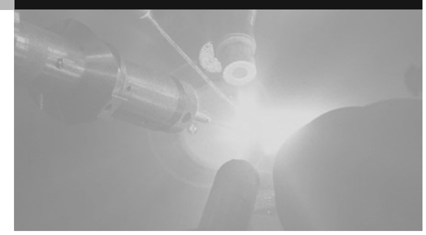
The Birth of Modern Physics

1

CHAPTER



The more important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote. . . . Our future discoveries must be looked for in the sixth place of decimals.

Albert A. Michelson, 1894

There is nothing new to be discovered in physics now. All that remains is more and more precise measurement.

William Thomson (Lord Kelvin), 1900

Although the Greek scholars Aristotle and Eratosthenes performed measurements and calculations that today we would call physics, the discipline of physics has its roots in the work of Galileo and Newton and others in the scientific revolution of the sixteenth and seventeenth centuries. The knowledge and practice of physics grew steadily for 200 to 300 years until another revolution in physics took place, which is the subject of this book. Physicists distinguish *classical physics*, which was mostly developed before 1895, from *modern physics*, which is based on discoveries made after 1895. The precise year is unimportant, but monumental changes occurred in physics around 1900.

The long reign of Queen Victoria of England, from 1837 to 1901, saw considerable changes in social, political, and intellectual realms, but perhaps none so important as the remarkable achievements that occurred in physics. For example, the description and predictions of electromagnetism by Maxwell are partly responsible for the rapid telecommunications of today. It was also during this period that thermodynamics rose to become an exact science. None of these achievements, however, have had the ramifications of the discoveries and applications of modern physics that would occur in the twentieth century. The world would never be the same.

In this chapter we briefly review the status of physics around 1895, including Newton's laws, Maxwell's equations, and the laws of thermodynamics. These results are just as important today as they were over a hundred years ago. Arguments by scientists concerning the interpretation of experimental data using wave and particle descriptions that seemed to have been resolved 200 years ago were reopened in the twentieth century. Today we look back on the evidence of the late nineteenth century and wonder how anyone could have doubted the validity of the atomic view of matter. The fundamental interactions of gravity, electricity, and magnetism were thought to be well understood in 1895. Physicists continued to be driven by the goal of understanding fundamental laws throughout the twentieth century. This is demonstrated by the fact that other fundamental forces (specifically the nuclear and weak interactions) have been added, and in some cases—curious as it may seem—various forces have even been combined. The search for the holy grail of fundamental interactions continues unabated today.

We finish this chapter with a status report on physics just before 1900. The few problems not then understood would be the basis for decades of fruitful investigations and discoveries continuing into the twenty-first century. We hope you find this chapter interesting both for the physics presented and for the historical account of some of the most exciting scientific discoveries of the modern era.

1.1 Classical Physics of the 1890s

Scientists and engineers of the late nineteenth century were indeed rather smug. They thought they had just about everything under control (see the quotes from Michelson and Kelvin on page 1). The best scientists of the day were highly recognized and rewarded. Public lectures were frequent. Some scientists had easy access to their political leaders, partly because science and engineering had benefited their war machines, but also because of the many useful technical advances. Basic research was recognized as important because of the commercial and military applications of scientific discoveries. Although there were only primitive automobiles and no airplanes in 1895, advances in these modes of transportation were soon to follow. A few people already had telephones, and plans for widespread distribution of electricity were under way.

Based on their success with what we now call macroscopic classical results, scientists felt that given enough time and resources, they could explain just about anything. They did recognize some difficult questions they still couldn't answer; for example, they didn't clearly understand the structure of matter—that was under intensive investigation. Nevertheless, on a macroscopic scale, they knew how to build efficient engines. Ships plied the lakes, seas, and oceans of the world. Travel between the countries of Europe was frequent and easy by train. Many scientists were born in one country, educated in one or two others, and eventually worked in still other countries. The most recent ideas traveled relatively quickly among the centers of research. Except for some isolated scientists, of whom Einstein is the most notable example, discoveries were quickly and easily shared. Scientific journals were becoming accessible.

Early successes of science

The ideas of classical physics are just as important and useful today as they were at the end of the nineteenth century. For example, they allow us to build automobiles and produce electricity. The conservation laws of energy, linear momentum, angular momentum, and charge can be stated as follows:

Classical conservation laws

Conservation of energy: The total sum of energy (in all its forms) is conserved in all interactions.

Conservation of linear momentum: In the absence of external forces, linear momentum is conserved in all interactions (vector relation).

Conservation of angular momentum: In the absence of external torque, angular momentum is conserved in all interactions (vector relation).

Conservation of charge: Electric charge is conserved in all interactions.

A nineteenth-century scientist might have added the **conservation of mass** to this list, but we know it not to be valid today (you will find out why in Chapter 2). These conservation laws are reflected in the laws of mechanics, electromagnetism, and thermodynamics. Electricity and magnetism, separate subjects for hundreds of years, were combined by James Clerk Maxwell (1831–1879) in his four equations. Maxwell showed optics to be a special case of electromagnetism. Waves, which permeated mechanics and optics, were known to be an important component of nature. Many natural phenomena could be explained by wave motion using the laws of physics.

Mechanics

The laws of mechanics were developed over hundreds of years by many researchers. Important contributions were made by astronomers because of the great interest in the heavenly bodies. Galileo (1564–1642) may rightfully be called the first great experimenter. His experiments and observations laid the groundwork for the important discoveries to follow during the next 200 years.

Galileo, the first great experimenter

Isaac Newton (1642–1727) was certainly the greatest scientist of his time and one of the best the world has ever seen. His discoveries were in the fields of mathematics, astronomy, and physics and include gravitation, optics, motion, and forces.

Newton, the greatest scientist of his time

We owe to Newton our present understanding of motion. He understood clearly the relationships among position, displacement, velocity, and acceleration. He understood how motion was possible and that a body at rest was just a special case of a body having constant velocity. It may not be so apparent to us today, but we should not forget the tremendous unification that Newton made when he pointed out that the motions of the planets about our sun can be understood by the same laws that explain motion on Earth, like apples falling from trees or a soccer ball being shot toward a goal. Newton was able to elucidate

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Galileo Galilei (1564–1642) was born, educated, and worked in Italy. Often said to be the "father of physics" because of his careful experimentation, he is shown here performing experiments by rolling balls on an inclined plane. He is perhaps best known for his experiments on motion, the development of the telescope, and his many astronomical discoveries. He came into disfavor with the Catholic Church for his belief in the Copernican theory. He was finally cleared of heresy by Pope John Paul II in 1992, 350 years after his death.

carefully the relationship between net force and acceleration, and his concepts were stated in three laws that bear his name today:

Newton's laws

Courtesy of Bausch & Lomb Optical Co. and the AIP Niels Bohr Library.

Isaac Newton (1642–1727), the great English physicist and mathematician, did most of his work at Cambridge where he was educated and became the Lucasian Professor of Mathematics. He was known not only for his work on the laws of motion but also as a founder of optics. His useful works are too numerous to list here, but it should be mentioned that he spent a considerable amount of his time on alchemy, theology, and the spiritual universe. His writings on these subjects, which were dear to him, were quite unorthodox. This painting shows him performing experiments with light.

Newton's first law: *An object in motion with a constant velocity will continue in motion unless acted upon by some net external force.* A body at rest is just a special case of Newton's first law with zero velocity. Newton's first law is often called the *law of inertia* and is also used to describe inertial reference frames.

Newton's second law: *The acceleration $\vec{a}$ of a body is proportional to the net external force $\vec{F}$ and inversely proportional to the mass m of the body. It is stated mathematically as*

$$\vec{F} = m\vec{a} \tag{1.1a}$$

A more general statement* relates force to the time rate of change of the linear momentum $\vec{p}$.

$$\vec{F} = \frac{d\vec{p}}{dt} \tag{1.1b}$$

Newton's third law: *The force exerted by body 1 on body 2 is equal in magnitude and opposite in direction to the force that body 2 exerts on body 1.* If the force on body 2 by body 1 is denoted by $\vec{F}_{21}$, then Newton's third law is written as

$$\vec{F}_{21} = -\vec{F}_{12} \tag{1.2}$$

It is often called the *law of action and reaction.*

These three laws develop the concept of force. Using that concept together with the concepts of velocity $\vec{v}$, acceleration $\vec{a}$, linear momentum $\vec{p}$, rotation (angular velocity $\vec{\omega}$ and angular acceleration $\vec{\alpha}$), and angular momentum $\vec{L}$, we can describe the complex motion of bodies.

Electromagnetism

Electromagnetism developed over a long period of time. Important contributions were made by Charles Coulomb (1736–1806), Hans Christian Oersted (1777–1851), Thomas Young (1773–1829), André Ampère (1775–1836), Michael Faraday (1791–1867), Joseph Henry (1797–1878), James Clerk Maxwell (1831–1879), and Heinrich Hertz (1857–1894). Maxwell showed that electricity and magnetism were intimately connected and were related by a change in the inertial frame of reference. His work also led to the understanding of electromagnetic radiation, of which light and optics are special cases. Maxwell's four equations, together with the Lorentz force law, explain much of electromagnetism.

Maxwell's equations

Gauss's law for electricity $$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0} \tag{1.3}$$

Gauss's law for magnetism $$\oint \vec{B} \cdot d\vec{A} = 0 \tag{1.4}$$

Faraday's law $$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \tag{1.5}$$

*It is a remarkable fact that Newton wrote his second law not as $\vec{F} = m\vec{a}$, but as $\vec{F} = d(m\vec{v})/dt$, thus taking into account mass flow and change in velocity. This has applications in both fluid mechanics and rocket propulsion.

$$\text{Generalized Ampere's law} \qquad \oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 I \tag{1.6}$$

$$\text{Lorentz force law} \qquad \vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \tag{1.7}$$

Maxwell's equations indicate that charges and currents create fields, and in turn, these fields can create other fields, both electric and magnetic.

Thermodynamics

Thermodynamics deals with temperature T, heat Q, work W, and the internal energy of systems U. The understanding of the concepts used in thermodynamics—such as pressure P, volume V, temperature, thermal equilibrium, heat, entropy, and especially energy—was slow in coming. We can understand the concepts of pressure and volume as mechanical properties, but the concept of temperature must be carefully considered. We have learned that the internal energy of a system of noninteracting point masses depends only on the temperature.

Important contributions to thermodynamics were made by Benjamin Thompson (Count Rumford, 1753–1814), Sadi Carnot (1796–1832), James Joule (1818–1889), Rudolf Clausius (1822–1888), and William Thomson (Lord Kelvin, 1824–1907). The primary results of thermodynamics can be described in two laws:

Laws of thermodynamics

First law of thermodynamics: *The change in the internal energy ΔU of a system is equal to the heat Q added to the system plus the work W done on the system.*

$$\Delta U = Q + W \tag{1.8}$$

The first law of thermodynamics generalizes the conservation of energy by including heat.

Second law of thermodynamics: *It is not possible to convert heat completely into work without some other change taking place.* Various forms of the second law state similar, but slightly different, results. For example, it is not possible to build a perfect engine or a perfect refrigerator. It is not possible to build a perpetual motion machine. Heat does not spontaneously flow from a colder body to a hotter body without some other change taking place. The second law forbids all these from happening. The first law states the conservation of energy, but the second law says what kinds of energy processes cannot take place. For example, it is possible to completely convert work into heat, but not vice versa, without some other change taking place.

Two other "laws" of thermodynamics are sometimes expressed. One is called the "zeroth" law, and it is useful in understanding temperature. It states that *if two thermal systems are in thermodynamic equilibrium with a third system, they are in equilibrium with each other.* We can state it more simply by saying that *two systems at the same temperature as a third system have the same temperature as each other.* This concept was not explicitly stated until the twentieth century. The "third" law of thermodynamics expresses that *it is not possible to achieve an absolute zero temperature.*

1.2 The Kinetic Theory of Gases

We understand now that gases are composed of atoms and molecules in rapid motion, bouncing off each other and the walls, but in the 1890s this had just gained acceptance. The kinetic theory of gases is related to thermodynamics and to the atomic theory of matter, which we discuss in Section 1.5. Experiments were relatively easy to perform on gases, and the Irish chemist Robert Boyle (1627–1691) showed around 1662 that the pressure times the volume of a gas was constant for a constant temperature. The relation PV = constant (for constant T) is now referred to as *Boyle's law*. The French physicist Jacques Charles (1746–1823) found that V/T = constant (at constant pressure), referred to as *Charles's law*. Joseph Louis Gay-Lussac (1778–1850) later produced the same result, and the law is sometimes associated with his name. If we combine these two laws, we obtain the ideal gas equation

Ideal gas equation

$$PV = nRT \tag{1.9}$$

where n is the number of moles and R is the ideal gas constant, 8.31 J/mol · K.

In 1811 the Italian physicist Amedeo Avogadro (1776–1856) proposed that equal volumes of gases at the same temperature and pressure contained equal numbers of molecules. This hypothesis was so far ahead of its time that it was not accepted for many years. The famous English chemist John Dalton opposed the idea because he apparently misunderstood the difference between atoms and molecules. Considering the rudimentary nature of the atomic theory of matter at the time, this was not surprising.

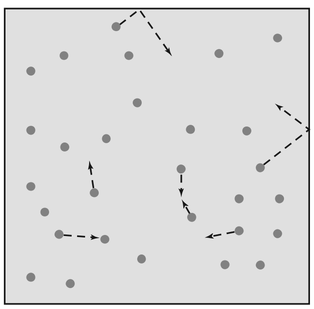


Figure 1.1 Molecules inside a closed container are shown colliding with the walls and with each other. The motions of a few molecules are indicated by the arrows. The number of molecules inside the container is huge.

Daniel Bernoulli (1700–1782) apparently originated the kinetic theory of gases in 1738, but his results were generally ignored. Many scientists, including Newton, Laplace, Davy, Herapath, and Waterston, had contributed to the development of kinetic theory by 1850. Theoretical calculations were being compared with experiments, and by 1895 the kinetic theory of gases was widely accepted. The statistical interpretation of thermodynamics was made in the latter half of the nineteenth century by Maxwell, the Austrian physicist Ludwig Boltzmann (1844–1906), and the American physicist J. Willard Gibbs (1839–1903).

In introductory physics classes, the kinetic theory of gases is usually taught by applying Newton's laws to the collisions that a molecule makes with other molecules and with the walls. A representation of a few molecules colliding is shown in Figure 1.1. In the simple model of an ideal gas, only elastic collisions are considered. By taking averages over the collisions of many molecules, the ideal gas law, Equation (1.9), is revealed. The average kinetic energy of the molecules is shown to be linearly proportional to the temperature, and the internal energy U is

Statistical thermodynamics

$$U = nN_A\langle K\rangle = \frac{3}{2}nRT \tag{1.10}$$

where n is the number of moles of gas, N_A is Avogadro's number, $\langle K\rangle$ is the average kinetic energy of a molecule, and R is the ideal gas constant. This relation ignores any nontranslational contributions to the molecular energy, such as rotations and vibrations.

However, energy is not represented only by translational motion. It became clear that all *degrees of freedom*, including rotational and vibrational, were also capable of carrying energy. The *equipartition theorem* states that each degree of freedom of a molecule has an average energy of $kT/2$, where k is the Boltzmann constant ($k = R/N_A$). Translational motion has three degrees of freedom, and rotational and vibrational modes can also be excited at higher temperatures. If there are f degrees of freedom, then Equation (1.10) becomes

Equipartition theorem

Internal energy

$$U = \frac{f}{2}nRT \tag{1.11}$$

The molar ($n = 1$) heat capacity c_V at constant volume for an ideal gas is the rate of change in internal energy with respect to change in temperature and is given by

Heat capacity

$$c_V = \frac{3}{2}R \tag{1.12}$$

The experimental quantity c_V/R is plotted versus temperature for hydrogen in Figure 1.2. The ratio c_V/R is equal to 3/2 for low temperatures, where only translational kinetic energy is important, but it rises to 5/2 at 300 K, where rotations occur for H_2, and finally reaches 7/2, because of vibrations at still higher temperatures, before the molecule dissociates. Although the kinetic theory of gases fails to predict specific heats for real gases, it leads to models that can be used on a gas-by-gas basis. Kinetic theory is also able to provide useful information on other properties such as diffusion, speed of sound, mean free path, and collision frequency.

In the 1850s Maxwell derived a relation for the distribution of speeds of the molecules in gases. The distribution of speeds $f(v)$ is given as a function of the speed and the temperature by the equation

Maxwell's speed distribution

$$f(v) = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT} \tag{1.13}$$

where m is the mass of a molecule and T is the temperature. This result is plotted for nitrogen in Figure 1.3 for temperatures of 300 K, 1000 K, and 4000 K. The peak of each distribution is the most probable speed of a gas molecule for the given temperature. In 1895 measurement was not precise enough to confirm Maxwell's distribution, and it was not confirmed experimentally until 1921.

By 1895 Boltzmann had made Maxwell's calculation more rigorous, and the general relation is called the *Maxwell-Boltzmann distribution*. The distribution can be used to find the *root-mean-square* speed v_{rms},

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3kT}{m}} \tag{1.14}$$

which shows the relationship of the energy to the temperature for an ideal gas:

$$U = nN_A\langle K\rangle = nN_A\frac{m\langle v^2\rangle}{2} = nN_A\frac{m3kT}{2m} = \frac{3}{2}nRT \tag{1.15}$$

This was the result of Equation (1.10).

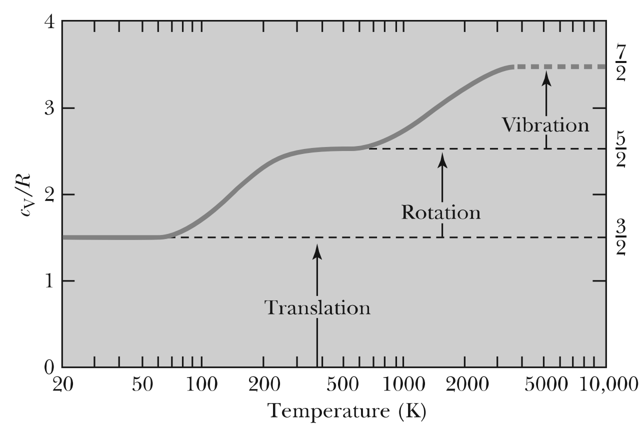


Figure 1.2 The molar heat capacity at constant volume (c_V) divided by R (c_V/R is dimensionless) is displayed as a function of temperature for hydrogen gas. Note that as the temperature increases, the rotational and vibrational modes become important. This experimental result is consistent with the equipartition theorem, which adds $kT/2$ of energy per molecule ($RT/2$ per mole) for each degree of freedom.

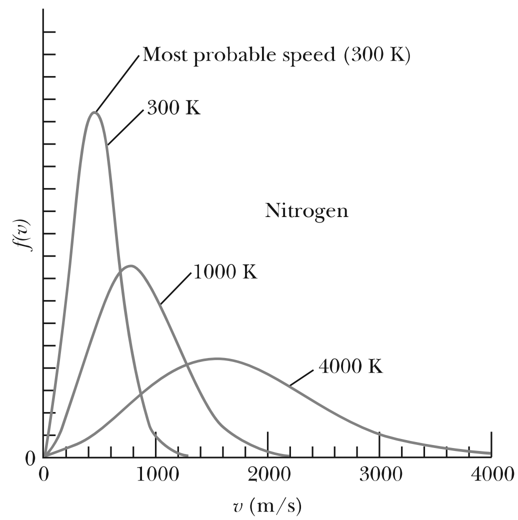


Figure 1.3 The Maxwell distribution of molecular speeds (for nitrogen), $f(v)$, is shown as a function of speed for three temperatures.

1.3 Waves and Particles

We first learned the concepts of velocity, acceleration, force, momentum, and energy in introductory physics by using a single particle with its mass concentrated in one small point. In order to adequately describe nature, we add two- and three-dimensional bodies and rotations and vibrations. However, many aspects of physics can still be treated as if the bodies are simple particles. In particular, the kinetic energy of a moving particle is one way that energy can be transported from one place to another.

But we have found that many natural phenomena can be explained only in terms of *waves*, which are traveling disturbances that carry energy. This description includes standing waves, which are superpositions of traveling waves. Most waves, like water waves and sound waves, need an elastic medium in which to move. Curiously enough, matter is not transported in waves—but energy is. Mass may oscillate, but it doesn't actually propagate along with the wave. Two examples are a cork and a boat on water. As a water wave passes, the cork gains energy as it moves up and down, and after the wave passes, the cork remains. The boat also reacts to the wave, but it primarily rocks back and forth, throwing around things that are not fixed on the boat. The boat obtains considerable kinetic energy from the wave. After the wave passes, the boat eventually returns to rest.

Energy transport

Nature of light: waves or particles?

Waves and particles were the subject of disagreement as early as the seventeenth century, when there were two competing theories of the nature of light. Newton supported the idea that light consisted of corpuscles (or particles). He performed extensive experiments on light for many years and finally published his book *Opticks* in 1704. *Geometrical optics* uses straight-line, particle-like trajectories called *rays* to explain familiar phenomena such as reflection and refraction. Geometrical optics was also able to explain the apparent observation of sharp shadows. The competing theory considered light as a wave phenomenon. Its strongest proponent was the Dutch physicist Christian Huygens (1629–1695), who presented his theory in 1678. The wave theory could also explain reflection and refraction, but it could not explain the sharp shadows observed. Experimental physics of the 1600s and 1700s was not able to discern between the two competing theories. Huygens's poor health and other duties kept him from working on optics much after 1678. Although Newton did not feel strongly about his corpuscular theory, the magnitude of his reputation caused it to be almost universally accepted for more than a hundred years and throughout most of the eighteenth century.

Finally, in 1802, the English physician Thomas Young (1773–1829) announced the results of his two-slit interference experiment, indicating that light behaved as a wave. Even after this singular event, the corpuscular theory had its supporters. During the next few years Young and, independently, Augustin Fresnel (1788–1827) performed several experiments that clearly showed that light behaved as a wave. By 1830 most physicists believed in the wave theory—some 150 years after Newton performed his first experiments on light.

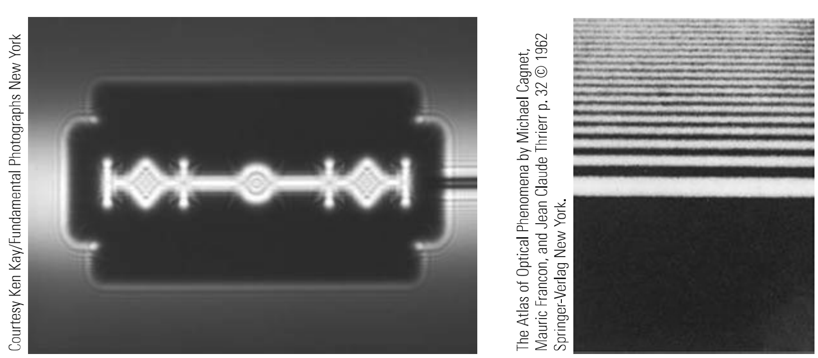
One final experiment indicated that the corpuscular theory was difficult to accept. Let c be the speed of light in vacuum and v be the speed of light in another medium. If light behaves as a particle, then to explain refraction, light must speed up when going through denser material ($v > c$). The wave theory of Huygens predicts just the opposite ($v < c$). The measurements of the speed of light in various media were slowly improving, and finally, in 1850, Foucault showed that *light traveled more slowly in water than in air*. The corpuscular theory seemed incorrect. Newton would probably have been surprised that his weakly held beliefs lasted as long as they did. Now we realize that geometrical optics is correct only if the wavelength of light is much smaller than the size of the obstacles and apertures that the light encounters.

Figure 1.4 shows the "shadows" or *diffraction patterns* from light falling on sharp edges. In Figure 1.4a the alternating black and white lines can be seen all around the razor blade's edges. Figure 1.4b is a highly magnified photo of the diffraction from a sharp edge. The bright and dark regions can be understood only if light is a wave and not a particle. The physicists of 200 to 300 years ago apparently did not observe such phenomena. They believed that shadows were sharp, and only the particle nature of light could explain their observations.

In the 1860s Maxwell showed that electromagnetic waves consist of oscillating electric and magnetic fields. Visible light covers just a narrow range of the total electromagnetic spectrum, and all electromagnetic radiation travels at the speed of light c in free space, given by

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \lambda f \tag{1.16}$$

where λ is the wavelength and f is the frequency. The fundamental constants μ_0 and ϵ_0 are defined in electricity and magnetism and reveal the connection to the speed of light. In 1887 the German physicist Heinrich Hertz (1857–1894) succeeded in generating and detecting electromagnetic waves having wavelengths



(a) (b)

Figure 1.4 In contradiction to what scientists thought in the seventeenth century, shadows are not sharp, but show dramatic diffraction patterns—as seen here (a) for a razor blade and (b) for a highly magnified sharp edge.

far outside the visible range ($\lambda \approx 5$ m). The properties of these waves were just as Maxwell had predicted. His results continue to have far-reaching effects in modern telecommunications: cable TV, cell phones, lasers, fiber optics, wireless Internet, and so on.

Some unresolved issues about electromagnetic waves in the 1890s eventually led to one of the two great modern theories, the *theory of relativity* (see Section 1.6 and Chapter 2). Waves play a central and essential role in the other great modern physics theory, *quantum mechanics*, which is sometimes called *wave mechanics*. Because waves play such a central role in modern physics, we review their properties in Chapter 5.

1.4 Conservation Laws and Fundamental Forces

Conservation laws are the guiding principles of physics. The application of a few laws explains a vast quantity of physical phenomena. We listed the conservation laws of classical physics in Section 1.1. They include energy, linear momentum, angular momentum, and charge. Each of these is extremely useful in introductory physics. We use linear momentum when studying collisions, and the conservation laws when examining dynamics. We have seen the concept of the conservation of energy change. At first we had only the conservation of kinetic energy in a force-free region. Then we added potential energy and formed the conservation of mechanical energy. In our study of thermodynamics, we added internal energy, and so on. The study of electrical circuits was made easier by the conservation of charge flow at each junction and the conservation of energy throughout all the circuit elements.

Much of what we know about conservation laws and fundamental forces has been learned within the last hundred years. In our study of modern physics we will find that mass is added to the conservation of energy, and the result is sometimes called the *conservation of mass-energy*, although the term *conservation of energy* is still sufficient and generally used. When we study elementary particles we will add the conservation of baryons and the conservation of leptons. Closely related to conservation laws are invariance principles. Some parameters are invariant in some interactions or in specific systems but not in others. Examples include time reversal, parity, and distance. We will study the Newtonian or Galilean invariance and find it lacking in our study of relativity; a new invariance principle will be needed. In our study of nuclear and elementary particles, conservation laws and invariance principles will often be used (see Figure 1.5).

