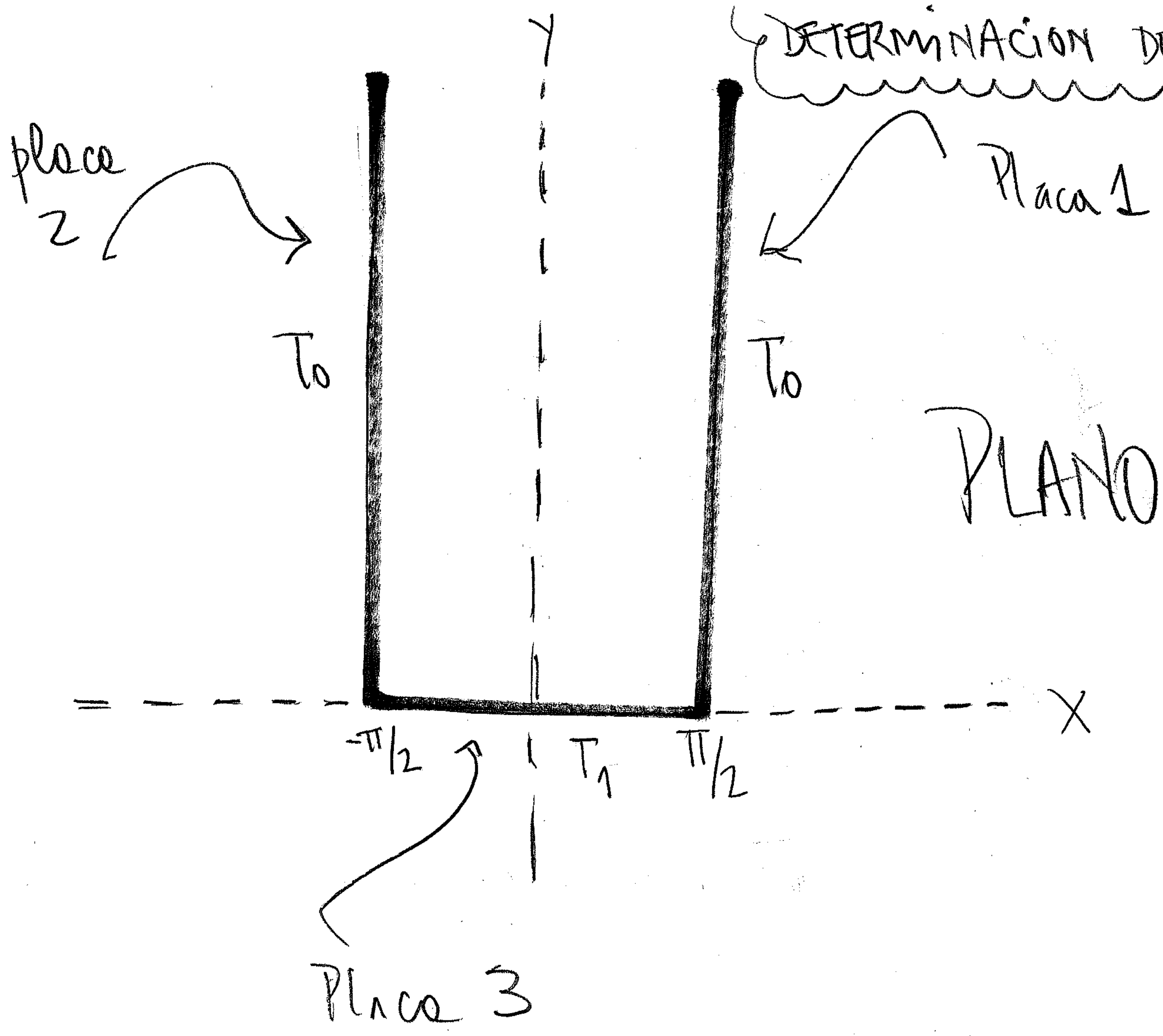
{DETERMINACION DE $T(x,y)$}

1



PLANO z

TRANSF. I

$$f(z) = \operatorname{sen} z$$

a) Determinación de $u(x,y)$, $v(x,y)$

$$\Downarrow$$

$$\operatorname{sen} z = u(x,y) + i\,v(x,y)$$

Entonces.

$$\operatorname{sen} z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\operatorname{sen} z = \frac{e^{i(x+iy)} - e^{-i(x+iy)}}{2i}$$

$$= \frac{e^{ix}e^{-y} - e^{-ix}e^{y}}{2i}$$

$$= \frac{(\cos x + i \operatorname{sen} x)e^{-y} - (\cos x - i \operatorname{sen} x)e^{y}}{2i}$$

$$= \frac{1}{2i}\left[\cos x\left(e^{-y} - e^{y}\right) + i \operatorname{sen} x\left(e^{-y} + e^{y}\right)\right]$$

Pero $\quad \cosh(y) = \dfrac{e^{y} + e^{-y}}{2}$

$\qquad \operatorname{senh}(y) = \dfrac{e^{y} - e^{-y}}{2}$

Entonces:

$$\operatorname{sen} z = \frac{1}{\cancel{2}i}\left[-\cancel{2}\cos x \operatorname{senh} y + \cancel{2}\operatorname{sen} x \cosh y\right]$$

$\therefore$

$$\operatorname{sen} z = \operatorname{sen} x \cosh y + i \cos x \operatorname{senh} y$$

$$\Downarrow$$

Esto es $\left.\begin{aligned} u &= \operatorname{sen} x \cosh y \\ v &= \cos x \operatorname{senh} y \end{aligned}\right\} \to$ Regla de transf.

para x, y arbitrarios

b) Transf. de los lugares geométricos que posicionen las placas:

<u>PLACA 1</u>

$$\begin{matrix} x = \pi/2 \\ y \geq 0 \end{matrix} \implies \begin{aligned} u &= \overset{1}{\cancel{\operatorname{sen} \pi/2}} \cosh y = \cosh y \\ v &= \cos \pi/2 \operatorname{senh} y = 0 \,// . \end{aligned}$$

