Prob 1

SE DESEA CALCULAR

$$\mathcal{K}_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right) \quad m = \frac{N}{V} \quad \mathcal{K}_T = \frac{1}{m}\left(\frac{\partial m}{\partial P}\right)_T$$

PARA UN GAS IDEAL DE BOSE TENEMOS,

$$\boxed{g_m(z) = \frac{1}{\Gamma(m)}\int_0^\infty \frac{x^{m-1}}{z^{-1}e^x - 1}\,dx \qquad 0 \le z \le 1 \ ; \ m \in \mathbb{R}}$$

$$m = \frac{1}{\lambda^3} g_{3/2}(z) + \cancel{\frac{N_o}{V}} \to 0 \ (N_o \ll N)$$

$$P = \frac{KT}{\lambda^3} g_{5/2}(z)$$

COMO $\lambda = \sqrt{\frac{h^2}{2\pi m K T}} \longrightarrow \lambda^{-3} = \alpha T^{3/2}$ CON $\alpha$: cte, ENTONCES

$$m = \alpha T^{3/2} g_{3/2}(z) \quad \wedge \quad P = \alpha K T^{5/2} g_{5/2}(z)$$

ENTONCES

$$dm = \frac{3}{2}\alpha T^{1/2} g_{3/2}(z)\, dT + \alpha T^{3/2} \frac{d g_{3/2}(z)}{dz}\, dz$$

$$\text{obs: } \frac{d g_\nu(z)}{dz} = \frac{1}{\Gamma(\nu)}\int_0^\infty \frac{d}{dz}\left(\frac{x^{\nu-1}}{z^{-1}e^x - 1}\right)dz = \frac{1}{\Gamma(\nu)} * \frac{1}{z^2}\int_0^\infty \frac{x^{\nu-1}e^x}{(z^{-1}e^x - 1)^2}\,dx$$

QUE AL SER INTEGRADA POR PARTES, NOS LLEVA A:

$$\frac{d g_\nu(z)}{dz} = \frac{1}{z} g_{\nu-1}(z)$$

ENTONCES:

$$dm = \frac{3}{2}\alpha T^{1/2} g_{3/2}(z)\, dT + \alpha T^{3/2} \frac{1}{z} g_{1/2}(z)\, dz$$

POR OTRO LADO

$$dp = \frac{5}{2}\alpha K T^{3/2} g_{5/2}(z)\, dT + \alpha K T^{5/2} \frac{1}{z} g_{3/2}(z)\, dz$$

CONSIDERANDO $dT=0$ (ISOTERMO)

$$\left.\begin{aligned} dn &= \alpha T^{3/2} \frac{1}{z} g_{1/2}(z)\, dz \\ dP &= \alpha K T^{5/2} \frac{1}{z} g_{3/2}(z)\, dz \end{aligned}\right\} \left(\frac{\partial n}{\partial P}\right)_T = \frac{1}{KT} * \frac{g_{1/2}(z)}{g_{3/2}(z)}$$

$$\Longrightarrow \boxed{\mathcal{K}_T = \frac{1}{nKT} * \frac{g_{1/2}(z)}{g_{3/2}(z)}}$$

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**Prob 2** PARA UN GAS IDEAL DE FERMI-DIRAC TENEMOS:

$$\left.\begin{aligned} n &= \frac{g}{\lambda^3} f_{3/2}(z) \\ \lambda &= \sqrt{\frac{h^2}{2\pi m K T}} \end{aligned}\right\} n = g * \left(\frac{2\pi m K T}{h}\right)^{3/2} f_{3/2}(z)$$

SI $\mu = 0 \Rightarrow z = 1$. ENTONCES

$$n = g\left(\frac{2\pi m K T_0}{h^2}\right)^{3/2} f_{3/2}(1)$$

$$\left(\frac{2\pi m K T_0}{h^2}\right)^{3/2} = \frac{n}{g f_{3/2}(1)} \Longrightarrow T_0 = \left(\frac{n}{g f_{3/2}(1)}\right)^{2/3} * \frac{h^2}{2\pi m k}$$

POR OTRO LADO, LA TEMPERATURA DE FERMI ESTÁ DADA POR

$$T_F = \frac{\varepsilon_F}{k} = \left(\frac{3n}{4\pi g}\right)^{2/3} \frac{h^2}{2mk}$$

ENTONCES

$$\frac{T_0}{T_F} = \frac{\left(\frac{n}{g f_{3/2}(1)}\right)^{2/3} * \frac{h^2}{2\pi m k}}{\left(\frac{3n}{4\pi g}\right)^{2/3} * \frac{h^2}{2mk}} = \left(\frac{4\pi}{3 f_{3/2}(1)}\right)^{2/3} * \frac{1}{\pi}$$

$$\Longrightarrow \boxed{T_0 = \left(\frac{4\pi}{3 f_{3/2}(1)}\right)^{2/3} * \frac{1}{\pi} * T_F}$$

