Clase 23 301121 1

Para la deflexión:

$$\left(\frac{1}{r^2}\frac{dr}{d\phi}\right)^2 = \frac{1}{b^2} - r^{-2} f(r)$$

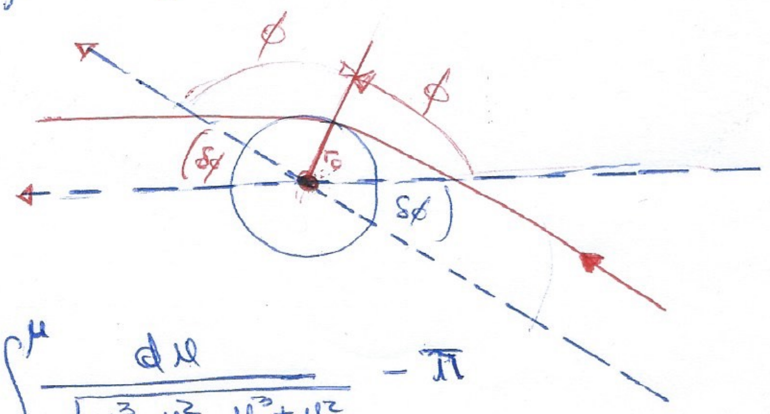
$$u = \frac{r_s}{r} \quad ; \quad du = \frac{-r_s}{r^2} dr$$

$$\left(-\frac{1}{r_s}\frac{du}{d\phi}\right)^2 = \frac{1}{b^2} - \frac{u^2}{r_s^2}(1-u)$$

$$\left(-\frac{du}{d\phi}\right)^2 = \left(\frac{r_s}{b}\right)^2 - u^2 + u^3$$

Hagamos:
$$\left(-\frac{du}{d\phi}\right)^2 = u^3 - u^2 - \mu^3 + \mu^2$$

donde $\mu = \frac{r_s}{r_o}$; r_o: distancia de máximo acercamiento.



$$\delta\phi = 2\int_0^{\mu} \frac{du}{\sqrt{u^3 - u^2 - \mu^3 + \mu^2}} - \pi$$

$$\delta\phi = 2I - \pi$$

Haciendo $x = \frac{u}{\mu}$ $\quad du = \mu dx$

$$u^3 - u^2 - \mu^3 + \mu^2 = x^3\mu^3 - x^2\mu^2 - \mu^3 + \mu^2$$
$$= \mu^2\left[\mu(x^3-1) + (1-x^2)\right]$$
$$= \mu^2\left[(1-x^2) - (1-x^3)\mu\right]$$

$$I = \int_0^1 \frac{dx}{\sqrt{(1-x^2)-(1-x^3)\mu}} =$$

$$I = \int_0^1 \frac{1}{\sqrt{1-\left(\frac{1-x^3}{1-x^2}\right)\mu}} \cdot \frac{dx}{\sqrt{1-x^2}}$$

Ya que $\mu << 1$ para el sistema solar.

$$* \quad \frac{1}{\sqrt{1-z}} = \sum_{m=0}^{\infty} \frac{1}{4^m}\binom{2m}{m} z^m$$

$$I = \sum_{m=0}^{\infty} \frac{1}{4^m}\binom{2m}{m}\left[\int_0^1 \left(\frac{1-x^3}{1-x^2}\right)^m \frac{dx}{\sqrt{1-x^2}}\right]\mu^m$$

$$I = \sum_{m=0}^{\infty} \frac{1}{4^m}\binom{2m}{m} I_m \mu^m$$

$$\therefore \ I \simeq I_0 + \frac{1}{2} I_1 \mu + \frac{3}{8} I_2 \mu^2 + \dots$$

$$I \simeq \frac{\pi}{2} + \mu + \frac{3}{8}\left[\frac{5\pi}{4} - \frac{4}{3}\right]\mu^2$$

CL23 (3)

$$\delta\phi = \pi + 2\mu + \frac{3}{4}\left[\frac{5\pi}{4} - \frac{4}{3}\right]\mu^2 - \pi$$

$$\delta\phi = 2\mu + \frac{3}{4}\left[\frac{5\pi}{4} - \frac{4}{3}\right]\mu^2$$

$$\delta\phi = \frac{2r_s}{r_0} + \frac{3}{4}\left[\frac{5\pi}{4} - \frac{4}{3}\right]\frac{r_s^2}{r_0^2}$$

Ahora, $\mu^2 - \mu^3 = \frac{r_s^2}{b^2} \equiv \nu^2$

$$\mu^2(1-\mu) = \nu^2$$

$$\mu = \frac{\nu}{\sqrt{1-\mu}} \simeq \nu\left(1 + \frac{1}{2}\mu\right) \simeq \nu$$

$$\mu \simeq \nu \rightarrow \frac{r_s}{r_0} \sim \frac{r_s}{b}$$

$$\therefore \boxed{\delta\phi \simeq \frac{2r_s}{b} + \frac{3}{4}\left[\frac{5\pi}{4} - \frac{4}{3}\right]\frac{r_s^2}{b^2} + \dots}$$

Corrección a 1^{er} orden:

Sol: $r_s \sim 2{,}94$ km ; $b \sim R_\odot \sim 6{,}96\times10^5$ km

$$\delta\phi_\odot \simeq \frac{2\cdot 2{,}94}{6{,}96\times10^5} \sim 8{,}45\times10^{-6}\ (\text{rad}) = 1''.74$$

Júpiter: $r_{s_J} \sim 2{,}24\times10^{-3}$ km ; $b \sim R_J \sim 6{,}98\times10^4$ km

$\delta\phi_J \sim 6{,}42\times10^{-8}$ rad $= 0''{,}013$

satélite Hipparcos.