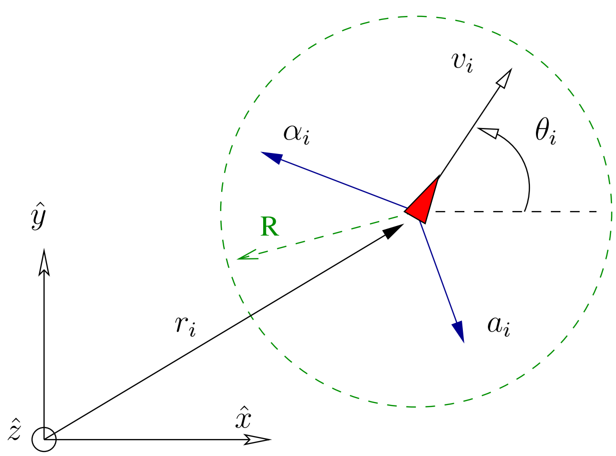

Fig. 1. *Control forces acting on agent i.*

The problem is to determine the input components so that the group exhibits a stable, collision free flocking motion. This is being understood technically as a convergence property on the agent velocity vectors and their relative distances.

III. Graph Theory Preliminaries

This section presents briefly the main graph theoretic terminology used in the paper. The interested reader is referred to [5].

An (undirected) graph $\mathcal{G}$ consists of a vertex set, $\mathcal{V}$, and an edge set $\mathcal{E}$, where an edge is an unordered pair of distinct vertices in $\mathcal{G}$. If $x, y \in \mathcal{V}$, and $(x, y) \in \mathcal{E}$, then x and y are said to be adjacent, or neighbors and we denote this by writing $x \sim y$. A path of length r from vertex x to vertex y is a sequence of $r+1$ distinct vertices starting with x and ending with y such that consecutive vertices are adjacent. If there is a path between any two vertices of a graph $\mathcal{G}$, then $\mathcal{G}$ is said to be connected. An orientation of a graph $\mathcal{G}$ is the assignment of a direction to each edge, so that the edge (i, j) is now an arc from vertex i to vertex j. We denote by $\mathcal{G}^\sigma$ the graph $\mathcal{G}$ with orientation σ. The incidence matrix $B(\mathcal{G}^\sigma)$ of an oriented graph $\mathcal{G}^\sigma$ is the matrix whose rows and columns are indexed by the vertices and edges of $\mathcal{G}$ respectively, such that the i, j entry of $B(\mathcal{G})$ is equal to 1 if the edge j is incoming to vertex i, -1 if edge j is outcoming from vertex i, and 0 otherwise.

The symmetric matrix defined as:

$$L(\mathcal{G}) = B(\mathcal{G}^\sigma) B(\mathcal{G}^\sigma)^T$$

is called the Laplacian of $\mathcal{G}$ and is independent of the choice of orientation σ. It is known that the Laplacian matrix captures many topological properties of the graph. Among those, is the fact that L is always positive semidefinite, it has zero as a single eigenvalue whenever the graph is connected and the associated eigenvector is the n-dimensional vector of ones, $\mathbf{1}_n$. The second largest eigenvalue, λ_2 is known to convey a lot of information about the structure of the graph and its connectivity, hence its name *"algebraic connectivity"*.

IV. Control Law with Dynamic Topology

In this section we present a realization of the control law (3) that achieves the control objective. The steering policy of each agent is based only on local state information from its nearest neighbors. The graph $\mathcal{G}$, represents the nearest neighboring relations:

Definition IV.1 (Neighboring graph) *The neighboring graph, $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, is an undirected graph consisting of:*

- *a set of vertices (nodes), $\mathcal{V} = \{n_1, \dots, n_N\}$, indexed by the agents in the group, and*
- *a set of edges, $\mathcal{E} = \{(n_i, n_j) \in \mathcal{V} \times \mathcal{V} \mid n_i \sim n_j\}$, containing unordered pairs of nodes that represent neighboring relations.*

Let $\mathcal{N}_i$ denote the index set of neighbors of i,

$$\mathcal{N}_i \triangleq \{j \mid \|r_{ij}\| \leq R\} \subseteq \{1, \dots, N\}.$$

Since the agents are in motion, their relative distances can change with time, affecting their neighboring sets.