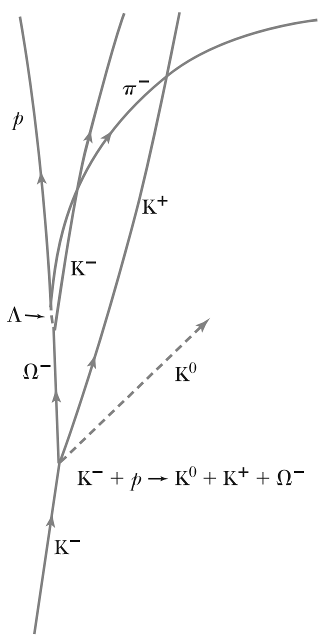


Figure 1.5 The conservation laws of momentum and energy are invaluable in untangling complex particle reactions like the one shown here, where a 5-GeV K^- meson interacts with a proton at rest to produce an Ω^- in a bubble chamber. The uncharged K^0 is not observed. Notice the curved paths of the charged particles in the magnetic field. Such reactions are explained in Chapter 14.

Fundamental Forces

In introductory physics, we often begin our study of forces by examining the reaction of a mass at the end of a spring, because the spring force can be easily calibrated. We subsequently learn about tension, friction, gravity, surface, electrical, and magnetic forces. Despite the seemingly complex array of forces, we presently believe there are only three fundamental forces. All the other forces can be derived from them. These three forces are the **gravitational**, **electroweak**, and **strong** forces. Some physicists refer to the electroweak interaction as separate electromagnetic and weak forces because the unification occurs only at very high energies. The approximate strengths and ranges of the three fundamental forces are listed in Table 1.1. Physicists sometimes use the term *interaction* when

Table 1.1 Fundamental Forces

Interaction		Relative Strength*	Range
Strong		1	Short, $\sim 10^{-15}$ m
Electroweak	Electromagnetic	10^{-2}	Long, $1/r^2$
	Weak	10^{-9}	Short, $\sim 10^{-15}$ m
Gravitational		10^{-39}	Long, $1/r^2$

*These strengths are quoted for neutrons and/or protons in close proximity.

referring to the fundamental forces because it is the overall interaction among the constituents of a system that is of interest.

The gravitational force is the weakest. It is the force of mutual attraction between masses and, according to Newton, is given by

$$\vec{F}_g = -G\frac{m_1 m_2}{r^2}\hat{r} \tag{1.17}$$

Gravitational interaction

where m_1 and m_2 are two point masses, G is the gravitational constant, r is the distance between the masses, and $\hat{r}$ is a unit vector directed along the line between the two point masses (attractive force). The gravitational force is noticeably effective only on a macroscopic scale, but it has tremendous importance: it is the force that keeps Earth rotating about our source of life energy—the sun—and that keeps us and our atmosphere anchored to the ground. Gravity is a long-range force that diminishes as $1/r^2$.

Weak interaction

The primary component of the electroweak force is *electromagnetic*. The other component is the *weak* interaction, which is responsible for beta decay in nuclei, among other processes. In the 1970s Sheldon Glashow, Steven Weinberg, and Abdus Salam predicted that the electromagnetic and weak forces were in fact facets of the same force. Their theory predicted the existence of new particles, called *W* and *Z* bosons, which were discovered in 1983. We discuss bosons and the experiment in Chapter 14. For all practical purposes, the weak interaction is effective in the nucleus only over distances the size of 10^{-15} m. Except when dealing with very high energies, physicists mostly treat nature as if the electromagnetic and weak forces were separate. Therefore, you will sometimes see references to the *four* fundamental forces (gravity, strong, electromagnetic, and weak).

Electromagnetic interaction

The electromagnetic force is responsible for holding atoms together, for friction, for contact forces, for tension, and for electrical and optical signals. It is responsible for all chemical and biological processes, including cellular structure and nerve processes. The list is long because the electromagnetic force is responsible for practically all nongravitational forces that we experience. The electrostatic, or Coulomb, force between two point charges q_1 and q_2, separated by a distance r, is given by

$$\vec{F}_C = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \tag{1.18}$$

Coulomb force

The easiest way to remember the vector direction is that like charges repel and unlike charges attract. Moving charges also create and react to magnetic fields [see Equation (1.7)].

Strong interaction

The third fundamental force, the strong force, is the one holding the nucleus together. It is the strongest of all the forces, but it is effective only over short distances—on the order of 10^{-15} m. The strong force is so strong that it easily binds two protons inside a nucleus even though the electrical force of repulsion over the tiny confined space is huge. The strong force is able to contain dozens of protons inside the nucleus before the electrical force of repulsion becomes strong enough to cause nuclear decay. We study the strong force extensively in this book, learning that neutrons and protons are composed of *quarks*, and that the part of the strong force acting between quarks has the unusual name of *color* force.

Unification of forces

Physicists strive to combine forces into more fundamental ones. Centuries ago the forces responsible for friction, contact, and tension were all believed to be different. Today we know they are all part of the electroweak force. Two hundred years ago scientists thought the electrical and magnetic forces were independent, but after a series of experiments, physicists slowly began to see their connection. This culminated in the 1860s in Maxwell's work, which clearly showed they were but part of one force and at the same time explained light and other radiation. Figure 1.6 is a diagram of the unification of forces over time. Newton certainly had an inspiration when he was able to unify the planetary motions with the apple falling from the tree. We will see in Chapter 15 that Einstein was even able to link gravity with space and time.

The further unification of forces currently remains one of the most active research fields. Considerable efforts have been made to unify the electroweak and strong forces through the *grand unified theories,* or GUTs. A leading GUT is the mathematically complex *string theory*. Several predictions of these theories have not yet been verified experimentally (for example, the instability of the proton and the existence of magnetic monopoles). We present some of the exciting research areas in present-day physics throughout this book, because these

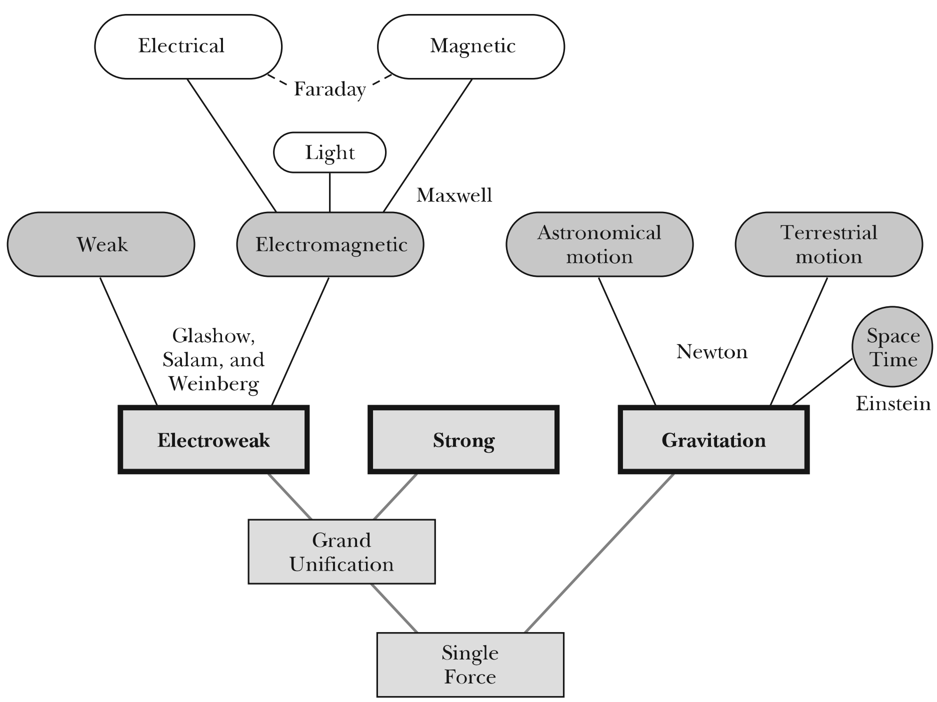


Figure 1.6 The three fundamental forces (shown in the heavy boxes) are themselves unifications of forces that were once believed to be fundamental. Present research is under way (see blue lines) to further unify the fundamental forces into a single force.

topics are the ones you will someday read about on the front pages of newspapers and in the weekly news magazines and perhaps will contribute to in your own careers.

1.5 The Atomic Theory of Matter

Today the idea that matter is composed of tiny particles called *atoms* is taught in elementary school and expounded throughout later schooling. We are told that the Greek philosophers Democritus and Leucippus proposed the concept of atoms as early as 450 B.C. The smallest piece of matter, which could not be subdivided further, was called an *atom*, after the Greek word *atomos*, meaning "indivisible." Physicists do not discredit the early Greek philosophers for thinking that the basic entity of life consisted of atoms. For centuries, scientists were called "natural philosophers," and in this tradition the highest university degree American scientists receive is a Ph.D., which stands for doctor of philosophy.

Not many new ideas were proposed about atoms until the seventeenth century, when scientists started trying to understand the properties and laws of gases. The work of Boyle, Charles, and Gay-Lussac presupposed the interactions of tiny particles in gases. Chemists and physical chemists made many important advances. In 1799 the French chemist Proust (1754–1826) proposed the *law of definite proportions*, which states that when two or more elements combine to form a compound, the proportions by weight (or mass) of the elements are always the same. Water (H_2O) is always formed of one part hydrogen and eight parts oxygen by mass.

Dalton, the father of the atomic theory

The English chemist John Dalton (1766–1844) is given most of the credit for originating the modern atomic theory of matter. In 1803 he proposed that the atomic theory of matter could explain the law of definite proportions if the elements are composed of atoms. Each element has atoms that are physically and chemically characteristic. The concept of atomic weights (or masses) was the key to the atomic theory.

In 1811 the Italian physicist Avogadro proposed the existence of molecules, consisting of individual or combined atoms. He stated without proof that *all gases contain the same number of molecules in equal volumes at the same temperature and pressure*. Avogadro's ideas were ridiculed by Dalton and others who could not imagine that atoms of the same element could combine. If this could happen, they argued, then all the atoms of a gas would combine to form a liquid. The concept of molecules and atoms was indeed difficult to imagine, but finally, in 1858, the Italian chemist Cannizzaro (1826–1910) solved the problem and showed how Avogadro's ideas could be used to find atomic masses. Today we think of an atom as the smallest unit of matter that can be identified with a particular element. A molecule can be a single atom or a combination of two or more atoms of either like or dissimilar elements. Molecules can consist of thousands of atoms.

Avogadro's number

The number of molecules in one gram-molecular weight of a particular element (6.023×10^{23} molecules/mol) is called Avogadro's number (N_A). For example, one mole of hydrogen (H_2) has a mass of about 2 g and one mole of carbon has a mass of about 12 g; one mole of each substance consists of 6.023×10^{23} atoms. Avogadro's number was not even estimated until 1865, and it was finally accurately measured by Perrin, as we discuss at the end of this section.

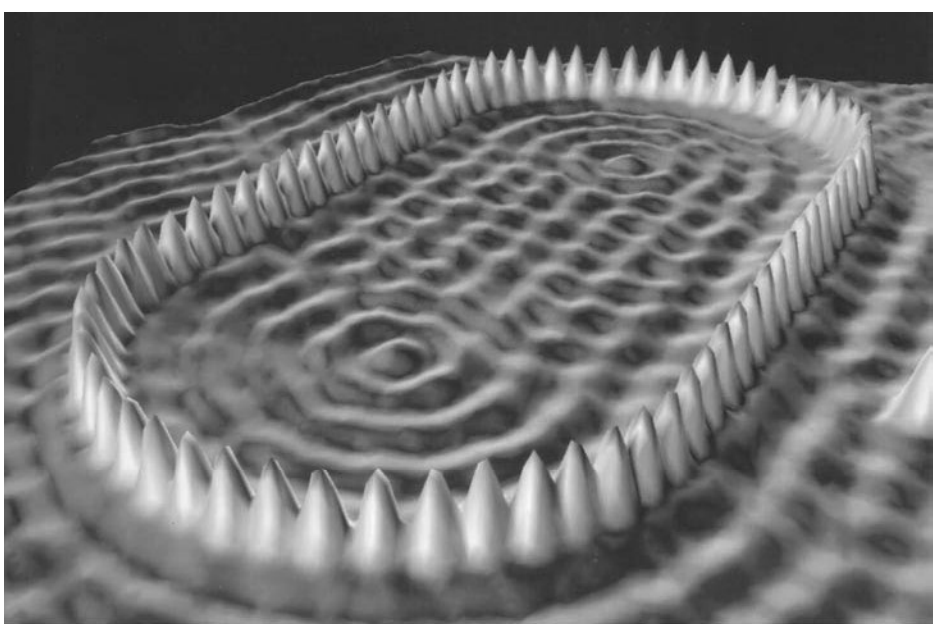
During the mid-1800s the kinetic theory of gases was being developed, and because it was based on the concept of atoms, its successes gave validity to the atomic theory. The experimental results of specific heats, Maxwell speed distribution, and transport phenomena (see the discussion in Section 1.2) all supported the concept of the atomic theory.

In 1827 the English botanist Robert Brown (1773–1858) observed with a microscope the motion of tiny pollen grains suspended in water. The pollen appeared to dance around in random motion, while the water was still. At first the motion (now called *Brownian motion*) was ascribed to convection or organic matter, but eventually it was observed to occur for any tiny particle suspended in liquid. The explanation according to the atomic theory is that the molecules in the liquid are constantly bombarding the tiny grains. A satisfactory explanation was not given until the twentieth century (by Einstein).

Although it may appear, according to the preceding discussion, that the atomic theory of matter was universally accepted by the end of the nineteenth century, that was not the case. Certainly most physicists believed in it, but there was still opposition. A principal leader in the antiatomic movement was the renowned Austrian physicist Ernst Mach. Mach was an absolute positivist, believing in the reality of nothing but our own sensations. A simplified version of his line of reasoning would be that because we have never *seen* an atom, we cannot say anything about its reality. The Nobel Prize–winning German physical chemist Wilhelm Ostwald supported Mach philosophically but also had more practical arguments on his side. In 1900 there were difficulties in understanding radioactivity, x rays, discrete spectral lines, and how atoms formed molecules and solids. Ostwald contended that we should therefore think of atoms as hypothetical constructs, useful for bookkeeping in chemical reactions.

Opposition to atomic theory

On the other hand, there were many believers in the atomic theory. Max Planck, the originator of quantum theory, grudgingly accepted the atomic theory of matter because his radiation law supported the existence of submicroscopic quanta. Boltzmann was convinced that atoms must exist, mainly because they were necessary in his statistical mechanics. It is said that Boltzmann committed suicide in 1905 partly because he was despondent that so many people rejected his theory. Today we have pictures of the atom (see Figure 1.7) that would



Courtesy of International Business Machines.

Figure 1.7 This scanning tunneling microscope photo, called the "stadium corral," shows 76 individually placed iron atoms on a copper surface. The IBM researchers were trying to contain and modify electron density, observed by the wave patterns, by surrounding the electrons inside the quantum "corral." Researchers are thus able to study the quantum behavior of electrons. See also the Special Topic on Scanning Probe Microscopes in Chapter 6.

undoubtedly have convinced even Mach, who died in 1916 still unconvinced of the validity of the atomic theory.

Overwhelming evidence of atomic theory

Overwhelming evidence for the existence of atoms was finally presented in the first decade of the twentieth century. First, Einstein, in one of his three famous papers published in 1905 (the others were about special relativity and the photoelectric effect), provided an explanation of the Brownian motion observed almost 80 years earlier by Robert Brown. Einstein explained the motion in terms of molecular motion and presented theoretical calculations for the *random walk* problem. A random walk (often called the *drunkard's walk*) is a statistical process that determines how far from its initial position a tiny grain may be after many random molecular collisions. Einstein was able to determine the approximate masses and sizes of atoms and molecules from experimental data.

Finally, in 1908, the French physicist Jean Perrin (1870–1942) presented data from an experiment designed using kinetic theory that agreed with Einstein's predictions. Perrin's experimental method of observing many particles of different sizes is a classic work, for which he received the Nobel Prize for Physics in 1926. His experiment utilized four types of measurements. Each was consistent with the atomic theory, and each gave a quantitative determination of Avogadro's number—the first accurate measurements that had been made. Since 1908 the atomic theory of matter has been accepted by practically everyone.

1.6 Unresolved Questions of 1895 and New Horizons

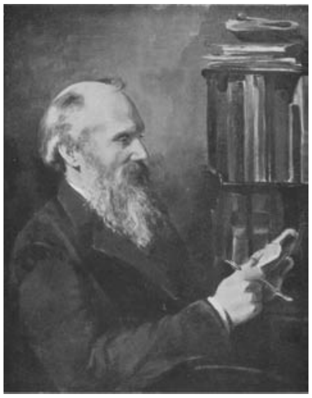
We choose 1895 as a convenient time to separate the periods of classical and modern physics, although this is an arbitrary choice based on discoveries made in 1895–1897. The thousand or so physicists living in 1895 were rightfully proud of the status of their profession. The precise experimental method was firmly established. Theories were available that could explain many observed phenomena. In large part, scientists were busy measuring and understanding such physical parameters as specific heats, densities, compressibility, resistivity, indices of refraction, and permeabilities. The pervasive feeling was that, given enough time, everything in nature could be understood by applying the careful thinking and experimental techniques of physics. The field of mechanics was in particularly good shape, and its application had led to the stunning successes of the kinetic theory of gases and statistical thermodynamics.

Experiment and reasoning

In hindsight we can see now that this euphoria of success applied only to the macroscopic world. Objects of human dimensions such as automobiles, steam engines, airplanes, telephones, and electric lights either existed or were soon to appear and were triumphs of science and technology. However, the atomic theory of matter was not universally accepted, and what made up an atom was purely conjecture. The structure of matter was unknown.

Clouds on the horizon

There were certainly problems that physicists could not resolve. Only a few of the deepest thinkers seemed to be concerned with them. Lord Kelvin, in a speech in 1900 to the Royal Institution, referred to "two clouds on the horizon." These were the electromagnetic medium and the failure of classical physics to explain blackbody radiation. We mention these and other problems here. Their solutions were soon to lead to two of the greatest breakthroughs in human thought ever recorded—the theories of quantum physics and of relativity.



AIP Emilio Segrè Visual Archives, Brittle Books Collection.

William Thomson (Lord Kelvin, 1824–1907) was born in Belfast, Ireland, and at age 10 entered the University of Glasgow in Scotland where his father was a professor of mathematics. He graduated from the University of Cambridge and, at age 22, accepted the chair of natural philosophy (later called physics) at the University of Glasgow, where he finished his illustrious 53-year career, finally resigning in 1899 at age 75. Lord Kelvin's contributions to nineteenth-century science were far reaching, and he made contributions in electricity, magnetism, thermodynamics, hydrodynamics, and geophysics. He was involved in the successful laying of the transatlantic cable. He was arguably the preeminent scientist of the latter part of the nineteenth century. He was particularly well known for his prediction of the Earth's age, which would later turn out to be inaccurate (see Chapter 12).

Electromagnetic Medium. The waves that were well known and understood by physicists all had media in which the waves propagated. Water waves traveled in water, and sound waves traveled in any material. It was natural for nineteenth-century physicists to assume that electromagnetic waves also traveled in a medium, and this medium was called the *ether*. Several experiments, the most notable of which were done by Michelson, had sought to detect the ether without success. An extremely careful experiment by Michelson and Morley in 1887 was so sensitive, it should have revealed the effects of the ether. Subsequent experiments to check other possibilities were also negative. In 1895 some physicists were concerned that the elusive ether could not be detected. Was there an alternative explanation?

Electrodynamics. The other difficulty with Maxwell's electromagnetic theory had to do with the electric and magnetic fields as seen and felt by moving bodies. What appears as an electric field in one reference system may appear as a magnetic field in another system moving with respect to the first. Although the relationship between electric and magnetic fields seemed to be understood by using Maxwell's equations, the equations do not keep the same form under a Galilean transformation [see Equations (2.1) and (2.2)], a situation that concerned both Hertz and Lorentz. Hertz unfortunately died in 1894 at the young age of 36 and never experienced the modern physics revolution. The Dutch physicist Hendrik Lorentz (1853–1928), on the other hand, proposed a radical idea that solved the electrodynamics problem: space was contracted along the direction of motion of the body. George FitzGerald in Ireland independently proposed the same concept. The Lorentz-FitzGerald hypothesis, proposed in 1892, was a precursor to Einstein's theory advanced in 1905 (see Chapter 2).

Blackbody Radiation. In 1895 thermodynamics was on a strong footing; it had achieved much success. One of the interesting experiments in thermodynamics concerns an object, called a *blackbody*, that absorbs the entire spectrum of electromagnetic radiation incident on it. An enclosure with a small hole serves as a blackbody, because all the radiation entering the hole is absorbed. A blackbody also emits radiation, and the emission spectrum shows the electromagnetic power emitted per unit area. The radiation emitted covers all frequencies, each with its own intensity. Precise measurements were carried out to determine the spectrum of blackbody radiation, such as that shown in Figure 1.8. Blackbody radiation was a fundamental issue, because the emission spectrum is independent of the body itself—it is characteristic of all blackbodies.

Many physicists of the period—including Kirchhoff, Stefan, Boltzmann, Rubens, Pringsheim, Lummer, Wien, Lord Rayleigh, Jeans, and Planck—had worked on the problem. It was possible to understand the spectrum both at the low-frequency end and at the high-frequency end, but no single theory could account for the entire spectrum. When the most modern theory of the day (the equipartition of energy applied to standing waves in a cavity) was applied to the problem, the result led to an *infinite* emissivity (or energy density) for high frequencies. The failure of the theory was known as the "ultraviolet catastrophe." The solution of the problem by Max Planck in 1900 would shake the very foundations of physics.

Ultraviolet catastrophe: infinite emissivity

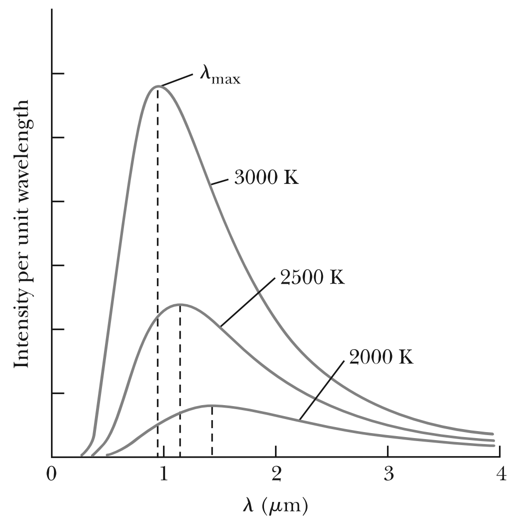


Figure 1.8 The blackbody spectrum, showing the emission spectrum of radiation emitted from a blackbody as a function of the radiation wavelength. Different curves are produced for different temperatures, but they are independent of the type of blackbody cavity. The intensity peaks at λ_{max}.

On the Horizon

Discovery of x rays

Discovery of radioactivity

During the years 1895–1897 there were four discoveries that were all going to require deeper understanding of the atom. The first was the discovery of x rays by the German physicist Wilhelm Röntgen (1845–1923) in November 1895. Next came the accidental discovery of radioactivity by the French physicist Henri Becquerel (1852–1908), who in February 1896 placed uranium salt next to a carefully wrapped photographic plate. When the plate was developed, a silhouette of the uranium salt was evident—indicating the presence of a very penetrating ray.

Discovery of the electron

The third discovery, that of the electron, was actually the work of several physicists over a period of years. Michael Faraday, as early as 1833, observed a gas discharge glow—evidence of electrons. Over the next few years, several scientists detected evidence of particles, called *cathode rays*, being emitted from charged cathodes. In 1896 Perrin proved that cathode rays were negatively charged. The discovery of the electron, however, is generally credited to the British physicist J. J. Thomson (1856–1940), who in 1897 isolated the electron (cathode ray) and measured its velocity and its ratio of charge to mass.

Discovery of the Zeeman effect

The final important discovery of the period was made by the Dutch physicist Pieter Zeeman (1865–1943), who in 1896 found that a single spectral line was sometimes separated into two or three lines when the sample was placed in a magnetic field. The (normal) *Zeeman effect* was quickly explained by Lorentz as the result of light being emitted by the motion of electrons inside the atom. Zeeman and Lorentz showed that the frequency of the light was affected by the magnetic field according to the classical laws of electromagnetism.

The unresolved issues of 1895 and the important discoveries of 1895–1897 bring us to the subject of this book, *Modern Physics*. In 1900 Max Planck completed his radiation law, which solved the blackbody problem but required that energy be quantized. In 1905 Einstein presented his three important papers on Brownian motion, the photoelectric effect, and special relativity. While the work of Planck and Einstein may have solved the problems of the nineteenth-century physicists, they broadened the horizons of physics and have kept physicists active ever since.

Summary

Physicists of the 1890s felt that almost anything in nature could be explained by the application of careful experimental methods and intellectual thought. The application of mechanics to the kinetic theory of gases and statistical thermodynamics, for example, was a great success.

The particle viewpoint of light had prevailed for over a hundred years, mostly because of the weakly held belief of the great Newton, but in the early 1800s the nature of light was resolved in favor of waves. In the 1860s Maxwell showed that his electromagnetic theory predicted a much wider frequency range of electromagnetic radiation than the visible optical phenomena. In the twentieth century, the question of waves versus particles was to reappear.

The conservation laws of energy, momentum, angular momentum, and charge are well established. The three fundamental forces are gravitational, electroweak, and strong. Over the years many forces have been unified into these three. Physicists are actively pursuing attempts to unify these three forces into only two or even just one single fundamental force.

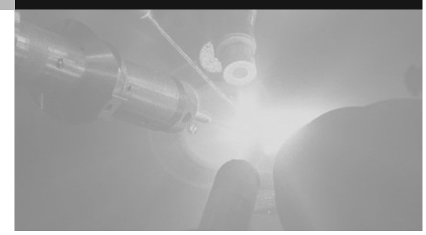
The atomic theory of matter assumes atoms are the smallest unit of matter that is identified with a characteristic element. Molecules are composed of atoms, which can be from different elements. The kinetic theory of gases assumes the atomic theory is correct, and the development of the two theories proceeded together. The atomic theory of matter was not fully accepted until around 1910, by which time Einstein had explained Brownian motion and Perrin had published overwhelming experimental evidence.

The year 1895 saw several outstanding problems that seemed to worry only a few physicists. These problems included the inability to detect an electromagnetic medium, the difficulty in understanding the electrodynamics of moving bodies, and blackbody radiation. Four important discoveries during the period 1895–1897 were to signal the atomic age: x rays, radioactivity, the electron, and the splitting of spectral lines (Zeeman effect). The understanding of these problems and discoveries (among others) is the object of this book on modern physics.

