Introduction to Cosmology

True progress in cosmology began in the 20th century:

- General Relativity
- Powerful tools of observational astronomy

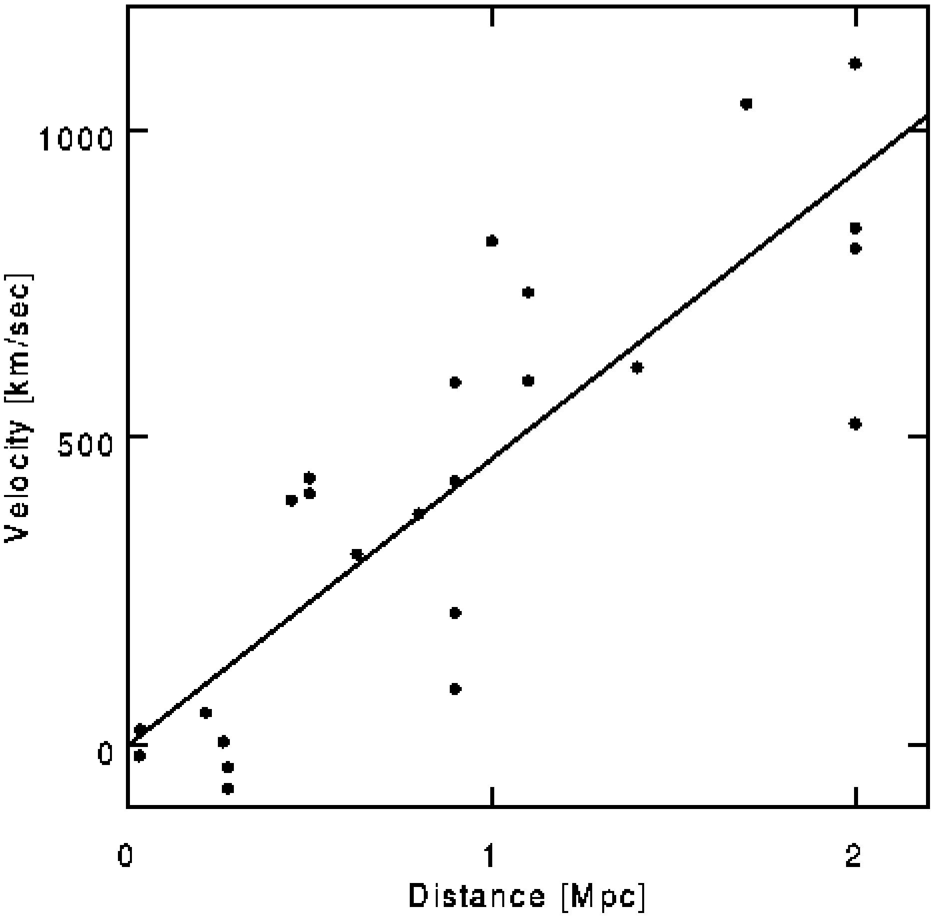
Two critical observations:

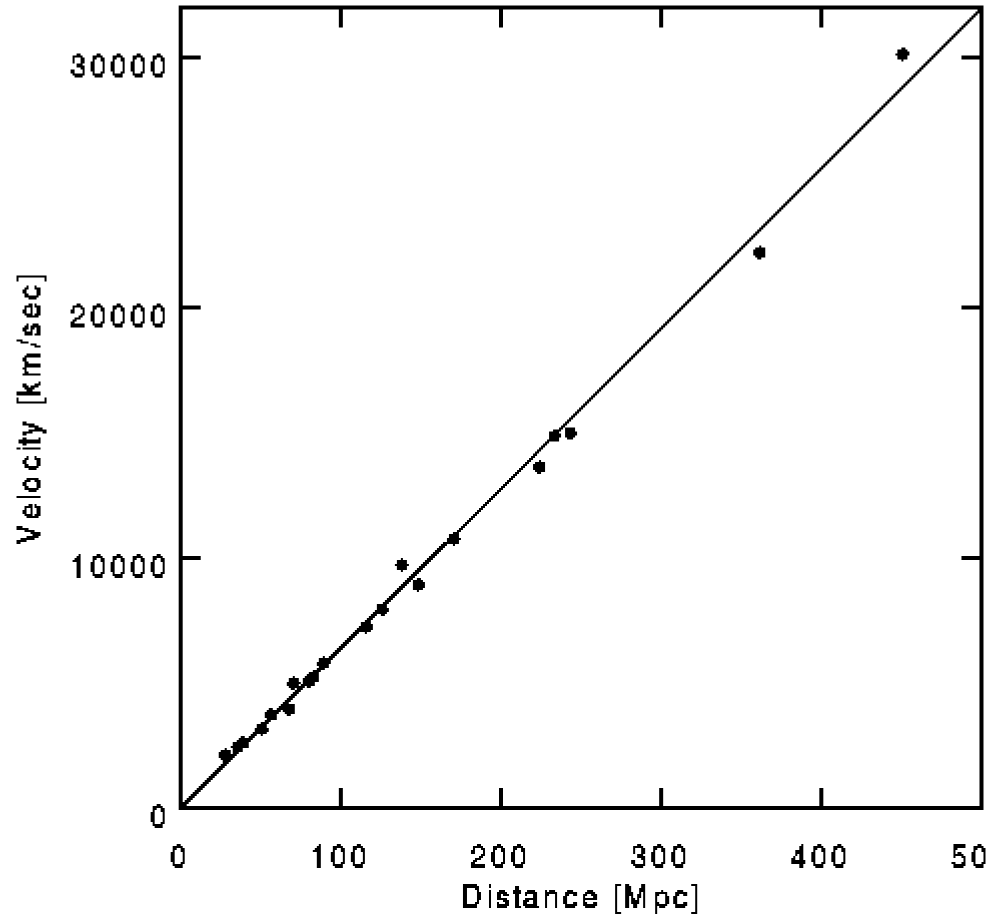
1. Hubble's Law (1929): Measure both the distance and radial velocity of a set of galaxies.

Radial velocity is easy to find, by measuring the redshift z and using the first-order Doppler shift:

$$1 + z \equiv \frac{\lambda_o}{\lambda_e} \approx 1 + \frac{v}{c}$$

Distance is much harder to measure. Need to use "cosmic distance ladder", with uncertainties at each step.





(1 Mpc = 3.086×10^{24} cm)

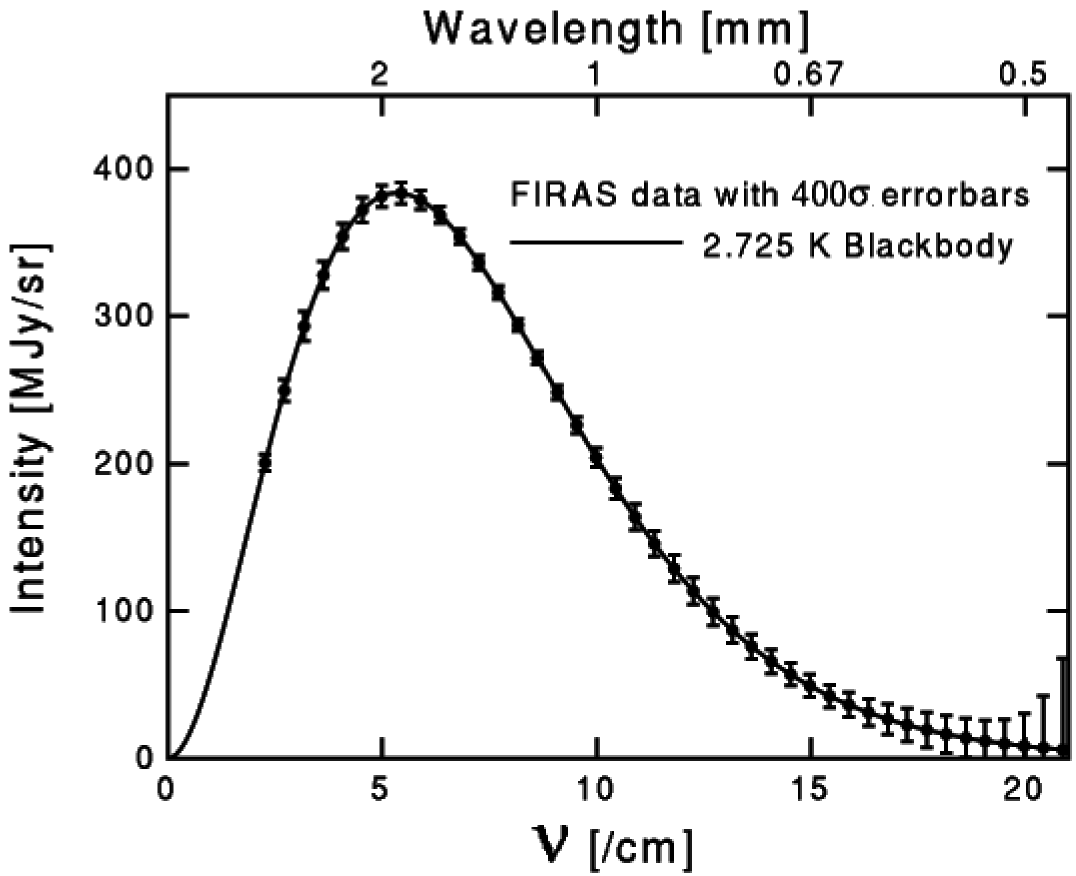
Hubble's Law: $v = H_0 d$

Recent determination (Riess et al. 2019, ApJ, 876, 85):

$H_0 = 74.03 \pm 1.42$ km s^{-1} Mpc^{-1}

Hubble constant is often expressed as $H_0 = 100\, h$ km s^{-1} Mpc^{-1}

2. The cosmic microwave background (CMB): Discovered by Penzias & Wilson in 1965. Blackbody radiation with T = 2.725 K observed (with very little variation) on all directions on the sky.



Hubble and COBE FIRAS plots are from Ned Wright's cosmology tutorial (www.astro.ucla.edu/~wright/cosmolog.htm)

Need to introduce a tremendous simplification to make the theoretical study of the universe as a whole feasible:

The cosmological principle: the universe is spatially homogeneous and isotropic

The CP is clearly ridiculous on small distance scales, but galaxy surveys (e.g., the Sloan Digital Sky Survey) suggest that homogeneity applies on scales $\sim 10^9$ lyr.

Isotropy: CMB and Hubble expansion (as well as galaxy distribution)

Isotropy for us plus Copernican principle => isotropy everywhere
=> homogeneity

We'll assume the CP in constructing cosmological models and hope that the results are also relevant to the real universe. This hope can be put to the test!

60,000 SDSS galaxies

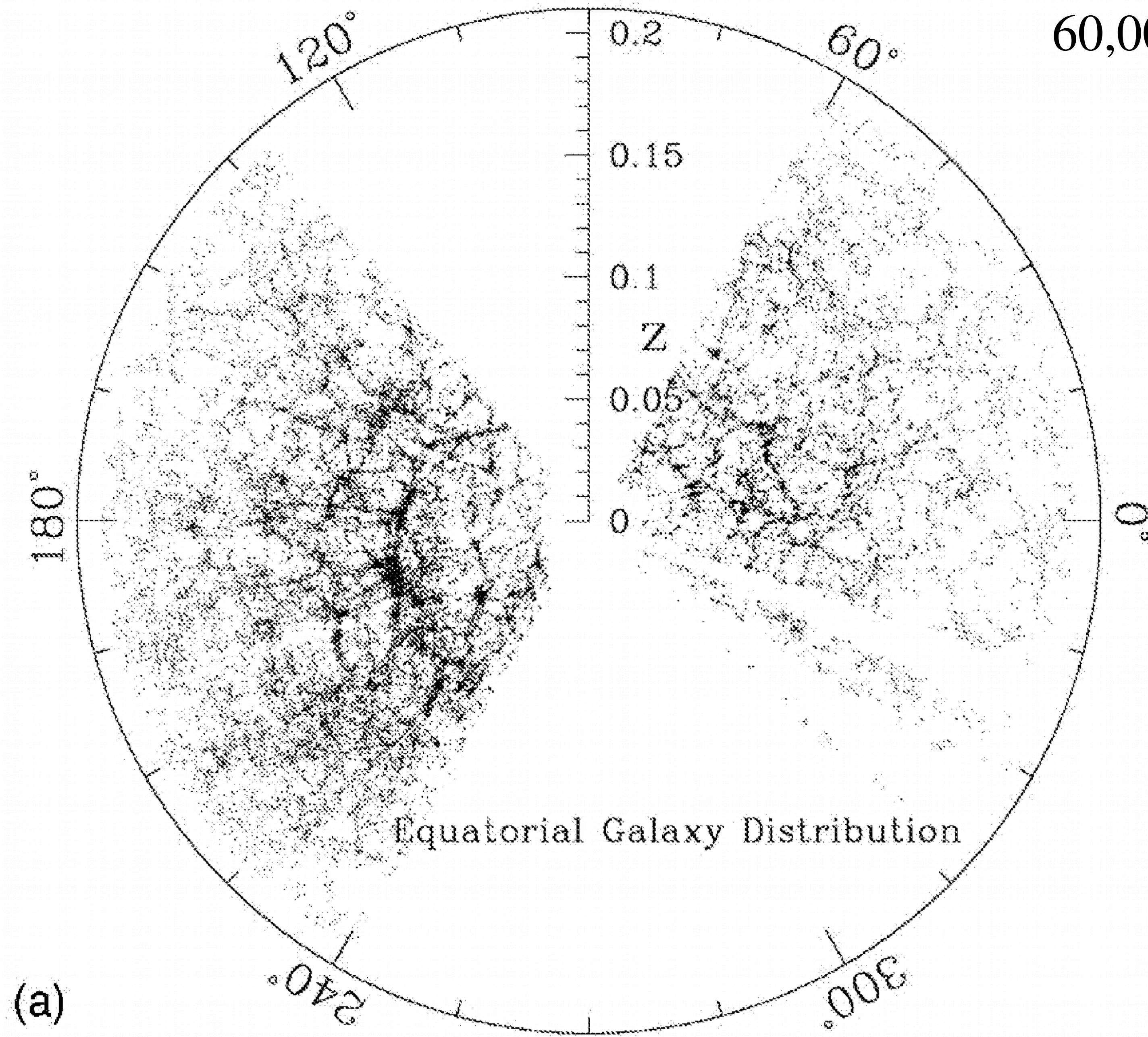


Fig from Freedman & Turner (2003, Rev Mod Phys, 75, 1433)

