Chapter 2. Electrostatics

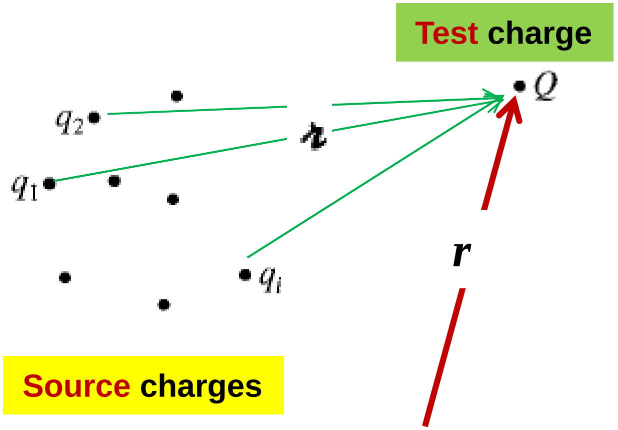
Introduction to Electrodynamics, David J. Griffiths

2.1 The Electric Field

The electromagnetic theory hopes to solve is this:

→ **What force** do the **source charges (q_1, q_2, …)** exert on the **test charge (Q)?**

→ In general, **both the source charges and the test charge are in motion.**



To begin with, **consider the special case of ELECTROSTATICS**

→ All the source charges are STATIONARY

→ The test charge may be MOVING.

The solution to this problem is facilitated by the ***principle of superposition***

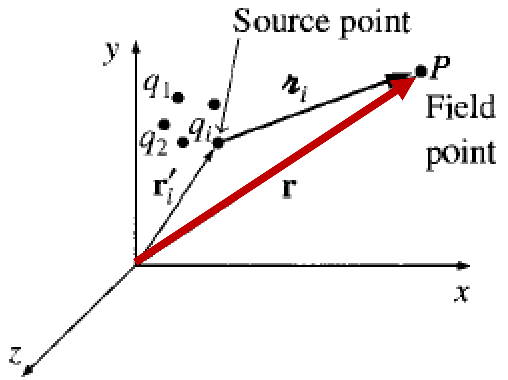
→ The interaction between any two charges is **completely unaffected by the others**.

→ To determine the force on Q, we can first compute the force F_1, due to q_1 alone (ignoring all the others); then we compute the force F_2, due to q_2 alone; and so on.

→ Finally, we take the **vector sum** of all these individual forces: $F = F_I + F_2 + F_3 + \ldots$

2.1.3 The Electric Field

If we have *several* point charges $q_1, q_2, \ldots, q_n$, at distances $\mathscr{r}_1, \mathscr{r}_2, \ldots, \mathscr{r}_n$ from Q, the total force on Q is evidently



$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \ldots = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 Q}{\mathscr{r}_1^2}\hat{\boldsymbol{\mathscr{r}}}_1 + \frac{q_2 Q}{\mathscr{r}_2^2}\hat{\boldsymbol{\mathscr{r}}}_2 + \ldots\right)$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{q_1\hat{\boldsymbol{\mathscr{r}}}_1}{\mathscr{r}_1^2} + \frac{q_2\hat{\boldsymbol{\mathscr{r}}}_2}{\mathscr{r}_2^2} + \frac{q_3\hat{\boldsymbol{\mathscr{r}}}_3}{\mathscr{r}_3^2} + \ldots\right)$$

$$\mathbf{F} = Q\mathbf{E}$$

$$\boxed{\mathbf{E}(\mathbf{r}) \equiv \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i^2}\hat{\boldsymbol{\mathscr{r}}}_i}$$

The experimental law of Coulomb (1785)

