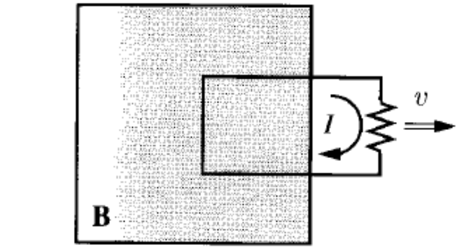
Chapter 7. Electrodynamics

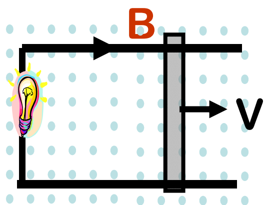
7.2 Electromagnetic Induction

7.2.1 Faraday's Law

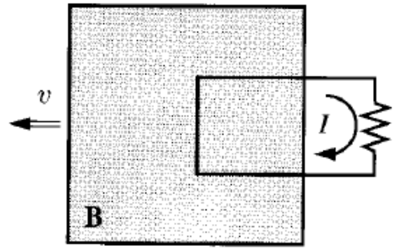
In 1831 Michael Faraday reported on a series of experiments:

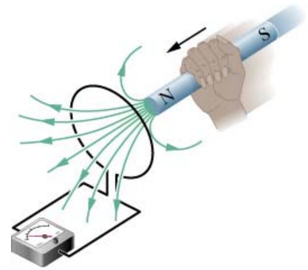
Experiment 1. He pulled a loop of wire to the right through a magnetic field.
➔ **A current flowed in the loop.**



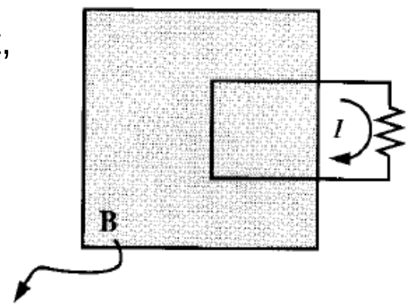


Experiment 2. He moved the *magnet* to the *left,* holding the loop still.
➔ **Again, a current flowed in the loop.**

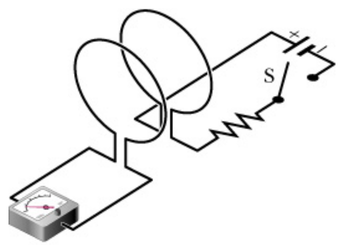




Experiment 3. With both the loop and the magnet at rest, he changed the *strength of the field* *(with varied current in the coil)*
➔ **Once again, current flowed in the loop.**

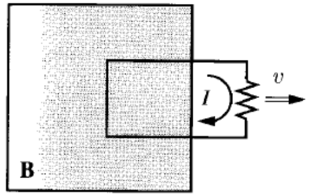


changing
magnetic field



Faraday's Law

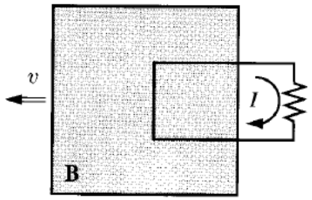
He pulled a loop of wire to the right through a magnetic field in the **Experiment 1**.

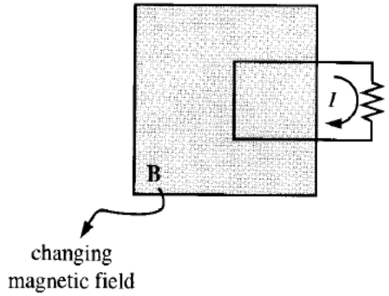


→ **It is just an example of motional emf:** $\mathcal{E} = -\frac{d\Phi}{dt}$

→ ***The emf is magnetic*** **(Lorentz force law at work).**

But if the loop is ***stationary, as in the Experiments 2 and 3,***
the force cannot be magnetic: stationary charges experience no magnetic forces.





What is responsible?
What sort of field exerts a force on charges at rest?

→ ***The force must be electric.***
but in this case there doesn't seem to be any electric field in sight.

Faraday had an ingenious inspiration:

➔ ***A changing magnetic field induces an electric field.***

If the emf is again equal to the rate of change of the flux, $\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$

then E is related to the change in B by

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}$$

Or, in differential form by

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Faraday's Law

Faraday's Law

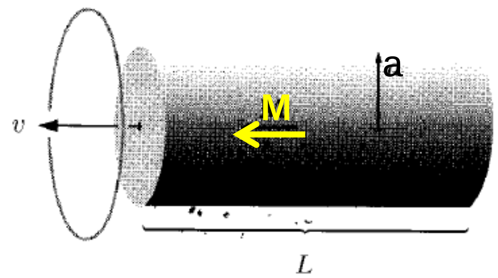
$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Faraday's Law

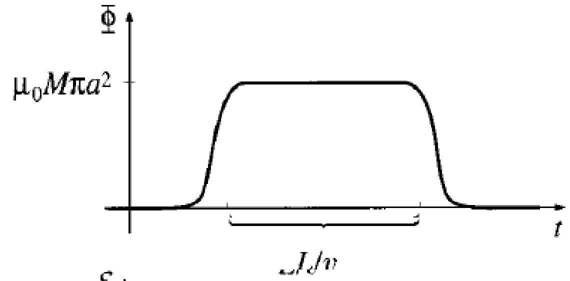
➔ It is the law ***for electric fields induced by changing magnetic fields***

Example 7.5 A long cylindrical magnet of length *L* and radius ***a*** carries a uniform magnetization **M** parallel to its axis.
It passes at constant velocity ***v*** through a circular wire ring of slightly larger diameter.
Graph the emf induced in the ring, as a function of time.



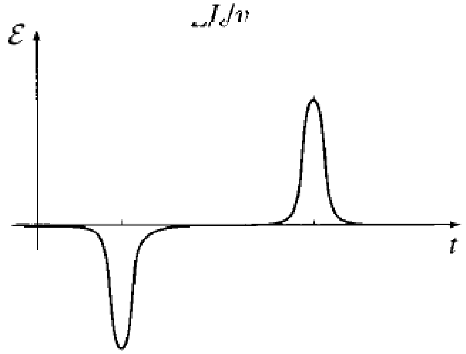
The magnetic field is the same as that of a long solenoid with surface current, $\mathbf{K}_b = M\hat{\boldsymbol{\phi}}$.

the field inside is $\mathbf{B} = \mu_0 \mathbf{M}$



The flux through the ring is zero when the magnet is far away;
it builds up to a maximum of $\mu_0 M\pi a^2$ as the leading end passes through;
and it drops back to zero as the trailing end emerges.

