Problem 6.1.
We know that the free energy $F(T,V,N)$ of a thermodynamic system is extensive. Show that

$$N\left(\frac{\partial F}{\partial N}\right)_{T,V}+V\left(\frac{\partial F}{\partial V}\right)_{T,N}=Nf=F$$

with f the free energy density expressed in suitable variables. Given this result, from the differential properties of $F(T,V,N)$, show that

$$\Phi=N\mu$$

with Φ the Gibbs potential defined as $\Phi=F+PV$. In the above expression, μ is the chemical potential properly defined in terms of $F(T,V,N)$.

$$dU = TdS - PdV + \mu dN \quad / \quad F = U - TS$$

$$dF = -SdT - PdV + \mu dN$$

$$dF = \left(\frac{\partial F}{\partial T}\right)_{V,N} dT + \left(\frac{\partial F}{\partial V}\right)_{T,N} dV + \left(\frac{\partial F}{\partial N}\right)_{T,V} dN$$

Comparando se obtiene $\left(\frac{\partial F}{\partial T}\right)_{V,N} = -S$; $\left(\frac{\partial F}{\partial V}\right)_{T,N} = -P$; $\left(\frac{\partial F}{\partial N}\right)_{T,V} = \mu$ $(*)$

del calculo previo

$$N\left(\frac{\partial F}{\partial N}\right)_{T,V} = \qquad \underbrace{N\left(\frac{\partial F}{\partial N}\right)_{T,V}}_{*} + \underbrace{V\left(\frac{\partial F}{\partial V}\right)_{T,N}}_{*}$$

$$Nf - V\left(\frac{\partial f}{\partial v}\right)_{T} + V\left(\frac{\partial f}{\partial v}\right)_{T} = Nf \quad // \quad \text{demostrado.}$$

pero si usamos

$$N\left(\frac{\partial F}{\partial N}\right)_{T,V} + V\left(\frac{\partial F}{\partial V}\right)_{T,N} = Nf = F$$

$$N\mu - VP = F$$

$$N\mu = F + PV$$

$$N\mu = G \quad // \text{ demostrado}$$

$dU = TdS - PdV + \mu dN$ $\quad (H = U + PV)$

$U = TS - PV + \mu N$ $\quad (F = U - TS)$

$H = TS + \mu N$ $\quad (G = H - TS)$

$F = -PV + \mu N$

$G = \mu N$ //