PROBLEMA GUÍA II | #3

$\vec{E}(\vec{r}) = -\nabla\phi(\vec{r})$, la componente i-ésima de esta ecuación esta dada por:

$$E_i = -\partial_i \phi(\vec{r}) = -\frac{1}{4\pi\varepsilon_0}\,\partial_i \int_{V'} \frac{1}{2}\left(3\frac{(\vec{r}\cdot\vec{r}')^2}{r^5} - \frac{r'^2}{r^3}\right)\rho(\vec{r}')\,dV'$$

Obs. ∂_i actua sobre la posición $\vec{r}$

$\therefore$

$$E_i = -\frac{1}{4\pi\varepsilon_0}\int_{V'} \frac{1}{2}\left(3\,\partial_i\left(\frac{(\vec{r}\cdot\vec{r}')^2}{r^5}\right) - r'^2\,\partial_i\left(\frac{1}{r^3}\right)\right)\rho(\vec{r}')\,dV'$$

Obs. $\partial_i\left[\frac{(\vec{r}\cdot\vec{r}')^2}{r^5}\right] = \frac{1}{r^5}\left(\partial_i(\vec{r}\cdot\vec{r}')^2\right) + (\vec{r}\cdot\vec{r}')^2\,\partial_i(r^{-5})$

$$= \frac{1}{r^5}\,2(\vec{r}\cdot\vec{r}')\,\partial_i(\vec{r}\cdot\vec{r}') + (\vec{r}\cdot\vec{r}')^2\frac{(-5)}{r^6}(\partial_i r)$$

$$= \frac{2(\vec{r}\cdot\vec{r}')}{r^5}\,\partial_i(x_\ell x'_\ell) - 5\frac{(\vec{r}\cdot\vec{r}')^2}{r^6}\,\frac{x_i}{r}$$

$$= \frac{2(\vec{r}\cdot\vec{r}')}{r^5}\,x'_\ell\,\partial_i x_\ell - 5\frac{(\vec{r}\cdot\vec{r}')^2}{r^7}x_i$$

$$= \frac{2(\vec{r}\cdot\vec{r}')}{r^5}\,x'_i - 5\frac{(\vec{r}\cdot\vec{r}')^2}{r^7}x_i$$

Obs. $\partial_i\left(\frac{1}{r^3}\right) = \partial_i(r^{-3}) = -\frac{3}{r^4}(\partial_i r) = -3\frac{x_i}{r^5}$

$\therefore$

$$E_i = -\frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{1}{2}\left(6\frac{(\vec{r}\cdot\vec{r}\,')}{r^5}x_i' - 15\frac{(\vec{r}\cdot\vec{r}\,')^2}{r^7}x_i + 3\frac{r'^2}{r^5}x_i\right)\rho(\vec{r}\,')dV'$$

$$\Downarrow$$

Finalmente:

$$\vec{E}(\vec{r}) = -\frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{1}{2}\left[6\frac{(\vec{r}\cdot\vec{r}\,')\vec{r}\,'}{r^5} - 15\frac{(\vec{r}\cdot\vec{r}\,')^2\vec{r}}{r^7} + 3\frac{r'^2}{r^5}\vec{r}\right]\rho(\vec{r}\,')dV'$$

$$\Downarrow$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{1}{2}\left[15\frac{(\vec{r}\cdot\vec{r}\,')^2\vec{r}}{r^7} - 6\frac{(\vec{r}\cdot\vec{r}\,')\vec{r}\,'}{r^5} - 3\frac{r'^2}{r^5}\vec{r}\right]\rho(\vec{r}\,')dV'$$

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