EJEMPLO

$$J = \int_0^\infty \int_0^\infty y e^{-xy} e^{-y} \, dx \, dy$$

PASO 1 Expansión del integrando

$$y e^{-xy} e^{-y} = \sum_n \sum_m \phi_{n,m} x^n y^{n+m+1}$$

PASO 2 Serie de brackets de J

$$J = \sum_n \sum_m \phi_{n,m} \langle n+1 \rangle \langle n+m+2 \rangle$$

PASO 3 Solución

$$J = \Gamma(-n)\Gamma(-m) \Big|_{\substack{n=-1 \\ m=-2-n=-1}}$$

$$J = \Gamma(\cancel{1})\Gamma(\cancel{1})$$

$$J = 1$$

EJEMPLO

$$J = \int_0^\infty \int_0^\infty x y^2 e^{-xy} e^{-\frac{y^2}{x^2}} \, dx\, dy$$

PASO 1 : Expansión del integrando

$$x y^2 e^{-xy} e^{-\frac{y^2}{x^2}} = \sum_n \sum_m \phi_{n,m} \, x^{n-2m+1} \, y^{n+2m+2}$$

PASO 2 Serie de brackets de J.

$$J = \sum_n \sum_m \phi_{n,m} \langle n-2m+2 \rangle \langle n+2m+3 \rangle$$

$$\Downarrow$$

PASO 3 : SOLUCIÓN

$$J = \frac{1}{|\det M|} \Gamma(-n)\Gamma(-m) \Big|_{\substack{n=-5/2 \\ m=-1/4}} \Leftarrow \underbrace{\begin{pmatrix} 1 & -2 \\ 1 & 2 \end{pmatrix}}_{M} \begin{pmatrix} n \\ m \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$

con $\det M = 4$.

$\therefore$

$$J = \frac{1}{4} \Gamma(5/2)\Gamma(1/4) \;//$$

EJEMPLO

$$J = \int_0^\infty \int_0^\infty \sqrt{x}\, y^2 e^{-\frac{x^2}{y}} e^{-\frac{y^3}{x}} \, dx\, dy$$

PASO 1: Expansión del integrando

$$\sqrt{x}\, y^2 e^{-\frac{x^2}{y}} e^{-\frac{y^3}{x}} = \sum_n \sum_m \phi_{n,m}\, x^{2n-m+\frac{1}{2}}\, y^{-n+3m+2}$$

PASO 2: serie de brackets de J

$$J = \sum_n \sum_m \phi_{n,m} \left\langle 2n-m+\tfrac{3}{2} \right\rangle \left\langle -n+3m+3 \right\rangle$$

$$\Downarrow$$

$$\underbrace{\begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}}_{M} \begin{pmatrix} n \\ m \end{pmatrix} = \begin{pmatrix} -3/2 \\ -3 \end{pmatrix}$$

$$\det M = 5 \qquad \begin{pmatrix} n \\ m \end{pmatrix} = \begin{pmatrix} -3/2 \\ -3/2 \end{pmatrix}$$

$$\therefore \quad J = \frac{1}{|\det M|} \Gamma(-n)\Gamma(-m) \Big|_{n=m=-3/2}$$

$$J = \frac{1}{5} \Gamma(3/2)\Gamma(3/2) = \frac{1}{5} \cdot \frac{1}{2}\Gamma(1/2)\, \frac{1}{2}\Gamma(1/2) = \frac{1}{20}\pi \;//$$

EJEMPLO

$$J = \int_0^\infty\int_0^\infty\int_0^\infty \frac{xy^2z\, e^{-\frac{x^2z}{y}}\, e^{-\frac{y^3}{x}}}{z^2+1}\, dx\, dy\, dz$$

PASO 1: Expansión del integrando

$$e^{-\frac{x^2z}{y}} = \sum_n \phi_n\, x^{2n} y^{-n} z^n$$

$$e^{-\frac{y^3}{x}} = \sum_m \phi_m\, y^{3m} x^{-m}$$

$$\frac{1}{z^2+1} = \sum_\ell \sum_k \phi_{\ell,k}\, z^{2\ell} \langle 1+\ell+k \rangle$$

