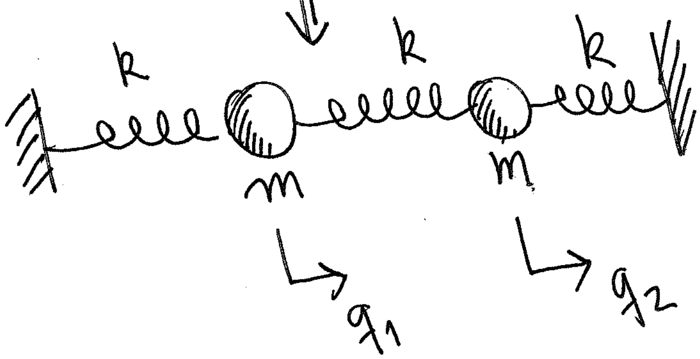
COMPLEMENTO II

B1

Aplicación

$\Downarrow$



$\Downarrow$

$$L = \frac{1}{2}(\dot{q}_1 \ \dot{q}_2)\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \end{pmatrix} - \frac{1}{2}(q_1 \ q_2)\begin{pmatrix} 2k & -k \\ -k & 2k \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

$\Downarrow$

$$\hat{M} = m\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \ ; \ \hat{K} = k\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

$\Downarrow$

Hallando valores y vectores propios de $\hat{M}^{-1}\hat{K}$

$\Downarrow$

donde $\hat{M}^{-1}\hat{K} = \frac{k}{m}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} = \frac{k}{m}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$

y se resuelve el problema de valores propios $(\hat{M}^{-1}\hat{K} - \omega^2)\,q = 0$

B2

$$\Downarrow$$

Valores propios : $\omega_1^2 = \frac{k}{m}$

$\omega_2^2 = \frac{3k}{m}$

vectores propios normalizados : $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \longleftrightarrow \omega_1^2$

$\frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix} \longleftrightarrow \omega_2^2$

Construcción de $\hat{U}$:

$$\hat{U} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{-\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$\Downarrow$$

$$\hat{U}^T = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

Se comprueba que $\hat{U}^T\hat{U} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \hat{\mathbb{I}}$

$\therefore \quad \hat{U}^T = \hat{U}^{-1}$ B3

Entonces la solución final para las coord. $q_1(t)$ y $q_2(t)$ es:

$$\begin{pmatrix} q_1(t) \\ q_2(t) \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \cos(w_1 t) & 0 \\ 0 & \cos(w_2 t) \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} q_1(0) \\ q_2(0) \end{pmatrix}$$

$$+ \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{w_1} & 0 \\ 0 & \frac{1}{w_2} \end{pmatrix}\begin{pmatrix} \operatorname{sen}(w_1 t) & 0 \\ 0 & \operatorname{sen}(w_2 t) \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} \dot{q}_1(0) \\ \dot{q}_2(0) \end{pmatrix}$$

$$\Downarrow$$

$$\begin{pmatrix} q_1(t) \\ q_2(t) \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \cos(w_1 t) + \cos(w_2 t) & \cos(w_1 t) - \cos(w_2 t) \\ \cos(w_1 t) - \cos(w_2 t) & \cos(w_1 t) + \cos(w_2 t) \end{pmatrix}\begin{pmatrix} q_1(0) \\ q_2(0) \end{pmatrix}$$

$$+ \frac{1}{2}\begin{pmatrix} \frac{\operatorname{sen}(w_1 t)}{w_1} + \frac{\operatorname{sen}(w_2 t)}{w_2} & \frac{\operatorname{sen}(w_1 t)}{w_1} - \frac{\operatorname{sen}(w_2 t)}{w_2} \\ \frac{\operatorname{sen}(w_1 t)}{w_1} - \frac{\operatorname{sen}(w_2 t)}{w_2} & \frac{\operatorname{sen}(w_1 t)}{w_1} + \frac{\operatorname{sen}(w_2 t)}{w_2} \end{pmatrix}\begin{pmatrix} \dot{q}_1(0) \\ \dot{q}_2(0) \end{pmatrix}$$

//