Zemansky Termo - Problemas cap2

Friday, March 10, 2023 11:26 PM

2.1. The equation of state of an ideal gas is $PV = nRT$, where n and R are constants.
(*a*) Show that the volume expansivity β is equal to $1/T$.
(*b*) Show that the isothermal compressibility κ is equal to $1/P$.

a) expansividad $\quad \beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \quad \Big| \quad V = \frac{nRT}{P}$

$$\beta = \left(\frac{P}{nRT}\right)\left(\frac{\partial}{\partial T}\frac{nRT}{P}\right)_P = \left(\frac{P}{nRT}\right)\frac{nR}{P} = \frac{1}{T}$$

$$\boxed{\beta = \frac{1}{T}}$$

b) compresibilidad isotermica $\quad \kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$

$$\kappa = -\left(\frac{P}{nRT}\right)\left(\frac{\partial}{\partial P}\frac{nRT}{P}\right)_T = -\left(\frac{P}{nRT}\right)\left(-\frac{nRT}{P^2}\right)$$

$$\kappa = \frac{P}{P^2} = \frac{1}{P} \quad \rightarrow \quad \boxed{\kappa = \frac{1}{P}}$$

2.2. The equation of state of a van der Waals gas is given as

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT,$$

where a, b, and R are constants. Calculate the following quantities:
(*a*) $(\partial P/\partial v)_T$;
(*b*) $(\partial P/\partial T)_v$.
From parts (*a*) and (*b*) calculate $(\partial v/\partial T)_P$.

a) $(\partial P$

a) $\left(\frac{\partial P}{\partial \nu}\right)_T$ | $P = \frac{RT}{\nu - b} - \frac{a}{\nu^2}$

$$\rightarrow \frac{\partial}{\partial \nu}\left(\frac{RT}{\nu-b} - \frac{a}{\nu^2}\right)_T = -\frac{RT}{(\nu-b)^2} + 2\frac{a}{\nu^3}$$

$$\boxed{\left(\frac{\partial P}{\partial \nu}\right)_T = -\frac{RT}{(\nu-b)^2} + 2\frac{a}{\nu^3}}$$

b) $\left(\frac{\partial P}{\partial T}\right)_v = \frac{1}{\left(\frac{\partial T}{\partial P}\right)_v}$ | $T = \frac{1}{R}\left(P + \frac{a}{\nu^2}\right)(\nu - b)$

$$\left(\frac{\partial T}{\partial P}\right)_\nu = \frac{\nu - b}{R}$$

$$\therefore \boxed{\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{\nu - b}} //$$

c) calcular

$\left(\frac{\partial \nu}{\partial T}\right)_P$ $\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$

$$\boxed{\left(\frac{\partial P}{\partial \nu}\right)_T = -\frac{RT}{(\nu-b)^2} + 2\frac{a}{\nu^3}}$$

$$\boxed{\left(\frac{\partial P}{\partial T}\right)_v = \frac{R}{\nu - b}}$$

$$\left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = \frac{-1}{\left(\frac{\partial x}{\partial y}\right)_z} = -\left(\frac{\partial y}{\partial x}\right)_z$$

$$\left(\frac{\partial \nu}{\partial T}\right)_p$$

$y = \nu$
$x = T$
$z = p$

$$\left(\frac{\partial \nu}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_\nu = -\left(\frac{\partial \nu}{\partial T}\right)_p$$

$$\frac{1}{\left(\frac{\partial p}{\partial \nu}\right)_T} \left(\frac{R}{\nu - b}\right) = -\left(\frac{\partial \nu}{\partial T}\right)_p$$

$$\frac{1}{-\frac{RT}{(\nu-b)^2} + 2\frac{a}{\nu^3}} \left(\frac{R}{\nu - b}\right) = -\left(\frac{\partial \nu}{\partial T}\right)_p \quad / \cdot (-1)$$

$$\left(\frac{\partial \nu}{\partial T}\right)_p = \frac{-\nu^3 (\nu-b)^{\cancel{2}}}{2a(\nu-b)^2 - RT(\nu)^3} \left(\frac{R}{\cancel{\nu - b}}\right)$$

$$\boxed{\left(\frac{\partial \nu}{\partial T}\right)_p = \frac{-\nu^3 (\nu-b)}{2a(\nu-b)^2 - RT(\nu)^3} R}$$

2.4. (*a*) A block of copper at a pressure of 1 atm (approximately 100 kPa) and a temperature of 5°C is kept at constant volume. If the temperature is raised to 10°C, what will be the final pressure?

