COMPLEMENTO I

1 Funciones Hipergeométricas conocidas (Vers. 1.0)

1.1 Funciones elementales

$$\left[\frac{1}{2}(1-x)^{\frac{1}{2}}+\frac{1}{2}\right]^{1-2a} = {}_2F_1\left(\begin{array}{c} 2a \quad , \quad a+1 \\ a \end{array}\middle| x\right) \tag{1}$$

$$(1+x)^a = {}_2F_1\left(\begin{array}{c} -a \quad , \quad b \\ b \end{array}\middle| -x\right) = {}_1F_0\left(\begin{array}{c} -a \\ - \end{array}\middle| -x\right) \tag{2}$$

$$\exp(x) = \lim_{a\longrightarrow\infty} {}_2F_1\left(\begin{array}{c} a \quad , \quad b \\ b \end{array}\middle| \frac{x}{a}\right) = {}_0F_0\left(\begin{array}{c} - \\ - \end{array}\middle| x\right) \tag{3}$$

$$(1-x)^{-2a-1}(1+x) = {}_2F_1\left(\begin{array}{c} 2a \quad , \quad a+1 \\ a \end{array}\middle| x\right) \tag{4}$$

$$\cos(x) = {}_0F_1\left(\begin{array}{c} - \\ \frac{1}{2} \end{array}\middle| -\frac{1}{4}x^2\right) \tag{5}$$

$$\sin(x) = x\ {}_0F_1\left(\begin{array}{c} - \\ \frac{3}{2} \end{array}\middle| -\frac{1}{4}x^2\right) \tag{6}$$

$$\ln(1+x) = x\ {}_2F_1\left(\begin{array}{c} 1 \quad , \quad 1 \\ 2 \end{array}\middle| -x\right) \tag{7}$$

$$\ln\left[\frac{(1+x)}{(1-x)}\right] = 2x\ {}_2F_1\left(\begin{array}{c} \frac{1}{2} \quad , \quad 1 \\ \frac{3}{2} \end{array}\middle| x^2\right) \tag{8}$$

$$\arcsin(x) = x\ {}_2F_1\left(\begin{array}{c} \frac{1}{2} \quad , \quad \frac{1}{2} \\ \frac{3}{2} \end{array}\middle| x^2\right) \tag{9}$$

$$\arctan(x) = x\ {}_2F_1\left(\begin{array}{c} \frac{1}{2} \quad , \quad 1 \\ \frac{3}{2} \end{array}\middle| -x^2\right) \tag{10}$$

1.2 Funciones Especiales

1.2.1 Polinomios

$$C_n^{\lambda}(x) = \frac{(2\lambda)_n}{n!}\ {}_2F_1\left(\begin{array}{c} -n \quad , \quad n+2\lambda \\ \lambda+\frac{1}{2} \end{array}\middle| \frac{(1-x)}{2}\right) \qquad \text{Gegenbauer} \tag{11}$$

$$P_n(x) = {}_2F_1\left(\begin{array}{c} -n \quad , \quad n+1 \\ 1 \end{array}\middle| \frac{1}{2}(1-x)\right) \qquad \text{Legendre} \tag{12}$$

$$H_n(x) = (2x)^n\ {}_2F_0\left(\begin{array}{c} -\frac{n}{2} \quad , \quad \frac{(1-n)}{2} \\ - \end{array}\middle| -\frac{1}{x^2}\right) \qquad \text{Hermite} \tag{13}$$

$$L_n^{\alpha}(x) = \frac{(1+\alpha)_n}{n!}\ {}_1F_1\left(\begin{array}{c} -n \\ 1+\alpha \end{array}\middle| x\right) \qquad \text{Laguerre} \tag{14}$$

$$P_n^{(\alpha,\beta)}(x) = \frac{(1+\alpha)_n}{n!}\ {}_2F_1\left(\begin{array}{ccc} -n & , & n+\alpha+\beta+1 \\ & 1+\alpha & \end{array}\middle| \frac{1-x}{2}\right) \qquad \text{Jacobi} \tag{15}$$

