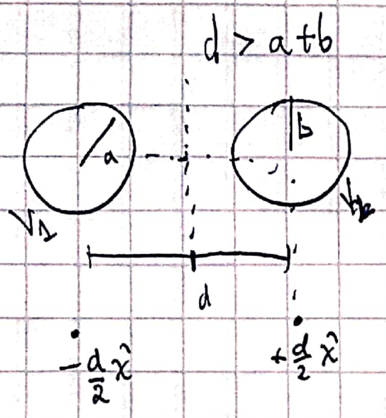
Problema 5
Fabian Trigo



para una sola esfera

$$\nabla^2 V = 0$$

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta} = 0$$

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} = 0 \quad \Big| \; u = \frac{\partial V}{\partial r}$$

$\frac{\partial V}{\partial \theta} = 0$

$$\frac{d^2 V}{dr^2} + \frac{1}{r}\frac{dV}{dr} = \frac{du}{dr} + \frac{1}{r}u = 0$$

cond contorno

$V(r=a) = V_1$

$$\int \frac{du}{u} = \int -\frac{dr}{r} \quad \Big\{ \; C + \ln(u) = -\ln(r) \quad / e^{u}$$

$$\frac{C}{2}\ln\left[\left(a+\frac{d}{2}\right)^2 + (0)^2\right] = V_1$$

$$> \; u = \frac{C}{r} \quad \Big| \; u = \frac{dV}{dr} \quad \Big\{ \int dV = C\int \frac{dr}{r}$$

$$C = \frac{2V_1}{2\ln\left[\left(a+\frac{d}{2}\right)\right]} = \frac{V_1}{\ln\left[a+\frac{d}{2}\right]}$$

$$V = C\ln[r)$$

$$r = \sqrt{(x-x_0)^2 + (y-y_0)^2}$$

$$r_a = \sqrt{\left(x+\frac{d}{2}\right)^2 + y^2} \quad !!$$

$$r_b = \sqrt{\left(x-\frac{d}{2}\right)^2 + y^2} \quad !!$$

Para una sola esfera (a)

$$V_{a(x,y)}[r) = \frac{V_1}{\ln\left[a+\frac{d}{2}\right]}\ln\left[r_a\right)$$

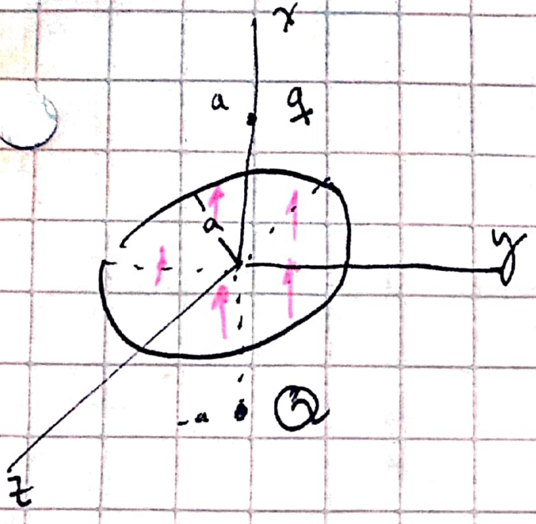
desarrollando para la otra

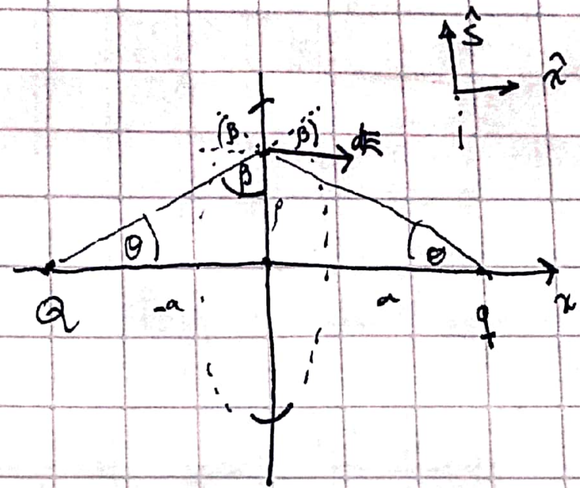
$$V_b[r) = \frac{V_2}{\ln\left[b-\frac{d}{2}\right)}\ln\left[r_b\right)$$