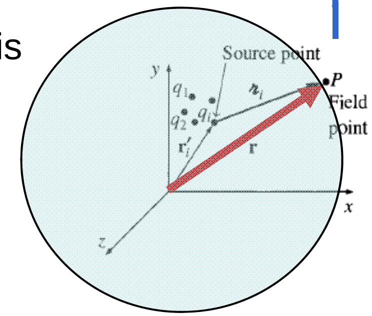
$$dq = \lambda\, dl' \longrightarrow \mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{P}}\frac{\lambda(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, dl'$$

$$dq = \sigma\, da' \longrightarrow \mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{S}}\frac{\sigma(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, da'$$

$$dq = \rho\, d\tau' \longrightarrow \mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}}\frac{\rho(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\, d\tau'$$

Flux and Gauss's Law

In the case of **a point charge *q*** at the origin, the flux of E through a sphere of radius *r* is



$$\Phi_E \equiv \oint \mathbf{E} \cdot d\mathbf{a} = \int \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2}\hat{\mathbf{r}}\right) \cdot (r^2 \sin\theta\, d\theta\, d\phi\, \hat{\mathbf{r}}) = \frac{1}{\epsilon_0} q$$

➔ *The flux through any surface enclosing the charge is q/ε_0.*

Now suppose **a bunch of charges** scattered about.

➔ According to the **principle of superposition**, the total field is the (vector) sum of all the individual fields:

$$\mathbf{E} = \sum_{i=1}^{n} \mathbf{E}_i$$

$$\Phi_E \equiv \oint \mathbf{E} \cdot d\mathbf{a} = \sum_{i=1}^{n}\left(\oint \mathbf{E}_i \cdot d\mathbf{a}\right) = \sum_{i=1}^{n}\left(\frac{1}{\epsilon_0} q_i\right) \Rightarrow \boxed{\oint_S \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{enc}}}$$

Gauss's Law

By applying the divergence theorem:

$$\oint_S \mathbf{E} \cdot d\mathbf{a} = \int_{\mathcal{V}} (\nabla \cdot \mathbf{E})\, d\tau$$

$$Q_{\text{enc}} = \int_{\mathcal{V}} \rho\, d\tau$$

$$\Rightarrow \int_{\mathcal{V}} (\nabla \cdot \mathbf{E})\, d\tau = \int_{\mathcal{V}} \left(\frac{\rho}{\epsilon_0}\right) d\tau \Rightarrow \boxed{\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho}$$

Gauss's law in differential form

2.2.2 The Divergence of E

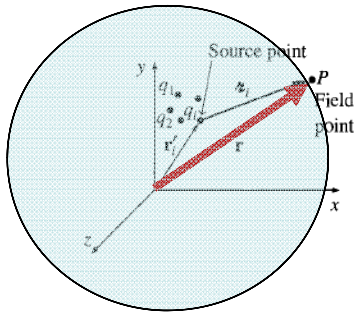
Let's calculate the **divergence of E directly from the Coulomb' s Law** of

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\rho(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\,d\tau' \longrightarrow \nabla\cdot\mathbf{E} \quad \text{(divergence in terms of } \boldsymbol{r}\text{)}$$

Since the **r**-dependence is contained in $\boldsymbol{\mathscr{r}}$ **= r - r'**, we have

$$\nabla\cdot\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int \nabla\cdot\left(\frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\right)\rho(\mathbf{r}')\,d\tau'$$

$$\nabla\cdot\left(\frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\right) = 4\pi\delta^3(\boldsymbol{\mathscr{r}})$$



$$\nabla\cdot\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int 4\pi\delta^3(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}')\,d\tau' = \frac{1}{\epsilon_0}\rho(\mathbf{r})$$

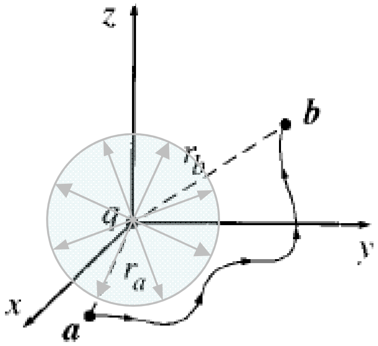
$$\boxed{\nabla\cdot\mathbf{E} = \frac{\rho(r)}{\varepsilon_0}}$$