The CP severely constrains the metric:

a) Each point in space is characterized by physical and metric conditions (e.g., energy density, velocity, Riemann tensor).
CP => it must be possible to define a time coord such that at any given instant, these conditions are identical everywhere in the universe => the existence of a preferred cosmic time

b) The only possible type of relative motion btwn points in the universe is an expansion or contraction of the universe as a whole. Otherwise, homogeneity would be spoiled.
Note: obviously, this doesn't apply on small scales!

c) Fundamental observer: an observer located at some point in the universe and at rest wrt the local material. Homogeneity requires that proper time ticks at the same rate for each of these observers. Also, it's possible for them to synchronize their clocks.
We will take our cosmic time coord to be the synchronized proper time of the fundamental observers.

So far, we know the metric can be written: $ds^2 = c^2dt^2 - [f(t)]^2dl^2$

$f(t)$ is a function of time and dl^2 is a spatial metric, independent of time

CP also severely constrains dl^2. Now consider just the spatial geometry:

Adopting geodesic coords, the metric is that of Euclidean 3-space:

$$g_{ij} = \delta_{ij}$$

Isotropy of space => the components of R_{hijk} cannot depend on how we orient the axes of our geodesic coord system. Otherwise, the curvature would have a directionality in space.

=> R_{hijk} is expressible in terms of tensors whose components don't change on spatial rotations. The only such tensor is δ_{ij}.

=> $$R_{hijk} = k\,\delta_{hj}\,\delta_{ik} + k_1\,\delta_{hk}\,\delta_{ij} + k_2\,\delta_{hi}\,\delta_{jk}$$

(in each term, h pairs with either i, j, or k; the remaining 2 indices then pair)

$R_{hijk} = - R_{hikj}$ => $k_1 = - k$ and $k_2 = 0$

=> $$R_{hijk} = k\,(\delta_{hj}\,\delta_{ik} - \delta_{hk}\,\delta_{ij})$$

For arbitrary coords:

$$R_{hijk} = k\,(g_{hj}\,g_{ik} - g_{hk}\,g_{ij})$$

=> $R_{ij} = -2\,k\,g_{ij}$ and $R = -6\,k$

The spatial metric dl^2 must satisfy the above diff eqn for g_{ij}, which has only one free parameter; k must be the same everywhere.

We know that a 2-sphere has constant curvature everywhere, so let's look at a 3-sphere (living in Euclidean 4-space):

$$x^2 + y^2 + z^2 + w^2 = a^2$$

$$dl^2 = dx^2 + dy^2 + dz^2 + dw^2$$

$$= dx^2 + dy^2 + dz^2 + \frac{(x\,dx + y\,dy + z\,dz)^2}{a^2 - r^2}$$

$$x\,dx + y\,dy + z\,dz = r\,dr$$

=> $$dl^2 = \frac{dr^2}{1 - r^2/a^2} + r^2(d\theta^2 + \sin^2\theta\,d\phi^2)$$

This metric satisfies the equation for g_{ij} with $k = a^{-2}$

$$dl^2 = \frac{dr^2}{1 - r^2/a^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

Define η = *r/a* and factor out an a^2.

=> $$dl^2 = a^2\left[\frac{d\eta^2}{1-\eta^2} + \eta^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

a can be a function of time, while η, θ, and ϕ are constant for a fundamental observer. Such coord systems are known as "comoving". These coords for a point in the universe do not change, even as the universe expands or contracts. With η rather than *r*, $k = 1$ rather than $k = a^{-2}$.

Alternatively, with η = sin χ (0 ≤ χ < π):

$$dl^2 = a^2[d\chi^2 + \sin^2\chi\,(d\theta^2 + \sin^2\theta\, d\phi^2)]$$

Total volume of the space is:

$$V = \int_0^{\pi} d\chi \int_0^{\pi} d\theta \int_0^{2\pi} d\phi \, a^3 \sin^2\chi \, \sin\theta$$

$$= 4\pi a^3 \int_0^{\pi} d\chi \, \sin^2\chi = 4\pi a^3 \left[\frac{\chi}{2} - \frac{1}{4}\sin(2\chi)\right]_0^{\pi}$$

$= 2\pi^2 a^3$ (finite, as expected in analogy with the 2-sphere)

The metric for the 3-sphere is $dl^2 = a^2\left[\frac{d\eta^2}{1-k\eta^2} + \eta^2(d\theta^2 + \sin^2\theta \, d\phi^2)\right]$

with $k = 1$. When $k = 0$ or $k = -1$, this metric also satisfies the requirement on g_{ij}. The resulting spaces are said to have positive (negative) curvature when $k = +1$ ($k = -1$) and are flat when $k = 0$.

$$dl^2 = a^2\left[\frac{d\eta^2}{1-k\eta^2} + \eta^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

Clearly, $k = 0$ corresponds to Euclidean 3-space, which is infinite and open. The lack of spatial curvature does not exclude it from consideration, given that there is matter in the universe. Even if the spatial part is always flat, there can still be spacetime curvature.

$k = -1$ is an infinite, open space. With $\eta = \sinh\chi$,

$$dl^2 = a^2[d\chi^2 + \sinh^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)]$$

The resulting spacetime metrics are

$$d\mathbf{s}^2 = c^2dt^2 - a^2(t)\left[\frac{d\eta^2}{1-k\eta^2} + \eta^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

with $k = \pm 1$ or 0. These are known as the "Robertson-Walker" metrics, and they are the only ones consistent with the CP.

Using only the CP (no GR at all!), we've found metrics with just one free parameter (k) and one function of time, $a(t)$, known as the "scale factor".

Note that the metric is the same everywhere in the universe at a given cosmological time. To find $a(t)$, we need to solve Einstein's equations.

Einstein's equations involve the stress tensor. What types of material constituents are possible?

CP => only perfect fluids, in which the only stress is isotropic pressure P

=> $T_{\mu\nu} = \mathrm{diag}\,(-g_{ii}P\,,\ \rho c^2\,)$ (ρc^2 is the relativistic energy density)

For idealized non-relativistic particles (like galaxies), $P = 0$.
For idealized relativistic particles (like photons), $P = \rho c^2/\,3$.

1. Baryons: proton ($mc^2 = 938.3$ MeV) and neutron ($mc^2 = 939.6$ MeV) electron ($mc^2 = 0.511$ MeV), though not technically a baryon, is included here. Today, baryons are predominantly non-relativistic.

2. Radiation: photon has $mc^2 = 0$; $E = h\nu$

3. Neutrinos: small but non-zero mass; relativistic (sometimes included with the radiation)

4. Other (dark matter...)

Solution of Einstein's equations: (dots denote diff wrt t)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho - \frac{kc^2}{a^2}$$ (the Friedmann eqn)

$$\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = -\frac{4\pi GP}{c^2} - \frac{kc^2}{2a^2}$$

$$\Rightarrow \qquad \frac{\ddot{a}}{a} = -\frac{4}{3}\pi G\left(\rho + \frac{3P}{c^2}\right)$$ (the acceleration eqn)

=> a static universe, with *a* constant, is not possible!

The acceleration is negative because of the gravitational force. If the material has pressure, then this increases the deceleration, because in GR there is gravitation associated with pressure. Since the universe is homogeneous, there is no force associated with pressure gradients.

Einstein obtained this result shortly after introducing GR, over a decade before Hubble's work. At that time, there was a strong bias for a static universe (a = const). So, Einstein added a new term to his equation of GR, introducing a new fundamental constant Λ with units of inverse area (the "cosmological constant"):

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -\frac{8\pi G}{c^4}T_{\mu\nu}$$

This is equivalent to adding a new, ubiquitous source component with

$$T_{\mu\nu} = \frac{\Lambda c^4}{8\pi G}g_{\mu\nu}$$

=> a perfect fluid with $\rho_\Lambda = \dfrac{\Lambda c^2}{8\pi G}$ and $P_\Lambda = -\rho_\Lambda c^2$

Following Hubble's discovery, Einstein called the introduction of the cosmological constant his greatest blunder, but there is evidence today that the expansion is accelerating, so we'll keep Λ.

The cosmological eqns become:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

$$\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = -\frac{4\pi GP}{c^2} - \frac{kc^2}{2a^2} + \frac{\Lambda c^2}{2}$$

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G\left(\rho + \frac{3P}{c^2}\right) + \frac{\Lambda c^2}{3}$$

SP 11.1-2

=> static universe is possible. Λ yields a net acceleration because the acceleration associated with the negative pressure term exceeds the deceleration associated with the density term.

$$\dot{a} = \ddot{a} = 0 \quad \text{and} \quad P = 0 \qquad \Rightarrow \qquad \frac{kc^2}{a^2} = \Lambda c^2 = 4\pi G\rho$$

$\rho > 0$ => $k = 1$ and $\Lambda > 0$ (Einstein universe)

The required value of Λ is so small that it wouldn't affect the results of any of the solar system tests of GR. But: the equilibrium is unstable!

Can we find a physical interpretation for the cosmological constant?

It could arise as a vacuum energy density, e.g. as a quantum-mechanical zero-point energy. Simple attempts to quantify this yield a value of Λ which is vastly larger than observationally allowed; known as the "cosmological constant problem." Other types of "fluid" with negative pressure, but without constant density (in time) are possible (e.g., "quintessence").

Multiply the Friedmann eqn by a^3: $$a\dot{a}^2 + kc^2 a - \frac{\Lambda c^2 a^3}{3} = \frac{8}{3}\pi G\rho a^3$$

Now differentiate: $$2a\dot{a}\ddot{a} + \dot{a}^3 + kc^2\dot{a} - \Lambda c^2 a^2 \dot{a} = \frac{8}{3}\pi G\left(\dot{\rho}a^3 + 3\rho a^2 \dot{a}\right)$$

Mult 2nd cosmological eqn by $2a^2\dot{a}$: $$2a\dot{a}\ddot{a} + \dot{a}^3 + kc^2\dot{a} - \Lambda c^2 a^2 \dot{a} = -\frac{8\pi G P}{c^2}a^2\dot{a}$$

Subtract eqns: $$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{P}{c^2}\right)$$

Compare to the 1st Law of Thermodynamics for a perfect fluid and an adiabatic process (since the CP prohibits heat flow):

$$dU = -PdV \qquad \Rightarrow \qquad d\left(\rho c^2 a^3\right) = -Pda^3 \qquad => \qquad \dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{P}{c^2}\right)$$

This results automatically in GR since Einstein's eqns imply $T^{\mu\nu}{}_{;\nu} = 0$.

The Hubble Law revisited

Cosmological redshifts are due to the expansion of space, not to the motion of galaxies within space. The redshift arises not from the Doppler effect, but because the wavelength is stretched in proportion to $a(t)$:

$$1 + z \equiv \frac{\lambda_0}{\lambda_e} = \frac{a(t_0)}{a(t_e)}$$

Subscripts 0 and e refer to today and the time of emission.

