Vector Identities

Gradient

1. $\vec{\nabla}(f+g) = \vec{\nabla}f + \vec{\nabla}g$
2. $\vec{\nabla}(cf) = c\vec{\nabla}f$, for any **constant** c
3. $\vec{\nabla}(fg) = f\vec{\nabla}g + g\vec{\nabla}f$
4. $\vec{\nabla}(f/g) = \big(g\vec{\nabla}f - f\vec{\nabla}g\big)/g^2$ at points $\vec{\mathbf{x}}$ where $g(\vec{\mathbf{x}}) \neq 0$.
5. $\vec{\nabla}(\vec{\mathbf{F}} \cdot \vec{\mathbf{G}}) = \vec{\mathbf{F}} \times (\vec{\nabla} \times \vec{\mathbf{G}}) - (\vec{\nabla} \times \vec{\mathbf{F}}) \times \vec{\mathbf{G}} + (\vec{\mathbf{G}} \cdot \vec{\nabla})\vec{\mathbf{F}} + (\vec{\mathbf{F}} \cdot \vec{\nabla})\vec{\mathbf{G}}$

Divergence

6. $\vec{\nabla} \cdot (\vec{\mathbf{F}} + \vec{\mathbf{G}}) = \vec{\nabla} \cdot \vec{\mathbf{F}} + \vec{\nabla} \cdot \vec{\mathbf{G}}$
7. $\vec{\nabla} \cdot (c\vec{\mathbf{F}}) = c\vec{\nabla} \cdot \vec{\mathbf{F}}$, for any **constant** c
8. $\vec{\nabla} \cdot (f\vec{\mathbf{F}}) = f\vec{\nabla} \cdot \vec{\mathbf{F}} + \vec{\mathbf{F}} \cdot \vec{\nabla}f$
9. $\vec{\nabla} \cdot (\vec{\mathbf{F}} \times \vec{\mathbf{G}}) = \vec{\mathbf{G}} \cdot (\vec{\nabla} \times \vec{\mathbf{F}}) - \vec{\mathbf{F}} \cdot (\vec{\nabla} \times \vec{\mathbf{G}})$

Curl

10. $\vec{\nabla} \times (\vec{\mathbf{F}} + \vec{\mathbf{G}}) = \vec{\nabla} \times \vec{\mathbf{F}} + \vec{\nabla} \times \vec{\mathbf{G}}$
11. $\vec{\nabla} \times (c\vec{\mathbf{F}}) = c\vec{\nabla} \times \vec{\mathbf{F}}$, for any **constant** c
12. $\vec{\nabla} \times (f\vec{\mathbf{F}}) = f\vec{\nabla} \times \vec{\mathbf{F}} + \vec{\nabla}f \times \vec{\mathbf{F}}$
13. $\vec{\nabla} \times (\vec{\mathbf{F}} \times \vec{\mathbf{G}}) = \vec{\mathbf{F}}(\vec{\nabla} \cdot \vec{\mathbf{G}}) - (\vec{\nabla} \cdot \vec{\mathbf{F}})\vec{\mathbf{G}} + (\vec{\mathbf{G}} \cdot \vec{\nabla})\vec{\mathbf{F}} - (\vec{\mathbf{F}} \cdot \vec{\nabla})\vec{\mathbf{G}}$

Laplacian

14. $\vec{\nabla}^2(f+g) = \vec{\nabla}^2 f + \vec{\nabla}^2 g$
15. $\vec{\nabla}^2(cf) = c\vec{\nabla}^2 f$, for any **constant** c
16. $\vec{\nabla}^2(fg) = f\vec{\nabla}^2 g + 2\vec{\nabla}f \cdot \vec{\nabla}g + g\vec{\nabla}^2 f$

Degree Two

17. $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{\mathbf{F}}) = 0$
18. $\vec{\nabla} \times (\vec{\nabla}f) = 0$
19. $\vec{\nabla} \cdot (\vec{\nabla}f \times \nabla g) = 0$
20. $\vec{\nabla} \cdot (f\vec{\nabla}g - g\vec{\nabla}f) = f\vec{\nabla}^2 g - g\vec{\nabla}^2 f$
21. $\vec{\nabla} \times (\vec{\nabla} \times \vec{\mathbf{F}}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{\mathbf{F}}) - \vec{\nabla}^2\vec{\mathbf{F}}$