The time dependence of the neighboring relations gives rise to a switching graph. For each edge incident to agent i, we define an inter-agent potential function, U_{ij} which should satisfy:

Definition IV.2 (Potential function) *The potential function U_{ij} is a nonnegative function of the distance $\|r_{ij}\|$ between agents i and j, such that*

1. *$U_{ij}(\|r_{ij}\|) \to \infty$ as $\|r_{ij}\| \to 0$,*

2. *U_{ij} attains its unique minimum when agents i and j are located at a desired distance.*

3. *U_{ij} is increasing near $\|r_{ij}\| = R$.*

Function U_{ij} can be nonsmooth at distance $\|r_{ij}\| = R$, and constant $U_{ij} = V_R$ for $\|r_{ij}\| > R$, to capture the fact that beyond this distance there is no agent interaction. One example of such a nonsmooth potential function is the following, depicted in Figure 2:

$$U_{ij} = \begin{cases} \frac{1}{\|r_{ij}\|^2} + \log \|r_{ij}\|^2, & \|r_{ij}\| < R, \\ V_R, & \|r_{ij}\| \geq R \end{cases}$$

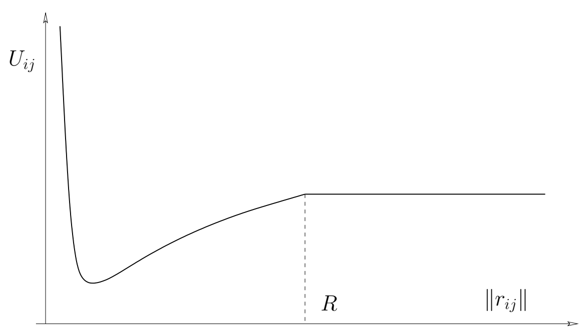

Fig. 2. *A nonsmooth inter-agent potential function.*

For agent i the (total) potential U_i is formed by summing the potentials due to each of its neighbors:

$$U_i \triangleq (N - |\mathcal{N}_i|)V_R + \sum_{j \in \mathcal{N}_i} U_{ij}(\|r_{ij}\|)$$

where $N_i = |\mathcal{N}_i|$. The control law u_i is defined as:

$$u_i = \underbrace{-\sum_{j \in \mathcal{N}_i} (v_i - v_j)}_{\alpha_i} \underbrace{- \sum_{j \in \mathcal{N}_i} \nabla_{r_i} U_{ij}}_{a_i} \tag{4}$$

Changes in the neighboring set $\mathcal{N}_i$, introduce discontinuities in the control law (4). The stability of the discontinuous dynamics should be analyzed using differential inclusions [4] and nonsmooth analysis [3].

V. Nonsmooth Analysis Preliminaries

This section introduces briefly concepts from nonsmooth analysis and stability of nonsmooth systems.

Definition V.1 ([7]) *Consider the following differential equation in which the right hand side can be discontinuous:*

$$\dot{x} = f(x) \tag{5}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ is measurable and essentially locally bounded and n is finite. A vector function $x(\cdot)$ is called a solution of (5) on $[t_0, t_1]$, where if $x(\cdot)$ is absolutely continuous on $[t_0, t_1]$ and for almost all $t \in [t_0, t_1]$

$$\dot{x} = K[f](x) \triangleq \overline{co}\{\lim_{x_i \to x} f(x_i) \mid x_i \notin M_f \cup M\}$$

where $M_f \subset \mathbb{R}^n$, $\mu(M_f) = 0$ and $M \subset \mathbb{R}^n$, $\mu(M) = 0$.

The above definition of solutions, along with the assumption that the vector field f is measurable, guarantees the uniqueness of solutions of (5) [4].

Lyapunov stability has been extended to nonsmooth systems [9, 1]. Establishing stability results in this framework requires working with generalized derivatives, in all cases where classical derivatives cannot be defined.

