Prueba2_Prob1_pt2

Tuesday, July 11, 2023 10:59 PM

$$d\theta' = \left[\frac{1}{R^2} + \frac{\alpha}{R^2} + \frac{5\gamma}{6}\frac{1}{R} + \frac{r_s}{R^3}\left(\frac{3}{2}+\frac{\alpha}{4}\right) - \frac{\alpha^2}{4R^2} - \frac{\gamma^2}{3} - \frac{r_s\gamma}{3R^2} - \frac{7\alpha\gamma}{12}\frac{1}{R} \right] d\theta$$

$$R = \frac{\ell}{1+\epsilon\cos\theta} \qquad \frac{1}{R} = \frac{1+\epsilon\cos\theta}{\ell}$$

$$d\theta' = \left[\frac{1}{R^2}\left(1+\alpha-\frac{\alpha^2}{4}-\frac{r_s\gamma}{3}\right) + \frac{1}{R}\left(\frac{5\gamma}{6} - \frac{7\alpha\gamma}{12}\right) + \frac{r_s}{R^3}\left(\frac{3}{2}+\frac{\alpha}{4}\right) - \frac{\gamma^3}{3}\right] d\theta$$

$$\int^{\Delta\theta'} d\theta' = \int_0^{2\pi}\left[\frac{1+\cancel{2\epsilon\cos\theta}+\epsilon^2\cos^2\theta}{\ell^2}\eta + \frac{1+\cancel{\epsilon\cos\theta}}{\ell}\nu + \frac{1+\cancel{3\epsilon\cos\theta}+3\epsilon^2\cos^2\theta+\epsilon^3\cos^3\theta}{\ell^3}\xi - \beta\right] d\theta$$

con $\eta = \left(1+\alpha-\frac{\alpha^2}{4}-\frac{r_s\gamma}{3}\right)$; $\nu = 5\gamma - \frac{7\alpha\gamma}{12}$; $\xi = r_s\left(\frac{3}{2}+\frac{\alpha}{4}\right)$; $\beta = \frac{\gamma^3}{3}$

$$\int_0^{2\pi}\cos\theta\, d\theta = 0 \quad ; \quad \int_0^{2\pi}\cos^2\theta = \pi \quad ;$$

$$\int_0^{2\pi}\cos^3\theta\, d\theta = \int_0^{2\pi}\cos\theta\,(1-\sin^2\theta)\,d\theta = \underbrace{\cancel{\int_0^{2\pi}\cos\theta\, d\theta}}_{0} - \int_0^{2\pi}\cos\theta\,\sin^2\theta\, d\theta$$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$$-\int u^2\, du = -\frac{\sin^3\theta}{3}\Big|_0^{2\pi} = 0$$

$$\Delta\theta' = \int_0^{2\pi}\left[\frac{1+\epsilon^2\cos^2\theta}{\ell^2}\eta + \frac{\nu}{\ell} + \frac{1+3\epsilon^2\cos^2\theta}{\ell^3}\xi - \beta\right] d\theta$$

$$\Delta\theta' = \underbrace{\frac{2\pi + \epsilon^2\pi}{\ell^2}\eta + 2\pi\frac{\nu}{\ell} + \frac{2\pi + 3\epsilon^2\pi}{\ell^3}\xi}_{\text{avance del perihelio} \quad \Delta\phi} - \beta 2\pi - 2\pi + 2\pi$$

$$\Delta\phi = \pi\left\{\left(\frac{2+\epsilon^2}{\ell^2}\right)\left(1+\alpha-\frac{\alpha^2}{4}-\frac{r_s\gamma}{3}\right) + \frac{2}{\ell}\left(5\gamma - \frac{7\alpha\gamma}{12}\right) + \left(\frac{2+3\epsilon^2}{\ell^3}\right)r_s\left(\frac{3}{2}+\frac{\alpha}{4}\right) - \frac{2}{3}\gamma^3 - 2\right\}$$

$$= \pi\left\{ \frac{2+\epsilon^2}{\ell^2} + \alpha\left(1-\frac{\alpha}{4}\right)\frac{2+\epsilon^2}{\ell^2} - \gamma\frac{r_s}{3}\frac{2+\epsilon^2}{\ell^2} + \frac{10\gamma}{\ell} - \frac{7}{6}\frac{\alpha\gamma}{\ell} \right.$$