➔ ***This is Gauss's law in differential form***

2.2.4 The Curl of E

Consider the electric field from a point charge **q** at the origin: $\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$

Now let's **calculate the line Integral** of this field from some point **a** to some other point **b:** $\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E}\cdot d\mathbf{l}$



In spherical coordinates, $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$

$$\mathbf{E}\cdot d\mathbf{l} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}dr \longrightarrow \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E}\cdot d\mathbf{l} = \frac{1}{4\pi\epsilon_0}\int_{\mathbf{a}}^{\mathbf{b}} \frac{q}{r^2}dr = \frac{-1}{4\pi\epsilon_0}\frac{q}{r}\bigg|_{r_a}^{r_b} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_a} - \frac{q}{r_b}\right)$$

The integral around a *closed* path is evidently zero (for then $r_a = r_b$):

$\oint \mathbf{E}\cdot d\mathbf{l} = 0$ ➔ Applying Stokes' theorem, $\nabla\times\mathbf{E} = 0$

If we have many charges, the principle of superposition states that the total field is a vector sum of their individual fields: $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \ldots,$

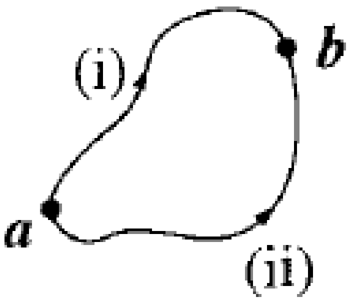
$$\nabla\times\mathbf{E} = \nabla\times(\mathbf{E}_1 + \mathbf{E}_2 + \ldots) = (\nabla\times\mathbf{E}_1) + (\nabla\times\mathbf{E}_2) + \ldots = 0$$

$\nabla\times\mathbf{E} = 0$ ➔ *For any **static charge distribution** whatever*

2.3 Electric Potential

We're going to reduce a vector problem (finding **E** from $\nabla \times \mathbf{E} = 0$) down to a much simpler scalar problem.

$\nabla \times \mathbf{E} = 0 \Rightarrow \oint \mathbf{E} \cdot d\mathbf{l} = 0$ ➔ **the line integral of E from point a to point b is the same for all paths (independent of path)**



⇒ Because the line integral of **E** is independent of path, we can define a function called the **Electric Potential**:

$$\boxed{V(\mathbf{r}) \equiv -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l}}$$: ***O*** is some standard reference point

➔ The ***potential difference*** *between two points a and b is*

$$V(\mathbf{b}) - V(\mathbf{a}) = -\int_{\mathcal{O}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l} + \int_{\mathcal{O}}^{\mathbf{a}} \mathbf{E} \cdot d\mathbf{l} = -\int_{\mathcal{O}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l} - \int_{\mathbf{a}}^{\mathcal{O}} \mathbf{E} \cdot d\mathbf{l} = -\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l}$$

➔ The fundamental theorem for gradients states that $V(\mathbf{b}) - V(\mathbf{a}) = \int_{\mathbf{a}}^{\mathbf{b}} (\nabla V) \cdot d\mathbf{l}$

$$\Rightarrow \int_{\mathbf{a}}^{\mathbf{b}} (\nabla V) \cdot d\mathbf{l} = -\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l} \Rightarrow \boxed{\mathbf{E} = -\nabla V}$$

➔ **The electric field is the gradient of scalar potential**

2.3.3 Poisson's Equation and Laplace's Equation

➔ **What do the fundamental equations for E look like, in terms of *V*?**

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \xrightarrow[\nabla \cdot \mathbf{E} = \nabla \cdot (-\nabla V) = -\nabla^2 V]{\mathbf{E} = -\nabla V} \nabla^2 V = -\frac{\rho}{\epsilon_0}$$

