1. La solución para el espacio-tiempo estático, simétricamente esférico en un *fondo de quintesencia*, que se encuentra rodeado por una *nube de cuerdas*, viene dado por el siguiente elemento de línea ($c = G = 1$)

$$\mathrm{d}s^2 = -B(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{B(r)} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2, \tag{1}$$

donde la función $B(r)$ es dada, para un *parámetro de la ecuación de estado de la quintesencia* $w_q = -2/3$, por

$$B(r) = 1 - \alpha - \frac{2M}{r} - \gamma r, \tag{2}$$

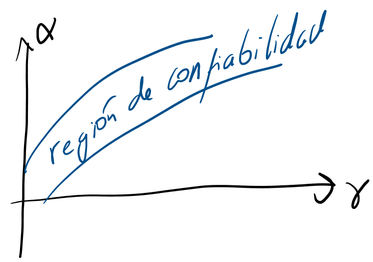
en la cual M es la masa del agujero negro, α es el parámetro adimensional de la nube de cuerdas, y γ es el parámetro de quintesencia.

(a) Determine la anomalía en la precesión de las órbitas. Utilice los datos de [1] para los distintos planetas del sistema solar, y así determine una región de confiabilidad en el plano $\alpha - \gamma$.

α cuerdas

γ quintasencia

plano $\alpha - \gamma$



Desarollo de:

Elementary derivation of the advance of the perihelion of a planetary orbit

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An elementary derivation of the law for the advance of the perihelion of a planet in orbit about the sun is given. This is obtained by comparing a Kepler ellipse in a Lorentz coordinate system, with one in Schwarzschild coordinates, related by the areal constant, and attributing the variation entirely to an increase in the angular coordinate. The result is shown to be entirely in agreement with the classical value.

areal constant:

II. Transformada de coordenadas

$$ds'^2 = dt'^2 - dr'^2 - r^2 d\theta' - r^2 \sin\theta \, d\phi'$$

(1) $$ds^2 = c^2dt^2 - dr^2 - r^2 d\vartheta^2 - r^2\sin^2\vartheta \, d\varphi^2,$$ flat

(2) $$\boxed{ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} - r^2 d\theta^2 - r^2 \sin\theta \, d\phi^2}$$

schwarz.

comparando

$$dt' = \sqrt{1 - 2M} \, dt = (1 - 2M)^{1/2} dt$$

schwarz.

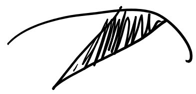
$$dt' = \sqrt{1-\frac{2M}{r}}\,dt = \left(1-\frac{2M}{r}\right)^{1/2} dt$$

donde $(1-x)^{1/2} = 1 - \frac{1}{2}\frac{1}{(1-x)^{3/2}}\Big|_{x=0} x + \mathcal{O}(x^2) \approx 1-\frac{1}{2}x$

$$\Delta\ dt' \approx \left(1-\frac{M}{r}\right)dt$$

$$\Delta\ dr' = \left(1-\frac{2M}{r}\right)^{-1/2} dr \approx \left(1+\frac{M}{r}\right)dr \quad \Big| \quad (1-x)^{-1/2} = 1+\frac{1}{2}\frac{1}{(1-x)^{3/2}}\Big|_{x=0} x + \mathcal{O}(x^2) \approx 1+\frac{1}{2}x$$

Se tiene en r', t' una aproximación de Schwarzschild con $\frac{2GM}{c^2 r} \Rightarrow \frac{2M}{r} \ll 1$ así que