$$\left. \frac{2+3\epsilon^2}{\ell^3}\cdot\frac{3}{2}r_s + \alpha\frac{2+3\epsilon^2}{4\ell^3}r_s - \frac{2}{3}\gamma^3 - 2 \right\}$$

$$= \pi\left\{ \frac{1}{\ell^2}\left(2+\epsilon^2 + \frac{(2+3\epsilon^2)}{\ell}\frac{3}{2}r_s - 2\ell^2\right) + \frac{\alpha}{\ell^2}\left(\left(1-\frac{\alpha}{4}\right)\left(2+\epsilon^2\right) + \frac{(2+3\epsilon^2)}{\ell}\frac{r_s}{4}\right) \right.$$

$$\left. + \frac{\gamma}{\ell}\left(10 - \frac{2+\epsilon^2}{\ell}\frac{r_s}{3} - \frac{2}{3}\gamma^2\right) - \frac{1}{\ell}\frac{7}{6}\alpha\gamma \right\} \left[\frac{\text{rad}}{\text{periodo}}\right]$$

$$\Delta\phi = \pi\left\{ \frac{\Delta\phi_r}{\pi} + \frac{\Delta\phi_a}{\pi} + \frac{\Delta\phi_\gamma}{\pi} + \frac{\Delta\phi_{\alpha\gamma}}{\pi} \right\}$$

Pasando a arcosegundo por siglo

$$\frac{180}{\cancel{\pi}}\left[\frac{\text{grados}}{\text{rad}}\right]\cdot 3600\left[\frac{''}{\text{grado}}\right]\cdot\tau\left[\frac{\text{periodo}}{\text{año}}\right] 10^2\left[\frac{\text{año}}{\text{siglo}}\right] = \frac{\tau\cdot 6{,}48\times10^7}{\pi}\left[\frac{''}{\text{siglo}}\right]$$

$$\Delta\phi_r = \frac{1}{\ell^2}\left(2+\epsilon^2 + \frac{(2+3\epsilon^2)}{\ell}\frac{3}{2}r_s - 2\ell^2\right)\tau\cdot 6{,}48\times10^7\left[\frac{''}{\text{siglo}}\right]$$

$$\Delta\phi_a = \frac{\alpha}{\ell^2}\left((1-\alpha)\left(2+\epsilon^2\right) + \frac{(2+3\epsilon^2)}{\ell}\frac{r_s}{4}\right)\tau\cdot 6{,}48\times10^7\left[\frac{''}{\text{siglo}}\right]$$

$$\Delta\phi_\gamma = \frac{\gamma}{\ell}\left(10 - \frac{2+\epsilon^2}{\ell}\frac{r_s}{3} - \frac{2}{3}\gamma^2\right)\tau \cdot 6{,}48\times10^7 \left[\frac{''}{\text{siglo}}\right]$$

$$\Delta\phi_{\gamma\alpha} = -\frac{1}{\ell}\frac{7}{6}\alpha\gamma \quad \tau\cdot 6{,}48\times10^7 \left[\frac{''}{\text{siglo}}\right]$$

los arcosegundos por año que entrega son demasiado grandes

```
# el termino más grande con los datos de mercurio da un resultado que parece explotar numeric
# posiblemente debido a la precisión numerica o a algo que habria sido ignorado en la aproxim
phi_r(np.float64(0.206), np.float64(55.3), np.float64(4.15))
```

✓ 0.0s

```
-537839999.9999999
```

```
# los demas componentes son mucho más pequeños
phi_alpha_gamma(np.float64(0.206),227e11,np.float64(4.15),1e-11,1e-11)
```

✓ 0.0s

```
-1.3821145374449338e-38
```

```
# debido a esto, se retrocederá para reaproximar
Phi(1e-11,1e-11,np.float64(0.206),np.float64(55.3),np.float64(4.15))
```

✓ 0.0s

```
-537839999.9999999
```

Por tanto se considerará esto una mala aproximación, se reiniciara el problema en la siguiente sección: