$\boxed{\text{Formulario III}}$

Método de Brackets: Definiciones, reglas y teoremas

Def. 1: El bracket

$$\boxed{\langle \alpha \rangle \equiv \int_0^{\infty} x^{\alpha-1}\, dx} \tag{1}$$

Teor. 1: Teorema de escalamiento

$$\boxed{\langle \alpha\beta \rangle = \frac{1}{|\beta|} \langle \alpha \rangle = \frac{1}{|\alpha|} \langle \beta \rangle} \tag{2}$$

Teor. 2: Teorema de simetría

$$\boxed{\langle -\alpha \rangle = \langle \alpha \rangle} \tag{3}$$

Def. 2: El indicador

$$\boxed{\phi_n = \frac{(-1)^n}{n!} = \frac{(-1)^n}{\Gamma(n+1)}} \tag{4}$$

Regla 1: Regla de sumación

$$\boxed{\sum_{n\geq 0} \phi_n\ C(n) \langle n+\alpha \rangle = C(-\alpha)\,\Gamma(\alpha)} \tag{5}$$

Teor. 3: Teorema de escalamiento generalizado

$$\boxed{\langle \alpha n+\beta \rangle = \frac{1}{|\alpha|} \left\langle n+\frac{\beta}{\alpha} \right\rangle} \tag{6}$$

Teor. 4: Teorema de absorción

$$\boxed{F(n) \langle n+\alpha \rangle = F(-\alpha) \langle n+\alpha \rangle} \tag{7}$$

Notación 1: Producto de indicadores

$$\boxed{\phi_{n_1} = \phi_1} \tag{8}$$

$$\boxed{\phi_{n_1}\phi_{n_2}...\phi_{n_N} = \phi_{1,...,N}} \tag{9}$$

Teor. 5: Teorema de sumación múltiple

$$\boxed{\begin{aligned}&\sum_{n_1\geq 0}...\sum_{n_r\geq 0}\phi_1...\phi_r\ C\left(n_1,...,n_r\right)\left\langle a_{11}n_1+...+a_{1r}n_r+c_1\right\rangle...\left\langle a_{11}n_1+...+a_{1r}n_r+c_1\right\rangle\\ &=\frac{1}{\left|\det A\right|}C\left(n_1^*,...,n_r^*\right)\Gamma\left(-n_1^*\right)...\Gamma\left(-n_r^*\right)\end{aligned}} \tag{10}$$

con:

$$A=\begin{pmatrix} a_{11} & \cdots & a_{1r}\\ \vdots & \ddots & \vdots\\ a_{r1} & \cdots & a_{rr}\end{pmatrix} \tag{11}$$

y el vector:

$$\mathbf{n}^*=\begin{pmatrix} n_1^*\\ \vdots\\ n_r^*\end{pmatrix} \tag{12}$$

solución de la ecuación:

$$\mathbf{n}^*=A^{-1}\mathbf{c} \tag{13}$$

siendo:

$$\mathbf{c}=\begin{pmatrix} -c_1\\ \vdots\\ -c_r\end{pmatrix} \tag{14}$$

Teor. 6: Teorema de expansión multinomial

$$\boxed{\frac{1}{\left(A_1+...+A_n\right)^{\alpha}}=\sum_{k_1\geq 0}...\sum_{k_n\geq 0}\phi_1...\phi_r\ A_1^{k_1}...A_n^{k_n}\frac{\left\langle\alpha+k_1+...+k_n\right\rangle}{\Gamma\left(\alpha\right)}} \tag{15}$$

Teor. 7: teorema de integración Si:

$$f\left(x\right)=\sum_{n\geq 0}\phi_n\ F\left(n\right)x^{\alpha n+\beta-1}$$

entonces se cumple que:

$$\boxed{\int\limits_0^{\infty}f\left(x\right)\ dx=\frac{1}{\left|\alpha\right|}F\left(-\frac{\beta}{\alpha}\right)\Gamma\left(\frac{\beta}{\alpha}\right)} \tag{16}$$