Chapter 6. Magnetic Fields in Matter

6.3 The Auxiliary Field H

6.3.1 Ampere's law in Magnetized Materials

$$\mathbf{J} = \mathbf{J}_b + \mathbf{J}_f. \qquad \mathbf{J}_b = \nabla \times \mathbf{M}$$

$$\frac{1}{\mu_0}(\nabla \times \mathbf{B}) = \mathbf{J} = \mathbf{J}_f + \mathbf{J}_b = \mathbf{J}_f + (\nabla \times \mathbf{M}) \longrightarrow \nabla \times \left(\frac{1}{\mu_0}\mathbf{B} - \mathbf{M}\right) = \mathbf{J}_f$$

$$\boxed{\mathbf{H} \equiv \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \ \left(\mathrm{A/m}\right)}$$

$$\boxed{\begin{aligned} &\nabla \times \mathbf{H} = \mathbf{J}_f \ \left(\mathbf{A/m^2}\right) \\ &\oint \mathbf{H} \cdot d\mathbf{l} = I_{f_{enc}} \end{aligned}}$$

: Ampere's law

➔ where $I_{f_{enc}}$ is the total *free* current passing through the Amperian loop.

➔ ***H permits us to express Ampere's law in terms of the free current alone.***

(p. 271)

*Many authors call **H,** not **B,** the "magnetic field."*
*Then they have to invent a new word for **B**: the "flux density," or magnetic "induction"*
(an absurd choice, since that term already has at least two other meanings in electrodynamics.)

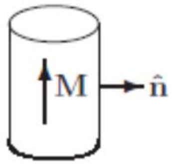
*Anyway, **B** is indisputably the fundamental quantity, so I shall continue to call it the "magnetic field,"*
as everyone does in the spoken language.
***H** has no sensible name: just call it "**H**".*

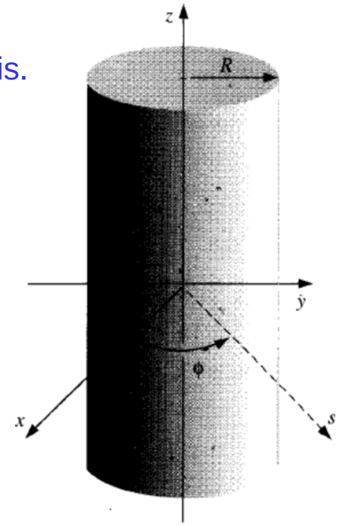
The unhappy term 'magnetic field' for H should be avoided as far as possible.
It seems to us that this term has led into error none less than Maxwell himself.

Ampere's law in Magnetized Materials

$$\nabla \times \mathbf{H} = \mathbf{J}_f \qquad \mathbf{H} \equiv \mathbf{B}/\mu_0 - \mathbf{M}$$

Problem 6.7 An infinitely long circular cylinder carries a uniform magnetization **M** parallel to its axis. Find the magnetic field (due to **M**) inside and outside the cylinder.



