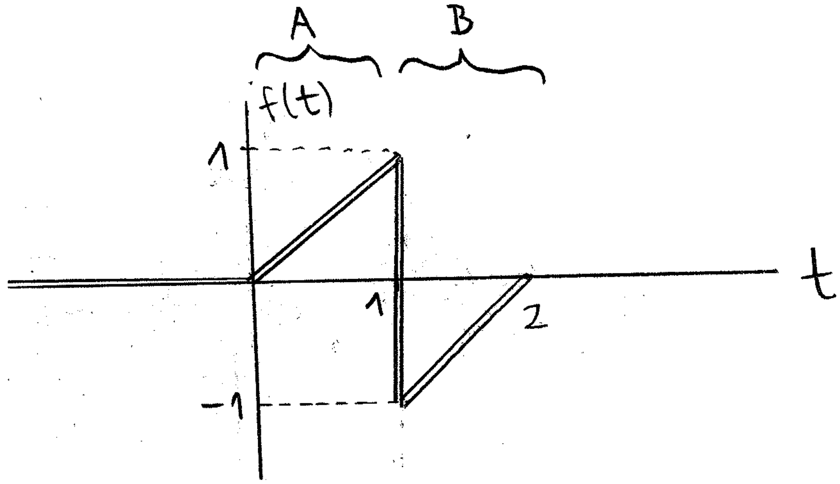
1)



a) Escribimos la función por intervalo (convencional):

$$f(t) = \begin{cases} 0; & -\infty < t < 0 \\ t; & 0 \leq t \leq 1 \\ t-2; & 1 < t \leq 2 \\ 0; & 2 < t < \infty \end{cases}$$

Usando funciones de Heaviside:

$$f(t) = t\,[H(t) - H(t-1)] + (t-2)[H(t-1) - H(t-2)]$$

$$= t\,H(t) - \cancel{t\,H(t-1)} + \cancel{t\,H(t-1)} - t\,H(t-2) - 2H(t-1) + 2H(t-2)$$

$$= t\,H(t) - t\,H(t-2) - 2H(t-1) + 2H(t-2)$$ //

b) De la gráfica es directo: $\int_{-\infty}^{\infty} f(t)\,dt = 0$

Haciendo el cálculo:

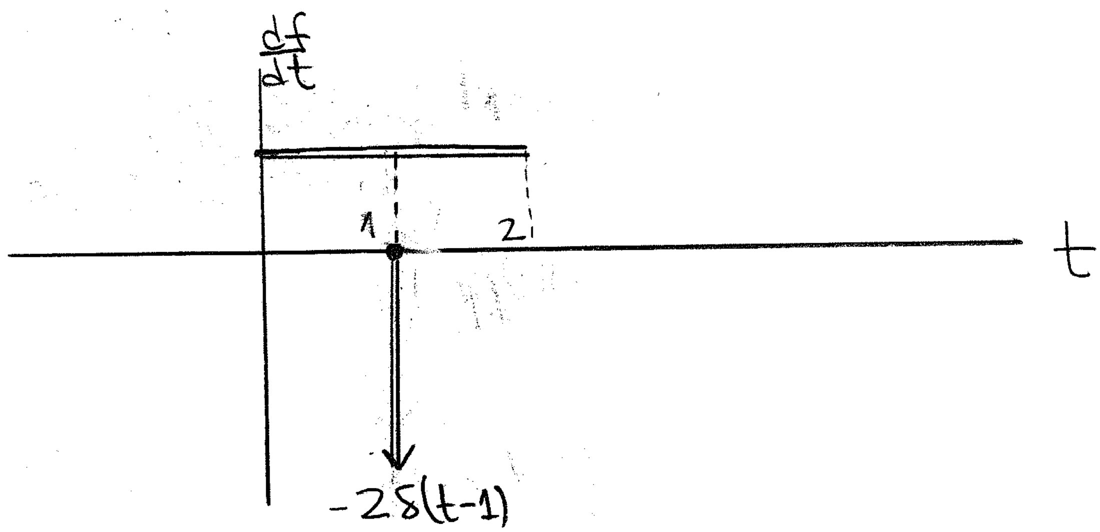
$$\int_{-\infty}^{\infty} f(t)\,dt = \int_{-\infty}^{\infty} t\,[H(t) - H(t-2)]\,dt - 2\int_{-\infty}^{\infty} [H(t-1) - H(t-2)]\,dt$$

$$\int_{-\infty}^{\infty} f(t)\,dt = \int_0^2 t\,dt - 2\int_1^2 dt$$

$$= \frac{1}{2}t^2\Big|_0^2 - 2t\Big|_1^2 = \frac{4}{2} - 2 = 0$$

c) $$\frac{df(t)}{dt} = \frac{d}{dt}\Big[t\,[H(t)-H(t-2)] - 2\,[H(t-1)-H(t-2)]\Big]$$

$$= H(t)-H(t-2) + t\big(\delta(t)-\delta(t-2)\big) - 2\big(\delta(t-1)-\delta(t-2)\big)$$



