PROBLEMA GUÍA I | #11

$$\nabla^2\left(\frac{1}{r}\right) = \nabla \cdot \nabla\left(\frac{1}{r}\right)$$

$$= \partial_i \partial_i\left(\frac{1}{r}\right) = \partial_i\left(\partial_i r^{-1}\right)$$

$$= \partial_i\left[(-1) r^{-2}\left(\partial_i r\right)\right]$$

$$= -\partial_i\left(\frac{1}{r^2}\left(\partial_i r\right)\right)$$

$$= -\partial_i\left(\frac{1}{r^2}\,\frac{x_i}{r}\right) = -\partial_i\left(\frac{x_i}{r^3}\right)$$

$$= -\left(\left(\partial_i x_i\right)\frac{1}{r^3} + x_i\left(\partial_i r^{-3}\right)\right)$$

$$= -\left(\frac{\delta_{ii}}{r^3} + x_i\,\frac{(-3)}{r^4}\left(\partial_i r\right)\right)$$

$$= -\left(\frac{\delta_{ii}}{r^3} - 3\,\frac{x_i}{r^4}\,\frac{x_i}{r}\right) = -\left(\frac{\delta_{ii}}{r^3} - 3\,\frac{r^2}{r^5}\right)$$

$$= -\left(\frac{3}{r^3} - \frac{3}{r^3}\right) = 0 \quad //$$