The emf is (minus) the derivative of Φ with respect to time,



Which way around the ring does the induced current flow?

In principle, the right-hand rule does the job.

There's a handy rule, called Lenz's law.

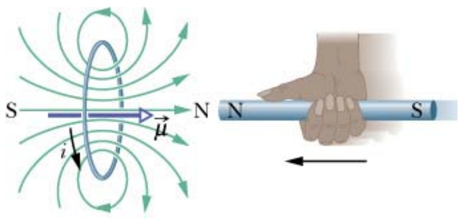
Lenz's Law

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

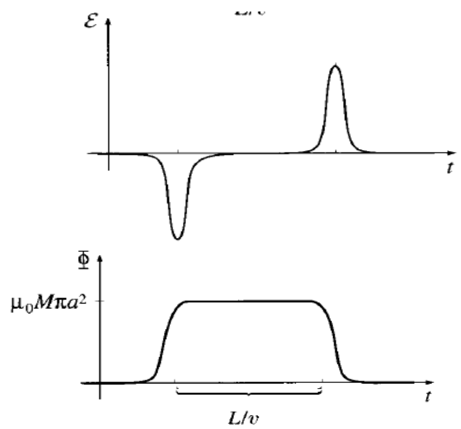
Faraday's Law

➔ It is the law ***for electric fields induced by changing magnetic fields***



$$\mathcal{E} = -\frac{d\Phi}{dt}$$

Keeping track of the ***signs in Faraday's law*** can be a real headache.

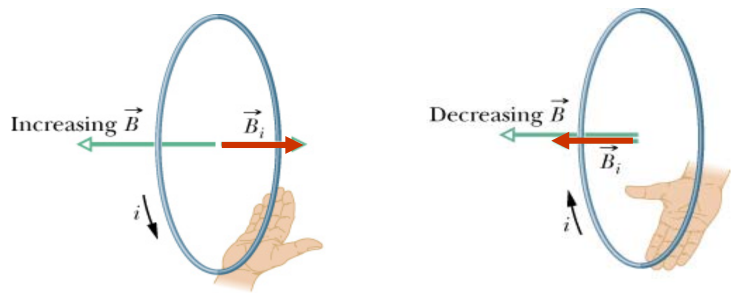


There's a handy rule, called **Lenz's law**,
whose sole purpose is to help you get the directions right.

➔ ***Nature abhors (hates) a change in flux.***

➔ *The induced current will flow in such a direction that the flux it produces tends to cancel the change:* ***Lenz's Law***

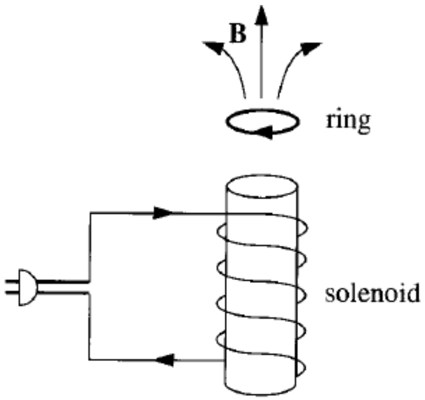
➔ *All Lenz's law tells you is the direction of the flow.*



Faraday's Law

Example 7.6

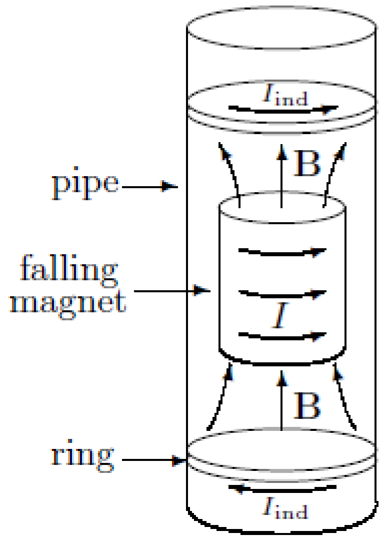
The "jumping ring" demonstration.
If you wind a solenoidal coil around an iron core (the iron is there to beef up the magnetic field), place a metal ring on top, and plug it in, the ring will jump several feet in the air.
Why?



Before you turned on the current. the flux through the ring was *zero.*
Afterward a flux appeared (upward),
→ the emf generated in the ring led to a current (in the ring).
→ According to Lenz's law, the emf was in such a direction that *its* field tended to cancel this new flux.
→ This means that the current in the loop is *opposite* to the current in the solenoid.
➔ And opposite currents repel, so the ring flies off.

Problem 7.14

A short cylindrical bar magnet is dropped down a vertical aluminum pipe of slightly larger diameter, about 2 meters long. It takes several seconds to emerge at the bottom, Whereas an otherwise identical piece of unmagnetized iron makes the trip in a fraction of a second.
Explain why the magnet falls more slowly.

