PROBLEMA II

$$I = \int_0^\infty e^{-Ax} \sin(x)\, dx$$

$$\Downarrow$$

$$e^{-Ax} = \sum_n \phi_n A^n x^n$$

$$\sin(x) = \sum_m \frac{(-1)^m x^{2m+1}}{\Gamma(2m+2)} = \sum_m \phi_m \frac{\Gamma(m+1)}{\Gamma(2m+2)} x^{2m+1}$$

$$\Downarrow$$

$$I = \sum \phi_{m,n} \frac{A^n \Gamma(m+1)}{\Gamma(2m+2)} \langle n+2m+2 \rangle$$

$$\Downarrow$$

Se generan los 2 términos:

$$I_1 = \frac{1}{A^2} \sum_{k\geq 0} \left(-\frac{1}{A^2}\right)^k \,//$$

$$I_2 = \frac{1}{2} \sum_{k\geq 0} \frac{\Gamma\left(\frac{k}{2}+1\right)\Gamma\left(-\frac{k}{2}\right)}{\Gamma(-k)} \frac{(-A)^k}{k!} \,//$$

$$\therefore \quad I = \begin{cases} I_1 \\ \text{ó} \\ I_2 \end{cases}$$

del término I_2 solo los términos pares de la serie contribuyen, esto es:

$$\Downarrow$$

$$I_2 = \frac{1}{2} \sum_{k \geq 0} \frac{\Gamma(k+1)\,\Gamma(-k)}{\Gamma(-2k)} \frac{A^{2k}}{\Gamma(2k+1)}$$

$$= \frac{1}{2} \sum_{k \geq 0} \frac{(1)_k}{(1)_{2k}} \frac{\Gamma(-k)}{\Gamma(-2k)} A^{2k}$$

$\underline{\text{Obs.}}$ $$\frac{\Gamma(-k)}{\Gamma(-2k)} = \lim_{\epsilon \to 0} \frac{\Gamma(-\epsilon)}{\Gamma(-2\epsilon)} \frac{(1)_{2k}}{(1)_k} (-1)^k = 2 \frac{(1)_{2k}}{(1)_k} (-1)^k$$

$\therefore$ $$I_2 = \sum_{k \geq 0} (-A^2)^k = \frac{1}{1+A^2} \quad //$$

o también

$$I_1 = \frac{1}{A^2} \sum_{k \geq 0} \left(-\frac{1}{A^2}\right)^k = \frac{1}{A^2} \frac{1}{1+\frac{1}{A^2}} = \frac{1}{1+A^2} \quad //$$

Obviamente $I = \begin{cases} I_1 \\ \text{ó} \\ I_2 \end{cases}$ //