Prueba2_Prob1_pt4

$$\left(\frac{dr}{d\phi}\right)^2 \frac{h^2}{r^4} = \varepsilon^2 - \frac{h^2}{r^2} B[r]$$

$$u = \frac{1}{r} \quad ; \quad du = -\frac{dr}{r^2} \quad ; \quad \left(\frac{dr}{d\phi}\right)^2 = \left(-\frac{du}{d\phi}\right)^2 r^4$$

$$\left(-\frac{du}{d\phi}\right)^2 r^4 \frac{h^2}{r^4} = \left(-\frac{du}{d\phi}\right)^2 h^2 = \varepsilon^2 - \frac{h^2}{r^2} B[r]$$

$$\Rightarrow \left(-\frac{du}{d\phi}\right)^2 = \frac{\varepsilon^2}{h^2} - \frac{B[r]}{r^2} = \frac{\varepsilon^2}{h^2} - \frac{1}{r^2} + \frac{\alpha}{r^2} + \frac{2M}{r^3} + \frac{\gamma}{r}$$

$$\left(-\frac{du}{d\phi}\right)^2 = \frac{\varepsilon^2}{h^2} - u^2 + \alpha u^2 + 2M u^3 + \gamma u \qquad // \ \frac{1}{b} = \frac{\varepsilon}{L}$$

$$\boxed{\left(-\frac{du}{d\phi}\right)^2 = 2M u^3 + (\alpha - 1) u^2 + \gamma u + \frac{1}{b^2}}$$

es posible obtener información con una integración

$$\left(-\frac{du}{d\phi}\right)^2 = \frac{1}{b^2}\left(2mb^2 u^3 + (\alpha-1) b^2 u^2 + \gamma b^2 u + 1\right)$$

$$-\frac{du}{d\phi} = \pm\frac{1}{b}\sqrt{P_3[u]}$$

$$\int d\phi = \pm \int \frac{b\, du}{\sqrt{2Mu^3b^2 + (\alpha-1)b^2u^2 + \gamma b^2 u + 1}}$$

para analizar el problema de la curva de luz

$$\left(\frac{du}{d\phi}\right)^2 = \left(2Mu^3 + (\alpha-1)u^2 + \gamma u + \frac{1}{b^2}\right) \qquad / \frac{d}{d\phi}$$

$$2\left(\cancel{\frac{du}{d\phi}}\right)\left(\frac{d^2u}{d\phi^2}\right) = \left(2\cdot 3Mu^2 + 2(\alpha-1)u + \gamma\right)\left(\cancel{\frac{du}{d\phi}}\right)$$

$$\frac{d^2u}{d\phi^2} = 3Mu^2 + (\alpha-1)u + \frac{1}{2}\gamma$$

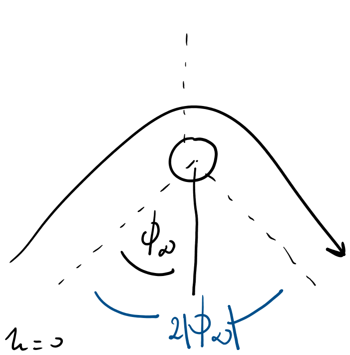
$$\frac{d^2u}{d\phi^2} + u = 3Mu^2 + \alpha u + \frac{1}{2}\gamma$$

utilicen el anzats.

$$u = \frac{1}{b}\sin\phi + \frac{3M}{2b^2} + \frac{\alpha\sqrt{2}}{2b} + \frac{\gamma}{2} + \left(\frac{M}{2b^2} + \frac{\alpha\sqrt{2}}{12b}\right)\cos(2\phi)$$

$\lim_{u\to 0}$ //

$$-\frac{1}{b}\sin\phi - \left(\frac{M}{2b^2} + \frac{\alpha\sqrt{2}}{12b}\right)\cos(2\phi) = \frac{3M}{2b^2} + \frac{\alpha\sqrt{2}}{2b} + \frac{\gamma}{2} \qquad /\cdot b$$



$$-\sin\phi - \left(\frac{M}{2b} + \frac{\alpha\sqrt{2}}{12}\right)\cos(2\phi) = \frac{3M}{2b} + \frac{\alpha\sqrt{2}}{2} + \frac{\gamma b}{2}$$

$$\sin\phi + \left(\frac{M}{2b} + \frac{\alpha\sqrt{2}}{12}\right)\cos(2\phi) = -\frac{3M}{2b} - \frac{\alpha\sqrt{2}}{2} - \frac{\gamma b}{2}$$

$$\sin\phi + \eta \cos 2\phi = -\omega$$

$$\sin(\phi + 2\phi) = \sin\phi \cos(2\phi) + \sin(2\phi)\cos\phi$$

$$\sin(2\phi) = 2\sin\phi\cos\phi$$

$$\sin(3\phi) = \sin\phi \cos(2\phi) + 2\sin\phi \sin\phi \cos\phi$$

$$= \sin\phi \cos(2\phi) + \sin\phi \sin\phi \cos\phi + \sin^2\phi \cos\phi$$

$$= \sin\phi \cos(2\phi) + \sin\phi \sin\phi \cos\phi + \sin^2\phi \cos\phi$$

$$\sin 3\phi = \sin\phi \left(\cos 2\phi + \sin\phi\cos\phi \right) + \sin^2\phi \cos\phi$$