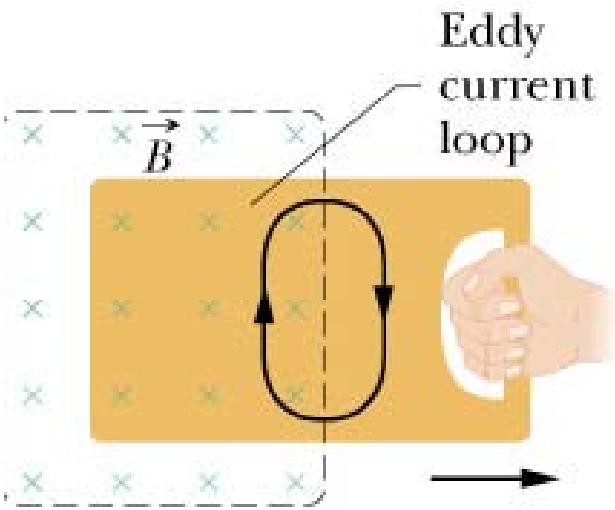


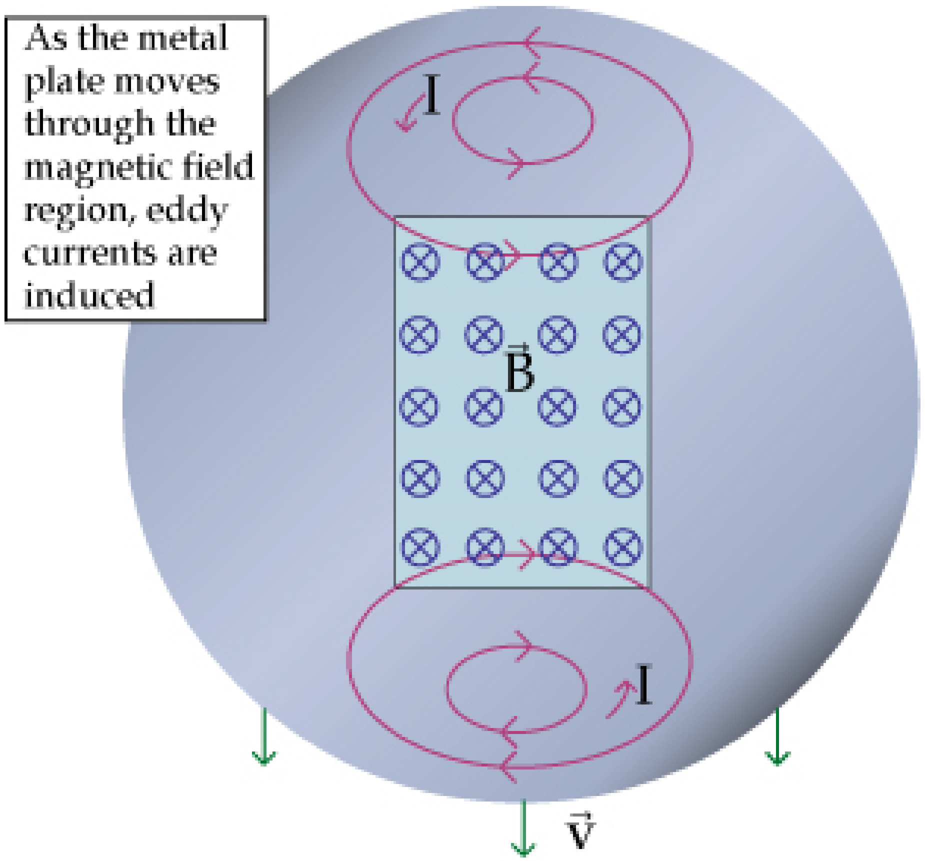
A ring of pipe below the magnet experiences an increasing upward flux as the magnet approaches, and hence (by Lenz's law) a current (I_{ind}) will be induced in it such as to produce a downward flux.
Thus I_{ind} must flow clockwise, which is opposite to the current in the magnet.
Since opposite currents repel, the force on the magnet is upward.
Meanwhile, a ring above the magnet experiences a decreasing (upward) flux, so its induced current is parallel to I, and it attracts the magnet upward.

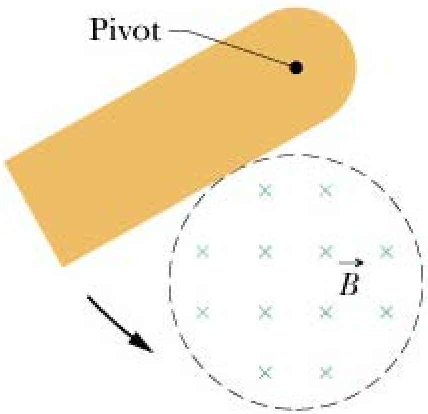
➔ The delay is due to forces exerted on the magnet by induced **eddy currents** in the pipe.

소용돌이 전류 (Eddy current, or Foucault current)

http://www.ndt-ed.org/EducationResources/HighSchool/Electricity/eddycurrents.htm







Braking in Roller coaster

Vending machines (detection of coins)

7.2.2 The Induced Electric Field

There are really two distinct kinds of electric fields:

E-fields, produced by static electric charges from Coulomb's Law: $\nabla \cdot \mathbf{E} = (1/\epsilon_0)\rho, \ \nabla \times \mathbf{E} = 0$

➔ **Static electric fields**

E-fields, produced by changing magnetic fields from Faraday's law: $\nabla \cdot \mathbf{E} = 0, \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

➔ **Induced electric fields**

Let's exploit analogy between induced electric fields and magnetic fields:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \cdot \mathbf{E} = 0$$

← Induced field by exclusively changing in B (with $\rho = 0$),

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \qquad \nabla \cdot \mathbf{B} = 0$$

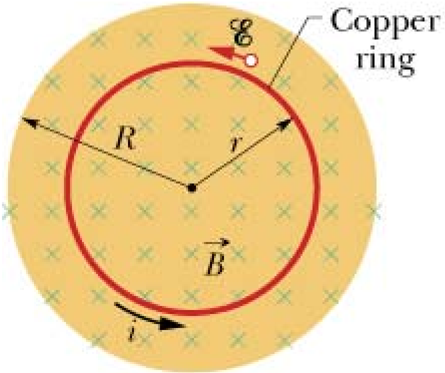
Faraday-induced electric fields are determined by $-(\partial \mathbf{B}/\partial t)$ *in exactly the same way as magnetostatic fields are determined by* $\mu_0 \mathbf{J}$

We can use all the tricks associated with Ampere's law in integral form, to find the induced field by Faraday's law:

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt} \longleftrightarrow \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$$

➔ ***The rate of change of (magnetic) flux plays the role of*** $\mu_0 I_{enc}$ ***in exactly the same way.***

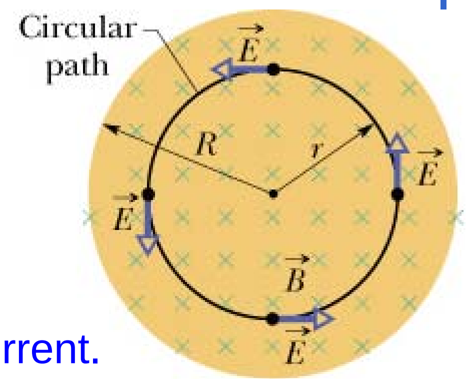
Induced electric fields



$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$$

The loop does not have to be a wire!
The emf exists even in vacuum!

When we put a wire there,
the electrons respond to the emf → circular current.

