PROBLEMA GUÍA I | #8 y un extra

Sea la componente i-ésima de ∇r^n (con $r = |\vec{r}|$):

$$(\nabla r^n)_i = \partial_i(r^n) = n r^{n-1}(\partial_i r) = n r^{n-1}\frac{x_i}{r}$$

$$(\nabla r^n)_i = n r^{n-2} x_i$$

$$\Downarrow$$

$$\nabla r^n = n\frac{\vec{r}}{r^{2-n}}$$

$n=-1 \Rightarrow \nabla\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$

$n=1 \Rightarrow \nabla r = \frac{\vec{r}}{r}$

$\vdots$

etc.

Evaluando $\nabla^2(r^n) = \nabla\cdot\nabla(r^n) = \nabla\cdot\left[n\frac{\vec{r}}{r^{2-n}}\right]$

luego $\nabla\cdot\left[n\frac{\vec{r}}{r^{2-n}}\right] = n\,\partial_i\left(\frac{x_i}{r^{2-n}}\right)$

$$= n\left[(\partial_i x_i)\frac{1}{r^{2-n}} + x_i\,\partial_i(r^{n-2})\right]$$

$$= n\left[\frac{\delta_{ii}}{r^{2-n}} + x_i\frac{(n-2)}{r^{3-n}}(\partial_i r)\right]$$

$$= n\left[\frac{\delta_{ii}}{r^{2-n}} + (n-2)\frac{x_i}{r^{3-n}}\frac{x_i}{r}\right]$$

$$= n\left[\frac{3}{r^{2-n}} + \frac{(n-2)}{r^{2-n}}\right] = \frac{n(1+n)}{r^{2-n}}$$

Problema #12 se resuelve con esta fórmula, al igual que el problema #11