— Bases continuas — Complemento III

$\Downarrow$

conexión con magnitudes físicas (caso 1D)

i) x = posición , $x \in]-\infty, \infty[$ $(x \in \mathbb{R})$

$\sqcap$ $\hat{X}|x\rangle = x|x\rangle \Rightarrow \hat{X}^{\dagger} = \hat{X} \wedge \{|x\rangle\}$

suponemos que x es un autovalor

$\Downarrow$

$* \ \langle x|x'\rangle = \delta(x-x')$

$* \ \int_{-\infty}^{\infty} |x\rangle\langle x| dx = \hat{\mathbb{1}}$

* Sea un vector $|\phi\rangle$ arbitrario:

$\Downarrow$

$$|\phi\rangle = \int_{-\infty}^{\infty} \phi(x)\, |x\rangle\, dx \quad \text{con } \phi(x) = \langle x|\phi\rangle$$

ii) $\vec{k}$ = vector n° de onda $\sqcap$ $|k| = \frac{2\pi}{\lambda} = k$

con $k \in \mathbb{R} \Rightarrow k \in]-\infty, \infty[$

$\sqcap$ $\hat{k}|k\rangle = k|k\rangle \Rightarrow \hat{k}^{\dagger} = \hat{k}$

y $\{|k\rangle\}$ una base ortogonal completa

$$\Downarrow$$

$$\langle k|k'\rangle = \delta(k-k')$$

$$\int_{-\infty}^{\infty} |k\rangle\langle k|\, dk = \hat{\mathbb{1}}$$

* para un vector arbitrario $|\phi\rangle$ se cumple que:

$$|\phi\rangle = \int_{-\infty}^{\infty} \tilde{\phi}(k)\, |k\rangle\, dk \quad \text{con} \quad \tilde{\phi}(k) = \langle k|\phi\rangle$$

— Relación entre $\phi(x)$ y $\tilde{\phi}(k)$ ——

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ikx}\, \tilde{\phi}(k)\, dk$$

$$\tilde{\phi}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx}\, \phi(x)\, dx$$

— Conmutador ——

$$[\hat{x}, \hat{k}] = i\,\hat{\mathbb{1}}$$

fórmulas útiles

Si $\langle x|k\rangle = \dfrac{e^{ikx}}{\sqrt{2\pi}}$

$$\Downarrow$$

$$k^n \langle x|k\rangle = (-i)^n \frac{d^n}{dx^n}\langle x|k\rangle$$

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$$\Downarrow$$

$$i^{2n} = (-1)^n$$

$$i^{2n+1} = i(-1)^n$$