Review of dielectric and magnetic materials

Dielectric properties of materials are due to atomic-scale electric dipoles. Atoms and molecules have induced dipoles so that when an electric field is applied they have a dipole moment. Often the response is linear a low fields so that $\vec{p} = \alpha \vec{E}$, where α is the electric polarizability. Molecules may have permanent dipoles, for example water has a permanent dipole. Permanent dipoles are in random directions at high temperature and low electric fields however they align at low temperatures and high electric fields. The degree of alignment or polarization is given by the Langevin formula, $< p_z >= p_0[coth(a) - 1/a]$ where $a = p_0 E/k_B T$. This expression is linear for small a. The density of aligned atomic-scale electric dipoles is the polarization, $\vec{P}$ that is treated in Maxwell's equations.

Magnetic properties of materials are due to atomic-scale magnetic dipoles. Atoms and molecules have induced orbital magnetic dipoles so that when a magnetic field is applied they have a dipole moment. Often the response is linear a low fields so that $\vec{m} = \alpha_m \vec{B}$, where α_m is the magnetic polarizability. Atoms and molecules may also have permanent magnetic dipoles primarily due to unpaired electrons. Electrons have an intrinsic magnetic dipole moment that is almost the same as the Bohr magneton. Permanent dipoles are in random directions at high temperature and low magnetic fields however they align at low temperatures and high magnetic fields. The degree of alignment or magnetization is given by the Langevin formula, $< m_z >= p_0[coth(a) - 1/a]$ where $a = m_0 B/k_B T$. This expression is linear for small a. The density of aligned atomic-scale magnetic dipoles is the magnetization, $\vec{M}$ that is treated in Maxwell's equations.

The electric fields produced by polarization are calculated by finding bulk and surface charges that may be used to produce the observed polarization,

$$\sigma_b = \vec{P} \cdot \hat{n}; \quad \rho_b = -\vec{\nabla} \cdot \vec{P} \tag{1}$$

The source terms are called bound charges to distinguish them from excess charges in materials. These excess charges are called free charges. If we sum up all of the bound charge in a material it must sum to zero. Maxwell's equations for electrostatics still apply, but now we must make the replacement $q \rightarrow q_f + q_b$. In addition it is useful to introduce the displacement field $\vec{D}$ that is produced by the free charges alone. Maxwell's equations for the electrostatics of dielectric materials are then,

$$\oint \vec{E} \cdot d\vec{a} = \frac{q_f + q_b}{\epsilon_0}; \quad \oint \vec{D} \cdot d\vec{a} = q_f; \quad \oint \vec{E} \cdot d\vec{l} = 0 \tag{2}$$

From these equations we find that the boundary conditions across an interface that has surface charge densities σ_b and σ_f are,

$$E_{above}^{\perp} - E_{below}^{\perp} = \frac{\sigma_f + \sigma_b}{\epsilon_0}; \quad D_{above}^{\perp} - D_{below}^{\perp} = \sigma_f \quad \vec{E}_{above}^{\parallel} - \vec{E}_{below}^{\parallel} = 0 \tag{3}$$

The electrostatic potential is continuous across the surface. In differential form Maxwell's equations for electrostatics in materials are,

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0}(\rho_f + \rho_b); \quad \vec{\nabla} \cdot \vec{D} = \rho_f; \quad \vec{\nabla} \wedge \vec{E} = 0 \tag{4}$$

Using the second of these equations in the first, along with $\rho_b = -\vec{\nabla} \cdot \vec{P}$ yields

$$\epsilon_0 \vec{E} = \vec{D} - \vec{P} \tag{5}$$

Standard problems are the elecric field, potential, displacement field and bound charge density for uniformly polarized sphere, cylinder and disc or slab (Fields can be calculated inside and far away from the object). Note that $\vec{D}$ is not zero even when ρ_f is zero as only the surface integral of $\vec{D}$ has to be zero in that case. Once $\vec{E}$ has been found then $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$.

The magnetic fields produced by magnetization are calculated by finding bulk and surface currents that may be used to produce the observed magnetization,

$$\vec{K}_b = \vec{M} \wedge \hat{n}; \quad \vec{j}_b = \vec{\nabla} \wedge \vec{M}. \tag{6}$$

The source terms are called bound currents to distinguish them from excess currents in materials. These excess currents are called free currents. If we sum up all of the bound currents crossing a surface of separation in the material they must sum to zero. Maxwell's equations for magnetostatics still apply, but now we must make the

replacement $i \rightarrow i_f + i_b$. In addition it is useful to introduce the auxilliary field $\vec{H}$ that is produced by the free currents alone. Maxwell's equations for the magnetostatics in magnetic materials are then,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0(i_f + i_b); \quad \oint \vec{H} \cdot d\vec{l} = i_f; \quad \oint \vec{B} \cdot d\vec{a} = 0 \tag{7}$$