así el potencial total para este caso:

$$V[r) = \frac{V_1 \ln[r_a)}{\ln\left[a+\frac{d}{2}\right)} + \frac{V_2 \ln[r_b)}{\ln\left[b-\frac{d}{2}\right|} \quad //$$

Problema 1:



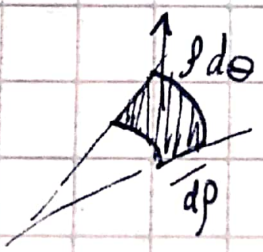


$$d\vec{E} = k_0 \frac{Q}{(\rho^2 + a^2)^{3/2}} (\rho\hat{s} + a\hat{x}) + k_0 \frac{q}{(\rho^2+a^2)^{3/2}} (\rho\hat{s} - a\hat{x})$$

$$d\vec{E} = \frac{k_0}{(\rho^2+a^2)^{3/2}} \left[\rho\hat{s}\,(Q+q) + a\hat{x}\,(-q+Q) \right]$$

Para el flujo atraves del circulo

$$F = \iint \vec{E} \cdot \rho\, d\theta\, d\rho\, \hat{x} \quad \Big| \begin{array}{l} \hat{x}\cdot\hat{x} = 1 \\ \hat{s}\cdot\hat{x} = 0 \end{array} \Big\} \int$$



prob 1 parte 2

$$\vec{E} = \int_0^a\int_0^{2\pi} \frac{k_0}{(\rho^2+a^2)^{3/2}} \left[\rho\hat{s}(Q+q) + a\hat{x}(Q-q)\right]\rho\, d\theta\, d\rho$$

$$\int_0^{2\pi} \hat{s} f(\rho) + \hat{x} g(\rho)\, d\theta = 0 + \hat{x} g(\rho)\int_0^{2\pi} d\theta = \hat{x} g(\rho)\, 2\pi$$

$$\vec{E} = \hat{x}\int_0^a k_0\, a(Q-q)\frac{\rho}{(\rho^2+a^2)^{3/2}} 2\pi\, d\rho = \hat{x}\, 2\pi\, k_0\, a(Q-q)\int_0^a \frac{\rho}{(\rho^2+a^2)^{3/2}} d\rho$$

$$\int_0^a \frac{\rho}{(\rho^2+a^2)^{3/2}} d\rho = -\frac{1}{\sqrt{a^2+\rho^2}}\Bigg|_0^a = -\frac{1}{\sqrt{2a^2}} + \frac{1}{\sqrt{a^2}}$$

$$\vec{E} = 2\pi k_0 a(Q-q)\left[\frac{1}{\sqrt{a}} - \frac{1}{|\sqrt{2}a|}\right]\hat{x} = |E_0|\hat{x}$$

← cte

← no es un val absoluto

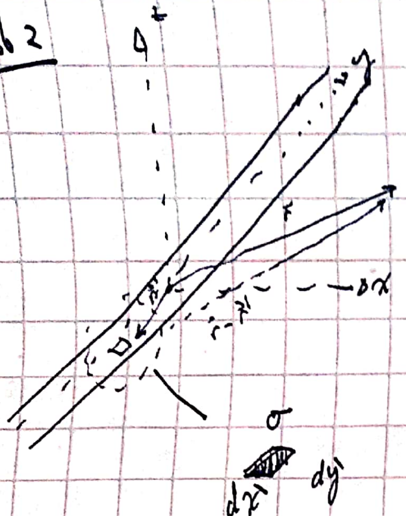
$$\text{flux} = \iint \vec{E}\cdot\rho\, d\theta\, d\rho\, \hat{x} = |E_0|\int_0^a\int_0^{2\pi}\rho\, d\theta\, d\rho = |E_0|\, 2\pi \frac{\rho^2}{2}\Bigg|_0^a$$