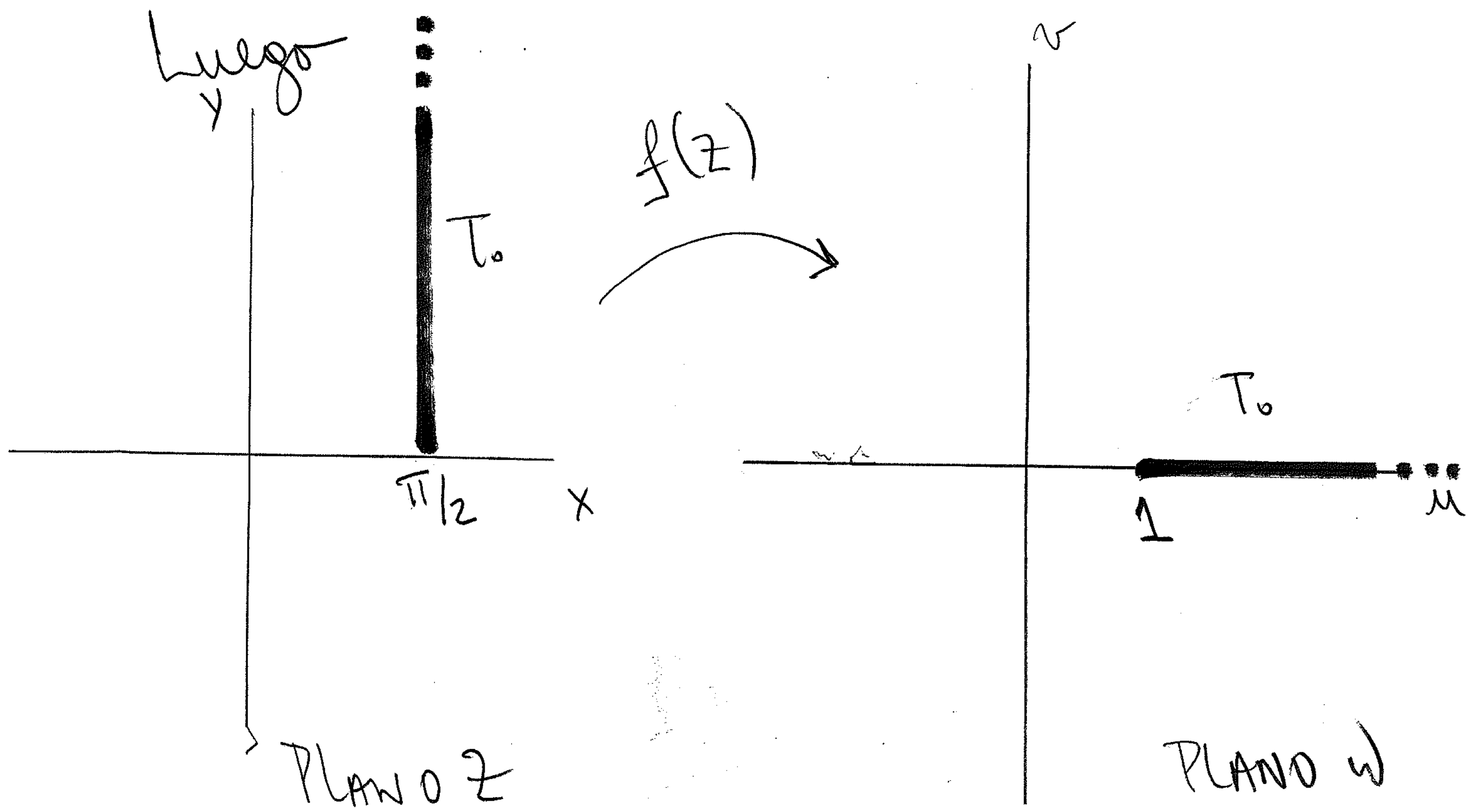
$$\Downarrow$$

Placa coincide con el eje u (en parte al menos) para $y=0 \Rightarrow u=1, \ v=0$

$y \to \infty \Rightarrow u \to \infty, \ v=0.$

Obs. bastan estos puntos para fijar los límites en u. No es necesaria más info para el resto de los ptos., están todos sobre el eje u.

Luego



PLACA 2

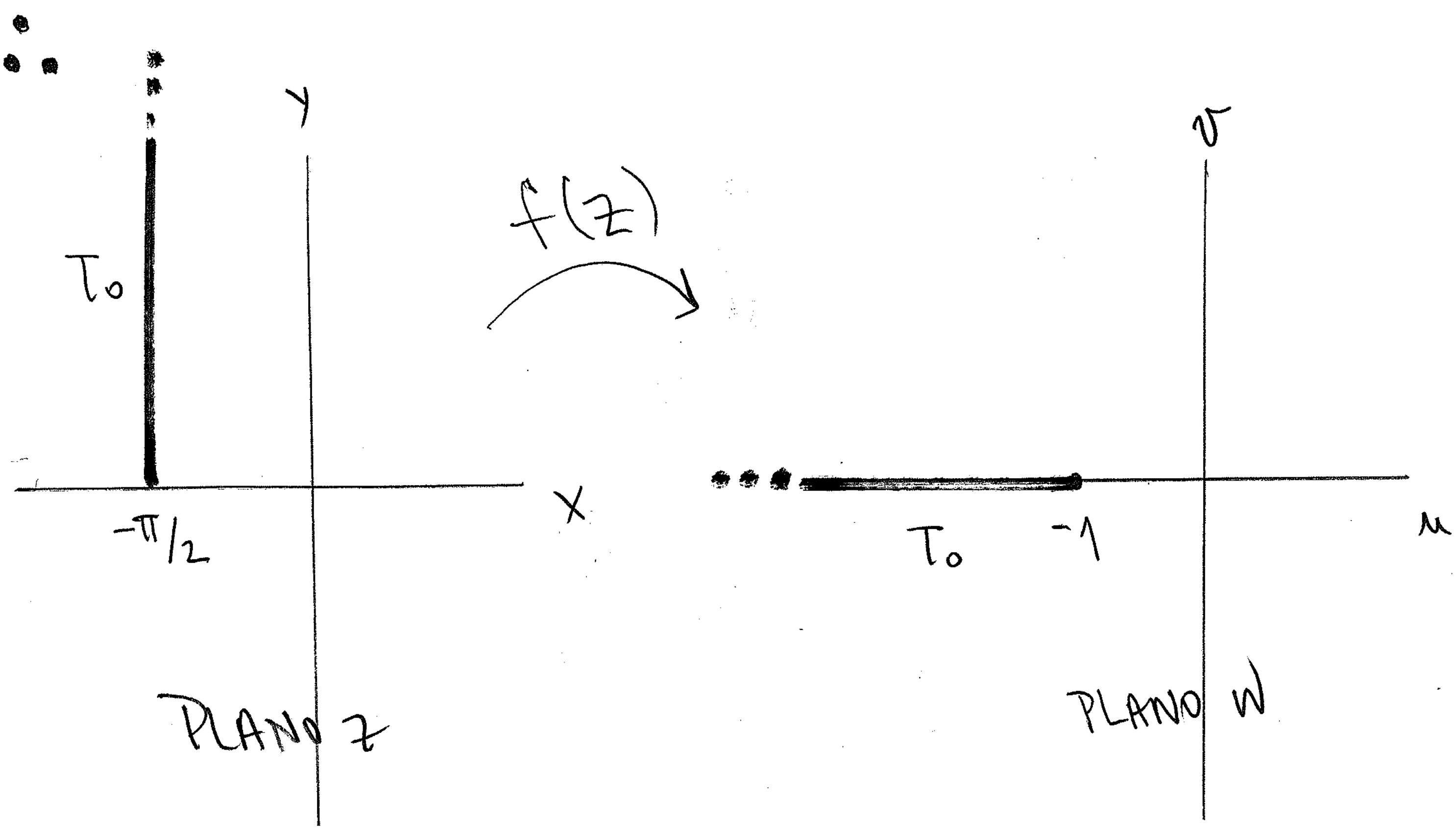
$$X = -\pi/2 \qquad Y \geq 0 \implies u = \underbrace{\cancel{\operatorname{sen}(-\pi/2)}}_{-1} \cosh Y = -\cosh Y$$