- area en espacio plano  $dA = \int_0^R r\,dr\,d\theta = \frac{R^2}{2}d\theta$

$$\frac{dA}{dt} = \frac{R^2}{2}\frac{d\theta}{dt}$$

- area schw. $dA' = \int_0^R r\,dr'\,d\theta = \int_0^R r\left(1+\frac{M}{r}\right)dr = \left(\frac{R^2}{2}+RM\right)d\theta$

$$\frac{dA'}{dt'} = \left(\frac{R^2}{2}+RM\right)\frac{d\theta}{dt'} \quad \Big| \quad \frac{1}{dt'} = \left(1-\frac{2M}{r}\right)^{-1/2}\frac{1}{dt} \approx \left(1+\frac{M}{r}\right)\frac{1}{dt}$$

$$\frac{dA'}{dt'} = \left(\frac{R^2}{2}+RM\right)\left(1+\frac{M}{R}\right)\frac{d\theta}{dt}$$

$\{M^2 \ll 1\}$ por $\left(\frac{GM}{c^2}\right)^2 \ll 1$

$$\frac{dA'}{dt'} = \left(\frac{R^2}{2}+\frac{RM}{2}+RM+M^2\right)\frac{d\theta}{dt} \approx \left(\frac{R^2}{2}+3\frac{MR}{2}\right)\frac{d\theta}{dt}$$

Comparando las dos orbitas $\frac{dA'}{dt'} = \frac{R^2}{2}\frac{d\theta'}{dt} = \left(\frac{R^2}{2}+\frac{3}{2}MR\right)\frac{d\theta}{dt}$

$$\frac{R^2}{2}d\theta' = \left(\frac{R^2}{2}+\frac{3}{2}MR\right)d\theta$$

$$d\theta' = \left(1+\frac{3M}{R}\right)d\theta \longrightarrow \Delta\theta' = \int_0^{2\pi}\left(1+\frac{3M}{R}\right)d\theta \quad \Big| \quad R = \frac{\ell}{1+\epsilon\cos\theta}$$

$$\Delta\theta' = \int_0^{2\pi}d\theta + \int_0^{2\pi}\frac{3M}{\ell}(1+\epsilon\cos\theta)\,d\theta = 2\pi + \frac{6\pi M}{\ell} + \frac{3M}{\ell}\epsilon\underbrace{\cancel{\int_0^{2\pi}\cos\theta\,d\theta}}_{0}$$

integrar esta [illegible]ción periódica [illegible]completos

$$\Delta\Theta = \int_0 d\theta + \int_0 \frac{6M}{\ell}(1+e\cos\theta)\,d\theta = 2\pi + \frac{6\pi M}{\ell} + \frac{6M}{\ell}e\cancel{\int_0 \cos\theta\, d\theta}$$

integrar esta función periódica en periodos completos se anula.

$$\Delta\Theta' = 2\pi + \frac{6\pi M}{\ell}$$

$\Delta\phi$ avance del perihelio por periodo.

units.

$$\left[\frac{GM}{c^2}\right] = \frac{\left[\frac{N}{kg^2}m^2\right][kg]}{\left[\frac{m^2}{s^2}\right]} = \left[\frac{N}{kg}s^2\right] = \left[\frac{m}{s^2}s^2\right] = [m]$$

entonces $[\ell] = [m]$

$$\Delta\phi = 2\pi \cdot \frac{3M}{\ell} = 2\pi \cdot \frac{3}{2}\frac{2M}{\ell} = 2\pi \cdot \frac{3}{2}\frac{r_s}{\ell}\left[\frac{rad}{periodo}\right]$$

$r_s = 2{,}95 \times 10^3\,[m] \cdot \frac{10^2\,[cm]}{1\,[m]}$

$r_s = 2{,}95 \times 10^5\,[cm]$

$$\Delta\phi = 3\cancel{\pi}\cdot\frac{2{,}95\times10^5\,[cm]}{(\ell)\times10^{11}\,[cm]}\left[\frac{rad}{periodo}\right]\cdot\frac{180}{\cancel{\pi}}\left[\frac{grados}{rad}\right]\cdot 3600\left[\frac{''}{grado}\right]\cdot \tau\left[\frac{periodo}{año}\right]10^2\left[\frac{año}{siglo}\right]$$

orbitas por año

$$\Delta\phi = 573{,}48 \times \frac{\tau}{\ell}\left[\frac{''}{siglo}\right]$$

Mercurio: $\tau = 4{,}15$, $\ell = 55{,}3$ $\Big\}$ $\Delta\phi = 43{,}037\left[\frac{segundos}{siglo}\right]$

Venus: $\tau = 1{,}622$, $\ell = 108$ $\Big\}$ $\Delta\phi = 8{,}61$ ''/siglo

Tierra: $\tau = 1$, $\ell = 149$ $\Big\}$ $\Delta\phi = 3{,}84$ ''/siglo.

