Integrales de Mecanica estadistica

0628#1

todo comienza a partir de la integral gaussiana

$$\int_{-\infty}^{\infty} dx \cdot e^{-\lambda x^2} = \sqrt{\frac{\pi}{\lambda}}$$

ya que es par $\left\{ \boxed{2\int_{0}^{\infty} dx \cdot e^{-\lambda x^2} = \sqrt{\frac{\pi}{\lambda}}} \right.$ (1)

• derivando ambos lados con respecto a λ.

$$\int_{-\infty}^{\infty} dx \cdot x^2 e^{-\lambda x^2} = \frac{1}{2}\sqrt{\frac{\pi}{\lambda^3}}$$

$$\int_{-\infty}^{\infty} dx \cdot x^4 e^{-\lambda x^2} = \frac{3}{4}\sqrt{\frac{\pi}{\lambda^5}}$$

$$\int_{-\infty}^{\infty} dx \cdot x^6 e^{-\lambda x^2} = \frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}\sqrt{\frac{\pi}{\lambda^7}}$$

$$\boxed{\int_{0}^{\infty} dx \cdot x^2 e^{-\lambda x^2} = \frac{1}{2}\cdot\frac{1}{2}\sqrt{\frac{\pi}{\lambda^3}}} \quad (2)$$

$$\boxed{\int_{0}^{\infty} dx \cdot x^4 e^{-\lambda x^2} = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{3}{2}\sqrt{\frac{\pi}{\lambda^5}}} \quad (3)$$

$$\boxed{\int_{0}^{\infty} dx \cdot x^6 e^{-\lambda x^2} = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}\sqrt{\frac{\pi}{\lambda^7}}} \quad (4)$$

uso de coordenadas polares simplifica el desarrollo

$$\int_{-\infty}^{\infty} d^3p \cdot f(p) \quad \left| \begin{array}{l} p_1 = p\cos\phi\sin\theta \\ p_2 = p\sin\phi\sin\theta \\ p_3 = p\cos\theta \end{array} \right. \quad \int_{p=0}^{\infty}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} p^2 dp \sin\theta \, d\theta \, d\phi \cdot f(p) = \boxed{\int_{0}^{\infty} 4\pi \, dp \cdot p^2 f(p) = \frac{N}{V}} = n$$

$d^3p = p^2 \sin\theta \, dp \, d\theta \, d\phi$

se reduce a una sola integral

$$\langle \epsilon \rangle = \left\langle \frac{p^2}{2m} \right\rangle = c\int_{0}^{\infty} 4\pi \, dp \cdot p^2 \, \frac{p^2}{2m} e^{-\lambda p^2} = \frac{4\pi}{2m} c \int_{0}^{\infty} dp \cdot p^4 e^{-\lambda p^2} = \frac{4\pi}{2m}\cdot\frac{3}{8}\sqrt{\frac{\pi}{\lambda^5}} \, c$$

constante normalización

$$\boxed{f(p) = e^{-\lambda p^2} = e^{-\frac{p^2}{2m}\frac{1}{KT}}}$$

Usar la distribución de boltzman como una probabilidad. (densidad)

$$\iiint \rho(\vec{p})d\vec{p}^3 = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^3p \cdot e^{-\lambda p^2}}{\int d^3p\, e^{-\lambda p^2}} \quad \left\{ \; \rho(\vec{p})\, d^3p = \frac{e^{-\lambda p^2} d^3p}{\int d^3p\, e^{-\lambda p^2}} = \frac{e^{-\lambda p^2} dp \cdot p^2 \sin\theta\, d\theta\, d\phi}{\int dp\, p^2 \sin\theta\, d\theta\, d\phi\, e^{-\lambda p^2}} \right.$$

un ejercicio es obtener la energía media por partícula, sin distribución espacial.

$$\langle \epsilon \rangle = \frac{\int_0^\infty \cancel{4\pi} p^2 \cdot \epsilon\, e^{-\lambda p^2} dp}{\int_0^\infty \cancel{4\pi} p^2 e^{-\lambda p^2} dp} = \frac{1}{2m} \frac{\int_0^\infty p^4 e^{-\lambda p^2} dp}{\int_0^\infty p^2 e^{-\lambda p^2} dp} = \frac{1}{2m} \frac{\frac{3}{8}\sqrt{\frac{\pi}{\lambda^5}}}{\frac{1}{4}\sqrt{\frac{\pi}{\lambda^3}}}$$

