Problema I

$$(a\hat{k}^3 + \beta\hat{x})|\phi\rangle = 0$$

$$\beta\hat{x}|\phi\rangle = -a\hat{k}^3|\phi\rangle \quad / \quad \hat{x}|\phi\rangle = \underbrace{\frac{-a}{\beta}}_{\gamma}\hat{k}^3|\phi\rangle$$

escalar $\gamma = \frac{-a}{\beta}$

a) $\rightarrow \hat{x}|\phi\rangle = \gamma\hat{k}^3|\phi\rangle \quad /\cdot \langle x|(\cdot)$

$\rightarrow \langle x|\hat{x}|\phi\rangle = \langle x|\gamma\hat{k}^3|\phi\rangle \quad /\cdot \langle x|\hat{x} = \langle x|x$

$\hat{k}$ es el operador n° de onda $\hat{k} = -i\frac{d}{dx}$ $\therefore \hat{k}^3 = \hat{k}^2\hat{k}^1 = -i\frac{d}{dx}\left(-i\frac{d}{dx}\left(-i\frac{d}{dx}\right.\right.$

ya que $\frac{di}{dx} = 0$ $\quad \hat{k}^3 = (-i)^3\frac{d^3}{dx^3} = i\frac{d^3}{dx^3}$

$\rightarrow x\langle x|\phi\rangle = \gamma\langle x|i\frac{d^3}{dx^3}|\phi\rangle = i\gamma\frac{d^3}{dx^3}\langle x|\phi\rangle \quad /\cdot \langle x|\phi\rangle = \phi(x)$

$\rightarrow x\phi(x) = i\gamma\frac{d^3}{dx^3}\phi(x) \rightarrow \boxed{x\phi(x) = -i\frac{a}{\beta}\frac{d^3}{dx^3}\phi(x)}$

cuántica

b) $\hat{x}|\phi\rangle = \gamma\hat{k}^3|\phi\rangle \quad /\cdot \langle k|(\cdot)$

$\circledast \cdot \langle k|\hat{x}|\phi\rangle = \gamma\langle k|\hat{k}^3|\phi\rangle \quad /\cdot \langle k|\hat{k}^3 = \langle k|k^3 = k^3\langle k|$

escalar

espacio de Hilbert	espacio n° onda k
$\hat{k}$	k
$\ldots(\hat{\ })$	$g(k)$

espacio de Hilbert	espacio nº onda k
$\hat{k}$	k
$g(\hat{k})$	$g(k)$
$\hat{x}$	$i\frac{d}{dk}$
$f(\hat{x})$	$f\left(i\frac{d}{dk}\right)$

$$\langle k|\hat{x}|\phi\rangle = \langle k|\, i\frac{d}{dk}\,|\phi\rangle = i\frac{d}{dk}\langle k|\phi\rangle = i\frac{d}{dk}\tilde{\phi}(k)$$

así la ecuación ⊛ $\quad i\frac{d}{dk}\tilde{\phi}(k) = \gamma k^3\tilde{\phi}(k)$

$$\boxed{i\frac{d}{dk}\tilde{\phi}(k) = -\frac{\alpha}{\beta}k^3\tilde{\phi}(k)}$$ ecuación

ec) $\left[\frac{d}{dk}\tilde{\Phi}(k) = \frac{i\alpha}{\beta}k^3\tilde{\phi}(k)\right]$ por metodos numericos o usa Mathematica

$$\frac{d}{dx}y + Cx^3y = 0 \rightarrow y = C_1 e^{-\frac{cx^4}{4}} \quad / \quad C = -\frac{i\alpha}{\beta}$$

La sol. es $\tilde{\Phi}(k) = C_1 e^{\frac{i\alpha k^4}{4\beta}}$ para C_1 requerimos cond. de borde

la funcion $\phi(x) = \langle x|\phi\rangle = \langle x|\hat{\mathbb{1}}|\phi\rangle = \int_{-\infty}^{\infty}\langle x|k\rangle\langle k|\phi\rangle\, dk$

$$\phi(x) = \int_{-\infty}^{\infty}\langle x|k\rangle\,\tilde{\phi}(k)\,dk$$ ← Fourier Transform

donde $\langle x|k\rangle = \frac{1}{\sqrt{2\pi}}e^{ikx}$ & $\tilde{\phi}(k) = Ce^{\frac{i\alpha k^4}{4\beta}}$

así $$\phi(x) = \int_{-\infty}^{\infty}\frac{C}{\sqrt{2\pi}}e^{ikx}e^{i\frac{\alpha k^4}{4\beta}}$$

$$\phi(x) = \frac{c}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{i\left(kx + \frac{\alpha k^4}{4\beta}\right)}dk$$