1.- $m\ddot{\vec{r}}' - K\vec{r}' = 0$; $\vec{r}' = \vec{r} - \vec{S}$

$\underbrace{m(\ddot{\vec{r}}) - K\vec{r}}\; + \underbrace{K\dot{\vec{S}}}_{C} = 0$ $\quad \dot{\vec{r}}' = \dot{\vec{r}}$

$\ddot{\vec{r}}' = \ddot{\vec{r}}$

2.- $m\ddot{\vec{r}} = f(x)\,\vec{r}$ $\quad \vec{r} \to \vec{r}' = \hat{R}\vec{r}$

$m\ddot{\vec{r}}' = f(x)\,\vec{r}'$ $\quad \dot{\vec{r}} \to \dot{\vec{r}}' = \hat{R}\dot{\vec{r}}$

$\ddot{\vec{r}} \to \ddot{\vec{r}}' = \hat{R}\ddot{\vec{r}}$

$m\hat{R}\ddot{\vec{r}} = f(x)\,\hat{R}\vec{r} \,/\, \hat{R}^{-1}$

$m\ddot{\vec{r}} = f(x)\,\vec{r}$ //

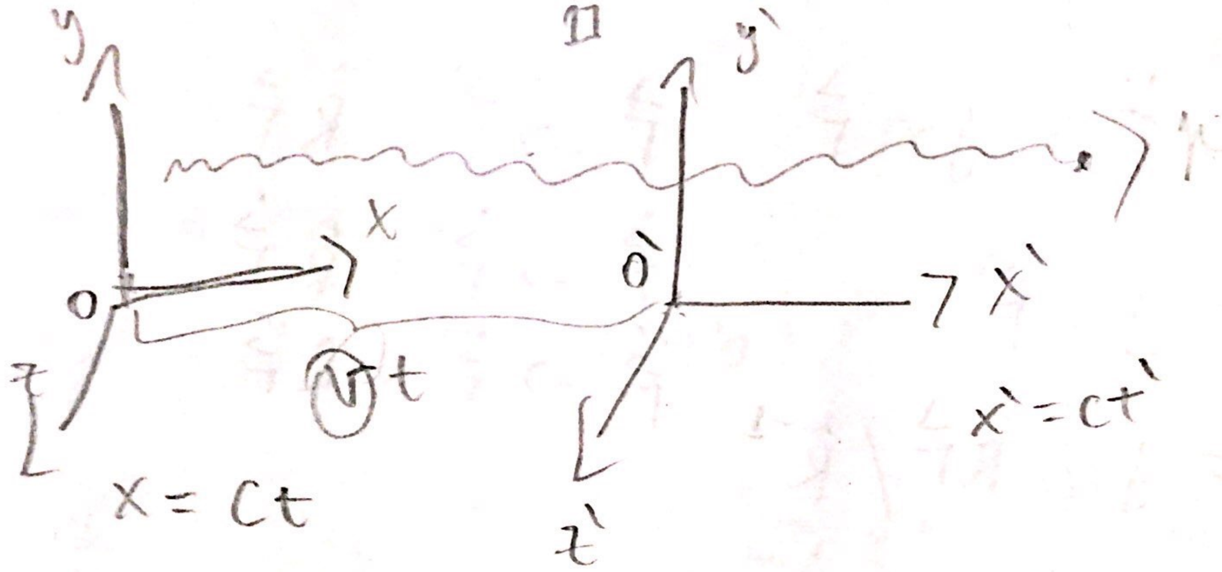
3.- $\dfrac{\partial^2}{\partial x_i' \partial x^{i'}} = \dfrac{\partial^2}{\partial x_i \partial x^i}$ //

$\partial x^{i'} = \hat{R}\,\partial x^i$

$\partial x_i' = (\partial x^{i'})^T = (\hat{R}\,\partial x^i)^T = \partial x^{i\,T}\,\hat{R}^T = \partial x_i\,\hat{R}^T$