$$= \frac{3}{4m}\sqrt{\frac{\lambda^3}{\lambda^5}} = \frac{3}{4m}\sqrt{\frac{1}{\lambda^2}} = \frac{3}{4m}\left(\frac{1}{2m}\frac{1}{kT}\right)^{-1} = \frac{3}{4m} 2mkT = \frac{3}{2}kT \;//$$

otro calculo util

$$\langle \epsilon^2 \rangle = \frac{\int_0^\infty p^2 \cdot \epsilon^2 e^{-\lambda p^2} dp}{\int_0^\infty p^2 e^{-\lambda p^2} dp} = \left(\frac{1}{2m}\right)^2 \frac{\int_0^\infty p^2 \cdot p^4 e^{-\lambda p^2} dp}{\frac{1}{4}\sqrt{\frac{\pi}{\lambda^3}}} = \left(\frac{1}{2m}\right)^2 4\sqrt{\frac{\lambda^3}{\pi}} \int_0^\infty p^6 e^{-\lambda p^2} dp$$

$$= \left(\frac{1}{2m}\right)^2 \cancel{4}\sqrt{\frac{\lambda^3}{\cancel{\pi}}} \cdot \frac{1}{\cancel{4}} \cdot \frac{15}{4} \cdot \sqrt{\frac{\cancel{\pi}}{\lambda^7}} = \left(\frac{1}{2m}\right)^2 \cdot \frac{15}{4}\sqrt{\frac{1}{\lambda^4}} = \left(\frac{1}{\cancel{2m}}\right)^{\cancel{2}} \frac{15}{4}(\cancel{2m})^{\cancel{2}}(kT)^2 = \frac{15}{4}(kT)^2$$

se puede obtener

$$\Delta^2_\epsilon = \langle (\epsilon - \langle \epsilon \rangle)^2 \rangle = \langle \epsilon^2 \rangle - \langle \epsilon \rangle^2 = \frac{15}{4}(kT)^2 - \frac{9}{4}(kT)^2 = \frac{6}{4}(kT)^2 = \frac{3}{2}(kT)^2 \;//$$

Problema 6.2 Huang: encuentrese una densidad de probabilidad $P(E)dE$ para un gas ideal no relativista. Obl2#3

$$d^3p \cdot e^{-\lambda p^2} = dp \cdot 4\pi \cdot p^2 e^{-\lambda p^2} =$$ densidad de partículas (N/V) con p y $p+dp$

la energía es $E = \frac{p^2}{2m}$; $dE = \frac{2p}{2m}dp = \frac{p}{m}dp$ $\left\{ dp = \frac{m}{p}dE \wedge p = \sqrt{2mE} \right.$

$$dp\, 4\pi p^2 e^{-\lambda p^2} = \frac{m}{p} 4\pi p^2 e^{-\frac{E}{kT}} dE = 4m\pi p\, e^{-E/kT} dE = 4m\pi\sqrt{2mE}\, e^{-E/kT} dE$$

$$= \underbrace{2\pi \cdot (2m)^{3/2}}\sqrt{E}\, e^{-E/kT} dE$$

probabilidad de encontrar la partícula entre E y $E+dE$
celda

$$\boxed{P(E)dE = C_0 \sqrt{E}\, e^{-E/kT} dE}$$

← constante normalizadora.

$$\frac{N}{V} = \int_0^\infty P[E]dE = C_0 \int_0^\infty \sqrt{E}\, e^{-E/kT} dE \quad \left| \begin{array}{l} u = E/kT \\ du \cdot kT = dE \\ E = \sqrt{kTu} \end{array} \right| = C_0 \int_0^\infty \sqrt{kT}\, u^{1/2} e^{-u} kT\, du = C_0 (kT)^{3/2} \int_0^\infty u^{1/2} e^{-u} du$$

$$\left\{ \Gamma(x) = \int_0^\infty u^{x-1} e^{-u} du \quad ; \quad \int_0^\infty u^{\frac{3}{2}-1} e^{-u} du = \Gamma\left(\tfrac{3}{2}\right) = \tfrac{1}{2}\Gamma\left(\tfrac{1}{2}\right) = \frac{\sqrt{\pi}}{2} \right\}$$

$$= C_0 (kT)^{3/2} \frac{\sqrt{\pi}}{2} = \frac{N}{V} = n \Rightarrow \boxed{C_0 = 2\pi^{-1/2} n (kT)^{-3/2} = 2n\sqrt{\frac{1}{\pi (kT)^3}}}$$

al integrar todo el sistema encontraremos el nº de partículas en el volumen.