1.2.2 Series Infinitas

$$K(x) = \tfrac{1}{2}\pi\ {}_2F_1\left(\begin{array}{ccc} \frac{1}{2} & , & \frac{1}{2} \\ & 1 & \end{array}\middle| x^2\right) \qquad \text{Elíptica tipo I} \tag{16}$$

$$E(x) = \tfrac{1}{2}\pi\ {}_2F_1\left(\begin{array}{ccc} -\frac{1}{2} & , & \frac{1}{2} \\ & 1 & \end{array}\middle| x^2\right) \qquad \text{Elíptica tipo II} \tag{17}$$

$$J_\alpha(x) = \left(\frac{x}{2}\right)^{\alpha} \frac{1}{\Gamma(1+\alpha)}\ {}_0F_1\left(\begin{array}{c} - \\ 1+\alpha \end{array}\middle| -\tfrac{1}{4}x^2\right) \qquad \text{Bessel} \tag{18}$$

$$I_\alpha(x) = \left(\frac{x}{2}\right)^{\alpha} \frac{1}{\Gamma(1+\alpha)}\ {}_0F_1\left(\begin{array}{c} - \\ 1+\alpha \end{array}\middle| \tfrac{1}{4}x^2\right) \qquad \text{Bessel} \tag{19}$$

$$H_\alpha(x) = \left(\frac{x}{2}\right)^{1-\alpha} \frac{1}{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{3}{2}+\alpha\right)}\ {}_1F_2\left(\begin{array}{c} 1 \\ \frac{3}{2},\ \ \frac{3}{2}+\alpha \end{array}\middle| -\tfrac{1}{4}x^2\right) \qquad \text{Struve} \tag{20}$$

$$s_{\mu,\nu}(x) = \frac{x^{1+\mu}}{(1+\mu-\nu)(1+\mu+\nu)}\ {}_1F_2\left(\begin{array}{c} 1 \\ \frac{1}{2}(3+\mu-\nu),\ \ \frac{1}{2}(3+\mu+\nu) \end{array}\middle| -\tfrac{1}{4}x^2\right) \qquad \text{Lommel} \tag{21}$$

$$J_\mu(z)\,J_\nu(z) = \left(\frac{z}{2}\right)^{\mu+\nu} \frac{1}{\Gamma(\mu+1)\Gamma(\nu+1)}\ {}_2F_3\left(\begin{array}{c} \frac{1}{2}(\mu+\nu+1),\ \ \frac{1}{2}(\mu+\nu+2) \\ \mu+1,\ \ \nu+1,\ \ \mu+\nu+1 \end{array}\middle| -z^2\right) \tag{22}$$

$$\operatorname{erf}(x) = \tfrac{2}{\sqrt{\pi}} \int_0^x dt\ \exp\left(-t^2\right) = \tfrac{2x}{\sqrt{\pi}}\ {}_1F_1\left(\begin{array}{c} \frac{1}{2} \\ \\ \frac{3}{2} \end{array}\middle| -x^2\right) = \tfrac{2x}{\sqrt{\pi}} \exp\left(-x^2\right)\ {}_1F_1\left(\begin{array}{c} 1 \\ \frac{3}{2} \end{array}\middle| x^2\right) \quad \text{Función Error} \tag{23}$$

$$\gamma(a,x) \quad = \int_0^x t^{a-1}\exp(-t)\ dt = \frac{x^a}{a}\ {}_1F_1\left(\begin{array}{c} 1 \\ \\ 1+a \end{array}\middle| x\right) \quad \text{Gamma Incompleta Inferior} \tag{24}$$

$$\Gamma(a,x) \quad = \int_x^\infty t^{a-1}\exp(-t)\ dt = \Gamma(a) - \frac{x^a}{a}\ {}_1F_1\left(\begin{array}{c} 1 \\ \\ 1+a \end{array}\middle| x\right) \quad \text{Gamma Incompleta Superior} \tag{25}$$

1.2.3 Otras series útiles

$$Li_s(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^s} \quad \text{Función polilogaritmo} \tag{26}$$

$$\zeta_s = \sum_{n=1}^{\infty} \frac{1}{n^s} \quad \text{Función Zeta de Riemann} \tag{27}$$

$$\zeta(s,a) = \sum_{n=0}^{\infty} \frac{1}{(n+a)^s} \quad \text{Función Zeta de Hurwitz} \tag{28}$$

$$x^{-\alpha}(1-x)^{-\beta} \int_0^x dt\, t^{\alpha-1}(1-t)^{\beta-1} = \frac{1}{\alpha}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\, {}_2F_1\left(\begin{array}{c} 1 \quad , \quad \alpha+\beta \\ \alpha+1 \end{array} \middle| x\right) \tag{29}$$

2 Identidades importantes

$${}_2F_1\left(\begin{array}{c} -n \quad , \quad b \\ c \end{array} \middle| 1\right) = \frac{(c-b)_n}{(c)_n} \tag{30}$$

$${}_2F_1\left(\begin{array}{c} a \quad , \quad b \\ c \end{array} \middle| 1\right) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} \tag{31}$$