

PROBLEMA GUÍA II | #2

$$\nabla^2 \phi(\vec{r}) = \nabla \cdot \nabla \phi(\vec{r}) = \nabla \cdot \nabla \left(\frac{1}{r^{1+2\beta}} \right)$$

$$= \partial_i \partial_i \left(\frac{1}{r^{1+2\beta}} \right) = \partial_i \left[\partial_i (r^{-1-2\beta}) \right]$$

$$= \partial_i \left[(-1-2\beta) r^{-2-2\beta} \partial_i (r) \right] ; \text{ con } \partial_i (r) = \frac{x_i}{r}$$

$$= -(1+2\beta) \partial_i \left[r^{-3-2\beta} x_i \right]$$

$$= -(1+2\beta) \left[x_i \left(\partial_i (r^{-3-2\beta}) \right) + \frac{(\partial_i x_i)}{r^{3+2\beta}} \right]$$

$$= -(1+2\beta) \left[x_i \frac{(-3-2\beta)}{r^{4+2\beta}} \partial_i (r) + \frac{\delta_{ii}}{r^{3+2\beta}} \right]$$

$$= -(1+2\beta) \left[-(3+2\beta) \frac{x_i}{r^{4+2\beta}} \frac{x_i}{r} + \frac{\delta_{ii}}{r^{3+2\beta}} \right] ; \text{ donde } \begin{aligned} x_i x_i &= r^2 \\ \delta_{ii} &= 3 \end{aligned}$$

$$= -(1+2\beta) \left[-\frac{(3+2\beta)}{r^{3+2\beta}} + \frac{3}{r^{3+2\beta}} \right]$$

$$\nabla^2 \phi = (1+2\beta) 2\beta \left(\frac{1}{r^{3+2\beta}} \right) \Rightarrow \text{si } \nabla^2 \phi = 0 \Rightarrow \beta = -\frac{1}{2} \text{ o } \beta = 0 //$$