1) $\langle \alpha \rangle_B = \int_B^{\infty} x^{\alpha - 1}\, dx$ $\quad / \; x = \frac{1}{u} + B$

$\therefore \; u = \frac{1}{x - B}$

$$\langle \alpha \rangle_B = \int_0^{\infty} \left(\frac{1}{u} + B\right)^{\alpha-1} \frac{du}{u^2}$$

$x = B \to u = \infty$

$x = \infty \to u = 0$

$dx = -\frac{du}{u^2}$

$\therefore \int_B^{\infty} \dots \to -\int_{\infty}^{0} \dots \to \int_0^{\infty} \dots$

$$\left(\frac{1}{u}\right)^2 \left(\frac{1}{u} + B\right)^{-\gamma} = \left(\frac{1}{u}\right)^2 \sum_{k_1} \sum_{k_2} \phi_{k_1} \phi_{k_2} \frac{\left(\frac{1}{u}\right)^{k_1} (B)^{k_2} \langle \gamma + k_1 + k_2 \rangle}{\Gamma(\gamma)}$$

$/ -\gamma = \alpha - 1 \quad \therefore \quad \gamma = 1 - \alpha$

$$\therefore \left(\frac{1}{u}\right)^2 \left(\frac{1}{u} + B\right)^{\alpha - 1} = \sum_{k_1} \sum_{k_2} \phi_{k_1} \phi_{k_2} \frac{\left(\frac{1}{u}\right)^{k_1+2} (B)^{k_2} \langle 1 - \alpha + k_1 + k_2 \rangle}{\Gamma(1 - \alpha)}$$

$\hookrightarrow \; = \eta \left(\frac{1}{u}\right)^{k_1 + 2}$

$$\langle \alpha \rangle_B = \eta \int_0^{\infty} \left(\frac{1}{u}\right)^{k_1+2} du$$

via $\frac{1}{u} = x$ $\quad u = 0 \to x = \infty$, $u = \infty \to x = 0$ $\quad du = -\frac{dx}{x^2}$

$$\int_0^{\infty} \left(\frac{1}{u}\right)^{k_1+2} du = \int_0^{\infty} x^{k_1 + \cancel{2}} \frac{dx}{\cancel{x^2}} = \int_0^{\infty} x^{k_1}\, dx$$

comparab a $\int_0^{\infty} x^{\alpha-1}\, dx \;\square\; \int_0^{\infty} x^{k_1}\, dx$

$\square$ comparar forma

$$\langle \alpha \rangle \cdot \overbrace{\langle k_1 + 1 \rangle}$$

hoja (2

$$\therefore \langle \alpha \rangle_B = \eta \int_0^\infty x^{k_1} dx = \eta \langle k_1 + 1 \rangle$$

expandiendo η

$$\langle \alpha \rangle_B = \sum_{k_1} \sum_{k_2} \phi_{k_1} \phi_{k_2} \langle k_1 + 1 \rangle \frac{(B)^{k_2} \langle 1 - \alpha + k_1 + k_2 \rangle}{\Gamma(1-\alpha)}$$

Serie de Bracket :)

2) teniendo casibracket inferior

$$\langle \alpha \rangle_B = \int_B^{\infty} x^{\alpha-1} dx$$

queremos casibracket superior

$$\langle \alpha \rangle^B = \int_0^B x^{\alpha-1} dx$$

sabemos

$$\int_0^B f(x) dx + \int_B^{\infty} f(x) dx = \int_0^{\infty} f(x) dx$$

reordenando

$$-\int_B^{\infty} f(x) dx + \int_0^{\infty} f(x) dx = \int_0^B f(x) dx \qquad / \text{con } f(x) = x^{\alpha-1}$$

$$\int_0^{\infty} x^{\alpha-1} dx - \int_B^{\infty} x^{\alpha-1} dx = \int_0^B x^{\alpha-1} dx$$

$$\langle \alpha \rangle - \langle \alpha \rangle_B = \langle \alpha \rangle^B \quad //$$