$$v = \underbrace{\cancel{\cos(-\pi/2)}}_{0} \operatorname{senh} Y$$

Apane $y = 0 \implies u = -1,\ v = 0$

$y \to \infty \implies u \to -\infty,\ v = 0.$

(todo y lleve a $v = 0$).



PLACA 3

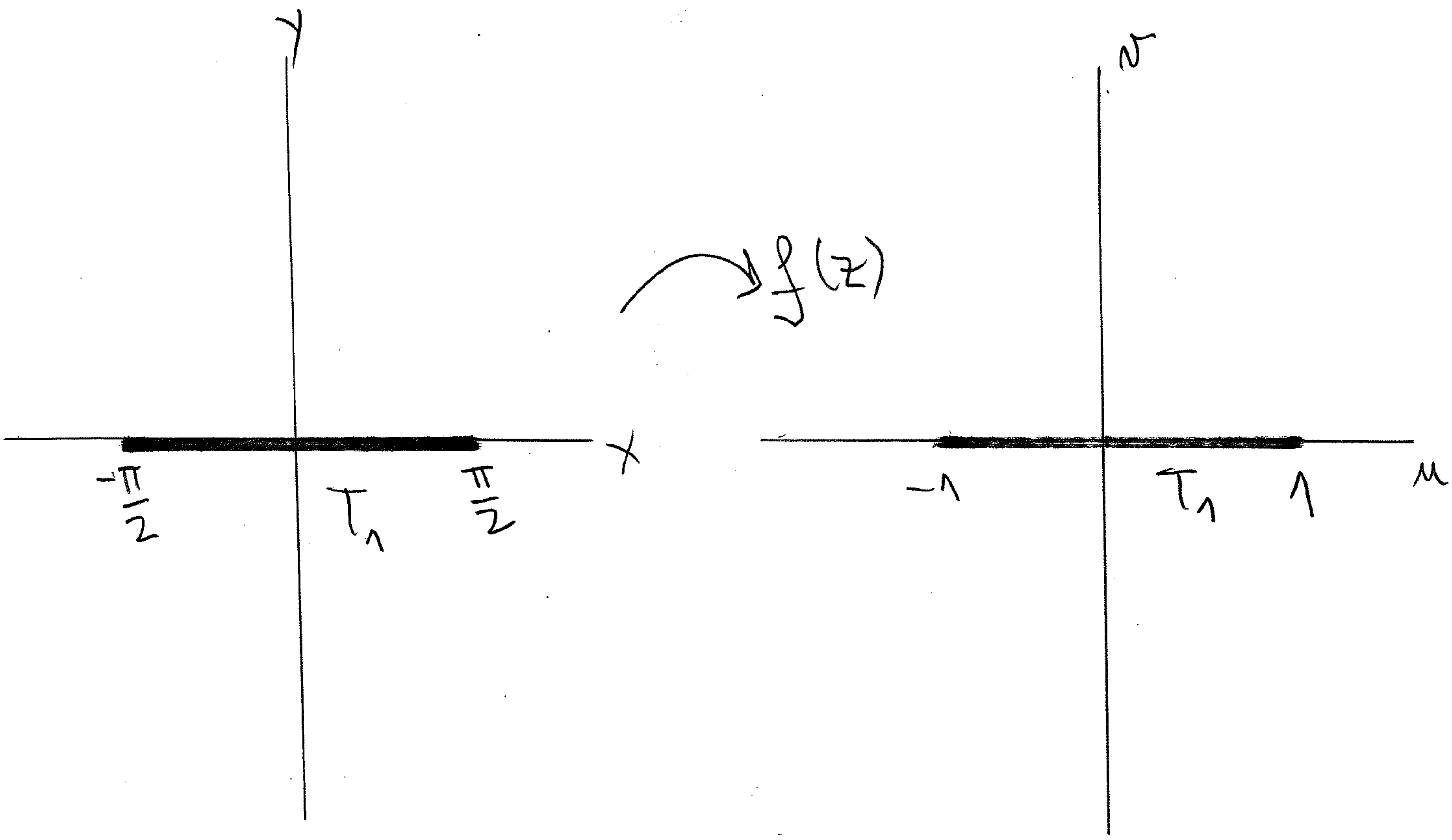
$$-\frac{\pi}{2} \le x \le \frac{\pi}{2} \qquad \Longrightarrow \qquad u = \operatorname{sen} x \underbrace{\cosh(0)}_{1} = \operatorname{sen} x .$$