Special Theory of Relativity

2

CHAPTER



It was found that there was no displacement of the interference fringes, so that the result of the experiment was negative and would, therefore, show that there is still a difficulty in the theory itself. . . .

Albert Michelson, Light Waves and Their Uses, 1907

One of the great theories of physics appeared early in the twentieth century when Albert Einstein presented his special theory of relativity in 1905. We learned in introductory physics that Newton's laws of motion must be measured relative to some reference frame. A reference frame is called an **inertial frame** if Newton's laws are valid in that frame. If a body subject to no net external force moves in a straight line with constant velocity, then the coordinate system attached to that body defines an inertial frame. If Newton's laws are valid in one reference frame, then they are also valid in a reference frame moving at a uniform velocity relative to the first system. This is known as the **Newtonian principle of relativity** or **Galilean invariance**.

Inertial frame

Galilean invariance

Newton showed that it was not possible to determine absolute motion in space by any experiment, so he decided to use relative motion. In addition, the Newtonian concepts of time and space are completely separable. Consider two inertial reference frames, K and K′, that move along their x and x' axes, respectively, with uniform relative velocity $\vec{v}$ as shown in Figure 2.1. We show system K′ moving to the right with velocity $\vec{v}$ with respect to system K, which is fixed or

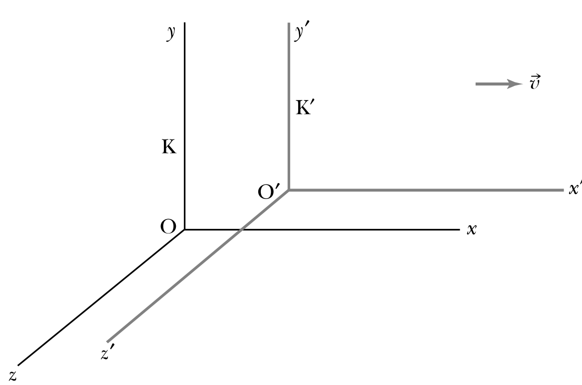


Figure 2.1 Two inertial systems are moving with relative speed v along their x axes. We show the system K at rest and the system K′ moving with speed v relative to the system K.

stationary somewhere. One result of the relativity theory is that there are no fixed, absolute frames of reference. We use the term *fixed* to refer to a system that is fixed on a particular object, such as a planet, star, or spaceship that itself is moving in space. The transformation of the coordinates of a point in one system to the other system is given by

$$\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \end{aligned} \tag{2.1}$$

Similarly, the inverse transformation is given by

$$\begin{aligned} x &= x' + vt \\ y &= y' \\ z &= z' \end{aligned} \tag{2.2}$$

Galilean transformation

where we have set $t = t'$ because Newton considered time to be absolute. Equations (2.1) and (2.2) are known as the **Galilean transformation**. Newton's laws of motion are invariant under a Galilean transformation; that is, they have the same form in both systems K and K′.

In the late nineteenth century Albert Einstein was concerned that although Newton's laws of motion had the same form under a Galilean transformation, Maxwell's equations did not. Einstein believed so strongly in Maxwell's equations that he showed there was a significant problem in our understanding of the Newtonian principle of relativity. In 1905 he published ideas that rocked the very foundations of physics and science. He proposed that space and time are not separate and that Newton's laws are only an approximation. This special theory of relativity and its ramifications are the subject of this chapter. We begin by presenting the experimental situation historically—showing why a problem existed and what was done to try to rectify the situation. Then we discuss Einstein's two postulates on which the special theory is based. The interrelation of space and time is discussed, and several amazing and remarkable predictions based on the new theory are shown.

As the concepts of relativity became used more often in everyday research and development, it became essential to understand the transformation of momentum, force, and energy. Here we study relativistic dynamics and the relationship between mass and energy, which leads to one of the most famous equations in physics and a new conservation law of mass-energy. Finally, we return to electromagnetism to investigate the effects of relativity. We learn that Maxwell's equations don't require change, and electric and magnetic effects are relative, depending on the observer. We leave until Chapter 15 our discussion of Einstein's general theory of relativity.

2.1 The Apparent Need for Ether

Thomas Young, an English physicist and physician, performed his famous experiments on the interference of light in 1802. A decade later, the French physicist and engineer Augustin Fresnel published his calculations showing the detailed understanding of interference, diffraction, and polarization. Because all known waves (other than light) require a medium in which to propagate (water waves have water, sound waves have, for example, air, and so on), it was naturally assumed that light also required a medium, even though light was apparently able to travel in vacuum through outer space. This medium was called the *luminiferous ether* or just **ether** for short, and it must have some amazing properties. The ether had to have such a low density that planets could pass through it, seemingly for eternity, with no apparent loss of orbit position. Its elasticity must be strong enough to pass waves of incredibly high speeds!

The concept of ether

The electromagnetic theory of light (1860s) of the Scottish mathematical physicist James Clerk Maxwell shows that the speed of light in different media depends only on the electric and magnetic properties of matter. In vacuum, the speed of light is given by $v = c = 1/\sqrt{\mu_0\epsilon_0}$, where μ_0 and ϵ_0 are the permeability and permittivity of free space, respectively. The properties of the ether, as proposed by Maxwell in 1873, must be consistent with electromagnetic theory, and the feeling was that to be able to discern the ether's various properties required only a sensitive enough experiment. The concept of ether was well accepted by 1880.

When Maxwell presented his electromagnetic theory, scientists were so confident in the laws of classical physics that they immediately pursued the aspects of Maxwell's theory that were in contradiction with those laws. As it turned out, this investigation led to a new, deeper understanding of nature. Maxwell's equations predict the velocity of light in a vacuum to be c. If we have a flashbulb go off in the moving system K′, an observer in system K′ measures the speed of the light pulse to be c. However, if we make use of Equation (2.1) to find the relation between speeds, we find the speed measured in system K to be $c + v$, where v is the relative speed of the two systems. However, Maxwell's equations don't differentiate between these two systems. Physicists of the late nineteenth century proposed that there must be one preferred inertial reference frame in which the ether was stationary and that in this system the speed of light was c. In the other systems, the speed of light would indeed be affected by the relative speed of the reference system. Because the speed of light was known to be so enormous, 3×10^8 m/s, no experiment had as yet been able to discern an effect due to the relative speed v. The ether frame would in fact be an absolute standard, from which other measurements could be made. Scientists set out to find the effects of the ether.


AIP/Emilio Segrè Visual Archives.

Albert A. Michelson (1852–1931) shown at his desk at the University of Chicago in 1927. He was born in Prussia but came to the United States when he was two years old. He was educated at the U.S. Naval Academy and later returned on the faculty. Michelson had appointments at several American universities including the Case School of Applied Science, Cleveland, in 1883; Clark University, Worcester, Massachusetts, in 1890; and the University of Chicago in 1892 until his retirement in 1929. During World War I he returned to the U.S. Navy, where he developed a rangefinder for ships. He spent his retirement years in Pasadena, California, where he continued to measure the speed of light at Mount Wilson.

2.2 The Michelson-Morley Experiment

The Earth orbits around the sun at a high orbital speed, about $10^{-4}c$, so an obvious experiment is to try to find the effects of the Earth's motion through the ether. Even though we don't know how fast the sun might be moving through the ether, the Earth's orbital *velocity* changes significantly throughout the year because of its change in direction, even if its orbital *speed* is nearly constant.

Albert Michelson (1852–1931) performed perhaps the most significant American physics experiment of the 1800s. Michelson, who was the first U.S. citizen to receive the Nobel Prize in Physics (1907), was an ingenious scientist who built an extremely precise device called an *interferometer*, which measures the phase difference between two light waves. Michelson used his interferometer to detect the difference in the speed of light passing through the ether in different directions. The basic technique is shown in Figure 2.2. Initially, it is assumed that one of the interferometer arms (AC) is parallel to the motion of the Earth through the ether. Light leaves the source S and passes through the glass plate at A. Because the back of A is partially silvered, part of the light is reflected,

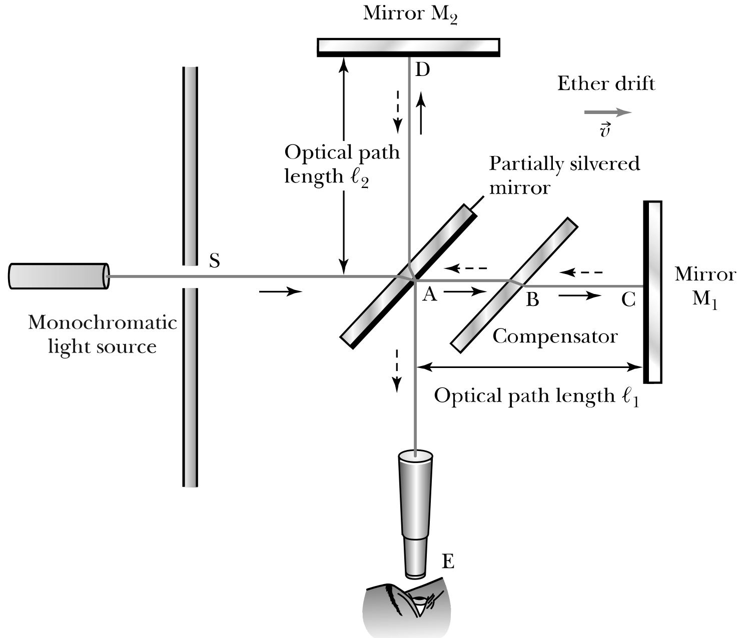
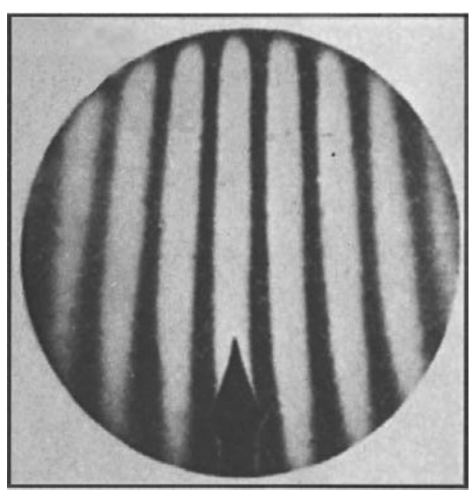


Figure 2.2 A schematic diagram of Michelson's interferometer experiment. Light of a single wavelength is partially reflected and partially transmitted by the glass at A. The light is subsequently reflected by mirrors at C and D, and, after reflection or transmission again at A, enters the telescope at E. Interference fringes are visible to the observer at E.

eventually going to the mirror at D, and part of the light travels through A on to the mirror at C. The light is reflected at the mirrors C and D and comes back to the partially silvered mirror A, where part of the light from each path passes on to the telescope and eye at E. The compensator is added at B to make sure both light paths pass through equal thicknesses of glass. Interference fringes can be found by using a bright light source such as sodium, with the light filtered to make it monochromatic, and the apparatus is adjusted for maximum intensity of the light at E. We will show that the fringe pattern should shift if the apparatus is rotated through 90° such that arm AD becomes parallel to the motion of the Earth through the ether and arm AC is perpendicular to the motion.

We let the optical path lengths of AC and AD be denoted by ℓ_1 and ℓ_2, respectively. The observed interference pattern consists of alternating bright and dark bands, corresponding to constructive and destructive interference, respectively (Figure 2.3). For constructive interference, the difference between the two



From L. S. Swenson, Jr., *Invention and Discovery* 43 (Fall 1987).

Figure 2.3 Interference fringes as they would appear in the eyepiece of the Michelson-Morley experiment.

path lengths (to and from the mirrors) is given by some number of wavelengths, $2(\ell_1 - \ell_2) = n\lambda$, where λ is the wavelength of the light and n is an integer.

The expected shift in the interference pattern can be calculated by determining the time difference between the two paths. When the light travels from A to C, the velocity of light according to the Galilean transformation is $c + v$, because the ether carries the light along with it. On the return journey from C to A the velocity is $c - v$, because the light travels opposite to the path of the ether. The total time for the round-trip journey to mirror M_1 is t_1:

$$t_1 = \frac{\ell_1}{c+v} + \frac{\ell_1}{c-v} = \frac{2c\ell_1}{c^2 - v^2} = \frac{2\ell_1}{c}\left(\frac{1}{1 - v^2/c^2}\right)$$

Now imagine what happens to the light that is reflected from mirror M_2. If the light is pointed directly at point D, the ether will carry the light with it, and the light misses the mirror, much as the wind can affect the flight of an arrow. If a swimmer (who can swim with speed v_2 in still water) wants to swim across a swiftly moving river (speed v_1), the swimmer must start heading upriver, so that when the current carries her downstream, she will move directly across the river. Careful reasoning shows that the swimmer's velocity is $\sqrt{v_2^2 - v_1^2}$ throughout her journey (Problem 4). Thus the time t_2 for the light to pass to mirror M_2 at D and back is

$$t_2 = \frac{2\ell_2}{\sqrt{c^2 - v^2}} = \frac{2\ell_2}{c}\frac{1}{\sqrt{1 - v^2/c^2}}$$

The time difference between the two journeys Δt is

$$\Delta t = t_2 - t_1 = \frac{2}{c}\left(\frac{\ell_2}{\sqrt{1 - v^2/c^2}} - \frac{\ell_1}{1 - v^2/c^2}\right) \tag{2.3}$$

We now rotate the apparatus by 90° so that the ether passes along the length ℓ_2 toward the mirror M_2. We denote the new quantities by primes and carry out an analysis similar to that just done. The time difference $\Delta t'$ is now

$$\Delta t' = t_2' - t_1' = \frac{2}{c}\left(\frac{\ell_2}{1 - v^2/c^2} - \frac{\ell_1}{\sqrt{1 - v^2/c^2}}\right) \tag{2.4}$$

Michelson looked for a shift in the interference pattern when his apparatus was rotated by 90°. The time difference is

$$\Delta t' - \Delta t = \frac{2}{c}\left(\frac{\ell_1 + \ell_2}{1 - v^2/c^2} - \frac{\ell_1 + \ell_2}{\sqrt{1 - v^2/c^2}}\right)$$

Because we know $c \gg v$, we can use the binomial expansion* to expand the terms involving v^2/c^2, keeping only the lowest terms.

$$\begin{aligned}\Delta t' - \Delta t &= \frac{2}{c}(\ell_1 + \ell_2)\left[\left(1 + \frac{v^2}{c^2} + \cdots\right) - \left(1 + \frac{v^2}{2c^2} + \cdots\right)\right] \\ &\approx \frac{v^2(\ell_1 + \ell_2)}{c^3} \end{aligned} \tag{2.5}$$

Michelson left his position at the U.S. Naval Academy in 1880 and took his interferometer to Europe for postgraduate studies with some of Europe's best physi-

*See Appendix 3 for the binomial expansion.

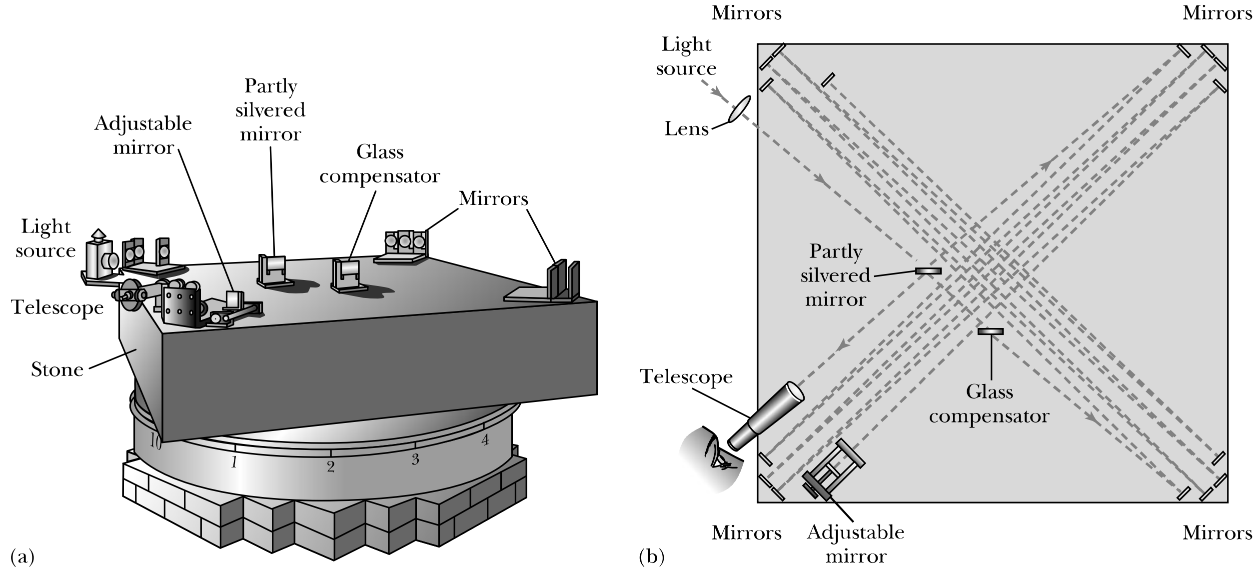


Figure 2.4 An adaptation of the Michelson and Morley 1887 experiment taken from their publication [A. A. Michelson and E. M. Morley, *Philosophical Magazine* **190**, 449 (1887)]. (a) A perspective view of the apparatus. To reduce vibration, the experiment was done on a massive soapstone, 1.5 m square and 0.3 m thick. This stone was placed on a wooden float that rested on mercury inside the annular piece shown underneath the stone. The entire apparatus rested on a brick pier. (b) The incoming light is focused by the lens and is both transmitted and reflected by the partly silvered mirror. The adjustable mirror allows fine adjustments in the interference fringes. The stone was rotated slowly and uniformly on the mercury to look for the interference effects of the ether.

Michelson in Europe

cists, particularly Hermann Helmholtz in Berlin. After a few false starts he finally was able to perform a measurement in Potsdam (near Berlin) in 1881. In order to use Equation (2.5) for an estimate of the expected time difference, the value of the Earth's orbital speed around the sun, 3×10^4 m/s, was used. Michelson's apparatus had $\ell_1 \approx \ell_2 \approx \ell = 1.2$ m. Thus Equation (2.5) predicts a time difference of 8×10^{-17} s. This is an exceedingly small time, but for a visible wavelength of 6×10^{-7} m, the period of one wavelength amounts to $T = 1/f = \lambda/c = 2 \times 10^{-15}$ s. Thus the time period of 8×10^{-17} s represents 0.04 fringes in the interference pattern. Michelson reasoned that he should be able to detect a shift of at least half this value but found none. Although disappointed, Michelson concluded that the hypothesis of the stationary ether must be incorrect.

The result of Michelson's experiment was so surprising that he was asked by several well-known physicists to repeat it. In 1882 Michelson accepted a position at the then-new Case School of Applied Science in Cleveland. Together with Edward Morley (1838–1923), a professor of chemistry at nearby Western Reserve College who had become interested in Michelson's work, he put together the more sophisticated experiment shown in Figure 2.4. The new experiment had an optical path length of 11 m, created by reflecting the light for eight round trips. The new apparatus was mounted on soapstone that floated on mercury to eliminate vibrations and was so effective that Michelson and Morley believed they could detect a fraction of a fringe shift as small as 0.005. With their new apparatus they expected the ether to produce a shift as large as 0.4 of a fringe. They reported in 1887 a *null result*—no effect whatsoever! The ether

Null result of Michelson-Morley experiment

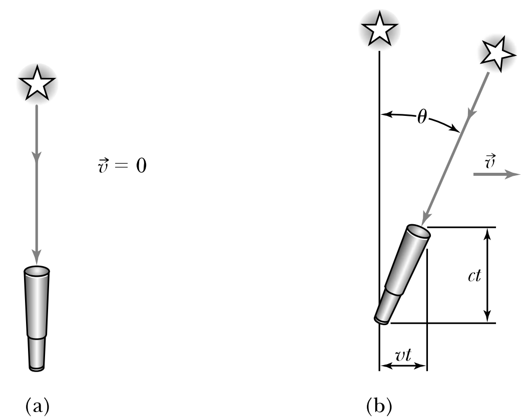


Figure 2.5 The effect of stellar aberration. (a) If a telescope is at rest, light from a distant star will pass directly into the telescope. (b) However, if the telescope is traveling at speed v (because it is fixed on the Earth, which has a motion about the sun), it must be slanted slightly to allow the starlight to enter the telescope. This leads to an apparent circular motion of the star as seen by the telescope, as the motion of the Earth about the sun changes throughout the solar year.

does not seem to exist. It is this famous experiment that has become known as the *Michelson-Morley experiment.*

The measurement so shattered a widely held belief that many suggestions were made to explain it. What if the Earth just happened to have a zero motion through the ether at the time of the experiment? Michelson and Morley repeated their experiment during night and day and for different seasons throughout the year. It is unlikely that at least sometime during these many experiments, the Earth would not be moving through the ether. Michelson and Morley even took their experiment to a mountaintop to see if the effects of the ether might be different. There was no change.

Ether drag

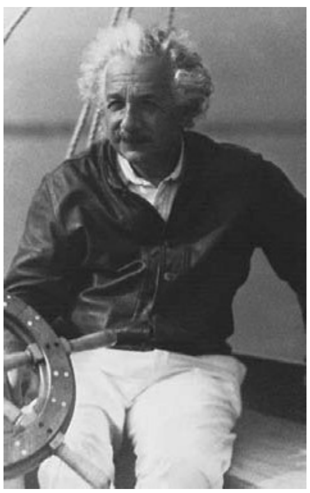
Of the many possible explanations of the null ether measurement, the one taken most seriously was the *ether drag* hypothesis. Some scientists proposed that the Earth somehow dragged the ether with it as the Earth rotates on its own axis and revolves around the sun. However, the ether drag hypothesis contradicts results from several experiments, including that of *stellar aberration* noted by the British astronomer James Bradley in 1728. Bradley noticed that the apparent position of the stars seems to rotate in a circular motion with a period of one year. The angular diameter of this circular motion with respect to the Earth is 41 seconds of arc. This effect can be understood by an analogy. From the viewpoint of a person sitting in a car during a rainstorm, the raindrops appear to fall vertically when the car is at rest but appear to be slanted toward the windshield when the car is moving forward. The same effect occurs for light coming from stars directly above the Earth's orbital plane. If the telescope and star are at rest with respect to the ether, the light enters the telescope as shown in Figure 2.5a. However, because the Earth is moving in its orbital motion, the apparent position of the star is at an angle θ as shown in Figure 2.5b. The telescope must actually be slanted at an angle θ to observe the light from the overhead star. During a time period t the starlight moves a vertical distance ct while the telescope moves a horizontal distance vt, so that the tangent of the angle θ is

Stellar aberration

$$\tan\theta = \frac{vt}{ct} = \frac{v}{c}$$

The orbital speed of the Earth is about 3×10^4 m/s; therefore, the angle θ is 10^{-4} rad or 20.6 seconds of arc, with a total opening of $2\theta = 41$ s as the Earth rotates—in agreement with Bradley's observation. The aberration reverses itself over the course of six months as the Earth orbits about the sun, in effect giving a circular motion to the star's position. This observation is in disagreement with the hypothesis of the Earth dragging the ether. If the ether were dragged with the Earth, there would be no need to tilt the telescope! The experimental observation of stellar aberration together with the null result of the Michelson and Morley experiment is enough evidence to refute the suggestions that the ether exists. Many other experimental observations have now been made that also confirm this conclusion.

The inability to detect the ether was a serious blow to reconciling the invariant form of the electromagnetic equations of Maxwell. There seems to be no single reference inertial system in which the speed of light is actually c. H. A. Lorentz and G. F. FitzGerald suggested, apparently independently, that the results of the Michelson-Morley experiment could be understood if length is contracted by the factor $\sqrt{1 - v^2/c^2}$ in the direction of motion, where v is the speed in the direction of travel. For this situation, the length ℓ_1, in the direction of motion, will be contracted by the factor $\sqrt{1 - v^2/c^2}$, whereas the length ℓ_2, perpendicular to v, will not. The result in Equation (2.3) is that t_1 will have the extra factor $\sqrt{1 - v^2/c^2}$, making Δt precisely zero as determined experimentally by Michelson. This contraction postulate, which became known as the *Lorentz-FitzGerald contraction,* was not proven from first principles using Maxwell's equations, and its true significance was not understood for several years until Einstein presented his explanation. An obvious problem with the Lorentz-FitzGerald contraction is that it is an ad hoc assumption that cannot be directly tested. Any measuring device would presumably be shortened by the same factor.



AIP/Emilio Segrè Visual Archives.

Albert Einstein (1879–1955), shown here sailing on Long Island Sound, was born in Germany and studied in Munich and Zurich. After having difficulty finding a position, he served seven years in the Swiss Patent Office in Bern (1902–1909), where he did some of his best work. He obtained his doctorate at the University of Zurich in 1905. His fame quickly led to appointments in Zurich, Prague, back to Zurich, and then to Berlin in 1914. In 1933, after Hitler came to power, Einstein left for the Institute for Advanced Study at Princeton University, where he became a U.S. citizen in 1940 and remained until his death in 1955. Einstein's total contributions to physics are rivaled only by those of Isaac Newton.

2.3 Einstein's Postulates

At the turn of the twentieth century, the Michelson-Morley experiment had laid to rest the idea of finding a preferred inertial system for Maxwell's equations, yet the Galilean transformation, which worked for the laws of mechanics, was invalid for Maxwell's equations. This quandary represented a turning point for physics.

Albert Einstein (1879–1955) was only two years old when Michelson reported his first null measurement for the existence of the ether. Einstein said that he began thinking at age 16 about the form of Maxwell's equations in moving inertial systems, and in 1905, when he was 26 years old, he published his startling proposal* about the principle of relativity, which he believed to be fundamental. Working without the benefit of discussions with colleagues outside his small circle of friends, Einstein was apparently unaware of the interest concerning the null result of Michelson and Morley.† Einstein instead looked at the problem in a more formal manner and believed that Maxwell's equations must be valid in

*In one issue of the German journal *Annalen der Physik* **17**, No. 4 (1905), Einstein published three remarkable papers. The first, on the quantum properties of light, explained the photoelectric effect; the second, on the statistical properties of molecules, included an explanation of Brownian motion; and the third was on special relativity. All three papers contained predictions that were subsequently confirmed experimentally.

†The question of whether Einstein knew of Michelson and Morley's null result before he produced his special theory of relativity is somewhat uncertain. For example, see J. Stachel, "Einstein and Ether Drift Experiments," *Physics Today* (May 1987), p. 45.

all inertial frames. With piercing insight and genius, Einstein was able to bring together seemingly inconsistent results concerning the laws of mechanics and electromagnetism with two postulates (as he called them; today we would call them laws). These postulates are

Einstein's two postulates

1. **The principle of relativity:** The laws of physics are the same in all inertial systems. There is no way to detect absolute motion, and no preferred inertial system exists.
2. **The constancy of the speed of light:** Observers in all inertial systems measure the same value for the speed of light in a vacuum.