Definition V.2 ([3]) *Let f be Lipschitz near a given point x and let w be any vector in a Banach space X. The generalized directional derivative of f at x in the direction w, denoted $f^\circ(x; w)$ is defined as follows:*

$$f^\circ(x; w) \triangleq \limsup_{\substack{y \to x \\ t \downarrow 0}} \frac{f(y + tw) - f(y)}{t}$$

The generalized gradient, on the other hand, is generally a set of vectors, which reduces to the single classical gradient in the case where the function is differentiable:

Definition V.3 ([3]) *The generalized gradient of f at x, denoted $\partial f(x)$, is the subset of X^* given by:*

$$\partial f(x) \triangleq \{\zeta \in X^* \mid f^\circ(x; w) \geq \langle \zeta, w \rangle, \ \forall w \in X\}$$

In the special case where X is finite dimensional, we have the following convenient characterization of the generalized gradient:

Theorem V.4 ([2]) *Let $x \in \mathbb{R}^n$ and let $f : \mathbb{R}^n \to \mathbb{R}$ be Lipschitz near x. Let Ω be any subset of zero measure in $\mathbb{R}^n$, and let Ω_f be the set of points in $\mathbb{R}^n$ at which f fails to be differentiable. Then*

$$\partial f(x) \triangleq \text{co}\{\lim_{x_i \to x} \nabla f(x_i) \mid x_i \notin \Omega, x_i \notin \Omega_f\}$$

Calculus based on generalized derivatives usually involves set inclusions. When functions are regular, these inclusions can be turned to equalities.

Definition V.5 ([3]) *A function f is said to be regular at x provided,*

1. *For all w, the usual one-sided directional derivative $f'(x; w)$ exists, and*
2. *for all w, $f'(x; w) = f^{\circ}(x; w)$.*

The time (generalized) derivative of a function that is either nonsmooth or the dynamics of its arguments is discontinuous, is given by this special case of the nonsmooth case of the chain rule:

Theorem V.6 ([9]) *Let $x(\cdot)$ be a Filippov solution to $\dot{x} = f(x)$ on an interval containing t and $V : \mathbb{R}^n \to \mathbb{R}$ be a Lipschitz and in addition, regular function. Then $V(x(t))$ is absolutely continuous, $\frac{d}{dt}V(x(t))$ exists almost everywhere and*

$$\frac{d}{dt}V\big(x(t)\big) \in^{\text{a.e.}} \dot{\tilde{V}}(x) \triangleq \bigcap_{\xi \in \partial V(x(t))} \xi^T K[f](x(t))$$

It can easily be shown that the (global) Lipschitz continuity requirement for $V(x)$ can be relaxed to local. In what follows, we are going to use the following nonsmooth version of LaSalle's invariant principle:

Theorem V.7 ([9]) *Let Ω be a compact set such that every Filippov solution to the autonomous system $\dot{x} = f(x)$, $x(0) = x(t_0)$ starting in Ω is unique and remains in Ω for all $t \geq t_0$. Let $V : \Omega \to \mathbb{R}$ be a time independent regular function such that $v \leq 0$ for all $v \in \dot{\tilde{V}}$ (if $\dot{\tilde{V}}$ is the empty set then this is trivially satisfied). Define $S = \{x \in \Omega \mid 0 \in \dot{\tilde{V}}\}$. Then every trajectory in Ω converges to the largest invariant set, M, in the closure of S.*

VI. Stability Analysis

In this section we show how the decentralized control laws (4) give rise to a coordinated flocking behavior. Specifically, we prove that all agents of the closed loop system (1)- (4) asymptotically attain a common velocity vector, minimize their artificial potential and avoid collisions with their flockmates. This happens regardless of switching in the neighboring graph, as long as the graph remains connected at all times:

Assumption VI.1 *The neighboring graph $\mathcal{G}$ remains connected.*

Our main result is formally stated as follows:

Theorem VI.2 (Flocking in switching networks) *Consider a system of N mobile agents with dynamics (1), each steered by control law (4) and assume that the neighboring graph is connected. Then all pairwise velocity differences converge asymptotically to zero, collisions between the agents are avoided, and the system approaches a configuration that minimizes all agent potentials.*

Proof: Consider the following function:

$$Q = \frac{1}{2}\sum_{i=1}^{N}\Big(\sum_{j=1}^{N} U_{ij} + v_i^T v_i\Big). \tag{6}$$

