COMPLEMENTO CLASE 4

JU 08/09/2016.

$$\sum_{k=0}^{n} \cos kx = Re\left(\sum_{k=0}^{n} e^{ikx}\right)$$

$\boxed{1}$

<u>Obs</u>. $$\sum_{k=0}^{n} y^n = \frac{1-y^{n+1}}{1-y}$$

evaluemos primero

$$\sum_{k=0}^{n} (e^{ix})^k = \frac{1-(e^{ix})^{n+1}}{1-(e^{ix})}$$

$$= \frac{1-e^{ix(n+1)}}{1-e^{ix}}$$

pero $1-e^{ix} = e^{i\frac{x}{2}}\left(e^{-i\frac{x}{2}} - e^{i\frac{x}{2}}\right)$.

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El viejo truco de factorizar por la mitad

donde $e^{ix/2} - e^{-ix/2} = 2i\,\mathrm{sen}(x/2)$

$\therefore\ 1-e^{ix} = e^{ix/2}\,2i\,\mathrm{sen}\left(\frac{x}{2}\right)$.

por otro lado y similarmente a lo anterior:

$$1-e^{ix(n+1)} = e^{ix\frac{(n+1)}{2}}\,2i\,\mathrm{sen}\left[\frac{(n+1)}{2}x\right]$$

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luego

$$\sum_{k=0}^{n} e^{ikx} = \frac{1 - e^{ix(n+1)}}{1 - e^{ix}}$$

$$= \frac{e^{\frac{ix(n+1)}{2}} \cancel{2i} \operatorname{sen}\left(\frac{n+1}{2} x\right)}{e^{\frac{ix}{2}} \cancel{2i} \operatorname{sen}\left(\frac{x}{2}\right)}$$

$$= \frac{e^{\frac{inx}{2}} \operatorname{sen}\left[\frac{n+1}{2} x\right]}{\operatorname{sen}\left[\frac{x}{2}\right]}$$

$$= \frac{\cos\left(\frac{nx}{2}\right) \operatorname{sen}\left(\frac{n+1}{2} x\right)}{\operatorname{sen}\left(\frac{x}{2}\right)}$$

$$+ i \operatorname{sen}\left(\frac{x}{2}\right) \operatorname{sen}\left(\frac{n+1}{2} x\right) \Big/ \operatorname{sen}\left(\frac{x}{2}\right)$$

finalmente

$$1 + \cos x + \dots + \cos nx = \mathrm{Re}\left(\sum_{k=0}^{n} e^{ikx}\right)$$

$$= \frac{\cos\left(\frac{nx}{2}\right) \operatorname{sen}\left(\frac{n+1}{2}x\right)}{\operatorname{sen}(x/2)}$$

$\therefore$

$$\mathrm{Im}\left(\sum_{k=0}^{n} e^{ikx}\right) = \sum_{k=0}^{n} \operatorname{sen} kx$$

$$= \operatorname{sen} x + \operatorname{sen} 2x + \dots + \operatorname{sen} nx$$

$$= \frac{\operatorname{sen}\left(\frac{n}{2}x\right) \operatorname{sen}\left(\frac{n+1}{2}x\right)}{\operatorname{sen}\left(\frac{x}{2}\right)}$$