: Poisson's equation

$$\nabla^2 V = 0$$

: Laplace's equation

➔ **Gauss's law on E can be converted to Poisson’s equation on V**

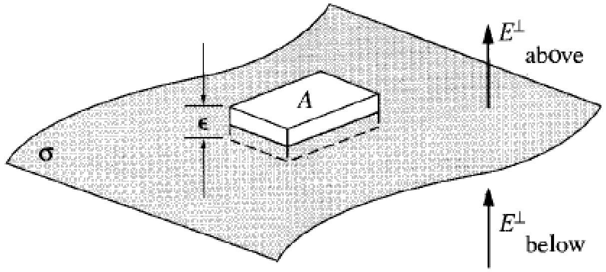
➔ ***It takes only one differential equation (Poisson's) to determine V, because V is a scalar;***
(for E we needed two, the divergence and the curl.)

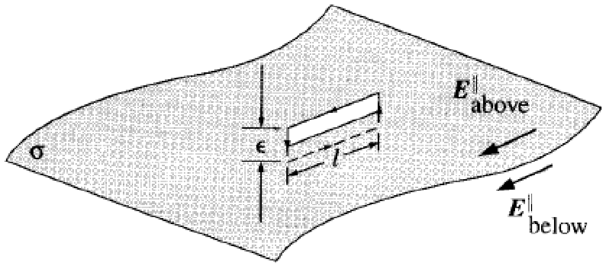
For a volume, surface, or line charge →

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}')}{\mathscr{r}}\, d\tau' \qquad \frac{1}{4\pi\epsilon_0}\int \frac{\sigma(\mathbf{r}')}{\mathscr{r}}\, da' \qquad \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(\mathbf{r}')}{\mathscr{r}}\, dl'$$

Electrostatic Boundary Conditions

Notice that *the electric field always undergoes a discontinuity* when you cross a surface charge σ.





$$\oint_S \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{enc}} = \frac{1}{\epsilon_0} \sigma A$$

As the thickness ε goes to zero,

$$E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}} = \frac{1}{\epsilon_0}\sigma$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0 \longrightarrow \mathbf{E}^{\parallel}_{\text{above}} = \mathbf{E}^{\parallel}_{\text{below}}$$

➔ ***The normal component of E is discontinuous by σ/ε_0 at any boundary.***

➔ If $\sigma = 0$, / e_0 it is continuous: $E^{\perp}_{\text{above}} = E^{\perp}_{\text{below}}$

➔ ***The parallel (tangential) component of E is always continuous.***

➔ **The boundary conditions on E into a single formula:** $\boxed{\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}}}$

➔ **The potential, meanwhile, is continuous across any boundary:**

$$V_{\text{above}} - V_{\text{below}} = -\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l} \xrightarrow{\text{as the path length shrinks to zero}} \boxed{V_{\text{above}} = V_{\text{below}}}$$

2.3.5 Summary; Relations of E – ρ – V

From just two experimental observations:

(1) ***the principle of superposition*** - a broad general rule
(2) ***Coulomb's law*** - the fundamental law of electrostatics.