At small distances, we can Taylor expand $a(t)$:

$$\frac{a(t_0)}{a(t_e)} \approx a(t_0)\left[a(t_0) + \left(\frac{da}{dt}\right)_{t_0}(t_e - t_0)\right]^{-1}$$ (for small $t_0 - t_e$)

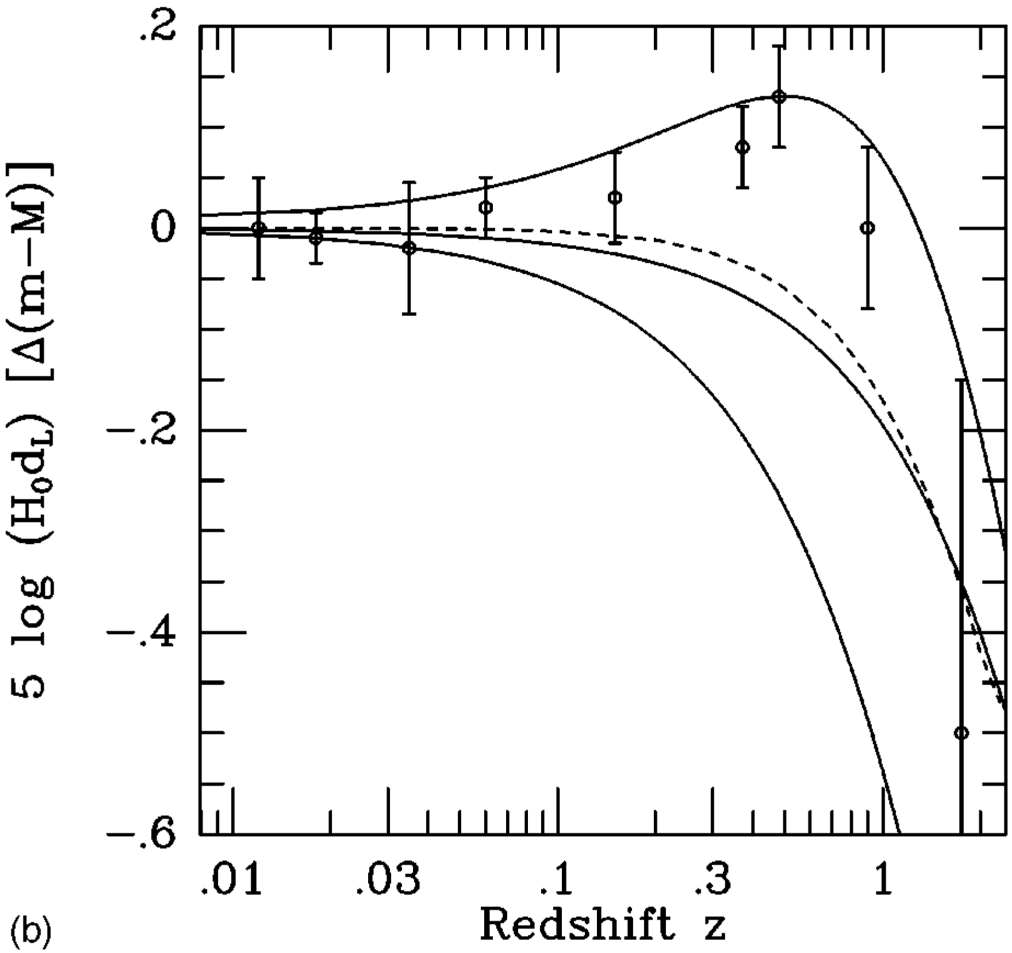
$$\approx 1 + \left(\frac{\dot{a}}{a}\right)_{t_0}(t_0 - t_e) = 1 + \frac{H_0 d}{c}$$ (d = distance to the galaxy)

Doppler shift: $\frac{\lambda_0}{\lambda_e} \approx 1 + \frac{v}{c}$ => $v = H_0 d$ (Hubble's Law)

The Hubble parameter $H \equiv \dot{a}/a$; it varies with time.

Its value today is called the Hubble constant, H_0.

Deviations from Hubble Law at large redshift tell us about the acceleration or deceleration of the expansion. Using Type Ia SNe, Riess et al. (1998, AJ, 116, 1009) and Perlmutter et al. (1999, ApJ, 517, 565) found that the expansion is accelerating!



Differential Hubble plot

Ω_Λ	Ω_m
0.7	0.3
0	0.3
0	1

Points above the dashed curve indicate acceleration.

(from Freedman & Turner 2003)

Consider a universe consisting of a combination of matter and radiation, and perhaps with a cosmological constant. How does it evolve?

1. Solve the fluid eqn for matter.

$$P = 0 \quad \Rightarrow \quad \frac{\dot{\rho}}{\rho} = -3\frac{\dot{a}}{a} \quad \Rightarrow \quad \frac{d\rho}{\rho} = -3\frac{da}{a} \quad \Rightarrow \quad \rho \propto a^{-3}$$

This is as expected; since $P = 0$, total energy is conserved, and the volume of the universe $\propto a^3$.

2. For radiation, $P = \rho c^2/3 \quad \Rightarrow \quad \dfrac{\dot{\rho}}{\rho} = -4\dfrac{\dot{a}}{a} \quad \Rightarrow \quad \rho \propto a^{-4}$

There are 2 ways to understand why ρ drops off faster than a^{-3}:

a) Since $P \neq 0$, the gas does PdV work as the universe expands.

b) The wavelength of the radiation $\propto a$ => the energy per photon $\propto a^{-1}$. This is on top of the decrease in photon number density.

3. The Friedmann eqn is: $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho_r + \frac{8}{3}\pi G\rho_m - \frac{kc^2}{a^2} + \frac{1}{3}\Lambda c^2$

The 4 terms on the RHS $\propto$ a^{-4}, a^{-3}, a^{-2}, and a^0.

If all 4 are present and the universe expands forever, then radiation first dominates the expansion. Then matter, curvature, cosmological constant in sequence. A major goal of observational cosmology is to determine the role of these 4 terms today.

4. As a concrete example, suppose $k = 0$, $\Lambda = 0$, and matter has dominated over radiation for most of the history of the universe. Then

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho \propto a^{-3} \quad \Rightarrow \quad \frac{da}{dt} \propto a^{-0.5} \quad \Rightarrow \quad a(t) = a_0 t^{2/3} \quad ; \quad \dot{a}(t) = \frac{2}{3}a_0 t^{-1/3}$$

$$\Rightarrow \quad H_0 = \left(\frac{\dot{a}}{a}\right)_0 = \frac{2}{3}t_0^{-1}$$ (t_0 is the age of the universe)

$$\Rightarrow \quad t_0 = \frac{2}{3}H_0^{-1} \approx \frac{2}{3}(73\,\mathrm{km\,s^{-1}\,Mpc^{-1}})^{-1} = \frac{2}{3}(13.4\,\mathrm{Gyr}) = 8.9\,\mathrm{Gyr}$$

Best estimate of the ages of the oldest stars is 10 to 13 Gyr.

In the case of a radiation-dominated universe, $\rho \propto a^{-4} \quad \Rightarrow \quad a(t) \propto t^{1/2}$

The expansion is slower than in the matter-dominated case, due to the extra deceleration provided by the pressure.

5. Suppose only matter and curvature are present. Friedmann eqn is:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G \rho_0 a^{-3} - \frac{kc^2}{a^2}$$

We already saw that for $k = 0$, the expansion continues forever but $\dot{a} \to 0$.

For $k = -1$, expansion is even faster: $\dot{a} \to \text{const} > 0$.

For $k = +1$, eventually $\dot{a} = 0$.

Time reversibility of the eqn yields a subsequent collapse.

With $M \equiv \frac{4}{3}\pi\rho_0$: $\frac{1}{2}\dot{a}^2 - \frac{GM}{a} = -\frac{kc^2}{2}$

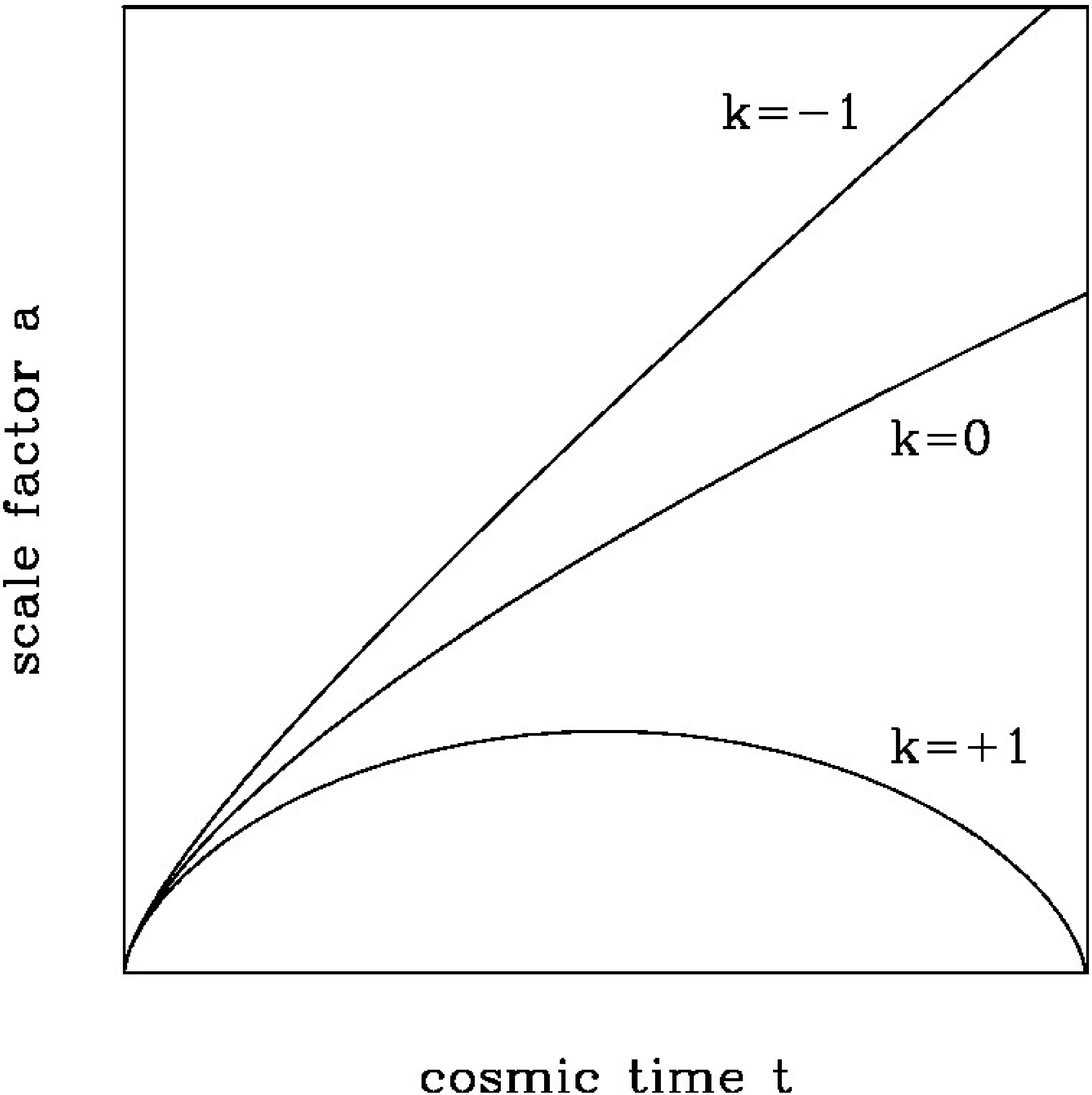
This is equivalent to the Newtonian energy equation for a particle moving in a gravitational field. If k = -1, the universe expands forever, just as the particle escapes to infinity. If k = +1, the universe collapses, just as the particle would fall back to the "ground".

For $k = \pm 1$, the solution to the Friedmann eqn can be expessed parametrically. With $C = 4\pi G\rho_0/3c^2$:

k = +1 : $a = C(1 - \cos\xi)$ $ct = C(\xi - \sin\xi)$

k = -1 : $a = C(\cosh\xi - 1)$ $ct = C(\sinh\xi - \xi)$

When k = -1, $a \approx ct$ for large t.


k=−1
k=0
k=+1
scale factor a
cosmic time t

de-Sitter universe: empty, with $k = 0$, $\Lambda > 0$.

The Friedmann eqn becomes:

$$\frac{da}{dt} = \left(\frac{\Lambda c^2}{3}\right)^{1/2} a$$

Solution:

$$a \propto \exp\left[\left(\frac{\Lambda c^2}{3}\right)^{1/2} t\right]$$

Any indefinitely expanding universe with $\Lambda > 0$ ultimately tends to this solution, since the Λ term dominates in the Friedmann eqn as $a \to \infty$.

Observational parameters

Astronomers try to determine the values of the following quantitites in order to constrain the cosmological model:

1. The Hubble constant H_0 (or, more generally, parameter H)

2. The densities of the constituents. These are generally expressed relative to the critical density ρ_c, which, for a given value of H, is the density of matter + radiation required to have $k = 0$:

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G} \qquad \begin{aligned} \rho_c(t_0) &= 1.00 \times 10^{-29}\,\mathrm{g\,cm^{-3}} \\ &= 2.86 \times 10^{11}\,M_\odot\,\mathrm{Mpc^{-3}} \end{aligned} \qquad (\text{for } h = 0.73)$$

Typical mass of a galaxy is $\sim 10^{11}\,M_\odot$ and typical separation is ~ 1 Mpc => matter density is not wildly off from critical density.

