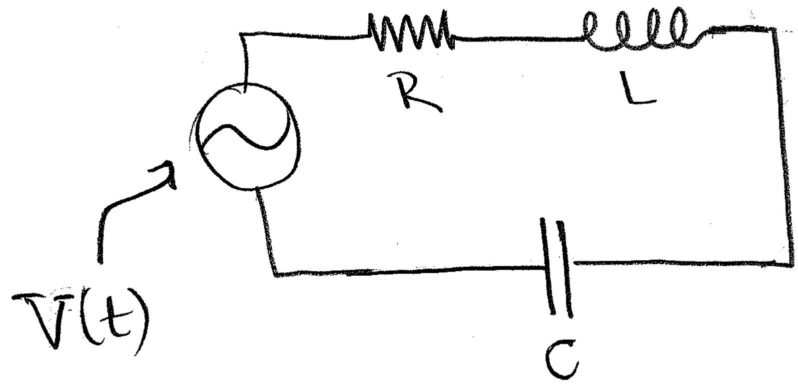
— CIRCUITO EN SERIE RLC —

↓

FUENTE ALTERNA



la ec. de malla es la siguiente:

$$V(t) = \frac{q}{C} + iR + L\frac{di}{dt}$$

; en un circuito la variable de interés es la corriente i

↓ derivamos esta ecuación

$$\frac{dV(t)}{dt} = \frac{i}{C} + R\frac{di}{dt} + L\frac{d^2 i}{dt^2}$$

Si $V(t) = V_0 \operatorname{sen} \omega t$

$= V_0 \operatorname{Im}(e^{i\omega t})$

Propiedades

(I) $\left.\begin{aligned} \operatorname{Im}(\alpha z) &= \alpha \operatorname{Im}(z) \\ \operatorname{Re}(\alpha z) &= \alpha \operatorname{Re}(z) \end{aligned}\right\}$ para $\alpha \in \mathbb{R}$

II

$$\left.\begin{aligned} Im\left(\frac{dz}{dt}\right) &= \frac{d}{dt} Im(z) \\ Re\left(\frac{dz}{dt}\right) &= \frac{d}{dt} Re(z) \end{aligned}\right\} \text{ para } t \in \mathbb{R}$$

luego $V(t) = Im\left(V_0 e^{i\omega t}\right)$

y $\dfrac{dV(t)}{dt} = Im\left(\dfrac{d}{dt}\left(V_0 e^{i\omega t}\right)\right) = Im\left(i\omega V_0 e^{i\omega t}\right)$

definamos además que:

$i = Im(I)$ e $I = I_0 e^{i\omega t}$

complejo indep. de t.

$$\frac{di}{dt} = Im\left(i\omega I_0 e^{i\omega t}\right)$$

$$\frac{d^2 i}{dt^2} = Im\left(-\omega^2 L I_0 e^{i\omega t}\right)$$

$\therefore$ la ec. del circuito está dado por:

$$Im\left(i\omega V_0 e^{i\omega t}\right) = Im\left(\frac{I}{C}\right) + Im\left(R i\omega I\right) + Im\left(-\omega^2 L I\right)$$

$$\Downarrow$$

$$i\omega V_0 e^{i\omega t} = \frac{I}{C} + i\omega R I - \omega^2 L I$$

; pero $I = I_0 e^{i\omega t}$

$$\therefore\ i\omega V_0 \cancel{e^{i\omega t}} = \frac{I_0}{C}\cancel{e^{i\omega t}} + i\omega R I_0 \cancel{e^{i\omega t}} - \omega^2 L I_0 \cancel{e^{i\omega t}}$$

$$i\omega V_0 = \left(\frac{1}{C} + i\omega R - \omega^2 L\right) I_0$$

$$\cancel{i\omega} V_0 = \cancel{i\omega}\left(\frac{1}{i\omega C} + R - \frac{\omega^2 L}{i\omega}\right) I_0$$

$$V_0 = \underbrace{\left(R + i\omega L - \frac{i}{\omega C}\right)}_{Z\ =\ \text{impedancia}} I_0$$

$Z_R = R$
$Z_L = i\omega L$
$Z_C = -\frac{i}{\omega C}$

Resistencia compleja

$\therefore$ elementos pasivos se comportan algebraicamente como resistencias reales.