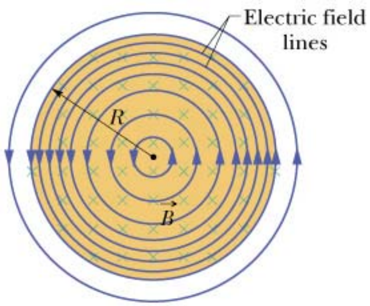


The work done by the induced electric field for a charge ***q*$_0$** after one circulation:

$$W = \oint \vec{F} \cdot d\vec{s} = q_0 \left(\oint \vec{E} \cdot d\vec{s} \right) = -q_0 \left(\frac{d\Phi_B}{dt} \right) \neq 0$$

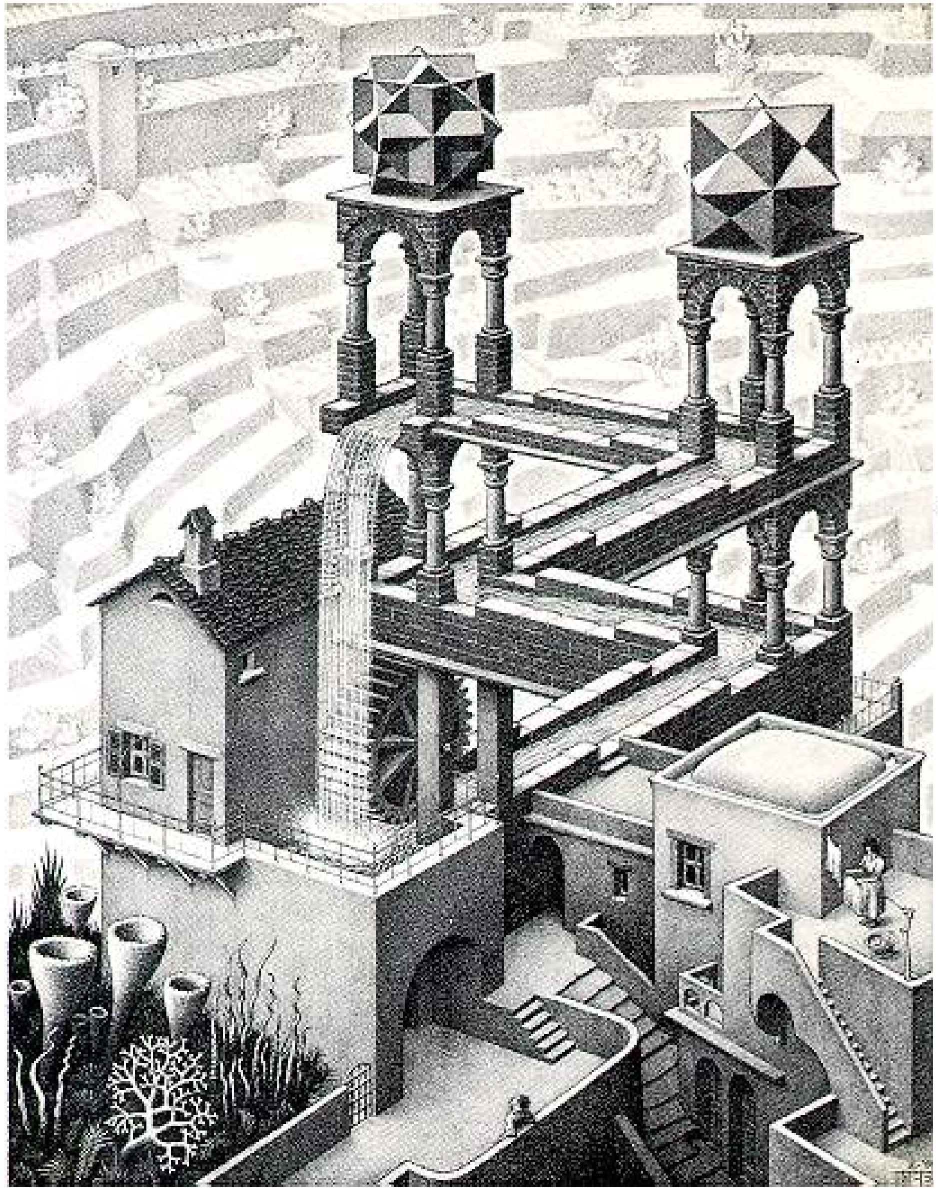
The induced fields (forces) from changing magnetic fields are *nonconservative!*

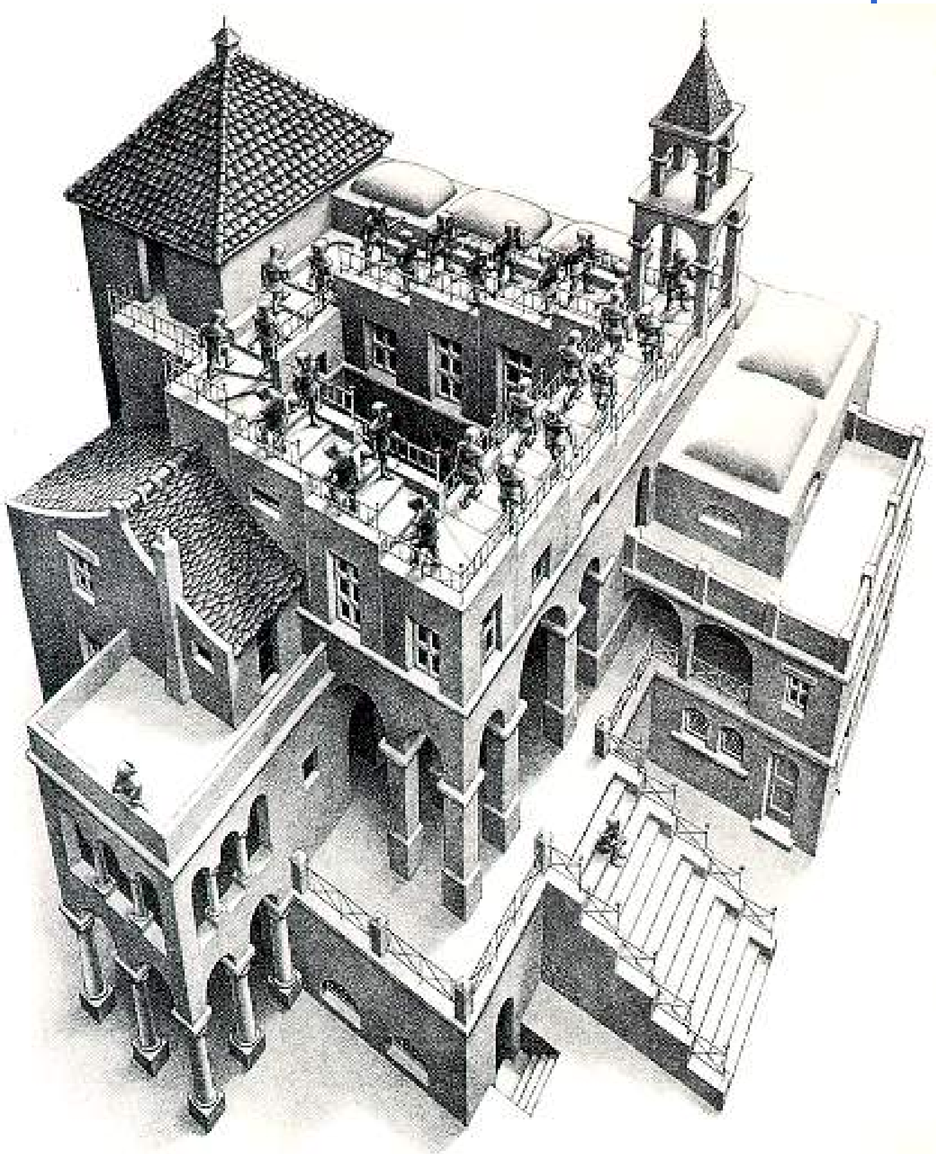
No potential can be defined!