$\Omega_m \equiv \rho_m / \rho_c$ (and similarly for radiation)

Also define $\Omega_k \equiv -\frac{kc^2}{a^2 H^2}$ and $\Omega_\Lambda \equiv \rho_\Lambda / \rho_c$

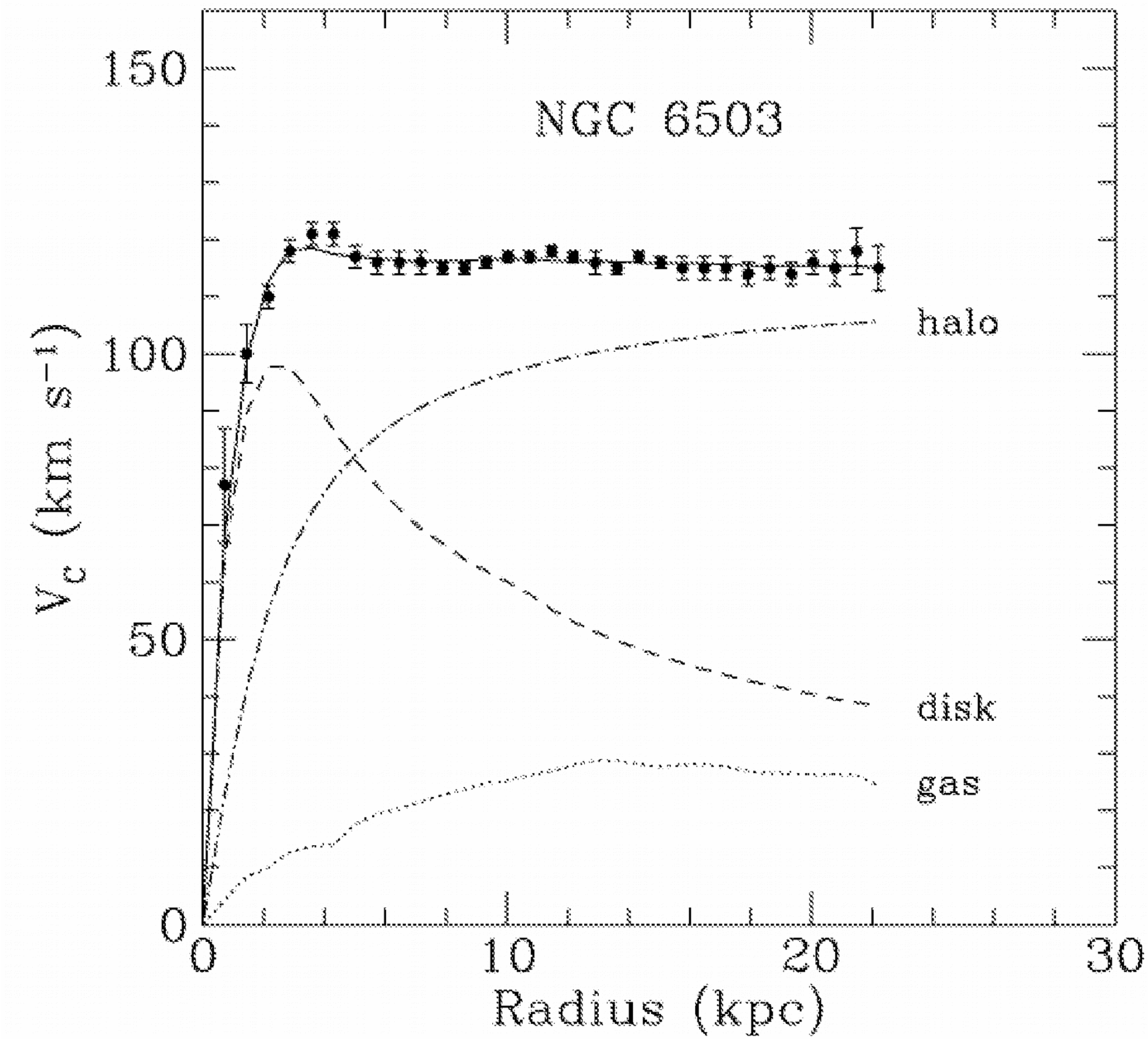
Friedmann eqn becomes: $\Omega_r + \Omega_m + \Omega_k + \Omega_\Lambda = 1$

How can we estimate the density of matter in the universe?

1. Mass in stars (can be estimated reasonably well since we understand stellar structure theory). $\Omega_{\text{stars}} \approx 0.005$ to 0.01.
2. What about other baryons that are not in stars? (e.g., brown dwarfs, hot gas in galaxy clusters) The theory of Big Bang nucleosynthesis (more later...) => $\Omega_b > \approx 0.02$ and $< \approx 0.06$.

Apparently stars do not account for nearly all of the baryons.

3. Galaxy rotation curves => galaxies contain more matter than observed as baryons, in a "dark matter halo". $\Omega_{halo} \approx 0.1$.



Begemann et al. (1991, MNRAS, 249, 439)

4. X-ray observations of hot gas in galaxy clusters => there is several times more mass in cluster and intergalactic gas than in stars; agrees with nucleosynthesis result

5. The mass required to gravitationally hold the hot gas to the galaxy cluster ~10 times the observed baryon mass

=> $\Omega_m \approx 0.3$ (dominated by non-baryonic dark matter)

6. What is the non-baryonic dark matter? Neutrinos? These have low mass and so would be relativistic for at least part of the universe's history (called "hot dark matter"). Simulations of the formation of structure in the universe strongly favor "cold dark matter", i.e., particles that are non-relativistic throughout the universe's history. Perhaps the lightest supersymmetric particle, if it exists.

Back to the CMB

The CMB is a blackbody radiation field with $T = 2.725$ K, permeating the universe.

The energy density $u = aT^4 = 4.17 \times 10^{-13}\,\mathrm{erg\,cm^{-3}} = 0.26\,\mathrm{eV\,cm^{-3}}$

$$\rho_r = u/c^2 \quad \Rightarrow \quad \Omega_{\rm CMB} \approx 4.6 \times 10^{-5}$$

As the universe expands, a blackbody radiation field retains its blackbody spectrum, but $T \propto a^{-1}$, since $\rho_r \propto a^{-4}$ and $u \propto T^4$.

The CMB was hotter in the past.

Why do we have a CMB?

Consider an early time when $T_{\rm CMB}$ was sufficiently high that most photons have enough energy to ionize an H atom (13.6 eV). The baryons would be in the form of a plasma of free electrons and protons, rather than as atoms.

Photons interact strongly with free electrons => short mean free path; i.e., the matter is opaque to radiation. As the universe expands and the CMB cools, eventually the photons can no longer ionize H atoms => atoms form. Now the universe is transparent to the CMB and CMB photons travel virtually unimpeded for the rest of the history of the universe.

The switch from opaque to transparent is called "decoupling".

Simplest way to estimate the value of T_{CMB} when decoupling occurred:

Set the mean photon energy = 13.6 eV => T_{CMB} = 50,000 K

But: the number density of photons $\gg$ that of the baryons

=> photons in high-energy tail of Planck dist. ionize atoms

=> lower T_{CMB}

Detailed calculation yields T_{CMB} = 3000 K at decoupling

=> CMB formed at a redshift of $z \approx 1100$.

(Universe was about a thousandth of its present size.)

Hot vs. Cold Big Bang vs. Steady State

Following the discovery of Hubble's Law, 2 model cosmologies were considered.

"Big Bang Cosmology": assumes conventional fluid eqns
=> universe was denser in the past

"Steady State Cosmology": assumes that particles can be created from nothing, filling in the gaps as the universe expands
=> properties of the universe do not evolve

The discovery of the CMB killed the steady state theory. Today, the CMB and matter hardly interact, so how could the radiation achieve thermal equilibrium? (The photons don't interact with one another.) In the steady state model, the radiation originates in stars and somehow has to be "thermalized" (i.e., converted into a blackbody field). No successful mechanism has been identified.

The Big Bang naturally accounts for the CMB, since radiation and matter interacted strongly when the universe was denser and hotter. The discovery of the CMB also established the "Hot Big Bang" in preference to the "Cold Big Bang". In the latter, matter dominates radiation throughout the history of the universe. This simpler model was preferred in the early days but it was found that nucleosynthesis required a Hot Big Bang. Thus, the CMB was predicted!

The Early Universe

The history of the early universe can be simply described if we assume:

- $k = 0$ and $\Lambda = 0$ (good approx early on)
- an abrupt transition from radiation to matter domination (we can estimate when this occurred given estimates of Ω_r and Ω_m today)

Results

1. Decoupling occurred when age of universe, $t \approx 350{,}000$ yrs.

2. Transition from radiation to matter domination occurred when $t \sim$ 50,000 yrs.

3. When radiation dominated, $kT \approx 2\,\mathrm{MeV}\left(\frac{1\,\mathrm{s}}{t}\right)^{1/2}$

4. At early times, T was sufficiently high to prevent the formation of composite particles.

5. Highest particle energy achieved in lab $\approx$ 100 GeV = 10^{15} K; occurred when $t \sim 10^{-10}$ s. We don't understand physics earlier than that.

Nucleosynthesis

Nuclear binding energies ~1 MeV => nucleosynthesis started at $t \sim 1$ s.

Important facts: n is slightly more massive than p and free n decays with half-life of 614 s.

As long as the weak interactions that convert p ↔ n are rapid, p and n are in thermal equilibrium => their number densities are nearly the same (due to the small mass diff).

These reactions are rapid until $T \approx 0.8$ MeV, and very slow for lower T; $n_n/n_p \approx 1/5$ at that T.

When $T \approx 0.1$ MeV, nuclei are no longer destroyed by radiation; occurs when $t \approx 400$ s. Quite close to n decay time! => substantial n decay, but most survive => n_n/n_p in nuclei $\approx 1/8$.

The most stable light nucleus is ^{4}He, so it and H are the dominant products. 16 p per 2 n => $m_{He}/m_{tot} \approx 4/18 = 0.22$

Detailed analysis predicts primordial abundances of all light nuclei. Results depend on the number of low-mass neutrino species (since this affects T vs. t) and the baryon density. Primordial abundances have been observed and are only consistent with 3 neutrino species. Only 3 have been observed, and, following the nucleosynthesis calculations, the measured decay rate of the Z^0 particle also indicates only 3 neutrino species. Remarkable confirmation of Hot Big Bang. Observations also constrain Ω_b, as we saw earlier.

Inflation

There are some major mysteries associated with the Hot Big Bang model:

1. The flatness problem: $\Omega_r + \Omega_m + \Omega_\Lambda$ is close to 1 => Ω_k is close to zero today. So, the universe is either flat or close to it. But:

$$|\Omega_k| = \frac{|k|c^2}{a^2 H^2} = \frac{|k|c^2}{\dot{a}^2} \quad \text{and} \quad \dot{a}^{-2} \propto t \ , \ t^{2/3} \quad \text{during rad, matter domination}$$

=> $\Omega_k = 0$ if it starts out exactly zero. Otherwise, Ω_k increases with time. Fact that Ω_k is close to zero today => it was extremely close to zero in the early universe (e.g., $|\Omega_k| < \sim 10^{-18}$ at nucleosynthesis). Does this mean $k = 0$? If so, why?

2. The horizon problem: T_{CMB} is nearly identical for all directions on the sky. Most natural explanation is that all the regions from which we receive CMB photons today were in thermal equilibrium in the past. But, the photons we receive from opposite sides of the sky come from regions so far apart that they could not have interacted with one another during the life of the universe (much less by the time of decoupling). Also, there are small fluctuations in T_{CMB}, which presumably gave rise to structure in the universe; what produced these fluctuations?

These problems can be solved if the universe underwent a period of inflation very early in its history. For example, suppose the universe was dominated by a cosmological constant very early on.

Friedmann eqn yields: $a(t) \propto \exp(\sqrt{\frac{\Lambda}{3}} ct)$

The idea is that inflation ends long before 10^{-10} s (when known physics breaks down; usually taken to be ~ 10^{-34} s). At that time, the field that was responsible for the inflationary cosmological constant decays into conventional particles and the exponential expansion stops. None of the successes of the Hot Big Bang model are compromised. But the 2 problems above are solved.

