NOTA 2.

Suma de oscilaciones.

(II)

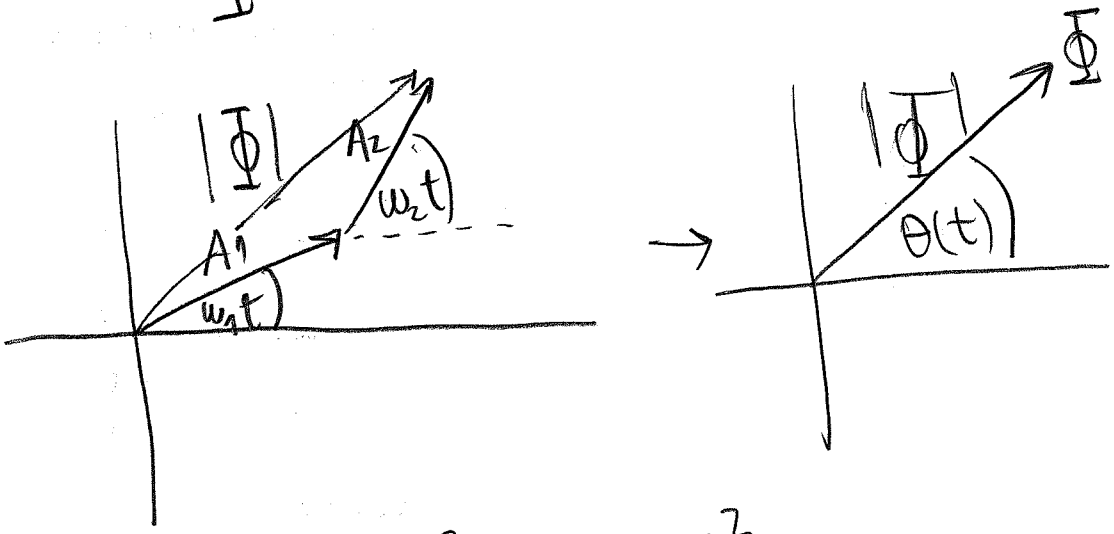
$$\phi_1 = A_1 \cos(\omega_1 t) = Re(A_1 e^{i\omega_1 t}) \quad (A_1, A_2 \in \mathbb{R})$$

$$\phi_2 = A_2 \cos(\omega_2 t) = Re(A_2 e^{i\omega t})$$

$$\phi = \phi_1 + \phi_2 = Re(A_1 e^{i\omega_1 t}) + Re(A_2 e^{i\omega_2 t})$$

$$\phi = Re(A_1 e^{i\omega_1 t} + A_2 e^{i\omega_2 t})$$

sea $\Phi = A_1 e^{i\omega t} + A_2 e^{i\omega t} \Rightarrow \phi = Re(\Phi)$



$$|\Phi|^2 = Re(\Phi)^2 + Im(\Phi)^2$$

$$= (A_1 \cos\omega_1 t + A_2 \cos\omega_2 t)^2 + (A_1 \operatorname{sen}\omega_1 t + A_2 \operatorname{sen}\omega_2 t)^2$$

$$= A_1^2 + A_2^2 + 2A_1A_2(\cos\omega_1 t \cos\omega_2 t + \operatorname{sen}\omega_1 t \operatorname{sen}\omega_2 t)$$

$$= A_1^2 + A_2^2 + 2A_1A_2 \cos((\omega_1 - \omega_2)t)$$

$$|\Phi| = [A_1^2 + A_2^2 + 2A_1A_2 \cos[(\omega_1 - \omega_2)t]]^{1/2}$$

$$\theta(t) = \text{Arg}(\Phi) = tg^{-1}\left(\frac{Im(\Phi)}{Re(\Phi)}\right)$$

$$= tg^{-1}\left[\frac{A_1 \operatorname{sen} w_1 t + A_2 \operatorname{sen} w_2 t}{A_1 \cos w_2 t + A_2 \cos w_2 t}\right]$$

Finalmente

$$\Phi(t) = |\Phi| \, e^{i\theta(t)}$$

y $$\phi(t) = Re(\Phi)$$

$$= |\Phi| \cos(\theta(t))$$

$$\phi(t) = \left[A_1^2 + A_2^2 + 2A_1A_2 \cos[(w_1 - w_2)t]\right]^{1/2}$$

$$\cdot \cos\left[tg^{-1}\left(\frac{A_1 \operatorname{sen} w_1 t + A_2 \operatorname{sen} w_2 t}{A_1 \cos w_1 t + A_2 \cos w_2 t}\right)\right]$$

VER MAPLE

(suma oscilacion mmf.mw)