Function Q is continuous everywhere but nonsmooth whenever $\|r_{ij}\| = R$ for some $(i, j) \in N \times N$. Whenever the neighboring graph is connected, the level sets of Q define compact sets in the space of agent velocities and relative distances. The set $\{r_{ij}, v_i\}$ such that $Q \leq c$, for $c > 0$ is closed by continuity. Boundedness follows from connectivity: from $Q \leq c$ we have that $U_{ij} \leq c$. Connectivity ensures that a path connecting nodes i and j has length at most $N-1$. Thus $\|r_{ij}\| \leq U_{ij}^{-1}\big(c(N-1)\big)$. Similarly, $v_i^T v_i \leq c$ yielding $\|v\|_i \leq \sqrt{c}$. Thus, the set

$$\Omega = \{(v_i, r_{ij}) \mid Q \leq c\} \tag{7}$$

is compact. The restriction of Q in Ω ensures, besides collision avoidance, the differentiability of $\|r_i - r_j\|$, $\forall i, j \in \{1, \ldots, N\}$. Since U_{ij} is continuous at R, it is locally Lipschitz. It is shown that U_{ij} is regular [3]:

Lemma VI.3 *The function U_{ij} is regular everywhere in its domain.*

Proof: It suffices to show regularity at of U_{ij} at R. To simplify notation we will drop the subscripts ij and denote $U_{ij}(R) \equiv V_R$. It is reasonable to assume that the desired distance between two agents is smaller than the neighborhood range, R. By Definition IV.2 therefore, U_{ij} will be increasing at R. For the classical directional derivative we have: $U'(R; w) = \lim_{t\downarrow 0} \frac{U(R+tw)-U(R)}{t}$, and for the derivative to make sense, let $w \neq 0$. If $w > 0$ then, $U'(R; w) = \lim_{t\downarrow 0} \frac{U(R+tw)-V_R}{t} = \lim_{t\downarrow 0} \frac{V_R-V_R}{t} = 0$. If $w < 0$ then $U'(R; w) = \lim_{t\downarrow 0} \frac{V(R+tw)-V_R}{t} \equiv c < 0$, where c is used to denote the directional derivative of V_{ij} at R, in a negative direction ($w < 0$).

For the generalized directional derivative, we distinguish the same two cases: If $w \geq 0$, then $U^{\circ}(R; w) = \limsup_{\substack{y\to R\\ t\downarrow 0}} \frac{U(y+tw)-U(y)}{t} \leq \limsup_{\substack{y'\to R\\ t\downarrow 0}} \frac{V_R - V\left(y'-tw\right)}{t} = -\lim_{t\downarrow 0} \frac{V_R - V_R}{t} = 0$. If $w < 0$, then, $U^{\circ}(R; w) = \limsup_{\substack{y\to R\\ t\downarrow 0}} \frac{U(y+tw)-U(y)}{t} = \limsup_{\substack{y\to R\\ t\downarrow 0}} \frac{V(y+tw)-U(y)}{t} = \lim_{t\downarrow 0} \frac{V(R+tw)-V_R}{t} = c$. ■

Regularity of each potential function U_{ij} is required to ensure the regularity of U_i, as a linear combination of a finite number of regular functions [3]. The latter is a necessary condition for all nonsmooth stability theorems. The following Corollary is an immediate consequence of Lemma VI.3.

Corollary VI.4 *The generalized gradient of U_{ij} at R is empty:*

$$\partial U_{ij}(R) = \emptyset. \tag{8}$$

Thus, Q is regular as a sum of regular functions. Another interesting fact that results from U_{ij} being increasing at R is the following, which is useful in computing the generalized time derivative of Q:

Lemma VI.5 *The (partial) generalized gradient of U_{ij} with respect to r_i at R is empty:*

$$\partial_{r_i} U_{ij}(R) = \emptyset. \tag{9}$$

Proof: of Lemma VI.5 The generalized derivative of U_{ij} at R along w, namely $U^{\circ}_{ij}(R)$, is determined by the expression: $U^{\circ}_{ij}(R;w) \triangleq \max\{\langle \zeta, w\rangle \mid \zeta \in \partial U_{ij}(R)\}$. Depending on the sign of w we distinguish the two cases:

1. if $w > 0$ then $0 \geq \zeta w$, which means that all $\zeta \in \partial U_{ij}(R)$ have to be non positive;
2. if $w < 0$ then $\zeta w \leq c < 0$ which means that all $\zeta \in \partial U_{ij}(R)$ have to be positive.