$$= |E_0|\, \underbrace{2\pi\frac{a^2}{2}}_{\oplus \text{ positivo}}$$

$$\text{flux} > 0 \quad \therefore \quad E_0 > 0 \quad \left\{ 0 < \underbrace{2\pi k_0 a\left[\frac{1}{|a|} - \frac{1}{|\sqrt{2}a|}\right]}_{\oplus}(Q-q)\right.$$

$$0 < Q-q \rightarrow \boxed{q < Q}$$

Prob 2



$$\vec{r}' = x'\hat{x} + y'\hat{y}$$
$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$K_0 = \frac{1}{4\pi\epsilon_0}$; $\int = \iint$

$$\vec{r} - \vec{r}' = (x-x')\hat{x} + (y-y')\hat{y} + z\hat{z}$$

$$\Phi_{(r)} = K_0 \int \frac{\sigma}{|\vec{r}-\vec{r}'|} da = K_0\sigma \int_{-\infty}^{\infty}\int_{-\frac{a}{2}}^{a/2} \frac{dx'\,dy'}{\left([x-x']^2 + [y-y']^2 + z^2\right)^{\frac{1}{2}}} \quad (1)$$

$\Phi_{(r)}$ no converge en los límites $(-\infty, +\infty)$; y

potencial por ecuación de Poisson

$$\nabla^2 \Phi = -\frac{\rho}{\epsilon_0} \quad ;$$

$$\vec{E} = K_0 \int_{-\infty}^{+\infty}\int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{(x-x')\hat{x} + (y-y')\hat{y} + z\hat{z}}{\left((x-x')^2 + (y-y')^2 + z^2\right)^{3/2}} dx\,dy$$

$$\Phi_{(r)} = \int_{\infty}^{-\infty} \frac{1}{2} Ln\left[1 + \frac{u}{\sqrt{u^2+v^2+z^2}}\right) - \frac{1}{2} Ln\left[1 - \frac{u}{\sqrt{u^2+v^2+z^2}}\right)\Bigg|^{u_2} d\cancel{y}$$

sin embargo rapidamente se puede notar que, no conviene seguir resolviendo

$\lim_{r\to\infty} \Phi_{(r)} \neq 0$ al ser infinito no hay punto 0 el <u>potencial es divergente</u>.

Prob 2. desarrollo Parte 2

$$\iint \frac{dx'\,dy'}{((x-x')^2+(y-y')^2+z^2)^{1/2}}$$

$u = x - x'$; $du = -dx'$

$v = y - y'$; $dv = -dy'$

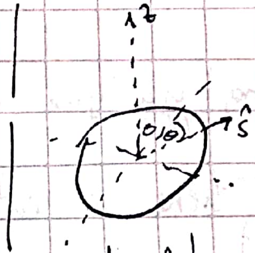
$\therefore\ dx'dy' = du\,dv$

$u[\frac{a}{2}] = x - \frac{a}{2}$; $v_{(\infty)} = -\infty$

$u[-\frac{a}{2}] = x + \frac{a}{2}$; $v_{(-\infty)} = +\infty$

limites.

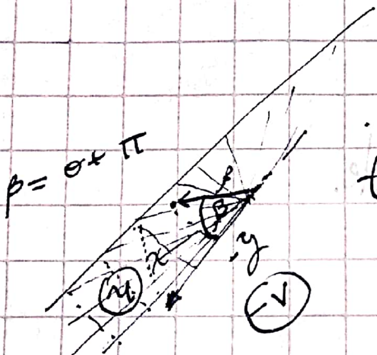
$$\iint \frac{du\,dv}{(u^2+v^2+\cancel{z}^2)^{1/2}}$$



coord polares. cilindricas.