$\underline{\text{obs}}$: $\hat{R}^T \hat{R} = \hat{1} \Rightarrow \hat{R}^T = \hat{R}^{-1}$

$$\frac{\partial^2}{\partial x_i' \partial x^{i'}} = \frac{\partial^2}{\partial x_i \underbrace{\hat{R}^T \hat{R}}_{\hat{1}} \partial x^{i}} = \frac{\partial^2}{\partial x_i \partial x^{i}} \quad //$$



$x = a x' + b t' \wedge x' = a x - b t$

$O \rightarrow O' \rightarrow v$

$O' \rightarrow O \rightarrow v' = -v$

$x' = 0 \Rightarrow bt = ax \Rightarrow b\,dt = a\,dx \rightarrow v = \frac{b}{a}$

$x = 0 \Rightarrow -bt' = ax' \Rightarrow -b\,dt' = a\,dx'$

$-\frac{b}{a} = v' = -v$.

UN RAYO DE luz visto DESDE $O = x = ct$

$O' = x' = ct'$

$ct = act' + bt' \Rightarrow \boxed{ct = (ac+b)t'}$ (2)

$\boxed{ct' = (ac-b)t}$ (1)

(2) EN (1)

$c\cancel{t} = (ac-b)(ac+b)\frac{\cancel{t}}{c}$

$c^2 = ((ac)^2 - b^2)\frac{1}{c} \Rightarrow c^2 = (c^2 - \frac{b^2}{a^2})a^2$

$\rightarrow c^2 = (c^2 - v^2)a^2$

$a^2 = \frac{1}{1-\left(\frac{v}{c}\right)^2} \Rightarrow a = \left(\frac{1}{1-\left(\frac{v}{c}\right)^2}\right)^{1/2} = \gamma$

$b = \left(\frac{c^2}{1-\left(\frac{v}{c}\right)^2} - c^2\right)^{1/2} = \left(\frac{(v/c)^2}{1-\left(\frac{v}{c}\right)^2}\right)^{1/2} c = v\gamma$

$$x = a x' + b t'$$

$$x = \gamma x' + v \gamma t' = \gamma (x' + v t')$$

$$x' = \gamma (x - v t)$$

5.- DEM: $\Box' = \Box$; $\partial_\mu = \frac{\partial}{\partial x_\mu}$

$$\Box' = \partial'_\mu \partial'^\mu = \eta^{\mu\nu} \partial'_\nu \partial'_\mu$$

$$ds^2 = \eta^{\mu\nu} dx_\nu dx_\mu = \underbrace{c^2 dt^2}_{dx_0^2} - dx^2 - dy^2 - dz^2$$

$$\Box = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$$

Escaneado con CamScanner

$$\square' = \partial'_\mu \partial^{\mu'} = \frac{\partial}{\partial x'_\mu} \frac{\partial}{\partial x^{\mu'}}$$

$$\partial x'_\mu \partial x^{\mu'} = \partial x'_\nu \partial x^{\nu'} = \partial x_\mu \Lambda^\mu_{\ \nu'} \Lambda^{\nu'}_{\ \mu} \partial x^\mu$$

obs $\Lambda^{\nu'}_{\ \mu} = (\Lambda^{-1})^\mu_{\ \nu'}$; $\Lambda^{\nu'}_{\ \mu} = (\Lambda^\mu_{\ \nu'})^T$

$$\partial x'_\mu \partial x^{\mu'} = \partial x_\mu (\Lambda^{-1})^\mu_{\ \nu'} (\Lambda^\mu_{\ \nu'})^T \partial x^\mu$$

obs : $(\Lambda^\mu_{\ \nu'})^T = \Lambda^\mu_{\ \nu'}$

$$\partial x'_\mu \partial x^{\mu'} = \partial x_\mu \underbrace{(\Lambda^{-1})^\mu_{\ \nu'} \Lambda^\mu_{\ \nu'}}_{\hat{1}} \partial x^\mu$$

