Problema, Guía II | #4

1) Sea la componente i-ésima del vector $\nabla f(r)$:

$$(\nabla f(r))_i = \partial_i f(r) = \frac{\partial}{\partial x_i} f(r) = \frac{\partial r}{\partial x_i} \frac{\partial}{\partial r} f(r)$$

$$= (\partial_i r) \frac{d}{dr} f(r)$$

$$(\nabla f(r))_i = \frac{x_i}{r} \frac{d}{dr} f(r)$$

$\therefore$

$$\nabla f(r) = \frac{\vec{r}}{r} \frac{d}{dr} f(r) \quad //$$

2) $$\nabla \cdot \vec{F}(r) = \partial_i F_i(r) = \frac{\partial}{\partial x_i} F_i(r) = \frac{\partial r}{\partial x_i} \frac{\partial}{\partial r} F_i(r)$$

$$= (\partial_i r) \frac{d}{dr} F_i(r)$$

$$= \frac{x_i}{r} \frac{d}{dr} F_i(r) = \frac{1}{r} \vec{r} \cdot \frac{d\vec{F}(r)}{dr}$$

3) $\nabla f(\xi) = \nabla f(\vec{A} \cdot \vec{r})$, la componente i-ésima está dada por la expresión:

$$(\nabla f(A \cdot \vec{r}))_i = \partial_i f(\vec{A} \cdot \vec{r}) = \partial_i f(\xi)$$

$$= \frac{\partial}{\partial x_i} f(\xi) = \frac{\partial \xi}{\partial x_i} \frac{\partial}{\partial \xi} f(\xi)$$

$$= \partial_i \xi \; \frac{d}{d\xi} f(\xi)$$

$$= \partial_i (\vec{A} \cdot \vec{r}) \; \frac{df(\xi)}{d\xi}$$

$$= \partial_i (A_\ell x_\ell) \; \frac{df(\xi)}{d\xi}$$

$$= A_\ell (\partial_i x_\ell) \; \frac{df(\xi)}{d\xi}$$

$$= A_\ell \delta_{i\ell} \; \frac{df(\xi)}{d\xi}$$

$$(\nabla f(\xi))_i = A_i \frac{df(\xi)}{d\xi}$$

$$\Downarrow$$

$$\nabla f(\xi) = \vec{A} \frac{df(\xi)}{d\xi} \; //$$