## Ejercicio 1 Respuesta impulsiva

Considere el sistema en tiempo discreto cuya entrada es $x(n)$ y la salida $y(n)$. Sabemos que

- $y(n) = g(n) * z(n)$, donde $g(n) = \beta^n$ para $n \geq 0$.
- $z(n) = z_1(n) + z_2(n)$
- $z_1 = f_1(n) * x(n)$, donde $f_1(n) = \alpha_1^n$ para $n \geq 0$.
- $z_2 = f_2(n) * x(n)$, donde $f_2(n) = \alpha_2 \delta(n-\gamma)$.

1. Halle la respuesta impulsiva $h(n)$ tal que $y(n) = h(n) * x(n)$.

$$y[n] = g[n] * z[n] = g[n] * (z_1[n] + z_2[n]) = g[n] * (f_1[n] * x[n] + f_2[n] * x[n])$$

$$= g[n] * (f_1[n] + f_2[n]) * x[n]$$

$$= \{g[n] * (f_1[n] + f_2[n])\} * x[n]$$

$$\therefore \boxed{h[n] = g[n] * (f_1[n] + f_2[n])}$$

Para $n \geq 0$:

$$g[n] * f_1[n] = \sum_{k=-\infty}^{+\infty} \beta^k \alpha_1^{(n-k)} u[k] u[n-k] = \alpha_1^n \sum_{k=0}^{n} \left(\frac{\beta}{\alpha_1}\right)^k$$

$$\therefore (g * f_1)[n] = \alpha_1^n \frac{1 - \left(\frac{\beta}{\alpha_1}\right)^{n+1}}{1 - \frac{\beta}{\alpha_1}} u[n]$$

si $n \geq \gamma$

Para $n \geq 0$:

$$g[n] * f_2[n] = \sum_{k=-\infty}^{+\infty} \beta^{n-k} u[n-k] \alpha_2 \delta[k-\gamma] = \beta^{n-\gamma} \alpha_2$$

$$\therefore (g * f_2)[n] = \beta^{n-\gamma} \alpha_2 u[n-\gamma], \quad \gamma \geq 0.$$

2. Grafique $h(n)$ para

a) $\beta = \frac{1}{2}, \alpha_1 = \frac{1}{5}, \alpha_2 = -3, \gamma = 2$

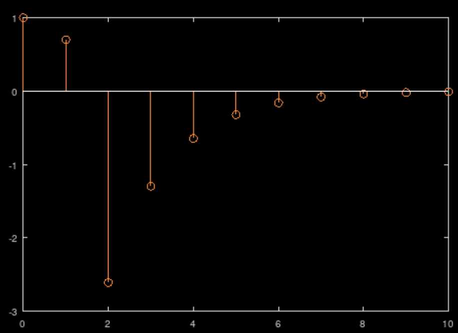
b) $\beta = -\frac{1}{2}, \alpha_1 = \frac{1}{5}, \alpha_2 = -3, \gamma = 2$

c) $\beta = \frac{1}{2}, \alpha_1 = -\frac{1}{5}, \alpha_2 = -3, \gamma = 2$

a) $$(g * f_1)[n] = \left(\frac{1}{5}\right)^n \frac{1 - \left(\frac{1}{10}\right)^{n+1}}{1 - \frac{1}{10}} = \left(\frac{1}{5}\right)^n \frac{10}{9} \left(1 - \left(\frac{1}{10}\right)^{n+1}\right) = 2^n \left(\frac{1}{10}\right)^n \left(10 - \left(\frac{1}{10}\right)^n\right)$$

$(n \geq 0)$,

$$(g * f_2)[n] = \left(\frac{1}{2}\right)^{n-2} (-3) u[n-2]$$



## Ejercicio 2 Respuesta en frecuencia

Considere el sistema en tiempo discreto cuya transferencia es

$$H(z) = 1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}.$$

1. Obtenga la respuesta en frecuencia del sistema $H(\omega)$.

$$\boxed{H(\omega) = 1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}}$$

2. Halle la respuesta impulsiva $h(n)$.

$$\boxed{h[n] = \delta[n] - \frac{3}{4}\delta[n-1] + \frac{1}{8}\delta[n-2].}$$

$$1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2} = 0$$

$$z^{-1} = u$$

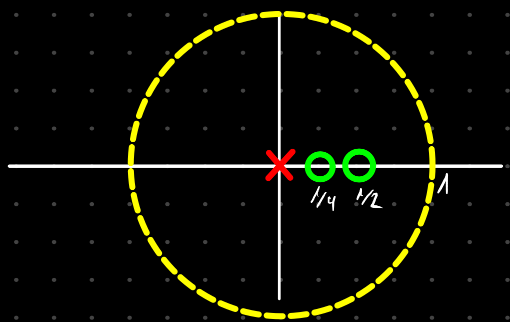
$$1 - \frac{3}{4}u + \frac{1}{8}u^2 = 0$$

