Ejercicios Churchill 9ed para 1era Prueba

Seccion 5

8. Let z_1 and z_2 denote any complex numbers

$$z_1 = x_1 + iy_1 \quad \text{and} \quad z_2 = x_2 + iy_2.$$

Use simple algebra to show that

$$|(x_1 + iy_1)(x_2 + iy_2)| \quad \text{and} \quad \sqrt{(x_1^2 + y_1^2)(x_2^2 + y_2^2)}$$

are the same and then point out how the identity

$$|z_1 z_2| = |z_1||z_2|$$

follows.

9. Use the final result in Exercise 8 and mathematical induction to show that

$$|z^n| = |z|^n \qquad (n = 1, 2, \ldots),$$

where z is any complex number. That is, after noting that this identity is obviously true when $n = 1$, assume that it is true when $n = m$ where m is any positive integer and then show that it must be true when $n = m + 1$.

Seccion 6

1. Use properties of conjugates and moduli established in Sec. 6 to show that

(*a*) $\overline{\overline{z} + 3i} = z - 3i$; (*b*) $\overline{iz} = -i\overline{z}$;

(*c*) $\overline{(2 + i)^2} = 3 - 4i$; (*d*) $|(2\overline{z} + 5)(\sqrt{2} - i)| = \sqrt{3}\,|2z + 5|$.

7. Show that

$$|\text{Re}(2 + \overline{z} + z^3)| \leq 4 \qquad \text{when } |z| \leq 1.$$

Seccion 11

1. Find the square roots of (a) $2i$; (b) $1-\sqrt{3}i$ and express them in rectangular coordinates.

Ans. (a) $\pm(1+i)$; (b) $\pm\dfrac{\sqrt{3}-i}{\sqrt{2}}$.

2. Find the three cube roots $c_k (k = 0, 1, 2)$ of $-8i$, express them in rectangular coordinates, and point out why they are as shown in Fig. 15.

Ans. $\pm\sqrt{3}-i, 2i$.

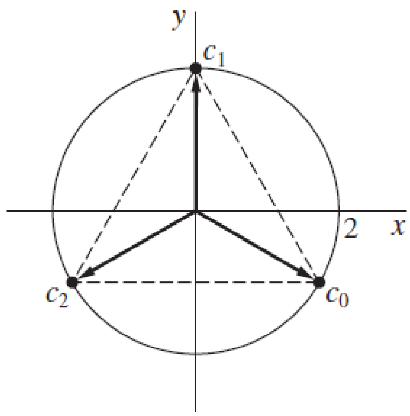

FIGURE 15

Seccion 12

9. Show that any point z_0 of a domain is an accumulation point of that domain.

10. Prove that a finite set of points $z_1, z_2, \ldots, z_n$ cannot have any accumulation points.

Seccion 24

4. Use the theorem in Sec. 24 to show that each of these functions is differentiable in the indicated domain of definition, and also to find $f'(z)$:

(a) $f(z) = 1/z^4 \quad (z \neq 0)$;

(b) $f(z) = e^{-\theta}\cos(\ln r) + i\,e^{-\theta}\sin(\ln r) \quad (r > 0, 0 < \theta < 2\pi)$.

Ans. (b) $f'(z) = i\,\dfrac{f(z)}{z}$.

8. (*a*) Recall (Sec. 6) that if $z = x + iy$, then

$$x = \frac{z + \overline{z}}{2} \quad \text{and} \quad y = \frac{z - \overline{z}}{2i}.$$

By *formally* applying the chain rule in calculus to a function $F(x, y)$ of two real variables, derive the expression

$$\frac{\partial F}{\partial \overline{z}} = \frac{\partial F}{\partial x}\frac{\partial x}{\partial \overline{z}} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial \overline{z}} = \frac{1}{2}\left(\frac{\partial F}{\partial x} + i\frac{\partial F}{\partial y}\right).$$

(*b*) Define the operator

$$\frac{\partial}{\partial \overline{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right),$$

suggested by part (*a*), to show that if the first-order partial derivatives of the real and imaginary components of a function $f(z) = u(x, y) + iv(x, y)$ satisfy the Cauchy–Riemann equations, then

$$\frac{\partial f}{\partial \overline{z}} = \frac{1}{2}[(u_x - v_y) + i(v_x + u_y)] = 0.$$

Thus derive the ***complex form*** $\partial f/\partial \overline{z} = 0$ of the Cauchy–Riemann equations.

Seccion 30

6. Show that $|\exp(z^2)| \leq \exp(|z|^2)$.

7. Prove that $|\exp(-2z)| < 1$ if and only if $\operatorname{Re} z > 0$.

8. Find all values of z such that

(*a*) $e^z = -2$; (*b*) $e^z = 1 + i$; (*c*) $\exp(2z - 1) = 1$.

Ans. (*a*) $z = \ln 2 + (2n + 1)\pi i \ (n = 0, \pm 1, \pm 2, \ldots)$;

(*b*) $z = \dfrac{1}{2}\ln 2 + \left(2n + \dfrac{1}{4}\right)\pi i \ (n = 0, \pm 1, \pm 2, \ldots)$;

(*c*) $z = \dfrac{1}{2} + n\pi i \ (n = 0, \pm 1, \pm 2, \ldots)$.

Seccion 33

1. Show that

(*a*) $\mathrm{Log}(-ei) = 1 - \frac{\pi}{2}i$; (*b*) $\mathrm{Log}(1-i) = \frac{1}{2}\ln 2 - \frac{\pi}{4}i$.

2. Show that

(*a*) $\log e = 1 + 2n\pi i \quad (n = 0, \pm 1, \pm 2, \ldots)$;

(*b*) $\log i = \left(2n + \frac{1}{2}\right)\pi i \quad (n = 0, \pm 1, \pm 2, \ldots)$;

(*c*) $\log(-1 + \sqrt{3}i) = \ln 2 + 2\left(n + \frac{1}{3}\right)\pi i \quad (n = 0, \pm 1, \pm 2, \ldots)$.

Seccion 38

14. Show that

(*a*) $\overline{\cos(iz)} = \cos(i\bar{z})$ for all z;

(*b*) $\overline{\sin(iz)} = \sin(i\bar{z})$ if and only if $z = n\pi i \ (n = 0, \pm 1, \pm 2, \ldots)$.

15. Find all roots of the equation $\sin z = \cosh 4$ by equating the real parts and then the imaginary parts of $\sin z$ and $\cosh 4$.

Ans. $\left(\frac{\pi}{2} + 2n\pi\right) \pm 4i \quad (n = 0, \pm 1, \pm 2, \ldots)$.

Seccion 39

16. By using one of the identities (9) and (10) in Sec. 39 and then proceeding as in Exercise 15, Sec. 38, find all roots of the equation

(*a*) $\sinh z = i$; (*b*) $\cosh z = \frac{1}{2}$.

Ans. (*a*) $z = \left(2n + \frac{1}{2}\right)\pi i \quad (n = 0, \pm 1, \pm 2, \ldots)$;

(*b*) $z = \left(2n \pm \frac{1}{3}\right)\pi i \quad (n = 0, \pm 1, \pm 2, \ldots)$.

17. Find all roots of the equation $\cosh z = -2$. (Compare this exercise with Exercise 16, Sec. 38.)

Ans. $z = \pm \ln(2 + \sqrt{3}) + (2n + 1)\pi i \ (n = 0, \pm 1, \pm 2, \ldots)$.