+ simplificado usando la identidad $g(t)\,\delta(t-\alpha) = g(\alpha)\,\delta(t-\alpha)$

$$\frac{df}{dt} = H(t)-H(t-2) + \cancel{0\,\delta(t)} - \cancel{2\delta(t-2)} - 2\delta(t-1) + \cancel{2\delta(t-2)}$$

$$= H(t) - H(t-2) - 2\delta(t-1)$$

2) Sea $F(\hat{D})\,x(t) = f(t)$.

La solución particular está dada por la siguiente expresión:

$$X_p(t) = F^{-1}(\hat{D})\,f(t)$$

si suponemos que $f(t) = \sum_n a_m t^n$ y por otro lado se tiene que:

$$t^n = t^n e^{\alpha t}\Big|_{\alpha=0} = \lim_{\alpha \to 0} t^n e^{\alpha t}$$

$$\Downarrow$$

$$t^n = \frac{d^n}{d\alpha^n} e^{\alpha t}\Big|_{\alpha=0} = \left(\frac{d}{d\alpha}\right)^n e^{\alpha t}\Big|_{\alpha=0}$$

luego $f(t) = \left[\sum_n a_n \frac{d^n}{d\alpha^n}\right] e^{\alpha t}\Big|_{\alpha=0} = f\left(\frac{d}{d\alpha}\right) e^{\alpha t}\Big|_{\alpha=0}$

$\therefore$

$$X_p(t) = F^{-1}(\hat{D})\,f\left(\frac{d}{d\alpha}\right) e^{\alpha t}\Big|_{\alpha=0} = f\left(\frac{d}{d\alpha}\right) F^{-1}(\hat{D})\,e^{\alpha t}\Big|_{\alpha=}$$

CONMUTAN ENTRE SI

3) La densidad de carga está dada por la siguiente expresión en coord. cilíndricas:

$$\rho(\vec{r}') = A\,\delta(r'-R)\,\delta(z')\left[H(\theta')-H(\theta'-\pi)\right]$$

z = coord. axial
r' = " radial
θ = " azimutal.

Hallando A

$$Q = \int \rho(\vec{r}')\,dV' = A\int_{r'=0}^{\infty}\int_{\theta'=0}^{2\pi}\int_{z'=-\infty}^{\infty}\delta(r'-R)\,\delta(z')\left[H(\theta')-H(\theta'-\pi)\right]\underbrace{dV'}_{r'dr'dz'd\theta'}$$

$$= A\underbrace{\int_{r'=0}^{\infty} r'\,\delta(r'-R)\,dr'}_{R}\;\underbrace{\int_{z'=-\infty}^{\infty}\delta(z')\,dz'}_{1}\;\underbrace{\int_{\theta'=0}^{2\pi}\left[H(\theta')-H(\theta'-\pi)\right]d\theta'}_{\pi}$$

$$= AR\pi$$

$$\Downarrow$$

$$A = \frac{Q}{\pi R} = \lambda_0.$$

$$\therefore\quad \rho(\vec{r}') = \lambda_0\,\delta(r'-R)\,\delta(z')\left[H(\theta')-H(\theta'-\pi)\right]$$

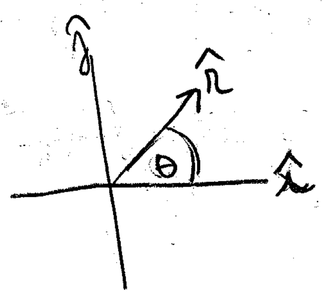
Para el potencial eléctrico se debe evaluar la siguiente integral:

$$\Phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}\, dV' .$$

en coord. cilíndricas se tiene que:

$$\left.\begin{aligned}\vec{r} &= z\hat{k} + \pi\hat{\pi}\\ \vec{r}' &= z'\hat{k} + \pi'\hat{\pi}'\end{aligned}\right\}$$ No tienen componentes en la dirección $\hat{\theta}$ y $\hat{\theta}'$

