

3.0 $f(x_1, x_2) = (x_1 - 1.5)^2 + (x_2 - 0.75)^4$

$$-1 + x_1 + x_2 \leq 0 \quad (g_1)$$

$$-1 + x_1 - x_2 \leq 0 \quad (g_2)$$

$$-1 - x_1 + x_2 \leq 0 \quad (g_3)$$

$$-1 - x_1 - x_2 \leq 0 \quad (g_4)$$

$$\begin{aligned} L(x_1, x_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4) &= L(x, \lambda) = (x_1 - 1.5)^2 + (x_2 - 0.75)^4 \\ &\quad + \lambda_1(-1 + x_1 + x_2) + \lambda_2(-1 + x_1 - x_2) \\ &\quad + \lambda_3(-1 - x_1 + x_2) + \lambda_4(-1 - x_1 - x_2) \end{aligned}$$

$$\nabla_x L(x, \lambda) = \begin{pmatrix} 2(x_1 - 1.5) + \lambda_1 + \lambda_2 - \lambda_3 - \lambda_4 \\ 4(x_2 - 0.75)^3 + \lambda_1 - \lambda_2 + \lambda_3 - \lambda_4 \end{pmatrix} =$$

$$\nabla f + \nabla g_1 + \nabla g_2 + \nabla g_3 + \nabla g_4 =$$

$$\begin{pmatrix} 2(x_1 - 1.5) \\ 4(x_2 - 0.75)^3 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \lambda_3 \begin{pmatrix} -1 \\ 1 \end{pmatrix} + \lambda_4 \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$B(x^*, \lambda) = \nabla_{xx}^2 L(x^*, \lambda) = H_f(x^*) + \sum_{i=1}^4 \lambda_i H_{g_i}(x^*)$$

$$\begin{aligned} &= \begin{pmatrix} 2 & 0 \\ 0 & 12(x_2 - 0.75)^2 \end{pmatrix} + \lambda_1 \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \lambda_3 \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ &\quad + \lambda_4 \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{aligned}$$

$$= \begin{pmatrix} 2 & 0 \\ 0 & 12(x_2 - 0.75)^2 \end{pmatrix}$$