	정전기장	유도전기장
근원	• 전하 (쿨롱 법칙) $\oint_S \mathrm{E}_{전하} \cdot d\mathrm{A} = \frac{q}{\varepsilon_0}$	• 자기장의 변화 (파라데이 법칙) $\oint_C \mathrm{E}_{유도} \cdot d\mathrm{s} = -\frac{d\Phi_B}{dt}$
전위	• 정의됨 $\oint_C \mathrm{E}_{전하} \cdot d\mathrm{s} = 0$ $\Rightarrow V = -\int_{r_{ref}}^{r} \mathrm{E}_{전하} \cdot d\mathrm{s}$	• 정의할 수 없음 $\oint_C \mathrm{E}_{유도} \cdot d\mathrm{s} \neq 0$

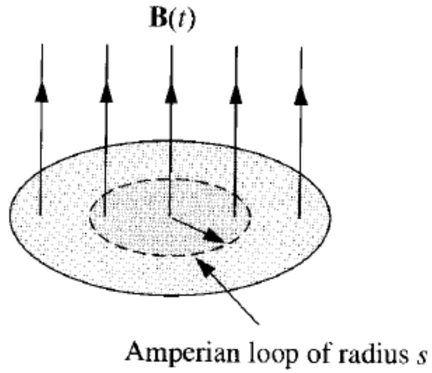
Escher depiction of nonconservative *emf*





Induced electric fields

Example 7.7 A uniform magnetic field *B(t),* pointing straight up, fills the shaded circular region. If B is changing with time, what is the induced electric field?



We can use all the tricks associated with Ampere's law in integral form, to find the induced field by Faraday's law:

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt} \longleftarrow \mu_0 I_{\text{enc}}$$

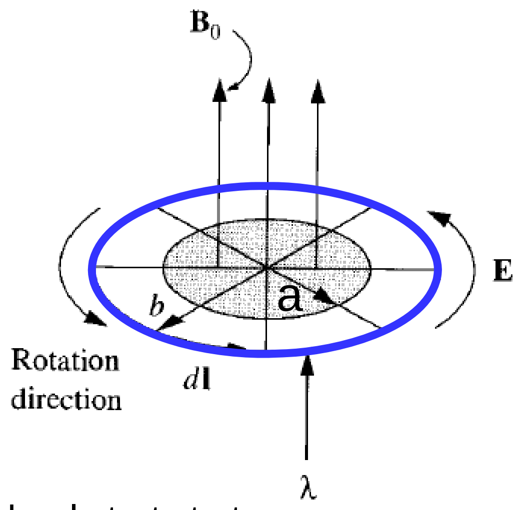
Draw an Amperian loop of radius *s,* and apply Faraday's law:

→ If B is decreasing, E runs counterclockwise, as viewed from above.

Induced electric fields

Example 7.8

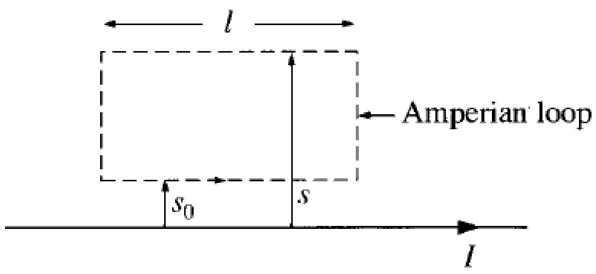
A line charge λ is glued onto the rim of a wheel of radius ***b***, which is then suspended horizontally, so that it is free to rotate.
In the central region, out to radius ***a***, there is a uniform magnetic field $\mathbf{B_0}$, pointing up.
Now someone turns the field off.
What happens?



The changing magnetic field will induce an electric field,
This electric field exerts a force on the charges at the rim, and the wheel starts to tum.
According to Lenz's law, it will rotate in counterclockwise direction.

Induced electric fields

Example 7.9 An infinitely long straight wire carries a slowly varying current *I(t).*
Determine the induced electric field,
as a function of the distance s from the wire.



The magnetic field is **($\mu_0 I/2\pi s$).**
The induced E here runs parallel to the axis.

Like the B-field of a solenoid, E here runs parallel to the axis.
For the rectangular "Amperian loop", Faraday's law gives:

→ *E blows up as s goes to infinity. That can't be true . What's gone wrong?*

→ Electromagnetic "news" travels at the speed of light (Chapter 9),
→ At large distances **B** depends not on the current *now,* but on the current *as it was* at some earlier time.
→ Hence the result of E field in the limit of ***quasistatic approximation*** simply does not apply,
at extremely large s.

Induced electric fields

Problem 7.15

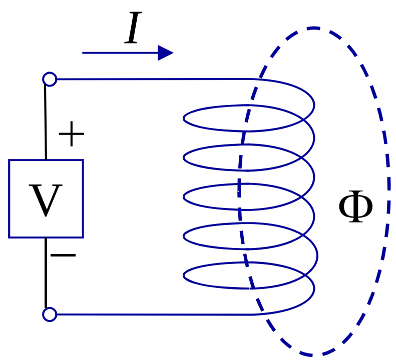
A long solenoid **with radius *a* and *n* turns** per unit length carries a time-dependent current $I(t)$ in the ϕ direction. Find the electric field (magnitude and direction) at a distance s from the axis (both inside and outside the solenoid), in the *quasistatic approximation*.

