

COMPLEMENTO CLASE ~~4~~

TO 08/09/2016.

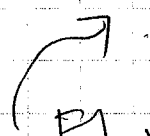
$$\sum_{k=0}^n \cos kx = \operatorname{Re} \left(\sum_{k=0}^n e^{ikx} \right) \quad \underline{\underline{1}}$$

Obs. $\sum_{k=0}^n y^k = \frac{1 - y^{n+1}}{1 - y}$

Evaluemos primero

$$\begin{aligned} \sum_{k=0}^n (e^{ix})^k &= \frac{1 - (e^{ix})^{n+1}}{1 - (e^{ix})} \\ &= \frac{1 - e^{ix(n+1)}}{1 - e^{ix}} \end{aligned}$$

pero $1 - e^{ix} = e^{ix/2} (e^{-ix/2} - e^{ix/2})$



El viejo truco de
factorizar por la
mitad

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donde $e^{ix/2} - e^{-ix/2} = 2i \sin(x/2)$

$\therefore 1 - e^{ix} = e^{ix/2} 2i \sin\left(\frac{x}{2}\right)$

por otro lado y similarmente a lo
anterior:

$1 - e^{ix(n+1)} = e^{ix \frac{(n+1)}{2}} 2i \sin\left[\frac{(n+1)x}{2}\right]$

luego

$$\sum_{k=0}^n e^{ikx} = \frac{1 - e^{ix(n+1)}}{1 - e^{ix}}$$

$$= \frac{e^{i\frac{x(n+1)}{2}} \cancel{2i} \operatorname{sen}\left(\frac{n+1}{2}x\right)}{e^{i\frac{x}{2}} \cancel{2i} \operatorname{sen}\left(\frac{x}{2}\right)}$$

$$= \frac{e^{i\frac{nx}{2}} \operatorname{sen}\left[\frac{n+1}{2}x\right]}{\operatorname{sen}\left[\frac{x}{2}\right]}$$

$$= \frac{\cos\left(\frac{nx}{2}\right) \operatorname{sen}\left(\frac{n+1}{2}x\right)}{\operatorname{sen}\left(\frac{x}{2}\right)}$$

$$+ i \operatorname{sen}\left(\frac{x}{2}\right) \operatorname{sen}\left(\frac{n+1}{2}x\right) / \operatorname{sen}\left(\frac{x}{2}\right)$$

finalmente

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$$1 + \cos x + \dots + \cos nx = \operatorname{Re} \left(\sum_{k=0}^n e^{ikx} \right)$$

$$= \frac{\cos\left(\frac{nx}{2}\right) \sin\left(\frac{n+1}{2}x\right)}{\sin\left(\frac{x}{2}\right)}$$

...

$$\operatorname{Im} \left(\sum_{k=0}^n e^{ikx} \right) = \sum_{k=0}^n \sin kx$$

$$= \sin x + \sin 2x + \dots + \sin nx$$

$$= \frac{\sin\left(\frac{nx}{2}\right) \sin\left(\frac{n+1}{2}x\right)}{\sin\left(\frac{x}{2}\right)}$$