

### PROBLEMA GUÍA II | #3

$\vec{E}(\vec{r}) = -\nabla \phi(\vec{r})$  , la componente  $i$ -ésima de esta ecuación está dada por:

$$E_i = -\partial_i \phi(\vec{r}) = -\frac{1}{4\pi\epsilon_0} \partial_i \int_{V'} \frac{1}{2} \left( 3 \frac{(\vec{r} \cdot \vec{r}')^2}{r^5} - \frac{r'^2}{r^3} \right) \rho(\vec{r}') dV'$$

Obs.  $\partial_i$  actúa sobre la posición  $\vec{r}$

∴

$$E_i = -\frac{1}{4\pi\epsilon_0} \int_{V'} \frac{1}{2} \left( 3 \partial_i \left( \frac{(\vec{r} \cdot \vec{r}')^2}{r^5} \right) - r'^2 \partial_i \left( \frac{1}{r^3} \right) \right) \rho(\vec{r}') dV'$$

Obs.  $\partial_i \left[ \frac{(\vec{r} \cdot \vec{r}')^2}{r^5} \right] = \frac{1}{r^5} \left( \partial_i (\vec{r} \cdot \vec{r}')^2 \right) + (\vec{r} \cdot \vec{r}')^2 \partial_i (r^{-5})$

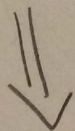
$$\begin{aligned} &= \frac{1}{r^5} 2(\vec{r} \cdot \vec{r}') \partial_i (\vec{r} \cdot \vec{r}') + (\vec{r} \cdot \vec{r}')^2 \frac{(-5)}{r^6} (\partial_i r) \\ &= \frac{2(\vec{r} \cdot \vec{r}')}{r^5} \partial_i (x_\ell x'_\ell) - 5 \frac{(\vec{r} \cdot \vec{r}')^2}{r^6} \frac{x_i}{r} \\ &= \frac{2(\vec{r} \cdot \vec{r}')}{r^5} x'_\ell \partial_i x_\ell - 5 \frac{(\vec{r} \cdot \vec{r}')^2}{r^7} x_i \\ &= \frac{2(\vec{r} \cdot \vec{r}')}{r^5} x'_i - 5 \frac{(\vec{r} \cdot \vec{r}')^2}{r^7} x_i \end{aligned}$$



Obs.  $\partial_i \left( \frac{1}{r^3} \right) = \partial_i (r^{-3}) = -\frac{3}{r^4} (\partial_i r) = -3 \frac{x_i}{r^5}$

∴

$$E_i = -\frac{1}{4\pi\epsilon_0} \int_{V'} \frac{1}{2} \left( \frac{6(\vec{r} \cdot \vec{r}')}{r^5} x'_i - \frac{15(\vec{r} \cdot \vec{r}')^2}{r^7} x_i + 3 \frac{r'^2}{r^5} x_i \right) \rho(\vec{r}') dV'$$



Finalmente:

$$\vec{E}(\vec{r}) = -\frac{1}{4\pi\epsilon_0} \int_{V'} \frac{1}{2} \left[ \frac{6(\vec{r} \cdot \vec{r}') \vec{r}'}{r^5} - \frac{15(\vec{r} \cdot \vec{r}')^2 \vec{r}}{r^7} + 3 \frac{r'^2 \vec{r}}{r^5} \right] \rho(\vec{r}') dV'$$



$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{V'} \frac{1}{2} \left[ \frac{15(\vec{r} \cdot \vec{r}')^2 \vec{r}}{r^7} - \frac{6(\vec{r} \cdot \vec{r}') \vec{r}'}{r^5} - 3 \frac{r'^2 \vec{r}}{r^5} \right] \rho(\vec{r}') dV'$$