7.2.3 Inductance

- An **inductor** is a two-terminal device that consists of a coiled conducting wire wound around a core.
- A current flowing through the device produces a magnetic flux Φ.
- For a linear inductor, *Φ is proportional to I* : $\Phi = LI$

  → ***$L = \Phi / I$* : Inductance (self-inductance)**

  : Unit- Henry (H) or (V•s/A)



Suppose you have two loops of wire, at rest:

A steady current $\mathbf{I_1}$ around loop **1,** it produces a magnetic field $\mathbf{B_1}$.

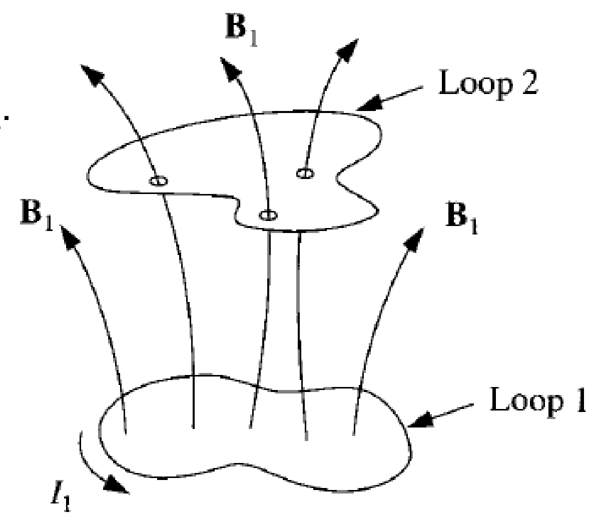
$$\mathbf{B}_1 = \frac{\mu_0}{4\pi} I_1 \oint \frac{d\mathbf{l}_1 \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} \quad \leftarrow \text{Biot-Savart law}$$

The flux through loop 2: $\Phi_2 = \int \mathbf{B}_1 \cdot d\mathbf{a}_2$

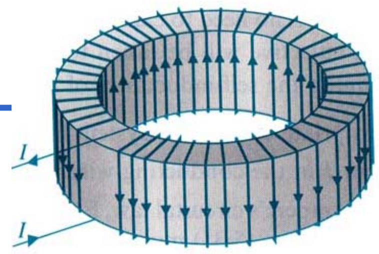
$$\longrightarrow \Phi_2 = M_{21} I_1$$

M_{21} is the constant of proportionality

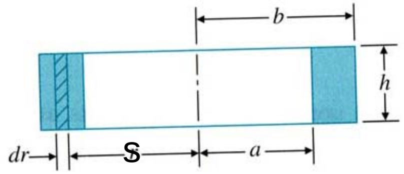
→ **Mutual inductance** of the two loops.



Self-inductance



Example 7.11 Find the self-inductance of a toroidal coil with rectangular cross section (inner radius ***a***, outer radius ***b***, height ***h***), which carries a total of ***N* turns**.



The magnetic field inside the toroid is

$$a < s < b;$$

$$\oint_C \mathbf{B} \bullet dl = \mu_0 \int_S \mathbf{J} \bullet d\mathbf{s} \rightarrow \int_0^{2\pi} Bs d\phi = 2\pi r B = \mu_0 NI \rightarrow \mathbf{B} = \frac{\mu_0 NI}{2\pi s} \hat{\phi}$$

The flux through a single tum is →

The *total* flux is *N* times this, so the self-inductance is →

Problem 7.22 Find the self-inductance per unit length of a long solenoid, of radius *R*, carrying ***n* turns per unit length**.

Mutual Inductance

$$\boxed{\Phi_2 = M_{21} I_1}$$

$$\Phi_2 = \int \mathbf{B}_1 \cdot d\mathbf{a}_2$$

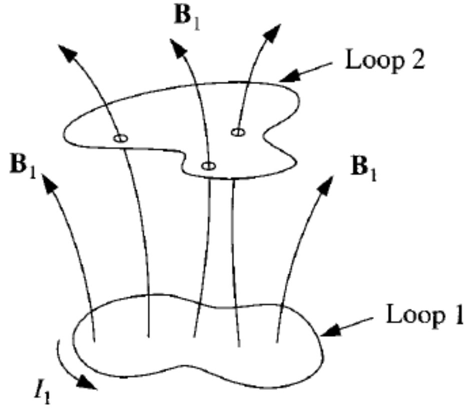
$$= \int (\nabla \times \mathbf{A}_1) \cdot d\mathbf{a}_2 = \oint \mathbf{A}_1 \cdot d\mathbf{l}_2 .$$ ← Stokes' theorem

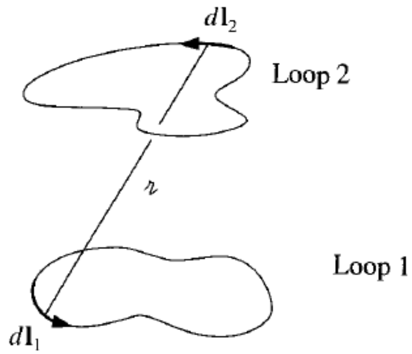
$$\mathbf{A}_1 = \frac{\mu_0 I_1}{4\pi} \oint \frac{d\mathbf{l}_1}{\mathscr{r}}$$

$$\Phi_2 = \frac{\mu_0 I_1}{4\pi} \oint \left(\oint \frac{d\mathbf{l}_1}{\mathscr{r}} \right) \cdot d\mathbf{l}_2$$

$$M_{21} = \frac{\mu_0}{4\pi} \oint \oint \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{\mathscr{r}}$$

➔ **Neumann formula**





1. ***M_{21}*** is a purely geometrical quantity;
   having to do with the sizes, shapes, and relative positions of the two loops.