The first postulate indicates that the laws of physics are the same in all coordinate systems moving with uniform relative motion to each other. Einstein showed that postulate 2 actually follows from the first one. He returned to the principle of relativity as espoused by Newton. Although Newton's principle referred only to the laws of mechanics, Einstein expanded it to include all laws of physics—including those of electromagnetism. We can now modify our previous definition of *inertial frames of reference* to be those frames of reference in which *all the laws of physics* are valid.

Inertial frames of reference revisited

Einstein's solution requires us to take a careful look at time. Return to the two systems of Figure 2.1 and remember that we had previously assumed that $t = t'$. We assumed that events occurring in system K′ and in system K could easily be synchronized. Einstein realized that each system must have its own observers with their own clocks and metersticks. *An event in a given system must be specified by stating both its space and time coordinates*. Consider the flashing of two bulbs fixed in system K as shown in Figure 2.6a. **M**ary, in system K′ (the **M**oving system) is beside **F**rank, who is in system K (the **F**ixed system), when the bulbs flash. As seen in Figure 2.6b the light pulses travel the same distance in system K and arrive at Frank *simultaneously*. Frank sees the two flashes at the same time. However, the two light pulses do not reach Mary simultaneously, because system K′ is moving to the right, and she has moved closer to the bulb on the right by the time the flash reaches her. The light flash coming from the left will reach her at some later time. Mary thus determines that the light on the right flashed before the one on the left, because she is at rest in her frame and both flashes approach her

Simultaneity

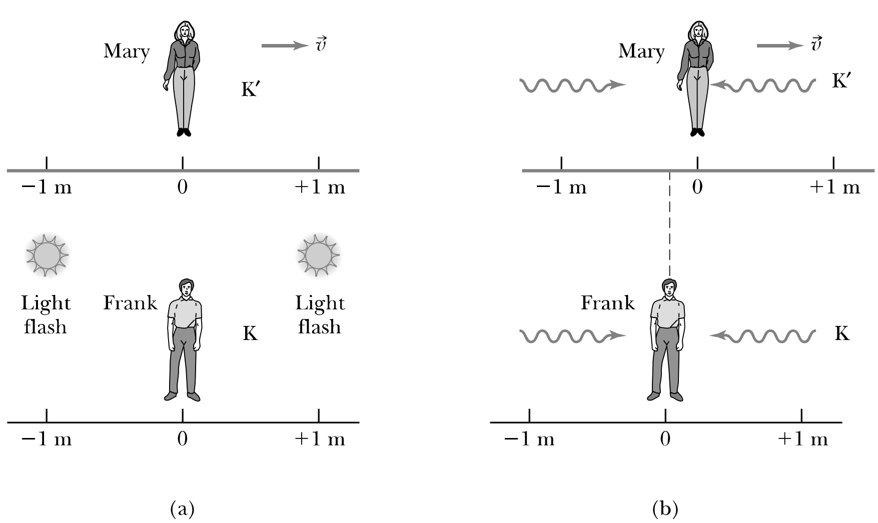


Figure 2.6 The problem of simultaneity. Flashbulbs positioned in system K at one meter on either side of Frank go off simultaneously in (a). Frank indeed sees both flashes simultaneously in (b). However, Mary, at rest in system K′ moving to the right with speed v, does not see the flashes simultaneously despite the fact that she was alongside Frank when the flashbulbs went off. During the finite time it took light to travel the one meter, Mary has moved slightly, as shown in exaggerated form in (b).

at speed c. We conclude that

Two events that are simultaneous in one reference frame (K) are not necessarily simultaneous in another reference frame (K′) moving with respect to the first frame.

We must be careful when comparing the same event in two systems moving with respect to one another. Time comparison can be accomplished by sending light signals from one observer to another, but this information can travel only as fast as the finite speed of light. It is best if each system has its own observers with clocks that are synchronized. How can we do this? We place observers with clocks throughout a given system. If, when we bring all the clocks together at one spot at rest, all the clocks agree, then the clocks are said to be **synchronized**. However, we have to move the clocks relative to each other to reposition them, and this might affect the synchronization. A better way would be to flash a bulb halfway between each pair of clocks at rest and make sure the pulses arrive simultaneously at each clock. This will require many measurements, but it is a safe way to synchronize the clocks. We can determine the time of an event occurring far away from us by having a colleague at the event, with a clock fixed at rest, measure the time of the particular event, and send us the results, for example, by telephone or even by mail. If we need to check our clocks, we can always send light signals to each other over known distances at some predetermined time.

Synchronization of clocks

In the next section we derive the correct transformation, called the **Lorentz transformation**, that makes the laws of physics invariant between inertial frames of reference. We use the coordinate systems described by Figure 2.1. At $t = t' = 0$, the origins of the two coordinate systems are coincident, and the system K′ is traveling along the x and x' axes. For this special case, the Lorentz transformation equations are

Lorentz transformation equations

$$\begin{aligned} x' &= \frac{x - vt}{\sqrt{1 - v^2/c^2}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t - (vx/c^2)}{\sqrt{1 - v^2/c^2}} \end{aligned} \tag{2.6}$$

Relativistic factor

We commonly use the symbols β and the *relativistic factor* γ to represent two longer expressions:

$$\beta = \frac{v}{c} \tag{2.7}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{2.8}$$

which allows the Lorentz transformation equations to be rewritten in compact form as

$$\begin{aligned} x' &= \gamma(x - \beta ct) \\ y' &= y \\ z' &= z \\ t' &= \gamma(t - \beta x/c) \end{aligned} \tag{2.6}$$

Note that $\gamma \geq 1$ ($\gamma = 1$ when $v = 0$).

2.4 The Lorentz Transformation

In this section we use Einstein's two postulates to find a transformation between inertial frames of reference such that all the physical laws, including Newton's laws of mechanics and Maxwell's electrodynamics equations, will have the same form. We use the fixed system K and moving system K′ of Figure 2.1. At $t = t' = 0$ the origins and axes of both systems are coincident, and system K′ is moving to the right along the x axis. A flashbulb goes off at the origins when $t = t' = 0$. According to postulate 2, the speed of light will be c in both systems, and the wavefronts observed in both systems must be spherical and described by

$$x^2 + y^2 + z^2 = c^2t^2 \tag{2.9a}$$

$$x'^2 + y'^2 + z'^2 = c^2t'^2 \tag{2.9b}$$

These two equations are inconsistent with a Galilean transformation because a wavefront can be spherical in only one system when the second is moving at speed v with respect to the first. The Lorentz transformation *requires* both systems to have a spherical wavefront centered on each system's origin.

Another clear break with Galilean and Newtonian physics is that we do not assume that $t = t'$. Each system must have its own clocks and metersticks as indicated in a two-dimensional system in Figure 2.7. Because the systems move only along their x axes, observers in both systems agree by direct observation that

$$y' = y$$

$$z' = z$$

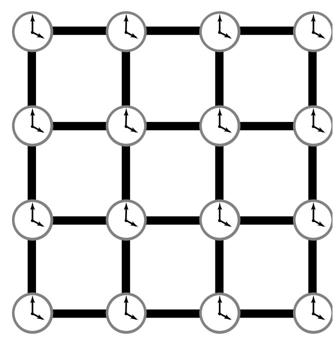


Figure 2.7 In order to make sure accurate event measurements can be obtained, synchronized clocks and uniform measuring sticks are placed throughout a system.

We know that the Galilean transformation $x' = x - vt$ is incorrect, but what is the correct transformation? We require a linear transformation so that each event in system K corresponds to one, and only one, event in system K′. The simplest *linear* transformation is of the form

$$x' = \gamma(x - vt) \tag{2.10}$$

We will see if such a transformation suffices. The parameter γ cannot depend on x or t because the transformation must be linear. The parameter γ must be close to 1 for $v \ll c$ in order for Newton's laws of mechanics to be valid for most of our measurements. We can use similar arguments from the standpoint of an observer stationed in system K′ to obtain an equation similar to Equation (2.10).

$$x = \gamma'(x' + vt') \tag{2.11}$$

Because postulate 1 requires that the laws of physics be the same in both reference systems, we demand that $\gamma' = \gamma$. Notice that the only difference between Equations (2.10) and (2.11) other than the primed and unprimed quantities being switched is that $v \to -v$, which is reasonable because according to the observer in each system, the other observer is moving either forward or backward.

According to postulate 2, the speed of light is c in both systems. Therefore, in each system the wavefront of the flashbulb light pulse along the respective x axes must be described by $x = ct$ and $x' = ct'$, which we substitute into Equations (2.10) and (2.11) to obtain

$$ct' = \gamma(ct - vt) \tag{2.12a}$$

and

$$ct = \gamma(ct' + vt') \tag{2.12b}$$

We divide each of these equations by c and obtain

$$t' = \gamma t\left(1 - \frac{v}{c}\right) \tag{2.13}$$

and

$$t = \gamma t'\left(1 + \frac{v}{c}\right) \tag{2.14}$$

We substitute the value of t from Equation (2.14) into Equation (2.13).

$$t' = \gamma^2 t'\left(1 - \frac{v}{c}\right)\left(1 + \frac{v}{c}\right) \tag{2.15}$$

We solve this equation for γ^2 and obtain

$$\gamma^2 = \frac{1}{1 - v^2/c^2}$$

or

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{2.16}$$

In order to find a transformation for time t', we rewrite Equation (2.13) as

$$t' = \gamma\left(t - \frac{vt}{c}\right)$$

We substitute $t = x/c$ for the light pulse and find

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = \frac{t - vx/c^2}{\sqrt{1 - \beta^2}}$$

We are now able to write the complete Lorentz transformations as

$$\begin{aligned} x' &= \frac{x - vt}{\sqrt{1 - \beta^2}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t - (vx/c^2)}{\sqrt{1 - \beta^2}} \end{aligned} \tag{2.17}$$

The inverse transformation equations are obtained by replacing v by $-v$ as discussed previously and by exchanging the primed and unprimed quantities.

Inverse Lorentz transformation equations

$$\begin{aligned} x &= \frac{x' + vt'}{\sqrt{1 - \beta^2}} \\ y &= y' \\ z &= z' \\ t &= \frac{t' + (vx'/c^2)}{\sqrt{1 - \beta^2}} \end{aligned} \tag{2.18}$$

Notice that Equations (2.17) and (2.18) both reduce to the Galilean transformation when $v \ll c$. It is only for speeds that approach the speed of light that the Lorentz transformation equations become significantly different from the Galilean equations. In our studies of mechanics we normally do not consider such high speeds, and our previous results probably require no corrections. The laws of mechanics credited to Newton are still valid over the region of their applicability. Even for a speed as high as the Earth orbiting about the sun, 30 km/s, the value of the relativistic factor γ is 1.000000005. We show a plot of the relativistic parameter γ versus speed in Figure 2.8. As a rule of thumb, we should consider using the relativistic equations when $v/c > 0.1$ ($\gamma \approx 1.005$).

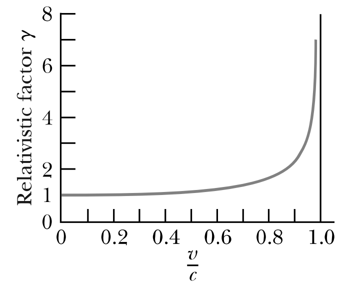


Figure 2.8 A plot of the relativistic factor γ as a function of speed v/c, showing that γ becomes large quickly as v approaches c.

Finally, consider the implications of the Lorentz transformation. The linear transformation equations ensure that a single event in one system is described by a single event in another inertial system. However, space and time are not separate. In order to express the position of x in system K′, we must use both x' and t'. We have also found that the Lorentz transformation does not allow a speed greater than c; the relativistic factor γ becomes imaginary in this case. We show later in this chapter that no object of nonzero mass can have a speed greater than c.

2.5 Time Dilation and Length Contraction

The Lorentz transformations have immediate consequences with respect to time and length measurements made by observers in different inertial frames. We shall consider time and length measurements separately and then see how they are related to one another.

Time Dilation

Consider again our two systems K and K′ with system K fixed and system K′ moving along the x axis with velocity $\vec{v}$ as shown in Figure 2.9a (p. 32). Frank lights a sparkler at position x_1 in system K. A clock placed beside the sparkler indicates the time to be t_1 when the sparkler is lit and t_2 when the sparkler goes out (Figure 2.9b). The sparkler burns for time T_0, where $T_0 = t_2 - t_1$. The time difference between two events occurring at the same position in a system as measured by a clock at rest in the system is called the **proper time**. We use the subscript zero on the time difference T_0 to denote the proper time.

Proper time

Now what is the time as determined by Mary who is passing by (but at rest in her own system K′)? All the clocks in both systems have been synchronized when the systems are at rest with respect to one another. The two events (sparkler lit and then going out) do not occur at the same place according to Mary. She is beside the sparkler when it is lit, but she has moved far away from the sparkler when it goes out (Figure 2.9b). Her friend Melinda, also at rest in system K′, is beside the sparkler when it goes out. Mary and Melinda measure the two times for the sparkler to be lit and to go out in system K′ as times t'_1 and t'_2. The Lorentz transformation relates these times to those measured in system K as

$$t'_2 - t'_1 = \frac{(t_2 - t_1) - (v/c^2)(x_2 - x_1)}{\sqrt{1 - v^2/c^2}}$$

In system K the clock is fixed at x_1, so $x_2 - x_1 = 0$; that is, the two events occur at the same position. The time $t_2 - t_1$ is the proper time T_0, and we denote the time difference $t'_2 - t'_1 = T'$ as measured in the moving system K′:

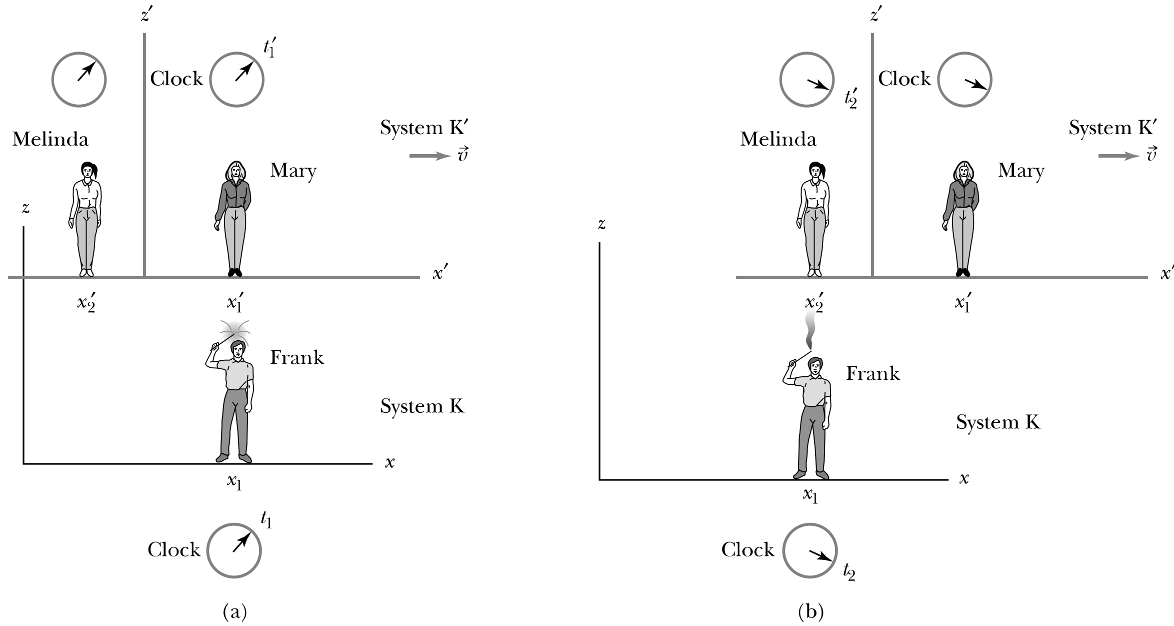


Figure 2.9 Frank measures the proper time for the time interval that a sparkler stays lit. His clock is at the same position in system K when the sparkler is lit in (a) and when it goes out in (b). Mary, in the moving system K′, is beside the sparkler at position x'_1 when it is lit in (a), but by the time it goes out in (b), she has moved away. Melinda, at position x'_2, measures the time in system K′ when the sparkler goes out in (b).

Time dilation

$$T' = \frac{T_0}{\sqrt{1 - v^2/c^2}} = \gamma T_0 \tag{2.19}$$

Thus the time interval measured in the moving system K′ is greater than the time interval measured in system K where the sparkler is at rest. This effect is known as **time dilation** and is a direct result of Einstein's two postulates. The time measured by Mary and Melinda in their system K′ for the time difference was greater than T_0 by the relativistic factor γ ($\gamma > 1$). The two events, sparkler being lit and then going out, did not occur at the same position ($x'_2 \neq x'_1$) in system K′ (see Figure 2.9b). This result occurs because of the absence of simultaneity. The events do not occur at the same space and time coordinates in the two systems. It requires three clocks to perform the measurement: one in system K and two in system K′.

Moving clocks run slow

The time dilation result is often interpreted by saying that *moving clocks run slow* by the factor γ^{-1}, and sometimes this is a useful way to remember the effect. The moving clock in this case can be any kind of clock. It can be the time that sand takes to pass through an hourglass, the time a sparkler stays lit, the time between heartbeats, the time between ticks of a clock, or the time spent in a class lecture. In all cases, the actual time interval on a moving clock is greater than the proper time as measured on a clock at rest. The proper time is always the smallest possible time interval between two events.

Each person will claim the clock in the other (moving) system is running slow. If Mary had a sparkler in her system K′ at rest, Frank (fixed in system K) would also measure a longer time interval on his clock in system K because the sparkler would be moving with respect to his system.

EXAMPLE 2.1

Show that Frank in the fixed system will also determine the time dilation result by having the sparkler be at rest in the system K′.

Strategy We should be able to proceed similarly to the derivation we did before when the sparkler was at rest in system K. In this case Mary lights the sparkler in the moving system K′. The time interval over which the sparkler is lit is given by $T'_0 = t'_2 - t'_1$, and the sparkler is placed at the position $x'_1 = x'_2$ so that $x'_2 - x'_1 = 0$. In this case T'_0 is the proper time. We use the Lorentz transformation from Equation (2.18) to determine the time difference $T = t_2 - t_1$ as measured by the clocks of Frank and his colleagues.

Solution We use Equation (2.18) to find $t_2 - t_1$:

$$T = t_2 - t_1 = \frac{(t'_2 - t'_1) + (v/c^2)(x'_2 - x'_1)}{\sqrt{1 - v^2/c^2}}$$
$$= \frac{T'_0}{\sqrt{1 - v^2/c^2}} = \gamma T'_0$$

The time interval is still smaller in the system where the sparkler is at rest.

***Gedanken* experiments**

The preceding results naturally seem a little strange to us. In relativity we often carry out thought (or *gedanken* from the German word) experiments, because the actual experiments would be somewhat impractical. Consider the following *gedanken* experiment. Mary, in the moving system K′, flashes a light at her origin along her y' axis (Figure 2.10). The light travels a distance L, reflects off a mirror, and returns. Mary says that the total time for the journey is $T'_0 = t'_2 - t'_1 = 2L/c$, and this is indeed the proper time, because the clock in K′ beside Mary is at rest.

What do Frank and other observers in system K measure? Let T be the round-trip time interval measured in system K for the light to return to the x axis. The light is flashed when the origins are coincident, as Mary passes by Frank with relative velocity v. When the light reaches the mirror in the system K′ at time $T/2$, the system K′ will have moved a distance $vT/2$ down the x axis. When the light is reflected back to the x axis, Frank will not even see the light return, because it will return a distance vT away, where another observer, Fred, is positioned. Because observers Frank and Fred have previously synchronized their clocks, they can still measure the total elapsed time for the light to be reflected from the mirror and return. According to observers in the K system, the total distance the light travels (as shown in Figure 2.10) is $2\sqrt{(vT/2)^2 + L^2}$. And according to postulate 2, the light must travel at the speed of light, so the total time interval T measured in system K is

$$T = \frac{\text{distance}}{\text{speed}} = \frac{2\sqrt{(vT/2)^2 + L^2}}{c}$$

As can be determined from above, $L = cT'_0/2$, so we have

$$T = \frac{2\sqrt{(vT/2)^2 + (cT'_0/2)^2}}{c}$$

which reduces to

$$T = \frac{T'_0}{\sqrt{1 - v^2/c^2}} = \gamma T'_0$$

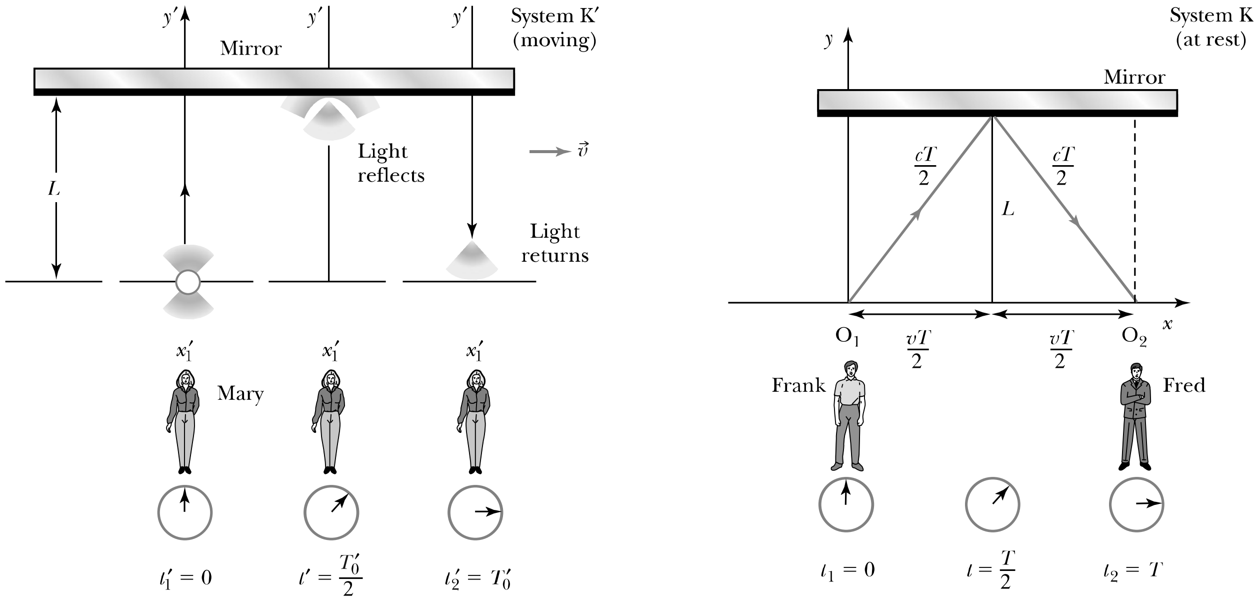


Figure 2.10 Mary, in system K′, flashes a light along her y' axis and measures the proper time $T'_0 = 2L/c$ for the light to return. In system K Frank will see the light travel partially down his x axis, because system K′ is moving. Fred times the arrival of the light in system K. The time interval T that Frank and Fred measure is related to the proper time by $T = \gamma T'_0$.

This is consistent with the earlier result. In this case $T > T'_0$. The proper time is always the shortest time interval, and we find that the clock in Mary's system K′ is "running slow."

EXAMPLE 2.2

It is the year 2150 and the United Nations Space Federation has finally perfected the storage of antiprotons for use as fuel in a spaceship. (Antiprotons are the antiparticles of protons. We discuss antiprotons in Chapter 3.) Preparations are under way for a manned spacecraft visit to possible planets orbiting one of the three stars in the star system Alpha Centauri, some 4.30 lightyears away. Provisions are placed on board to allow a trip of 16 years' total duration. How fast must the spacecraft travel if the provisions are to last? Neglect the period of acceleration, turnaround, and visiting times, because they are negligible compared with the actual travel time.

Strategy The time interval as measured by the astronauts on the spacecraft can be no longer than 16 years, because that is how long the provisions will last. However, from Earth we realize that the spacecraft will be moving at a high relative speed v to us, and that according to our clock in the stationary system K, the trip will last $T = 2L/v$, where L is the distance to the star.

Because provisions on board the spaceship will last for only 16 years, we let the proper time T'_0 in system K′ be 16 years. Using the time dilation result, we determine the relationship between T, the time measured on Earth, and the proper time T'_0 to be

$$T = \frac{2L}{v} = \frac{T'_0}{\sqrt{1 - v^2/c^2}} \tag{2.20}$$

We then solve this equation for the required speed v.

Solution A lightyear is a convenient way to measure large distances. It is the distance light travels in one year and is denoted by ly:

$$1\ \text{ly} = \left(3.00 \times 10^8\ \frac{\text{m}}{\text{s}}\right)(1\ \text{year})\left(365\ \frac{\text{days}}{\text{year}}\right)\left(24\ \frac{\text{h}}{\text{day}}\right)\left(3600\ \frac{\text{s}}{\text{h}}\right)$$
$$= 9.46 \times 10^{15}\ \text{m}$$

Note that the distance of one lightyear is the speed of light, c, multiplied by the time of one year. The dimension of a lightyear works out to be length. In this case, the result is $4.30\ \text{ly} = c(4.30\ \text{y}) = 4.07 \times 10^{16}$ m.

We insert the appropriate numbers into Equation (2.20) and obtain

$$\frac{2(4.30\ \text{ly})(9.46 \times 10^{15}\ \text{m/ly})}{v} = \frac{16\ \text{y}}{\sqrt{1 - v^2/c^2}}$$

The solution to this equation is $v = 0.473c = 1.42 \times 10^8$ m/s. The time interval as measured on Earth will be $\gamma T_0' = 18.2$ y. Notice that the astronauts will age only 16 years (their clocks run slow), whereas their friends remaining on Earth will age 18.2 years. Can this really be true? We shall discuss this question again in Section 2.8.

Length Contraction

Now let's consider what might happen to the length of objects in relativity. Let an observer in each system K and K′ have a meterstick at rest in his or her own respective system. Each observer lays the stick down along his or her respective x axis, putting the left end at x_ℓ (or x_ℓ') and the right end at x_r (or x_r'). Thus, Frank in system K measures his stick to be $L_0 = x_r - x_\ell$. Similarly, in system K′, Mary measures her stick at rest to be $L_0' = x_r' - x_\ell' = L_0$. Every observer measures a meterstick at rest in his or her own system to have the same length, namely one meter. The length as measured at rest is called the **proper length**.

