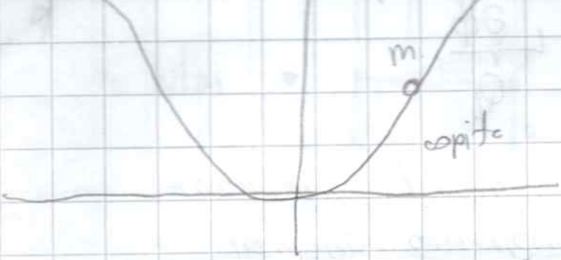
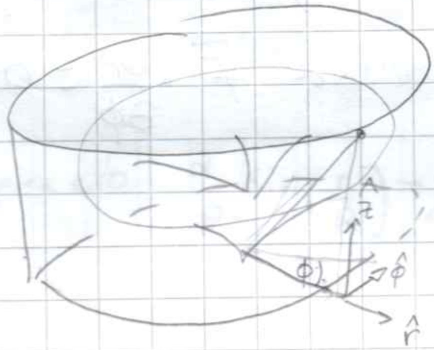


$\downarrow \vec{g}$ crea un potencial $U = mgh$

h siendo la dif. de altura en este potencial



el problema en coord cilíndricas

$\triangleright\ T = \frac{1}{2} m \left(r^2 \dot\phi^2 + \dot r^2 + \dot z^2 \right)$

$\triangleright\ q = \{ r, \phi \}$ $\triangle$ $\varphi = z - \alpha \sin(r/R) = 0$

$\triangleright\ U = mgh = mgz$

▷ el Lagrangiano $L = T - U = \frac{1}{2} m \left(r^2 \dot\phi^2 + \dot r^2 + \dot z^2 \right) - mgz$ (*1)

▷ ecuación para minimizar acción $\delta S = \delta \int L\, dt = 0$

↳ EL $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q_\sigma}\right) - \frac{\partial L}{\partial q_\sigma} = \lambda \frac{\partial \varphi}{\partial q_\sigma}$ / $T(\dot q_\sigma)$ ∧ $U(q_\sigma)$

$\triangleright$ (EL) $\frac{d}{dt}\left(\frac{\partial}{\partial \dot q_\sigma} T\right) + \frac{\partial U}{\partial q_\sigma} = \lambda \frac{\partial \varphi}{\partial q_\sigma}$ /. (*1)

▷ $\underline{\underline{q = r}}$ ||

$\triangleright\ \frac{d}{dt}\left(\frac{\partial}{\partial r}\left(\frac{1}{2} m \dot r^2 + \frac{1}{2} m r^2 \dot\phi^2\right)\right) + \frac{\partial}{\partial r}(mgz) = \lambda \frac{\partial}{\partial r}\left(-\alpha \sin(r/R)\right)$

$\triangleright\ \frac{d}{dt}\left(m\dot r + m r \dot\phi^2 \right) + \cancel{\frac{\partial}{\partial r}(mgz)}_{0} = -\lambda\alpha \frac{\partial \sin(r/R)}{\partial r/R} \frac{1}{R} = -\frac{\lambda\alpha}{R} \cos\left(\frac{r}{R}\right)$

// $m = cte$ $\Rightarrow$ $m\ddot r + m \dot r \dot\phi^2 + m r \frac{d\dot\phi^2}{d\dot\phi}\frac{d\dot\phi}{dt} = -\frac{\lambda\alpha}{R}\cos(r/R)$

$\triangleright\ \underbrace{m\left(\ddot r + \dot r \dot\phi^2 + r\, 2\dot\phi \ddot\phi \right)}_{} = -\frac{\lambda\alpha}{R}\cos(r/R)$ (*2)

ecuación de mov de r $\boxed{m\left(\ddot r + \dot r \dot\phi^2 + 2 r \dot\phi \ddot\phi\right) = -\left(m(\ddot z + g)\right)\frac{\alpha}{R}\cos(r/R)}$

con (*4)