Problema III

$$[\hat{X}, \hat{k}] = i\hat{\mathbb{1}} \quad \text{ou} \quad [\hat{k}, \hat{X}] = -[\hat{x}, \hat{k}] = -i\hat{\mathbb{1}}$$

$$\exp(\beta\hat{k}) = \sum_n^{\infty} \frac{\beta^n}{n!}\hat{k}^n \quad ; \quad \langle x|k\rangle = \frac{1}{\sqrt{2\pi}} e^{ikx}$$

- $$e^{\beta\hat{k}}|x\rangle = \sum_n^{\infty} \frac{\beta^n}{n!}\hat{k}^n|x\rangle = \sum_n^{\infty} \frac{\beta^n}{n!}\hat{k}^n \hat{\mathbb{1}} |x\rangle$$

$$\hat{\mathbb{1}} = \int_{-\infty}^{\infty} |k\rangle\langle k| \, dk$$

- $$\sum_n^{\infty} \frac{\beta^n}{n!}\hat{k}^n \int_{-\infty}^{\infty} |k\rangle \langle k|x\rangle \, dk$$

- donde $(\langle x|k\rangle)^{\dagger} = (\langle k|x\rangle) = \frac{1}{\sqrt{2\pi}} e^{-ikx}$ & $\hat{k} = -i\frac{d}{dx}$

$$e^{\beta \hat{k}} |x\rangle = \sum_n^{\infty} \frac{\beta^n}{n!} \hat{k}^n |x\rangle \qquad / \langle k| \ ① \qquad / \langle x| \ ②$$

1) $\langle k| \sum_n \frac{\beta^n}{n!} \hat{k}^n |x\rangle = \sum \frac{\beta^n}{n!} \langle k| \hat{k}^n |x\rangle = \sum \frac{\beta^n k^n}{n!} \langle k|x\rangle$

$/ \ \langle k| \hat{k}^n = \langle k| k^n$

2) $\langle x| \sum \frac{\beta^n}{n!} \hat{k}^n |x\rangle = \sum \frac{\beta^n}{n!} \langle x| \left(i \frac{d}{dx}\right)^n |x\rangle =$

$e^{\beta k} \langle k|x\rangle$ donde $\langle x|k\rangle = \frac{1}{\sqrt{2\pi}} e^{ikx}$ $\quad / (\ )^{\dagger}$

$$(\langle x|k\rangle)^{\dagger} = \langle k|x\rangle = \frac{1}{\sqrt{2\pi}} e^{-ikx}$$

así $e^{\beta k} \frac{e^{-ikx}}{\sqrt{2\pi}}$ es el vector resultante proyectado a k $= \phi(k)$

el proyectado a x podemos llegar mediante tran Fourier

$$\phi(x) = \int_{-\infty}^{\infty} \cancel{e^{ikx}} \, \cancel{e^{-ikx}} \, \frac{e^{\beta k}}{\sqrt{2\pi}} \, dk$$

$$\phi(x) = \int_{-\infty}^{\infty} \frac{e^{\beta k}}{\sqrt{2\pi}} \, dk$$