$$\vdots$$

no seguiré este, mejor aproximar pensando en $\phi \ll 1$
para $u \ll 1$ o $r \gg 1$

$$-\omega = \sin\phi + \eta \cos 2\phi \simeq \phi + \eta\left(1 - \frac{4\phi^2}{2}\right) = \phi + \eta\left(1 - \frac{\phi^2}{2}\right)$$

$$\boxed{-\frac{\eta}{2}\phi^2 + \phi + \omega = 0} \rightarrow \frac{\eta\phi^2}{2} - \phi - \omega = 0 \qquad / \phi = \phi_\infty$$

solución

$$\phi_\infty = \frac{1 \pm \sqrt{1 + 4\frac{\eta}{2}\omega}}{\eta} = \frac{1 \pm \sqrt{1 + 2\eta\omega}}{\eta}$$

$$\eta = \left(\frac{M}{2b} + \frac{\alpha\sqrt{2}}{12} \right) \quad // \qquad ; \qquad \omega = \left(\frac{3M}{2b} + \frac{\alpha\sqrt{2}}{2} + \frac{\gamma b}{2} \right) \quad //$$

$$\Delta\phi = 2|\phi_B| \quad //$$

al apagar α y γ (hacerlos tender a 0) deberíamos recuperar [illegible] al apagar α y γ (hacerlos tender a 0) deberíamos recuperar Schwarzschild

$$\phi_\infty = \frac{1 \pm \sqrt{1 + 2\eta^* \omega^*}}{\eta^*} \quad ; \quad \eta^* = \frac{M}{2b} \quad ; \quad \omega^* = \frac{3M}{2b}$$

$$\phi_\infty = \frac{2b \pm \sqrt{1 + \frac{3}{4b^2} M^2}}{M} \neq \phi^*_\infty = \frac{2M}{b}$$

Schwarzschild version

quizás ϕ_∞ no era tan chico después de todo.

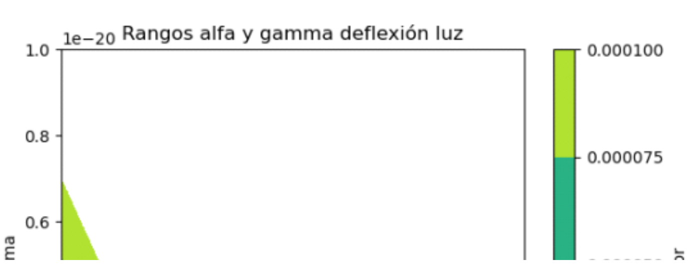
mientras que en el paper citado [2], la solución sí aproxima a Schwarzschild al apagar α y γ.

$$\hat{\vartheta} = 2\left|-\phi_\infty\right| = \frac{4M}{b} + \frac{7\alpha\sqrt{2}}{6} + \gamma b,$$

Si es que se compara usando el código:

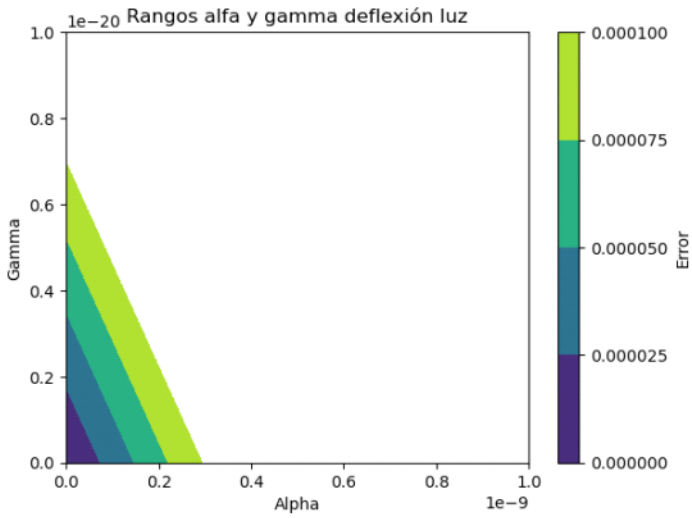
```
rs = 2.95e5 # cm proviene de rs = 2.95e3 # metros
M = rs/2 # M en cm
R = 696340000*1e2 # radio del sol en cm

rad_to_sec = 180/np.pi * 3600
0.1s

error_bound = 1e-4 # diferencia prograda y retrograda para el sol
def schw_deflection(M,R):
    return 4*M/R * rad_to_sec

schw_deflection(M,R)
0.1s
76553937126529

def quintestring_deflection(M,R,alpha,gamma):
    return (4*M/R + 7 * np.sqrt(2) * alpha / 6 + gamma * R) * rad_to_sec
0.0s
```

```
alpha_range = np.linspace(0,1e-9,1000)
gamma_range = np.linspace(0,1e-20,1000)
A,G = np.meshgrid(alpha_range, gamma_range)

error = quintestring_deflection(M,R,A,G) - schw_deflection(M,R)

# Testing de codigo
condicion = error_bound

masked_output = np.ma.masked_where(error >= condicion , error) # no grafica donde el erro es mayor a condicion
plt.contourf(A,G,masked_output, levels=3)
plt.colorbar(label='Error')
plt.xlabel('Alpha')
plt.ylabel('Gamma')
plt.title("Rangos alfa y gamma deflexión luz")
plt.show()
```



$\alpha \sim 10^{-9}$

$\gamma \sim 10^{-20}$ //



$\alpha \sim 10^{-9}$

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