Proper length

Let system K be at rest and system K′ move along the x axis with speed v. Frank, who is at rest in system K, measures the length of the stick moving in K′. The difficulty is to measure the ends of the stick simultaneously. We insist that Frank measure the ends of the stick at the same time so that $t = t_r = t_\ell$. The events denoted by (x, t) are (x_ℓ, t) and (x_r, t). We use Equation (2.17) and find

$$x_r' - x_\ell' = \frac{(x_r - x_\ell) - v(t_r - t_\ell)}{\sqrt{1 - v^2/c^2}}$$

The meterstick is at rest in system K′, so the length $x_r' - x_\ell'$ must be the proper length L_0'. Denote the length measured by Frank as $L = x_r - x_\ell$. The times t_r and t_ℓ are identical, as we insisted, so $t_r - t_\ell = 0$. Notice that the times of measurement by Mary in her system, t_ℓ' and t_r', are *not* identical. It makes no difference when Mary makes the measurements in her own system, because the stick is at rest. However, it makes a big difference when Frank makes his measurements, because the stick is moving with speed v with respect to him. The measurements must be done simultaneously! With these results, the previous equation becomes

$$L_0' = \frac{L}{\sqrt{1 - v^2/c^2}} = \gamma L$$

or, because $L_0' = L_0$,

$$L = L_0\sqrt{1 - v^2/c^2} = \frac{L_0}{\gamma} \qquad (2.21)$$

Length contraction

Notice that $L_0 > L$, so the moving meterstick shrinks according to Frank. This effect is known as **length** or **space contraction** and is characteristic of relative motion. This effect is also sometimes called the *Lorentz-FitzGerald contraction* because Lorentz and FitzGerald independently suggested the contraction as a way to solve the electrodynamics problem. This effect, like time dilation, is also reciprocal. Each observer will say that the other moving stick is shorter. There is no length contraction perpendicular to the relative motion, however, because $y' = y$ and $z' = z$. Observers in both systems can check the length of the other meterstick placed perpendicular to the direction of motion as the metersticks pass each other. They will agree that both metersticks are one meter long.

We can perform another *gedanken* experiment to arrive at the same result. This time we lay the meterstick along the x' axis in the moving system K′ (Figure 2.11a). The two systems K and K′ are aligned at $t = t' = 0$. A mirror is placed at the end of the meterstick, and a flashbulb goes off at the origin at $t = t' = 0$, sending a light pulse down the x' axis, where it is reflected and returned. Mary sees the stick at rest in system K′ and measures the proper length L_0 (which should of course be one meter). Mary uses the same clock fixed at $x' = 0$ for the time measurements. The stick is moving at speed v with respect to Frank in the fixed system K. The clocks at $x = x' = 0$ both read zero when the origins are aligned just when the flashbulb goes off. Notice the situation shown in system K

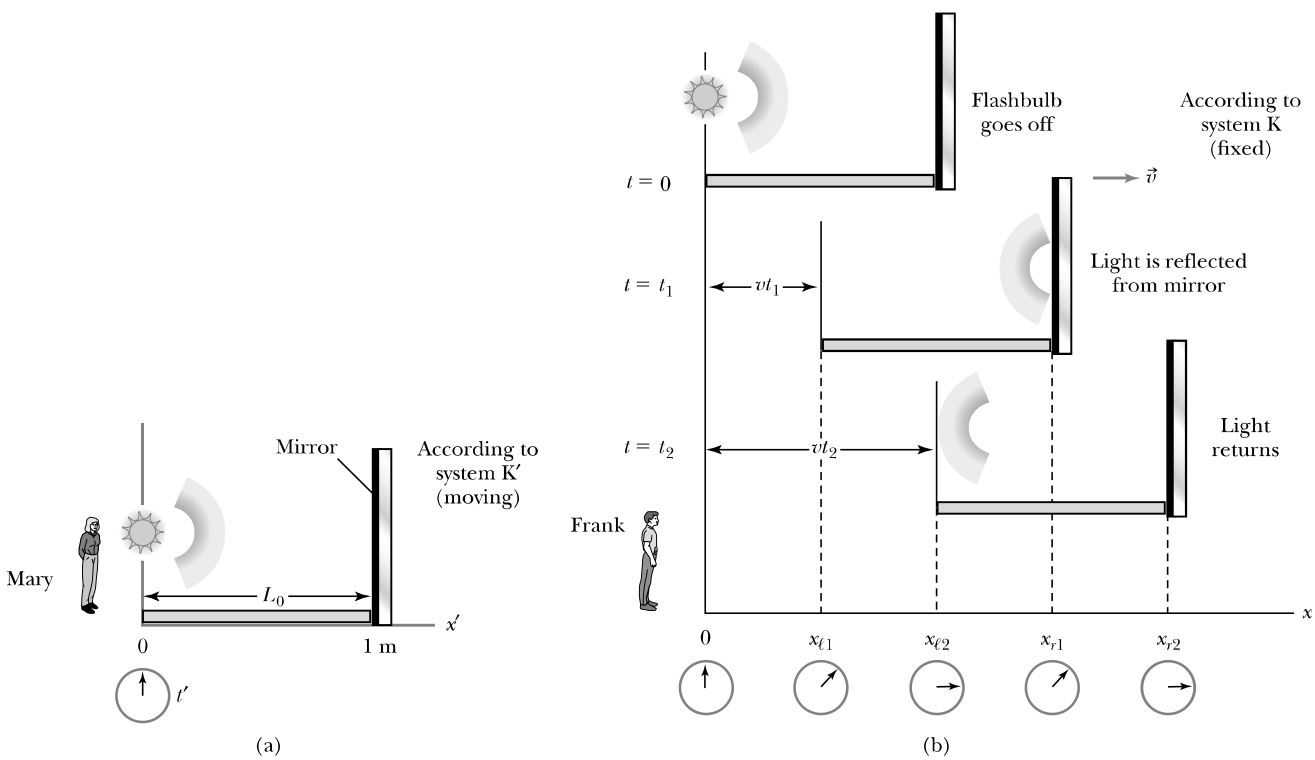


Figure 2.11 (a) Mary, in system K′, flashes a light down her x' axis along a stick at rest in her system of length L_0, which is the proper length. The time interval for the light to travel down the stick and back is $2L_0/c$. (b) Frank, in system K, sees the stick moving, and the mirror has moved a distance vt_1 by the time the light is reflected. By the time the light returns to the beginning of the stick, the stick has moved a total distance of vt_2. The times can be compared to show that the moving stick has been length contracted by $L = L_0\sqrt{1 - v^2/c^2}$.

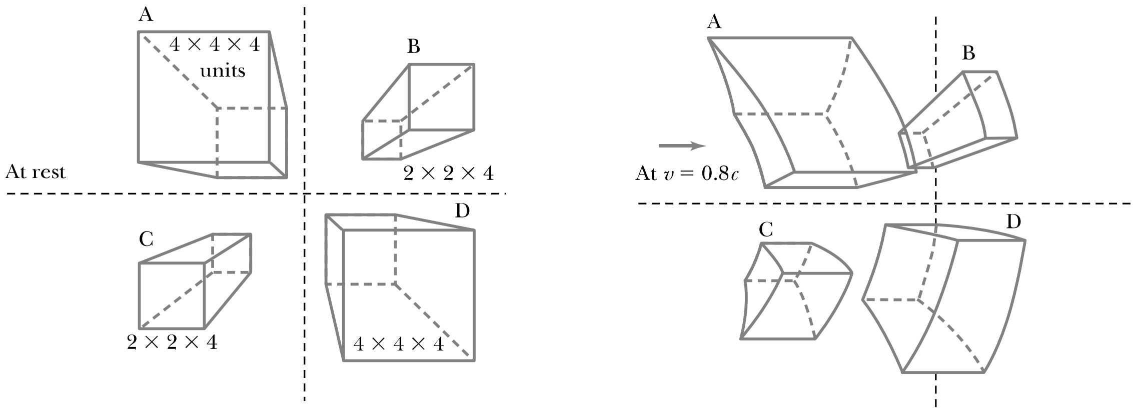


Figure 2.12 In this computer simulation, the rectangular boxes are drawn as if the observer were 5 units in front of the near plane of the boxes and directly in front of the origin. The boxes are shown at rest on the left. On the right side, the boxes are moving to the right at a speed of $v = 0.8c$. The horizontal lines are only length contracted, but notice that the vertical lines become hyperbolas. The objects appear to be slightly rotated in space. The objects that are further away from the origin appear earlier because they are photographed at an earlier time and because the light takes longer to reach the camera (or our eyes). *Reprinted with permission from* American Journal of Physics ***33,*** *534 (1965), G. D. Scott and M. R. Viner. © 1965, American Association of Physics Teachers.*

(Figure 2.11b), where by the time the light reaches the mirror, the entire stick has moved a distance vt_1. By the time the light has been reflected back to the front of the stick again, the stick has moved a total distance vt_2. We leave the solution in terms of length contraction to Problem 18.

The effect of length contraction along the direction of travel may strongly affect the appearances of two- and three-dimensional objects. We see such objects when the light reaches our eyes, not when the light actually leaves the object. Thus, if the objects are moving rapidly, we will not see them as they appear at rest. Figure 2.12 shows the appearance of several such objects as they move. Note that not only do the horizontal lines become contracted, but the vertical lines also become hyperbolas. We show in Figure 2.13 a row of bars moving to the right with speed $v = 0.9c$. The result is quite surprising.

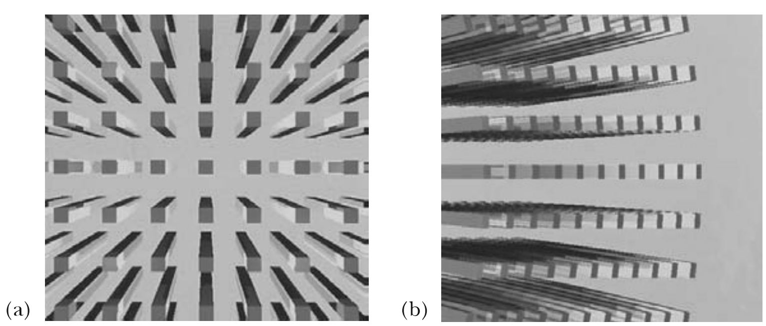


Figure 2.13 (a) An array of rectangular bars is seen from above at rest. (b) The bars are moving to the right at $v = 0.9c$. The bars appear to contract and rotate. *Quoted from P.-K. Hsuing and R. H. P. Dunn,* Science News ***137,*** *232 (1990).*

EXAMPLE 2.3

Consider the solution of Example 2.2 from the standpoint of length contraction.

Strategy The astronauts have only enough provisions for a trip lasting 16 years. Thus they expect to travel for 8 years each way. If the star system Alpha Centauri is 4.30 lightyears away, it may appear that they need to travel at a velocity of $0.5c$ to make the trip. We want to consider this example as if the astronauts are at rest. Alpha Centauri will appear to be moving toward them, and the distance to the star system is length contracted. The distance measured by the astronauts will be less than 4.30 ly.

Solution The contracted distance according to the astronauts in motion is $(4.30 \text{ ly})\sqrt{1 - v^2/c^2}$. The velocity they need to make this journey is the contracted distance divided by 8 years.

$$v = \frac{\text{distance}}{\text{time}} = \frac{(4.30 \text{ ly})\sqrt{1 - v^2/c^2}}{8 \text{ y}}$$

If we divide by c, we obtain

$$\beta = \frac{v}{c} = \frac{(4.30 \text{ ly})\sqrt{1 - v^2/c^2}}{c(8 \text{ y})} = \frac{(4.30 \text{ ly})\sqrt{1 - v^2/c^2}}{(8 \text{ ly})}$$

$$8\beta = 4.30\sqrt{1 - \beta^2}$$

which gives

$$\beta = 0.473$$

$$v = 0.473c$$

which is just what we found in the previous example. The effects of time dilation and length contraction give identical results.

2.6 Addition of Velocities

A spaceship launched from a space station (see Figure 2.14) quickly reaches its cruising speed of $0.60c$ with respect to the space station when a band of asteroids is observed straight ahead of the ship. Mary, the commander, reacts quickly and orders her crew to blast away the asteroids with the ship's proton gun to avoid a catastrophic collision. Frank, the admiral on the space station, listens with apprehension to the communications because he fears the asteroids may eventually destroy his space station as well. Will the high-energy protons of speed $0.99c$ be able to successfully blast away the asteroids and save both the spaceship and

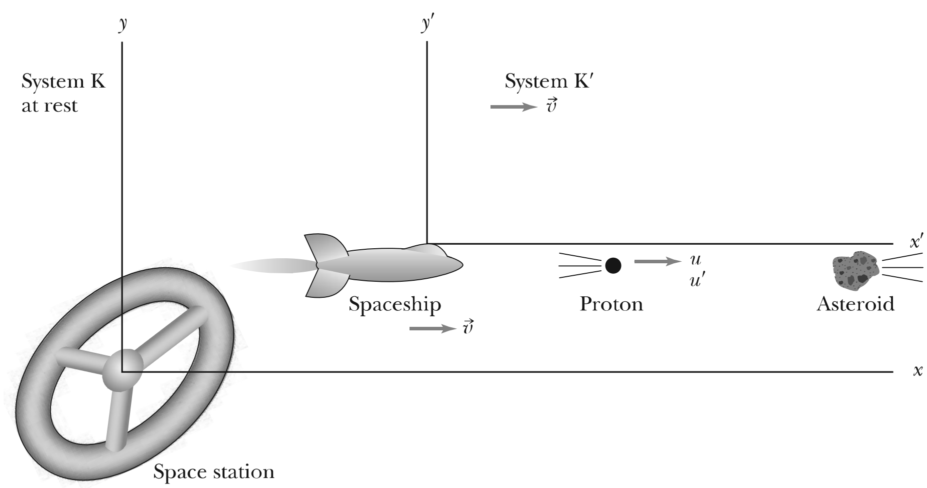


Figure 2.14 The space station is at rest at the origin of system K. The spaceship is moving to the right with speed v with respect to the space station and is in system K′. An asteroid is moving to the left toward both the spaceship and space station, so Mary, the commander of the spaceship, orders that the proton gun shoot protons to break up the asteroid. The speed of the protons is u and u' with respect to systems K and K′, respectively.

space station? If $0.99c$ is the speed of the protons with respect to the spaceship, what speed will Frank measure for the protons?

We will use the letter u to denote velocity of objects as measured in various coordinate systems. In this case, Frank (in the fixed, stationary system K on the space station) will measure the velocity of the protons to be u, whereas Mary, the commander of the spaceship (the moving system K′), will measure $u' = 0.99c$. We reserve the letter v to express the velocity of the coordinate systems with respect to each other. The velocity of the spaceship with respect to the space station is $v = 0.60c$.

Newtonian mechanics teaches us that to find the velocity of the protons with respect to the space station, we simply add the velocity of the spaceship with respect to the space station ($0.60c$) to the velocity of the protons with respect to the spaceship ($0.99c$) to determine the result $u = v + u' = 0.60c + 0.99c = 1.59c$. However, this result is not in agreement with the results of the Lorentz transformation. We use Equation (2.18), letting x be along the direction of motion of the spaceship (and high-speed protons), and take the differentials, with the results

$$\begin{aligned} dx &= \gamma(dx' + v\,dt') \\ dy &= dy' \\ dz &= dz' \\ dt &= \gamma[dt' + (v/c^2)\,dx'] \end{aligned} \tag{2.22}$$

Velocities are defined by $u_x = dx/dt$, $u_y = dy/dt$, $u'_x = dx'/dt'$, and so on. Therefore we determine u_x by

Relativistic velocity addition

$$u_x = \frac{dx}{dt} = \frac{\gamma(dx' + v\,dt')}{\gamma[dt' + (v/c^2)\,dx']} = \frac{u'_x + v}{1 + (v/c^2)u'_x} \tag{2.23a}$$

Similarly, u_y and u_z are determined to be

$$u_y = \frac{u'_y}{\gamma[1 + (v/c^2)u'_x]} \tag{2.23b}$$

$$u_z = \frac{u'_z}{\gamma[1 + (v/c^2)u'_x]} \tag{2.23c}$$

Equations (2.23) are referred to as the **Lorentz velocity transformations**. Notice that although the relative motion of the systems K and K′ is only along the x direction, the velocities along y and z are affected as well. This contrasts with the Lorentz transformation equations, where $y = y'$ and $z = z'$. However, the difference in velocities is simply ascribed to the transformation of time, which depends on v and x'. Thus, the transformations for u_y and u_z depend on v and u'_x. The inverse transformations for u'_x, u'_y, and u'_z can be determined by simply switching primed and unprimed variables and changing v to $-v$. The results are

$$\begin{aligned} u'_x &= \frac{u_x - v}{1 - (v/c^2)u_x} \\ u'_y &= \frac{u_y}{\gamma[1 - (v/c^2)u_x]} \\ u'_z &= \frac{u_z}{\gamma[1 - (v/c^2)u_x]} \end{aligned} \tag{2.24}$$

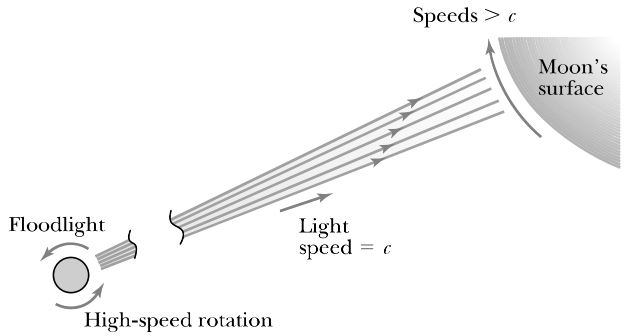


Figure 2.15 A floodlight revolving at high speeds can sweep a light beam across the surface of the moon at speeds exceeding c, but the speed of the light still does not exceed c.

Note that we found the velocity transformation equations for the situation corresponding to the inverse Lorentz transformation, Equations (2.18), before finding the velocity transformation for Equations (2.17).

What is the correct result for the speed of the protons with respect to the space station? We have $u'_x = 0.99c$ and $v = 0.60c$, so Equation (2.23a) gives us the result

$$u_x = \frac{0.990c + 0.600c}{1 + \dfrac{(0.600c)(0.990c)}{c^2}} = 0.997c$$

where we have assumed we know the speeds to three significant figures. Therefore, the result is a speed only slightly less than c. The Lorentz transformation does not allow a material object to have a speed greater than c. Only massless particles, such as light, can have speed c. If the crew members of the spaceship spot the asteroids far enough in advance, their reaction times should allow them to shoot down the uncharacteristically swiftly moving asteroids and save both the spaceship and the space station.

Although no particle with mass can carry energy faster than c, we can imagine a signal being processed faster than c. Consider the following *gedanken* experiment. A giant floodlight placed on a space station above the Earth revolves at 100 Hz, as shown in Figure 2.15. Light spreads out in the radial direction from the floodlight at speeds of c. On the surface of the moon, the light beam sweeps across at speeds far exceeding c (Problem 36). However, the light itself does not reach the moon at speeds faster than c. No energy is associated with the beam of light sweeping across the moon's surface. The energy (and linear momentum) is only along the radial direction from the space station to the moon.

EXAMPLE 2.4

Mary, the commander of the spaceship just discussed, is holding target practice for junior officers by shooting protons at small asteroids and space debris off to the side (perpendicular to the direction of spaceship motion) as the spaceship passes by. What speed will an observer in the space station measure for these protons?

Strategy We use the coordinate systems and speeds of the spaceship and proton gun as described previously. Let the direction of the protons now be perpendicular to the direction of the spaceship—along the y' direction. We already know in the spaceship's K′ system that $u'_y = 0.99c$ and $u'_x =$

$u'_z = 0$, and that the speed of the K′ system (spaceship) with respect to the space station is $v = 0.60c$. We use Equations (2.23) to determine u_x, u_y, and u_z and finally the speed u.

Solution To find the speeds in the system K, we first need to find γ.

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.600^2}} = 1.25$$

Next we are able to determine the components of $\vec{u}$.

$$u_x(\text{protons}) = \frac{0 + 0.600c}{[1 + (0.600c)(0c)/c^2]} = 0.600c$$

$$u_y(\text{protons}) = \frac{0.990c}{1.25[1 + (0.600c)(0c)/c^2]} = 0.792c$$

$$u_z(\text{protons}) = \frac{0}{1.25[1 + (0.600c)(0c)/c^2]} = 0$$

$$u(\text{protons}) = \sqrt{u_x^2 + u_y^2 + u_z^2} = \sqrt{(0.600c)^2 + (0.792c)^2} = 0.994c$$

We have again assumed we know the velocity components to three significant figures. Mary and her junior officers only observe the protons moving perpendicular to their motion. However, because there are both u_x and u_y components, Frank (on the space station) sees the protons moving at an angle with respect to both his x and his y directions.

EXAMPLE 2.5

By the early 1800s experiments had shown that light slows down when passing through liquids. A. J. Fresnel suggested in 1818 that there would be a partial drag on light by the medium through which the light was passing. Fresnel's suggestion explained the problem of stellar aberration if the Earth was at rest in the ether. In a famous experiment in 1851, H. L. Fizeau measured the "ether" drag coefficient for light passing in opposite directions through flowing water. Let a moving system K′ be at rest in the flowing water and let v be the speed of the flowing water with respect to a fixed observer in K (see Figure 2.16). The speed of light in the water at rest (that is, in system K′) is u', and the speed of light as measured in K is u. If the index of refraction of the water is n, Fizeau found experimentally that

$$u = u' + \left(1 - \frac{1}{n^2}\right)v$$

which was in agreement with Fresnel's prediction. This result was considered an affirmation of the ether concept. The factor $1 - 1/n^2$ became known as *Fresnel's drag coefficient*. Show that this result can be explained using relativistic velocity addition *without the ether concept*.

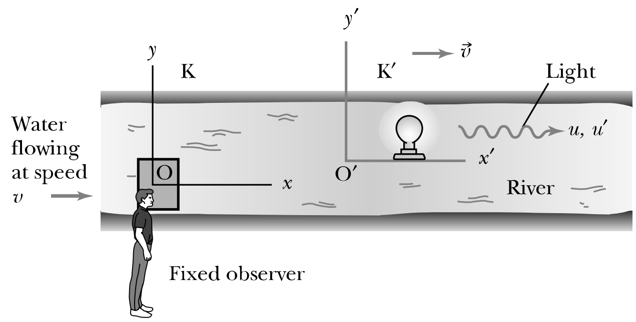


Figure 2.16 A stationary system K is fixed on shore, and a moving system K′ floats down the river at speed v. Light emanating from a source under water in system K′ has speed u, u' in systems K, K′, respectively.

Strategy We note from introductory physics that the velocity of light in a medium of index of refraction n is $u' = c/n$. We use Equation (2.23a) to solve for u.

Solution We have to calculate the speed only in the x-direction, so we dispense with the subscripts. We utilize Equation (2.23a) to determine

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{c/n + v}{1 + v/nc} = \frac{c}{n}\frac{\left(1 + \dfrac{nv}{c}\right)}{\left(1 + \dfrac{v}{nc}\right)}$$

Because $v \ll c$ in this case, we can expand the denominator $(1 + x)^{-1} = 1 - x + \cdots$ keeping only the lowest term in $x = v/c$. The above equation becomes

$$\begin{aligned} u &= \frac{c}{n}\left(1 + \frac{nv}{c}\right)\left(1 - \frac{v}{nc} + \cdots\right) \\ &= \frac{c}{n}\left(1 + \frac{nv}{c} - \frac{v}{nc} + \cdots\right) \\ &= \frac{c}{n} + v - \frac{v}{n^2} = u' + \left(1 - \frac{1}{n^2}\right)v \end{aligned}$$

which is in agreement with Fizeau's experimental result and Fresnel's prediction given earlier. This relativistic calculation is another stunning success of the special theory of relativity. There is no need to consider the existence of the ether.

2.7 Experimental Verification

We have used the special theory of relativity to describe some unusual phenomena. The special theory has also been used to make some startling predictions concerning length contraction, time dilation, and velocity addition. In this section we discuss only a few of the many experiments that have been done to confirm the special theory of relativity.

Muon Decay

When high-energy particles called *cosmic rays* enter the Earth's atmosphere from outer space, they interact with particles in the upper atmosphere (see Figure 2.17), creating additional particles in a *cosmic shower*. Many of the particles in the shower are π-*mesons* (pions), which decay into other unstable particles called *muons*. The properties of muons are described later when we discuss nuclear and particle physics. Because muons are unstable, they decay according to the radioactive decay law

Radioactive decay law

$$N = N_0 \exp\left(-\frac{(\ln 2)t}{t_{1/2}}\right) = N_0 \exp\left(-\frac{0.693t}{t_{1/2}}\right)$$

where N_0 and N are the number of muons at times $t = 0$ and $t = t$, respectively, and $t_{1/2}$ is the half-life of the muons. This means that in the time period $t_{1/2}$ half of the muons will decay to other particles. The half-life of muons (1.52×10^{-6} s) is long enough that many of them survive the trip through the atmosphere to the Earth's surface.

We perform an experiment by placing a muon detector on top of a mountain 2000 m high and counting the number of muons traveling at a speed near $v = 0.98c$ (see Figure 2.18a). Suppose we count 10^3 muons during a given time period t_0. We then move our muon detector to sea level (see Figure 2.18b), and we determine experimentally that approximately 540 muons survive the trip without decaying. We ignore any other interactions that may remove muons.

Classically, muons traveling at a speed of $0.98c$ cover the 2000-m path in 6.8×10^{-6} s, and according to the radioactive decay law, only 45 muons should survive the trip. There is obviously something wrong with the classical calculation, because we counted a factor of 12 more muons surviving than the classical calculation predicts.

Figure 2.17 Much of what we know about muons in cosmic rays was learned from balloon flights carrying sophisticated detectors. This balloon is being prepared for launch in NASA's Ultra Long Duration Balloon program for a mission that may last up to 100 days. The payload will hang many meters below the balloon. Victor Hess began the first such balloon flights in 1912 (when he discovered cosmic rays), and much improved versions are still launched today from all over the world to study cosmic rays, the atmosphere, the sun, and the universe.

Photo courtesy of NASA.