Marte: $\tau = 0{,}531$, $\ell = 227$ $\Big\}$ $\Delta\phi = 1{,}34$ ''/siglo

Classical value	
43.03	Mercurio ✓
8.6	Venus ✓
3.8	Tierra ✓
1,34	Marte

Repitiendo con:

$$ds^2 = -B(r)dt^2 + \frac{dr^2}{\ldots} + r^2d\theta^2 + r^2\sin^2\theta\, d\phi^2$$

Repitiendo con:
y comparando con espacio plano

$$\mathrm{d}s^2 = -B(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{B(r)} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2,$$

$$dt' = \left(1-\alpha-\frac{r_s}{r}-\gamma r\right)^{1/2} dt$$

$$B(r) = 1-\alpha-\frac{2M}{r}-\gamma r,$$

$$dr' = \left(1-\alpha-\frac{r_s}{r}-\gamma r\right)^{-1/2} dr \qquad \& \qquad \frac{1}{dt'} = \left(1-\alpha-\frac{r_s}{r}-\gamma r\right)^{-1/2}\frac{1}{dt}$$

$$dA = \int_0^R r\,dr\,d\theta \rightarrow \frac{dA}{dt} = \frac{1}{2}R^2\frac{d\theta}{dt}$$

$$dA' = \int_0^R r\,dr'\,d\theta = \int_0^R r\left(1-\alpha-\frac{r_s}{r}-\gamma r\right)^{-1/2} dr\,d\theta = \int_0^R r\left[\left(\frac{1}{r}\right)\left(r-\alpha r-\gamma r^2-r_s\right)\right]^{-1/2} dr\,d\theta$$

$$= \int_0^R r\,(r)^{1/2}\left(r-\alpha r-\gamma r^2-r_s\right)^{-1/2} dr\,d\theta = \int_0^R \frac{r^{3/2}}{\sqrt{(r-\alpha r-\gamma r^2-r_s)}}\,dr\,d\theta$$

Seria ideal aproximarla, en este caso podemos pensar

$$1 \gg \alpha+\frac{r_s}{r}+\gamma r$$

$$\left(1-\alpha-\frac{r_s}{r}-\gamma r\right)^{-1/2} = (1-y)^{-1/2} \approx 1+\frac{1}{2}y = 1+\frac{1}{2}\left(\alpha+\frac{r_s}{r}+\gamma r\right)$$

$$\left(1-\alpha-\frac{r_s}{r}-\gamma r\right)^{1/2} \approx 1-\frac{1}{2}\left(\alpha+\frac{r_s}{r}+\gamma r\right)$$

$$dA' = \int_0^R r\,dr'\,d\theta = \int_0^R r\left(1-\alpha-\frac{r_s}{r}-\gamma r\right)^{-1/2} dr\,d\theta \approx \int_0^R r\left(1+\frac{1}{2}\left(\alpha+\frac{r_s}{r}+\gamma r\right)\right)dr\,d\theta$$

$$= \int_0^R r\left(1+\frac{\alpha}{2}\right)dr\,d\theta + \int_0^R \frac{1}{2}r_s\,dr\,d\theta + \int_0^R \frac{1}{2}\gamma r^2\,dr\,d\theta$$

$$dA' = \left\{ \frac{R^2}{2}\left(1+\frac{\alpha}{2}\right) + \frac{R}{2} r_s + \frac{R^3}{6}\gamma \right\} d\theta$$

