PROBLEMA GUÍA I | #15

$|\vec{A}| = 1 \wedge \vec{A} = cte$

$\vec{A} \cdot \left(\nabla(\vec{v} \cdot \vec{A}) - \nabla \times (\vec{v} \times \vec{A}) \right) = A_i \left[\partial_i(\vec{v} \cdot \vec{A}) - \epsilon_{ijk} \partial_j (\vec{v} \times \vec{A})_k \right]$

$= A_i \partial_i (v_\ell A_\ell) - \epsilon_{ijk} \partial_j \epsilon_{k\ell m} v_\ell A_m$

$= A_i A_\ell \partial_i v_\ell - \epsilon_{kij} \epsilon_{k\ell m} A_m \partial_j v_\ell A_i$

$= A_i A_\ell \partial_i v_\ell - (\delta_{i\ell} \delta_{jm} - \delta_{im} \delta_{j\ell}) A_m \partial_j v_\ell A_i$

$= A_i A_\ell \partial_i v_\ell - A_i \delta_{i\ell} \delta_{jm} A_m \partial_j v_\ell + A_i \delta_{im} \delta_{j\ell} A_m \partial_j v_\ell$

$= A_i A_\ell \partial_i v_\ell - A_i A_j \partial_j v_i + A_i A_i \partial_j v_j$

$= \cancel{A_\ell A_i \partial_i v_\ell} - \cancel{A_\ell A_i \partial_i v_\ell} + (\vec{A} \cdot \vec{A}) \nabla \cdot \vec{v}$

$\Downarrow$ renombramos los índices

$\Downarrow$ $A^2 = 1$ (es $\vec{A}$ unitario)

$\therefore$

$\vec{A} \cdot \left[\nabla(\vec{v} \cdot \vec{A}) - \nabla \times (\vec{v} \times \vec{A}) \right] = \nabla \cdot \vec{v}$

QED.