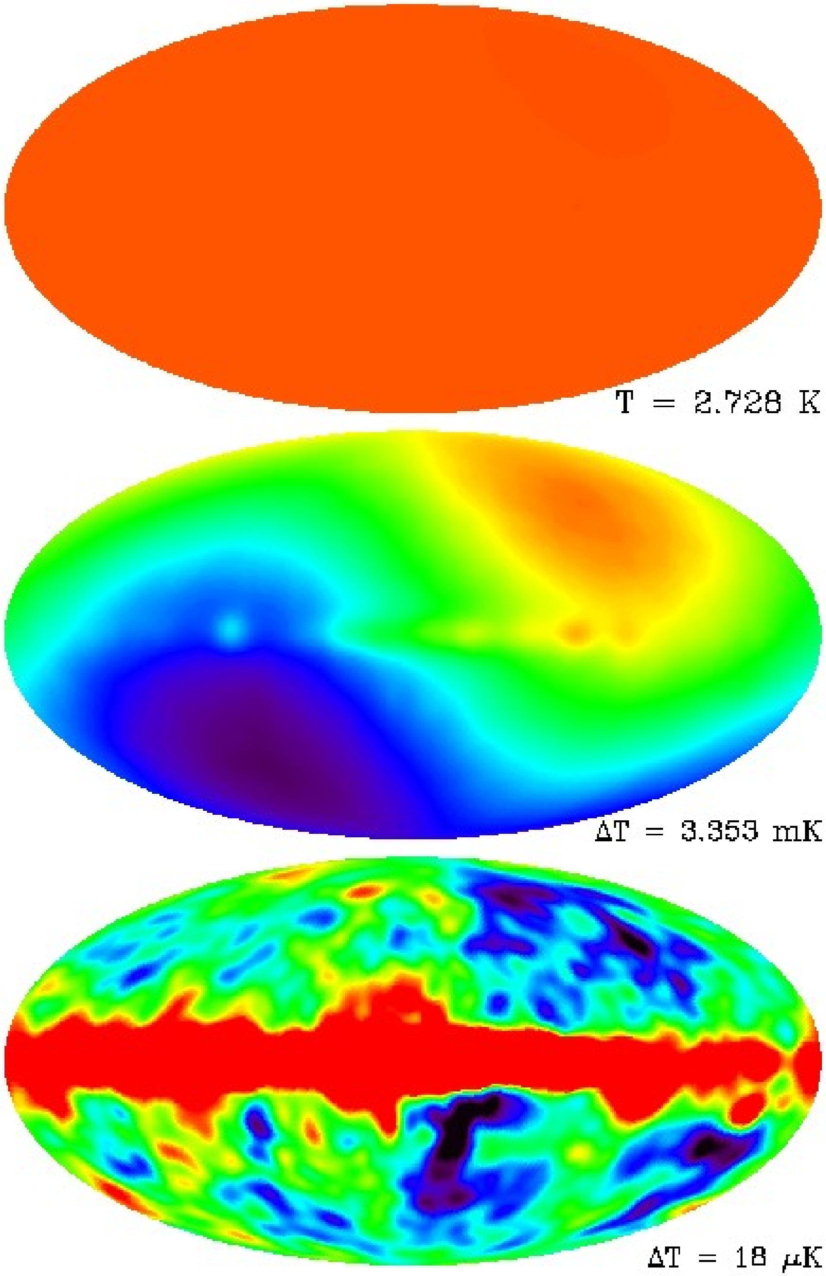
1. Flatness: since the expansion is accelerating rather than decelerating, Ω_k is driven towards zero rather than away from it, and exponentially fast. So any initial deviation from flatness is thoroughly ironed out by inflation.

2. Horizon: a small region that is thermalized before inflation can grow to be larger than the size of the observable universe. Also, some process prior to inflation could produce small deviations from homogeneity that grow with inflation.

At the close of the 20th century, a consistent cosmological model emerged, with a flat geometry, $\Omega_m \approx 0.3$ (baryons account for $\approx$10%) and $\Omega_\Lambda \approx 0.7$. Observations of anisotropies in the CMB (i.e., fluctuations in T_{CMB} as a function of direction on the sky), especially by WMAP and Planck, have confirmed this picture and improved the precision with which the parameters are known.

CMB anisotropies

The most prominent feature is a dipole, due to the motion of the solar system wrt the CMB. It was accurately measured by the Cosmic Background Explorer (COBE), launched in 1989. The magnitude of the dipole (3.353 mK) => $v_{ss} = 369 \pm 3$ km s^{-1}. Velocity of the SS wrt to the CM of the Local Group is in opposite direction; $v_{LG} = 600 \pm 45$ km s^{-1}.



COBE maps

1. No contrast enhancement
2. Contrast enhanced by a factor ≈ 400
3. Dipole removed and additional contrast enhancement; Galaxy emission remains.

Suppose T_{CMB} for direction (θ, ϕ) differs from the average, T_0, by ΔT.
We can expand the T fluctuation in spherical harmonics:

$$\frac{\Delta T(\theta,\phi)}{T_0} = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta,\phi)$$

If the fluctuations are Gaussian, then a full statistical description is provided by the radiation angular power spectrum:

$$C_l \equiv \langle |a_{lm}|^2 \rangle$$

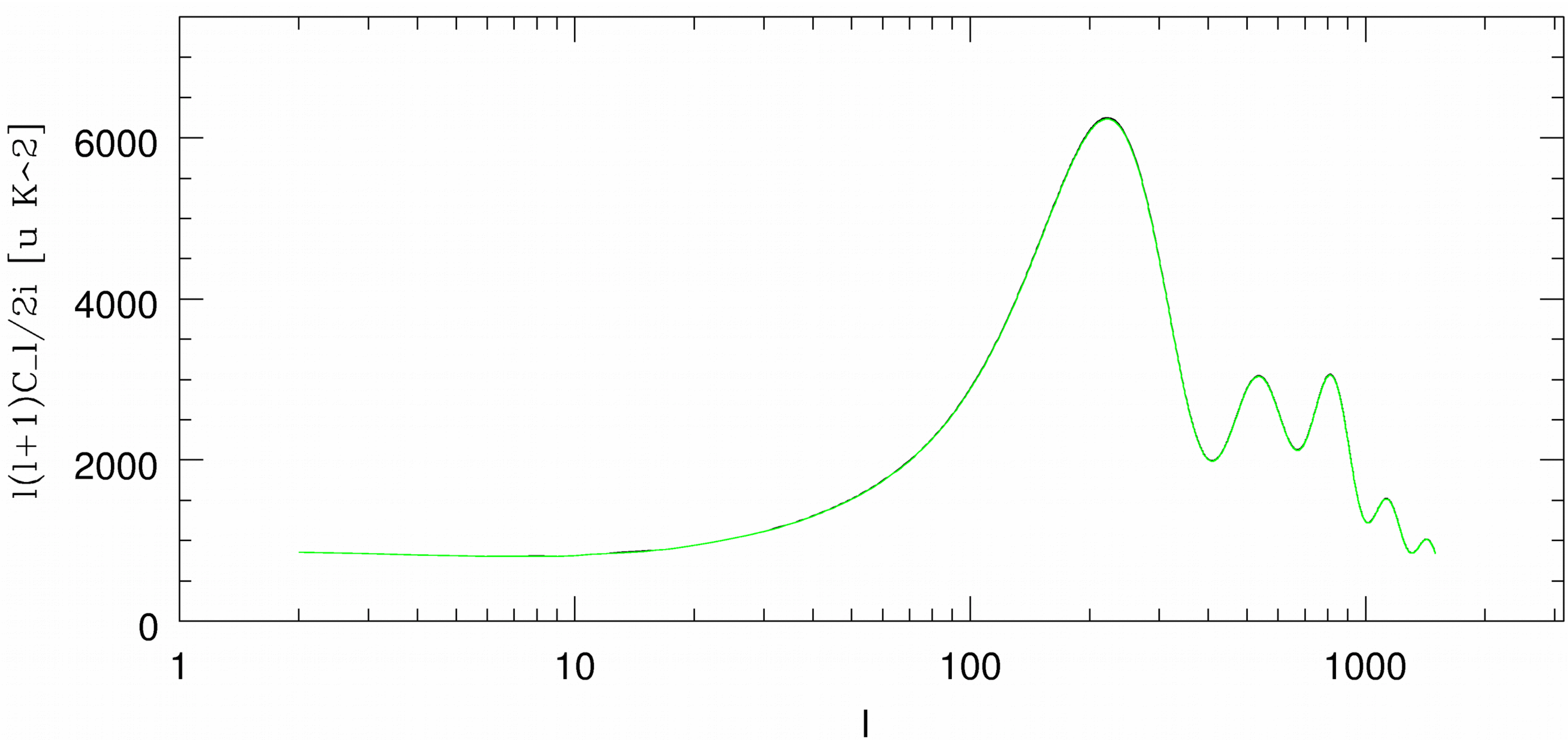
The brackets denote an average over an ensemble of realizations of the fluctuations (e.g., over all observers in the universe).

$$\frac{\langle (\Delta T)^2 \rangle}{T_0^2} = \sum_l \frac{2l+1}{4\pi} C_l \approx \int \frac{l(l+1)}{2\pi} C_l d \ln l$$

Thus, $l(l+1)C_l / 2\pi$ is the power per logarithmic interval (for large l).
Observations of CMB anisotropies are usually presented as a plot of this quantity vs. l. The angular scale associated with l is $\theta \approx 180^\circ / l$.

Since we can only observe the universe from one location, the only averaging we can do is over *m*. We can also bin over *l*, with $\Delta l \sim l$. The uncertainty associated with this limitation is called "cosmic variance" and is very important for low *l*.

Here's a predicted power spectrum, for a particular cosmological model:



Why does the spectrum look like this?

The anisotropies have their origin in fluctuations in the density and gravitational potential (i.e., spacetime curvature). In the inflation scenario, these were originally quantum fluctuations that were greatly stretched during inflation. Early on, the universe was dominated by radiation, and the baryons were tied to the radiation field. Since radiation has pressure, the initial density fluctuations drove acoustic waves in the radiation/baryon plasma; the "sound" speed was $v_s \approx 3^{-1/2}c$. The fundamental distance scale at the time of decoupling is the "sound horizon"; the sound speed times the age of the universe.

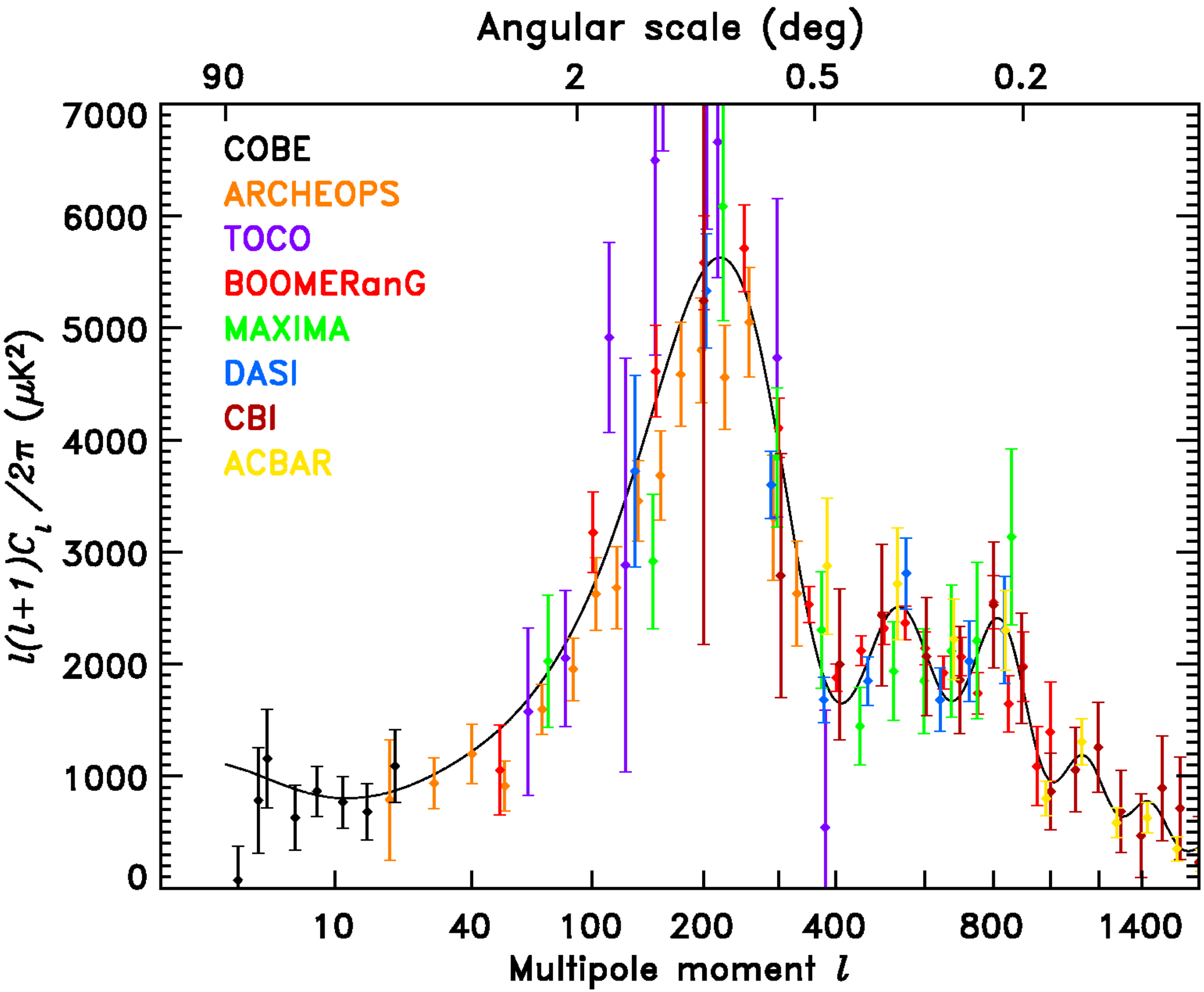
1. Two regions separated by more than the sound horizon do not interact before decoupling. Thus, on angular scales larger than that associated with the sound horizon, the CMB anisotropies are determined primarily by the primordial fluctuations.

