Formulario II

1 Funciones Hipergeométricas conocidas (Ver.2.0)

1.1 Funciones elementales

$$\left[\frac{1}{2}(1-x)^{\frac{1}{2}}+\frac{1}{2}\right]^{1-2a} = {}_2F_1\left(\begin{array}{c} 2a \quad , \quad a+1 \\ a \end{array}\middle| x\right) \tag{1}$$

$$(1+x)^a = {}_2F_1\left(\begin{array}{c} -a \quad , \quad b \\ b \end{array}\middle| -x\right) = {}_1F_0\left(\begin{array}{c} -a \\ - \end{array}\middle| -x\right) \tag{2}$$

$$\exp(x) = \lim_{a\longrightarrow\infty} {}_2F_1\left(\begin{array}{c} a \quad , \quad b \\ b \end{array}\middle| \frac{x}{a}\right) = {}_0F_0\left(\begin{array}{c} - \\ - \end{array}\middle| x\right) \tag{3}$$

$$(1-x)^{-2a-1}(1+x) = {}_2F_1\left(\begin{array}{c} 2a \quad , \quad a+1 \\ a \end{array}\middle| x\right) \tag{4}$$

$$\cos(x) = {}_0F_1\left(\begin{array}{c} - \\ \frac{1}{2} \end{array}\middle| -\tfrac{1}{4}x^2\right) \tag{5}$$

$$\sin(x) = x\; {}_0F_1\left(\begin{array}{c} - \\ \frac{3}{2} \end{array}\middle| -\tfrac{1}{4}x^2\right) \tag{6}$$

$$\ln(1+x) = x\; {}_2F_1\left(\begin{array}{c} 1 \quad , \quad 1 \\ 2 \end{array}\middle| -x\right) \tag{7}$$

$$\ln\left[\frac{(1+x)}{(1-x)}\right] = 2x\; {}_2F_1\left(\begin{array}{c} \frac{1}{2} \quad , \quad 1 \\ \frac{3}{2} \end{array}\middle| x^2\right) \tag{8}$$

$$\arcsin(x) = x\; {}_2F_1\left(\begin{array}{c} \frac{1}{2} \quad , \quad \frac{1}{2} \\ \frac{3}{2} \end{array}\middle| x^2\right) \tag{9}$$

$$\arctan(x) = x\; {}_2F_1\left(\begin{array}{c} \frac{1}{2} \quad , \quad 1 \\ \frac{3}{2} \end{array}\middle| -x^2\right) \tag{10}$$

1.2 Funciones Especiales

1.2.1 Polinomios

$$C_n^{\lambda}(x) = \frac{(2\lambda)_n}{n!}\; {}_2F_1\left(\begin{array}{c} -n \quad , \quad n+2\lambda \\ \lambda+\frac{1}{2} \end{array}\middle| \frac{(1-x)}{2}\right) \qquad \text{Gegenbauer} \tag{11}$$

$$P_n(x) = {}_2F_1\left(\begin{array}{c} -n \quad , \quad n+1 \\ 1 \end{array}\middle| \tfrac{1}{2}(1-x)\right) \qquad \text{Legendre} \tag{12}$$

$$H_n(x) = (2x)^n\; {}_2F_0\left(\begin{array}{c} -\frac{n}{2} \quad , \quad \frac{(1-n)}{2} \\ - \end{array}\middle| -\frac{1}{x^2}\right) \qquad \text{Hermite} \tag{13}$$

$$L_n^{\alpha}\left(x\right)=\frac{\left(1+\alpha\right)_n}{n!}\ {}_1F_1\left(\begin{matrix}-n\\1+\alpha\end{matrix}\middle|\,x\right)\qquad \text{Laguerre} \tag{14}$$

$$P_n^{(\alpha,\beta)}\left(x\right)=\frac{\left(1+\alpha\right)_n}{n!}\ {}_2F_1\left(\begin{matrix}-n\quad , \quad n+\alpha+\beta+1\\1+\alpha\end{matrix}\middle|\,\frac{1-x}{2}\right)\qquad \text{Jacobi} \tag{15}$$

1.2.2 Series Infinitas

$$K\left(x\right)=\tfrac{1}{2}\pi\ {}_2F_1\left(\begin{matrix}\frac{1}{2}\quad , \quad \frac{1}{2}\\1\end{matrix}\middle|\,x^2\right)\qquad \text{Elíptica tipo I} \tag{16}$$

$$E\left(x\right)=\tfrac{1}{2}\pi\ {}_2F_1\left(\begin{matrix}-\frac{1}{2}\quad , \quad \frac{1}{2}\\1\end{matrix}\middle|\,x^2\right)\qquad \text{Elíptica tipo II} \tag{17}$$

$$J_\alpha\left(x\right)=\left(\frac{x}{2}\right)^{\alpha}\frac{1}{\Gamma\left(1+\alpha\right)}\ {}_0F_1\left(\begin{matrix}-\\1+\alpha\end{matrix}\middle|\,-\tfrac{1}{4}x^2\right)\qquad \text{Bessel de primer tipo} \tag{18}$$