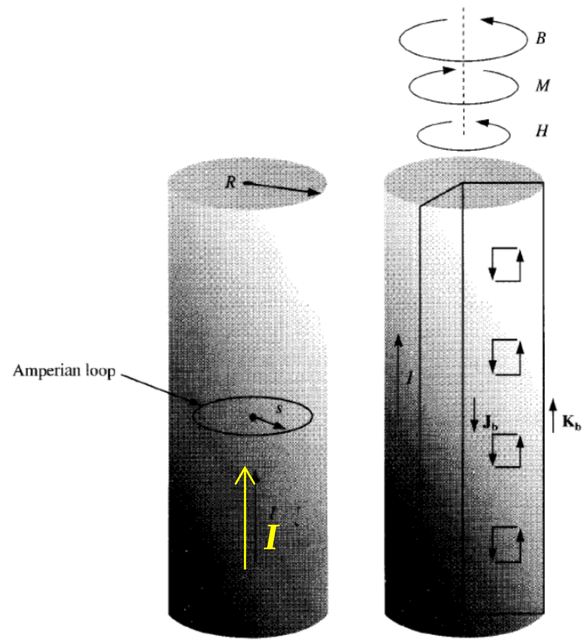


Problem 6.8 A long circular cylinder of radius *R* carries a magnetization $\mathbf{M} = ks^2\hat{\boldsymbol{\phi}}$, where *k is a constant, s is the distance from the axis.* Find the magnetic field (due to **M**) inside and outside the cylinder.

Ampere's law in Magnetized Materials

$$\nabla \times \mathbf{H} = \mathbf{J}_f \qquad \mathbf{H} \equiv \mathbf{B}/\mu_0 - \mathbf{M}$$

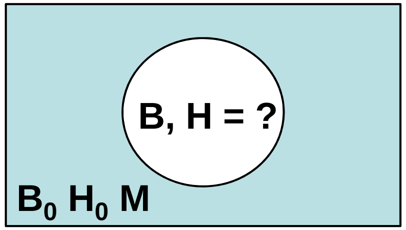
Example 6.2 A long copper rod of radius *R* carries a uniformly distributed (free) current ***I*** .
Find ***H*** inside and outside the rod.



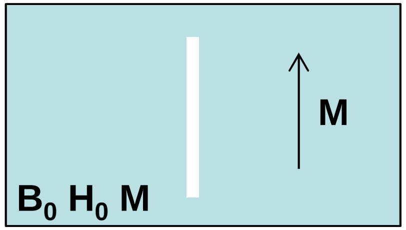
Ampere's law in Magnetized Materials $\nabla \times \mathbf{H} = \mathbf{J}_f \quad \mathbf{H} \equiv \mathbf{B}/\mu_0 - \mathbf{M}$

Problem 6.13 Suppose the field inside a large piece of magnetic material is Bo, so that $\mathbf{H_0 = (1/\mu_0)B_0 - M}$.

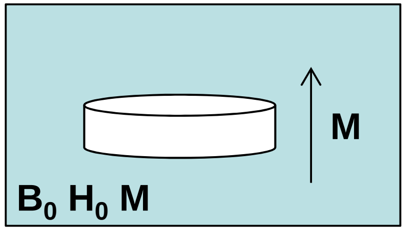
(a) Now a small spherical cavity is hollowed out of the material.
Find the field at the center of the cavity, in terms of $\mathbf{B_0}$ **and M**.
Also find **H** at the center of the cavity, in terms of $\mathbf{H_0}$ **and M.**



(b) Do the same for a long needle-shaped cavity running parallel to M.



(c) Do the same for a thin wafer-shaped cavity perpendicular to M.



6.3.2 A Deceptive Parallel

$$\nabla \times \mathbf{B} = \mathbf{J} = \mathbf{J}_f + \mathbf{J}_b \longrightarrow \mathbf{H} \equiv \mathbf{B}/\mu_0 - \mathbf{M} \longrightarrow \nabla \times \mathbf{H} = \mathbf{J}_f$$

$$\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M}$$

➔ It does *not* say that μ_0**H** is "just like **B**, only its source is $\mathbf{J}_f$ instead of **J**."

$$\nabla \cdot \mathbf{B} = 0 \longrightarrow \nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M} \neq \mathbf{0}$$

➔ The divergence of **H** is *not,* in general, zero.
➔ Only when the divergence of M vanishes is the parallel between B and μ_0**H** faithful.

When you are asked to find B or H in a problem involving magnetic materials,
first look for symmetry.