$\therefore$

$$\frac{xy^2z\, e^{-\frac{x^2z}{y}}\, e^{-\frac{y^3}{x}}}{z^2+1} = \sum_n\sum_m\sum_\ell\sum_k \phi_{n,m,\ell,k}\, x^{2n-m+1}\, y^{-n+3m+2}\, z^{n+2\ell+1} \langle 1+\ell+k \rangle$$

PASO 2: Serie de brackets de J

$$J = \sum_n\sum_m\sum_\ell\sum_k \phi_{n,m,\ell,k} \langle 1+\ell+k \rangle \langle 2n-m+2 \rangle \langle -n+3m+3 \rangle \langle n+2\ell+2 \rangle$$

PASO 3: Solución

$$\underbrace{\begin{pmatrix} 0 & 0 & 1 & 1 \\ 2 & -1 & 0 & 0 \\ -1 & 3 & 0 & 0 \\ 1 & 0 & 2 & 0 \end{pmatrix}}_{M} \begin{pmatrix} n \\ m \\ \ell \\ k \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ -3 \\ -2 \end{pmatrix}$$

donde $\det M = -10$ y $\begin{pmatrix} n \\ m \\ \ell \\ k \end{pmatrix} = \begin{pmatrix} -9/5 \\ -8/5 \\ -1/10 \\ -9/10 \end{pmatrix}$

$\therefore$

$$J = \frac{1}{|\det M|} \Gamma(-n)\Gamma(-m)\Gamma(-\ell)\Gamma(-k)$$

$$= \frac{1}{10} \Gamma(9/5)\Gamma(8/5)\Gamma(1/10)\Gamma(9/10) \;//$$

EJEMPLO

$$J=\int_0^\infty\int_0^\infty\int_0^\infty \frac{xzy^2\, e^{-\frac{x^2z}{y}}\, J_0\left(\frac{y^3}{x}\right)}{z^2+1}\,dx\,dy\,dz$$

PASO 1: Expansión del integrando

$$e^{-\frac{x^2z}{y}}=\sum_n \phi_n\, x^{2n} z^n y^{-n}$$

$$J_0\left(\frac{y^3}{x}\right)=\sum_m \phi_m \frac{1}{\Gamma(m+1)}\frac{1}{4^m}\, y^{6m} x^{-2m}$$

$$(z^2+1)^{-1}=\sum_l \sum_k \phi_{l,k}\, z^{2l}\langle 1+l+k\rangle$$

$\therefore$

$$\frac{xy^2z\, e^{-\frac{x^2z}{y}}\, J_0\left(\frac{y^3}{x}\right)}{z^2+1}=\sum_n\sum_m\sum_l\sum_k \phi_{n,m,l,k}\frac{x^{2n-2m+1}\, y^{-n+6m+2}\, z^{n+2l+1}}{4^m\,\Gamma(m+1)}$$

$$\times\langle 1+l+k\rangle$$

PASO 2: Serie de brackets de J

$$J=\sum_m\sum_n\sum_l\sum_k \phi_{n,m,l,k}\frac{1}{4^m\,\Gamma(m+1)}\langle 1+l+k\rangle\langle 2n-2m+2\rangle\langle -n+6m+3\rangle\langle n+2l+2\rangle$$

$$\underbrace{\begin{pmatrix}0&0&1&1\\2&-2&0&0\\-1&6&0&0\\1&0&2&0\end{pmatrix}}_{M}\begin{pmatrix}n\\m\\l\\k\end{pmatrix}=\begin{pmatrix}-1\\-2\\-3\\-2\end{pmatrix}$$

luego $\det M = -20$ y $\begin{pmatrix} n \\ m \\ \ell \\ k \end{pmatrix} = \begin{pmatrix} -9/5 \\ -4/5 \\ -1/10 \\ -9/10 \end{pmatrix}$

$\therefore$

$$J = \frac{1}{|\det M|} \frac{\Gamma(-n)\Gamma(-m)\Gamma(-\ell)\Gamma(-k)}{4^{m}\,\Gamma(m+1)}$$ o

equivalentemente:

$$J = \frac{1}{20} \frac{\Gamma(9/5)\,\Gamma(4/5)\,\Gamma(1/10)\,\Gamma(9/10)}{4^{-4/5}\,\Gamma(1/5)} \; //$$