$\rho^2 = u^2 + v^2$

$du\,dv = \rho\, d\theta\, d\rho$



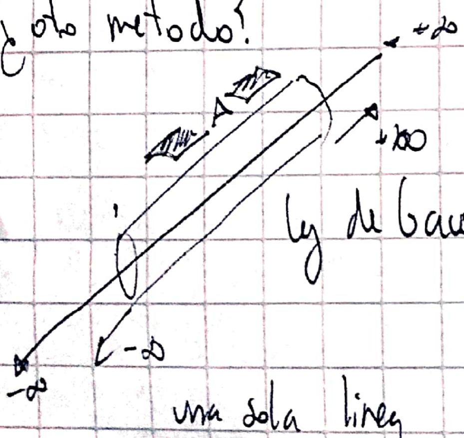
$\beta = \theta + \pi$

$\tan[\beta] = -\frac{u}{v}$

$\cos[\beta] = \frac{-v}{\rho}$ $\quad \sin[\beta] = \frac{u}{\rho}$

$$\vec{E} = k_0 \int_{+\infty}^{-\infty}\int_{x+\frac{a}{2}}^{x-\frac{a}{2}} \frac{u\hat{x} + v\hat{y} + z}{(u^2+v^2+z^2)^{3/2}}\, du\,dv$$

¿otro metodo?



ley de Gauss

una sola linea de carga

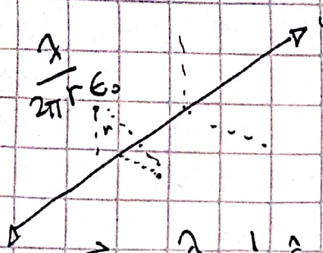
el area $A = 2\pi r L$ / Lim. $L \to \infty$

flux $\int \vec{E}\cdot d\vec{a} = E\, 2\pi r L = \frac{q_{enc}}{\epsilon_0}$ $\quad \frac{q_{enc}}{L} = \lambda \to q_{en}$

$$E\, 2\pi r \cancel{L} = \frac{\cancel{L}\lambda}{\epsilon_0}$$

$$\vec{E}_{(r)} = \frac{\lambda}{2\pi r \epsilon_0}\hat{s}$$

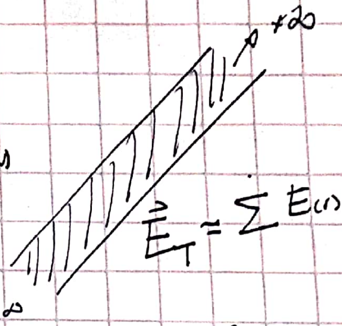
probl 2 deber Parte 3.



$\vec{E}(r) = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{r}\hat{s}$

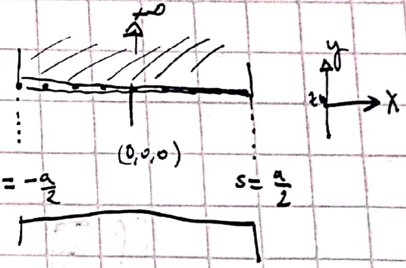
para un cable infinito

de juntar suficientes obtendremos



$\vec{E}_T = \sum E(r)$

el centro $r = \mathscr{r} = \left((x-s)^2 + z^2\right)^{1/2}$



tenemos una integración

$$\vec{E}_T = \frac{\lambda\,\hat{s}}{2\pi\epsilon_0}\underbrace{\int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{ds}{((x-s)^2+z^2)^{1/2}}}_{I}$$

$v = (x-s)$
$dv = -ds$

$$I = \int_{x+\frac{a}{2}}^{x-\frac{a}{2}} \frac{dv}{(v^2+z^2)^{1/2}}$$

$v = z\tan\theta$
$dv = z\sec^2\theta\, d\theta$

$$= \int_{v(x+\frac{a}{2})}^{v(x-\frac{a}{2})} \frac{z\sec^2\theta\, d\theta}{(z^2)^{1/2}(1+\tan^2\theta)^{1/2}}$$