$$\frac{dA'}{dt'} = \underbrace{\left\{ R^3 \frac{\gamma}{6} + R^2\left(\frac{1}{2}+\frac{\alpha}{4}\right) + R\,\frac{r_s}{2} \right\}}_{g(R)} \frac{d\theta}{dt'}$$

$$= g(R)\,\frac{d\theta}{dt'} = g(R)\left(1-\alpha-\frac{r_s}{R}-\gamma R\right)^{-1/2} \frac{d\theta}{dt}$$

$$= \left\{ R^3 \frac{\gamma}{6} + R^2\left(\frac{1}{2}+\frac{\alpha}{4}\right) + R\,\frac{r_s}{2} \right\} \left(1+ \frac{1}{2}\left[\alpha + \frac{r_s}{R} + \gamma R\right]\right) \frac{d\theta}{dt}$$

donde

$$g(R) = R^2\left(\frac{\gamma}{6}R + \frac{1}{2} + \frac{\alpha}{4} + \frac{r_s}{2R}\right) = R^2 \Big($$

$$= \left(\frac{3\gamma R}{6} - \frac{2\gamma R}{6}\right) + \left(1 - \frac{1}{2}\right) + \left(\frac{\alpha}{2} - \frac{\alpha}{4}\right) + \frac{r_s}{2R}$$

$$= \left(1 + \frac{\alpha}{2} + \frac{r_s}{2R} + \frac{\gamma R}{2} - \frac{\gamma R}{3} - \frac{1}{2} - \frac{\alpha}{4}\right)$$

$$g(R) = \left(1+\frac{1}{2}\left[\alpha + \frac{r_s}{R} + \gamma R\right] - \left(\frac{1}{2} + \frac{\alpha}{4} + \frac{\gamma R}{3}\right)\right)$$

entonces dado $1 \gg \alpha + \frac{r_s}{R} + \gamma R \quad \rightarrow \quad \left(\alpha + \frac{r_s}{R} + \gamma R\right)^2 \approx 0$

$$\Phi = g(R)\cdot\left(1+ \frac{1}{2}\left[\alpha + \frac{r_s}{R} + \gamma R\right]\right) \qquad \Big| \text{ con } \quad \alpha + \frac{r_s}{R} + \gamma R = y$$

$$\hookrightarrow \left(1+\frac{1}{2}y - \left(\frac{1}{2} + \frac{\alpha}{4} + \frac{\gamma R}{3}\right)\right)\left(1+\frac{1}{2}y\right)$$

$$= 1 + \frac{1}{2}y + \frac{1}{2}y + \underbrace{\cancel{\frac{1}{4}y^2}}_{0} - \left(\frac{1}{2} + \frac{\alpha}{4} + \frac{\gamma R}{3}\right) - \frac{1}{2}y\left(\frac{1}{2} + \frac{\alpha}{4} + \frac{\gamma R}{3}\right)$$

$$\Phi = 1 + \alpha + \frac{r_s}{R} + \gamma R - \left(\frac{1}{2} + \frac{\alpha}{4} + \frac{\gamma R}{3}\right) - \frac{1}{2}\left(\alpha + \frac{r_s}{R} + \gamma R\right)\left(\frac{1}{2} + \frac{\alpha}{4} + \frac{\gamma R}{3}\right)$$

// resolviendo

$$\frac{1}{2}\left(\alpha + \frac{r_s}{R} + \gamma R\right)\left(\frac{1}{2} + \frac{\alpha}{4} + \frac{\gamma R}{3}\right) = \frac{1}{2}\left\{\frac{\alpha}{2} + \frac{\alpha^2}{4} + \frac{\alpha\gamma R}{3} + \frac{r_s}{2R} + \frac{\alpha r_s}{4R} + \frac{r_s\gamma}{3} + \frac{\gamma R}{2} + \frac{\alpha\gamma R}{4} + \frac{\gamma^2 R^2}{3}\right\}$$