If the problem exhibits cylindrical, plane, solenoidal, or toroidal symmetry,
*Then you can get **H** directly from* $\nabla \times \mathbf{M} = \mathbf{J}_f$ *by the usual Ampere's law methods.*
(Evidently, in such cases $\nabla \cdot$*M is automatically zero, since the free current alone determines the answer.)*

If the requisite symmetry is absent,
*you **must not assume** that **H** is zero just because you see no free current.*

6.3.3 Boundary conditions

$$\nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M} \longrightarrow H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}} = -(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}})$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f \longrightarrow \mathbf{H}^{\parallel}_{\text{above}} - \mathbf{H}^{\parallel}_{\text{below}} = \mathbf{K}_f \times \hat{\mathbf{n}}.$$

$$\mathbf{D}^{\parallel}_{\text{above}} - \mathbf{D}^{\parallel}_{\text{below}} = \mathbf{P}^{\parallel}_{\text{above}} - \mathbf{P}^{\parallel}_{\text{below}}$$

$$D^{\perp}_{\text{above}} - D^{\perp}_{\text{below}} = \sigma_f$$

$$\mathbf{E}^{\parallel}_{\text{above}} - \mathbf{E}^{\parallel}_{\text{below}} = 0$$

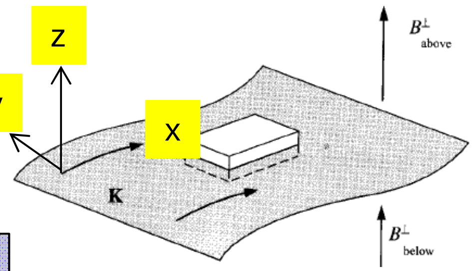
$$E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}} = \frac{1}{\epsilon_0}\sigma$$

$$(\sigma = \sigma_f + \sigma_b)$$

$$B^{\perp}_{\text{above}} = B^{\perp}_{\text{below}}$$

$$\mathbf{B}^{\parallel}_{\text{above}} - \mathbf{B}^{\parallel}_{\text{below}} = \mu_0(\mathbf{K} \times \hat{\mathbf{n}})$$

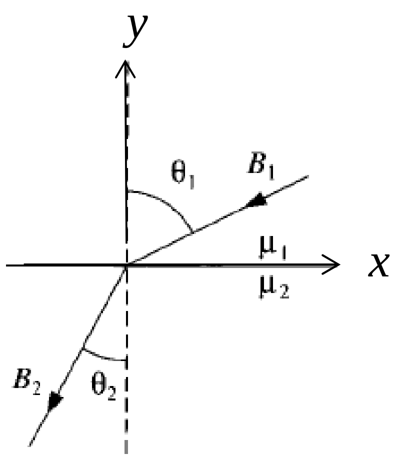
$$\mathbf{A}_{above} = \mathbf{A}_{below}$$



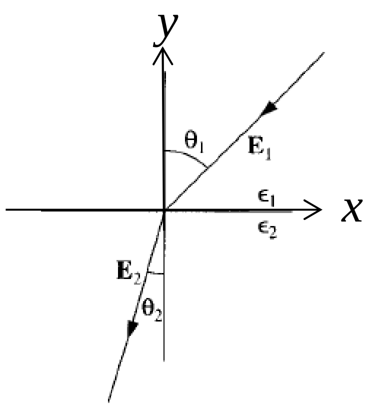
Boundary conditions

Problem 6.26 At the interface between two linear magnetic materials, the magnetic field lines bend.

Show that $\mu_1 \tan\theta_2 = \mu_2 \tan\theta_1$ (Assume there is no free current at the boundary.)



