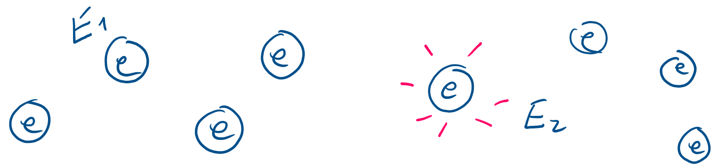
0630_mecest_4

Saturday, July 1, 2023 12:08 AM

Problem 6.9.
A statistical system is composed of N independent distinguishable particles. Each one of these particles has only two energy levels, E_1 and E_2, such that $E_2 - E_1 = \varepsilon > 0$. Choose a suitable ground state for the energy and write down the total energy as a function of the temperature T. Finally, discuss the limits $T \to 0$ and $T \to +\infty$.

$$E_2 - E_1 = 0$$



$$\Omega = \binom{N}{n}$$

set el ground state $E_1 = 0$; como consecuencia $E_2 = \varepsilon$

y tenemos el set $\{n_i\}$ $i = 1, \ldots, N$ ($n_j = 0$ la partícula j en E_1

$n_j = 1$ part. j en E_2)

se cumplirá $E = \sum_{j=1}^{N} n_j \varepsilon = m\varepsilon$

$$\boxed{dE = TdS + \mu dN}$$

y se tiene $TdS = (dE)_N \rightarrow \left(\frac{\partial S}{\partial E}\right)_N = \frac{1}{T}$

$S = k \ln \Omega$; $\Omega = \frac{N!}{m!\,(N-m)!}$ cuenta m part. en E_2 distinguibles.

$$S = k \ln(\Omega) = k\Big[N \ln N - N - (m \ln m - m) - \big[(N-m)\ln(N-m) - N + m\big]\Big]$$

$$= k\left[N\ln N - \cancel{N} + \cancel{m} - m\ln m - (N-m)\ln(N-m) + \cancel{N} - \cancel{m} \right]$$

$$\boxed{S = k\left\{ N\ln N - m\ln m - (N-m)\ln(N-m)\right\}}$$

$$\frac{\partial}{\partial m}(N\log(N) - m\log(m) - (N-m)\log(N-m)) = \log(N-m) - \log(m)$$

$E = m\varepsilon$

$E = E(m)$

$$\left(\frac{\partial S}{\partial E}\right)_N = \frac{1}{T} = \left(\frac{\partial S}{\partial m}\right)_N \left(\frac{\partial m}{\partial E}\right)_N = \left(\frac{\partial S}{\partial m}\right)_N \frac{1}{\left(\frac{\partial E}{\partial m}\right)_N} = \left(\frac{\partial S}{\partial m}\right)_N \frac{1}{\varepsilon}$$

$$\rightarrow \frac{1}{T} = \frac{1}{\varepsilon}\left(\frac{\partial S}{\partial m}\right)_N = \frac{k}{\varepsilon}\left\{ -\left(\ln m - \frac{m}{m}\right) + \frac{\partial}{\partial m}\left(m\ln(N-m)\right)_N \right\}$$

$$= \frac{k}{\varepsilon}\left\{ 1 - \ln m + \ln(N-m) + m\frac{1}{N-m}\cdot\frac{\partial(N-m)}{\partial m}\right.$$

$$1 - \ln m + \ln(N-m) - \frac{m}{N-m}$$

$$S = k\left\{ N\ln N - m\ln m - (N-m)\ln(N-m)\right\}$$

$$\frac{1}{k}\left(\frac{\partial S}{\partial m}\right)_N = \frac{\partial}{\partial m}\left(N\ln N - m\ln m - (N-m)\ln(N-m)\right)$$

$$= -\left(\ln m + \frac{m}{m}\right) - \frac{\partial}{\partial m}(N-m)\left(\ln(N-m)\right)$$

$$= -1 - \ln m \qquad - \left\{ -1 \ln(N-m) + \frac{N-m}{N-m}(-1) \right\}$$

$$= -1 - \ln m + \ln(N-m) + 1$$

$$= \ln(N-m) - \ln m$$

$$\left(\frac{\partial S}{\partial m}\right)_N = K \ln\left(\frac{N-m}{m}\right)$$

$$\rightarrow \frac{1}{T} = \frac{1}{\varepsilon}\left(\frac{\partial S}{\partial m}\right)_N = \boxed{\frac{K}{\varepsilon}\ln\left(\frac{N-m}{m}\right) = \frac{1}{T}}$$

$$\lim_{T\to\infty} \frac{1}{T} = 0 = \frac{k}{\varepsilon}\ln\left[\frac{N-m}{m}\right) \rightarrow \frac{N-m}{m} = 1 \qquad / \cdot m$$

$$N - m = m$$

$$N = 2m \rightarrow \boxed{m = \frac{N}{2}}$$

$$\lim_{T\to 0} \frac{1}{T} = \infty = \frac{k}{\varepsilon}\ln\left[\frac{N-m}{m}\right] \qquad / e^{()}$$

$$\infty = \frac{N-m}{m} \qquad \boxed{m = 0} \qquad \boxed{T = 0}$$