2) coord. elíptica (ζ, σ, z)

$$x = \frac{1}{2} a \cosh(\zeta) \cos\sigma \qquad y = \frac{1}{2} a \sinh\zeta \sin\sigma$$

$$\frac{dx}{dt} = \frac{1}{2} a \left[\sinh(\zeta) \dot{\zeta} \cos(\sigma) - \cosh(\zeta) \sin(\sigma) \dot{\sigma} \right]$$

$$\frac{d^2x}{dt^2} = \frac{1}{2} a \Big[\cosh(\zeta) (\dot{\zeta})^2 \cos\sigma + \sinh(\zeta) \ddot{\zeta} \cos\sigma$$

$$- \sinh(\zeta) \dot{\zeta} \sin(\sigma) \dot{\sigma}$$

$$- \sinh(\zeta) \dot{\zeta} \sin(\sigma)(\dot{\sigma}) - \cosh(\zeta) \cos\sigma (\dot{\sigma})^2$$

$$- \cosh(\zeta) \sin(\sigma) \ddot{\sigma} \Big]$$

$$\frac{d x^2}{dt^2} = \frac{1}{2} a \Big[\cosh(\zeta) \left[\cos\sigma \left((\dot{\zeta})^2 - (\dot{\sigma})^2 \right) - \ddot{\sigma} \sin\sigma \right]$$

$$- \sinh(\zeta) \left[\sin\sigma (2 \dot{\zeta} \dot{\sigma}) - \ddot{\zeta} \cos\sigma \right] \Big]$$

2) coor. elípticas (ζ, σ, z)

$$x = \frac{1}{2} a \cosh(\zeta) \cos\sigma \qquad y = \frac{1}{2} a \sinh\zeta \sin\sigma$$

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ los vectores unitarios se consiguen $\frac{\partial \bar{r}}{\partial x} = \hat{e}_x$ no siempre normalizados

$$\bar{r} = \frac{1}{2}a\left(\cosh\zeta \cos\sigma\,\hat{\imath} + \sinh\zeta \sin\sigma\,\hat{\jmath}\right) + z\hat{k}$$

$$\frac{\partial \bar{r}}{\partial \zeta} = \frac{1}{2}a\left(\sinh\zeta \cos\sigma\,\hat{\imath} + \cosh\zeta \sin\sigma\,\hat{\jmath}\right) = \bar{e}_\zeta$$

$$\frac{\partial \bar{r}}{\partial \sigma} = \frac{1}{2}a\left(-\cosh\zeta \sin\sigma\,\hat{\imath} + \sinh\zeta \cos\sigma\,\hat{\jmath}\right) = \bar{e}_\sigma$$

$$\frac{\partial \bar{r}}{\partial z} = \hat{k}$$

las magnitudes

$$|e_\xi| = \frac{1}{2}a(\sinh^2\xi\cos^2\sigma + \cosh^2\xi\sin^2\sigma)$$

$$|e_\sigma| = \frac{1}{2}a(\quad \parallel \quad)$$

$$|e_\xi| = |e_\sigma| = |e|$$

los vectores normalizados

$$\hat{e}_\xi = \frac{\bar{e}_\xi}{|e|} \qquad \hat{e}_\sigma = \frac{\bar{e}_\sigma}{|e|} \qquad \hat{e}_z = \frac{\hat{e}_z}{|1|} = \hat{k}$$

$$\hat{e}_\xi = \frac{\sinh\xi\cos\sigma\,\hat{i} + \cosh\xi\sin\sigma\,\hat{j}}{(\sinh^2\xi\cos^2\sigma + \cosh^2\xi\sin^2\sigma)}$$

$$\hat{e}_\sigma = \frac{\sinh\xi\cos\sigma\,\hat{j} - \cosh\xi\sin\sigma\,\hat{j}}{|e|}$$

Escaneado con CamScanner

el vector velocidad $\dot{\bar{r}}$ no deja de apuntar en la misma dir. a pesar del sist. de coord.

$$\dot{\bar{r}} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k} = V_\zeta \hat{e}_\zeta + V_\sigma \hat{e}_\sigma + \dot{z}\hat{k}$$

$$\dot{x}\hat{i} + \dot{y}\hat{j} = V_\zeta \hat{e}_\zeta + V_\sigma \hat{e}_\sigma$$

$$\frac{1}{2}a\Big[\dot{\zeta}\sinh(\zeta)\cos\sigma - \dot{\sigma}\cosh\zeta\sin\sigma\Big]\hat{i}$$

$$+\Big[\dot{\zeta}\cosh(\zeta)\sin\sigma + \dot{\sigma}\sinh\zeta\cos\sigma\Big]\hat{j} = V_\zeta \hat{e}_\zeta + V_\sigma \hat{e}_\sigma$$

$$\dot{\zeta}\ \underbrace{\frac{a}{2}\left(\sinh(\zeta)\cos(\sigma)\hat{i} + \cosh\zeta\sin(\sigma)\hat{j}\right)}_{|e|\hat{e}_\zeta}$$

$$+\dot{\sigma}\ \underbrace{\frac{a}{2}\left(\sinh\zeta\cos\sigma\hat{j} - \cosh\zeta\sin\sigma\hat{i}\right)}_{|e|\hat{e}_\sigma} = V_\zeta \hat{e}_\zeta + V_\sigma \hat{e}_\sigma$$

$/\hat{e}_\zeta\}$ $\quad \dot{\zeta}|e| = V_\zeta$

$/\hat{e}_\sigma\}$ $\quad \dot{\sigma}|e| = V_\sigma$

$$\dot{z} = V_z$$

$$\bar{V} = \dot{\zeta}|e|\,\hat{e}_\zeta + \dot{\sigma}|e|\,\hat{e}_\sigma + \dot{z}\hat{k}$$

$$\frac{d\bar{v}}{dt} = \ddot{\zeta}\left(|e|\hat{e}_\zeta\right) + \dot{\zeta}\frac{a}{2}\Big[\left(\cosh(\zeta)\dot{\zeta}\cos\sigma - \sinh(\zeta)\sin(\sigma)\dot{\sigma}\right)\hat{i}$$

$$+\left(\sinh(\zeta)\dot{\zeta}\sin\sigma + \cosh(\zeta)\cos\sigma(\dot{\sigma})\right)\hat{j}\Big]$$

$$+\ddot{\sigma}\left(|e|\hat{e}_\sigma\right) + \dot{\sigma}\frac{a}{a}\Big[\left(\cosh(\zeta)\dot{\zeta}\cos\sigma - \dot{\sigma}\sinh(\zeta)\sin\sigma\right)\hat{i}$$

$$-\left(\sinh(\zeta)\dot{\zeta}\sin\sigma + \cosh(\zeta)\cos\sigma(\dot{\sigma})\right)\hat{j}\Big] = \bar{a}$$