E – ρ – V

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}')}{\mathscr{r}}\,d\tau'$$

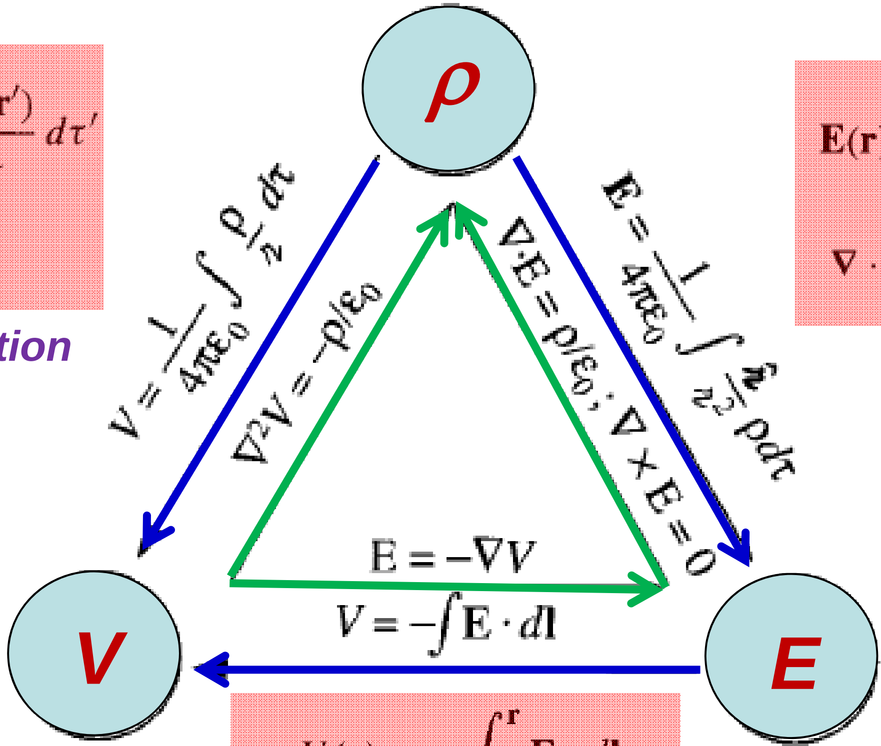
$$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$

Poisson's equation

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}')}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\,d\tau'$$

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \nabla\times\mathbf{E} = 0$$

Gauss Law



$$V(\mathbf{r}) \equiv -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E}\cdot d\mathbf{l}$$

$$\mathbf{E} = -\nabla V$$

Gradient theorem

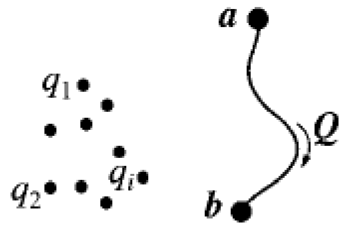
2.4 Work and Energy in Electrostatics

2.4.1 The Work Done to Move a Charge

To move a test charge **Q** from point **a** to point **b**, ***how much work*** will you have to do?

$$W = \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{F} \cdot d\mathbf{l} = -Q \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l} = Q[V(\mathbf{b}) - V(\mathbf{a})]$$

$$\mathbf{F} = -Q\mathbf{E}$$ (in opposite to electric force)



$$V(\mathbf{b}) - V(\mathbf{a}) = \frac{W}{Q}$$

➔ The *potential difference between points* a *and* b is *equal to the work per unit charge required to carry a particle from* a *to* b.

If you want to bring the charge Q in from far away and stick it at point **r**,

$$\xrightarrow{V(\mathbf{a}) = V(\infty) = 0} \quad \boxed{W = QV(\mathbf{r})}$$

➔ ***Potential is potential energy per unit charge** (just as the field is the force per unit charge).*

2.4.3 The Energy of a Continuous Charge Distribution

$$W = \frac{1}{2}\sum_{i=1}^{n} q_i V(\mathbf{r}_i) \longrightarrow \boxed{W = \frac{1}{2}\int \rho V \, d\tau} \qquad \int \lambda V \, dl \qquad \int \sigma V \, da$$

There is a lovely way to rewrite this result in terms of E.

$$\rho = \epsilon_0 \nabla \cdot \mathbf{E}, \quad \text{so} \quad W = \frac{\epsilon_0}{2}\int (\nabla \cdot \mathbf{E}) V \, d\tau = \frac{\epsilon_0}{2}\left[-\int \mathbf{E} \cdot (\nabla V) \, d\tau + \oint V\mathbf{E} \cdot d\mathbf{a}\right]$$ (Integration by parts)

$$= \frac{\epsilon_0}{2}\left(\int_{\mathcal{V}} E^2 \, d\tau + \oint_{\mathcal{S}} V\mathbf{E} \cdot d\mathbf{a}\right) \longleftarrow \quad \nabla V = -\mathbf{E}$$

Note that the energy *W can defined, whatever* volume you use (as long as it encloses all the charge),

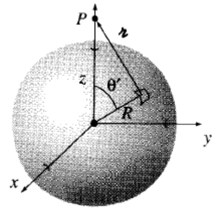
→ but the contribution from the ***volume integral of E^2* goes up**,

→ that of the ***surface integral of VE* goes down** since $E \sim 1/r^2$, $V \sim 1/r$, while da ~ r^2.