However, this is more than compensated by the gravitational redshift (due primarily to dark matter) as the photons climb out the potential well at decoupling. This part of the spectrum applied for $\theta > \approx 2°$ => $l < \approx 90$. Called the "Sachs-Wolfe plateau".

2. At smaller angular scales (higher l), acoustic waves yield more power than in the Sachs-Wolfe plateau. Consider an acoustic wave that has undergone one-half cycle during the age of the universe (at the time of decoupling). The dark matter interacts weakly with the photon-baryon plasma, so it is unaffected by the acoustic waves. Thus, regions of high gravitational potential coincide with rarefactions in the plasma and the grav redshift and photon densities effects add. This yields the large peak at $l \approx 200$. Other peaks are due to harmonics.

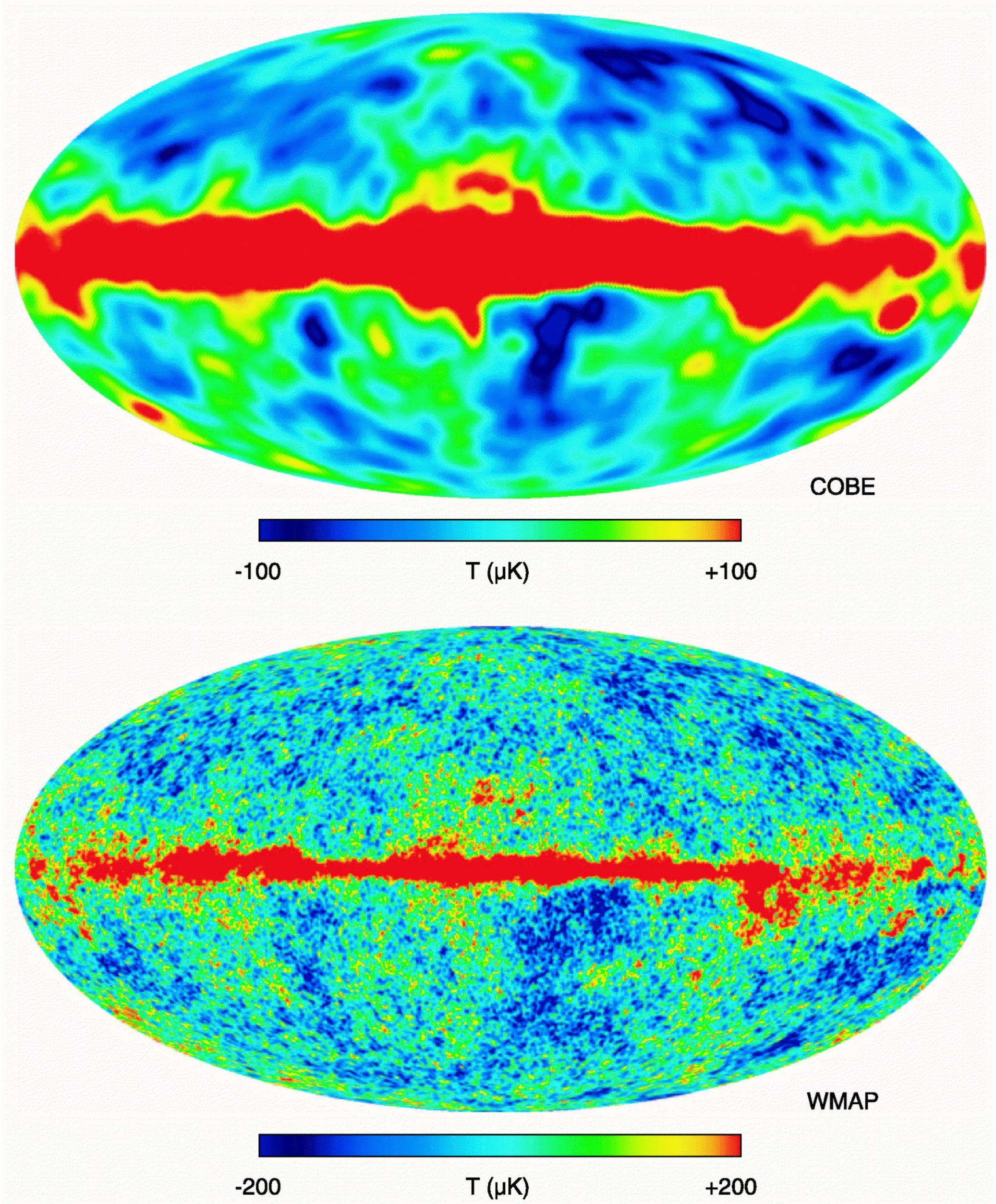
3. For $\theta < \approx 0.2$ degrees ($l > \approx 900$), fluctuations are damped by photon diffusion. Fluctuations are also suppressed because many small hot and cold spots lie along the line of sight. This part of the spectrum is called the "Silk damping tail".

The angular resolution of COBE was 7° ($l < \approx 26$) => it could only probe part of the Sachs-Wolfe plateau. Ground and balloon-borne instruments probed smaller angular scales, starting in the late 1990s.



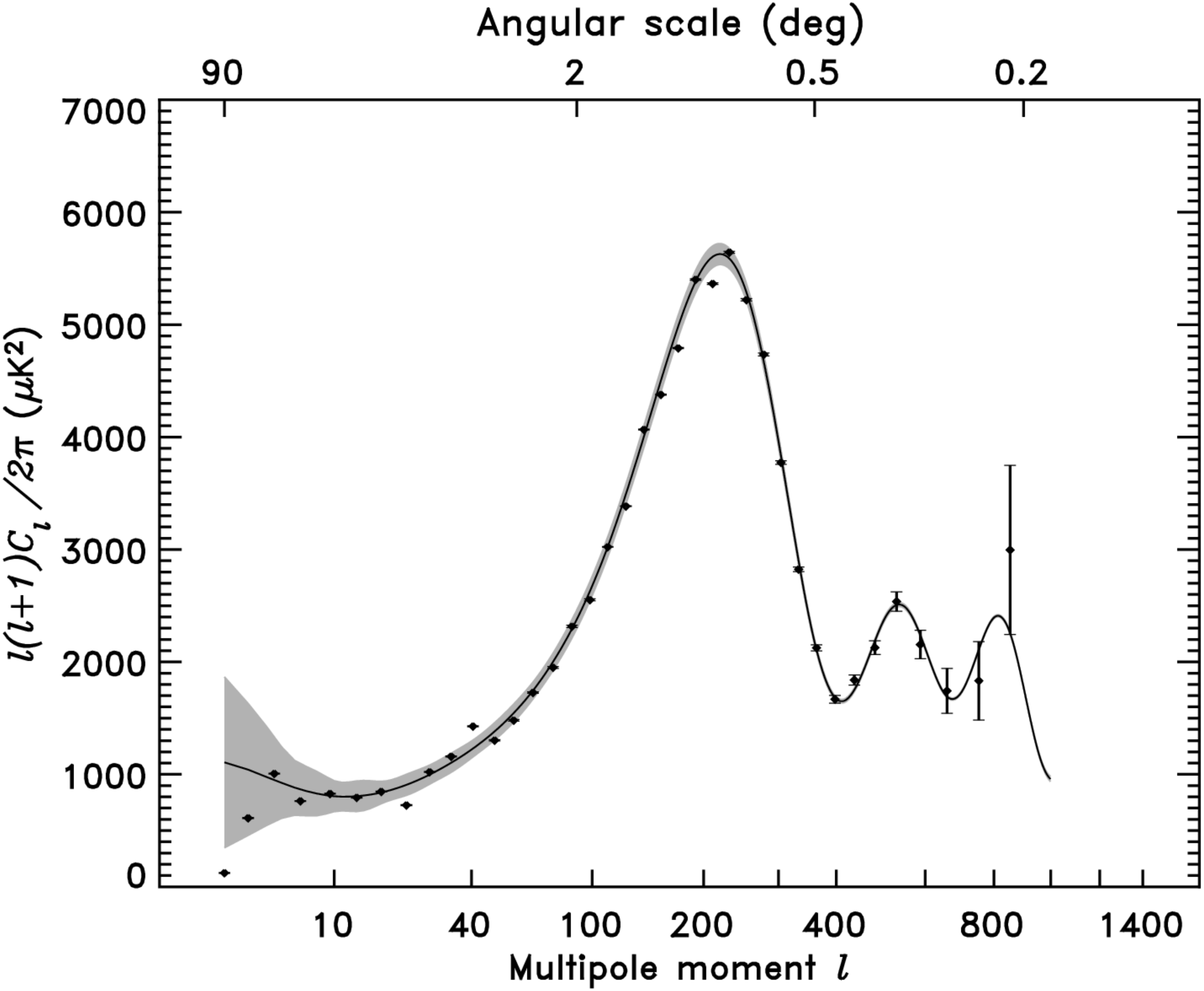
From Hinshaw et al. (2003, ApJS, 148, 135)

The Wilkinson Microwave Anisotropy Probe (WMAP) was launched in 2001, with 30 × the angular resolution and 45 × the sensitivity of COBE.



Results of the first year of data collection were presented in Sep 2003 (ApJS vol 148, issue 1).

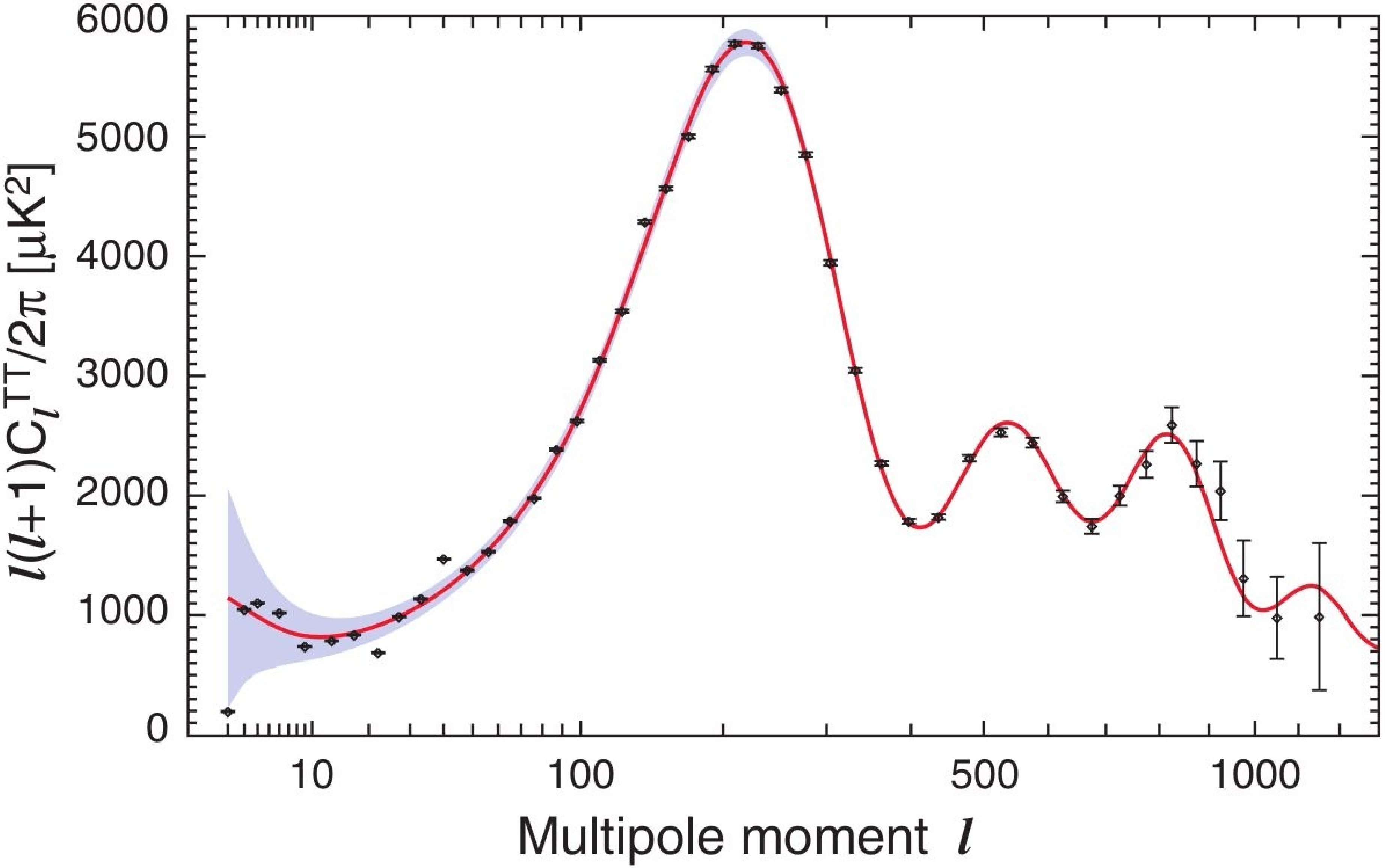
From Bennett et al. (2003, ApJS, 148, 1)



WMAP 1-year power spectrum

From Hinshaw et al. 2003

7-year WMAP data were presented in Feb, 2011 (ApJS vol 192, issue 2)



WMAP 7-year power spectrum

From Larson et al. (2011, ApJS, 192, 16)

Other instruments, including the South Pole Telescope, have probed higher multipoles (Keisler et al. 2011, ApJ, 743, 28).

