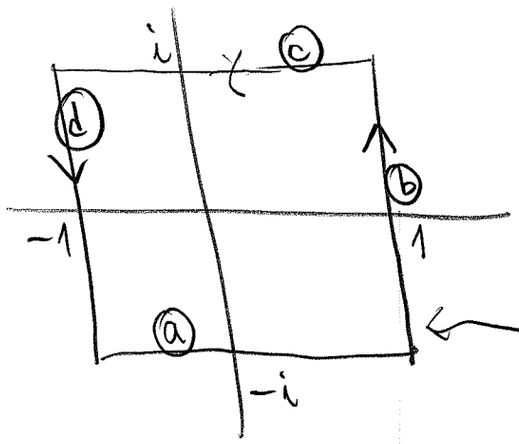

$f(z) = \frac{1}{z}$

trayectoria cerrada γ_1

Evaluar

$$\int_{\gamma_1} \frac{1}{z}\,dz = \int_{\gamma_a} \frac{1}{z}\,dz + \cdots + \int_{\gamma_d} \frac{1}{z}\,dz$$

Para γ_a

$z = x + iy$ con $y = -1$

$x = t \Rightarrow t: -1 \to 1$

$\Downarrow$

$z = t - i$

$\Downarrow$

$dz = dt$

$$\therefore \int_{\gamma_a} \frac{1}{z}\,dz = \int_{-1}^{1} \frac{1}{t-i}\,dt$$

Obs: $$\int_a^b \frac{1}{z+c}\,dt = \int_{a+c}^{b+c} \frac{1}{u}\,du = \ln u \Big|_{a+c}^{b+c} = \ln\left(\frac{b+c}{a+c}\right)$$

Luego $\int_{\gamma_a} \frac{1}{z}\,dz = \int_{-1}^{1} \frac{1}{t-i}\,dt = \ln\left(\frac{1-i}{-1-i}\right)$

Para γ_b

$z = x + iy$ con $x = 1$

$y = t \longrightarrow t: -1 \longrightarrow 1$

$\downarrow$

$z = 1 + it$

$\Downarrow$

$dz = i\,dt$

$\therefore \int_{\gamma_b} \frac{1}{z}\,dz = \int_{-1}^{1} \frac{1}{1+it}\,i\,dt = \int_{-1}^{1} \frac{1}{-i+t}\,dt$

$= \ln\left(\frac{1-i}{-1-i}\right)$

Para γ_c

$z = x + iy$ con $x = t$

$y = 1$

$\Downarrow$

$z = t + i \longrightarrow t: 1 \to -1$

$dz = dt$

$\therefore \int_{\gamma_c} \frac{1}{z}\,dz = \int_{1}^{-1} \frac{1}{t+i}\,dt = \ln\left(\frac{-1+i}{1+i}\right)$

Para γ_2

$z = x + iy$ con $x = -1$

$y = t \quad t: 1 \to -1$

$\Downarrow$

$z = -1 + it$

$\Downarrow$

$dz = i\,dt$

$$\int_{\gamma_2} \frac{1}{z}\,dz = \int_1^{-1} \frac{1}{-1+it}\,i\,dt = \int_1^{-1} \frac{1}{t+i}\,dt = \ln\left(\frac{-1+i}{1+i}\right)$$

Finalmente

$$\int_{\gamma_1} \frac{1}{z}\,dz = \ln\left(\frac{1-i}{-1-i}\right) + \ln\left(\frac{1-i}{-1-i}\right) + \ln\left(\frac{-1+i}{1+i}\right) + \ln\left(\frac{-1+i}{1+i}\right)$$

$$* = \ln\left(\frac{i-1}{1+i}\right) + \ln\left(\frac{i-1}{1+i}\right) + \ln\left(\frac{i-1}{1+i}\right) + \ln\left(\frac{i-1}{1+i}\right) = 4\ln\left(\frac{i-1}{1+i}\right)$$

Ahora $\ln\left(\frac{i-1}{1+i}\right) = \ln\left(\frac{i\left(1-\frac{1}{i}\right)}{1+i}\right)$

$= \ln\left(\frac{i(1+i)}{1+i}\right) = \ln i$

$= \ln|i| + i\,\mathrm{Arg}(i)$

$= \ln 1 + i\frac{\pi}{2}$. //

$$\int_{\gamma_1} \frac{1}{z}\,dz = 4\, i\frac{\pi}{2} = 2\pi i$$

FIN