➔ ***For all space (r goes infinite), the surface integral goes to zero!***

Energy of Continuous Charge Distribution ⇒ $\boxed{W = \frac{1}{2}\int \rho V \, d\tau}$ ⇒ $\boxed{W = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2 \, d\tau}$

Example 2.8 Find the energy of a uniformly charged spherical shell of total charge *q* and radius *R*.



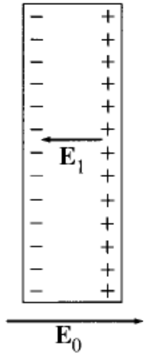
Solution 1: $W = \frac{1}{2}\int \sigma V \, da \xrightarrow{V = (1/4\pi\epsilon_0)q/R} W = \frac{1}{8\pi\epsilon_0}\frac{q}{R}\int \sigma \, da = \frac{1}{8\pi\epsilon_0}\frac{q^2}{R}$

Solution 2: Inside the sphere $\mathbf{E} = 0$; outside, $\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$ $\quad W = \frac{\epsilon_0}{2}\int_{\text{outside}} E^2 \, d\tau = \frac{\epsilon_0}{2(4\pi\epsilon_0)^2}\int_{\text{outside}} \left(\frac{q^2}{r^4}\right)(r^2 \sin\theta \, dr \, d\theta \, d\phi) = \frac{1}{8\pi\epsilon_0}\frac{q^2}{R}$

2.5 Conductors

2.4.1 Basic Properties

(i) E = 0 inside a conductor → Why?

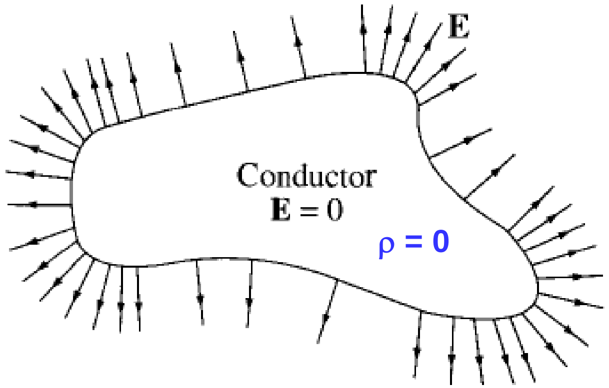


Put a conductor into an external electric field $\mathbf{E_o}$.
Induced charges produce a field of their own, E_1.
E_1 tends to cancel E_0. That's the crucial point.
The whole process is ***practically instantaneous***.
Outside the conductor the field is ***not zero***.

(ii) ρ = 0 inside a conductor

$\nabla \cdot \mathbf{E} = \rho/\epsilon_0.$ → **ρ = 0** because **E** = 0.

→ There is still charge around,
→ The **net charge density** in the interior ***is zero***.



(iii) Any net charge resides on the surface

(iv) A conductor is an equipotential

For if **a** and **b** are any two points **within** (or at the surface of) a given conductor

$$V(\mathbf{b}) - V(\mathbf{a}) = -\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l} = 0 \quad \text{(Since } \mathbf{E} = 0\text{)} \Rightarrow V(\mathbf{a}) = V(\mathbf{b})$$

(v) E is perpendicular to the surface, just outside a conductor

Otherwise, charge will immediately flow around the surface until it kills off the tangential component.

$$V(\mathbf{b}) - V(\mathbf{a}) = -\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l} = 0$$

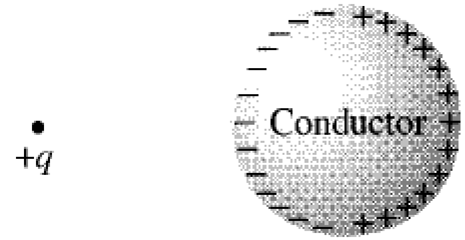
(a and b are outside the surface → **E** normal to the surface)

2.5.2 Induced Charges

If you hold a charge +*q* near an uncharged conductor, the two will attract one another.