From these equations we find that the boundary conditions across an interface that has surface current densities $\vec{K}_b$ and $\vec{K}_f$ are,

$$B^{\parallel}_{above} - B^{\parallel}_{below} = \mu_0(K_f + K_b); \quad H^{\parallel}_{above} - H^{\parallel}_{below} = K_f; \quad B^{\perp}_{above} - B^{\perp}_{below} = 0 \tag{8}$$

If we introduce a scalar magnetic potential it is continuous across the interface. In differential form Maxwell's equations for magnetostatics of materials are,

$$\vec{\nabla} \wedge \vec{B} = \mu_f(\vec{j}_f + \vec{j}_b); \quad \vec{\nabla} \wedge \vec{H} = \vec{j}_f; \quad \vec{\nabla} \cdot \vec{B} = 0 \tag{9}$$

Using the second of these equations in the first, along with $\vec{j}_b = \vec{\nabla} \wedge \vec{M}$ yields

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \tag{10}$$

Standard problems are the magnetic field, magnetic intensity and bound current density for uniformly magnetized sphere, cylinder and disc or slab (Fields can be calculated inside and far away from the object). Note that $\vec{H}$ is not zero even when $\vec{j}_f$ is zero as only the line integral of $\vec{H}$ has to be zero in that case. Once $\vec{B}$ has been found then $\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$.

In many cases instead of being given the polarization, we are instead given a relation between the electric field and the induced polarization. This is called a constitutive relation and in the simplest case is isotropic and linear, so that

$$\vec{P} = \epsilon_0 \chi_e \vec{E}; \quad \text{or} \quad \vec{D} = \epsilon_0(1 + \chi_e)\vec{E} = \epsilon \vec{E} \tag{11}$$

where $\epsilon = \epsilon_0(1 + \chi_e)$ is the permittivity and is found using the first of the above equations along with Eq. (5). The relative permittivity is $\epsilon_r = 1 + \chi_e = \epsilon/\epsilon_0$ is also often used. Typical problems in linear isotropic dielectric materials include: Electric field, potential and bound charges for a dielectric sphere with either a point charge or a dipole at the center. Capacitor problems (either slab, concentric cylinder or concentric sphere) where a dielectric material is used in the capacitor, including cases where the capacitor is partially filled with dielectric material. The energy density is $\frac{1}{2}\vec{D} \cdot \vec{E}$. The energy of a capacitor is still $\frac{1}{2}CV^2$ but the capacitance has to take into account the dielectric material. The lower energy state of any capacitor circuit is when the dielectric material (provided $\epsilon_r > 1$) is between the plates of the capacitor. Sometimes $\epsilon_0 \rightarrow \epsilon$ works, but this must be used with care. For example the capacitance of a capacitor that is completely filled with a dielectric material with dielectric constant ϵ is $\epsilon A/d$. Common boundary value problems are the electric field and potential inside and outside a sphere or cylinder of dielectric material that is placed in a uniform electric field. The image charge method can also be extended, for example to the case of a point charge above a dielectric half-space.

Similarly, in many cases instead of being given the magnetization, we are instead given a relation between the magnetic field and the induced magnetization. This is called a constitutive relation and in the simplest case is isotropic and linear, so that

$$\vec{M} = \chi_m \vec{H}; \quad \text{or} \quad \vec{B} = \mu_0(1 + \chi_m)\vec{H} = \mu \vec{H} \tag{12}$$

where $\mu = \mu_0(1 + \chi_m)$ is the permeability and is found using the first of the above equations along with Eq. (10). The relative permeability is $\mu_r = 1 + \chi_m = \mu/\mu_0$ is also often used. Typical problems in linear isotropic magnetic materials include: Inductor problems where a magnetic material is used in the inductor, including cases where the inductor is partially filled with magnetic material. The energy density is $\frac{1}{2}\vec{B} \cdot \vec{H}$. The energy of an inductor is still $\frac{1}{2}CV^2$ but the inductance has to take into account the magneric material. The lower energy state of any inductor circuit for a material with $\mu_r > 1$ material is when the material is inside the inductor. In contrast for cases where $\mu_r < 1$ the lower energy state is when the material is outside the inductor. Sometimes $\mu_0 \rightarrow \mu$ works, but this must be used with care. For example the inductance of a inductor that is completely filled with a magnetic material with permeability μ is $\mu N^2 A/l$.

The magnetic field inside magnetic materials with $\mu_r > 1$ is amplified, while the electric field inside materials with $\epsilon_r > 1$ is reduced even though in both cases the material is attracted to regions of high field. The correct correspondence is between the charge of a capacitor and the flux through an inductor. This correspondence shows that the two cases are actually analogous.