3) $f(x) = \sum_n \phi_n F(n) X^{\alpha n+\beta-1}$

$$J = \int_0^B f(x)\,dx = \sum_n \phi_n F(n) \int_0^B x^{\alpha n+\beta-1} dx = \sum_n \phi_n F(n) \langle \alpha n+\beta \rangle^B$$

$\langle \alpha n+\beta \rangle^B = \frac{1}{|\alpha|} \langle n+\frac{\beta}{\alpha} \rangle^B$ asumiendo cuasi bracket posee propd. bracket

$$J = \sum_n \phi_n F(n) \frac{1}{|\alpha|} \langle n+\frac{\beta}{\alpha} \rangle^B \quad / \langle \gamma \rangle^B = \langle \gamma \rangle - \langle \gamma \rangle_B \quad ②$$

$$J = \frac{1}{|\alpha|} \Big\{ \underbrace{\sum_n \phi_n F(n) \langle n+\frac{\beta}{\alpha} \rangle}_{I} - \sum_n \phi_n F(n) \langle n+\frac{\beta}{\alpha} \rangle_B \Big\}$$

$/ \bar{\alpha} = n + \beta/\alpha \quad \therefore \langle n+\frac{\beta}{\alpha} \rangle_B = \langle \bar{\alpha} \rangle_B$

$$J = I - \sum_n \frac{\phi_n F(n)}{|\alpha|} \sum_{k_1} \sum_{k_2} \phi_{k_1} \phi_{k_2} \frac{\langle k_1+1 \rangle \langle 1-\bar{\alpha}+k_1+k_2 \rangle}{\Gamma(1-\bar{\alpha})} B^{k_2}$$

hacer brackets 0 : $k_1 + 1 = 0 \quad \Rightarrow \quad \boxed{k_1 = -1}$

$k_1 + k_2 + 1 - \bar{\alpha} = 0 \qquad k_1 + k_2 = \bar{\alpha} - 1$

$\hookrightarrow \boxed{k_2 = \bar{\alpha}}$

$$J = I - \sum_n \frac{\phi_n F(n)}{|\alpha|} \sum_{k_2} \phi_{k_2} \frac{B^{k_2}}{\Gamma(1-\bar{\alpha})} \underbrace{\Big[\sum_{k_1} \phi_{k_1} \langle k_1+1 \rangle \langle 1-\bar{\alpha}+k_1+k_2 \rangle \Big]}$$

$$\Gamma(-n) \langle 1-\bar{\alpha}+n+k_2 \rangle \Big|_{n=-1}$$

$$\cancel{\Gamma(1)} \langle \cancel{0} - \bar{\alpha} + k_2 \rangle$$

$$J = I - \sum_n \frac{\phi_n F(n)}{\Gamma(1-\bar{\alpha})|\alpha|} \sum_{k_2} \phi_{k_2} B^{k_2} \langle k_2 - \bar{\alpha} \rangle$$

$$J = I - \sum_n \frac{\phi_n F(n)}{|\alpha|} \left[\sum_{k_2} \phi_{k_2} B^{k_2} \langle k_2 - \bar{\alpha} \rangle \right] \frac{1}{\Gamma(1-\bar{\alpha})}$$

$$\sum_{k_2} \phi_{k_2} B^{k_2} \langle k_2 - \frac{\alpha}{\beta} \rangle$$

$$/ \; I = \frac{1}{|\alpha|} \sum_n \phi_n F(n) \langle n + \frac{\beta}{\alpha} \rangle$$

$$I = \frac{1}{|\alpha|} F(\eta) \Gamma(-\eta) \Big|_{\eta = -\frac{\beta}{\alpha}}$$

$$I = \frac{1}{|\alpha|} F\left(-\frac{\beta}{\alpha}\right) \Gamma\left(\frac{\beta}{\alpha}\right)$$

$$J = I - \frac{\cancel{B}}{|\alpha|} \sum_n \phi_n F(n) \, B^{\alpha n+\beta} \frac{\Gamma(\alpha n+\beta)}{\Gamma(1-\alpha n+\beta)}$$