$$u = \frac{\frac{3}{4} \pm \sqrt{\frac{9}{16} - \frac{1}{2}}}{1/4}$$

$$u = 2, \quad u = 4$$

$$\Rightarrow z = \frac{1}{2}, \quad z = \frac{1}{4}$$

3. Obtenga el diagrama de polos y ceros




Polo en $z = 0$

Ceros en $z = \frac{1}{2}$, $z = \frac{1}{4}$

## Ejercicio 3 Respuesta en frecuencia

Repita el problema anterior con

$$H(z) = \frac{1}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}.$$

Explique las diferencias con el problema anterior.

1)

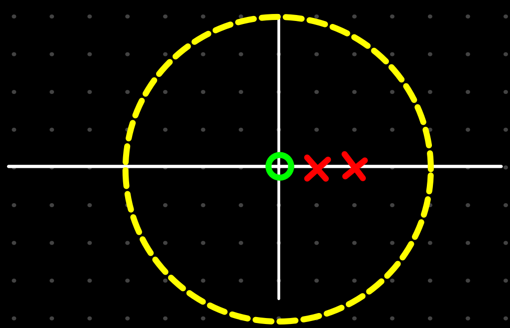
$$\boxed{H(\omega) = \frac{1}{1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}}}$$

2)

$$H(z) = \frac{1}{\frac{1}{8}(z^{-1}-2)(z^{-1}-4)} = 8\left[\frac{-1/2}{z^{-1}-2} + \frac{1/2}{z^{-1}-4}\right] = 8\left[\frac{1/4}{1-\frac{1}{2}z^{-1}} + \frac{-1/8}{1-\frac{1}{4}z^{-1}}\right]$$

$$\boxed{h[n] = \left\{2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right\}u[n]}$$

3)



Polos en $z = 1/2$, $z = 1/4$

Cero en $z = 0$

## Ejercicio 5 Ruido aditivo

Sea $Y = X + N$, con $X$ y $N$ variables aleatorias independientes.

1. Demostrar que $f_Y(y) = f_X(y) * f_N(y)$.

$$f_Y(y) \overset{\text{LPT}}{=} \int_{-\infty}^{+\infty} f_{Y|X}(y|x) f_X(x)\,dx = \int_{-\infty}^{+\infty} f_{N|X}(y-x|x)\, f_X(x)\,dx \overset{\text{indep.}}{=} \int_{-\infty}^{+\infty} f_N(y-x)\, f_X(x)\,dx$$

$$\therefore f_Y(y) = (f_N * f_X)(y) \quad \blacksquare$$

## Ejercicio 6 Cambio de variables

Sean $X$ e $Y$ dos variables exponenciales independientes de parámetros $\lambda_X$ y $\lambda_Y$ respectivamente. Hallar la función de densidad de probabilidad conjunta de $W = XY$ y $V = X/Y$.

$X \sim \mathcal{E}(\lambda_x)$, $Y \sim \mathcal{E}(\lambda_y)$ indep.

$W = XY$, $V = X/Y$.

$$J = \begin{vmatrix} y & x \\ 1/y & -\frac{x}{y^2} \end{vmatrix} = -\frac{x}{y} - \frac{x}{y} = -\frac{2x}{y} = -2v$$

$$f_{W,V}(w,v) = \frac{f_{X,Y}(\sqrt{wv}, \sqrt{w/v})}{|-2v|} \overset{\text{indep}}{=} \frac{f_X(\sqrt{wv})\, f_Y(\sqrt{w/v})}{2v} = \frac{\lambda_x e^{-\lambda_x\sqrt{wv}}\, \lambda_y e^{-\lambda_y\sqrt{w/v}}}{2v}$$

$$\boxed{f_{W,V}(w,v) = \frac{\lambda_x\lambda_y}{2v}\, e^{-\lambda_x\sqrt{wv}}\, e^{-\lambda_y\sqrt{w/v}}}$$