$$y = 0 \qquad\qquad v = \cos x \underbrace{\operatorname{senh}(0)}_{0} = 0$$

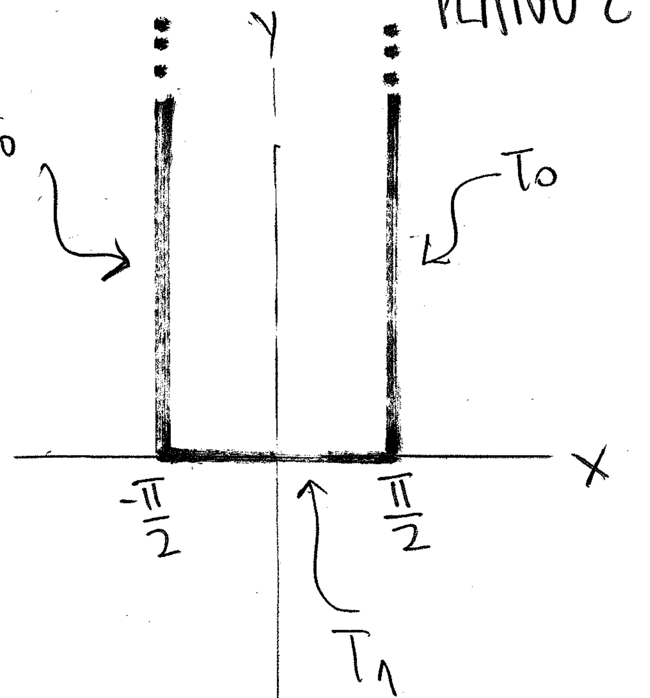
luego si $x \in [-\pi/2, \pi/2]$

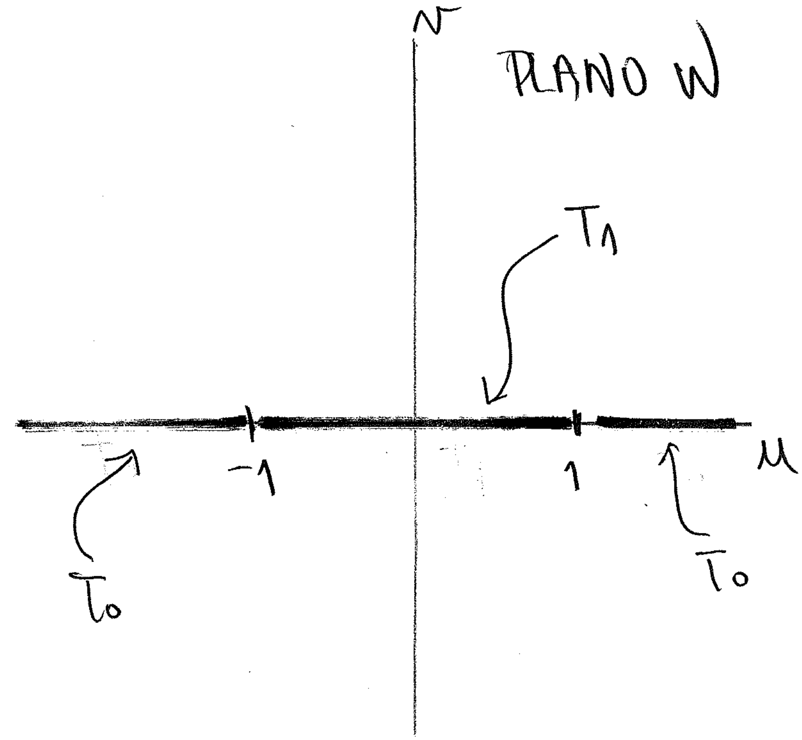
$$\Downarrow$$

$$u \in [-1, 1]$$



Finalmente





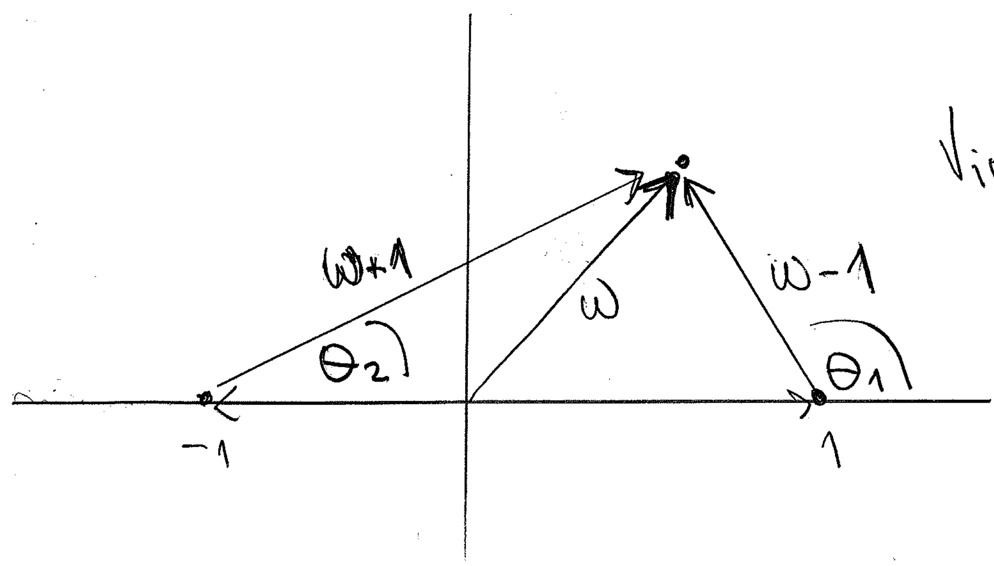
TRANSF. II

$$F(w) = \ln\left(\frac{w-1}{w+1}\right)$$

a) Determinación de u', v'.

w = punto arbitrario en plano w.

