

# Identidades relevantes COMPLEMENTO I

Sea  $X_i = X_i(\{q\}, t)$

$$\gamma \quad \frac{\partial q_i}{\partial q_e} = \delta_{ie} \Rightarrow \frac{\partial \dot{q}_i}{\partial \dot{q}_e} = \delta_{ie} ; \quad \frac{\partial q_e}{\partial \dot{q}_j} = 0 \wedge \frac{\partial \dot{q}_e}{\partial q_j} = 0$$

$$\therefore dX_i = \sum_{j=1}^s \frac{\partial X_i}{\partial q_j} dq_j + \frac{\partial X_i}{\partial t} dt \quad *$$

$$\dot{X}_i = \sum_{j=1}^s \frac{\partial X_i}{\partial q_j} \dot{q}_j + \frac{\partial X_i}{\partial t} \quad (i)$$

también:

$$\frac{\partial \dot{X}_i}{\partial \dot{q}_e} = \sum_{j=1}^s \frac{\partial}{\partial \dot{q}_e} \left( \frac{\partial X_i}{\partial q_j} \cdot \dot{q}_j \right) + \frac{\partial}{\partial \dot{q}_e} \frac{\partial X_i}{\partial t}$$

$$= \sum_{j=1}^s \frac{\partial}{\partial \dot{q}_e} \left( \frac{\partial X_i}{\partial q_j} \right) \dot{q}_j + \sum_{j=1}^s \frac{\partial X_i}{\partial q_j} \frac{\partial \dot{q}_j}{\partial \dot{q}_e} + \frac{\partial}{\partial \dot{q}_e} \left( \frac{\partial X_i}{\partial t} \right)$$

$$= \sum_{j=1}^s \frac{\partial}{\partial q_j} \left( \frac{\partial X_i}{\partial \dot{q}_e} \right) \dot{q}_j + \sum_{j=1}^s \frac{\partial X_i}{\partial q_j} \delta_{je}$$

$$\therefore \frac{\partial \dot{X}_i}{\partial \dot{q}_e} = \frac{\partial X_i}{\partial q_e} \quad (ii)$$

See  $\frac{\partial X_i}{\partial q_k} = \frac{\partial X_i}{\partial q_k}(\{q\}, t)$

II

∴

$$d\left(\frac{\partial X_i}{\partial q_k}\right) = \sum_{k=1}^s \frac{\partial}{\partial q_k} \left( \frac{\partial X_i}{\partial q_k} \right) dq_k + \frac{\partial}{\partial t} \left( \frac{\partial X_i}{\partial q_k} \right) dt$$

$$\frac{d}{dt} \left( \frac{\partial X_i}{\partial q_k} \right) = \sum_{k=1}^s \frac{\partial}{\partial q_k} \left( \frac{\partial X_i}{\partial q_k} \right) \dot{q}_k + \frac{\partial}{\partial t} \left( \frac{\partial X_i}{\partial q_k} \right)$$

però  $\frac{\partial}{\partial q_k} \frac{\partial X_i}{\partial q_k} = \frac{\partial}{\partial q_k} \frac{\partial}{\partial q_k} \wedge \frac{\partial}{\partial t} \frac{\partial X_i}{\partial q_k} = \frac{\partial}{\partial q_k} \frac{\partial X_i}{\partial t}$

∴  $\frac{d}{dt} \left( \frac{\partial X_i}{\partial q_k} \right) = \frac{\partial}{\partial q_k} \left( \underbrace{\sum_{k=1}^s \frac{\partial X_i}{\partial q_k} \dot{q}_k + \frac{\partial X_i}{\partial t}}_{\frac{dX_i}{dt}} \right)$

Finalmente

$$\frac{d}{dt} \left( \frac{\partial X_i}{\partial q_k} \right) = \frac{\partial \dot{X}_i}{\partial q_k} \quad (iii)$$