

PROBLEMA GUÍA II / #1

Para la i -ésima componente se tiene que:

$$\left(\nabla \times \nabla [e^{-\nabla \cdot (\alpha r^2 \vec{r})}] \right)_i = \epsilon_{ijk} \partial_j \partial_k (e^{-\nabla \cdot (\alpha r^2 \vec{r})})$$

$$= \epsilon_{ijk} \partial_j \partial_k e^{-\partial_x \alpha r^2 x_e}$$

$$= \epsilon_{ijk} \partial_j \partial_k e^{-\alpha \partial_x (r^2 x_e)}$$

Obs. $\partial_x (r^2 x_e) = (\partial_x r^2) x_e + r^2 \partial_x x_e$

$$\begin{aligned} &= 2r(\partial_x r) x_e + r^2 \delta_{xx} \\ &= 2r \frac{x_e}{r} x_e + 3r^2 \\ &= 2x_e x_e + 3r^2 \\ &= 2r^2 + 3r^2 = 5r^2. \end{aligned}$$

∴

$$\left(\nabla \times \nabla [e^{-\nabla \cdot (\alpha r^2 \vec{r})}] \right)_i = \epsilon_{ijk} \partial_j \partial_k e^{-5\alpha r^2}$$

Obs. sea $r^2 = u$

veamos

$$\partial_k e^{-5\alpha r^2} = \partial_k e^{-5\alpha u} \quad ; \text{ usando la regla de la cadena}$$

\Downarrow

$$\begin{aligned} \partial_k e^{-5\alpha r^2} &= \frac{\partial}{\partial x_k} e^{-5\alpha r^2} = \frac{\partial u}{\partial x_k} \frac{\partial}{\partial u} e^{-5\alpha u} \\ &= \frac{\partial u}{\partial x_k} (-5\alpha e^{-5\alpha u}) \\ &= -5\alpha e^{-5\alpha u} \partial_k u \\ &= -5\alpha e^{-5\alpha r^2} (\partial_k r^2) \\ &= -5\alpha e^{-5\alpha r^2} 2r (\partial_k r) \\ &= -5\alpha e^{-5\alpha r^2} 2r \frac{x_k}{r} \\ &= -10\alpha e^{-5\alpha r^2} x_k \end{aligned}$$

Entonces:

$$\begin{aligned} \left(\nabla \times \nabla [e^{-\nabla \cdot (\alpha r^2 \vec{F})}] \right)_i &= \epsilon_{ijk} \partial_j (-10\alpha e^{-5\alpha r^2} x_k) \\ &= -10\alpha \epsilon_{ijk} \partial_j (x_k e^{-5\alpha r^2}) \end{aligned}$$

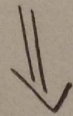
$$= -10\alpha \epsilon_{ijk} \left[e^{-5\alpha r^2} \underbrace{(\partial_j X_k)}_{\downarrow \delta_{jk}} + X_k \underbrace{(\partial_j e^{-5\alpha r^2})}_{\downarrow -10\alpha e^{-5\alpha r^2} X_j} \right]$$

$$= -10\alpha e^{-5\alpha r^2} \left[\cancel{\epsilon_{ijk} \delta_{jk}} - 10\alpha \underbrace{\epsilon_{ijk} X_j X_k}_{\downarrow 0} \right]$$

$(\vec{r} \times \vec{r})_i$

$$= 0 //$$

$$\therefore \nabla \times \nabla [e^{-\nabla \cdot (\alpha r^2 \vec{F})}] = \vec{0}$$



El resultado es obvio dado que el rotor de cualquier gradiente siempre es nulo.