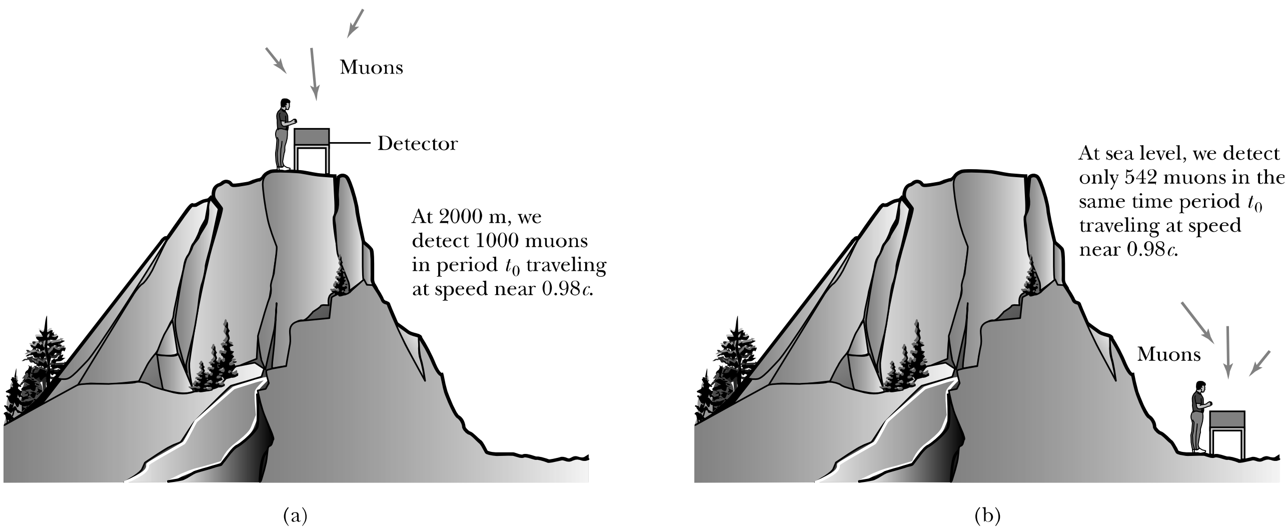


Figure 2.18 The number of muons detected with speeds near 0.98c is much different (a) on top of a mountain than (b) at sea level, because of the muon's decay. The experimental result agrees with our time dilation equation.

Because the classical calculation does not agree with the experimental result, we should consider a relativistic calculation. The muons are moving at a speed of 0.98c with respect to us on Earth, so the effects of time dilation will be dramatic. In the muon rest frame, the time period for the muons to travel 2000 m (on a clock fixed with respect to the mountain) is calculated from Equation (2.19) to be $(6.8/5.0) \times 10^{-6}$ s, because $\gamma = 5.0$ for $v = 0.98c$. For the time $t = 1.36 \times 10^{-6}$ s, the radioactive decay law predicts that 538 muons will survive the trip, in agreement with the observations. An experiment similar to this was performed by B. Rossi and D. B. Hall* in 1941 on the top of Mount Washington in New Hampshire.

It is useful to examine the muon decay problem from the perspective of an observer traveling with the muon. This observer would not measure the distance from the top of the 2000-m mountain to sea level to be 2000 m. Rather, this observer would say that the distance is contracted and is only (2000 m)/5.0 = 400 m. The time to travel the 400-m distance would be $(400 \text{ m})/0.98c = 1.36 \times 10^{-6}$ s according to a clock at rest with a muon. Using the radioactive decay law, an observer traveling with the muons would still predict 538 muons to survive. Therefore, we obtain the identical result whether we consider time dilation or space contraction, and both are in agreement with the experiment, thus confirming the special theory of relativity.

Atomic Clock Measurement

In an atomic clock, an extremely accurate measurement of time is made using a well-defined transition in the ^{133}Cs atom (f = 9,192,631,770 Hz). In 1971 two American physicists, J. C. Hafele and Richard E. Keating (Figure 2.19), used four

*B. Rossi and D. B. Hall, *Physical Review* **50**, 223 (1941). An excellent, though now dated, film recreating this experiment (*Time Dilation—An Experiment with μ-mesons* by D. H. Frisch and J. H. Smith) is available from the Education Development Center, Newton, Mass. See also D. H. Frisch and J. H. Smith, *American Journal of Physics* **31**, 342 (1963).

Figure 2.19 Joseph Hafele and Richard Keating are shown unloading one of their atomic clocks and the associated electronics from an airplane in Tel Aviv, Israel, during a stopover in November 1971 on their round-the-world trip to test special relativity.

cesium beam atomic clocks to test the time dilation effect. They flew the four portable cesium clocks eastward and westward on regularly scheduled commercial jet airplanes around the world and compared the time with a reference atomic time scale at rest at the U.S. Naval Observatory in Washington, D.C. (Figure 2.20).

The trip eastward took 65.4 hours with 41.2 flight hours, whereas the westward trip, taken a week later, took 80.3 hours with 48.6 flight hours. The comparison with the special theory of relativity is complicated by the rotation of the Earth and by a gravitational effect arising from the general theory of relativity. The actual relativistic predictions and experimental observations for the time differences* are

Travel	Predicted	Observed
Eastward	-40 ± 23 ns	-59 ± 10 ns
Westward	275 ± 21 ns	273 ± 7 ns

A negative time indicates that the time on the moving clock is less than the reference clock. The moving clocks lost time (ran slower) during the eastward trip, but gained time (ran faster) during the westward trip. This occurs because of the rotation of the Earth, indicating that the flying clocks ticked faster or slower than the reference clocks on Earth. The special theory of relativity is verified within the experimental uncertainties.

*See J. C. Hafele and R. E. Keating, *Science* **177**, 166–170 (1972).

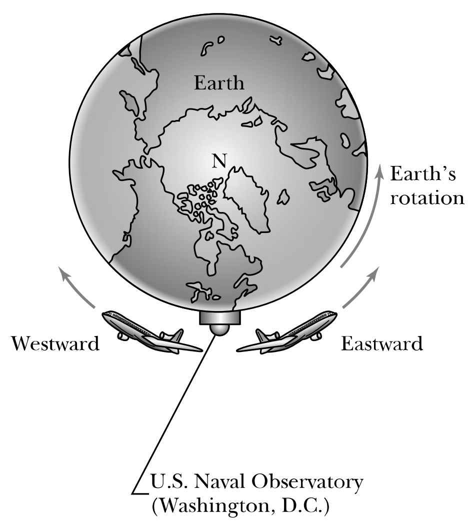


Figure 2.20 Two airplanes took off (at different times) from Washington, D.C., where the U.S. Naval Observatory is located. The airplanes traveled east and west around Earth as it rotated. Atomic clocks on the airplanes were compared with similar clocks kept at the observatory to show that the moving clocks in the airplanes ran slower.

EXAMPLE 2.6

In 1985 the space shuttle *Challenger* flew a cesium clock and compared its time with a fixed clock left on Earth. The shuttle orbited at approximately 330 km above Earth with a speed of 7712 m/s (~17,250 mph). (a) Calculate the expected time lost per second for the moving clock and compare with the measured result of -295.02 ± 0.29 ps/s, which includes a predicted effect due to general relativity of 35.0 ± 0.06 ps/s. (b) How much time would the clock lose due to special relativity alone during the entire shuttle flight that lasted for 7 days?

Strategy This should be a straightforward application of the time dilation effect, but we have the complicating fact that the space shuttle is moving in a noninertial system (orbiting around Earth). We don't want to consider this now, so we make the simplifying assumption that the space shuttle travels in a straight line with respect to Earth and the two events in the calculations are the shuttle passing the starting point (launch) and the ending point (landing). We are not including the effects of general relativity.

We know the orbital speed of the shuttle with respect to Earth, which allows us to determine β and the relativistic factor γ. We let T be the time measured by the clock fixed on Earth. Then we can use the time dilation effect given by Equation (2.19) to determine the proper time T_0' measured by the clock in the space shuttle. The time difference is $\Delta T = T - T_0'$. We have $T_0' = T\sqrt{1-\beta^2}$ and $\Delta T = T - T_0' = T(1-\sqrt{1-\beta^2})$. For part (b) we need to find the total time lost for the moving clock for 7 days.

Solution (a) We have $\beta = v/c = (7712 \text{ m/s})/(2.998 \times 10^8 \text{ m/s}) = 2.572 \times 10^{-5}$. Because β is such a small quantity, we can use a power series expansion of the square root $\sqrt{1-\beta^2}$, keeping only the lowest term in β^2 for ΔT.

$$\Delta T = T\left[1 - \left(1 - \frac{\beta^2}{2} + \cdots\right)\right] = \frac{\beta^2 T}{2}$$

Now we have

$$\frac{\Delta T}{T} = \frac{\beta^2}{2} = \frac{1}{2}(2.572 \times 10^{-5})^2 = 330.76 \times 10^{-12}$$

In this case ΔT is positive, which indicates that the space shuttle clock lost this fraction of time, so the moving clock lost 330.76 ps for each second of motion.

How does this compare with the measured time? The total measured result was a loss of 295.02 ± 0.29 ps/s, but we must add the general relativity prediction of 35.0 ± 0.06 ps/s to the measured value to obtain the result due only to special relativity. So the measured special relativity result is close to 330.02 ps/s, which differs from our calculated result by only 0.2%!

(b) The total time of the seven-day mission was 6.05×10^5 s, so the total time difference between clocks is $(330.76 \times 10^{-12})(6.05 \times 10^5 \text{ s}) = 0.2$ ms, which is easily detected by cesium clocks.

Velocity Addition

An interesting test of the velocity addition relations was made by T. Alväger and colleagues* at the CERN nuclear and particle physics research facility on the border of Switzerland and France. They used a beam of almost 20-GeV (20×10^9 eV) protons to strike a target to produce neutral pions (π^0) having energies of more than 6 GeV. The π^0 ($\beta \approx 0.99975$) have a very short half-life and soon decay into two γ rays. In the rest frame of the π^0 the two γ rays go off in opposite directions. The experimenters measured the velocity of the γ rays going in the forward direction in the laboratory (actually 6°, but we will assume 0° for purposes of calculation because there is little difference). The Galilean addition of velocities would require the velocity of the γ rays to be $u = 0.99975c + c = 1.99975c$, because the velocity of γ rays is already c. However, the relativistic velocity addition, in which

Pion decay experiment

*See T. Alväger, F. J. M. Farley, J. Kjellman, and I. Wallin, *Physics Letters* **12**, 260 (1964). See also article by J. M. Bailey, *Arkiv Fysik* **31**, 145 (1966).

$v = 0.99975c$ is the velocity of the π^0 rest frame with respect to the laboratory and $u' = c$ is the velocity of the γ rays in the rest frame of the π^0, predicts the velocity u of the γ rays measured in the laboratory to be, according to Equation (2.23a),

$$u = \frac{c + 0.99975c}{1 + \dfrac{(0.99975c)(c)}{c^2}} = c$$

The experimental measurement was accomplished by measuring the time taken for the γ rays to travel between two detectors placed about 30 m apart and was in excellent agreement with the relativistic prediction, but not the Galilean one. We again have conclusive evidence of the need for the special theory of relativity.

Testing Lorentz Symmetry

Although we have mentioned only three rather interesting experiments, physicists performing experiments with nuclear and particle accelerators have examined thousands of cases that verify the correctness of the concepts discussed here. Quantum electrodynamics (QED) includes special relativity in its framework, and QED has been tested to one part in 10^{12}.

Lorentz symmetry requires the laws of physics to be the same for all observers, and Lorentz symmetry is important at the very foundation of our description of fundamental particles and forces. Lorentz symmetry, together with the principles of quantum mechanics that are discussed in much of the remainder of this book, form the framework of relativistic quantum field theory. Many interactions that could be added to our best theories of physics (see the Standard Model in Chapter 14) are excluded, because they would violate Lorentz symmetry. In just the past two decades, physicists have conceived and performed many experiments that test Lorentz symmetry, but no violations have been discovered to date. For example, tests done with electrons have shown no violations to one part in 10^{32}, with neutrons one part in 10^{31}, and with protons one part in 10^{27}. These are phenomenal numbers, but many more experiments are currently underway, and more are planned. Several physicists have proposed in recent years that some theories of quantum gravity imply that Lorentz symmetry is not valid. They suggest a violation may occur at very small distances around 10^{-35} m. Direct investigation at these small distances is not now possible, because the energy required is huge (10^{28} eV), but such effects may be observed in highly energetic events in outer space. To date, no verified experiments have found a violation of Lorentz symmetry, but interest remains high.*

2.8 Twin Paradox

One of the most interesting topics in relativity is the twin (or clock) paradox. Almost from the time of publication of Einstein's famous paper in 1905, this subject has received considerable attention, and many variations exist. Let's summarize the paradox. Suppose twins, Mary and Frank, choose different career paths. Mary (the **M**oving twin) becomes an astronaut and Frank (the **F**ixed twin) a stockbroker. At age 30, Mary sets out on a spaceship to study a star system 8 ly from Earth. Mary travels at very high speeds to reach the star and returns during her life span.

*See "Lorentz Invariance on Trial," Maxim Pospelov and Michael Romalis, *Physics Today* (July 2004) p. 40. See also *Scientific American* (September 2004) Special Issue on "Beyond Einstein."

According to Frank's understanding of special relativity, Mary's biological clock ticks more slowly than his own, so he claims that Mary will return from her trip younger than he. The paradox is that Mary similarly claims that it is Frank who is moving rapidly with respect to her, so that when she returns, Frank will be the younger. To complicate the paradox further one could argue that because nature cannot allow both possibilities, it must be true that symmetry prevails and that the twins will still be the same age. Which is the correct solution?

Who is the younger twin?

The correct answer is that Mary returns from her space journey as the younger twin. According to Frank, Mary's spaceship takes off from Earth and quickly reaches its travel speed of $0.8c$. She travels the distance of 8 ly to the star system, slows down and turns around quickly, and returns to Earth at the same speed. The accelerations (positive and negative) take negligible times compared to the travel times between Earth and the star system. According to Frank, Mary's travel time to the star is 10 years $[(8 \text{ ly})/0.8c = 10 \text{ y}]$ and the return is also 10 years, for a total travel time of 20 years, so that Frank will be $30 + 10 + 10 \text{ y} = 50$ years old when Mary returns. However, because Mary's clock is ticking more slowly, her travel time to the star is only $10\sqrt{1 - 0.8^2} \text{ y} = 6$ years. Frank calculates that Mary will only be $30 + 6 + 6 \text{ y} = 42$ years old when she returns with respect to his own clock at rest.

The important fact here is that Frank's clock is in an inertial system* during the entire trip; however, Mary's clock is not. As long as Mary is traveling at constant speed away from Frank, both of them can argue that the other twin is aging less rapidly. However, when Mary slows down to turn around, she leaves her original inertial system and eventually returns in a completely different inertial system. Mary's claim is no longer valid, because she does not remain in the same inertial system. There is also no doubt as to who is in the inertial system. Frank feels no acceleration during Mary's entire trip, but Mary will definitely feel acceleration during her reversal time, just as we do when we step hard on the brakes of a car. The acceleration at the beginning and the deceleration at the end of her trip present little problem, because the fixed and moving clocks could be compared if Mary were just passing by Frank each way. It is Mary's acceleration at the star system that is the key. If we invoke the two postulates of special relativity, there is no paradox. The instantaneous rate of Mary's clock is determined by her instantaneous speed, but she must account for the acceleration effect when she turns around. A careful analysis of Mary's entire trip using special relativity, including acceleration, will be in agreement with Frank's assessment that Mary is younger. Mary returns to Earth rich as well as famous, because her stockbroker brother has invested her salary wisely during the 20-year period (for which she only worked 12 years!).

Mary is both younger and rich

We follow A. P. French's excellent book, *Special Relativity*, to present Table 2.1 (page 48), which analyzes the twin paradox. Both Mary and Frank send out signals at a frequency f (as measured by their own clock). We include in the table the various journey timemarks and signals received during the trip, with one column for the twin Frank who stayed at home and one for the astronaut twin Mary who went on the trip. Let the total time of the trip as measured on Earth be T. The speed of Mary's spaceship is v (as measured on Earth), which gives a relativistic factor γ. The distance Mary's spaceship goes before turning around (as measured on Earth) is L. Much of this table is best analyzed by using spacetime (see the next section) and the Doppler effect (see Section 2.10).

*The rotating and orbiting Earth is only an approximate inertial system.

Table 2.1 Twin Paradox Analysis

Item	Measured by Frank (remains on Earth)	Measured by Mary (traveling astronaut)
Time of total trip	$T = 2L/v$	$T' = 2L/\gamma v$
Total number of signals sent	$fT = 2fL/v$	$fT' = 2fL/\gamma v$
Frequency of signals received at beginning of trip f'	$f\sqrt{\dfrac{1-\beta}{1+\beta}}$	$f\sqrt{\dfrac{1-\beta}{1+\beta}}$
Time of detecting Mary's turnaround	$t_1 = L/v + L/c$	$t'_1 = L/\gamma v$
Number of signals received at the rate f'	$f't_1 = \dfrac{fL}{v}\sqrt{1-\beta^2}$	$f't'_1 = \dfrac{fL}{v}(1-\beta)$
Time for remainder of trip	$t_2 = L/v - L/c$	$t'_2 = L/\gamma v$
Frequency of signals received at end of trip f''	$f\sqrt{\dfrac{1+\beta}{1-\beta}}$	$f\sqrt{\dfrac{1+\beta}{1-\beta}}$
Number of signals received at rate f''	$f''t_2 = \dfrac{fL}{v}\sqrt{1-\beta^2}$	$f''t'_2 = \dfrac{fL}{v}(1+\beta)$
Total number of signals received	$2fL/\gamma v$	$2fL/v$
Conclusion as to other twin's measure of time taken	$T' = 2L/\gamma v$	$T = 2L/v$

After A. French, *Special Relativity,* New York: Norton (1968), p. 158.

2.9 Spacetime

When describing events in relativity, it is sometimes convenient to represent events on a **spacetime** diagram as shown in Figure 2.21. For convenience we use only one spatial coordinate x and specify position in this one dimension. We use ct instead of time so that both coordinates will have dimensions of length. Spacetime diagrams were first used by H. Minkowski in 1908 and are often called **Minkowski diagrams**. We have learned in relativity that we must denote both space and time to specify an event. This is the origin of the term *fourth dimension* for time. The events for A and B in Figure 2.21 are denoted by the respective coordinates (x_A, ct_A) and (x_B, ct_B), respectively. The line connecting events A and B is the path from A to B and is called a **worldline**. A spaceship launched from $x = 0$, $ct = 0$ with constant velocity v has the worldline shown in Figure 2.22: a straight line with slope c/v. For example, a light signal sent out from the origin with speed c is represented on a spacetime graph with a worldline that has a slope $c/c = 1$, so that line makes an angle of 45° with both the x and ct axes. Any real motion in the spacetime diagram cannot have a slope of less than 1 (angle with the x axis $< 45°$), because that motion would have a speed greater than c. The Lorentz transformation does not allow such a speed.

Spacetime (Minkowski) diagrams

Worldline

Let us consider two events that occur at the same time ($ct = 0$) but at different positions, x_1 and x_2. We denote the events (x, ct) as $(x_1, 0)$ and $(x_2, 0)$, and we show them in Figure 2.23 in an inertial system with an origin fixed at $x = 0$ and $ct = 0$. How can we be certain that the two events happen simultaneously if they occur at different positions? We must devise a method that will allow us to determine experimentally that the events occurred simultaneously. Let us place clocks at positions x_1 and x_2 and place a flashbulb at position x_3 halfway between x_1 and x_2. The two clocks have been previously synchronized and keep identical time. At time $t = 0$, the flashbulb explodes and sends out light signals from position x_3. The light signals proceed along their worldlines as shown in Figure 2.23. The two light signals arrive at positions x_1 and x_2 at identical times t as shown on the spacetime diagram. By using such techniques we can be sure that events occur simultaneously in our inertial reference system.

But what about other inertial reference systems? We realize that the two events will not be simultaneous in a reference system K′ moving at speed v with respect to our (x, ct) system. Because the two events have different spatial coordinates, x_1 and x_2, the Lorentz transformation will preclude them from occurring at the same time t' simultaneously in the moving coordinate systems. We can see this by supposing that events 1, 2, and 3 take place on a spaceship moving with velocity v. The worldlines for x_1 and x_2 are the two slanted lines beginning at x_1 and x_2 in Figure 2.24. However, when the flashbulb goes off, the light signals from x_3 still proceed at 45° in the (x, ct) reference system. The light signals intersect the worldlines from positions x_1 and x_2 at different times, so we do not see the events as being simultaneous in the moving system. Spacetime diagrams can be useful in showing such phenomena.

Anything that happened earlier in time than $t = 0$ is called the *past* and anything that occurs after $t = 0$ is called the *future*. The spacetime diagram in Figure 2.25a shows both the past and the future. Notice that only the events within the shaded area below $t = 0$ can affect the present. Events outside this area cannot affect the present because of the limitation $v \leq c$; this region is called *elsewhere*. Similarly, the present cannot affect any events occurring outside the shaded area above $t = 0$, again because of the limitation of the speed of light.

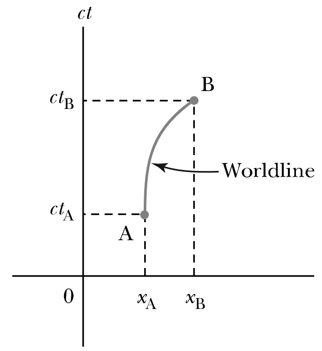


Figure 2.21 A spacetime diagram is used to specify events. The worldline denoting the path from event A to event B is shown.

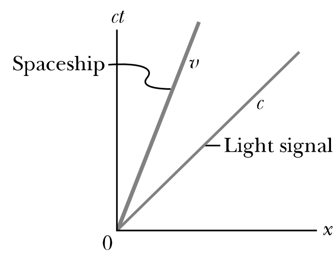


Figure 2.22 A light signal has the slope of 45° on a spacetime diagram. A spaceship moving along the x axis with speed v is a straight line on the spacetime diagram with a slope c/v.

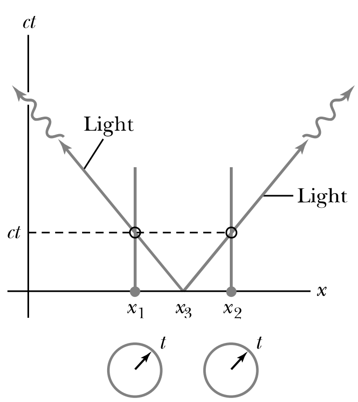


Figure 2.23 Clocks positioned at x_1 and x_2 can be synchronized by sending a light signal from a position x_3 halfway between. The light signals intercept the worldlines of x_1 and x_2 at the same time t.

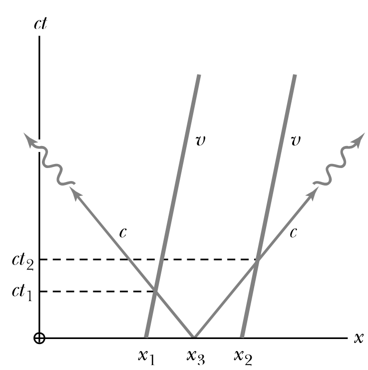


Figure 2.24 If the positions x_1 $(= x'_1)$ and x_2 $(= x'_2)$ of the previous figure are on a moving system K′ when the flashbulb goes off, the times will not appear simultaneous in system K, because the worldlines for x'_1 and x'_2 are slanted.

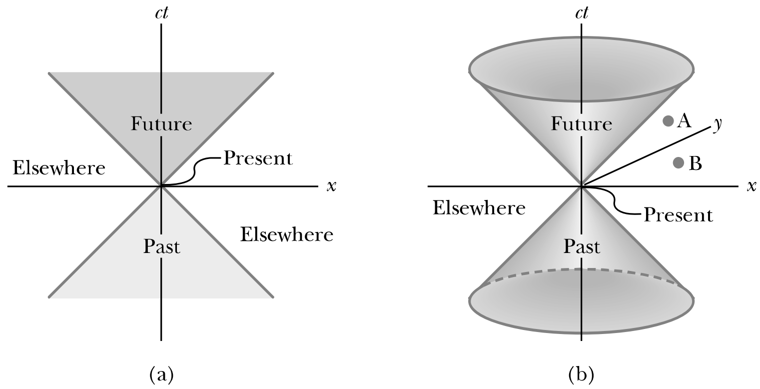


Figure 2.25 (a) The spacetime diagram can be used to show the past, present, and future. Only causal events are placed inside the shaded area. Events outside the shaded area below $t = 0$ cannot affect the present. (b) If we add an additional spatial coordinate y, a space cone can be drawn. The present cannot affect event A, but event B can.

Light cone

If we add another spatial coordinate y to our spacetime coordinates, we will have a cone as shown in Figure 2.25b, which we refer to as the **light cone**. All causal events related to the present ($x = 0$, $ct = 0$) must be within the light cone. In Figure 2.25b, anything occurring at present ($x = 0$, $ct = 0$) cannot possibly affect an event at position A; however, the event B can easily affect event A because A would be within the range of light signals emanating from B.

Invariant quantities

Invariant quantities have the same value in all inertial frames. They serve a special role in physics because their values do not change from one system to another. For example, the speed of light c is invariant. We are used to defining distances by $d^2 = x^2 + y^2 + z^2$, and in Euclidean geometry, we obtain the same result for d^2 in any inertial frame of reference. Is there a quantity, similar to d^2, that is also invariant in the special theory? If we refer to Equations (2.9), we have similar equations in both systems K and K′. Let us look more carefully at the quantity s^2 defined as

$$s^2 = x^2 - (ct)^2 \tag{2.25a}$$

and also

$$s'^2 = x'^2 - (ct')^2 \tag{2.25b}$$

If we use the Lorentz transformation for x and t, we find that $s^2 = s'^2$, so **s^2 is an invariant quantity.** This relationship can be extended to include the two other spatial coordinates, y and z, so that*

$$s^2 = x^2 + y^2 + z^2 - (ct)^2 \tag{2.26}$$

For simplicity, we will sometimes continue to use only the single spatial coordinate x.

If we consider two events, we can determine the quantity Δs^2 where

$$\Delta s^2 = \Delta x^2 - c^2\,\Delta t^2 \tag{2.27}$$

Spacetime interval

between the two events, and we find that it is invariant in any inertial frame. The quantity Δs is known as the **spacetime interval** between two events. There are three possibilities for the invariant quantity Δs^2.

Lightlike

1. **$\Delta s^2 = 0$:** In this case $\Delta x^2 = c^2\,\Delta t^2$, and the two events can be connected only by a light signal. The events are said to have a **lightlike** separation.
2. **$\Delta s^2 > 0$:** Here we must have $\Delta x^2 > c^2\,\Delta t^2$, and no signal can travel fast enough to connect the two events. The events are not causally connected

* Some authors use the negative of the expression here in Equation (2.26).

and are said to have a **spacelike** separation. In this case we can always find an inertial frame traveling at a velocity less than c in which the two events can occur simultaneously in time but at different places in space.

Spacelike

3. **$\Delta s^2 < 0$:** Here we have $\Delta x^2 < c^2\,\Delta t^2$, and the two events can be causally connected. The interval is said to be **timelike**. In this case we can find an inertial frame traveling at a velocity less than c in which the two events occur at the same position in space but at different times. The two events can never occur simultaneously.

Timelike

EXAMPLE 2.7

Draw the spacetime diagram for the motion of the twins discussed in Section 2.8. Draw light signals being emitted from each twin at annual intervals and count the number of light signals received by each twin from the other.

