

$$F(x) = 1 - e^{-\lambda x} = u \Rightarrow \begin{cases} 1-u = e^{-\lambda x} \\ \ln(1-u) = -\lambda x \\ -\frac{1}{\lambda} \ln(1-u) = x \end{cases}$$

$$F^{-1}(u) = -\frac{1}{\lambda} \ln(1-u)$$

$$\begin{cases} X_1 = 0.5 U_1 - 0.3 U_2 \\ X_2 = 0.7 U_1 + 0.2 U_2 \end{cases} \Rightarrow X = \begin{bmatrix} 0.5 & -0.3 \\ 0.7 & 0.2 \end{bmatrix} U$$

$$J = \begin{vmatrix} 0.5 & -0.3 \\ 0.7 & 0.2 \end{vmatrix} = 0.31$$

$$f_{X_1, X_2}(x_1, x_2) = \frac{f_{U_1, U_2}(0.2x_1 + 0.3x_2, -0.7x_1 + 0.5x_2)}{0.31} \stackrel{\text{indep}}{=} \frac{f_{U_1}(0.2x_1 + 0.3x_2) f_{U_2}(-0.7x_1 + 0.5x_2)}{0.31}$$

$$= \frac{\frac{1}{2} \mathbb{1}\{0.2x_1 + 0.3x_2 \in (0, 2)\} + \frac{1}{3} \mathbb{1}\{-0.7x_1 + 0.5x_2 \in (0, 3)\}}{0.31}$$

$$f_{X_1, X_2}(x_1, x_2) = \frac{1}{0.62} \mathbb{1}\{0.2x_1 + 0.3x_2 \in (0, 2)\} + \frac{1}{0.93} \mathbb{1}\{-0.7x_1 + 0.5x_2 \in (0, 3)\} \quad \text{graficar}$$

Sean U_1, U_2 dos variables aleatorias independientes uniformes $\sim U(0; 1)$.

1. Halle la densidad conjunta de las variables:

$$\begin{cases} R = \sqrt{-2 \ln(U_1)} \\ \Theta = 2\pi U_2 \end{cases}$$

Verifique que R tiene distribución Rayleigh, que Θ es uniforme y que son independientes (¿por qué?).

$$\begin{cases} U_1 \sim \mathcal{U}(0, 1) \\ U_2 \sim \mathcal{U}(0, 1) \end{cases}$$

$$\begin{cases} R = \sqrt{-2 \ln(U_1)} = g_1(U_1, U_2) \\ \Theta = 2\pi U_2 = g_2(U_1, U_2) \end{cases}$$

$$f_{R,\Theta}(r,\theta) = \frac{f_{u_1}\left(e^{-\frac{r^2}{2}}\right) f_{u_2}\left(\frac{\theta}{2\pi}\right)}{\left(\frac{-2\pi}{u_1 \sqrt{-2\ln(u_1)}}\right)}$$

$$= \frac{\mathbb{1}\left\{e^{-\frac{r^2}{2}} \in (0,1), \frac{\theta}{2\pi} \in (0,1)\right\}}{\left|\frac{-2\pi}{r e^{-\frac{r^2}{2}}}\right|}$$

$$\frac{\partial g_1}{\partial u_1} = \frac{-2}{2\sqrt{-2\ln(u_1)}} \frac{1}{u_1}$$

$$\frac{\partial g_2}{\partial u_2} = 2\pi$$

$$J = \begin{vmatrix} \frac{1}{u_1 \sqrt{-2\ln(u_1)}} & 0 \\ 0 & 2\pi \end{vmatrix}$$

$$= \frac{-2\pi}{u_1 \sqrt{-2\ln(u_1)}} = \frac{-2\pi}{e^{-\frac{r^2}{2}} r}$$

$$f_{R,\Theta}(r,\theta) = \frac{r e^{-\frac{r^2}{2}}}{2\pi} \mathbb{1}\left\{\theta \in (0, 2\pi), r > 0\right\}$$

$$f_R(r) = r e^{-\frac{r^2}{2}} \mathbb{1}\{r > 0\} \Rightarrow R \sim \text{Ray}(1)$$

$$f_\Theta(\theta) = \frac{1}{2\pi} \mathbb{1}\{\theta \in (0, 2\pi)\} \Rightarrow \Theta \sim \mathcal{U}(0, 2\pi)$$

Son independientes pues puedo separar $f_{R,\Theta}(r,\theta) = f_R(r) f_\Theta(\theta)$.

2. Halle la densidad conjunta de las variables:

$$\begin{cases} Z_1 = R \cos \Theta \\ Z_2 = R \sin \Theta \end{cases}$$

y demuestre que se trata de variables normales estándar independientes.

$$J = \begin{vmatrix} \cos \Theta & -R \sin \Theta \\ \sin \Theta & R \cos \Theta \end{vmatrix} = R$$

$$f_{Z_1, Z_2}(z_1, z_2) = \frac{f_R(\sqrt{z_1^2 + z_2^2}) f_\Theta(\arctan(\frac{z_2}{z_1}))}{\sqrt{z_1^2 + z_2^2}} = \frac{\cancel{\sqrt{z_1^2 + z_2^2}} e^{-\frac{(z_1^2 + z_2^2)}{2}} \frac{1}{2\pi}}{\cancel{\sqrt{z_1^2 + z_2^2}}}$$

$$f_{Z_1, Z_2}(z_1, z_2) = \frac{e^{-\frac{z_1^2}{2}}}{\sqrt{2\pi}} \frac{e^{-\frac{z_2^2}{2}}}{\sqrt{2\pi}}$$

$$\therefore z_1, z_2 \stackrel{\text{iid}}{\sim} \mathcal{N}(0, 1)$$

□