En términos de $u$

a) $r^2 \frac{d\phi}{d\tau} = L \rightarrow \boxed{\frac{d\tau}{d\phi} = \frac{1}{L u^2}}$ (1)

b) $\left(1 - \frac{r_s}{r}\right)\frac{dt}{d\tau} = E \rightarrow dt = \frac{E}{1 - r_s u} d\tau$

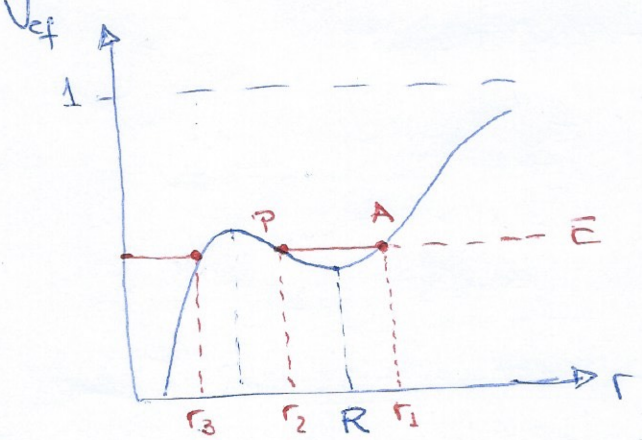
$$dt = \frac{E}{1 - r_s u} \cdot \frac{1}{L u^2} d\phi$$

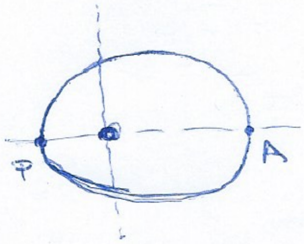
$$\boxed{\frac{dt}{d\phi} = \frac{E/L}{u^2(1 - r_s u)}} \quad (2)$$

junto con

$$\boxed{\frac{du}{d\phi} = \pm\sqrt{r_s}\,\sqrt{g(u)}}$$

$$\boxed{g(u) = u^3 - \frac{u^2}{r_s} + \frac{u}{L^2} - \frac{\mathcal{E}^2}{r_s L^2}}$$



cl 17 ②

Hagamos el siguiente cambio de variable

$$\boxed{u = \frac{1 + e\cos x}{R}}$$

El orden de jerarquía : $0 < r_3 < r_2 < r < r_1$

$$u_1 < u < u_2 < u_3 < \infty$$

- En el afelio : $x_A = \pi \Rightarrow u = u_1$

$$\boxed{u_1 = \frac{1-e}{R}}$$

- En el perihelio : $x_p = 0 \Rightarrow u = u_2$

$$\boxed{u_2 = \frac{1+e}{R}}$$

- $u_1 + u_2 + u_3 = \frac{1}{r_s} \Rightarrow \boxed{u_3 = \frac{1}{r_s} - \frac{2}{R}}$

Debemos escribir el polinomio :

$$g(u) = (u - u_1)(u_2 - u)(u_3 - u)$$

$$\Rightarrow g(x) = \left(\frac{\cancel{1} + e\cos x - \cancel{1} + e}{R}\right)\left(\frac{\cancel{1} + e - \cancel{1} - e\cos x}{R}\right) \times$$

$$\times \left(\frac{1}{r_s} - \frac{2 + 1 + e\cos x}{R}\right)$$

$$g(x) = \left(\frac{e}{R}\right)^2 (1 + \cos x)(1 - \cos x)\left(\frac{1}{r_s} - \frac{3 + e\cos x}{R}\right)$$

$$g(\chi) = \left(\frac{e}{R}\right)^2 \sin^2\chi \cdot \frac{1}{r_s}\left(1 - \frac{3r_s + r_s e\cos\chi}{R}\right)$$

Definamos la cantidad

$$\mu = \frac{r_s}{2R}$$

$$\boxed{g(\chi) = \frac{1}{r_s}\left(\frac{e}{R}\right)^2 \sin^2\chi \left(1 - 6\mu - 2\mu e\cos\chi\right)}$$

Por otro lado, $\frac{du}{d\phi} = \frac{du}{d\chi}\frac{d\chi}{d\phi}$

Pero $\frac{du}{d\chi} = -\frac{e}{R}\sin\chi \rightarrow \boxed{\frac{du}{d\phi} = -\left(\frac{e}{R}\right)\sin\chi\frac{d\chi}{d\phi}}$

$$\therefore -\cancel{\left(\frac{e}{R}\right)}\cancel{\sin\chi}\frac{d\chi}{d\phi} = \pm\cancel{\sqrt{r_s}}\cdot\frac{1}{\cancel{\sqrt{r_s}}}\cancel{\left(\frac{e}{R}\right)}\cancel{\sin\chi}\sqrt{1-6\mu-2\mu e\cos\chi}$$

$$\rightarrow \frac{d\chi}{d\phi} = \mp\sqrt{1-6\mu-2\mu e\cos\chi}$$

Nota: Desde aquí, siguiendo la regla de la cadena, podríamos integrar (1) y (2)