Since the direction of w is arbitrary, $\partial U_{ij}(R) = \emptyset$.

Function U_{ij} is a composition of a continuous function $U_{ij}(s)$ from the positive reals to the positive reals with $\|r_{ij}\|$. The norm $\|r_{ij}\|$ is a smooth (hence strictly differentiable) function of both position vectors r_i, r_j when $r_i \neq r_j$. Note that $r_i = r_j$ corresponds to collision configurations in the exterior of Ω, which are naturally excluded. Function, $U_{ij}(s)$ is locally Lipschitz and regular for all $s > 0$. Therefore [3]:

$$\partial_{r_i} U_{ij}(\|r_{ij}\|) = \partial_{r_{ij}} U_{ij}(\|r_{ij}\|) \cdot \frac{\partial \|r_{ij}\|}{\partial r_i}$$

At R where U_{ij} is not differentiable, $\partial_{r_{ij}} U_{ij}(R) = \emptyset$, and thus, $\partial_{r_i} U_{ij}(d) = \emptyset$. ■

Regularity of Q and the property of finite sums of generalized gradients ensures that:

$$\partial Q \subset \Big[\sum_{j=1}^{N} \partial_{r_1} U_{ij}^T, \cdots, \sum_{j=1}^{N} \partial_{r_N} U_{ij}^T, v_1^T, \cdots, v_N^T \Big]^T$$

Then for the generalized time derivative of Q,

$$\dot{\tilde{Q}} \subset \sum_{i=1}^{N} \left(\bigcap_{\xi_i} \xi_i^T v_i \right) - v^T K \left[(L_t \otimes I_2) v + \begin{pmatrix} \vdots \\ \nabla_{r_i} U_i \\ \vdots \end{pmatrix} \right]$$

where $\xi_i \in \sum_{j=1}^{N} \partial_{r_i} U_{ij}$, L_t is the (time-dependent) Laplacian of the neighboring graph and $\nabla_{r_i} U_i = \sum_{j \in \mathcal{N}_i} \nabla_{r_i} U_{ij}$. Both L_t and $\nabla_{r_i} U_i$ are switching over time, depending on the neighboring set $\mathcal{N}_i$ of each agent i. Recalling that $\partial U_{ij}(R) = \emptyset$ (Lemma VI.5) and using some differential inclusion algebra for sums, (finite) Cartesian products and multiplications with continuous matrices [7], we obtain

$$\begin{aligned} \dot{\tilde{Q}} &\subset \sum_{i=1}^{N} (\nabla_{r_i} U_i)^T v_i - v^T K [(L_t \otimes I_2) v] - \sum_{i=1}^{N} v_i^T \nabla_{r_i} U_i \\ &= -\overline{co}\{v_x^T L_t v_x + v_y^T L_t v_y\}. \end{aligned} \tag{10}$$

For any graph, the right hand of (10) will be an interval of the form $[e, 0]$, with $e < 0$. Therefore it is always $q \leq 0$, for all $q \in \dot{\tilde{Q}}$. If the graph is connected, then this interval contains 0 *only* when $v_x, v_y \in \text{span}\{\mathbf{1}\}$.

Applying the nonsmooth version of LaSalle's principle proposed by [9], it follows that for initial conditions in Ω, the Filippov trajectories of the system converge to a subset of $\{v \mid v_x, v_y \in \text{span}\{\mathbf{1}\}\}$ in which $\dot{r}_{ij} = v_i - v_j = 0, \quad \forall (i,j) \in N \times N$. In this set, the system dynamics reduces to $\dot{v} = -(B_t \otimes I_2)\left[\cdots\ (\nabla_{r_{ij}} V_{ij})^T\ \cdots\right]^T$ which implies that both $\dot{v}_x$ and $\dot{v}_y$ belong in the range of the switching incidence matrix B_t. For a connected graph, $\text{range}(B_t) = \text{span}\{\mathbf{1}\}^{\perp}$ and therefore

$$\dot{v}_x, \dot{v}_y \in \text{span}\{\mathbf{1}\} \cap \text{span}\{\mathbf{1}\}^{\perp} \equiv \{0\}. \tag{11}$$

From the above we conclude that

1. v does not change in steady state (and thus switching eventually stops), and
2. the potential V_i of each agent is minimized.