→ the **negative induced charge is closer to *q***,

→ There is a net force of attraction

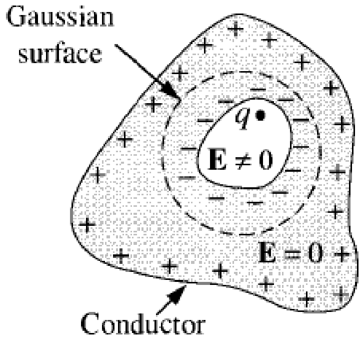


If there is some *cavity* in the conductor, within that cavity there is some charge,

→ the field *in the cavity* will *not* be zero.

→ No external fields penetrate the conductor; they are canceled at the outer surface by the **induced charge**.

→ $q_{\text{induced}} = -q$ since $\oint \mathbf{E} \cdot d\mathbf{a} = 0$ for E = 0 on a Gaussian surface.

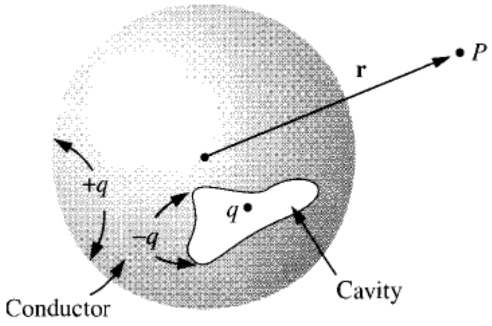


→ ***$q_{\text{induced}} = +q$ on the outside, uniformly distributed***

Example 2.9 What is the field outside the sphere?

The answer is, $\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$

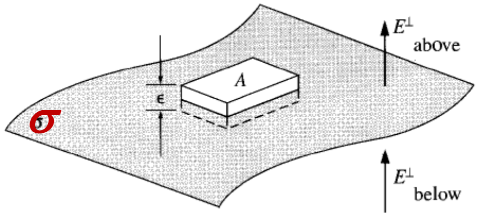
regardless **the shape of the cavity the placement of the charge.**

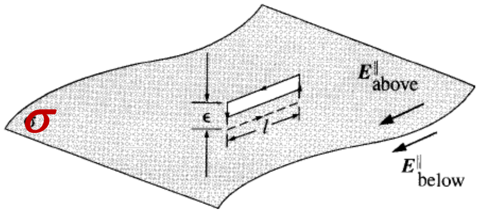


2.5.3 Surface Charge and the Force on a Conductor

Remember that the boundary condition for E at any interface in general was

$$E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}} = \frac{1}{\epsilon_0}\sigma \qquad \mathbf{E}^{\parallel}_{\text{above}} = \mathbf{E}^{\parallel}_{\text{below}} \longrightarrow \mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}}$$

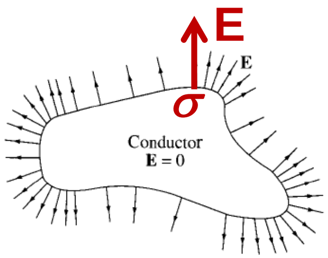




In the particular case of a conductor,

The field inside a conductor is zero, $\mathbf{E}_{\text{below}} = 0$

$\longrightarrow$ The **field immediately *outside*** is



$$\boxed{\mathbf{E} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}}}$$

(➔ Always normal to the surface)

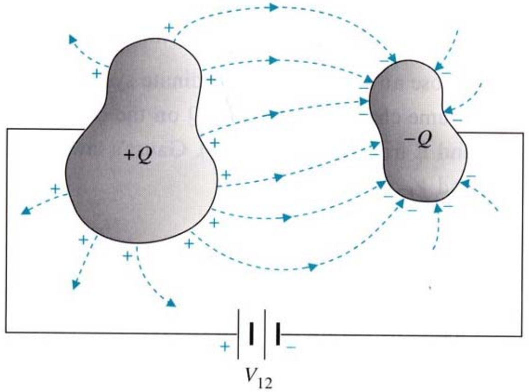
In terms of potential, $\mathbf{E} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}} \longrightarrow \boxed{\sigma = -\epsilon_0 \frac{\partial V}{\partial n}}$ ➔ **Surface charge on a conductor can be determined from E or *V*.**

2.5.4 Capacitors

Consider two conductors with +Q and –Q total charges.

