Teníamos que

$$\phi = 2\sqrt{\frac{P}{Q}}\left[K(k) - F(\tfrac{1}{2}\chi, k)\right] \quad (*)$$

$$\boxed{k^2 = \frac{Q-P+3r_s}{2Q}}$$

Aplicamos el sn a la ec. (*), después de ordenar

$$F(\tfrac{1}{2}\chi, k) = \underbrace{K(k)}_{\phi_0} - \underbrace{\frac{1}{2}\sqrt{\frac{Q}{P}}}_{\beta_0}\,\phi$$

$$F(\tfrac{1}{2}\chi, k) = \phi_0 - \beta_0\phi \quad / \operatorname{sn}$$

$$\boxed{\sin\tfrac{1}{2}\chi = \operatorname{sn}(\phi_0 - \beta_0\phi)} \quad (**)$$

De clase (13):

$$u + \frac{Q-P+r_s}{2r_sP} = \frac{Q-P+3r_s}{2r_sP}\cdot\sin^2\tfrac{1}{2}\chi$$

$$\rightarrow \boxed{\sin^2\tfrac{1}{2}\chi = u\cdot\frac{2r_sP}{Q-P+3r_s} + \sin^2\tfrac{1}{2}\chi_\infty}$$

$(**)^2$: $\sin^2\tfrac{1}{2}\chi = \operatorname{sn}^2(\phi_0 - \beta_0\phi)$

$$u\left(\frac{2r_sP}{Q-P+3r_s}\right) + \sin^2\tfrac{1}{2}\chi_\infty = \operatorname{sn}^2(\phi_0 - \beta_0\phi)$$

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$$u + \frac{Q-P+r_s}{2r_sP} = \frac{Q-P+3r_s}{2r_sP}\left[1-\mathrm{cn}^2(\phi_0-\beta_0\phi)\right]$$

$$u = \frac{\cancel{Q}-\cancel{P}+3r_s-\cancel{Q}+\cancel{P}-r_s}{2r_sP} - \frac{Q-P+3r_s}{2r_sP}\,\mathrm{cn}^2(\,)$$

$$u = \frac{1}{P} - \left(\frac{Q-P+3r_s}{2r_sP}\right)\mathrm{cn}^2(\phi_0-\beta_0\phi)$$

$$u = \frac{2r_s - (Q-P+3r_s)\,\mathrm{cn}^2(\phi_0-\beta_0\phi)}{2r_sP}$$

$$\rightarrow \quad r(\phi) = \frac{2r_sP}{2r_s - \left(Q-P+3r_s\right)\mathrm{cn}^2(\phi_0-\beta_0\phi)}$$

$$\rightarrow \quad \boxed{r(\phi) = \frac{P}{1-\epsilon\,\mathrm{cn}^2(\phi_0-\beta_0\phi)}}$$

$$\boxed{\epsilon = \frac{Q-P+3r_s}{2r_s}}$$

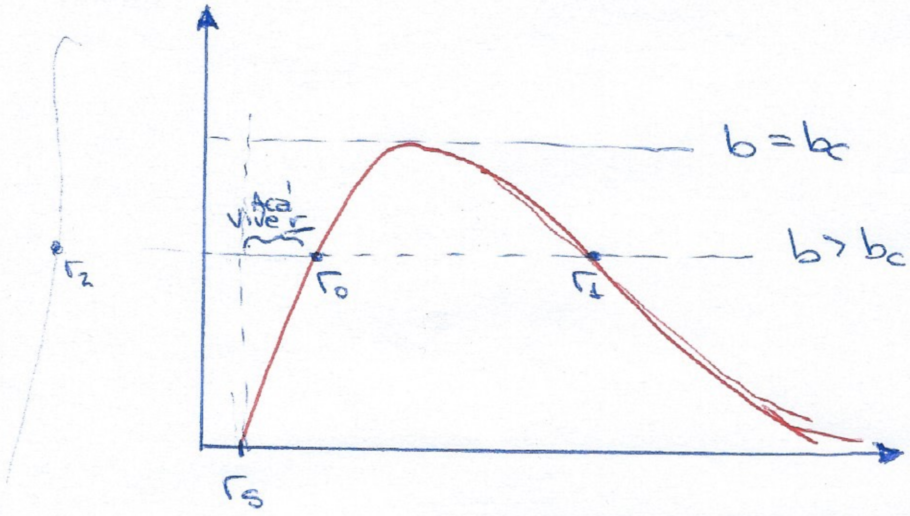
* En Mathematica: $F(\varphi|m) = \int_0^{\varphi}\frac{d\theta}{\sqrt{1-\cancel{k}m\sin^2\theta}}$

→ m es el módulo al cuadrado:

$$\underline{m = k^2}$$

cl 15 (3)

Geodésicas nulas de segunda especie



$$r_2 < 0 < r_s < r < r_0 < r_1$$

$$u_1 < u_0 < u < \infty \qquad \begin{matrix} u - u_0 > 0 \\ u - u_1 > 0 \end{matrix}$$

En este caso el cambio de variable.

$$\boxed{u = \frac{1}{P} + \frac{Q + P - 3r_s}{2r_s P} \sec^2 \frac{1}{2}\chi}$$

$$\chi = 0 \Rightarrow u = \frac{1}{P} + \frac{Q + P - 3r_s}{2r_s P} = \frac{2r_s + Q + P - 3r_s}{2r_s P}$$

$$= \frac{Q + P - r_s}{2r_s P} = u_0$$

es decir con $\chi = 0$ el fotón está en la distancia de afelio

Haciendo las sustituciones, se obtiene (TAREA)

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$$\boxed{\phi = 2\sqrt{\frac{P}{Q}}\, F\left(\tfrac{1}{2}\chi, k\right)}$$

$$\boxed{k^2 = \frac{Q - P + 3r_s}{2Q}}$$

$\therefore \quad F\left(\tfrac{1}{2}\chi, k\right) = \beta_0 \phi \qquad / \mathrm{sn}$

$$\sin\tfrac{1}{2}\chi = \mathrm{sn}\,\beta_0\phi$$

$$\rightarrow \quad \sin^2\tfrac{1}{2}\chi = \mathrm{sn}^2\,\beta_0\phi$$

$$\Rightarrow \quad 1 - \sin^2\tfrac{1}{2}\chi = 1 - \mathrm{sn}^2\,\beta_0\phi$$

$$\cos^2\tfrac{1}{2}\chi = \mathrm{cn}^2\,\beta_0\phi$$

$$\sec^2\tfrac{1}{2}\chi = \mathrm{nc}^2\,\beta_0\phi$$

$$\rightarrow \quad u = \frac{2r_s + (Q + P - 3r_s) \times \mathrm{nc}^2(\beta_0\phi)}{2r_s P}$$

$$\boxed{r(\phi) = \frac{P}{1 + \epsilon \times \mathrm{nc}^2(\beta_0\phi)}}$$

$$\boxed{\epsilon = \frac{Q + P - 3r_s}{2r_s}}$$