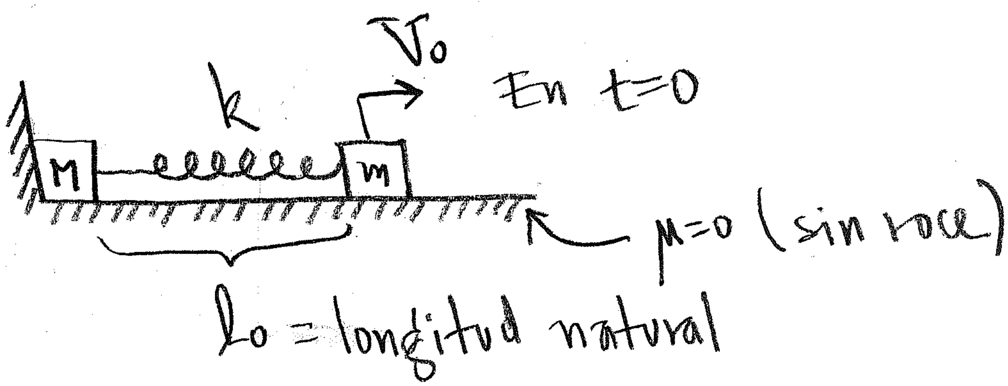
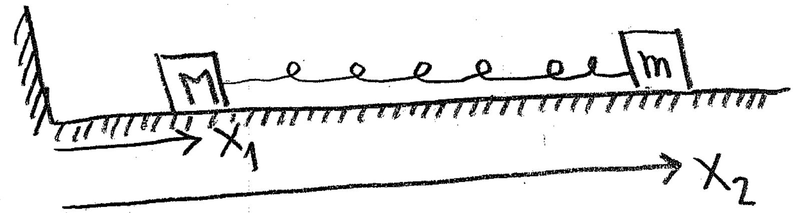
COMPLEMENTO II

Eg.



(fig. 1)



si $\dot{x}_2 > \dot{x}_1$

$\Downarrow$

Resorte estirado

a) Luego

$$L = \frac{1}{2} M \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2 - \frac{1}{2} k \eta^2$$

con η = Estiramiento $= x_2 - x_1 - l_0$

$$\therefore \quad L = \frac{1}{2} M \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2 - \frac{1}{2} k (x_2 - x_1 - l_0)^2 \quad //$$

b) Ecuaciones de movimiento

<u>Para coord. x_1</u>

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = M \ddot{x}_1 \quad ; \quad \frac{\partial L}{\partial x_1} = -k(x_2 - x_1 - l_0)(-1)$$

$\therefore$

$$M\ddot{x}_1 - k(x_2 - x_1 - l_0) = 0 \quad // \quad (i)$$

Para coord. x_2

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m\ddot{x}_2 \quad ; \quad \frac{\partial L}{\partial x_2} = -k(x_2 - x_1 - l_0)$$

$$\Downarrow$$

$$m\ddot{x}_2 + k(x_2 - x_1 - l_0) = 0 \quad // \quad (ii)$$

c) Relación entre $\ddot{x}_1$ y $\ddot{x}_2$

de (i) y (ii) se obtiene que

$$M\ddot{x}_1 + m\ddot{x}_2 = 0$$

de lo cual se deduce que $a_{cm} = 0$

Obs. $a_{cm} = \dfrac{M\ddot{x}_1 + m\ddot{x}_2}{m+M} = 0$

d) v_{cm} y x_{cm}

$$v_{cm} = \text{cte} \quad (*)$$

$$\Downarrow$$

$$x_{cm} = v_{cm}t + x_{0cm} \quad (**)$$

como $v_{cm} = \text{cte.}$ se evalúa en $t=0$

donde $x_1(0) = 0 \quad \dot{x}_1(0) = 0$

$x_2(0) = l_0 \quad \dot{x}_2(0) = v_0$

$$\therefore \quad X_{cm}(0) = \frac{M \cancel{x_1(0)}^{0} + m\, x_2(0)}{m+M} = \frac{m\, l_0}{m+M}$$

$$y \quad v_{cm}(t) = v_{cm}(0) = \frac{M \cancel{\dot{x}_1}^{0} + m\, \dot{x}_2}{m+M} = \frac{m\, v_0}{m+M}$$

Finalmente

$$X_{cm}(t) = \frac{m}{m+M} v_0\, t + \frac{m\, l_0}{m+M} \quad // \quad (*)$$

$$v_{cm} = \frac{m\, v_0}{m+M} \quad //$$

Obs. P_{sist} es otra cantidad conservada

e) x_1 y x_2 ligados

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Debido a $P_{sist} = cte.$

se tiene que

$$X_{cm} = \frac{M x_1 + m x_2}{m+M}$$; de ec. (*) se cumple

luego que:

$$m v_0 t + m l_0 = M x_1 + m x_2$$

α_4

despejamos luego X_2, esto es:

$$X_2 = V_0 t + l_0 - \frac{M}{m} X_1 \quad (**)$$

y reemplazamos en ec. de movimiento (i), entonces:

$$M\ddot{X}_1 - k\left(V_0 t + l_0 - \frac{M}{m} X_1\right) + kX_1 + kl_0 = 0$$

$$\Downarrow$$

$$M\ddot{X}_1 + k\left(1 + \frac{M}{m}\right) X_1 = kV_0 t$$

$$\Downarrow$$

$$\ddot{X}_1 + \underbrace{k\left(\frac{m+M}{mM}\right)}_{W_0^2} X_1 = \frac{kV_0 t}{M}$$

$$\Downarrow$$

$$\ddot{X}_1 + W_0^2 X_1 = \frac{kV_0}{M} t \quad \text{con } X_1(0) = 0 \text{ y } \dot{X}_1(0)$$

cuya solución es (usando software):

$$X(t) = \frac{kV_0 t}{MW_0^2} - \frac{kV_0}{MW_0^2} \operatorname{sen}(W_0 t)$$