PROBLEMA GUÍA II | #1

Para la i-ésima componente se tiene que:

$$\left(\nabla\times\nabla\left[e^{-\nabla\cdot(\alpha r^2\vec{r})}\right]\right)_i = \epsilon_{ijk}\,\partial_j\partial_k\left(e^{-\nabla\cdot(\alpha r^2\vec{r})}\right)$$

$$= \epsilon_{ijk}\,\partial_j\partial_k\, e^{-\partial_l \alpha r^2 x_l}$$

$$= \epsilon_{ijk}\,\partial_j\partial_k\, e^{-\alpha\,\partial_l(r^2 x_l)}$$

Obs. $\partial_l(r^2x_l) = (\partial_l r^2)\,x_l + r^2\partial_l x_l$

$$= 2r(\partial_l r)\,x_l + r^2\delta_{ll}$$

$$= 2r\frac{x_l}{r}x_l + 3r^2$$

$$= 2x_lx_l + 3r^2$$

$$= 2r^2 + 3r^2 = 5r^2.$$

$\therefore$

$$\left(\nabla\times\nabla\left[e^{-\nabla\cdot(\alpha r^2\vec{r})}\right]\right)_i = \epsilon_{ijk}\,\partial_j\partial_k\, e^{-5\alpha r^2}$$

Obs. sea $r^2 = u$

veamos

$$\partial_k e^{-5\alpha r^2} = \partial_k e^{-5\alpha u}$$; usando la regla de la cadena

$$\Downarrow$$

$$\partial_k e^{-5\alpha r^2} = \frac{\partial}{\partial x_k} e^{-5\alpha r^2} = \frac{\partial u}{\partial x_k}\frac{\partial}{\partial u} e^{-5\alpha u}$$

$$= \frac{\partial u}{\partial x_k}\left(-5\alpha e^{-5\alpha u}\right)$$

$$= -5\alpha e^{-5\alpha u}\, \partial_k u$$

$$= -5\alpha e^{-5\alpha r^2}\left(\partial_k r^2\right)$$

$$= -5\alpha e^{-5\alpha r^2}\, 2r\left(\partial_k r\right)$$

$$= -5\alpha e^{-5\alpha r^2}\, 2\cancel{r}\, \frac{x_k}{\cancel{r}}$$

$$= -10\alpha\, e^{-5\alpha r^2}\, x_k$$

Entonces:

$$\left(\nabla \times \nabla\left[e^{-\nabla\cdot(\alpha r^2 \vec{r})}\right]\right)_i = \epsilon_{ijk}\, \partial_j\left(-10\alpha\, e^{-5\alpha r^2} x_k\right)$$

$$= -10\alpha\, \epsilon_{ijk}\, \partial_j\left(x_k e^{-5\alpha r^2}\right)$$

$$= -10\alpha\, \epsilon_{ijk}\left[e^{-5\alpha r^2} \underbrace{(\partial_j x_k)}_{\delta_{jk}} + x_k \underbrace{\left(\partial_j e^{-5\alpha r^2}\right)}_{-10\alpha\, e^{-5\alpha r^2} x_j} \right]$$

$$= -10\alpha\, e^{-5\alpha r^2}\left[\cancelto{0}{\epsilon_{ijk}\delta_{jk}} - 10\alpha\, \underbrace{\cancelto{0}{\epsilon_{ijk} x_j x_k}}_{\cancelto{0}{(\vec{r}\times\vec{r})_i}} \right]$$

$$= 0 \;//$$

$$\therefore \quad \nabla \times \nabla\left[e^{-\nabla\cdot(\alpha r^2 \vec{r})} \right] = \vec{0}$$

$$\Downarrow$$

El resultado es obvio dado que el rotor de cualquier gradiente siempre es nulo.