$$F(x) = 1 - e^{-\lambda x} = u \Rightarrow \quad 1-u = e^{-\lambda x}$$

$$\ln(1-u) = -\lambda x$$

$$-\frac{1}{\lambda}\ln(1-u) = x$$

$$F^{-1}(u) = -\frac{1}{\lambda}\ln(1-u)$$

$$\begin{aligned} X_1 &= 0.5\,U_1 - 0.3\,U_2 \\ X_2 &= 0.7\,U_1 + 0.2\,U_2 \end{aligned} \Rightarrow X = \begin{bmatrix} 0.5 & -0.3 \\ 0.7 & 0.2 \end{bmatrix} U$$

$$J = \begin{vmatrix} 0.5 & -0.3 \\ 0.7 & 0.2 \end{vmatrix} = 0.31$$

$$f_{X_1,X_2}(x_1,x_2) = \frac{f_{U_1,U_2}(0.2x_1+0.3x_2, -0.7x_1+0.5x_2)}{0,31} \overset{\text{indep}}{=} \frac{f_{U_1}(0.2x_1+0.3x_2)\, f_{U_2}(0.7x_1+0.5x_2)}{}$$

$$= \frac{\frac{1}{2}\mathbb{1}\{0.2x_1+0.3x_2 \in (0,2)\} + \frac{1}{3}\mathbb{1}\{-0.7x_1+0.5x_2 \in (0,3)\}}{0,31}$$

$$\boxed{f_{X_1,X_2}(x_1,x_2) = \frac{1}{0.62}\mathbb{1}\{0.2x_1+0.3x_2 \in (0,2)\} + \frac{1}{0.93}\mathbb{1}\{-0.7x_1+0.5x_2 \in (0,3)\}}$$ graficar.

Sean U_1, U_2 dos variables aleatorias independientes uniformes ~U(0; 1).

1. Halle la densidad conjunta de las variables:

$$\begin{cases} R = \sqrt{-2\ln(U_1)} \\ \Theta = 2\pi U_2. \end{cases}$$

Verifique que R tiene distribución Rayleigh, que Θ es uniforme y que son independientes (¿por que?). ′

$$\begin{cases} U_1 \sim \mathcal{U}(0,1) \\ U_2 \sim \mathcal{U}(0,1) \end{cases} \qquad \begin{cases} R = \sqrt{-2\ln(U_1)} = g_1(U_1,U_2) \\ \Theta = 2\pi U_2 = g_2(U_1,U_2) \end{cases}$$

$$f_{R,\Theta}(r,\theta) = \frac{f_{U_1}\left(e^{-\frac{r^2}{2}}\right) f_{U_2}\left(\frac{\theta}{2\pi}\right)}{\left(\frac{-2\pi}{u_1\sqrt{-2\ln(u_1)}}\right)}$$

$$= \frac{\mathbb{1}\left\{e^{-\frac{r^2}{2}} \in (0,1),\ \frac{\theta}{2\pi} \in (0,1)\right\}}{\left|\frac{-2\pi}{r e^{-\frac{r^2}{2}}}\right|}$$

$$\frac{\partial g_1}{\partial u_1} = \frac{-2}{2\sqrt{-2\ln(u_1)}} \frac{1}{u_1}$$

$$\frac{\partial g_2}{\partial u_2} = 2\pi$$

$$J = \begin{vmatrix} \frac{-1}{u_1\sqrt{-2\ln(u_1)}} & 0 \\ 0 & 2\pi \end{vmatrix}$$

$$= \frac{-2\pi}{u_1\sqrt{-2\ln(u_1)}} = \frac{-2\pi}{e^{-\frac{r^2}{2}} r}$$

$$f_{R,\Theta}(r,\theta) = \frac{r e^{-\frac{r^2}{2}}}{2\pi} \mathbb{1}\{\theta \in (0,2\pi),\ r > 0\}$$

$$f_R(r) = r e^{-\frac{r^2}{2}} \mathbb{1}\{r > 0\} \Rightarrow R \sim Ray(1)$$

$$f_\Theta(\theta) = \frac{1}{2\pi} \mathbb{1}\{\theta \in (0,2\pi)\} \Rightarrow \Theta \sim \mathcal{U}(0,2\pi)$$

Son independientes pues puedo separar $f_{R,\Theta}(r,\theta) = f_R(r) f_\Theta(\theta)$.

2. Halle la densidad conjunta de las variables:

$$\begin{cases} Z_1 = R\cos\Theta \\ Z_2 = R\sin\Theta \end{cases}$$

y demuestre que se trata de variables normales estándar independientes.

$$J = \begin{vmatrix} \cos\theta & -R\sin\theta \\ \sin\theta & R\cos\theta \end{vmatrix} = R$$

$$f_{Z_1,Z_2}(z_1,z_2) = \frac{f_R\left(\sqrt{z_1^2+z_2^2}\right) f_\Theta\left(\operatorname{atan}\left(\frac{z_2}{z_1}\right)\right)}{\sqrt{z_1^2+z_2^2}} = \frac{\cancel{\sqrt{z_1^2+z_2^2}}\, e^{-\frac{(z_1^2+z_2^2)}{2}} \frac{1}{2\pi}}{\cancel{\sqrt{z_1^2+z_2^2}}}$$

$$f_{Z_1,Z_2}(z_1,z_2) = \frac{e^{-\frac{z_1^2}{2}}}{\sqrt{2\pi}} \frac{e^{-\frac{z_2^2}{2}}}{\sqrt{2\pi}}$$

$$\therefore\ Z_1, Z_2 \overset{iid}{\sim} \mathcal{N}(0,1) \quad \boxtimes$$