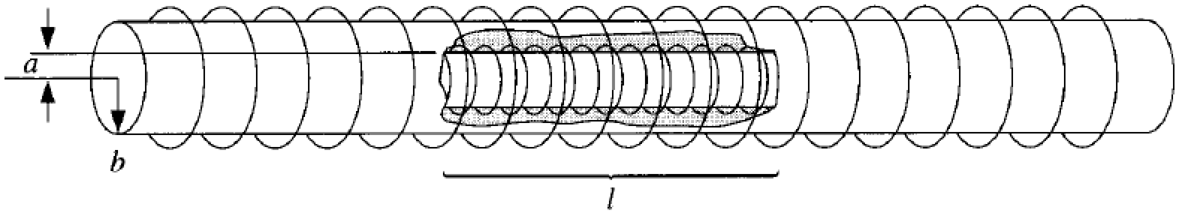
2. **The integral is unchanged** if we switch the roles of loops 1 and 2 → $M_{21} = M_{12}$

Whatever the shapes and positions of the loops,
The flux through 2 when we run a current I around 1 is *identical*
to the flux through 1 when we send the same current I around 2.

$$M_{21} = M_{12} = M$$

Mutual inductance

Example 7.10 A short solenoid (length *l* and radius *a*, with n_1 turns per unit length) lies on the axis of a very long solenoid (radius *b*, n_2 turns per unit length).
Current $\boldsymbol{I}$ flows in the short solenoid. What is the flux through the long solenoid?



Since the inner solenoid is short, it has a very complicated field; It would be a *miserable* task to compute the total flux this way.
However, It we exploit the equality of the mutual inductances, $M_{21} = M_{12} = M$
the problem becomes very easy.

Just look at the reverse situation:
run the current I through the outer solenoid, and calculate the flux through the inner one.

The field inside the long solenoid is constant →

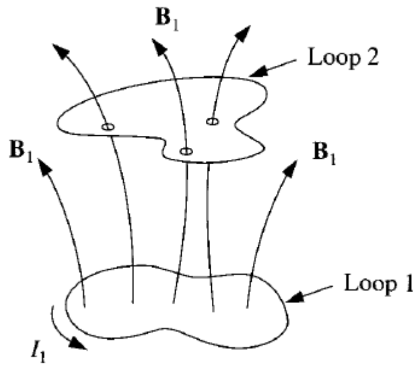
The flux through a single loop of the short solenoid is →

There are $\boldsymbol{n_1 l}$ t urns in all, so the total flux through the inner solenoid is →

Inductance and emf

$$\Phi_2 = MI_1$$

$$M = \frac{\mu_0}{4\pi} \oint \oint \frac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{\mathscr{r}} \longleftarrow \Phi_2 = \frac{\mu_0 I_1}{4\pi} \oint \left(\oint \frac{d\mathbf{l}_1}{\mathscr{r}} \right) \cdot d\mathbf{l}_2$$

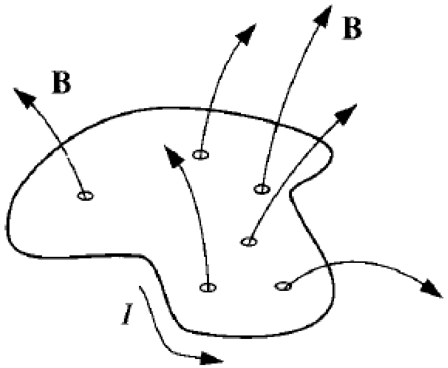


If you vary the current in loop 1,
→ The flux through loop 2 will vary accordingly,
→ The changing flux will induce an emf in loop 2:

$$\boxed{\mathcal{E}_2 = -\frac{d\Phi_2}{dt} = -M\frac{dI_1}{dt}}$$

→ Every time you change the current in loop 1, an induced current flows in loop 2,
→ even though there are no wires connecting them!

A changing current not only induces an emf in any nearby loops,
It also ***induces an emf in the source loop itself.***
Once again, the field (also the flux) is proportional to the current → $\Phi = LI$



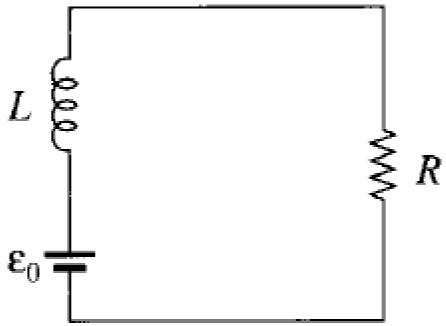
If the current changes,

the emf induced in the loop is

$$\boxed{\mathcal{E} = -L\frac{dI}{dt}}$$

Inductance → "*Back emf*"

$$\boxed{\mathcal{E} = -L\frac{dI}{dt}}$$



If you vary the current in a loop ,

→ ***The emf is in such a direction as to oppose any change in current →Lenz's law***

→ ***It may be called a "back emf".***

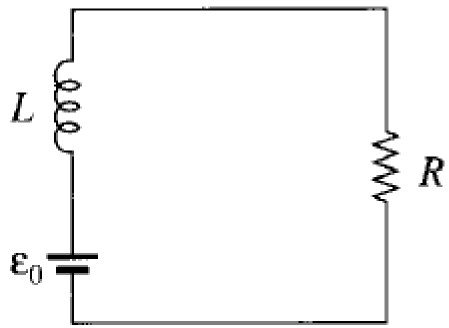
Whenever you try to alter the current in a wire,
you must fight against this back emf.
Thus inductor plays somewhat the same role as *mass* plays in mechanical systems:

The greater *L* is, the harder it is to change the current,
just as the larger the mass, the harder it is to change an object's velocity.

Inductor

Example 7.12 Consider a case when you plug *in* a toaster or iron.
Induction opposes the sudden increase in current, prescribing instead a smooth and continuous buildup.

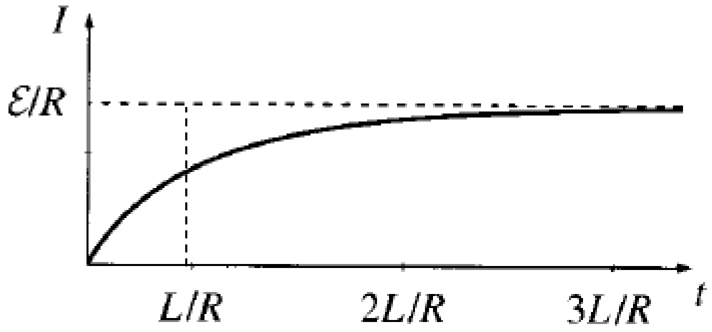
Suppose that a battery (a constant emf $\mathcal{E}_0$) is connected to a circuit of resistance *R* and inductance *L*.
What current flows?