For electric field lines



$$\mathbf{E}^{\parallel}_{\text{above}} - \mathbf{E}^{\parallel}_{\text{below}} = 0 \longrightarrow E_{x_1} = E_{x_2}$$

$$D^{\perp}_{\text{above}} - D^{\perp}_{\text{below}} = \sigma_f = 0 \longrightarrow D_{y_1} = D_{y_2} \longrightarrow \epsilon_1 E_{y_1} = \epsilon_2 E_{y_2}$$

$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{E_{x_2}/E_{y_2}}{E_{x_1}/E_{y_1}} = \frac{E_{y_1}}{E_{y_2}} = \frac{\epsilon_2}{\epsilon_1} \longrightarrow \varepsilon_1 \tan\theta_2 = \varepsilon_2 \tan\theta_1$$

6.4 Linear and Nonlinear Media

6.4.1 Magnetic Susceptibility and Permeability

For most substances the magnetization is *proportional* to the field, provided the field is not too strong.

$$\mathbf{M} = \chi_m \mathbf{H}$$: for linear media

χ_m : **magnetic susceptibility**

Material	Susceptibility	Material	Susceptibility
Diamagnetic:		*Paramagnetic:*	
Bismuth	-1.6×10^{-4}	Oxygen	1.9×10^{-6}
Gold	-3.4×10^{-5}	Sodium	8.5×10^{-6}
Silver	-2.4×10^{-5}	Aluminum	2.1×10^{-5}
Copper	-9.7×10^{-6}	Tungsten	7.8×10^{-5}
Water	-9.0×10^{-6}	Platinum	2.8×10^{-4}
Carbon Dioxide	-1.2×10^{-8}	Liquid Oxygen ($-200°$ C)	3.9×10^{-3}
Hydrogen	-2.2×10^{-9}	Gadolinium	4.8×10^{-1}

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) = \mu_0 (1 + \chi_m) \mathbf{H}$$

$$\mu \equiv \mu_0 (1 + \chi_m)$$: **permeability**

$$\mu / \mu_0 \equiv \mu_r = (1 + \chi_m)$$: **relative permeability**

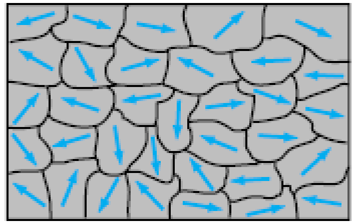
$$\mathbf{B} = \mu \mathbf{H}$$

Permeability

The permeability of most materials is very close to that of free space, μ_0
For ferromagnetic materials such as iron, nickel and cobalt, $(\mu / \mu_0) >> 1$

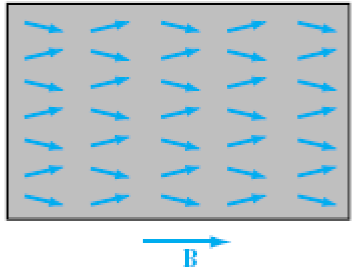
Magnetic Materials: Diamagnets, Paramagnets, Ferromagnets

- Diamagnetic, if $\mu_r \leq 1$ → magnetization *opposite* to B
  - χ_m ~ -10^{-5}
  - the orbital motion of the electrons
  - Copper, germanium, silver, gold



(a) Unmagnetized domains

- Paramagnetic, if $\mu_r \geq 1$ → magnetization *parallel* to B
  - χ_m ~ 10^{-5}
  - Magnetic dipole moments of the spinning electrons
  - Aluminum, magnesium, titanium and tungsten



(b) Magnetized domains

- Ferromagnetic, if μ_r >>1 → retain magnetization even aft
  - χ_m >> 1 (100~ 100,000)
  - Magnetized domains (strong coupling forces between the magnetic dipole moments of the atoms
  - Nickel, cobalt, iron (pure), mumetal

Analogous Relation

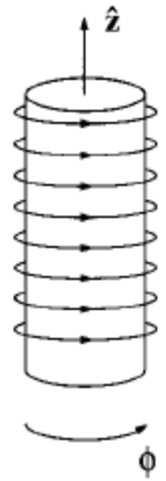
- Electrostatics and Magnetostatics

Electrostatics	Magnetostatics
E	B
D	H
ε	$1/\mu$
P	- M
ρ	J
V	A
$\bullet$	$\times$
$\times$	$\bullet$

Magnetic Susceptibility and Permeability in linear media

Example 6.3

An infinite solenoid (*n* turns per unit length, current *I*) is filled with linear material of susceptibility χ_m.
Find the magnetic field inside the solenoid and the bound current.



