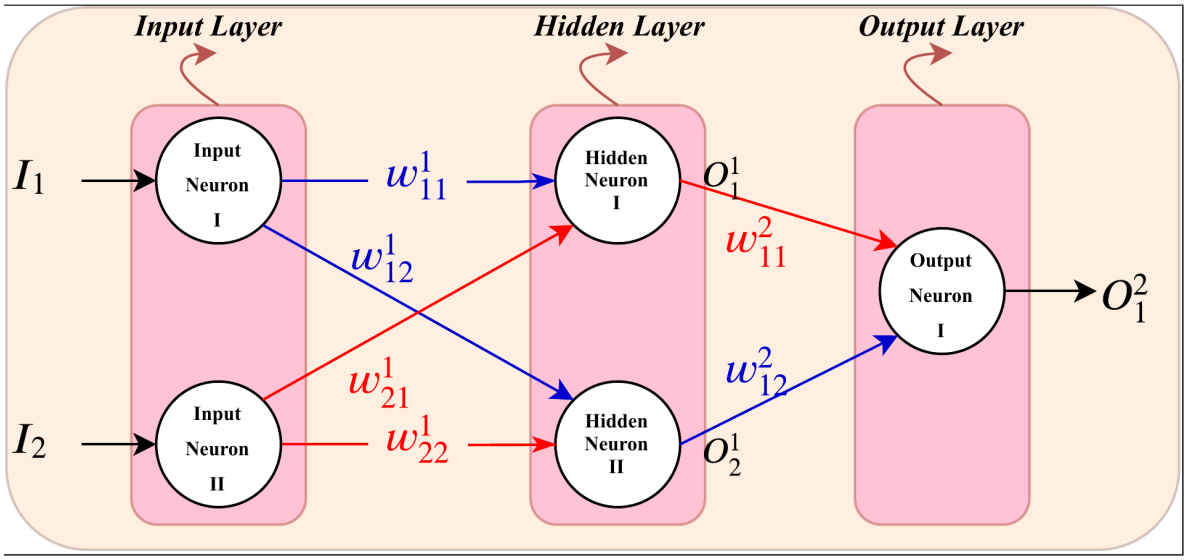
2-2-1 Neural Network formulas



Hidden -> Output

$$O^3_1 = \sigma(O^2_1 w^2_{11} + O^2_2 w^2_{21})$$
$$= \sigma(\begin{bmatrix} w^2_{11} \\ w^2_{21} \end{bmatrix}^T \begin{bmatrix} O^2_1 \\ O^2_2 \end{bmatrix})$$

Input -> Hidden

$$O^2_1 = \sigma(O^1_1 w^1_{11} + O^1_2 w^1_{21})$$
$$O^2_2 = \sigma(O^1_1 w^1_{12} + O^1_2 w^1_{22})$$
$$\begin{bmatrix} O^2_1 \\ O^2_2 \end{bmatrix} = \sigma(\begin{bmatrix} w^1_{11} & w^1_{12} \\ w^1_{21} & w^1_{22} \end{bmatrix}^T \begin{bmatrix} O^1_1 \\ O^1_2 \end{bmatrix})$$
$$\begin{bmatrix} O^1_1 \\ O^1_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Back propagation

$$Error \equiv \delta_O$$
$$= \Sigma(target - calculated)^2 = \Sigma(\Delta)^2$$

$$\delta_{O_1^3} = (\Delta_{O_1^3})^2$$

$$\frac{\partial \delta_{O_1^3}}{\partial w_{11}^2} = \frac{\partial \delta_{O_1^3}}{\partial O_1^3}\frac{\partial O_1^3}{\partial w_{11}^2}$$

$$\frac{\partial \delta_{O_1^3}}{\partial O_1^3} = \frac{\partial (target_1 - O_1^3)^2}{\partial O_1^3}$$

$$= -2\Delta_{O_1^3}$$

$$\frac{\partial O_1^3}{\partial w_{11}^2} = \frac{\partial (\sigma(O_1^2 w_{11}^2 + O_2^2 w_{21}^2))}{\partial w_{11}^2} = \frac{\partial (\sigma(I_{O_1^3}))}{\partial w_{11}^2}$$

$$= O_1^2 \sigma'(I_{O_1^3})$$

$$\frac{\partial O_1^3}{\partial w_{21}^2} = O_2^2 \sigma'(I_{O_1^3})$$

From hidden to input

w^{1}_{11}

$$\frac{\partial \delta_{O_1^3}}{\partial w_{11}^1} = \frac{\partial \delta_{O_1^3}}{\partial O_1^3}\frac{\partial O_1^3}{\partial w_{11}^1}$$

$$\frac{\partial O_1^3}{\partial w_{11}^1} = \frac{\partial (\sigma(O_1^2 w_{11}^2 + O_2^2 w_{21}^2))}{\partial w_{11}^1}$$