Obs. Usar por conveniencia coord. polares



Viendo como vectores 2D estos son los nºs

- $w+1$
- $w-1$

M

$$(w+1) = |w+1|\, e^{i\theta_1} = r_1 e^{i\theta_1} \quad ; r_1, r_2 \geq 0$$

$$(w-1) = |w-1|\, e^{i\theta_2} = r_2 e^{i\theta_2}$$

luego

$$F(w) = \ln\left(\frac{r_1 e^{i\theta_1}}{r_2 e^{i\theta_2}}\right)$$

$$= \ln\left(r_1/r_2\right) + i(\theta_1 - \theta_2)$$

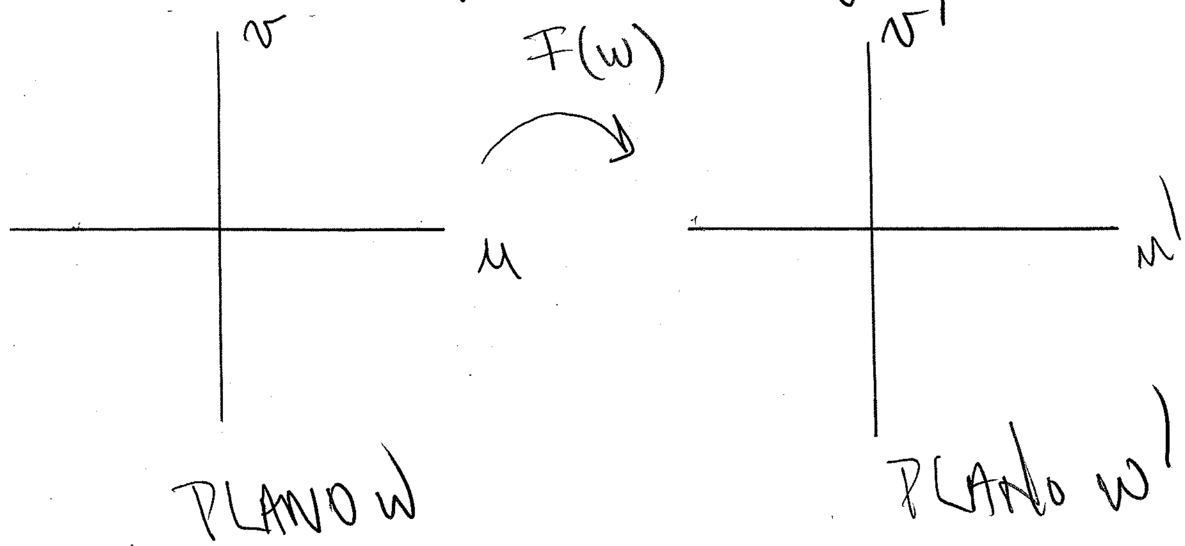
Rápidamente

$$F(w) = u' + i v'$$

$$\Rightarrow \quad u' = \ln\left(r_1/r_2\right)$$

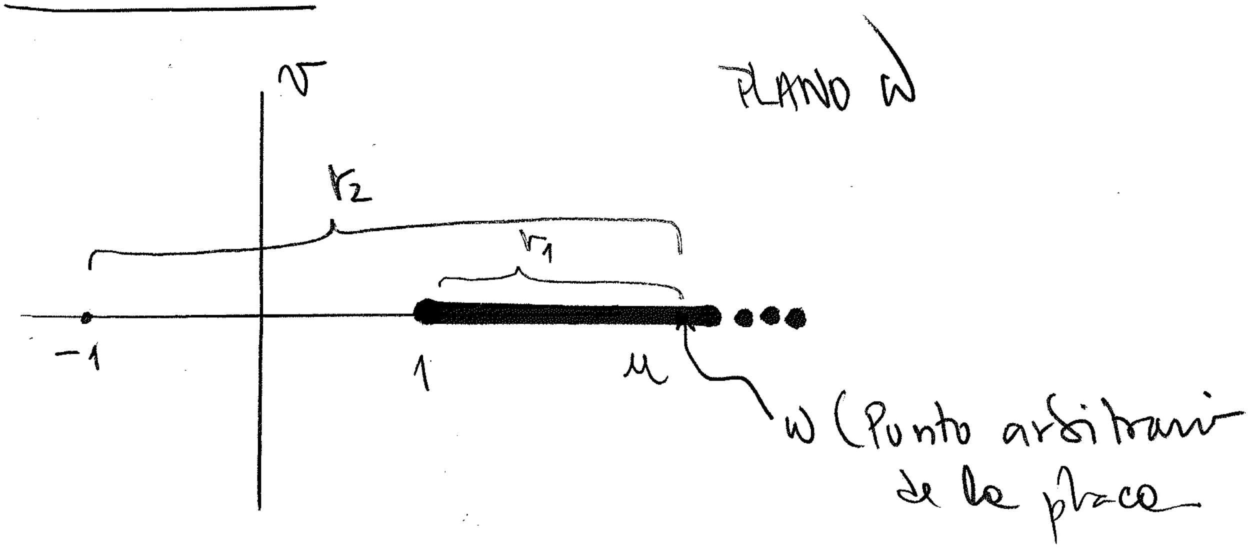
$$v' = \theta_1 - \theta_2$$

Obs. $F(w)$ define la siguiente transf.



b) Transf. de los lugares geométricos.

PLACA 1



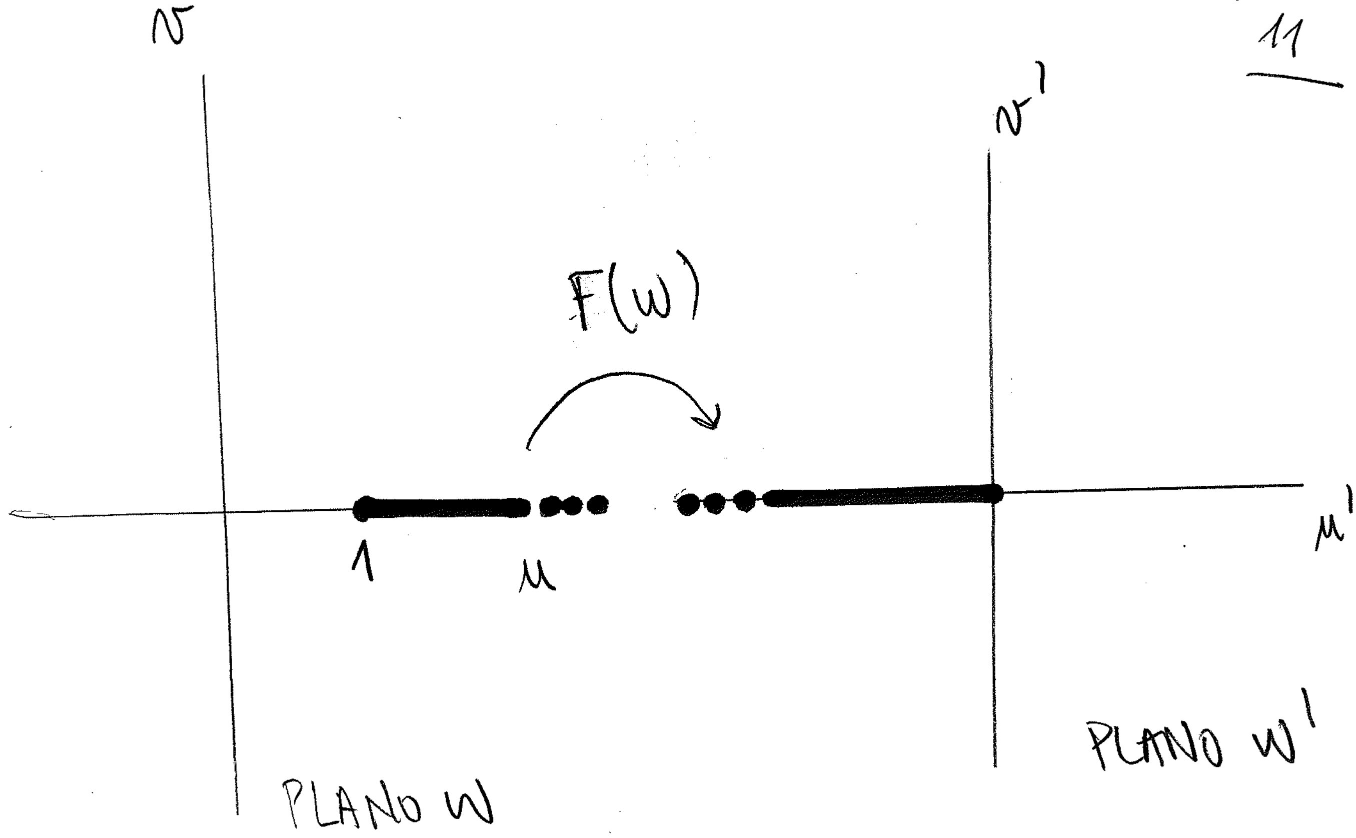
$r_1 \in [0,\infty]$; $r_2 = r_1+2$.