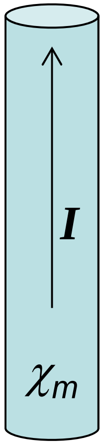
$$\nabla \times \mathbf{B} = \mathbf{J}_f + \mathbf{J}_b$$ → We cannot compute it directly due to $\mathbf{J_b}$.

→ This is one of those symmetrical cases in which we can get H from $\mathbf{J_f}$ alone.

$$\nabla \times \mathbf{H} = \mathbf{J}_f \quad \rightarrow \quad \oint \mathbf{H} \cdot d\mathbf{l} = I_{f_{enc}}$$

Magnetic Susceptibility and Permeability in linear media

Problem 6.17 A current $\boldsymbol{I}$ flows down a long straight wire of radius *a.*
If the wire is made of linear material with susceptibility χ_m and the current is distributed uniformly,
what is the magnetic field a distance *s* from the axis?
Find all the bound currents.
What is the *net* bound current flowing down the wire?



The magnetic field:

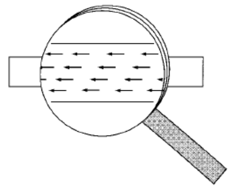
The bound currents:

The net bound current:

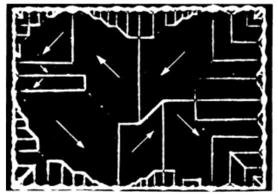
6.4.2 Ferromagnetism

Ferromagnets require no external fields to sustain the magnetization; the alignment is "frozen in.“

Like paramagnetism, ferromagnetism involves the magnetic dipoles associated with the spins of unpaired electrons.

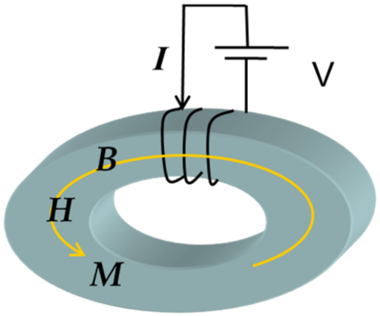


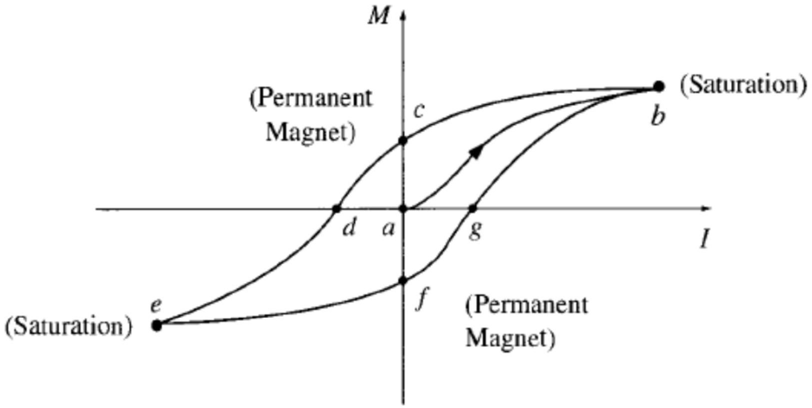
→ E*ach dipole* "likes" *to point in the same direction as its neighbors.*
→ It would look something with all the spins pointing the same way.



