Problem 6.1.
We know that the free energy $F(T,V,N)$ of a thermodynamic system is extensive. Show that

$$N\left(\frac{\partial F}{\partial N}\right)_{T,V} + V\left(\frac{\partial F}{\partial V}\right)_{T,N} = Nf = F$$

with f the free energy density expressed in suitable variables. Given this result, from the differential properties of $F(T,V,N)$, show that

$$\Phi = N\mu$$

with Φ the Gibbs potential defined as $\Phi = F + PV$. In the above expression, μ is the chemical potential properly defined in terms of $F(T,V,N)$.

(1) $F[T,V,N] = N f[T,v]$ // extensiva con $v = \frac{V}{N}$

(2) $N\left(\frac{\partial F}{\partial N}\right)_{T,V} + V\left(\frac{\partial F}{\partial V}\right)_{T,N} = Nf = F$

Δ (3) $G = N\mu$

(4) $G = F + PV$

reemplazando (1) en (2) derivada parcial $v = \frac{N}{V}$ osea $v[N,V]$

$$N\left(\frac{\partial F}{\partial N}\right)_{T,V} = N \frac{\partial}{\partial N}\left(N f[T,v]\right)_{T,V} = N\left(f + N\left(\frac{\partial f}{\partial N}\right)_{T,V}\right)$$

$$\left(\frac{\partial x}{\partial w}\right)_z = \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial w}\right)_z$$

$$\frac{\partial}{\partial N}\left(f[T,v]\right)_{T,V} = \left(\frac{\partial f}{\partial v}\right)_T \left(\frac{\partial v}{\partial N}\right)_{T,V} = \left(\frac{\partial f}{\partial v}\right)_T \left(\frac{\partial v}{\partial N}\right)_V = \left(\frac{\partial f}{\partial v}\right)_T \left(-\frac{V}{N^2}\right)$$

$v(V,N)$

$$\therefore N\left(\frac{\partial F}{\partial N}\right)_{T,V} = N\left(f - \frac{V}{N}\left(\frac{\partial f}{\partial v}\right)_T\right) = Nf - V\left(\frac{\partial f}{\partial v}\right)_T$$

Tambien

$$\# \quad V\left(\frac{\partial F}{\partial V}\right)_{T,N} = V\left(\frac{\partial Nf}{\partial V}\right)_{T,N} = VN\left(\frac{\partial f}{\partial V}\right)_{T,N} = VN\left(\frac{\partial f}{\partial v}\right)_T\left(\frac{\partial v}{\partial V}\right)_N$$

$$= VN\left(\frac{\partial f}{\partial v}\right)_T \cdot \frac{1}{N} = V\left(\frac{\partial f}{\partial v}\right)_T$$

entonces la ecuación a demostrar, reemplazd el L.I

$$* \quad \underbrace{N\left(\frac{\partial F}{\partial N}\right)_{T,V}}_{\#} + \underbrace{V\left(\frac{\partial F}{\partial V}\right)_{T,N}}_{\#} = Nf = F$$

$$Nf - V\left(\frac{\partial f}{\partial v}\right)_T + V\left(\frac{\partial f}{\partial v}\right)_T = Nf \quad //$$ demostrado.

extra; relaciones que no utilicé pero podrian servir en uno similar.

$$\left(\frac{\partial f}{\partial N}\right)_V = \left(\frac{\partial f}{\partial T}\right)_v\left(\frac{\partial T}{\partial N}\right)_V + \left(\frac{\partial f}{\partial v}\right)_T\left(\frac{\partial v}{\partial N}\right)$$

formula

$$\left(\frac{\partial A}{\partial y}\right)_z = \left(\frac{\partial A}{\partial y}\right)_x + \left(\frac{\partial A}{\partial x}\right)_y\left(\frac{\partial x}{\partial y}\right)_z$$

$$f[v,T] \rightarrow \left(\frac{\partial f}{\partial v}\right)_N = \left(\frac{\partial f}{\partial v}\right)_T + \left(\frac{\partial f}{\partial T}\right)_v\left(\frac{\partial T}{\partial v}\right)_N$$

$$\left(\frac{\partial f}{\partial T}\right)_N = \left(\frac{\partial f}{\partial T}\right)_\nu + \left(\frac{\partial f}{\partial \nu}\right)_T \left(\frac{\partial \nu}{\partial T}\right)_N$$