Prob 3 | CONSIDERANDO:

$$\varepsilon = \frac{1}{2} m u^2 \Rightarrow u = \sqrt{\frac{2}{m}\varepsilon}$$

POR TANTO, NECESITAMOS calcular $\langle \varepsilon^{1/2} \rangle$ Y $\langle \varepsilon^{-1/2} \rangle$, las QUE VIENEN DADAS POR:

$$\langle \varepsilon^{\pm 1/2} \rangle = \frac{\int_0^\infty \langle n \rangle \varepsilon^{\pm 1/2} * \frac{2\pi V}{h^3} (2m)^{3/2} \varepsilon^{1/2} d\varepsilon}{\int_0^\infty \langle n \rangle \frac{2\pi V}{h^3} (2m)^{3/2} \varepsilon^{1/2} d\varepsilon}$$

(a) PARA UN GAS DE BOSE TENEMOS.

$$\langle \varepsilon^{1/2} \rangle = \frac{\int_0^\infty \frac{\varepsilon}{z^{-1} e^{\beta\varepsilon} - 1} d\varepsilon}{\int_0^\infty \frac{\varepsilon^{1/2}}{z^{-1} e^{\beta\varepsilon} - 1} d\varepsilon} \xrightarrow{x = \beta\varepsilon} \langle \varepsilon^{1/2} \rangle = \frac{\frac{1}{\beta^2} \int_0^\infty \frac{x}{z^{-1} e^x - 1} dx}{\frac{1}{\beta^{3/2}} \int_0^\infty \frac{x^{1/2}}{z^{-1} e^x - 1} dx}$$

USANDO:

$$g_\nu(z) = \frac{1}{\Gamma(\nu)} \int_0^\infty \frac{x^{\nu-1}}{z^{-1} e^x - 1} dx$$

$$\langle \varepsilon^{1/2} \rangle = \frac{1}{\beta^{1/2}} * \frac{\Gamma(2)\, g_2(z)}{\Gamma(\frac{3}{2})\, g_{3/2}(z)} = (kT)^{1/2} \frac{\Gamma(2)\, g_2(z)}{\Gamma(\frac{3}{2})\, g_{3/2}(z)}$$

POR OTRO LADO:

$$\langle \varepsilon^{-1/2} \rangle = \frac{\int_0^\infty \frac{1}{z^{-1} e^{\beta\varepsilon} - 1} d\varepsilon}{\int_0^\infty \frac{\varepsilon^{1/2}}{z^{-1} e^{\beta\varepsilon} - 1} d\varepsilon} \xrightarrow{x = \beta\varepsilon} \langle \varepsilon^{-1/2} \rangle = \frac{\frac{1}{\beta} \int_0^\infty \frac{1}{z^{-1} e^x - 1} dx}{\frac{1}{\beta^{3/2}} \int_0^\infty \frac{x^{1/2}}{z^{-1} e^x - 1} dx}$$

$$\langle \varepsilon^{-1/2} \rangle = \beta^{1/2} * \frac{\Gamma(1)\, g_1(z)}{\Gamma(\frac{3}{2})\, g_{3/2}(z)} = \frac{1}{(kT)^{1/2}} * \frac{\Gamma(1)\, g_1(z)}{\Gamma(\frac{3}{2})\, g_{3/2}(z)}$$

ENTONCES:

$$\langle u \rangle \langle u^{-1} \rangle = \langle \varepsilon^{1/2} \rangle \langle \varepsilon^{-1/2} \rangle \Rightarrow \boxed{\langle u \rangle \langle u^{-1} \rangle = \frac{\Gamma(1)\Gamma(2)}{(\Gamma(\frac{3}{2}))^2} * \frac{g_1(z)\, g_2(z)}{(g_{3/2}(z))^2}}$$

(b) PARA UN GAS DE BOSONES LA SITUACIÓN ES SIMILAR.
USANDO.

$$f_\nu(z) = \frac{1}{\Gamma(\nu)} \int_0^\infty \frac{x^{\nu-1}}{z^{-1}e^x + 1}\, dx$$

TRAS REALIZAR EL ÁLGEBRA INVOLUCRADA SE LLEGA A,

$$\boxed{\langle u \rangle \langle u^{-1} \rangle = \frac{4}{\pi} * \frac{f_1(z)\, f_2(z)}{\left(f_{3/2}(z)\right)^2}}$$