Es necesario escribir $\hat{\pi}$ en función de $\hat{\imath}, \hat{\jmath}$:



$\hat{\pi} = \cos\theta\,\hat{\imath} + \operatorname{sen}\theta\,\hat{\jmath}$ análogamente:

$\hat{\pi}' = \cos\theta'\,\hat{\imath} + \operatorname{sen}\theta'\,\hat{\jmath}$.

luego

$$|\vec{r}-\vec{r}'| = \sqrt{\vec{r}\cdot\vec{r} - 2\vec{r}\cdot\vec{r}' + \vec{r}'\cdot\vec{r}'}$$

$$= \sqrt{r^2 - 2\vec{r}\cdot\vec{r}' + z'^2 + \pi'^2} \quad ; \quad \begin{aligned} r^2 &= z^2 + \pi^2 = |\vec{r}|^2 \\ r'^2 &= z'^2 + \pi'^2 \end{aligned}$$

$$= \sqrt{r^2 - 2rr'\hat{\pi}\cdot\hat{\pi}' + z'^2 + \pi'^2}$$

y $\hat{\pi}\cdot\hat{\pi}' = \cos\theta\cos\theta' + \operatorname{sen}\theta\operatorname{sen}\theta' = \cos(\theta-\theta')$

$\therefore$

$$\Phi(\vec{r}) = \frac{\lambda_0}{4\pi\varepsilon_0} \int_{r'=0}^{\infty}\int_{\theta'=0}^{2\pi}\int_{z'=-\infty}^{\infty} \frac{\delta(r'-R)\,\delta(z')\,[H(\theta')-H(\theta'-\pi)]\, r'\,dr'\,dz'\,d\theta'}{\sqrt{r^2-2rr'\cos(\theta-\theta')+z'^2+r'^2}}$$

en $z'=0$ y $r'=R \Rightarrow r'=R$

$$\Phi(\vec{r}) = \frac{\lambda_0}{4\pi\varepsilon_0} R \int_0^{\pi} \frac{d\theta'}{\left[r^2-2rR\cos(\theta-\theta')+R^2\right]^{1/2}}$$

4)
$$I = \int_0^{\infty} e^{-2x} \operatorname{sen} 3x \cos x \, dx$$

$$= \operatorname{sen}\left(-3\frac{d}{ds}\right)\cos\left(-\frac{d}{ds}\right)\frac{1}{s}\Bigg|_{s=2}$$

$$= -\operatorname{sen}\left(3\frac{d}{ds}\right)\cos\left(\frac{d}{ds}\right)\frac{1}{s}\Bigg|_{s=2}.$$

$$= -\frac{1}{2i}\left[e^{i3\frac{d}{ds}}-e^{-i3\frac{d}{ds}}\right]\frac{1}{2}\left[e^{i\frac{d}{ds}}+e^{-i\frac{d}{ds}}\right]\frac{1}{s}\Bigg|_{s=2}$$

$$= -\frac{1}{4i}\left[e^{4i\frac{d}{ds}}+e^{2i\frac{d}{ds}}-e^{-2i\frac{d}{ds}}-e^{-4i\frac{d}{ds}}\right]\frac{1}{s}\Bigg|_{s=2}$$

$$= -\frac{1}{4i}\left[2i\,\mathrm{Im}\left[e^{i4\frac{d}{ds}}\right] + 2i\,\mathrm{Im}\left[e^{i2\frac{d}{ds}}\right]\right]\frac{1}{s}\Bigg|_{s=2}$$

$$= -\frac{1}{2}\,\mathrm{Im}\left[e^{i4\frac{d}{ds}} + e^{i2\frac{d}{ds}}\right]\frac{1}{s}\Bigg|_{s=2}$$

$$= -\frac{1}{2}\,\mathrm{Im}\left[\frac{1}{s+4i} + \frac{1}{s+2i}\right]\Bigg|_{s=2}$$

$$= -\frac{1}{2}\,\mathrm{Im}\left[\frac{1}{2+4i} + \frac{1}{2+2i}\right]$$

$$= -\frac{1}{2}\,\mathrm{Im}\left[\frac{2-4i}{20} + \frac{2-2i}{8}\right]$$

$$= -\frac{1}{2}\left[-\frac{4}{20} - \frac{2}{8}\right] = \frac{2}{20} + \frac{1}{8} = \frac{4+5}{40} = \frac{9}{40}$$