Strategy We shall let Mary leave Earth at the origin $(x, ct) = (0, 0)$. She will return to Earth at $x = 0$, but at a later time $ct = 20$ ly. Her worldlines will be described by two lines of slope $+c/v$ and $-c/v$, whereas Frank's worldline remains fixed at $x = 0$. Frank's and Mary's signals have slopes of ± 1 on the spacetime diagram. We pay close attention to when the light signals sent out by Frank and Mary reach their twin's worldlines.

Solution We show in Figure 2.26 (page 52) the spacetime diagram. The line representing Mary's trip has a slope $c/0.8c = 1.25$ on the outbound trip and -1.25 on the return trip. During the trip to the star system, Mary does not receive the second annual light signal from Frank until she reaches the star system. This occurs because the light signal takes considerable time to catch up with Mary. However, during the return trip Mary receives Frank's light signals at a rapid rate, receiving the last one (number 20) just as she returns. Because Mary's clock is running slow, we see the light signals being sent less often on the spacetime diagram in the fixed system. Mary sends out her sixth annual light signal when she arrives at the star system. However, this signal does not reach Frank until the 18th year! During the remaining two years, however, Frank receives Mary's signals at a rapid rate, finally receiving all 12 of them. Frank receives the last 6 signals during a time period of only 2 years.

A 3-vector $\vec{R}$ can be defined using Cartesian coordinates x, y, z in three-dimensional Euclidean space. Another 3-vector $\vec{R}'$ can be determined in another Cartesian coordinate system using x', y', z' in the new system. So far in introductory physics we have discussed translations and rotations of axes between these two systems. We have learned that there are two geometries in Newtonian spacetime. One is the three-dimensional Euclidean geometry in which the space interval is $d\ell^2 = dx^2 + dy^2 + dz^2$, and the other is a one-dimensional time interval dt. Minkowski pointed out that both space and time by themselves will not suffice under a Lorentz transformation, and only a union of both will be independent and useful.

We can form a four-dimensional space or four-vector using the four components x, y, z, ict. The equivalent of Equation (2.27) becomes

$$\begin{aligned} ds^2 &= dx^2 + dy^2 + dz^2 - c^2dt^2 \\ ds'^2 &= dx'^2 + dy'^2 + dz'^2 - c^2dt'^2 \\ ds^2 &= ds'^2 \end{aligned} \tag{2.28}$$

We previously noted that ds^2 (actually Δs^2) can be positive, negative, or zero. With the four-vector formalism we only have the *spacetime* geometry, not separate geometries for space and time. The spacetime distances $ds^2 = ds'^2$ are invariant

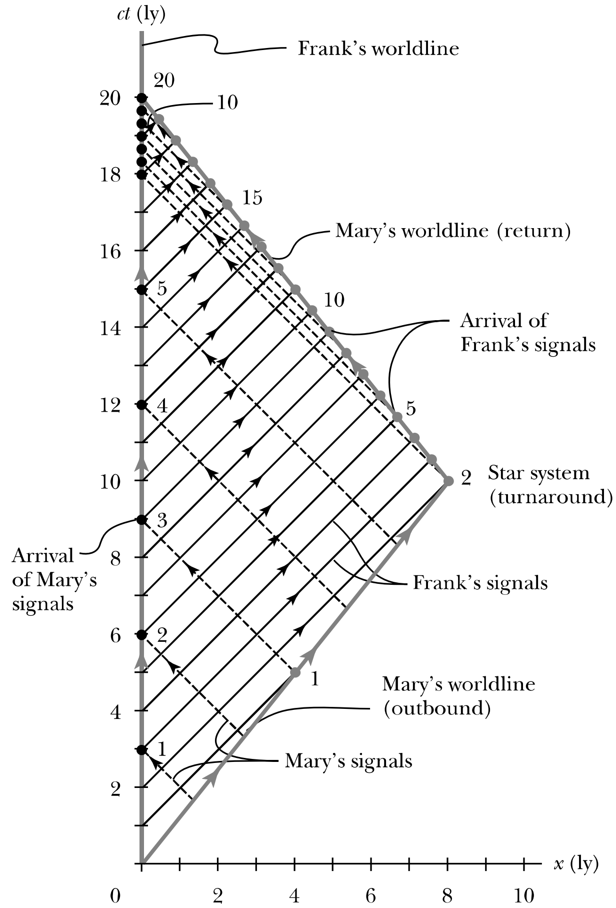


Figure 2.26 The spacetime diagram for Mary's trip to the star system and back. Notice that Frank's worldline is a vertical line at $x = 0$, and Mary's two worldlines have the correct slope given by the magnitude c/v. The black dashed lines represent light signals sent at annual intervals from Mary to Frank. Frank's annual signals to Mary are solid black. The solid dots denote the time when the light signals arrive.

under the Lorentz transformation. In Section 2.12 we will learn how the energy and momentum of a particle are connected. Similar to the spacetime four-vector, there is an energy-momentum four-vector, and the invariant quantity is the mass.

The four-vector formalism gives us equations that produce form-invariant quantities under appropriate Lorentz transformations. It allows the mathematical construction of relativistic physics to be somewhat easier. However, the penalty is that we would have to stop and learn matrix algebra and perhaps even about *tensors* and, eventually, *spinors*. At this point in our study there is little to be gained in understanding about relativity. Another disadvantage in utilizing four-vectors at this point is that there is no general agreement among authors as to terminology. Sometimes ict is term 0 of the four-vector (ict, x, y, z with x, y, z being terms 1, 2, 3), and sometimes it is described as term 4 (x, y, z, ict). Sometimes the formalism is arranged such that the imaginary number $i = \sqrt{-1}$ doesn't appear. We have chosen not to use four-vectors.

2.10 Doppler Effect

You may have already studied the Doppler effect of sound in introductory physics. It causes an increased frequency of sound as a source such as a train (with whistle blowing) approaches a receiver (our eardrum) and a decrease in frequency as the source recedes. A change in sound frequency also occurs when the source is fixed and the receiver is moving. The change in frequency of the sound wave depends on whether the source or receiver is moving. On first thought it seems that the Doppler effect in sound violates the principle of relativity, until we realize that there is in fact a special frame for sound waves. Sound waves depend on media such as air, water, or a steel plate to propagate. For light, however, there is no such medium. It is only relative motion of the source and receiver that is relevant, and we expect some differences between the relativistic Doppler effect for light waves and the normal Doppler effect for sound. It is not possible for a source of light to travel faster than light in a vacuum, but it is possible for a source of sound to travel faster than the speed of sound. Similarly, in a medium such as water in which light travels slower than c, a light source can travel faster than the speed of light.

Consider a source of light (for example, a star) and a receiver (an astronomer) approaching one another with a relative velocity v. First we consider the receiver fixed (Figure 2.27a) in system K and the light source in system K′ moving toward the receiver with velocity v. The source emits n waves during the time interval T. Because the speed of light is always c and the source is moving with velocity v, the total distance between the front and rear of the wave train emitted during the time interval T is

$$\text{Length of wave train} = cT - vT$$

Because there are n waves emitted during this time period, the wavelength must be

$$\lambda = \frac{cT - vT}{n}$$

and the frequency, $f = c/\lambda$, is

$$f = \frac{cn}{cT - vT} \tag{2.29}$$

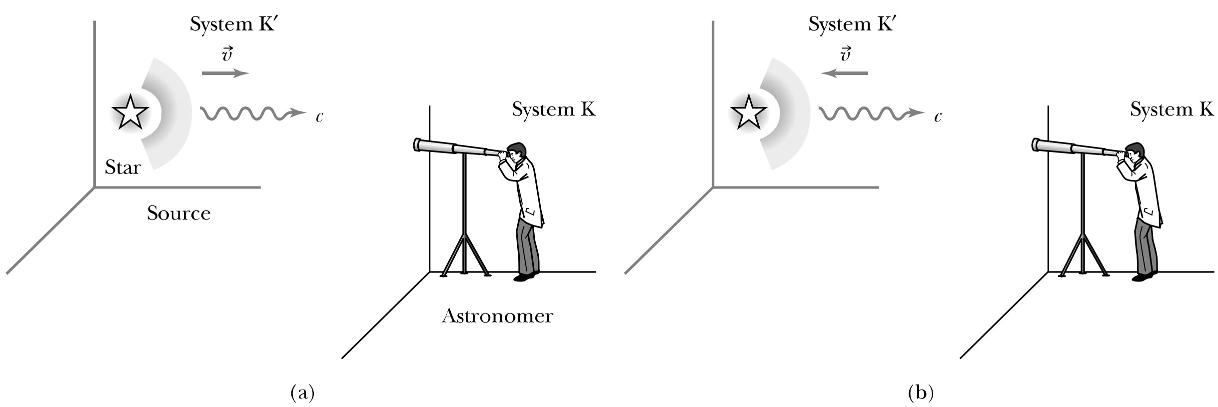
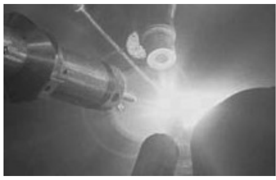


Figure 2.27 (a) The source (star) is approaching the receiver (astronomer) with velocity v while it emits starlight signals with speed c. (b) Here the source and receiver are receding with velocity v. The Doppler effect for light is different than that for sound, because of relativity and no medium to carry the light waves.



Special Topic

Applications of the Doppler Effect

The Doppler effect is not just a curious result of relativity. It has many practical applications, three of which are discussed here, and others are mentioned in various places in this text.

Astronomy

Perhaps the best-known application is in astronomy, where the Doppler shifts of known atomic transition frequencies determine the relative velocities of astronomical objects with respect to us. Such measurements continue to be used today to find the distances of such unusual objects as quasars (objects having incredibly large masses that produce tremendous amounts of radiation; see Chapter 16). The Doppler effect has been used to discover other effects in astronomy, for example, the rate of rotation of Venus and the fact that Venus rotates in the opposite direction of Earth—the sun rises in the west on Venus. This was determined by observing light reflected from both sides of Venus—on one side it is blueshifted and on the other side it is redshifted, as shown in Figure A. The same technique has been used to determine the rate of rotation of stars.

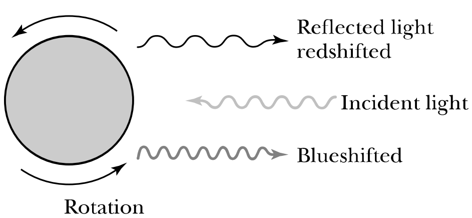


Figure A

Radar

The Doppler effect is nowhere more important than it is in radar. When an electromagnetic radar signal reflects off of a moving target, the so-called *echo* signal will be shifted in frequency by the Doppler effect. Very small frequency shifts can be determined by examining the beat frequency of the echo signal with a reference signal. The frequency shift is proportional to the radial component of the target's velocity. Navigation radar is quite complex, and ingenious techniques have been devised to determine the target position and velocity using multiple radar beams. By using pulsed Doppler radar it is possible to separate moving targets from stationary targets, called clutter.

Doppler radar is also extensively used in meteorology. Vertical motion of airdrafts, sizes and motion of raindrops, motion of thunderstorms, and detailed patterns of wind distribution have all been studied with Doppler radar.

X rays and gamma rays emitted from moving atoms and nuclei have their frequencies shifted by the Doppler effect. Such phenomena tend to broaden radiation frequencies emitted by stationary atoms and nuclei and add to the natural spectral widths observed.

In its rest frame, the source emits n waves of frequency f_0 during the proper time T_0'.

$$n = f_0 T_0' \tag{2.30}$$

The proper time interval T_0' measured on the clock at rest in the moving system is related to the time interval T measured on a clock fixed by the receiver in system K by

$$T_0' = \frac{T}{\gamma} \tag{2.31}$$

where γ is the relativistic factor of Equation (2.16). The clock moving with the source measures the proper time because it is present with both the beginning and end of the wave.

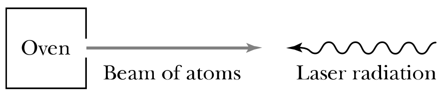


Figure B

Laser Cooling

In order to perform fundamental measurements in atomic physics, it is useful to limit the effects of thermal motion and to isolate single atoms. A method taking advantage of the Doppler effect can slow down even neutral atoms and eventually isolate them. Atoms emitted from a hot oven will have a spread of velocities. If these atoms form a beam as shown in Figure B, a laser beam impinging on the atoms from the right can slow them down by transferring momentum.

Atoms have characteristic energy levels that allow them to absorb and emit radiation of specific frequencies. Atoms moving with respect to the laser beam will "see" a shift in the laser frequency because of the Doppler effect. For example, atoms moving toward the laser beam will encounter light with high frequency, and atoms moving away from the laser beam will encounter light with low frequency. Even atoms moving in the same direction within the beam of atoms will see slightly different frequencies depending on the velocities of the various atoms. Now, if the frequency of the laser beam is tuned to the precise frequency seen by the faster atoms so that those atoms can be excited by absorbing the radiation, then those faster atoms will be slowed down by absorbing the momentum of the laser radiation. The slower atoms will "see" a laser beam that has been Doppler shifted to a lower frequency than is needed to absorb the radiation, and these atoms are not as likely to absorb the laser radiation. The net effect is that the atoms as a whole are *slowed down* and their *velocity spread is reduced*.

As the atoms slow down, they see that the Doppler-shifted frequencies of the laser change, and the atoms no longer absorb the laser radiation. They continue with the same lower velocity and velocity spread. The lower temperature limits reached by Doppler cooling depend on the atom, but typical values are on the order of hundreds of microkelvins. Doppler cooling is normally accompanied by intersecting laser beams at different angles; an "optical molasses" can be created in which atoms are essentially trapped. Further cooling is obtained by other techniques including "Sisyphus" and evaporative cooling, among others. In a remarkable series of experiments by various researchers, atoms have been cooled to temperatures approaching 10^{-10} K. The 1997 Nobel Prize in Physics was awarded to Steven Chu, Claude Cohen-Tannoudji, and William Phillips for these techniques. An important use of laser cooling is for atomic clocks. See http://www.nist.gov/physlab/div847/grp50/primary-frequency-standards.cfm for a good discussion. See also Steven Chu, "Laser Trapping of Neutral Particles," *Scientific American* **266**, 70 (February 1992). In Chapter 9 we will discuss how laser cooling is used to produce an ultracold state of matter known as a Bose-Einstein condensate.

We substitute the proper time T_0' from Equation (2.31) into Equation (2.30) to determine the number of waves n. Then n is substituted into Equation (2.29) to determine the frequency.

$$\begin{aligned} f &= \frac{cf_0 T/\gamma}{cT - vT} \\ &= \frac{1}{1 - v/c}\frac{f_0}{\gamma} = \frac{\sqrt{1 - v^2/c^2}}{1 - v/c} f_0 \end{aligned}$$

where we have inserted the equation for γ. If we use $\beta = v/c$, we can write the previous equation as

$$f = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} f_0 \quad \text{Source and receiver approaching} \tag{2.32}$$

It is straightforward to show that Equation (2.32) is also valid when the source is fixed and the receiver approaches it with velocity v. It is the relative velocity v, of course, that is important (Problem 49).

But what happens if the source and receiver are receding from each other with velocity v (see Figure 2.27b)? The derivation is similar to the one just done, except that the distance between the beginning and end of the wave train becomes

$$\text{Length of wave train} = cT + vT$$

because the source and receiver are receding rather than approaching. This change in sign is propagated throughout the derivation (Problem 50), with the final result

$$f = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} f_0 \quad \text{Source and receiver receding} \tag{2.33}$$

Equations (2.32) and (2.33) can be combined into one equation if we agree to use a + sign for β $(+v/c)$ when the source and receiver are approaching each other and a − sign for β $(-v/c)$ when they are receding. The final equation becomes

$$f = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} f_0 \quad \text{Relativistic Doppler effect} \tag{2.34}$$

The Doppler effect is useful in many areas of science including astronomy, atomic physics, and nuclear physics. One of its many applications includes an effective radar system for locating airplane position and speed (see Special Topic, "Applications of the Doppler Effect").

Elements absorb and emit characteristic frequencies of light due to the existence of particular atomic levels. We will learn more about this later. Scientists have observed these characteristic frequencies in starlight and have observed shifts in the frequencies. One reason for these shifts is the Doppler effect, and the frequency changes are used to determine the speed of the emitting object with respect to us.

Redshifts

This is the source of the **redshifts** of starlight caused by objects moving away from us. These data have been used to ascertain that the universe is expanding. The farther away the star, the higher the redshift. This observation is what led Harlow Shapley and Edwin Hubble to the idea that the universe started with a Big Bang.*

So far in this section we have only considered the source and receiver to be directly approaching or receding. Of course, it is also possible for the two to be moving at an angle with respect to one another, as shown in Figure 2.28. We omit the derivation here† but present the results. The angles θ and θ' are the angles the light signals make with the x axes in the K and K′ systems. They are related by

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*Excellent references are "The Cosmic Distance Scale" by Paul Hodge, *American Scientist* **72**, 474 (1984), and "Origins" by S. Weinberg, *Science* **230**, 15 (1985). This subject is discussed in Chapter 16.

†See Robert Resnick, *Introduction to Special Relativity*, New York: Wiley (1968).

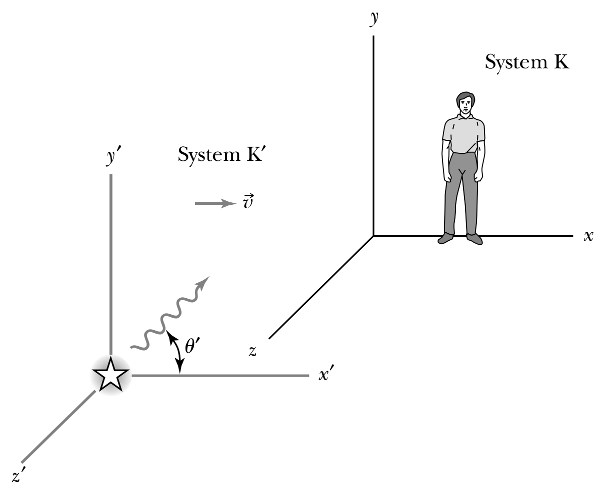


Figure 2.28 The light signals in system K′ are emitted at an angle θ' from the x' axis and remain in the $x'y'$ plane.

$$f \cos\theta = \frac{f_0(\cos\theta' + \beta)}{\sqrt{1-\beta^2}} \tag{2.35}$$

and

$$f \sin\theta = f_0 \sin\theta' \tag{2.36}$$

The generalized Doppler shift equation becomes

$$f = \frac{1 + \beta\cos\theta'}{\sqrt{1-\beta^2}} f_0 \tag{2.37}$$

Note that Equation (2.37) gives Equation (2.32) when $\theta' = 0$ (source and receiver approaching) and gives Equation (2.33) when $\theta' = 180°$ (source and receiver receding). This situation is known as the *longitudinal Doppler effect*.

When $\theta = 90°$ the emission is purely transverse to the direction of motion, and we have the *transverse Doppler effect*, which is purely a relativistic effect that does not occur classically. The transverse Doppler effect is directly due to time dilation and has been verified experimentally. Equations (2.35) through (2.37) can also be used to understand stellar aberration.

EXAMPLE 2.8

In Section 2.8 we discussed what happened when Mary traveled on a spaceship away from her twin brother Frank, who remained on Earth. Analyze the light signals sent out by Frank and Mary by using the relativistic Doppler effect.

Strategy We will use Equation (2.34) for both the outbound and return trip to analyze the frequency of the light signals sent and received. During the outbound trip the source (Frank) and receiver (Mary) are receding so that $\beta = -0.8$. For the return trip, we have $\beta = +0.8$. The frequency f_0 will be the signals that Frank sends; the frequency f will be those that Mary receives.

Solution First, we analyze the frequency of the light signals that Mary receives from Frank. Equation (2.34) gives

$$f = \frac{\sqrt{1+(-0.8)}}{\sqrt{1-(-0.8)}} f_0 = \frac{f_0}{3}$$

Because Frank sends out signals annually, Mary will receive the signals only every 3 years. Therefore during the 6-year trip in Mary's system to the star system, she will receive only 2 signals.

During the return trip, $\beta = 0.8$ and Equation (2.34) gives

$$f = \frac{\sqrt{1+0.8}}{\sqrt{1-0.8}} f_0 = 3f_0$$

so that Mary receives 3 signals each year for a total of 18 signals during the return trip. Mary receives a total of 20 annual light signals from Frank, and she concludes that Frank has aged 20 years during her trip.

Now let's analyze the light signals that Mary sends Frank. During the outbound trip the frequency at which Frank receives signals from Mary will also be $f_0/3$. During the 10 years that it takes Mary to reach the star system on his clock, he will receive 10/3 signals—3 signals plus 1/3 of the time to the next one. Frank continues to receive Mary's signals at the rate $f_0/3$ for another 8 years, because that is how long it takes the sixth signal she sent him to reach Earth. Therefore, for the first 18 years of her journey, according to his own clock he receives $18/3 = 6$ signals. Frank has no way of knowing that Mary has turned around and is coming back until he starts receiving signals at frequency $3f_0$. During Mary's return trip Frank will receive signals at the frequency $3f_0$ or 3 per year. However, in his system, Mary returns 2 years after he has received her sixth signal and turned around to come back. During this 2-year period he will receive 6 more signals, so he concludes she has aged a total of only 12 years.

Notice that this analysis is in total agreement with the spacetime diagram of Figure 2.26 and is somewhat easier to obtain. Although geometrical constructions like spacetime diagrams are sometimes useful, an analytical calculation is usually easier.

2.11 Relativistic Momentum

Newton's second law, $\vec{F} = d\vec{p}/dt$, keeps its same form under a Galilean transformation, but we might not expect it to do so under a Lorentz transformation. There may be similar transformation difficulties with the conservation laws of linear momentum and energy. We need to take a careful look at our previous definition of linear momentum to see whether it is still valid at high speeds. According to Newton's second law, for example, an acceleration of a particle already moving at very high speeds could lead to a speed greater than the speed of light. That would be in conflict with the Lorentz transformation, so we expect that Newton's second law might somehow be modified at high speeds.

Because physicists believe the conservation of linear momentum is fundamental, we begin by considering a collision that has no external forces. Frank (**F**ixed or stationary system) is at rest in system K holding a ball of mass m. Mary (**M**oving system) holds a similar ball in system K′ that is moving in the x direction with velocity v with respect to system K as shown in Figure 2.29a. Frank throws his ball along his y axis, and Mary throws her ball with exactly the same speed along her negative y' axis. The two balls collide in a perfectly elastic collision, and each of them catches their own ball as it rebounds. Each twin measures the speed of his or her own ball to be u_0 both before and after the collision.

We show the collision according to both observers in Figure 2.29. Consider the conservation of momentum according to Frank as seen in system K. The velocity of the ball thrown by Frank has components in his own system K of

$$\begin{aligned} u_{Fx} &= 0 \\ u_{Fy} &= u_0 \end{aligned} \tag{2.38}$$

If we use the definition of momentum, $\vec{p} = m\vec{v}$, the momentum of the ball thrown by Frank is entirely in the y direction:

$$p_{Fy} = mu_0 \tag{2.39}$$

Because the collision is perfectly elastic, the ball returns to Frank with speed u_0 along the $-y$ axis. The change of momentum of his ball as observed by Frank in system K is

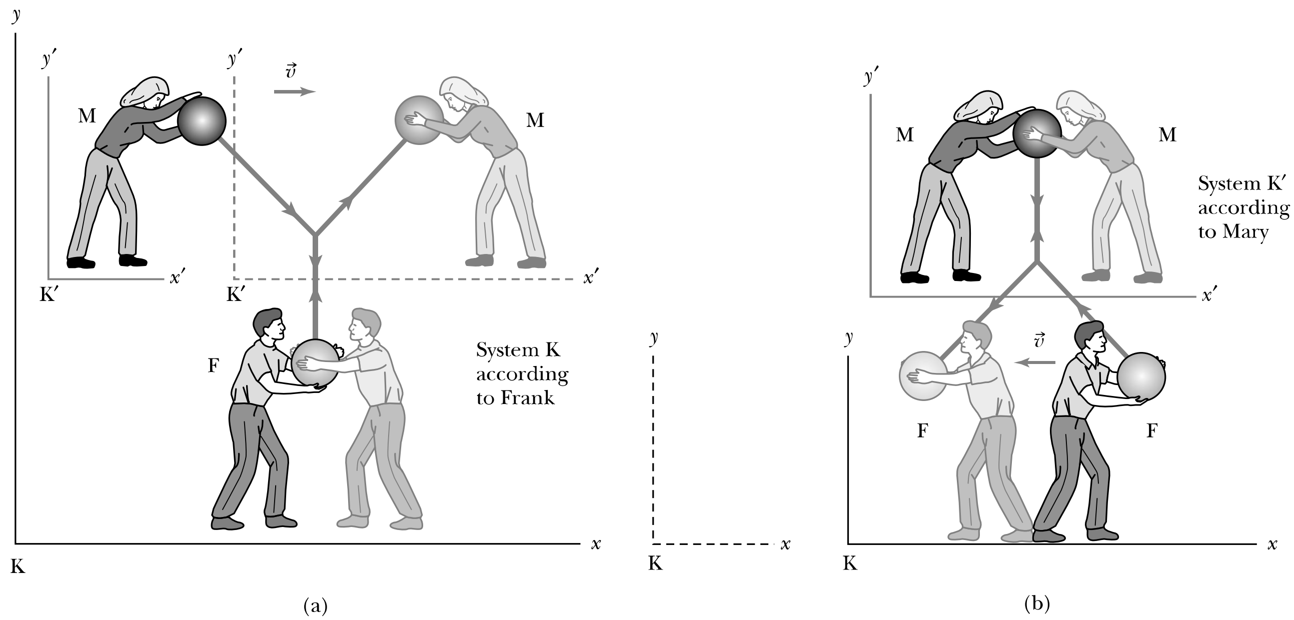


Figure 2.29 Frank is in the fixed K system, and Mary is in the moving K′ system. Frank throws his ball along his $+y$ axis, and Mary throws her ball along her $-y'$ axis. The balls collide. The event is shown in Frank's system in (a) and in Mary's system in (b). (Because it is awkward to show the twins as they catch the ball, we have drawn them faintly and in a reversed position.)

$$\Delta p_F = \Delta p_{Fy} = -2mu_0 \tag{2.40}$$

In order to confirm the conservation of linear momentum, we need to determine the change in the momentum of Mary's ball *as measured by Frank*. We will let the primed speeds be measured by Mary and the unprimed speeds be measured by Frank (except that u_0 is always the speed of the ball as measured by the twin in his or her own system). Mary measures the initial velocity of her own ball to be $u'_{Mx} = 0$ and $u'_{My} = -u_0$, because she throws it along her own $-y'$ axis. To determine the velocity of Mary's ball as measured by Frank, we need to use the velocity transformation equations of Equation (2.23). If we insert the appropriate values for the speeds just discussed, we obtain

$$\begin{aligned} u_{Mx} &= v \\ u_{My} &= -u_0\sqrt{1 - v^2/c^2} \end{aligned} \tag{2.41}$$

Before the collision, the momentum of Mary's ball as measured by Frank becomes

$$\begin{aligned} \text{Before} \quad p_{Mx} &= mv \\ \text{Before} \quad p_{My} &= -mu_0\sqrt{1 - v^2/c^2} \end{aligned} \tag{2.42}$$

For a perfectly elastic collision, the momentum after the collision is

$$\begin{aligned} \text{After} \quad p_{Mx} &= mv \\ \text{After} \quad p_{My} &= +mu_0\sqrt{1 - v^2/c^2} \end{aligned} \tag{2.43}$$

The change in momentum of Mary's ball according to Frank is

$$\Delta p_M = \Delta p_{My} = 2mu_0\sqrt{1 - v^2/c^2} \tag{2.44}$$

The conservation of linear momentum requires the total change in momentum of the collision, $\Delta p_F + \Delta p_M$, to be zero. The addition of Equations (2.40) and (2.44) clearly does not give zero. *Linear momentum is not conserved if we use the conventions for momentum from classical physics even if we use the velocity transformation equations from the special theory of relativity.* There is no problem with the x direction, but there is a problem with the y direction along the direction the ball is thrown in each system.