$$\frac{\partial O_1^3}{\partial w_{11}^1} = w_{11}^2 \frac{\partial O_1^2}{\partial w_{11}^1}\sigma'(I_{O_1^3})$$

$$\frac{\partial O_1^2}{\partial w_{11}^1} = \frac{\partial (\sigma(O_1^1 w_{11}^1 + O_2^1 w_{21}^1))}{\partial w_{11}^1} = O_1^1 \sigma'(I_{O_1^2})$$

$$\frac{\partial O_1^3}{\partial w_{11}^1} = w_{11}^2 O_1^1 \sigma'(I_{O_1^2})\sigma'(I_{O_1^3})$$

w^{1}_{12}

$$\frac{\partial \delta_{O_1^3}}{\partial w_{12}^1} = \frac{\partial \delta_{O_1^3}}{\partial O_1^3}\frac{\partial O_1^3}{\partial w_{12}^1}$$

$$\frac{\partial O_1^3}{\partial w_{12}^1} = \frac{\partial (\sigma(O_1^2 w_{11}^2 + O_2^2 w_{21}^2))}{\partial w_{12}^1}$$

$$\frac{\partial O_1^3}{\partial w_{12}^1} = w_{21}^2 \frac{\partial O_2^2}{\partial w_{12}^1}\sigma'(I_{O_1^3})$$

$$\frac{\partial O_2^2}{\partial w_{12}^1} = \frac{\partial (\sigma(O_1^1 w_{12}^1 + O_2^1 w_{22}^1))}{\partial w_{12}^1} = O_1^1 \sigma'(I_{O_2^2})$$

$$\frac{\partial O_1^3}{\partial w_{12}^1} = w_{21}^2 O_1^1 \sigma'(I_{O_2^2})\sigma'(I_{O_1^3})$$

w^{1}_{21}

$$\frac{\partial \delta_{O_1^3}}{\partial w_{21}^1} = \frac{\partial \delta_{O_1^3}}{\partial O_1^3}\frac{\partial O_1^3}{\partial w_{21}^1}$$

$$\frac{\partial O_1^3}{\partial w_{21}^1} = \frac{\partial(\sigma(O_1^2 w_{11}^2 + O_2^2 w_{21}^2))}{\partial w_{21}^1}$$

$$\frac{\partial O_1^3}{\partial w_{21}^1} = w_{11}^2 \frac{\partial O_1^2}{\partial w_{11}^1}\sigma'(I_{O_1^3})$$

$$\frac{\partial O_1^2}{\partial w_{21}^1} = \frac{\partial(\sigma(O_1^1 w_{11}^1 + O_2^1 w_{21}^1))}{\partial w_{21}^1} = O_2^1\sigma'(I_{O_1^2})$$

$$\frac{\partial O_1^3}{\partial w_{21}^1} = w_{11}^2 O_2^1 \sigma'(I_{O_1^2})\sigma'(I_{O_1^3})$$

w^{1}_{22}

$$\frac{\partial \delta_{O_1^3}}{\partial w_{22}^1} = \frac{\partial \delta_{O_1^3}}{\partial O_1^3}\frac{\partial O_1^3}{\partial w_{22}^1}$$

$$\frac{\partial O_1^3}{\partial w_{22}^1} = \frac{\partial(\sigma(O_1^2 w_{11}^2 + O_2^2 w_{21}^2))}{\partial w_{22}^1}$$

$$\frac{\partial O_1^3}{\partial w_{12}^1} = w_{21}^2 \frac{\partial O_2^2}{\partial w_{22}^1}\sigma'(I_{O_1^3})$$

$$\frac{\partial O_2^2}{\partial w_{22}^1} = \frac{\partial(\sigma(O_1^1 w_{12}^1 + O_2^1 w_{22}^1))}{\partial w_{22}^1} = O_2^1\sigma'(I_{O_2^2})$$

$$\frac{\partial O_1^3}{\partial w_{12}^1} = w_{21}^2 O_2^1 \sigma'(I_{O_2^2})\sigma'(I_{O_1^3})$$

$$\frac{\partial O_1^3}{\partial w_{11}^2} = O_1^2\sigma'(I_{O_1^3})$$

$$\frac{\partial O_1^3}{\partial w_{21}^2} = O_2^2\sigma'(I_{O_1^3})$$

$$\frac{\partial O_1^3}{\partial w_{11}^1} = w_{11}^2 O_1^1 \sigma'(I_{O_1^2})\sigma'(I_{O_1^3})$$

$$\frac{\partial O_1^3}{\partial w_{12}^1} = w_{21}^2 O_1^1 \sigma'(I_{O_2^2})\sigma'(I_{O_1^3})$$

$$\frac{\partial O_1^3}{\partial w_{21}^1} = w_{11}^2 O_2^1 \sigma'(I_{O_1^2})\sigma'(I_{O_1^3})$$

$$\frac{\partial O_1^3}{\partial w_{12}^1} = w_{21}^2 O_2^1 \sigma'(I_{O_2^2})\sigma'(I_{O_1^3})$$