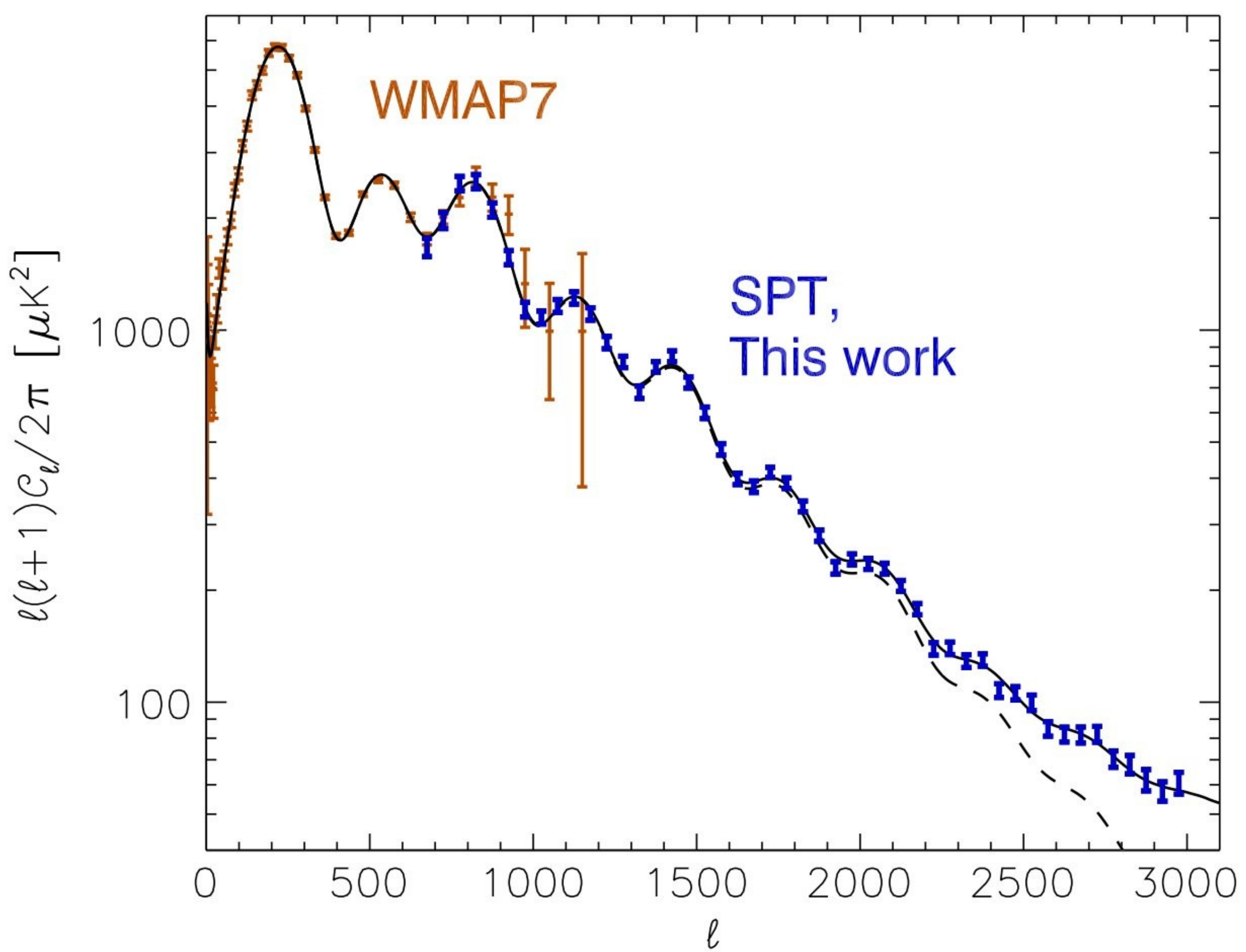
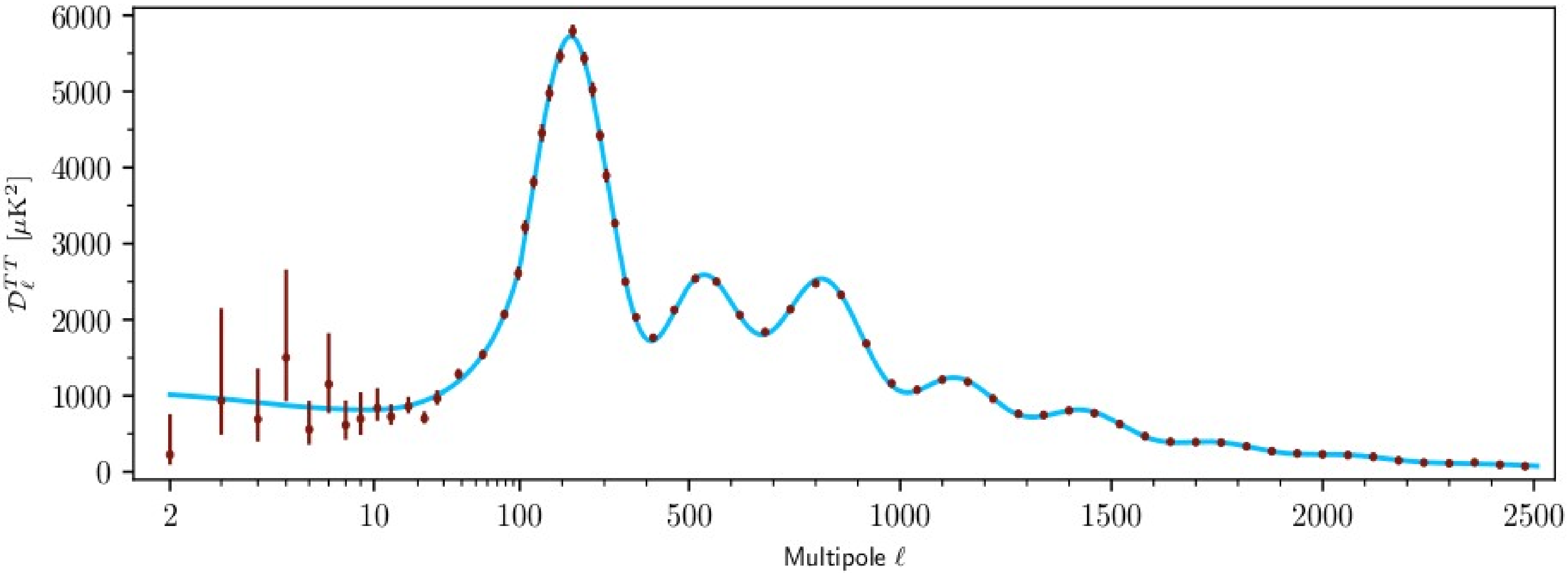
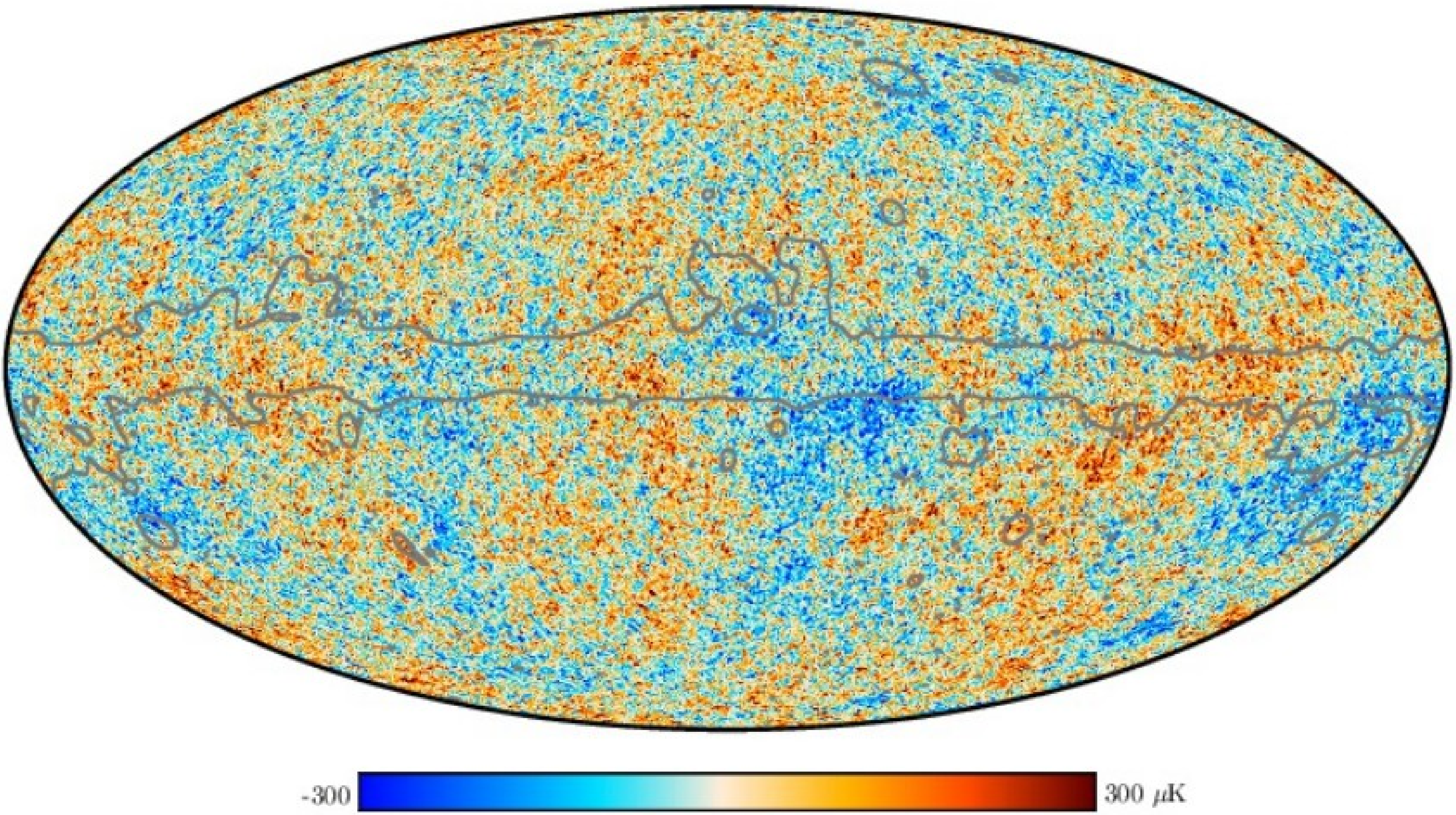


Figure 5. SPT bandpowers, *WMAP* bandpowers, and best-fit ΛCDM theory spectrum shown with dashed (CMB) and solid (CMB+foregrounds) lines. The bandpower errors do not include beam or calibration uncertainties.

Power spectrum from Planck space mission (Aug 2009 to Oct 2013)
Planck Collaboration 2020, A&A, 641, A1



Planck map



The CMB angular power spectrum is complicated, and depends on the values of the cosmological parameters, although in complicated ways. For example, the location (l) of the first acoustic peak is sensitive to the geometry of the universe (physical size is set by age of universe and sound speed at decoupling; angular size associated with that physical size depends on the geometry of the universe). Spergel et al. (2003, ApJS, 148, 175) computed power spectra for various combinations of parameters and looked for best match to observations (from WMAP, CBI, ACBAR, and astronomical measurements of the power spectrum: 2dFGRS and Ly α forest). Similar procedures were employed for later WMAP releases and for Planck.