■

VII. Simulations

In the simulation example, the group consists of ten mobile agents with identical second order dynamics. Initial positions were generated randomly within a ball of radius $R_0 = 2.5[\mathrm{m}]$ centered at the origin. Initial velocities were also selected randomly with arbitrary directions and magnitude in the $(0, 1)[m/s]$ range. The interconnection graph was also generated in random and the neighborhood radius was set to $R = 2[m]$. Figures 3-7 depict snapshots of the system's evolution within a time frame of 100 simulation seconds. The corresponding time instant is given below each Figure. The position of each agent is represented by a small dot and the neighboring relations by line segments connecting them. Velocity vectors are depicted as arrows, with their base point being the position of the corresponding agent. Dotted lines show the trajectory trails for each agent. The system converges to an invariant set that corresponds to a tight formation and a common heading direction, while avoiding collisions. The shape of the formation which the group converges to, is determined by the artificial potential functions.

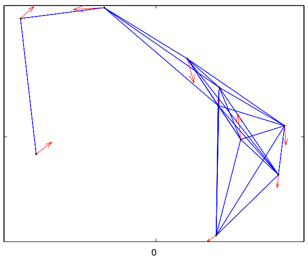

Fig. 3. *Initial configuration.*

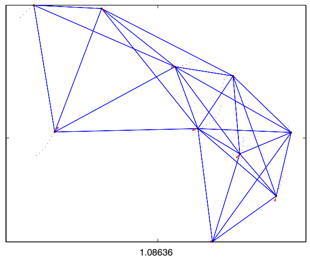

Fig. 4. *Cohesion forces increase connectivity.*

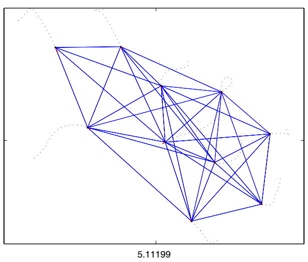

Fig. 5. *A tight formation is created.*

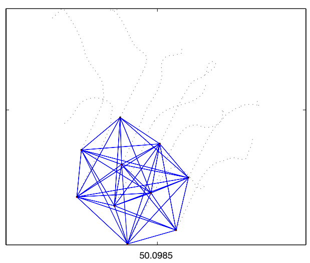

Fig. 6. *The group moves in the same direction.*

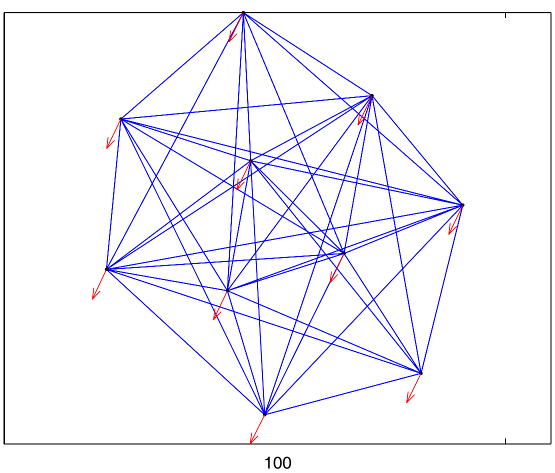

Fig. 7. *Steady state.*

VIII. Conclusions

In this paper we showed that a group of autonomous mobile agents, in which each agent is steered using local state information from its nearest neighbors, can asymptotically exhibit stable flocking behavior. Flocking is being understood as a collision free uniform motion in a tight formation with a common velocity vector. We introduced a set of control laws that guarantees flocking asymptotically, under the assumption that the graph representing agent interconnections remains connected at all times. Agent interconnections can be established and lost arbitrarily without affecting stability, although convergence is shown to be closely related to the algebraic connectivity properties of the graph.

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