PROBLEMA GUÍA I | #13

$$\nabla p(\vec{r}) = \rho(\vec{r})\vec{f}(\vec{r}) \implies f(\vec{r}) = \frac{1}{\rho(\vec{r})}\nabla p(\vec{r}) \implies f_k = \frac{1}{\rho}\partial_k p$$

$$\vec{f}\cdot\nabla\times\vec{f} = f_i(\nabla\times\vec{f})_i = f_i\,\epsilon_{ijk}\,\partial_j f_k$$

$$= f_i\,\epsilon_{ijk}\,\partial_j\left(\frac{1}{\rho}\nabla p\right)_k$$

$$= f_i\,\epsilon_{ijk}\,\partial_j\left(\frac{1}{\rho}\partial_k p\right)$$

$$= f_i\,\epsilon_{ijk}\left[\partial_j\left(\frac{1}{\rho}\right)\partial_k p + \frac{1}{\rho}\partial_j\partial_k p\right]$$

$$= f_i\,\epsilon_{ijk}\,\partial_j\left(\frac{1}{\rho}\right)\partial_k p + \frac{f_i}{\rho}\,\epsilon_{ijk}\,\partial_j\partial_k p$$

$$= \partial_j\left(\frac{1}{\rho}\right)\epsilon_{ijk}\,f_i\,\partial_k p + \frac{1}{\rho}f_i(\cancel{\nabla\times\nabla p})_i \qquad \circ\ \nabla\times\nabla\phi = \vec{0}$$

$$= -\partial_j\left(\frac{1}{\rho}\right)\epsilon_{jik}\,f_i\,\partial_k p$$

$$= -\left(\nabla\frac{1}{\rho}\right)_j\left(\vec{f}\times\nabla p\right)_j$$

$$= -\nabla\left(\frac{1}{\rho}\right)\cdot\vec{f}\times\nabla p = -\nabla\left(\frac{1}{\rho}\right)\cdot\cancel{\vec{f}\times(\rho\vec{f})}^{\,\vec{0}}$$

$$= 0 \quad \text{QED}$$