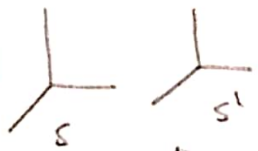
Problema 1

a) 

$\vec{V} = (V_0, 2V_0, 3V_0)$

con

$$|V| = V_0 (1+4+9)^{\frac{1}{2}}$$

$$|V| = V_0\sqrt{14} \rightarrow \beta = \frac{|V|}{c} = \frac{V_0}{c}\sqrt{14}$$

$$\beta_x = \frac{V_0}{c} \quad \beta_y = \frac{2V_0}{c} \quad \beta_z = \frac{3V_0}{c}$$

La transformada de Lorentz

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta_x & -\gamma\beta_y & -\gamma\beta_z \\ -\gamma\beta_x & 1+(\gamma-1)\frac{\beta_x^2}{\beta^2} & (\gamma-1)\frac{\beta_x\beta_y}{\beta^2} & (\gamma-1)\frac{\beta_x\beta_z}{\beta^2} \\ -\gamma\beta_y & (\gamma-1)\frac{\beta_y\beta_x}{\beta^2} & 1+(\gamma-1)\frac{\beta_y^2}{\beta^2} & (\gamma-1)\frac{\beta_y\beta_z}{\beta^2} \\ -\gamma\beta_z & (\gamma-1)\frac{\beta_z\beta_x}{\beta^2} & \frac{\beta_z\beta_y}{\beta^2}(\gamma-1) & 1+\frac{(\gamma-1)\beta_z^2}{\beta^2} \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

forma matriz

donde $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{V_0^2}{c^2}14}}$

$$ct' = \gamma\left(ct - \frac{V_0}{c}[x+y+z]\right)$$

$$x' = x + \left(\frac{x}{14} + \frac{2y}{14} + \frac{3z}{14}\right)(\gamma-1) - \gamma\frac{V_0}{c}ct$$

$$y' = y + (2x+4y+6z)\frac{(\gamma-1)}{14} - \gamma\frac{2V_0}{c}ct$$

$$z' = z + (3x+6y+9z)\frac{(\gamma-1)}{14} - \gamma\frac{3V_0}{c}ct$$

forma ecuación

Escaneado con CamScanner

b) $\begin{cases} A = (2a, a, 0, -a) \\ B = (\frac{3a}{2}, 2a, a, a) \end{cases}$ | distancia espacio temporal.

$$\Delta s^2 = c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

Para S. $\Delta s^2 = c^2\left(\frac{3a-4a}{2}\right)^2 - (a)^2 - a^2 - 4a^2$

$$\Delta s^2 = c^2\left(-\frac{a}{2}\right)^2 - 6a^2 = \frac{c^2a^2}{4} - 6a^2$$

Para S' $\Delta s'^2 = \Delta s^2$ ya que la distancia espacio temporal es una invariante de Lorentz

c) $\Delta \tau = \frac{1}{\gamma}\Delta t$ $\Delta \tau$ tiempo medido en S'
Δt tiempo medido en S

$\Delta \tau = 0 = \sqrt{1 - \frac{v^2}{c^2}}\left(\frac{a}{2}\right)$ | $v^2 = v_0^2 \cdot 14$ $\Big\{^*$ $v_0^2 \cdot 14 = c^2$

$$v_0 = \frac{c}{\sqrt{14}}$$

$* \ |v|^2 = c^2$

d) $\Delta x' = L_x + (L_x + 2L_y)\, g$

$\Delta y' = L_y + (2L_x + 4L_y)\, g$

$\Delta z' = (3L_x + 6L_y)\, g$

donde

$L_x = L\cos\theta$

$L_y = L\sin\theta$

$g = \dfrac{(\gamma - 1)}{14} = \dfrac{\left(\sqrt{\frac{14}{13}} - 1\right)}{14}$

