1a) representación del principio de Fermat $\quad t = \frac{x}{v}$ $\quad$ coord. cilíndricas

1

$$\delta T = \delta \int_a^b \frac{ds}{v} \leftarrow \text{velocidad}$$

(*1) $$\delta S = c\,\delta T = \delta \int_a^b \frac{c}{v}\, ds \qquad / \; |d\vec{s}| = \sqrt{dr^2 + r^2 d\phi^2 + dz^2}$$

velocidad de la luz

en nuestra imagen o modelo ocurre en un plano $z = cte$

$$|d\vec{s}| = ds = \sqrt{1 + r^2\left(\frac{d\phi}{dr}\right)^2}\, dr$$ de esa forma (*1) queda

$$\delta S = \delta \int_a^b \underbrace{\frac{c}{v}\sqrt{1 + r^2\left(\frac{d\phi}{dr}\right)^2}}_{\text{Lagrangiano, una funcional}}\, dr \leftarrow$$ representación integral del principio de Fermat

donde $\delta s = 0$ ocurrirán las Ec Euler Lagrange

$$\mathcal{L} = n\sqrt{1 + r^2\dot{\phi}^2} \qquad \left(n = \frac{c}{v} \; ; \; \dot{\phi} = \frac{d\phi}{dr}\right)$$

$$\frac{d}{dr}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_\sigma}\right) - \frac{\partial \mathcal{L}}{\partial q_\sigma} = 0 \quad \text{(EL)} \qquad \text{con } n = n(r)$$

con $q = \{\phi, \cancel{z}\}$

$$\frac{d}{dr}\left(\frac{n}{\sqrt{1 + r^2\dot{\phi}^2}}\frac{\partial (r^2\dot{\phi}^2)}{\partial \dot{\phi}}\right) = \frac{d}{dr}\left(\underbrace{\frac{n\, 2r^2\dot{\phi}}{\sqrt{1 + r^2\dot{\phi}^2}}}_{\text{cantidad conservada respecto a } r}\right) = 0$$

$$\mathcal{L}(r, \dot{\phi}) \therefore \frac{\partial L}{\partial r} \neq 0 \therefore H \neq T + U$$

$$p_\phi = \frac{2nr^2\dot{\phi}}{\sqrt{1 + r^2\dot{\phi}^2}} = cte = K \therefore \cancel{2}nr^2\dot{\phi} = K\,(1 + r^2\dot{\phi}^2)^{1/2} \quad / ()^2$$

se lo come la cte.

$$n^2 r^4 \dot{\phi}^2 = k(1 + r^2\dot{\phi}^2) \underset{(*)}{=} k + kr^2\dot{\phi}^2$$