Clase 16 261021 ①

Geodésicas tipo-tiempo

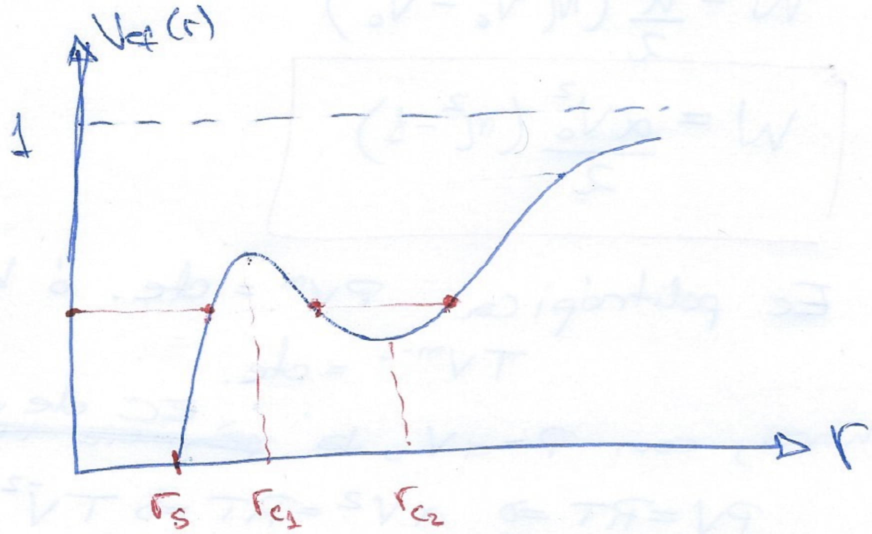
$$V_{ef}(r)=\left(1-\frac{r_s}{r}\right)\left(m+\frac{L^2}{r^2}\right)$$

Para partículas masivas $m=1$ (por normalización)

$$\dot{r}^2 = E^2 - V_{ef}(r)$$

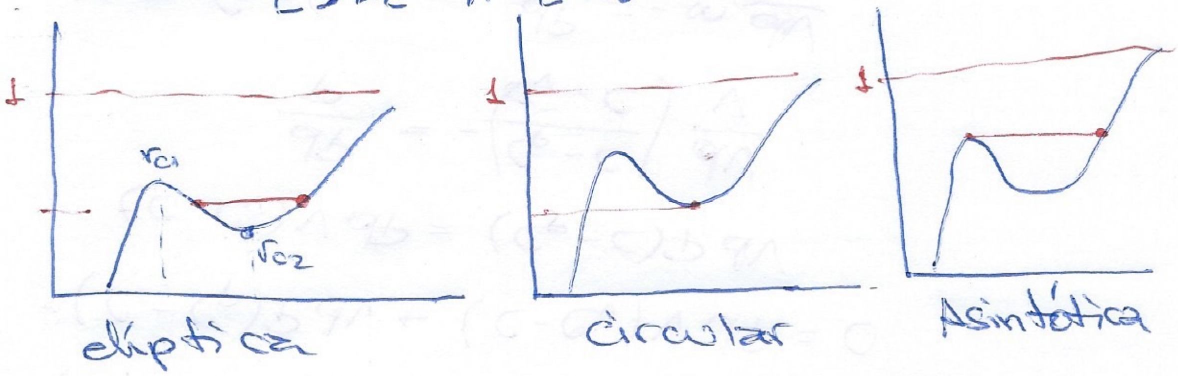
$$\dot{t} = \frac{E}{\left(1-\frac{r_s}{r}\right)}$$

El potencial



-. órbitas de 1ª especie

$L > L_c \wedge E < 1$



$$V_{ef}(r) = 1 + \frac{L^2}{r^2} - \frac{r_s}{r} - \frac{L^2 r_s}{r^3}$$

$$V'_{ef}(r) = -\frac{2L^2}{r^3} + \frac{r_s}{r^2} + \frac{3L^2 r_s}{r^4}$$

$$V'_{ef}(r_c) = 0 \Rightarrow r_{c1} \wedge r_{c2}$$

$$r_{c_{1,2}} = \frac{L^2}{r_s}\left(1 \mp \sqrt{1 - \frac{3r_s^2}{L^2}}\right)$$

El valor crítico para el momentum angular:

$$\boxed{L_c \equiv \sqrt{3}\, r_s}$$

CL 1B ③

Órbitas confinadas $\Rightarrow$ $L > L_c$ ($\wedge$ $E < 1$)

$$\mathring{r}^2 = E^2 - 1 - \frac{L^2}{r^2} + \frac{r_s}{r} + \frac{r_s L^2}{r^3}$$

$$\mathring{r}^2 = (E^2 - 1) + \frac{r_s}{r} - \frac{L^2}{r^2} + \frac{r_s L^2}{r^3}$$

Definamos $\cancel{\xi^2 = E^2 - 1 > 0}$ ($\xi \in \mathbb{R}$)

$$\xi^2 \equiv 1 - E^2 > 0$$

$$\boxed{\mathring{r}^2 = \frac{r_s L^2}{r^3} - \frac{L^2}{r^2} + \frac{r_s}{r} - \xi^2}$$

recordemos que $r^2 \mathring{\phi} = L$

$$\mathring{r} = \frac{d\phi}{d\tau}\frac{dr}{d\phi} = \frac{L}{r^2}\frac{dr}{d\phi} = -L\frac{d(1/r)}{d\phi}$$

Hacemos $u = 1/r$

$$+ L^2\left(\frac{du}{d\phi}\right)^2 = r_s L^2 u^3 - L^2 u^2 + r_s u - \xi^2$$

$$\boxed{\left(\frac{du}{d\phi}\right)^2 = r_s u^3 - u^2 + \frac{r_s}{L^2} u - \left(\frac{\xi}{L}\right)^2}$$

CL 16 (4)

o bien

$$\frac{du}{d\phi} = \pm\sqrt{g(u)}\,\sqrt{r_s}$$

$$\boxed{g(u) = u^3 - \frac{u^2}{r_s} + \frac{u}{L^2} - \frac{\varepsilon^2}{r_s L^2}}$$

TAREA: Encontrar, usando el método de cardano, los puntos de retorno, es decir, resolver

$$g(u) = 0 \qquad (u = 1/r)$$

Además, $g(u)$ puede ser escrito de forma factorizada:

$$g(u) = (u - u_0)(u - u_1)(u - u_2)$$

$$= u^3 - u^2(u_0 + u_1 + u_2) + u(u_0 u_1 + u_0 u_2 + u_1 u_2) - u_0 u_1 u_2$$

i) $u_0 + u_1 + u_2 = \frac{1}{r_s}$; ii) $u_0 u_1 + u_0 u_2 + u_1 u_2 = \frac{1}{L^2}$

iii) $u_0 u_1 u_2 = \frac{\varepsilon^2}{r_s L^2}$