The total emf in this circuit is

The general solution of this first-order differential equation, is

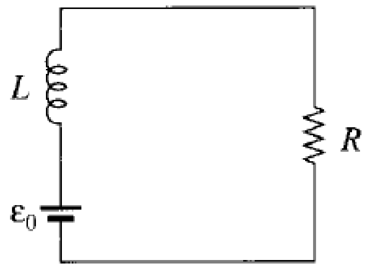
If the circuit is "plugged in" at time $t = 0$, so $I(0) = 0$ →



The quantity t = **τ = *L/ R* is called the time constant of LR circuit**;
It tells you how long the current takes to reach a substantial fraction.

7.2.4 Energy in Magnetic Fields

It takes an *energy (work)* to start a current flowing in a circuit, due to the "**back emf**" from L.



- ➔ you must do a work *against the back emf* to get the current going.
- ➔ This is *a fixed* amount, and it is ***recoverable.***
- ➔ You get it back when the current is turned off.
- ➔ In the meantime it represents energy latent in the circuit.
- ➔ ***it can be regarded as energy stored in the magnetic field.***

The work done on a unit charge, against the back emf, in one trip around the circuit is

→ $-\mathcal{E}$ (the minus sign means ***the work done by you against the emf***, not the work done by the emf).

The total work done per unit time →

$$\frac{dW}{dt} = -\mathcal{E}I = LI\frac{dI}{dt}$$

If the current is built up to a final value I, the work (or, the stored energy in ***L***) is

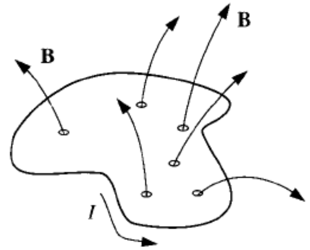
$$\Rightarrow \quad \boxed{W = \frac{1}{2}LI^2}$$

→ It does not depend on how *long* we take to crank up the current,
only on the geometry of the loop (in the form of ***L***) and the final current I.

Energy in Magnetic Fields

$\boxed{W = \frac{1}{2} L I^2}$ ➔ Let's convert the magnetic energy in terms of field quantity: ***B***

$$\Phi = LI \qquad \Phi = \int_{\mathcal{S}} \mathbf{B} \cdot d\mathbf{a} = \int_{\mathcal{S}} (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint_{\mathcal{P}} \mathbf{A} \cdot d\mathbf{l} \longrightarrow LI = \oint \mathbf{A} \cdot d\mathbf{l}$$



$$W = \frac{1}{2} I \oint \mathbf{A} \cdot d\mathbf{l} = \frac{1}{2} \oint (\mathbf{A} \cdot \mathbf{I})\, dl$$

The generalization to volume currents is → $W = \frac{1}{2} \int_{\mathcal{V}} (\mathbf{A} \cdot \mathbf{J})\, d\tau$

Ampere's law, $\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \longrightarrow W = \frac{1}{2\mu_0} \int \mathbf{A} \cdot (\nabla \times \mathbf{B})\, d\tau$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B}) \longrightarrow \mathbf{A} \cdot (\nabla \times \mathbf{B}) = \mathbf{B} \cdot \mathbf{B} - \nabla \cdot (\mathbf{A} \times \mathbf{B})$$

$$W = \frac{1}{2\mu_0} \left[\int B^2\, d\tau - \int \nabla \cdot (\mathbf{A} \times \mathbf{B})\, d\tau \right] = \frac{1}{2\mu_0} \left[\int_{\mathcal{V}} B^2\, d\tau - \oint_{\mathcal{S}} (\mathbf{A} \times \mathbf{B}) \cdot d\mathbf{a} \right]$$

The larger the region we pick the greater is the contribution from the volume integral, and therefore the smaller is that of the surface integral

$$A \propto \frac{1}{R}, H \propto \frac{1}{R^2}, S \propto R^2$$

→ Over *all* space, then the surface integral goes to zero,

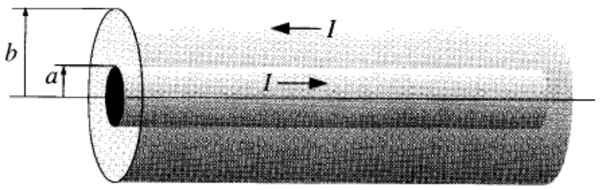
$$\boxed{W = \frac{1}{2\mu_0} \int_{\text{all space}} B^2\, d\tau}$$

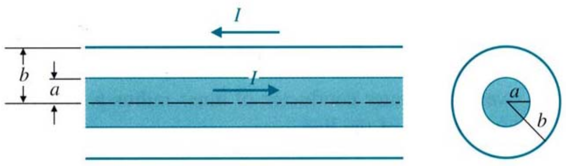
➔ ***Energy stored in the magnetic field***

$$\longleftrightarrow \quad W_{\text{elec}} = \frac{1}{2} \int (V\rho)\, d\tau = \frac{\epsilon_0}{2} \int E^2\, d\tau$$

Magnetic Energy $W = \frac{1}{2\mu_0}\int_{\text{all space}} B^2\, d\tau$

Example 7.13 A long coaxial cable carries current I
(the current flows down the surface of the inner cylinder, radius **a**, and back along the outer cylinder, radius **b**).
Find the magnetic energy stored in a section of length l.
Calculate the self-inductance of the cable.





According to Ampere's law,

the field between the cylinders →

Elsewhere, the field is zero.

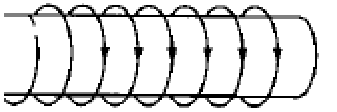
The energy per unit volume is

The energy in a cylinder shell of length l, radius s, and thickness ds, then, is

The energy can also be written as $W = \frac{1}{2}LI^2 \longrightarrow$

Magnetic Energy

Problem 7.26 Find the energy stored in a section of length l of a long solenoid (radius R, current I, n turns per unit length).



(a) Use $W = \frac{1}{2} L I^2$

(b) Use $W = \frac{1}{2} I \oint \mathbf{A} \cdot d\mathbf{l} = \frac{1}{2} \oint (\mathbf{A} \cdot \mathbf{I})\, dl$

(c) Use $W = \frac{1}{2\mu_0} \int_{\text{all space}} B^2\, d\tau$

(d) Use $W = \frac{1}{2\mu_0} \left[\int_{\mathcal{V}} B^2\, d\tau - \oint_{\mathcal{S}} (\mathbf{A} \times \mathbf{B}) \cdot d\mathbf{a} \right]$