9. **M. Caputo (1967):**

   The second popular definition is

$$
{}_a^C D_t^\alpha f(t) = \frac{1}{\Gamma(\alpha - n)} \int_a^t \frac{f^{(n)}(\tau)d\tau}{(t-\tau)^{\alpha+1-n}}, \quad (n-1 \le \alpha < n)
$$

10. **K. S. Miller, B. Ross (1993):**

    They used differential operator D as

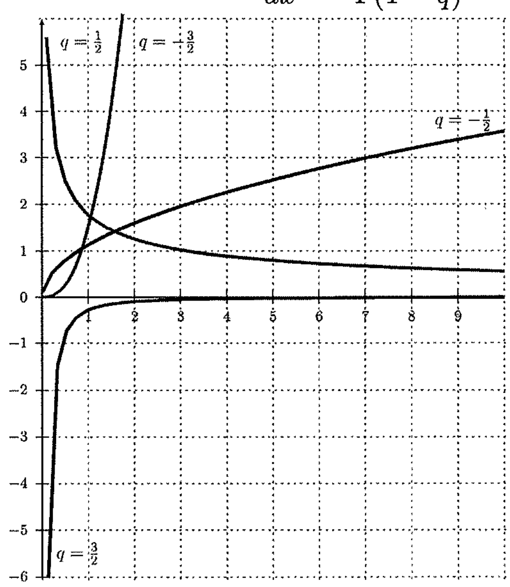
$$
D^{\bar{\alpha}} f(t) = D^{\alpha_1} D^{\alpha_2} \cdots D^{\alpha_n} f(t), \quad \bar{\alpha} = (\alpha_1, \alpha_2, \cdots, \alpha_n)
$$

    which D^{α_i} is Riemann-Liouvill or Caputo definitions.

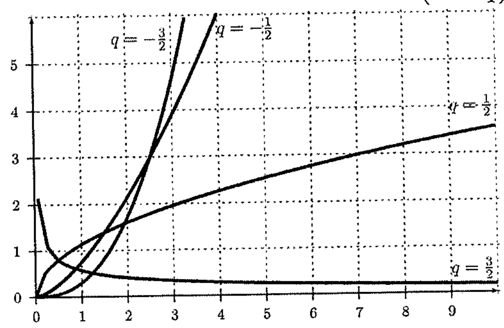
3 Fractional derivative of Some special Functions

In this section we give more explicit formulas of fractional derivative and integral of some special functions and then consider to there graph.

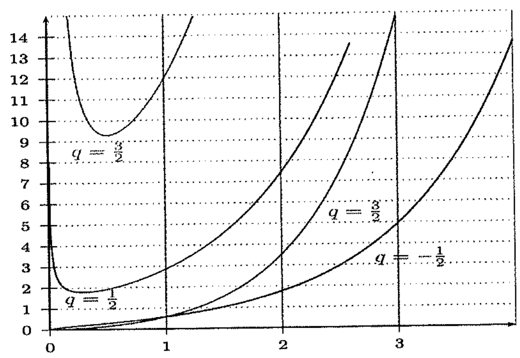
1. **Unit function:** For $f(x) = 1$ we have $\dfrac{d^q 1}{dx^q} = \dfrac{x^{-q}}{\Gamma(1-q)}$ for all q



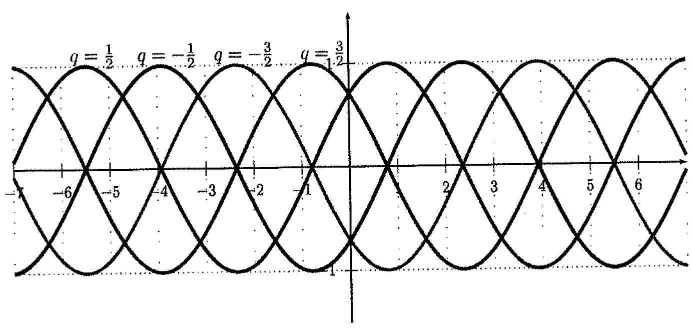
2. **Identity function:** For $f(x) = x$ we have $\dfrac{d^q x}{dx^q} = \dfrac{x^{1-q}}{\Gamma(2-q)}$



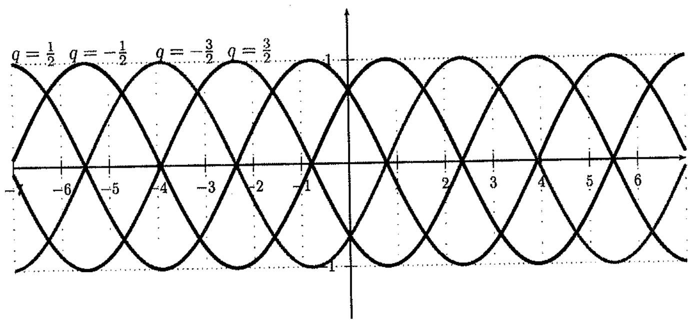
3. **Exponential function:** Fractional differintegral of the function $f(x) = e^x$ is $\dfrac{d^q e^{\pm x}}{dx^q} = \displaystyle\sum_{k=0}^{\infty} \dfrac{x^{k-q}}{\Gamma(k-q+1)}$



4. **Sin function:** If $f(x) = \sin x$ then $\dfrac{d^q \sin(x)}{dx^q} = \sin\left(x + \dfrac{q\pi}{2}\right)$



5. **Cosin function:** If $f(x) = \cos x$ then $\dfrac{d^q \cos(x)}{dx^q} = \cos\left(x + \dfrac{q\pi}{2}\right)$



4 Applications of Fractional Calculus

The basic mathematical ideas of fractional calculus (integral and differential operations of noninteger order)were developed long ago by the mathematicians Leibniz (1695), Liouville (1834), Riemann (1892), and others and brought to the attention of the engineering world by Oliver Heaviside in the 1890s, it was not until 1974 that the first book on the topic was published by Oldham and Spanier. Recent monographs and symposia proceedings have highlighted the application of fractional calculus in physics, continuum mechanics, signal processing, and electromagnetics. Here we state some of applications.

1. **First one**
   It may be important to point out that the first application of fractional calculus was made by Abel(1802-1829) in the solution of an integral equation that arises in the formulation of the *tautochronous problem.* This problem deals with the determination of the shape of a frictionless plane curve through the origin in a vertical plane along which a particle of mass m can fall in a time that is independent of the starting position. If the sliding time is constant T, then the Abel integral equation(1823) is

$$\sqrt{2g}T = \int_0^{\eta} (\eta - y)^{-\frac{1}{2}} f'(y)dy,$$

   where g is the acceleration due to gravity, (ξ, η) is the initial position and $s = f(y)$ is the equation of the sliding curve. It turns out that this equation is equivalent to the fractional integral equation

$$T\sqrt{2g} = \Gamma(\frac{1}{2})_0 D_\eta^{-\frac{1}{2}} f'(\eta)$$

   Indeed, Heaviside gave an interpretation of $\sqrt{p} = D^{\frac{1}{2}}$ so that ${}_0D_t^{\frac{1}{2}} 1 = \frac{1}{\sqrt{\pi t}}$.