LEY DE OHM COMPLEJA

$$V_0 = I_0 Z \implies Z = \frac{1}{\omega C}\left(\omega RC + i\omega^2 LC - i\right)$$

Ahora bien $I_0 = \frac{V_0}{Z} = \frac{V_0 \bar{Z}}{Z\bar{Z}}$

$$I_0 = \frac{V_0 \omega C\left(\omega RC + i(\omega^2 LC - 1)\right)}{\omega^2 R^2 C^2 + (\omega^2 LC - 1)^2}$$

luego

$$I = I_0 e^{i\omega t} = I_0 (\cos\omega t + i\,\mathrm{sen}\,\omega t)$$

$$\downarrow$$

$$I = \frac{V_0 \omega C}{\omega^2 R^2 C^2 + (\omega^2 LC - 1)^2}\left(\omega RC - i(\omega^2 LC - 1)\right)\left(\cos\omega t + i\,\mathrm{sen}\,\omega t\right)$$

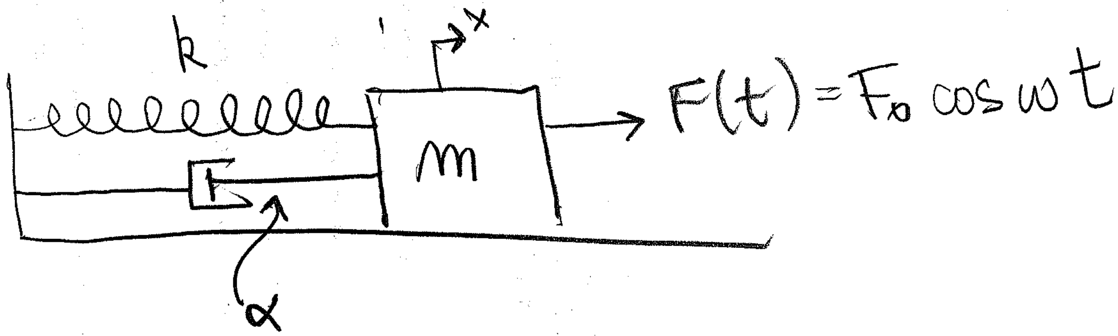
$$= \frac{V_0 \omega C}{\omega^2 R^2 C^2 + (\omega^2 LC - 1)^2}$$

$$\times \left(\omega RC\cos\omega t - (\omega^2 LC - 1)\,\mathrm{sen}\,\omega t + i\left[-(\omega^2 LC - 1)\cos\omega t + \omega RC\,\mathrm{sen}\,\omega t\right]\right)$$

Finalmente

$$i(t) = \frac{V_0 \omega C}{\omega^2 R^2 C^2 + (\omega^2 LC - 1)}\left((1 - \omega^2 LC)\cos\omega t + \omega RC\,\mathrm{sen}\,\omega t\right)$$

— Sistema oscilatorio mecánico forzado —



$$F(t) - kx - \alpha \dot{x} = m\ddot{x} \longrightarrow F(t) = kx + \alpha\dot{x} + m\ddot{x}$$

Sea $F(t) = Re(F_0 e^{i\omega t})$

def. $x(t) = Re(\mathbb{X}(t))$ $\quad \mathbb{X}(t) = \mathbb{X}_0 e^{i\omega t}$

$\uparrow$ complejo

$$Re(F_0 e^{i\omega t}) = Re(k\mathbb{X}) + Re\left(\alpha \frac{d\mathbb{X}}{dt}\right) + Re\left(m\frac{d^2\mathbb{X}}{dt^2}\right)$$

$$F_0 \cancel{e^{i\omega t}} = k\mathbb{X}_0 \cancel{e^{i\omega t}} + i\omega\alpha\mathbb{X}_0 \cancel{e^{i\omega t}} - m\omega^2 \mathbb{X}_0 \cancel{e^{i\omega t}}$$

$$F_0 = \underbrace{(k + i\omega\alpha - m\omega^2)}_{K \,=\, \text{cte. de resorte complejo}} \mathbb{X}_0$$

$\Downarrow$

$F_0 = K\mathbb{X}_0 \rightsquigarrow$ Ley de Hooke complejo.

$$\therefore \quad \mathbb{X}_0 = \frac{F_0}{K} = \frac{F_0 \overline{K}}{K\overline{K}} = \frac{F_0\left(k - m\omega^2 - i\alpha\omega\right)}{(k^2 - m\omega^2)^2 + \alpha^2\omega^2}$$

luego

$$\mathbb{X} = \mathbb{X}_0 e^{i\omega t} = \frac{F_0\left((k - m\omega^2) - i\alpha\omega\right)\left(\cos\omega t + i\,\mathrm{sen}\,\omega t\right)}{(k - m\omega^2)^2 + \alpha^2\omega^2}$$

$$= \frac{F_0}{(k - m\omega^2)^2 + \alpha^2\omega^2}\Big[(k - m\omega^2)\cos\omega t + \alpha\omega\,\mathrm{sen}\,\omega t$$

$$+\, i\left((k - m\omega^2)\,\mathrm{sen}\,\omega t - \alpha\omega\cos\omega t\right)\Big]$$

luego

$$x(t) = \mathrm{Re}(\mathbb{X})$$

$$= \frac{F_0\left[(k - m\omega^2)\cos\omega t + \alpha\omega\,\mathrm{sen}\,\omega t\right]}{(k - m\omega^2)^2 + \alpha^2\omega^2}$$