$(1+\tan^2\theta)^{\frac{1}{2}} = \sec\theta$

$$I = \int_{x+\frac{a}{2}}^{x-\frac{a}{2}} \sec\theta\, d\theta = \ln\left[\sec\theta + \tan\theta\right]$$

$\sec^2\theta = 1 + \tan^2\theta = 1 + \frac{v^2}{z^2}$

$\sec\theta = \sqrt{1+\frac{v^2}{z^2}}$

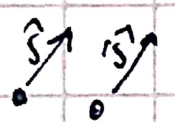
$\tan\theta = \frac{v}{z}$

$$I = \ln\left[\sqrt{1+\frac{v^2}{z^2}} + \frac{v}{z}\right)$$

prob. 2 parte 4

$$|\vec{E}_T| = \frac{\lambda}{2\pi\epsilon_0} \ln\left[\sqrt{1+\frac{v^2}{z^2}} + \frac{v}{z} \right)$$

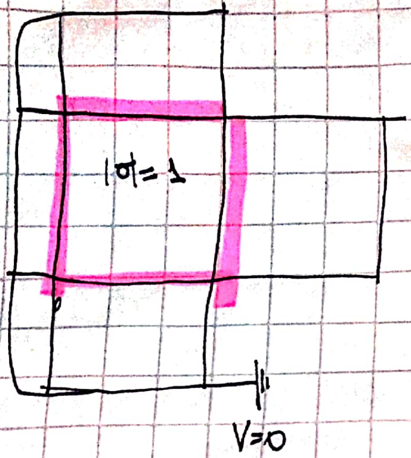
el vector $\hat{s}$ no seria el mismo, ya que con cada suma



cada linea tiene un centro polar distinto.

Sin embargo no se me ocurre una forma eficiente.

Probl 4



$$\nabla^2_{2D} V = -\frac{\sigma}{\epsilon_0} \quad ; \quad \nabla^2_{2D} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$\vec{E}_{out_\perp} - \vec{E}_{in_\perp} = \frac{\sigma}{\epsilon_0}\hat{n} \quad ; \quad \vec{E}_{in} = \vec{0} \quad \Big\{ \vec{E}_\perp = \frac{\sigma}{\epsilon_0}$$

$$\vec{E}_\perp = -\hat{n}\cdot\nabla V = -\frac{\partial V}{\partial n}$$

$$\sigma = -\epsilon_0 \frac{\partial V}{\partial n}$$

~~longitud~~ grosor infinitesimal $\delta(t)$

$$-\int_0^{0+dn} \frac{\sigma}{\epsilon_0}\,dn = \int_{V_1}^{V_2} dV \quad \Big\{ V_2 - V_1 = \cancel{\frac{\sigma}{\epsilon_0}} = 0 \rightarrow V_2 = V_1$$

sea cual sea, es continuo al atraviesa la superfi

Aproach ecuación de Poisson

$$\nabla^2_{2D} V = \frac{-\sigma}{\epsilon_0} \rightarrow \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{-\sigma}{\epsilon_0} \quad ; \quad V(x,y) = \psi(x)\,\phi(y)$$

$$\phi(y)\frac{d^2\psi}{dx^2} + \psi(x)\frac{d^2\phi}{dy^2} = \frac{-\sigma}{\epsilon_0} \quad \Big\{ \frac{1}{\psi}\frac{d^2\psi}{dx^2} + \frac{1}{\phi}\frac{d^2\phi}{dy^2} = -\frac{1}{V(x,y)}\frac{\sigma}{\epsilon_0}$$

$$\frac{1}{V(x,y)} = \frac{1}{\psi(x)\,\phi(y)} = \frac{A}{\psi(x)} + \frac{B}{\phi(x)}$$

$$\phi(y)A + \psi(x)B = 1$$

Sin sol. unica sin conocer ϕ, ψ