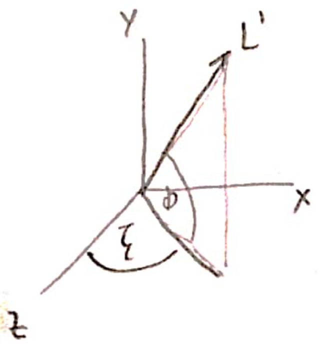
$v_0 = \dfrac{c}{\sqrt{14}}$

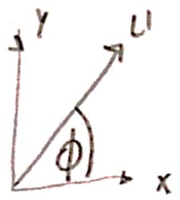
largo de barra en S' $= \sqrt{(\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2} = L'$

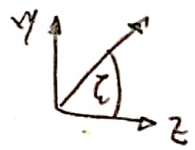
(valor numerico con $L = 1$ [m] $\theta = \frac{\pi}{4}$

$L' = 1{,}013$ [m]) * con Mathematica IL.

angulo que forma







$\phi = \cos^{-1}\left(\dfrac{\Delta x'}{L'}\right)$

$\zeta = \cos^{-1}\left(\dfrac{\Delta z'}{L'}\right)$

(valor numerico con $L = 1$ $\theta = \frac{\pi}{4}$

$\phi = 0{,}79$ [rad]

$\zeta = 1{,}55$ [rad])

Problema 2

la transformación de Lorentz de los campos de $S' \to S$ con S' moviéndose $\vec{v} = v_0 \hat{j}$ respecto a S.

$$E_x = \gamma(E_x' + vB_z') \qquad B_x = \gamma\left(B_x' - \frac{v}{c^2}E_z'\right)$$
$$E_y = E_y' \qquad B_y = B_y'$$
$$E_z = \gamma(E_z' - vB_x') \qquad B_z = \gamma\left(B_z' + \frac{v}{c^2}E_x'\right)$$

con

$$E_x' = \underbrace{k_e Q}_{K} \frac{x'}{r^3} \qquad E_y' = K\frac{y'}{r^3} \qquad E_z' = K\frac{z'}{r^3}$$

$K = k_e Q$

$$E_x = \gamma K \frac{x'}{[r^3]} \qquad B_x = -\frac{\gamma v}{c^2} K \frac{z'}{[r^3]}$$

$$E_y = K\frac{y'}{[r^3]} \qquad B_y = 0$$

$$E_z = \gamma K \frac{z'}{[r^3]} \qquad B_z = \frac{\gamma v}{c^2} K \frac{x'}{[r^3]}$$

$*$ $[r] = \sqrt{x^2 + (y')^2 + z^2}$

$[r] = \sqrt{x^2 + z^2 + \gamma^2(y - v_0 t)^2}$

r para S.

escribiendo en coord. para S.

$*$ $x' = x$

$y' = \gamma(y - \beta c t)$

$z' = z$

$$\boxed{\begin{aligned} E_x &= \frac{\gamma k_e Q x}{[(x^2+z^2+\gamma^2(y-v_0t)^2)^{3/2}]} & B_x &= -\frac{\gamma v_0}{c^2} k_e Q \frac{z}{[r^3]} \\ E_y &= \frac{k_e Q \gamma (y - v_0 t)}{[r^3]} & B_y &= 0 \\ E_z &= \frac{\gamma k_e Q z}{[r^3]} & B_z &= \frac{\gamma v_0}{c^2} k_e Q \frac{x}{[r^3]} \end{aligned}}$$