CMB measurements have largely confirmed and refined the standard cosmological model that evolved at the end of the 20th century.

TABLE I. Our 16 cosmological parameters.

Parameter value[a]		Description	WMAP[b]
		Ten global parameters	
h	0.72 ± 0.07	present expansion rate[c]	$0.71^{+0.04}_{-0.03}$
q_0	-0.67 ± 0.25	deceleration parameter[d]	-0.66 ± 0.10[e]
t_0	13 ± 1.5 Gyr	age of the universe[f]	13.7 ± 0.2 Gyr
T_0	2.725 ± 0.001 K	CMB temperature[g]	
Ω_0	1.03 ± 0.03	density parameter[h]	1.02 ± 0.02
Ω_B	0.039 ± 0.008	baryon density[i]	0.044 ± 0.004
Ω_{CDM}	0.29 ± 0.04	cold dark matter density[i]	0.23 ± 0.04
Ω_ν	$0.001 - 0.05$	massive neutrino density[j]	
Ω_X	0.67 ± 0.06	dark energy density[i]	0.73 ± 0.04
w	-1 ± 0.2	dark energy equation of state[k]	< -0.8 (95% cl)
		Six fluctuation parameters	
$\sqrt{S}$	$5.6^{+1.5}_{-1.0} \times 10^{-6}$	density perturbation amplitude[l]	
$\sqrt{T}$	$< \sqrt{S}$	gravity wave amplitude[m]	$T < 0.9S$ (95% cl)
σ_8	0.9 ± 0.1	mass fluctuations on 8 Mpc[n]	0.84 ± 0.04
n	1.05 ± 0.09	scalar index[h]	0.93 ± 0.03
n_T		tensor index	
$dn/d\ln k$	-0.02 ± 0.04	running of scalar index[o]	-0.03 ± 0.02

From Freedman & Turner (2003) review

Table 8
WMAP Seven-year Cosmological Parameter Summary

Description	Symbol	*WMAP*-only	*WMAP*+BAO+H_0
		Parameters for the Standard ΛCDM Model[a]	
Age of universe	t_0	13.75 ± 0.13 Gyr	13.75 ± 0.11 Gyr
Hubble constant	H_0	71.0 ± 2.5 km s^{-1} Mpc^{-1}	$70.4^{+1.3}_{-1.4}$ km s^{-1} Mpc^{-1}
Baryon density	Ω_b	0.0449 ± 0.0028	0.0456 ± 0.0016
Physical baryon density	$\Omega_b h^2$	$0.02258^{+0.00057}_{-0.00056}$	0.02260 ± 0.00053
Dark matter density	Ω_c	0.222 ± 0.026	0.227 ± 0.014
Physical dark matter density	$\Omega_c h^2$	0.1109 ± 0.0056	0.1123 ± 0.0035
Dark energy density	Ω_Λ	0.734 ± 0.029	$0.728^{+0.015}_{-0.016}$
Curvature fluctuation amplitude, $k_0 = 0.002$ Mpc^{-1}[b]	$\Delta^2_{\mathcal{R}}$	$(2.43 \pm 0.11) \times 10^{-9}$	$(2.441^{+0.088}_{-0.092}) \times 10^{-9}$
Fluctuation amplitude at $8h^{-1}$ Mpc	σ_8	0.801 ± 0.030	0.809 ± 0.024
Scalar spectral index	n_s	0.963 ± 0.014	0.963 ± 0.012
Redshift of matter–radiation equality	$z_{\rm eq}$	3196^{+134}_{-133}	3232 ± 87
Angular diameter distance to matter–radiation eq.[c]	$d_A(z_{\rm eq})$	14281^{+158}_{-161} Mpc	14238^{+128}_{-129} Mpc
Redshift of decoupling	z_*	$1090.79^{+0.94}_{-0.92}$	$1090.89^{+0.68}_{-0.69}$
Age at decoupling	t_*	379164^{+5187}_{-5243} yr	377730^{+3205}_{-3200} yr
Angular diameter distance to decoupling[c,d]	$d_A(z_*)$	14116^{+160}_{-163} Mpc	14073^{+129}_{-130} Mpc
Sound horizon at decoupling[d]	$r_s(z_*)$	$146.6^{+1.5}_{-1.6}$ Mpc	146.2 ± 1.1 Mpc
Acoustic scale at decoupling[d]	$l_A(z_*)$	302.44 ± 0.80	302.40 ± 0.73
Reionization optical depth	τ	0.088 ± 0.015	0.087 ± 0.014
Redshift of reionization	$z_{\rm reion}$	10.5 ± 1.2	10.4 ± 1.2
		Parameters for Extended Models[e]	
Total density[f]	$\Omega_{\rm tot}$	$1.080^{+0.093}_{-0.071}$	$1.0023^{+0.0056}_{-0.0054}$
Equation of state[g]	w	$-1.12^{+0.42}_{-0.43}$	-0.980 ± 0.053
Tensor-to-scalar ratio, $k_0 = 0.002$ Mpc^{-1}[b,h]	r	<0.36 (95% CL)	<0.24 (95% CL)
Running of spectral index, $k_0 = 0.002$ Mpc^{-1}[b,i]	$dn_s/d\ln k$	-0.034 ± 0.026	-0.022 ± 0.020
Neutrino density[j]	$\Omega_\nu h^2$	<0.014 (95% CL)	<0.0062 (95% CL)
Neutrino mass[j]	$\sum m_\nu$	<1.3 eV (95% CL)	<0.58 eV (95% CL)
Number of light neutrino families[k]	$N_{\rm eff}$	>2.7 (95% CL)	$4.34^{+0.86}_{-0.88}$

Jarosik et al. 2011, ApJS, 192, 14

Table 2
Maximum Likelihood ΛCDM Parameters[a]

Parameter	Symbol	*WMAP* Data	Combined Data[b]
Fit ΛCDM Parameters			
Physical baryon density	$\Omega_b h^2$	0.02256	0.02240
Physical cold dark matter density	$\Omega_c h^2$	0.1142	0.1146
Dark energy density ($w = -1$)	Ω_Λ	0.7185	0.7181
Curvature perturbations, $k_0 = 0.002$ Mpc^{-1}	$10^9 \Delta^2_{\mathcal{R}}$	2.40	2.43
Scalar spectral index	n_s	0.9710	0.9646
Reionization optical depth	τ	0.0851	0.0800
Derived Parameters			
Age of the universe (Gyr)	t_0	13.76	13.75
Hubble parameter, $H_0 = 100\,h$ km s^{-1} Mpc^{-1}	H_0	69.7	69.7
Density fluctuations @ 8 h^{-1} Mpc	σ_8	0.820	0.817
Baryon density/critical density	Ω_b	0.0464	0.0461
Cold dark matter density/critical density	Ω_c	0.235	0.236
Redshift of matter-radiation equality	z_{eq}	3273	3280
Redshift of reionization	z_{reion}	10.36	9.97

Notes.
[a] The maximum-likelihood ΛCDM parameters for use in simulations. Mean parameter values, with marginalized uncertainties, are reported in Table 4.
[b] "Combined data" refers to *WMAP*+eCMB+BAO+H_0.

WMAP 9-yr (full-mission) results (Hinshaw et al. 2013, ApJS, 208, 19)

Planck parameters

Table 6. 6-parameter ΛCDM model that best fits the combination of data from *Planck* CMB temperature and polarization power spectra (including lensing reconstruction), with and without BAO data (see text).

Parameter	*Planck* alone	*Planck* + BAO
$\Omega_b h^2$	0.022383	0.022447
$\Omega_c h^2$	0.12011	0.11923
$100\theta_{MC}$	1.040909	1.041010
τ	0.0543	0.0568
$\ln(10^{10}A_s)$	3.0448	3.0480
n_s	0.96605	0.96824
H_0 [km s^{-1}Mpc^{-1}] . .	67.32	67.70
Ω_Λ	0.6842	0.6894
Ω_m	0.3158	0.3106
$\Omega_m h^2$	0.1431	0.1424
$\Omega_m h^3$	0.0964	0.0964
σ_8	0.8120	0.8110
$\sigma_8(\Omega_m/0.3)^{0.5}$	0.8331	0.8253
z_{re}	7.68	7.90
Age [Gyr]	13.7971	13.7839

Notes. A number of convenient derived parameters are also given in the lower part of the table. These best fits can differ by small amounts from the central values of the confidence limits in Table 7.

The CP and GR seem to describe the universe very well! Different types of measurements, based on different physics, yield the same cosmological parameters. The model is known as "ΛCDM".

But, some big puzzles remain:

What is the dark matter?

What is the dark energy? If Λ, why is ρ_Λ 10^{120} times smaller than the Planck energy density? Since $\rho_\Lambda / \rho_m \propto a^3$, why do we live at a time when these are comparable?

Inflation explains flatness and the density perturbations. Can we test inflation?

Recently: "Hubble tension." Value of Hubble constant measured from "nearby" observations, like Hubble plot, disagree with value inferred from CMB (model-dependent).

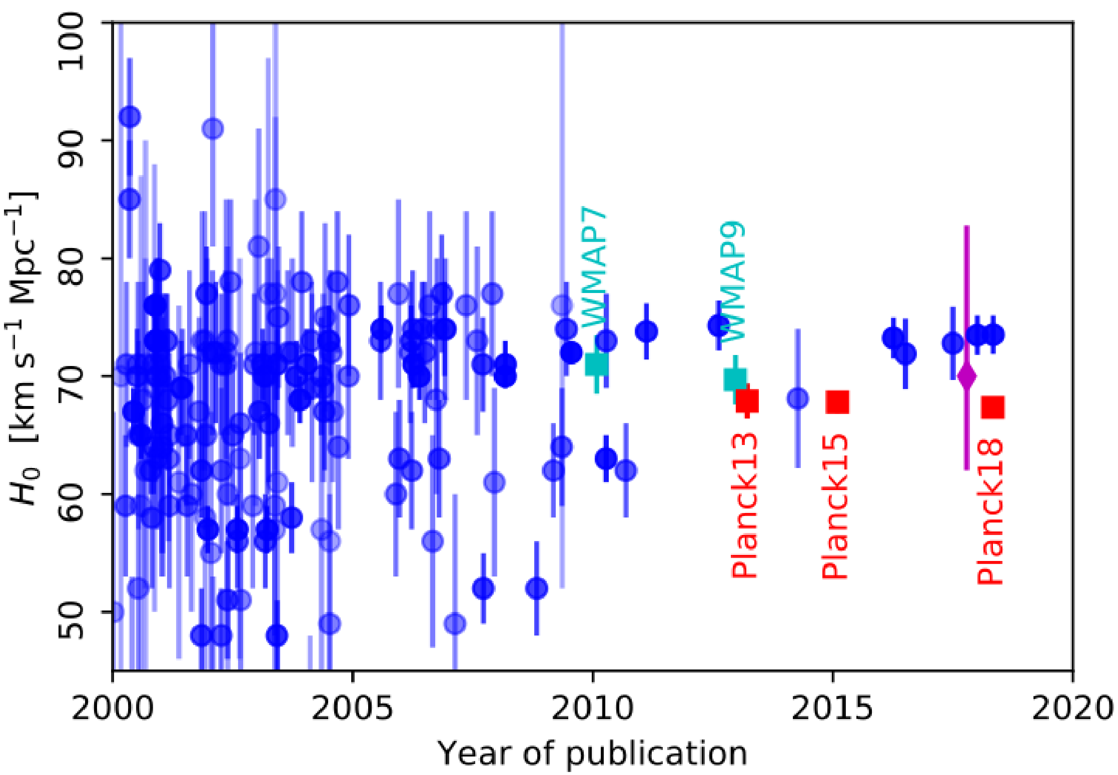


Fig. 21. Compilation of measurements of H_0 since 2000, based on the historical data assembled by J. Huchra for the NASA-HST Key Project on the Extragalactic Distance Scale. The additional points since 2010 are from Riess et al. (2011, 2016, 2018a,b), Freedman et al. (2012), Rathna Kumar et al. (2015), Bonvin et al. (2017), and Dhawan et al. (2018). Blue circles show "traditional" measures of H_0, while cyan and red squares show H_0 inferred from fits to CMB data from WMAP (Bennett et al. 2011; Hinshaw et al. 2013) and *Planck*. The magenta diamond shows the standard siren measurement from Abbott et al. (2017b). Inferences from the inverse distance ladder are discussed in the text and Fig. 22. We note the tremendous increase in precision with time, driven by improvements in methods and in data, and the narrowing of the difference between "high" and ''low" values of H_0.

Planck Collaboration, 2020

Does the Hubble tension suggest a problem with the ΛCDM model?
See Riess (2019, arXiv:2001.03624)