Since V is constant over a conductor,
we can speak unambiguously of the potential difference between them:

$$V = V_+ - V_- = -\int_{(-)}^{(+)} \mathbf{E} \cdot d\mathbf{l}$$



Since E is given by Coulomb's law: $\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho}{\mathscr{r}^2}\hat{\mathscr{r}}\, d\tau$

➔ Doubling Q *does* double ρ everywhere
➔ ***Therefore, E is proportional to Q, so also is V***

$$V \propto Q \quad \Rightarrow \quad \boxed{C \equiv \frac{Q}{V}}$$

: **Capacitance**

(In SI units) C is measured in **farads (F)**
→ A farad is a **coulomb-per-volt**.

To "charge up" a capacitor,
→ the work you must do to transport
the next piece of charge, *dq, on a positive plate q is* $dW = \left(\frac{q}{C}\right) dq$

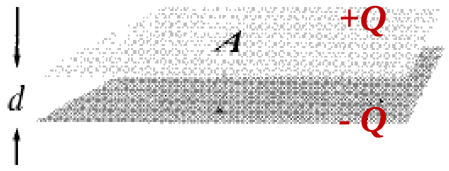
The total work necessary, then, to go from q = 0 to q = Q, is

$$W = \int_0^Q \left(\frac{q}{C}\right) dq = \frac{1}{2}\frac{Q^2}{C} \xrightarrow{Q = CV} W = \frac{1}{2}CV^2$$

: Energy stored in C

Capacitance $C \equiv \dfrac{Q}{V}$

Example 2.10 Parallel-plate capacitor

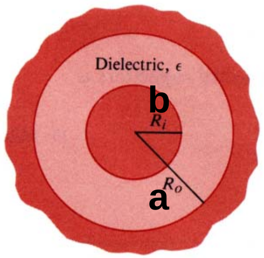


$$\sigma = Q/A$$

$$\mathbf{E} = (1/\epsilon_0)Q/A$$

$$V = \mathrm{E} \cdot d = \frac{Q}{A\epsilon_0}d \longrightarrow C = \frac{A\epsilon_0}{d}$$

Example 2.11 Two concentric spherical metal shells

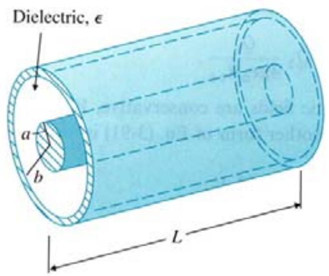


$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$$

$$V = -\int_b^a \mathbf{E} \cdot d\mathbf{l} = -\frac{Q}{4\pi\epsilon_0}\int_b^a \frac{1}{r^2}\,dr = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\longrightarrow C = \frac{Q}{V} = 4\pi\epsilon_0\frac{ab}{(b-a)}$$

Problem 2.39 Two coaxial metal cylindrical tubes



$$\rho_l = \frac{Q}{L} \rightarrow E = \frac{Q}{2\pi\varepsilon r L}\hat{r} \rightarrow V = -\int_{r=b}^{r=a}\left(\frac{Q}{2\pi\varepsilon r L}\hat{r}\right) \bullet (\hat{r}dr) = \frac{Q}{2\pi\varepsilon L}\ln\left(\frac{b}{a}\right)$$

$$\rightarrow C = \frac{Q}{V_{ab}} = \frac{2\pi\varepsilon L}{\ln\left(\dfrac{b}{a}\right)}$$