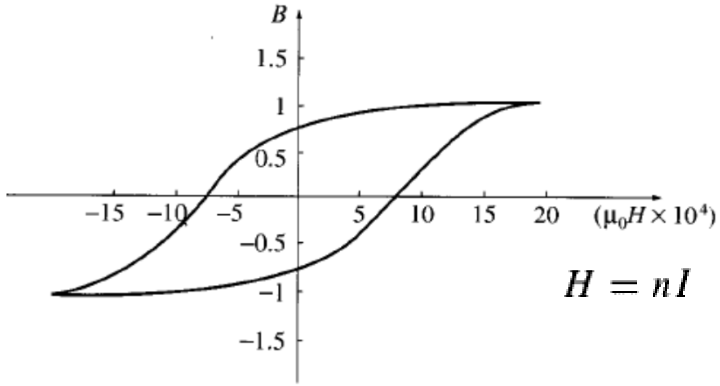
→ the alignment occurs in relatively small patches, called **domains.**
→ but the domains *themselves* are randomly oriented.

Hysteresis loops



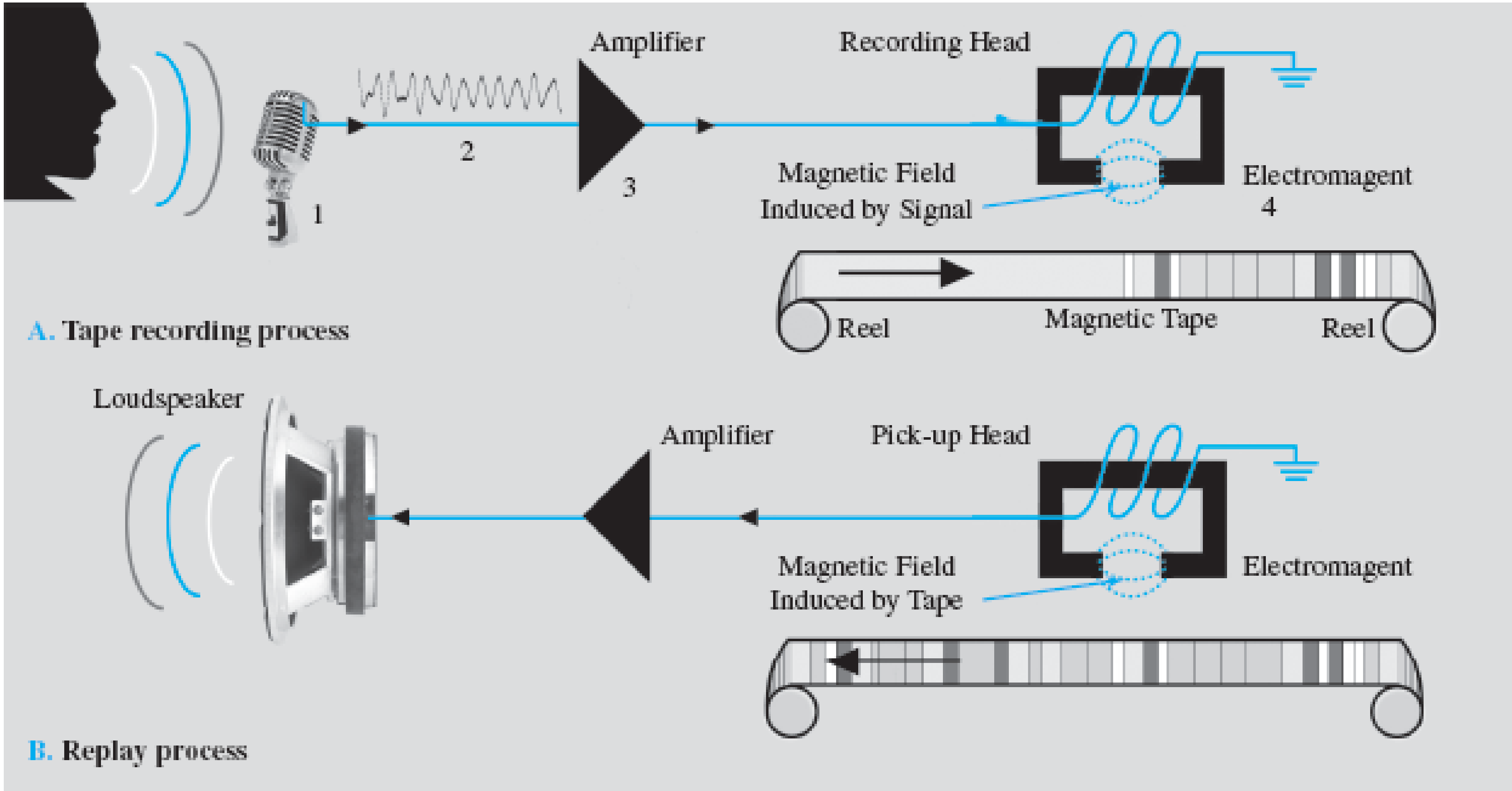


$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$



$$H = nI$$

Magnetic recording

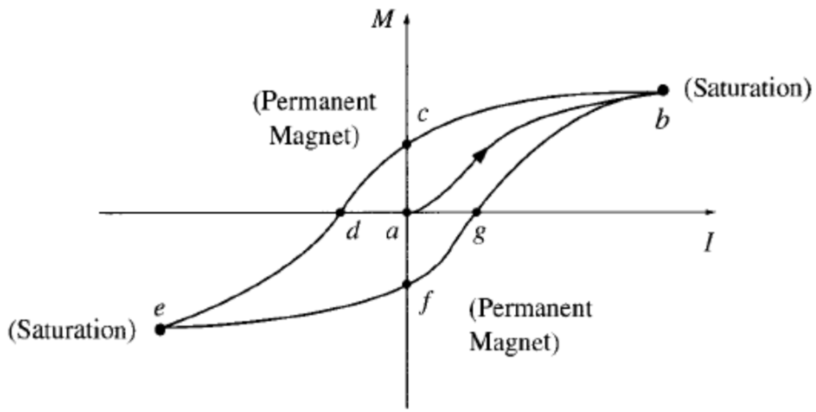


Ferromagnetic materials

Curie temperature (Curie Point): Demagnetization temperature (100 ~ 770◦C)

→ *Very* high temperatures do destroy the alignment.
→ What *is* surprising is that this occurs at a precise temperature (770ºC, for iron).
→ These abrupt changes in the properties of a substance, occurring at sharply defined temperatures, are known in statistical mechanics as **phase transitions.**

Problem 6.20 How would you go about demagnetizing a permanent magnet (at point c in the hysteresis loop)? That is, how could you restore it to its original state, with $M = 0$ at $I = 0$?



Parallelism between (E, D) and (B, H)