$$\partial x'_\mu \partial x^{\mu'} = \partial x_\mu \partial x^\mu = \square$$

$$\therefore \square' = \square$$

$$O \xrightarrow{\vec{v} = v_x \hat{x}}$$

$$\xrightarrow{\vec{E} = E_x \hat{x}}$$

$$a = \frac{qE}{m}\left(1-\frac{v^2}{c^2}\right)^{3/2}$$

$$f^\mu = \frac{d}{d\tau}p^\mu = m\frac{d}{d\tau}u^\mu = m\left(\frac{d}{d\tau}u^0 , \frac{d}{d\tau}u^i\right)$$

$$f^x = \cancel{\gamma} F_x = \cancel{\gamma}\, m \frac{d}{dt}(\gamma v_x)$$

$$F_x = m\left[\frac{d}{dt}(\gamma) v_x + \gamma \frac{d}{dt} v_x\right]$$

dos: $\boxed{\frac{d}{dt}(\gamma) = \gamma^3 \frac{\vec{v}\cdot\vec{a}}{c^2}}$

$$\| = m\left[\gamma^3 \frac{v^2}{c^2} a_x + \gamma a_x\right]$$

obs:

$$\gamma^2 = \frac{1}{1-\left(\frac{v}{c}\right)^2} \Rightarrow \gamma^2\left(1-\left(\frac{v}{c}\right)^2\right) = 1 \;/\gamma$$

$$\gamma^3 - \gamma^3\left(\frac{v}{c}\right)^2 = \gamma \Rightarrow \boxed{\gamma^3-\gamma = \gamma^3\left(\frac{v}{c}\right)^2}$$

$$\Rightarrow F_x = m\left[(\gamma^3-\cancel{\gamma})a_x + \cancel{\gamma}\, a_x\right]$$

$$F_x = m\,\gamma^3 a_x = qE_x$$

$$\therefore a = \frac{dv}{dt} = \frac{qE_x}{m}\left(1-\left(\frac{v}{c}\right)^2\right)^{3/2}$$

$$dv = \frac{qE}{m}\left(1-\frac{v^2}{c^2}\right)^{3/2} dt$$

$$\int_0^v \frac{dv}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} = \int_0 \frac{qE}{m} dt \Big/ \int$$

$$\frac{m}{qE}\int_0^{v(t)} \frac{dv}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} = \int_0^t dt\, \frac{qE}{m}$$

$$\frac{v(t)}{\sqrt{1-\frac{v(t)^2}{c^2}}} = t\frac{qE}{m} \Rightarrow v(t)^2 = t^2\left(\frac{qE}{m}\right)^2\left(1-\frac{v(t)^2}{c^2}\right)$$

$$\Rightarrow c^2 v(t)^2 = c^2t^2\left(\frac{qE}{m}\right)^2 - t^2\left(\frac{qE}{m}\right)^2 v(t)^2$$

$$\Rightarrow v(t) = \frac{ct\frac{qE}{m}}{\left(c^2+t^2\left(\frac{qE}{m}\right)^2\right)^{1/2}} = \frac{t\frac{qE}{m}}{\left(1+\frac{t^2}{c^2}\left(\frac{qE}{m}\right)^2\right)^{1/2}}$$

$$dx(t) = \frac{qE}{m}\frac{t\,dt}{\left(1+t^2\left(\frac{qE}{cm}\right)^2\right)^{1/2}} \Big/ \int$$

5) $$x(t) = \frac{qE}{m}\left[\frac{\sqrt{\left(\frac{qE}{cm}\right)^2 t^2 + 1}}{\left(\frac{qE}{cm}\right)^2} - \frac{1}{\left(\frac{qE}{cm}\right)^2}\right]$$

$$x(t) = \frac{mc^2}{qE}\left[\sqrt{1 + \left(\frac{qE}{cm}\right)^2 t^2} - 1\right]$$