$\theta_1 = 0$

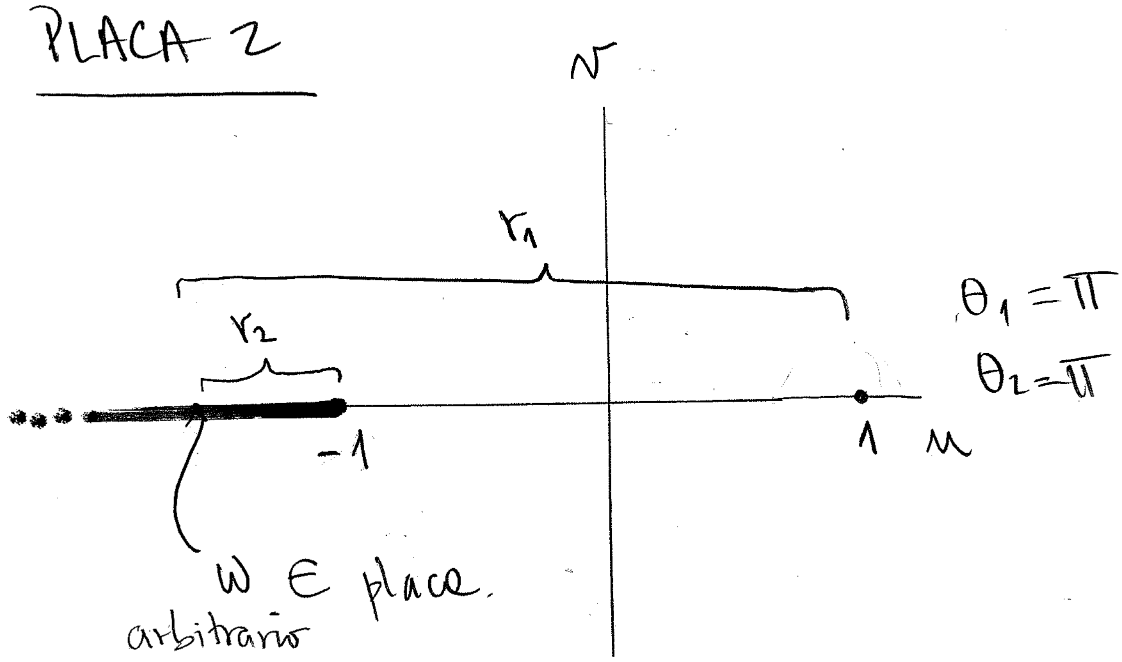
$\theta_2 = 0$

$\therefore \quad u' = \ln(r_1/r_2) = \ln\left(\frac{r_1}{r_1+2}\right) \in \left]-\infty, 0\right]$.

$v' = \theta_1 - \theta_2 = 0$ $\rightsquigarrow$ Transf. lleva a placa a coincidir con eje u'