③

foton $E = hf$

momento $p = \frac{E}{c}$ | debido a $E = pc$

$p = \frac{h}{\lambda}$ | $\lambda f = c$

① $E_\gamma + m_e c^2 = E'_\gamma + E'_e$

$E_e'^2 = (p'_e c)^2 + (m_e c^2)^2$

① conservación energía

② $p_{x\gamma} = p'_{\gamma x} + p'_{ex}$ ← conservación momentum

$p_{y\gamma} = p'_{\gamma y} + p'_{ey}$

① $$\left[(E_\gamma - E'_\gamma) + m_e c^2\right]^2 = (p'_e c)^2 + (m_e c^2)^2$$

$$(E_\gamma - E'_\gamma)^2 + 2(E_\gamma - E'_\gamma) m_e c^2 + \cancel{(m_e c^2)^2} = (p'_e c)^2 + \cancel{(m_e c^2)^2}$$

$\frac{E_\gamma}{c} = p_\gamma \quad \frac{E'_\gamma}{c} = p'_\gamma \qquad / \frac{1}{c^2}$

$$(p_\gamma - p'_\gamma)^2 + 2 m_e c (p_\gamma - p'_\gamma) = \cancel{p_e'^2}\ 0$$

② $p_{x\gamma} = p_\gamma$ y $p'_{\gamma x} = p'_\gamma \cos\theta$; $p'_{ex} \simeq 0$

$p_{y\gamma} = 0$ y $p'_{\gamma y} = p'_\gamma \sin\theta$ desprecíamos retroceso $p'_{ey} \simeq 0$

② $p_\gamma \simeq p'_\gamma \cos\theta$

$0 \simeq p'_\gamma \sin\theta$

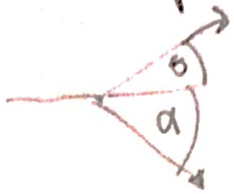
$\therefore (p_\gamma - p'_\gamma)^2 = (p'_\gamma \cos\theta - p'_\gamma)^2$

$= p_\gamma'^2 (\cos\theta - 1)^2$

$$(p_\gamma - p'_\gamma)^2 = -2 m_e c\, p'_\gamma (\cos\theta - 1)$$

$$\frac{\lambda'}{h}\left(\frac{h}{\lambda} - \frac{h}{\lambda'}\right)^2 = 2 m_e c (\cos\theta - 1) \quad // \ a$$

b) sin aprecia retroce



$$(P_\gamma - P'_\gamma)^2 + 2m_e c\,(P_\gamma - P'_\gamma) = P_e'^2$$

2) $$P_\gamma = P'_\gamma \cos\theta + P'_e \cos\alpha$$
$$0 = P'_\gamma \sin\theta + P'_e \sin\alpha \rightarrow P'_e = -\frac{P'_\gamma \sin\theta}{\sin\alpha}$$

o, $$(P'_e)^2 = (P_\gamma - P'_\gamma \cos\theta)^2 + (P'_\gamma \sin\theta)^2$$

$$P'_e = P_\gamma^2 - 2P_\gamma P'_\gamma \cos\theta + P'_\gamma \cos^2\theta + P'_\gamma \sin\theta$$

$$P'_e = P_\gamma^2 - 2P_\gamma P'_\gamma \cos\theta + P'_\gamma$$

↓ $$(P_\gamma - P'_\gamma)^2 + 2m_e c\,(P_\gamma - P'_\gamma) = P_\gamma^2 - 2P_\gamma P'_\gamma \cos\theta + P'_\gamma$$

$$\cancel{P_\gamma^2} - 2P_\gamma P'_\gamma + \cancel{P'_\gamma} + 2m_e c\,(P_\gamma - P'_\gamma) = \cancel{P_\gamma^2} - 2P_\gamma P'_\gamma \cos\theta + \cancel{P'_\gamma}$$

$$2m_e c\,(P_\gamma - P'_\gamma) = 2P_\gamma P'_\gamma (1 - \cos\theta) \quad / \frac{1}{2m_e c}\,\frac{1}{P_\gamma P'_\gamma}$$

$$\frac{1}{P'_\gamma} - \frac{1}{P_\gamma} = \frac{(1-\cos\theta)}{m_e c} \quad / P = \frac{h}{\lambda}$$

$$\boxed{\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)}$$