Ejercicio 1

Se desea un vector aleatorio $\mathbf{X} = [X_1, X_2, X_3]^t \sim \mathcal{N}(\mu_{\mathbf{X}}, C_{\mathbf{X}})$ donde

$$\mu_{\mathbf{X}} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$
 , $C_{\mathbf{X}} = \begin{bmatrix} 3/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$.

1. Hallar $f_{X_1}(x_1)$.

$$\chi_1 \sim \mathcal{N}(\mu_{\chi_1}, \sigma_{\chi_1}^2)$$
 con $\mu_{\chi_1} = 1$ y $\sigma_{\chi_1}^2 = \frac{3}{2}$

$$f_{\chi_1}(x_1) = \frac{1}{\sqrt{2\pi} \sigma_{\chi_1}} exp\left(-\frac{(x_1 - \mu_{\chi_1})^2}{2 \sigma_{\chi_2}}\right)$$

$$f_{\chi_1}(x) = \frac{1}{\sqrt{3\pi}} \exp\left(-\frac{(x_1 - 1)^2}{3}\right)$$

2. Se construye una nueva variable aleatoria, $Y = \mathbf{a}^t \mathbf{X}$, donde $\mathbf{a} \in \mathbb{R}^3$. Hallar \mathbf{a} tal que $\mathbb{E}[Y] = 0$. Obtenga $\mathbf{Var}[Y]$.

$$E[\gamma] = E[a^{T} \times] = a^{T} E[\times] = [a, a, a, a_{3}] \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = 0$$

:
$$a_1 = -2a_3$$
. $= [-2 \ o \ 1]$

$$Var(Y) = C_{Y} = \alpha^{T} C_{x} (\alpha^{T})^{T} = \alpha^{T} C_{x} \alpha = \begin{bmatrix} -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

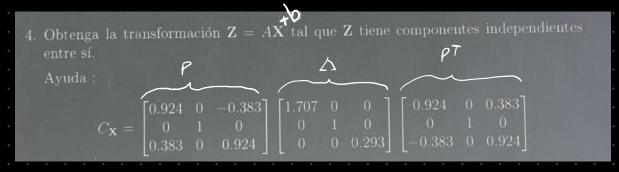
$$=\begin{bmatrix} -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} -9/2 \\ 0 \\ -1/2 \end{bmatrix} = \frac{9}{2}$$

$$Var(y) = \frac{9}{2}$$

. Escriba un pseudocódigo para obtener realizaciones de la variable aleatoria
$$Y$$
 obtenen el punto anterior, a partir de realizaciones del vector aleatorio \mathbf{X} .

Genero N mestras de un vector normal μ_X , χ y las guardo en $X=[\chi_1,\chi_2]$.

$$\mathcal{L} = -2\chi_1(i) + \chi_3(i) ,$$



$$C_{2} = \tilde{\Delta}^{'/2} \tilde{P}^{7} C_{X} (\tilde{\Delta}^{'/2} \tilde{P}^{T})^{T} = \tilde{\Delta}^{''} \tilde{P}^{T} \tilde{P} \tilde{\Delta} \tilde{P}^{T} (\tilde{P}^{T})^{T})^{T}$$

Como Pes ortogenal y
$$\tilde{\Delta}^{1/2}$$
 es diagonal, $P^TP = PP^T = \tilde{I}$ y $(\tilde{\Delta}^{1/2})^T = \tilde{\Delta}^{1/2}$

$$C_{\xi} = \Delta^{1/2} \Delta \Delta^{1/2} = 1$$

$$M_z = E[AX - A_{px}] = A \overline{E[X]} - A_{px} = \overline{O}$$

$$Z = \underbrace{\Lambda^{-1/2} P^{T}}_{A} \times - \underbrace{\Lambda^{-1/2} P^{T}}_{b} \times .$$

$$C_{z} = P^{T} P \Lambda P^{T} (P^{T})^{T} = \Lambda$$
 que ya da componentes independientes por ser Λ diagonal.