PROBLEMA GUÍA 2 | #18

$$\text{Tr}\,\hat{C} = C_{ii} = R_{ik}R_{ik} \qquad \left(C_{ij} = R_{ik}R_{jk}\right)$$

con

$$R_{ik} = \delta_{ik}\cos\theta + n_i n_k(1-\cos\theta) - \operatorname{sen}\theta\,\epsilon_{ikg}n_g$$

$$\therefore$$

$$\text{Tr}\,\hat{C} = \left(\delta_{ik}\cos\theta + n_i n_k(1-\cos\theta) - \operatorname{sen}\theta\,\epsilon_{ikg}n_g\right)$$

$$\times \left(\delta_{ik}\cos\theta + n_i n_k(1-\cos\theta) - \operatorname{sen}\theta\,\epsilon_{ike}n_e\right)$$

$$= \delta_{ik}\delta_{ik}\cos^2\theta + \delta_{ik}n_i n_k\cos\theta(1-\cos\theta)$$
$$- \operatorname{sen}\theta\cos\theta\,\delta_{ik}\epsilon_{ike}n_e + \delta_{ik}n_i n_k\cos\theta(1-\cos\theta)$$
$$+ n_i n_k n_i n_k(1-\cos\theta)^2 - (1-\cos\theta)\operatorname{sen}\theta\,\epsilon_{ike}n_e n_i n_k$$
$$- \operatorname{sen}\theta\cos\theta\,\delta_{ik}\epsilon_{ikg}n_g - (1-\cos\theta)\operatorname{sen}\theta\,\epsilon_{ikg}n_i n_g n_k$$
$$+ \operatorname{sen}^2\theta\,\epsilon_{ikg}\epsilon_{ike}n_g n_e$$

Obs.

$$\epsilon_{ike}\, n_e\, n_i\, n_k = \left(\epsilon_{ike}\, n_k\, n_e\right) n_i$$

$$= \left(\vec{n} \times \vec{n}\right)_i \overset{0}{} ; \quad n_i = \left(\vec{n} \times \vec{n}\right) \cdot \vec{n} \overset{0}{} = 0$$

De igual manera

$$\epsilon_{ikg}\, n_i\, n_g\, n_k = 0$$

Obs.

$$\delta_{ik}\, \delta_{ik} = \delta_{ii} = 3$$

Obs. $\quad n_i\, n_i\, n_k\, n_k = n^2\, n^2 = 1 \quad \left(\text{dado que por condición } |\vec{n}| = n = 1\right)$

Obs

$$\delta_{ik}\, \epsilon_{ikg}\, n_g = \epsilon_{iig}\, n_g = 0$$

y

$$\delta_{ik}\, \epsilon_{ike}\, n_e = \epsilon_{iie}\, n_e = 0$$

Obs. $\quad \delta_{ik}\, n_i\, n_k = n_i\, n_i = n^2 = 1.$

$\therefore$

$$Tr\, \hat{C} = 3\cos^2\theta + 2\cos\theta(1-\cos\theta) + (1-\cos\theta)^2$$

$$+ \operatorname{sen}^2\theta \underbrace{(\delta_{ke}\, \delta_{ge} - \delta_{kg}\, \delta_{ke})}_{3}\, n_g\, n_e$$

$$\text{Tr}\,\hat{C} = 3\cos^2\theta + 2\cos\theta(1-\cos\theta) + (1-\cos\theta)^2$$

$$+ 2\,\text{sen}^2\theta$$

$$= 2\cos^2\theta + 1 + 2\,\text{sen}^2\theta = 3 \; /\!/$$