* órbitas planetarias (elípticas)

Tenemos que $0 < e < 1$

CL17 (4)

Veamos que $\cos x = 2\cos^2 \frac{1}{2}x - 1$

$\therefore f(x) \equiv 1 - 6\mu - 2\mu e \cos x = 1 - 6\mu - 2\mu e\left(2\cos^2 \frac{1}{2}x - 1\right)$

$$f(x) = 1 - 6\mu + 2\mu e - 4\mu e \cos^2 \frac{1}{2}x$$

$$f(x) = (1 - 6\mu + 2\mu e)\left(1 - \frac{4\mu e}{1 - 6\mu + 2\mu e}\cos^2 \frac{1}{2}x\right)$$

Hagamos $\boxed{k^2 = \frac{4\mu e}{1 - 6\mu + 2\mu e} \equiv \frac{4\mu e}{A^2}}$

$$f(x) = A^2\left(1 - k^2 \cos^2 \frac{1}{2}x\right)$$

con $\boxed{\gamma = \frac{\pi}{2} - \frac{x}{2}} \Rightarrow \cos\frac{x}{2} = \cos\left(\frac{\pi}{2} - \gamma\right) = \sin\gamma$

$\boxed{dx = -2\,d\gamma}$

$$\boxed{\frac{d\gamma}{d\phi} = \pm \frac{A}{2}\sqrt{1 - k^2 \sin^2\gamma}}$$

- En el afelio $x_A = \pi \Rightarrow \gamma_A = 0$, y pongamos $\phi_A = 0$

$$\therefore \frac{A}{2}\int_0^{\phi} d\phi' = \int_0^{\gamma} \frac{d\gamma'}{\sqrt{1 - k^2 \sin^2\gamma'}}$$

c17 ⑤

$$\boxed{\frac{A}{2}\phi = F(\gamma, k)} \quad (*)$$

Antes de avanzar: $u_1 < \underbrace{u_2 < u_3}$

$$\frac{1+e}{R} < \frac{1}{r_s} - \frac{2}{R} \quad / r_s$$

$$2\mu + 2\mu e < 1 - 4\mu$$

$$6\mu < 1 - 2\mu e \quad / + 2\mu e$$

$$6\mu + 2\mu e < 1 - 4\mu e$$

$$4\mu e < 1 - 6\mu + 2\mu e$$

$$\frac{4\mu e}{1 - 6\mu + 2\mu e} < 1 \Rightarrow \underline{\underline{k^2 < 1}}$$

Sm en $(*)$

$$\sin\gamma = \operatorname{sm}\left(\tfrac{1}{2}A\phi\right)$$

$$\sin\left(\frac{\pi}{2} - \frac{\chi}{2}\right) = \operatorname{sm}\left(\tfrac{1}{2}A\phi\right)$$

$$\cos\tfrac{1}{2}\chi = \operatorname{sm}\left(\tfrac{1}{2}A\phi\right) \quad / e / + 1 / \frac{1}{R}$$

$$\frac{1 + e\cos\frac{1}{2}\chi}{R} = \frac{1 + e\operatorname{sm}\left(\frac{1}{2}A\phi\right)}{R} = u = \frac{1}{r}$$

$$\boxed{r(\phi) = \frac{R}{1 + e\operatorname{sm}\left(\frac{1}{2}A\phi\right)}}$$

~~Hagamos $\tilde{L} = L/r_s$~~

$$L_c^2 = 3 r_s^2$$

~~$R = r_s \tilde{L} \left(1 + \sqrt{1 - \frac{3}{\tilde{L}^2}}\right)$~~

$(L > L_c)$

$$\boxed{R = 3 r_s \left(\frac{L}{L_c}\right)^2 \left[1 + \sqrt{1 - \left(\frac{L_c}{L}\right)^2}\right]}$$

$$A = \sqrt{1 - 6\mu + 2\mu e}$$

$$k = \sqrt{\frac{4\mu e}{1 - 6\mu + 2\mu e}}$$

Veamos que $\quad u_1 u_2 u_3 = \dfrac{\varepsilon^2}{r_s L^2}$

$$\left(\frac{1 - e^2}{R^2}\right) \times \left(\frac{1}{r_s} - \frac{2}{R}\right) = \frac{\varepsilon^2}{r_s L^2}$$

$$(1 - e^2) \times \frac{1}{R^2} \times \left(1 - 2\frac{r_s}{R}\right) = \frac{\varepsilon^2}{L^2}$$

$$\rightarrow \quad 1 - e^2 = \frac{R^2}{1 - 4\mu} \cdot \frac{\varepsilon^2}{L^2} \qquad ; \quad b^2 = \frac{L^2}{\varepsilon^2}$$

$$e^2 = 1 - \frac{R^2/b^2}{1 - 4\mu}$$

$$\Rightarrow \quad \boxed{e = \sqrt{1 - \frac{R^2/b^2}{1 - 4\mu}}}$$