$$I_\alpha\left(x\right)=\left(\frac{x}{2}\right)^{\alpha}\frac{1}{\Gamma\left(1+\alpha\right)}\ {}_0F_1\left(\begin{matrix}-\\1+\alpha\end{matrix}\middle|\,\tfrac{1}{4}x^2\right)\qquad \text{Bessel modificada de primer tipo} \tag{19}$$

$$H_\alpha\left(x\right)=\left(\frac{x}{2}\right)^{1-\alpha}\frac{1}{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{3}{2}+\alpha\right)}\ {}_1F_2\left(\begin{matrix}1\\\frac{3}{2},\ \ \frac{3}{2}+\alpha\end{matrix}\middle|\,-\tfrac{1}{4}x^2\right)\qquad \text{Struve} \tag{20}$$

$$s_{\mu,\nu}\left(x\right)=\frac{x^{1+\mu}}{\left(1+\mu-\nu\right)\left(1+\mu+\nu\right)}\ {}_1F_2\left(\begin{matrix}1\\\frac{1}{2}\left(3+\mu-\nu\right),\ \ \frac{1}{2}\left(3+\mu+\nu\right)\end{matrix}\middle|\,-\tfrac{1}{4}x^2\right)\qquad \text{Lommel} \tag{21}$$

$$J_\mu\left(z\right)J_\nu\left(z\right)=\left(\frac{z}{2}\right)^{\mu+\nu}\frac{1}{\Gamma\left(\mu+1\right)\Gamma\left(\nu+1\right)}\ {}_2F_3\left(\begin{matrix}\frac{1}{2}\left(\mu+\nu+1\right),\ \ \frac{1}{2}\left(\mu+\nu+2\right)\\\mu+1,\ \ \nu+1,\ \ \mu+\nu+1\end{matrix}\middle|\,-z^2\right) \tag{22}$$

$$\operatorname{erf}\left(x\right)=\tfrac{2}{\sqrt{\pi}}\int_0^x dt\ \exp\left(-t^2\right)=\tfrac{2x}{\sqrt{\pi}}\ {}_1F_1\left(\begin{matrix}\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\,-x^2\right)=\tfrac{2x}{\sqrt{\pi}}\exp\left(-x^2\right)\ {}_1F_1\left(\begin{matrix}1\\\frac{3}{2}\end{matrix}\middle|\,x^2\right)\qquad \text{Función Error} \tag{23}$$

$$\gamma\left(a,x\right)\quad=\int_0^x t^{a-1}\exp\left(-t\right)\ dt=\frac{x^a}{a}\ {}_1F_1\left(\begin{matrix}1\\1+a\end{matrix}\middle|\,x\right)\qquad \text{Gamma Incompleta Inferior} \tag{24}$$

$$\Gamma\left(a,x\right)\quad=\int_x^\infty t^{a-1}\exp\left(-t\right)\ dt=\Gamma\left(a\right)-\frac{x^a}{a}\ {}_1F_1\left(\begin{matrix}1\\1+a\end{matrix}\middle|\,x\right)\qquad \text{Gamma Incompleta Superior} \tag{25}$$

1.2.3 Otras series útiles

$$Li_s\left(x\right)=\sum_{n=1}^{\infty}\frac{x^n}{n^s}\quad \text{Función polilogaritmo} \tag{26}$$

$$\zeta_s=\sum_{n=1}^{\infty}\frac{1}{n^s}\quad \text{Función Zeta de Riemann} \tag{27}$$

$$\zeta\left(s,a\right)=\sum_{n=0}^{\infty}\frac{1}{\left(n+a\right)^s}\quad \text{Función Zeta de Hurwitz} \tag{28}$$

$$x^{-\alpha}\left(1-x\right)^{-\beta}\int\limits_0^x dt\ t^{\alpha-1}\left(1-t\right)^{\beta-1}=\frac{1}{\alpha}\frac{\Gamma\left(\alpha+\beta\right)}{\Gamma\left(\alpha\right)\Gamma\left(\beta\right)}\ {}_2F_1\left(\begin{array}{c} 1\quad ,\quad \alpha+\beta \\ \alpha+1 \end{array}\middle|\, x\right) \tag{29}$$

2 Identidades importantes

$${}_2F_1\left(\begin{array}{c} -n\quad ,\quad b \\ c \end{array}\middle|\, 1\right)=\frac{\left(c-b\right)_n}{\left(c\right)_n} \tag{30}$$

$${}_2F_1\left(\begin{array}{c} a\quad ,\quad b \\ c \end{array}\middle|\, 1\right)=\frac{\Gamma\left(c\right)\Gamma\left(c-a-b\right)}{\Gamma\left(c-a\right)\Gamma\left(c-b\right)} \tag{31}$$