— Transformadas integrales — Complemento 2

$$\phi(x) = \int_{k_0}^{k_f} \eta(x,k)\, \tilde{\phi}(k)\, dk$$

$$\tilde{\phi}(k) = \int_{x_0}^{x_f} \xi(x,k)\, \phi(x)\, dx$$

$$\Downarrow$$

i) $$\phi(x) = \int_{k_0}^{k_f} \eta(x,k) \left[\int_{x_0}^{x_f} \xi(x',k)\, \phi(x')\, dx' \right] dk$$

$$= \int_{x_0}^{x_f} \phi(x') \left[\int_{k_0}^{k_f} \eta(x,k)\, \xi(x',k)\, dk \right] dx'$$

Se concluye que:

$$\int_{k_0}^{k_f} \eta(x,k)\, \xi(x',k)\, dk = \delta(x-x')$$

ii) Análogamente

$$\tilde{\phi}(k) = \int_{x_0}^{x_f} \xi(x,k) \left[\int_{k_0}^{k_f} \eta(x,k')\, \tilde{\phi}(k')\, dk' \right] dx$$

$$\tilde{\phi}(k) = \int_{k_0}^{k_f} \tilde{\phi}(k') \left[\int_{x_0}^{x_f} \eta(x,k')\,\xi(x,k)\,dx \right] dk'$$

lo que permite concluir que:

$$\boxed{\int_{x_0}^{x_f} \eta(x,k')\,\xi(x,k)\,dx = \delta(k'-k)}$$

Transformada de Fourier y $\delta(\cdot)$

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ikx} \tilde{\phi}(k)\, dk$$

$$\Downarrow$$

$$\therefore \quad \eta(x,k) = \frac{e^{ikx}}{\sqrt{2\pi}}$$

$$k_0 = -\infty$$

$$k_f = \infty$$

$$\text{y} \quad \tilde{\phi}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx} \phi(x)\, dx$$

$$\Downarrow$$

$$\therefore \quad \xi(x,k) = \frac{e^{-ikx}}{\sqrt{2\pi}}$$

$$x_0 = -\infty$$

$$x_f = \infty$$

$\therefore$ se concluye:

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ik(x-x')}\, dk = \delta(x-x') \quad //$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ix(k-k')}\, dx = \delta(k-k') \quad //$$