(*b*) If the vessel holding the block of copper has a negligibly small thermal expansivity and can withstand a maximum pressure of 1000 atm, what is the highest temperature to which the system may be raised?

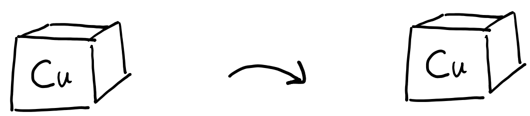
(*Note*: The volume expansivity β and isothermal compressibility κ are not always listed in handbooks of data. However, β is three times the linear expansion coefficient α, and κ is the reciprocal of the bulk modulus B. For this problem, assume that the volume expansivity and isothermal compressibility remain practically constant within the temperature range of 0 to 20°C at the values of $4.95 \times 10^{-5}\,\mathrm{K}^{-1}$ and $6.17 \times 10^{-12}\,\mathrm{Pa}^{-1}$, respectively.)

Cobre: datos

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = 4.95 \times 10^{-5} \left[\frac{1}{K}\right]$$

$$K = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T = 6.17 \times 10^{-12} \left[\frac{1}{Pa}\right]$$

a)



$P = 100\,[kPa]$ $\quad$ $P = \,?$

$T = 5\,[°C]$ $\quad$ $T = 10\,[°C]$

necesitamos

$$\left(\frac{\partial P}{\partial T}\right)_V$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{1}{\left(\frac{\partial T}{\partial P}\right)_V} \qquad \left\{ \left(\frac{\partial T}{\partial P}\right)_V \left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P = -1 \right.$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{1}{\left(\frac{\partial T}{\partial P}\right)_V} = -\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P$$

$$\left(\frac{\partial P}{\partial T}\right)_V = -\frac{1}{\left(\frac{\partial V}{\partial P}\right)_T} \left(\frac{\partial V}{\partial T}\right)_P \qquad \left| \left(\frac{\partial V}{\partial P}\right)_T = -VK \right.$$

$$\left| \left(\frac{\partial V}{\partial T}\right)_P = V\beta \right.$$

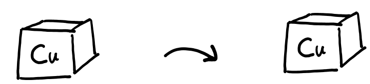
$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{\beta}{K} = \frac{4.95 \times 10^{-5}\left[\frac{1}{K}\right]}{6.17 \times 10^{-12}\left[\frac{1}{Pa}\right]} = 0.802 \times 10^{7}\left[\frac{Pa}{K}\right]$$

$$\left(\frac{\partial P}{\partial T}\right)_V = 8.02 \times 10^{6}\left[\frac{Pa}{K}\right]$$

Como cambia la presión al variar la temperatura

$$dP = \left(\frac{\partial P}{\partial T}\right)_V dT + \left(\frac{\partial P}{\partial V}\right)_T dV \quad / \text{ a volumen constant } dV = 0$$

$$\int_{P_0}^{P} dP = \int_{T_0}^{T}\left(\frac{\partial P}{\partial T}\right)_V dT$$



$P_0 = 100[kPa]$ $\quad$ $P = ?$

$T_0 = 5[°C]$ $\quad$ $T = 10\,[°C]$

$$P - P_0 = \left(\frac{\partial P}{\partial T}\right)_V (T - T_0)$$

$$P = P_0 + \left(\frac{\partial P}{\partial T}\right)_V (T - T_0)$$

$\left(\frac{\partial P}{\partial T}\right)_V = 8.02 \times 10^{6}\left[\frac{Pa}{K}\right]$

$T - T_0 = 5[K]$ ← es una diferencia d temp $\therefore °C = K$

$P_0 = 100[kPa]$

$$P = 100\,[kPa] + 40.1 \times 10^{6}\,[Pa] = 100\,[kPa] + 40.1 \times 10^{3}\,[kPa]$$

$$P = (100 + 40100)\,[kPa] = 40\,200\,[kPa] \quad //$$

b) dado que solo pueda haber un maximo de $1000\,[atm] = 10^3\,[atm] = 10^5\,[kPa]$ a que T_{max} puede llegar el sistema

$\{\; 5[°C] = 278.15\,[K]$

$$P = P_0 + \left(\frac{\partial P}{\partial T}\right)_V (T - T_0)$$

$$T = \frac{P - P_0}{\left(\frac{\partial P}{\partial T}\right)_V} + T_0 = \frac{10^5 - 10^2}{8.02 \times 10^6} + 278.15 \,[K]$$

$$T \approx 278.163 \,[K] \rightarrow T \approx 5.013 \,[°C]$$