PLACA 2



del dibujo $r_1 = r_2 + 1$ con $r_2 \in [0, \infty[$

luego $u' = \ln(r_1/r_2) = \ln\left(\frac{r_2+2}{r_2}\right)$

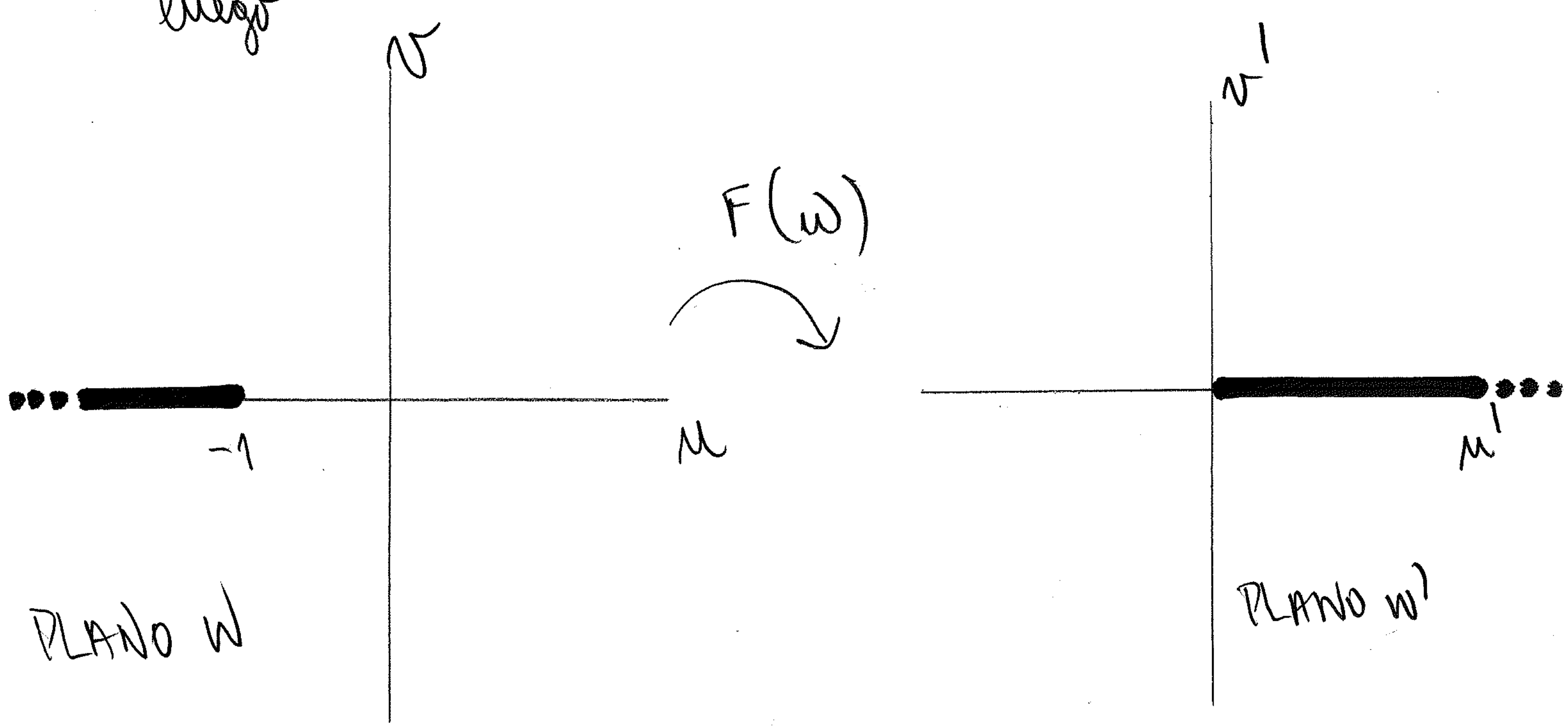
$v' = \pi - \pi = 0$

de lo anterior si $r_2 = 0 \Rightarrow u' = \infty$

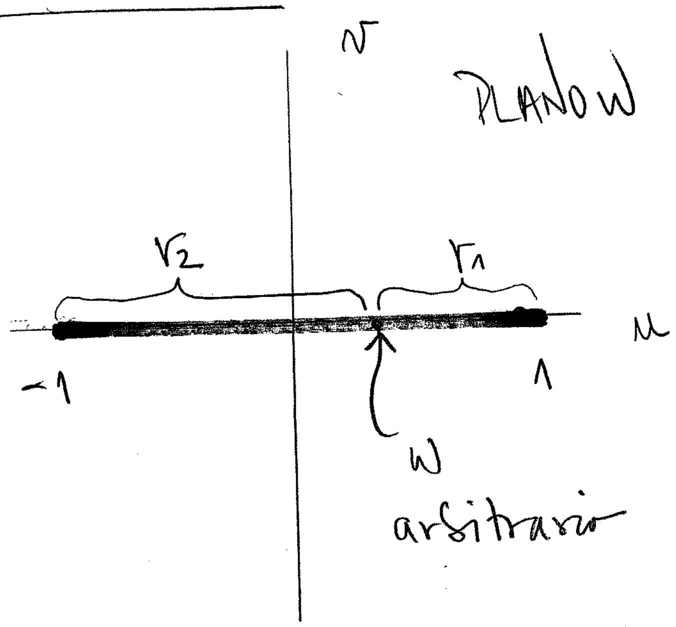
$r_2 = \infty \Rightarrow u' = 0$

$\Downarrow$

luego



PLACA 3



$$\theta_1 = \pi$$
$$\theta_2 = 0$$

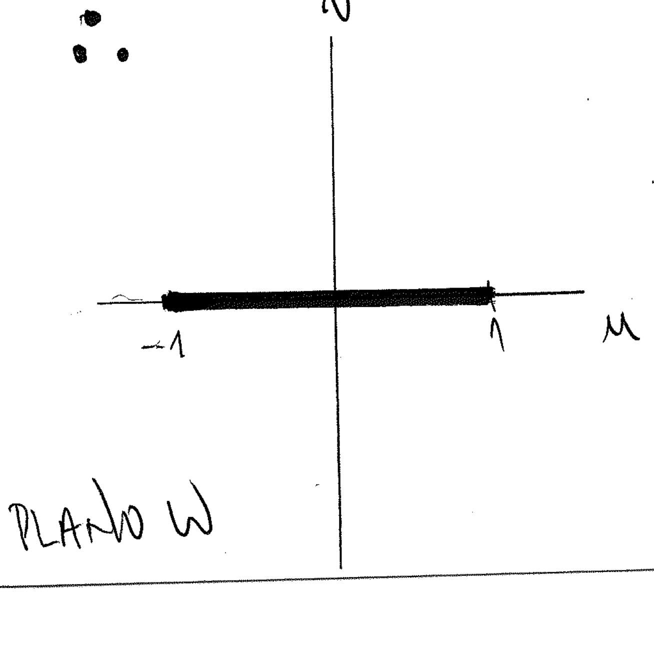
$$\therefore \; v' = \pi$$
$$u' = \ln(r_1/r_2)$$

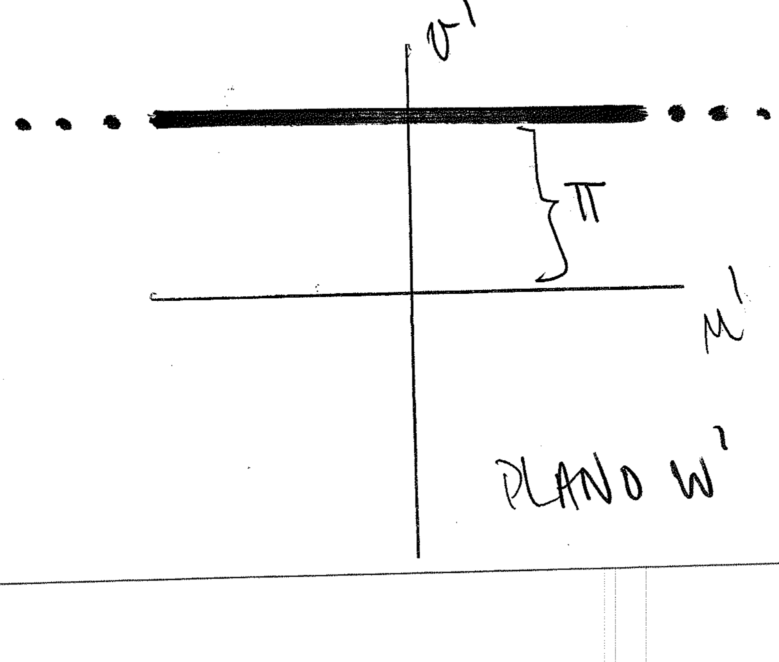
$$r_1 \in [0, 2]$$
$$r_2 \in [0, 2]$$

cuando $\left.\begin{matrix} r_1 = 0 \\ r_2 = 2 \end{matrix}\right\} u' \to -\infty$

