7.0

$$f(x_1, x_2) = (x_1 - 1{,}5)^2 + (x_2 - t)^4$$

u.d.N

$$-1 + x_1 + x_2 \leq 0 \quad (g_1)$$
$$-1 + x_1 - x_2 \leq 0 \quad (g_2)$$
$$-1 - x_1 + x_2 \leq 0 \quad (g_3)$$
$$-1 - x_1 - x_2 \leq 0 \quad (g_4)$$

Aus der Lagrange-Funktion:

$$L(x, \lambda) = (x_1 - 1{,}5)^2 + (x_2 - t)^4 + \lambda_1(-1 + x_1 + x_2) + \lambda_2(-1 + x_1 - x_2) + \lambda_3(-1 - x_1 + x_2) + \lambda_4(-1 - x_1 - x_2)$$

für $x^1 = (1, 0)$

$$g_1 = -1 \underbrace{+ x_1 + x_2}_{=0} \leq 0 \quad \checkmark \text{ (aktiv)}$$
$$g_2 = -1 \underbrace{+ x_1 - x_2}_{=0} \leq 0 \quad \checkmark \text{ (aktiv)}$$
$$g_3 = -1 \underbrace{- x_1 + x_2}_{-2} \leq 0 \quad \checkmark \text{ } (\cancel{\text{aktiv}}) \text{ (inaktiv)}$$
$$g_4 = -1 \underbrace{- x_1 - x_2}_{=-2} \leq 0 \quad \checkmark \text{ } (\cancel{\text{aktiv}}) \text{ (inaktiv)}$$

$$\nabla f = \begin{pmatrix} 2(x_1 - 1{,}5) \\ 4(x_2 - t)^3 \end{pmatrix} ; \quad \nabla f(1, 0) = \begin{pmatrix} -1 \\ -4t^3 \end{pmatrix} ; \quad \nabla g_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} ; \quad \nabla g_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\nabla L(x, \lambda) = \begin{pmatrix} -1 \\ -4t^3 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} \stackrel{!}{=} 0$$

$$-1 + \lambda_1 + \lambda_2 = 0 \qquad -4t^3 + \lambda_1 - \lambda_2 = 0$$

$$\lambda_1 = +1 - \lambda_2 \Rightarrow -4t^3 + 1 - \lambda_2 - \lambda_2 = 0$$

$$\lambda_2 = \frac{1 - 4t^3}{2}$$

$$\lambda_1 = \frac{1 + 4t^3}{2} \qquad \lambda_1, \lambda_2 \geq 0$$

$$\lambda_1 = \frac{1 + 4t^3}{2} \geq 0 \quad \Leftrightarrow \quad 1 + 4t^3 \geq 0 \quad | -1 \; | \, 4$$
$$t^3 \geq -\frac{1}{4}$$
$$t \geq -\sqrt[3]{\frac{1}{4}}$$

$$\lambda_2 = \frac{1 - 4t^3}{2} \geq 0$$
$$\Leftrightarrow 1 - 4t^3 \geq 0$$
$$t^3 \leq \frac{1}{4}$$
$$t \leq \sqrt[3]{\frac{1}{4}}$$

KKT-Bedingung für $t \in \left[-\sqrt[3]{\frac{1}{4}}, \sqrt[3]{\frac{1}{4}}\right]$