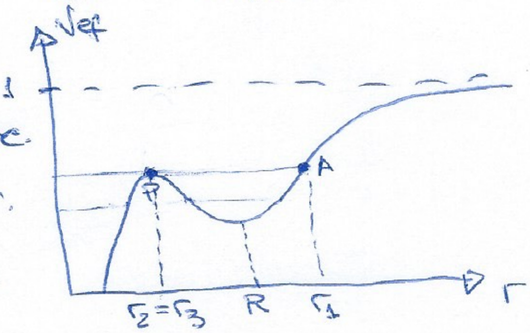
Lu 081121 (3) d 18.

Órbitas Asintóticas



- órbitas de primera especie

El orden de jerarquía:

$r_2 < r \leq r_1$; $u_2 > u \geq u_1$

$\rightarrow u_2 - u > 0 \wedge u - u_1 > 0$

$$u_1 = \frac{1-e}{R} \wedge u_2 = u_3 = \frac{1+e}{R}$$

i) $u_1 + u_2 + u_3 = \frac{1}{r_s} \Rightarrow u_1 + 2u_2 = \frac{1}{r_s}$

$$\frac{1-e+2+2e}{R} = \frac{1}{r_s}$$

$$\frac{3+e}{R} = \frac{1}{r_s} \Rightarrow \boxed{} \Rightarrow \boxed{R = r_s(3+e)}$$

$$r_1 \equiv A = \frac{R}{1-e} = r_s\left(\frac{3+e}{1-e}\right)$$

$$r_2 \equiv P = \frac{R}{1+e} = r_s\left(\frac{3+e}{1+e}\right)$$

Tenemos acotados estos valores:

- $e = 0 \rightarrow P = 3r_s \wedge A = 3r_s$
- $e = 1 \rightarrow P = 2r_s \wedge A \rightarrow \infty$

$\therefore$ $3r_s > P > 2r_s$

$\infty > A > 3r_s$

CL 18 (2)

ii) $u_1u_2 + u_1u_3 + u_2u_3 = 1/L^2$

$$2u_1u_2 + u_2^2 = 1/L^2$$

$$2\left(\frac{1-e}{R}\right)\left(\frac{1+e}{R}\right) + \left(\frac{1+e}{R}\right)^2 = \frac{1}{L^2}$$

$$\frac{2-2e^2+1+2e+e^2}{R^2} = \frac{1}{L^2}$$

$$\frac{3+2e-e^2}{R^2} = \frac{1}{L^2} = \frac{(1+e)(3-e)}{r_s^2\,(3+e)^2}$$

$$\Rightarrow \quad \boxed{\frac{L^2}{r_s^2} = \frac{(3+e)^2}{(1+e)(3-e)}}$$

iii) $u_1u_2u_3 = \dfrac{\mathcal{E}^2}{r_s L^2} = \dfrac{1-E^2}{r_s L^2} = u_1 u_2^2$

$$\frac{1}{\cancel{r_s}}\left(\frac{1-e}{3+e}\right)\cdot\frac{1}{r_s^2}\frac{(1+e)^2}{(3+e)^2} = \frac{\mathcal{E}^2}{\cancel{r_s}\,L^2}$$

$$\frac{L^2}{r_s^2}\,\frac{(1-e)}{(3+e)^3}\cdot(1+e)^2 = 1-E^2 = \frac{\cancel{(3+e)^2}}{\cancel{(1+e)}(3-e)}\,\frac{(1-e)(1+e)^2}{(3+e)^{\cancel{3}}}$$

$$\boxed{1-E^2 = \frac{1-e^2}{9-e^2}}$$

Ahora vamos a determinar la forma de la órbita:

$$\left(\mp\frac{du}{d\phi}\right)^2 = r_s\, g(u)$$

donde

$$g(u) = (u-u_1)(u_2-u)^2$$

$$g(u) = \left(\frac{1+e\cos\chi}{R} - \frac{1-e}{R}\right)\times\left(\frac{1+e}{R} - \frac{1+e\cos\chi}{R}\right)^2$$

$$g(u) = \frac{1}{R^3}\left(e(1+\cos\chi)\right)\times e^2(1-\cos\chi)^2$$

$$= \frac{e^3}{R^3}(1-\cos\chi)\sin^2\chi = \frac{1}{R}\cdot e\cdot\left(\frac{e}{R}\right)^2\sin^2\chi\,(1-\cos\chi)$$

$$r_s\, g(u) = 2\mu e\left(\frac{e}{R}\right)^2\sin^2\chi\,(1-\cos\chi)$$

y también $\left(\mp\frac{du}{d\phi}\right)^2 = \left(\frac{e}{R}\right)^2\sin^2\chi\left(\pm\frac{d\chi}{d\phi}\right)^2$

Finalmente: $\left(\pm\frac{d\chi}{d\phi}\right)^2 = 2\mu e\underbrace{(1-\cos\chi)}_{2\sin^2\frac{\chi}{2}}$

$$\boxed{\left(\pm\frac{d\chi}{d\phi}\right)^2 = 4\mu e\sin^2\frac{\chi}{2}}$$

Estudiemos la caída hacia la órbita asintótica (signo −)

$$\frac{d\chi}{d\phi} = -2\sqrt{\mu e}\,\sin\frac{\chi}{2}$$

Escogemos $\chi_A = \pi \rightarrow \phi_A = 0$.

CL 18 ④

$$\int_0^{\phi} d\phi' = -\frac{1}{2\sqrt{\mu e}} \int_{\pi}^{x} \frac{dx'}{\underbrace{\sin\frac{x'}{2}}_{2\sin\frac{x}{4}\cos\frac{x}{4}}} = -\frac{1}{4\sqrt{\mu e}} \int_{\pi}^{x} \frac{dx'}{\sin\frac{x'}{4}\cos\frac{x'}{4}}.$$

$$\phi = \frac{-1}{\sqrt{\mu e}} \int_{\pi}^{x} \frac{\sec^2\frac{x'}{4}\; d(x'/4)}{\tan\frac{x'}{4}} = -\frac{1}{\sqrt{\mu e}} \int_{\pi}^{x} \frac{d\left(\tan\frac{x'}{4}\right)}{\tan\frac{x'}{4}}$$

$$\Rightarrow \quad -\sqrt{\mu e}\,\phi = \ln \tan\frac{x'}{4}\Big|_{\pi}^{x} = \ln \tan\frac{x}{4}$$

$$\therefore \boxed{\tan\frac{x}{4} = \exp\left\{-\sqrt{\mu e}\,\phi\right\}}$$

$$* \quad \tan\frac{z}{4} = \frac{1-\cos\frac{z}{2}}{\sin\frac{z}{2}}$$