PROBLEMA GUÍA II | #13

a) $[\hat{A}, b\hat{B}+c\hat{C}] = \hat{A}(b\hat{B}+c\hat{C}) - (b\hat{B}+c\hat{C})\hat{A}$

$= b\hat{A}\hat{B} + c\hat{A}\hat{C} - b\hat{B}\hat{A} - c\hat{C}\hat{A}$

$= b(\hat{A}\hat{B} - \hat{B}\hat{A}) + c(\hat{A}\hat{C} - \hat{C}\hat{A})$

$= b[\hat{A},\hat{B}] + c[\hat{A},\hat{C}]$ QED.

b) Similar a item (a)

c) $[\hat{A},\hat{B}\hat{C}] = \hat{A}\hat{B}\hat{C} - \hat{B}\hat{C}\hat{A} + 0$

$= \hat{A}\hat{B}\hat{C} - \hat{B}\hat{C}\hat{A} + \hat{B}\hat{A}\hat{C} - \hat{B}\hat{A}\hat{C}$

$= \hat{A}\hat{B}\hat{C} - \hat{B}\hat{A}\hat{C} + \hat{B}\hat{A}\hat{C} - \hat{B}\hat{C}\hat{A}$

$= (\hat{A}\hat{B} - \hat{B}\hat{A})\hat{C} + \hat{B}(\hat{A}\hat{C} - \hat{C}\hat{A})$

$= [\hat{A},\hat{B}]\hat{C} + \hat{B}[\hat{A},\hat{C}]$ QED.

d) Similar a item (c)

e) Analicemos el término $[\hat{A},[\hat{B},\hat{C}]]$:

* $[\hat{A},[\hat{B},\hat{C}]] = \hat{A}[\hat{B},\hat{C}] - [\hat{B},\hat{C}]\hat{A}$

$$= \hat{A}(\hat{B}\hat{C} - \hat{C}\hat{B}) - (\hat{B}\hat{C} - \hat{C}\hat{B})\hat{A}$$

$$= \hat{A}\hat{B}\hat{C} - \hat{A}\hat{C}\hat{B} - \hat{B}\hat{C}\hat{A} + \hat{C}\hat{B}\hat{A}$$

De igual manera

* $[\hat{C}, [\hat{A}, \hat{B}]] = \hat{C}[\hat{A}, \hat{B}] - [\hat{A}, \hat{B}]\hat{C}$

$$= \hat{C}(\hat{A}\hat{B} - \hat{B}\hat{A}) - (\hat{A}\hat{B} - \hat{B}\hat{A})\hat{C}$$

$$= \hat{C}\hat{A}\hat{B} - \hat{C}\hat{B}\hat{A} - \hat{A}\hat{B}\hat{C} + \hat{B}\hat{A}\hat{C}$$

y

* $[\hat{B}, [\hat{C}, \hat{A}]] = \hat{B}[\hat{C}, \hat{A}] - [\hat{C}, \hat{A}]\hat{B}$

$$= \hat{B}(\hat{C}\hat{A} - \hat{A}\hat{C}) - (\hat{C}\hat{A} - \hat{A}\hat{C})\hat{B}$$

$$= \hat{B}\hat{C}\hat{A} - \hat{B}\hat{A}\hat{C} - \hat{C}\hat{A}\hat{B} + \hat{A}\hat{C}\hat{B}$$

Al sumar los tres conmutadores se obtiene sume 0.