PROBLEMA GUÍA 2 | #6

$$\nabla \cdot (r \nabla e^{i\vec{k}\cdot\vec{r}}) = \partial_j \left(r \partial_j e^{i\vec{k}\cdot\vec{r}} \right)$$

Obs. $\partial_j e^{i\vec{k}\cdot\vec{r}} = e^{i\vec{k}\cdot\vec{r}} \, \partial_j (i\vec{k}\cdot\vec{r})$

$= e^{i\vec{k}\cdot\vec{r}} \, i \, \partial_j (k_\ell x_\ell)$

$= i e^{i\vec{k}\cdot\vec{r}} \, k_\ell \, \delta_{j\ell}$

$= i k_j e^{i\vec{k}\cdot\vec{r}}$

Luego

$$\nabla \cdot (r \nabla e^{i\vec{k}\cdot\vec{r}}) = i k_j \partial_j \left(r e^{i\vec{k}\cdot\vec{r}} \right)$$

$= i k_j \left[(\partial_j r) e^{i\vec{k}\cdot\vec{r}} + r \partial_j e^{i\vec{k}\cdot\vec{r}} \right]$

$= i k_j \left[\frac{x_j}{r} e^{i\vec{k}\cdot\vec{r}} + r \left(i k_j e^{i\vec{k}\cdot\vec{r}} \right) \right]$

$= i \frac{k_j x_j}{r} e^{i\vec{k}\cdot\vec{r}} - k_j k_j r \, e^{i\vec{k}\cdot\vec{r}}$

$= i \frac{(\vec{k}\cdot\vec{r})}{r} e^{i\vec{k}\cdot\vec{r}} - k^2 r \, e^{i\vec{k}\cdot\vec{r}}$

$$\nabla \cdot (r \nabla e^{i\vec{k}\cdot\vec{r}}) = e^{i\vec{k}\cdot\vec{r}} \left[\frac{i(\vec{k}\cdot\vec{r})}{r} - k^2 r \right]$$