Problem 6.23 Notice the following parallel:

$$\begin{cases} \nabla \cdot \mathbf{D} = 0, & \nabla \times \mathbf{E} = 0, & \epsilon_0 \mathbf{E} = \mathbf{D} - \mathbf{P}, & \text{(no free charge);} \\ \nabla \cdot \mathbf{B} = 0, & \nabla \times \mathbf{H} = 0, & \mu_0 \mathbf{H} = \mathbf{B} - \mu_0 \mathbf{M}, & \text{(no free current).} \end{cases}$$

$\mathbf{D} \to \mathbf{B}$, $\mathbf{E} \to \mathbf{H}$, $\mathbf{P} \to \mu_0 \mathbf{M}$, $\epsilon_0 \to \mu_0$

: Use these relations to turn an electrostatic problem into an analogous magnetostatic one.

(a) the magnetic field inside a uniformly magnetized sphere

The electric field inside a uniformly polarized sphere,

$$\mathbf{E} = -\frac{1}{3\epsilon_0}\mathbf{P}$$

(b) the magnetic field inside a sphere of linear magnetic material in an otherwise uniform magnetic field

The electric field inside a sphere of linear dielectric in an otherwise uniform electric field is

$$\mathbf{E} = \frac{1}{1+\chi_e/3}\mathbf{E}_0$$