Prueba de Metodos - Fabian Trigo 20.183.107-5

IFA

universidad de valparaiso

lado izquierdo I lado derecho I^1

Problema I: demuestre que $\nabla \times [\vec{v} \times (\nabla \times \vec{v})] = -\nabla \times [(\vec{v}\cdot\nabla)\vec{v}]$

primero evaluar: $\vec{v} \times (\nabla \times \vec{v}) = I$

$$I = \epsilon_{ijk} V_j (\nabla \times \vec{v})_k \, \hat{e}_i = \epsilon_{ijk} V_j (\epsilon_{klm} \partial_l V_m) \hat{e}_i = I$$

$$/ \; \epsilon_{ijk}\epsilon_{klm} = \epsilon_{kji}\epsilon_{klm} \Big\{ = \delta_{jl}\delta_{im} - \delta_{jm}\delta_{il} \quad (*_1)$$

Levi civita

$$I \;/\; (*_1) \Big\{ (\delta_{jl}\delta_{im} - \delta_{jm}\delta_{il}) V_j \partial_l (V_m) \hat{e}_i = I$$

ahora $\nabla \times [I] = \epsilon_{ijk} \partial_j [I_k] \hat{e}_i =$

$$/ \; I_k = \left(V_\gamma (\partial_n V_\gamma) - V_m (\partial_l V_m)\right)$$

(I) mientra el otro lado $-\nabla \times [(v \cdot \nabla) v] = -\epsilon_{ijk} \partial_j [V_l \partial_l V_k] \hat{e}_i$

$$= -\epsilon_{ijk} \left((\partial_l V_k)(\partial_j V_l) + V_\sigma (\partial_j \partial_\sigma V_k)\right) \hat{e}_i = LD.$$

$$LI. \; \nabla \times (I) = \epsilon_{ijk} \Big[(\partial_n V_\gamma)(\partial_j V_\gamma) + V_\gamma (\partial_j \partial_n V_\gamma)$$
$$- (\partial_l V_m)(\partial_j V_m) - V_m (\partial_j \partial_l V_m) \Big] \hat{e}_i$$

$$-\nabla\times[(v\cdot\nabla)v] = -\nabla\times[v_a\partial_a v_b\hat{b}] = -\epsilon_{ijk}\partial_j[v_a\partial_a v_k]\hat{e}$$

$$= \epsilon_{ikj}\left[(\partial_j v_a)(\partial_a v_k) + v_a\partial_j\partial_a v_k\right]$$

$$\nabla\times\left(\underline{v\times(\nabla\times v)}\right) = -\nabla\times\left((v\cdot\nabla)\vec{v}\right)$$

$$\epsilon_{ijk}\partial_j(\vec{v}\times(\nabla\times\vec{v}))_k = -\epsilon_{ijk}\partial_j((v\cdot\nabla)\vec{v})_k$$

$$\epsilon_{fgh}\partial_g[v_j\partial_j v_k - v_m\partial_l v_m] = \epsilon_{ikj}\partial_j(v_\eta\partial_\eta v_k)$$

$$\epsilon_{fgh}\left[(\partial_g v_j)(\partial_j v_k) - (\partial_g v_m)(\partial_l v_m) + v_j\partial_{gj}v_k - v_m\partial_{gl}v_m\right] = LI$$

$$\epsilon_{ikj}\left[(\partial_j v_\eta)(\partial_\eta v_k) - v_\eta\,\partial_{j\eta}v_k\right] = LD$$

$$\epsilon_{ikj}\left[\frac{1}{v}(x_j)\frac{1}{v}(x_\eta) - v_\eta\,\partial_j\left(\frac{x_k}{v}\right)\right] \Big/ \partial_j x_k = \delta_{jk}$$

$$\epsilon_{ikj}\left[\frac{1}{v^2}(x_j x_\eta) - \frac{v_\eta}{v}\delta_{jk}\right] = LD$$

LI

$$G_{pgh}\left[\frac{x_g}{V^2}x_j - \frac{x_g x_l}{V^2} + \frac{V_j}{V}\delta_{gj} - \frac{V_m}{V}\delta_{gl}\right]$$

Escaneado con CamScanner