$$= \frac{\alpha^2}{8} + \frac{\gamma^2 R^2}{6} + \frac{\alpha}{4} + \frac{4\alpha\gamma R + 3\alpha\gamma R}{24} + \frac{r_s}{4R} + \frac{\alpha r_s}{8R} + \frac{r_s\gamma}{6} + \frac{\gamma R}{4}$$

$$= \frac{\alpha^2}{8} + \frac{\gamma^2 R^2}{6} + \frac{\alpha}{4} + \frac{\gamma R}{4} + \frac{7\alpha\gamma R}{24} + \frac{r_s}{2R}\left(\frac{1}{2} + \frac{\alpha}{4}\right) + \frac{r_s\gamma}{6}$$

entonces

$$\Phi = 1 + \alpha + \frac{r_s}{R} + \gamma R - \frac{1}{2} - \frac{\alpha}{4} - \frac{\gamma R}{3}$$

$$- \frac{\alpha^2}{8} - \frac{\gamma^2 R^2}{6} - \frac{\alpha}{4} - \frac{\gamma R}{4} - \frac{7\alpha\gamma R}{24} - \frac{r_s}{2R}\left(\frac{1}{2} + \frac{\alpha}{4}\right) - \frac{r_s\gamma}{6}$$

$$= 1 - \frac{1}{2} + \alpha - \frac{\alpha}{4} - \frac{\alpha}{4} + \gamma R - \frac{\gamma R}{3} - \frac{\gamma R}{4} + \frac{r_s}{R} - \frac{r_s}{2R}\left(\frac{1}{2} + \frac{\alpha}{4}\right)$$

$$- \frac{\alpha^2}{8} - \frac{\gamma^2 R^2}{6} - \frac{r_s\gamma}{6} - \frac{7\alpha\gamma R}{24}$$

$$= \frac{1}{2} + \frac{\alpha}{2} + \gamma R \cdot \frac{12 - 4 - 3}{12} + \frac{r_s}{2R}\left(2 - \frac{1}{2} + \frac{\alpha}{4}\right) - \frac{\alpha^2}{8} - \frac{\gamma^2 R^2}{6} - \frac{r_s\gamma}{6} - \frac{7\alpha\gamma R}{24}$$

$$\Phi = \frac{1}{2} + \frac{\alpha}{2} + \frac{5\gamma R}{12} + \frac{r_s}{2R}\left(\frac{3}{2} + \frac{\alpha}{4}\right) - \frac{\alpha^2}{8} - \frac{\gamma^2 R^2}{6} - \frac{r_s\gamma}{6} - \frac{7\alpha\gamma R}{24}$$

$\frac{dA'}{d\ldots} = \Phi \, \frac{d\theta}{\ldots}$ y entonces comparando

$$\frac{dA'}{dt'} = \Phi \frac{d\theta}{dt}$$ // y entonces comparando

$$\frac{R^2}{2}\, d\theta' = \left[\frac{1}{2} + \frac{\alpha}{2} + \frac{5\gamma R}{12} + \frac{r_s}{2R}\left(\frac{3}{2} + \frac{\alpha}{4}\right) - \frac{\alpha^2}{8} - \frac{\gamma^2 R^2}{6} - \frac{r_s \gamma}{6} - \frac{7\alpha\gamma R}{24}\right] d\theta$$

$$d\theta' = \left[\frac{1}{R^2} + \frac{\alpha}{R^2} + \frac{5\gamma}{6}\frac{1}{R} + \frac{r_s}{R^3}\left(\frac{3}{2} + \frac{\alpha}{4}\right) - \frac{\alpha^2}{4R^2} - \frac{\gamma^2}{3} - \frac{r_s\gamma}{3R^2} - \frac{7\alpha\gamma}{12}\frac{1}{R}\right] d\theta$$

$$R = \frac{\ell}{1+\epsilon\cos\theta} \qquad \frac{1}{R} = \frac{1+\epsilon\cos\theta}{\ell}$$