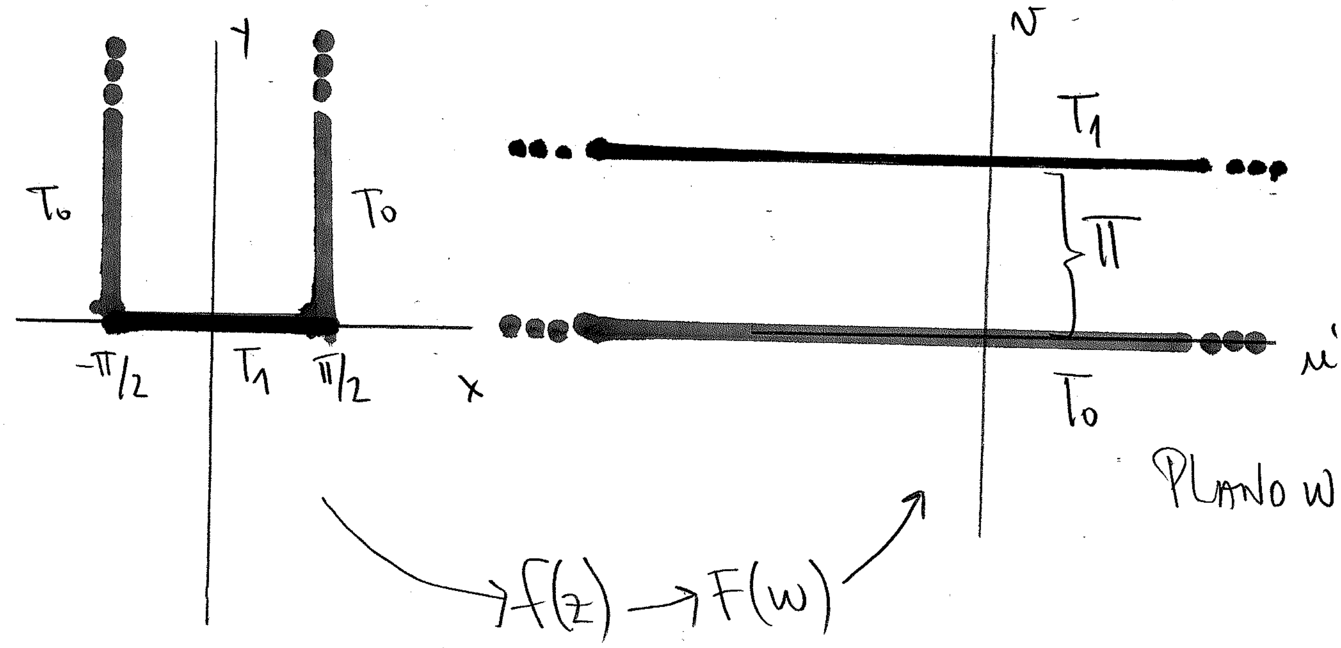
cuando $\left.\begin{matrix} r_1 = 2 \\ r_2 = 0 \end{matrix}\right\} u' \to \infty$

$\therefore$





∴ la transf. total hace lo siguiente



Luego en el plano w' la expresión para la temperatura está dado por:

$$T(v') = A v' + B = A(\theta_1 - \theta_2) + B$$

con $T(0) = T_0$
$T(\pi) = T_1$ $\Rightarrow$ $T_0 = B$ $\quad B = T_0$
$T_1 = A\pi + T_0$ $\quad A = \frac{T_1 - T_0}{\pi}$

$$\therefore \; T = \frac{(T_1 - T_0)}{\pi}(\theta_1 - \theta_2) + T_0$$

luego se tiene que la parte imaginaria de

$$F(w) = \log\left(\frac{w-1}{w+1}\right)$$ $Im[F(w)] = \theta_1 - \theta_2$.

<u>conectando con u y v.</u>

$$F(w) = \log\left(\frac{w-1}{w+1}\right) = \log\left(\frac{u+iv-1}{u+iv+1}\right)$$

$$= \log\left(\frac{(u-1)+iv}{(u+1)+iv}\right) = \log\left(\frac{(u-1)+iv}{(u+1)+iv}\cdot\frac{(u+1)-iv}{(u+1)-iv}\right)$$

$$= \log\left[\frac{(u-1)(u+1) - iv(u-1) + (u+1)iv + v^2}{(u+1)^2+v^2}\right]$$

$$= \log\left[\frac{u^2 - 1 - \cancel{ivu} + iv + \cancel{ivu} + iv + v^2}{(u+1)^2+v^2}\right]$$

$$= \log\left[\frac{u^2 - 1 + v^2 + 2iv}{(u+1)^2+v^2}\right]$$

en coord. polares la parte imag.
de log(·) es el argumento de log.

$\Downarrow$

$$\frac{u^2-1+v^2+2iv}{\underbrace{(u+1)^2+v^2}_{\in \mathbb{R}}} = r\, e^{i \arg(u^2-1+v^2+2iuv)}$$

$\Rightarrow$ argumento nulo.

donde

$$\arg(u^2-1+v^2+2iv) = tg^{-1}\left(\frac{2v}{u^2+v^2-1}\right)$$

luego se tenía que

$v = \cos x \operatorname{senh} y$

$u = \operatorname{sen} x \cosh y$

$$\therefore \operatorname{Im}[\log(w)] = \arg(u^2+v^2-1+2iv) = \theta_1 - \theta_2$$

$$= tg^{-1}\left(\frac{2v}{u^2+v^2-1}\right) = \theta_1 - \theta_2 \;//$$

$$\therefore \theta_1 - \theta_2 = \frac{2\cos x \operatorname{senh} y}{\operatorname{sen}^2 x \cosh^2 y + \cos^2 x \operatorname{senh}^2 y - 1}$$

Prop. $\cos^2 x + \operatorname{sen}^2 x = 1$

$\cosh^2 y - \operatorname{senh}^2 y = 1$

$$\therefore 1 = 1 \cdot 1 = (\cos^2 x + \operatorname{sen}^2 x)(\cosh^2 y - \operatorname{senh}^2 y)$$

$$= \cos^2 x \cosh^2 y - \cos^2 x \operatorname{senh}^2 y + \operatorname{sen}^2 x \cosh^2 y - \operatorname{sen}^2 x \operatorname{senh}^2 y$$

Veamos el denominador:

$$: \operatorname{sen}^2 x \cosh^2 y + \cos^2 x \operatorname{senh}^2 y - 1$$

$$: \operatorname{sen}^2 \cancel{x \cosh^2 y} + \cos^2 x \operatorname{senh}^2 y - (\cos^2 x \cosh^2 y - \cos^2 x \operatorname{senh}^2 y$$

$$+ \cancel{\operatorname{sen}^2 x \cosh^2 y} - \operatorname{sen}^2 x \operatorname{senh}^2 y)$$

$$: (\cos^2 x + \operatorname{sen}^2 x) \operatorname{senh}^2 y - \cos^2 x (\cosh^2 y - \operatorname{senh}^2 y)$$

$$: \operatorname{senh}^2 y - \cos^2 x .$$

∴ y finalmente.

$$T = T(x,y) = \frac{T_1 - T_0}{\pi} (\theta_1 - \theta_2) + T_0$$

$$T(x,y) = \frac{T_1 - T_0}{\pi} \operatorname{tg}^{-1} \left[\frac{2 \cos x \operatorname{senh} y}{\operatorname{senh}^2 y - \cos^2 x} \right] + T_0$$