Difficulty with classical linear momentum

Rather than abandon the conservation of linear momentum, let us look for a modification of the definition of linear momentum that preserves both it and Newton's second law. We follow a procedure similar to the one we used in deriving the Lorentz transformation; we assume the simplest, most reasonable change that may preserve the conservation of momentum. We assume that the classical form of momentum $m\vec{u}$ is multiplied by a factor that may depend on velocity. Let the factor be $\Gamma(u)$. Our trial definition for linear momentum now becomes

$$\vec{p} = \Gamma(u)m\vec{u} \tag{2.45}$$

In Example 2.9 we show that momentum is conserved in the collision just described for the value of $\Gamma(u)$ given by

$$\Gamma(u) = \frac{1}{\sqrt{1 - u^2/c^2}} \tag{2.46}$$

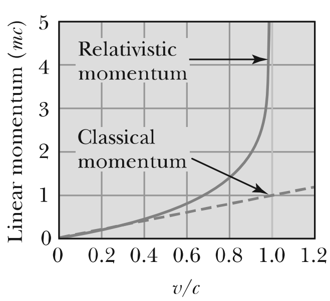


Figure 2.30 The linear momentum of a particle of mass m is plotted versus its velocity (v/c) for both the classical and relativistic momentum results. As $v \to c$ the relativistic momentum becomes quite large, but the classical momentum continues its linear rise. The relativistic result is the correct one.

Notice that the *form* of Equation (2.46) is the same as that found earlier for the Lorentz transformation. We even give $\Gamma(u)$ the same symbol: $\Gamma(u) = \gamma$. However, this γ is different; it contains the speed of the particle u, whereas the Lorentz transformation contains the relative speed v between the two inertial reference frames. This distinction should be kept in mind because it can cause confusion. Because the usage is so common among physicists, we will use γ for both purposes. However, when there is any chance of confusion, we will write out $1/\sqrt{1 - u^2/c^2}$ and use $\gamma = 1/\sqrt{1 - v^2/c^2}$ for the Lorentz transformation. We will write out $1/\sqrt{1 - u^2/c^2}$ often to avoid confusion.

We can make a plausible determination for the correct form of the momentum if we use the proper time discussed previously to determine the velocity. The momentum becomes

$$\vec{p} = m\frac{d\vec{r}}{d\tau} = m\frac{d\vec{r}}{dt}\frac{dt}{d\tau} \tag{2.47}$$

We retain the velocity $\vec{u} = d\vec{r}/dt$ as used classically, where $\vec{r}$ is the position vector. All observers do not agree as to the value of $d\vec{r}/dt$, but they do agree as to the value of $d\vec{r}/d\tau$, where $d\tau$ is the proper time measured in the moving system K′. The value of $dt/d\tau\ (= \gamma)$ is obtained from Equation (2.31), where the speed u is used in the relation for γ to represent the relative speed of the moving (Mary's) frame and the fixed (Frank's) frame.

Relativistic momentum

The definition of the **relativistic momentum** becomes, from Equation (2.47),

$$\vec{p} = m\frac{d\vec{r}}{dt}\gamma$$

$$\vec{p} = \gamma m\vec{u} \quad \text{Relativistic momentum} \tag{2.48}$$

where

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \tag{2.49}$$

This result for the relativistic momentum reduces to the classical result for small values of u/c. The classical momentum expression is good to an accuracy of 1% as long as $u < 0.14c$. We show both the relativistic and classical momentum in Figure 2.30.

Some physicists like to refer to the mass in Equation (2.48) as the *rest mass* m_0 and call the term $m = \gamma m_0$ the *relativistic mass*. In this manner the classical form of momentum, $m\vec{u}$, is retained. The mass is then imagined to increase at high speeds. Most physicists prefer to keep the concept of mass as an invariant, intrinsic property of an object. We adopt this latter approach and will use the term *mass* exclusively to mean *rest mass*. Although we may use the terms *mass* and *rest mass* synonymously, we will not use the term *relativistic mass*. The use of relativistic mass too often leads the student into mistakenly inserting the term into classical expressions where it does not apply.

Rest and relativistic mass

EXAMPLE 2.9

Show that linear momentum is conserved for the collision just discussed and shown in Figure 2.29.

Strategy We use the relativistic momentum to modify the expressions obtained for the momentum of the balls thrown by Frank and Mary. We will then check to see whether momentum is conserved according to Frank. We leave to Problem 62 the question of whether momentum is conserved according to Mary's system.

Solution From Equation (2.39), the momentum of the ball thrown by Frank becomes

$$p_{Fy} = \gamma m u_0 = \frac{m u_0}{\sqrt{1 - u_0^2/c^2}}$$

For an elastic collision, the magnitude of the momentum for this ball is the same before and after the collision. After the collision, the momentum will be the negative of this value, so the change in momentum becomes, from Equation (2.40),

$$\Delta p_F = \Delta p_{Fy} = -2\gamma m u_0 = -\frac{2 m u_0}{\sqrt{1 - u_0^2/c^2}} \tag{2.50}$$

Now we consider the momentum of Mary's ball as measured by Frank. Even with the addition of the γ factor for the momentum in the x direction, we still have $\Delta p_{Mx} = 0$. We must look more carefully at Δp_{My}. First, we find the speed of the ball thrown by Mary as measured by Frank. We use Equations (2.41) to determine

$$u_M = \sqrt{u_{Mx}^2 + u_{My}^2} = \sqrt{v^2 + u_0^2(1 - v^2/c^2)} \tag{2.51}$$

The relativistic factor γ for the momentum for this situation is

$$\gamma = \frac{1}{\sqrt{1 - u_M^2/c^2}}$$

The value of p_{My} is now found by modifying Equation (2.42) with this value of γ.

$$p_{My} = -\gamma m u_0 \sqrt{1 - v^2/c^2} = \frac{-m u_0 \sqrt{1 - v^2/c^2}}{\sqrt{1 - u_M^2/c^2}}$$

We insert the value of u_M from Equation (2.51) into this equation to give

$$p_{My} = \frac{-m u_0 \sqrt{1 - v^2/c^2}}{\sqrt{(1 - u_0^2/c^2)(1 - v^2/c^2)}} = \frac{-m u_0}{\sqrt{1 - u_0^2/c^2}} \tag{2.52}$$

The momentum after the collision will still be the negative of this value, so the change in momentum becomes

$$\Delta p_M = \Delta p_{My} = \frac{2 m u_0}{\sqrt{1 - u_0^2/c^2}} \tag{2.53}$$

The change in the momentum of the two balls as measured by Frank is given by the sum of Equations (2.50) and (2.53):

$$\Delta p = \Delta p_F + \Delta p_M = 0$$

Thus Frank indeed finds that momentum is conserved. Mary should also determine that linear momentum is conserved (see Problem 62).

2.12 Relativistic Energy

We now turn to the concepts of energy and force. When forming the new theories of relativity and quantum physics, physicists resisted changing the well-accepted ideas of classical physics unless absolutely necessary. In this same spirit we also choose to keep intact as many definitions from classical physics as possible and let experiment dictate when we are incorrect. In practice, the concept of force is best defined by its use in Newton's laws of motion, and we retain here the classical definition of force as used in Newton's second law. In the previous section we studied the concept of momentum and found a relativistic expression in Equation (2.48). Therefore, we modify Newton's second law to include our new definition of linear momentum, and force becomes

Relativistic force

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(\gamma m\vec{u}) = \frac{d}{dt}\left(\frac{m\vec{u}}{\sqrt{1 - u^2/c^2}}\right) \tag{2.54}$$

Aspects of this force will be examined in the problems (see Problems 55–58).

Introductory physics presents kinetic energy as the work done on a particle by a net force. We retain here the same definitions of kinetic energy and work. The work W_{12} done by a force $\vec{F}$ to move a particle from position 1 to position 2 along a path $\vec{s}$ is defined to be

$$W_{12} = \int_1^2 \vec{F}\cdot d\vec{s} = K_2 - K_1 \tag{2.55}$$

where K_1 is defined to be the kinetic energy of the particle at position 1.

For simplicity, let the particle start from rest under the influence of the force $\vec{F}$ and calculate the final kinetic energy K after the work is done. The force is related to the dynamic quantities by Equation (2.54). The work W and kinetic energy K are

$$W = K = \int \frac{d}{dt}(\gamma m\vec{u})\cdot \vec{u}\, dt \tag{2.56}$$

where the integral is performed over the differential path $d\vec{s} = \vec{u}\, dt$. Because the mass is invariant, it can be brought outside the integral. The relativistic factor γ depends on u and cannot be brought outside the integral. Equation (2.56) becomes

$$K = m\int dt\,\frac{d}{dt}(\gamma\vec{u})\cdot\vec{u} = m\int u\, d(\gamma u)$$

The limits of integration are from an initial value of 0 to a final value of γu.

$$K = m\int_0^{\gamma u} u\, d(\gamma u) \tag{2.57}$$

The integral in Equation (2.57) is straightforward if done by the method of integration by parts. The result, called the *relativistic kinetic energy*, is

Relativistic kinetic energy

$$\boxed{K = \gamma mc^2 - mc^2 = mc^2\left(\frac{1}{\sqrt{1 - u^2/c^2}} - 1\right) = mc^2(\gamma - 1)} \tag{2.58}$$

Equation (2.58) does not seem to resemble the classical result for kinetic energy, $K = \frac{1}{2}mu^2$. However, if it is correct, we expect it to reduce to the classical result for low speeds. Let's see whether it does. For speeds $u \ll c$, we expand γ in a binomial series as follows:

$$K = mc^2\left(1 - \frac{u^2}{c^2}\right)^{-1/2} - mc^2$$

$$= mc^2\left(1 + \frac{1}{2}\frac{u^2}{c^2} + \cdots\right) - mc^2$$

where we have neglected all terms of power $(u/c)^4$ and greater, because $u \ll c$. This gives the following equation for the relativistic kinetic energy at low speeds:

$$K = mc^2 + \frac{1}{2}mu^2 - mc^2 = \frac{1}{2}mu^2 \tag{2.59}$$

which is the expected classical result. We show both the relativistic and classical kinetic energies in Figure 2.31. They diverge considerably above a velocity of 0.6c.

A common mistake students make when first studying relativity is to use either $\frac{1}{2}mu^2$ or $\frac{1}{2}\gamma mu^2$ for the relativistic kinetic energy. It is important to *use only Equation (2.58) for the relativistic kinetic energy*. Although Equation (2.58) looks much different from the classical result, it is the only correct one, and *neither* $\frac{1}{2}mu^2$ *nor* $\frac{1}{2}\gamma mu^2$ *is a correct relativistic result.*

Equation (2.58) is particularly useful when dealing with particles accelerated to high speeds. For example, the fastest speeds produced in the United States have been in the 3-kilometer-long electron accelerator at the Stanford Linear Accelerator Laboratory. This accelerator produces electrons with a kinetic energy of 8×10^{-9} J (50 GeV) or 50×10^9 eV. The electrons have speeds so close to the speed of light that the tiny difference from c is difficult to measure directly. The speed of the electrons is inferred from the relativistic kinetic energy of Equation (2.58) and is given by 0.99999999995c. Such calculations are difficult to do with calculators because of significant-figure limitations. As a result, we use kinetic energy or momentum to express the motion of a particle moving near the speed of light and rarely use its speed.

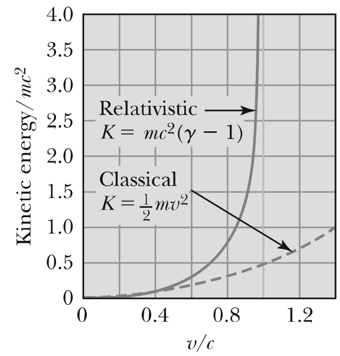


Figure 2.31 The kinetic energy as a fraction of rest energy (K/mc^2) of a particle of mass m is shown versus its velocity (v/c) for both the classical and relativistic calculations. Only the relativistic result is correct. Like the momentum, the kinetic energy rises rapidly as $v \to c$.

CONCEPTUAL EXAMPLE 2.10

Determine whether an object with mass can ever have the speed of light.

Solution If we examine Equation (2.58), we see that when $u \to c$, the kinetic energy $K \to \infty$. Because there is not an infinite amount of energy available, we agree that no object with mass can have the speed of light. The classical and relativistic speeds for electrons are shown in Figure 2.32 as a function of their kinetic energy. Physicists have found that experimentally it does not matter how much energy we give an object having mass. Its speed can never quite reach c.

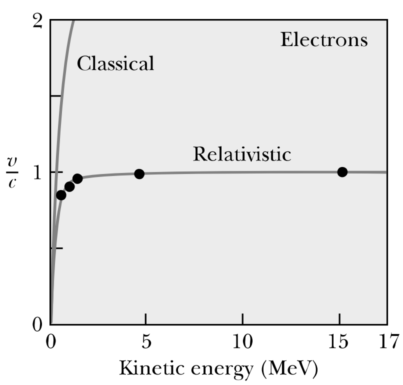


Figure 2.32 The velocity (v/c) of electrons is shown versus kinetic energy for both classical (incorrect) and relativistic calculations. The experimentally measured data points agree with the relativistic results. *Adapted with permission from* American Journal of Physics ***32****, 551 (1964), W. Bertozzi. © 1964 American Association of Physics Teachers.*

EXAMPLE 2.11

Electrons used to produce medical x rays are accelerated from rest through a potential difference of 25,000 volts before striking a metal target. Calculate the speed of the electrons and determine the error in using the classical kinetic energy result.

Strategy We calculate the speed from the kinetic energy, which we determine both classically and relativistically and then compare the results. In order to determine the correct speed of the electrons, we must use the relativistically correct kinetic energy given by Equation (2.58). The work done to accelerate an electron across a potential difference V is given by qV, where q is the charge of the particle. The work done to accelerate the electron from rest is the final kinetic energy K of the electron.

Solution The kinetic energy is given by

$$K = W = qV = (1.6 \times 10^{-19}\ \text{C})(25 \times 10^3\ \text{V})$$
$$= 4.0 \times 10^{-15}\ \text{J}$$

We first determine γ from Equation (2.58) and from that, the speed. We have

$$K = (\gamma - 1)mc^2 \tag{2.60}$$

From this equation, γ is found to be

$$\gamma = 1 + \frac{K}{mc^2} \tag{2.61}$$

The quantity mc^2 for the electron is determined to be

$$mc^2(\text{electron}) = (9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2$$
$$= 8.19 \times 10^{-14}\ \text{J}$$

The relativistic factor is then $\gamma = 1 + [(4.0 \times 10^{-15}\ \text{J})/(8.19 \times 10^{-14}\ \text{J})] = 1.049$. Equation (2.8) can be rearranged to determine β^2 as a function of γ^2, where $\beta = u/c$.

$$\beta^2 = \frac{\gamma^2 - 1}{\gamma^2} = \frac{(1.049)^2 - 1}{(1.049)^2} = 0.091 \tag{2.62}$$

The value of β is 0.30, and the correct speed, $u = \beta c$, is 0.90×10^8 m/s.

We determine the error in using the classical result by calculating the velocity using the nonrelativistic expression. The nonrelativistic expression is $K = \frac{1}{2}mu^2$, and the speed is given by

$$u = \sqrt{\frac{2(4.0 \times 10^{-15}\ \text{J})}{9.11 \times 10^{-31}\ \text{kg}}}$$
$$= 0.94 \times 10^8\ \text{m/s} \quad \text{(nonrelativistic)}$$

The (incorrect) classical speed is about 4% greater than the (correct) relativistic speed. Such an error is significant enough to be important in designing electronic equipment and in making test measurements. Relativistic calculations are particularly important for electrons, because they have such a small mass and are easily accelerated to speeds very close to c.

Total Energy and Rest Energy

We rewrite Equation (2.58) in the form

$$\gamma mc^2 = \frac{mc^2}{\sqrt{1 - u^2/c^2}} = K + mc^2 \tag{2.63}$$

The term mc^2 is called the **rest energy** and is denoted by E_0.

Rest energy

$$E_0 = mc^2 \tag{2.64}$$

This leaves the sum of the kinetic energy and rest energy to be interpreted as the **total energy** of the particle. The total energy is denoted by E and is given by

Total energy

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - u^2/c^2}} = \frac{E_0}{\sqrt{1 - u^2/c^2}} = K + E_0 \tag{2.65}$$

Equivalence of Mass and Energy

These last few equations suggest the equivalence of mass and energy, a concept attributed to Einstein. The result that energy $= mc^2$ is one of the most famous equations in physics. Even when a particle has no velocity, and thus no kinetic energy, we still believe that the particle has energy through its mass, $E_0 = mc^2$. Nuclear reactions are certain proof that mass and energy are equivalent. The concept of motion as being described by *kinetic energy* is preserved in relativistic dynamics, but a particle with no motion still has energy through its mass.

In order to establish the equivalence of mass and energy, we must modify two of the conservation laws that we learned in classical physics. Mass and energy are no longer two separately conserved quantities. We must combine them into one law of the **conservation of mass-energy**. We will see ample proof during the remainder of this book of the validity of this basic conservation law.

Conservation of mass-energy

Even though we often say "energy is turned into mass" or "mass is converted into energy" or "mass and energy are interchangeable," what we mean is that mass and energy are *equivalent*; this is important to understand. Mass is another form of energy, and we use the terms *mass-energy* and *energy* interchangeably. This is not the first time we have had to change our understanding of energy. In the late eighteenth century it became clear that heat was another form of energy, and the nineteenth-century experiments of James Joule showed that heat loss or gain was related to work.

Consider two blocks of wood, each of mass m and having kinetic energy K, moving toward each other as shown in Figure 2.33. A spring placed between them is compressed and locks in place as they collide. Let's examine the conservation of mass-energy. The energy before the collision is

$$\text{Mass-energy before:} \qquad E = 2mc^2 + 2K \tag{2.66}$$

and the energy after the collision is

$$\text{Mass-energy after:} \qquad E = Mc^2 \tag{2.67}$$

where M is the (rest) mass of the system. Because energy is conserved, we have $E = 2mc^2 + 2K = Mc^2$, and the new mass M is greater than the individual masses $2m$. The kinetic energy went into compressing the spring, so the spring has increased

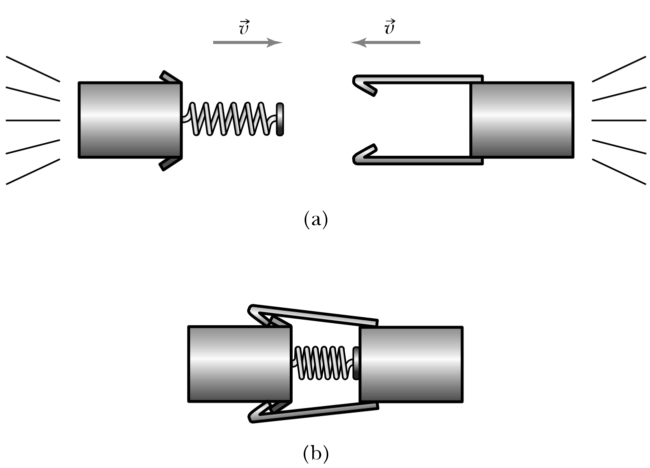


Figure 2.33 (a) Two blocks of wood, one with a spring attached and both having mass m, move with equal speeds v and kinetic energies K toward a head-on collision. (b) The two blocks collide, compressing the spring, which locks in place. The system now has increased mass, $M = 2m + 2K/c^2$, with the kinetic energy being converted into the potential energy of the spring.

potential energy. Kinetic energy has been converted into mass, the result being that the potential energy of the spring has caused the system to have more mass. We find the difference in mass ΔM by setting the previous two equations for energy equal and solving for $\Delta M = M - 2m$.

$$\Delta M = M - 2m = \frac{2K}{c^2} \tag{2.68}$$

Linear momentum is conserved in this head-on collision.

The fractional mass increase in this case is quite small and is given by $f_r = \Delta M/2m$. If we use Equation (2.68), we have

$$f_r = \frac{M - 2m}{2m} = \frac{2K/c^2}{2m} = \frac{K}{mc^2} \tag{2.69}$$

For typical masses and kinetic energies of blocks of wood, this fractional increase in mass is too small to measure. For example, if we have blocks of wood of mass 0.1 kg moving at 10 m/s, Equation (2.69) gives

$$f_r = \frac{\frac{1}{2}mv^2}{mc^2} = \frac{1}{2}\frac{v^2}{c^2} = \frac{1}{2}\frac{(10\text{ m/s})^2}{(3 \times 10^8\text{ m/s})^2} = 6 \times 10^{-16}$$

where we have used the nonrelativistic expression for kinetic energy because the speed is so low. This very small numerical result indicates that questions of mass increase are inappropriate for macroscopic objects such as blocks of wood and automobiles crashing into one another. Such small increases cannot now be measured, but in the next section, we will look at the collision of two high-energy protons, in which considerable energy is available to create additional mass. Mass-energy relations are essential in such reactions.

Relationship of Energy and Momentum

Physicists believe that linear momentum is a more fundamental concept than kinetic energy. There is no conservation of kinetic energy, whereas the conservation of linear momentum in isolated systems is inviolate as far as we know. A more fundamental result for the total energy in Equation (2.65) might include momentum rather than kinetic energy. Let's proceed to find a useful result. We begin with Equation (2.48) for the relativistic momentum written in magnitude form only.

$$p = \gamma mu = \frac{mu}{\sqrt{1 - u^2/c^2}}$$

We square this result, multiply by c^2, and rearrange the result.

$$\begin{aligned} p^2c^2 &= \gamma^2 m^2 u^2 c^2 \\ &= \gamma^2 m^2 c^4 \left(\frac{u^2}{c^2}\right) = \gamma^2 m^2 c^4 \beta^2 \end{aligned}$$

We use Equation (2.62) for β^2 and find

$$\begin{aligned} p^2c^2 &= \gamma^2 m^2 c^4 \left(1 - \frac{1}{\gamma^2}\right) \\ &= \gamma^2 m^2 c^4 - m^2 c^4 \end{aligned}$$

The first term on the right-hand side is just E^2, and the second term is E_0^2. The last equation becomes

$$p^2c^2 = E^2 - E_0^2$$

We rearrange this last equation to find the result we are seeking, a relation between energy and momentum.

$$\boxed{E^2 = p^2c^2 + E_0^2} \tag{2.70}$$

Momentum-energy relation

or

$$\boxed{E^2 = p^2c^2 + m^2c^4} \tag{2.71}$$

Equation (2.70) is a useful result to relate the total energy of a particle with its momentum. The quantities $(E^2 - p^2c^2)$ and m are invariant quantities. Note that when a particle's velocity is zero and it has no momentum, Equation (2.70) correctly gives E_0 as the particle's total energy.

Massless Particles

Equation (2.70) can also be used to determine the total energy for particles having zero mass. For example, Equation (2.70) predicts that the total energy of a photon is

$$E = pc \qquad \text{Photon} \tag{2.72}$$

The energy of a photon is completely due to its motion. It has no rest energy, because it has no mass.

We can show that the previous relativistic equations correctly predict that the speed of a photon must be the speed of light c. We use Equations (2.65) and (2.72) for the total energy of a photon and set the two equations equal.

$$E = \gamma mc^2 = pc$$

If we insert the value of the relativistic momentum from Equation (2.48), we have

$$\gamma mc^2 = \gamma muc$$

The fact that $u = c$ follows directly from this equation after careful consideration of letting $m \to 0$ and realizing that $\gamma \to \infty$.

$$u = c \qquad \text{Massless particle} \tag{2.73}$$

Massless particles must travel at the speed of light

CONCEPTUAL EXAMPLE 2.12

Tachyons are postulated particles that travel faster than the speed of light. (The word tachyon is derived from the Greek word *tachus*, which means "speedy.") They were first seriously proposed and investigated in the 1960s. Use what we have learned thus far in this chapter and discuss several properties that tachyons might have.

Solution Let's first examine Equation (2.65) for energy:

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - u^2/c^2}} \tag{2.65}$$

Because $u > c$, the energy must be imaginary if the mass is real, or conversely, if we insist that energy be real, we must have an imaginary mass! For purposes of discussion, we will henceforth assume that energy is real and tachyon mass is imaginary. Remember that ordinary matter must always travel at speed less than c, light must travel at the speed of light, and tachyons must always have speed greater than c. In order to slow down a tachyon, we must give it *more* energy, according to Equation (2.65). Note that the energy must become infinite if we want to slow down a tachyon to speed c. If the tachyon's energy is reduced, it speeds up!

Because tachyons travel faster than c, we have a problem with causality. Consider a tachyon leaving Earth at time $t = 0$ that arrives at a distant galaxy at time T. A spaceship traveling at speed less than c from Earth to the galaxy could conceivably find that the tachyon arrived at the galaxy before it left Earth!

It has been proposed that tachyons might be created in high-energy particle collisions or in cosmic rays. No confirming evidence has been found. Tachyons, if charged, could also be detected from *Cerenkov radiation*. When we refer to speed c, we always mean in a vacuum. When traveling in a medium, the speed must be less than c. When particles have speed greater than light travels in a medium, characteristic electromagnetic radiation is emitted. The effect of the blue glow in swimming pool nuclear reactors is due to this Cerenkov radiation.

2.13 Computations in Modern Physics

We were taught in introductory physics that the international system of units is preferable when doing calculations in science and engineering. This is generally true, but in modern physics we sometimes use other units that are more convenient for atomic and subatomic scales. In this section we introduce some of those units and demonstrate their practicality through several examples. Recall that the work done in accelerating a charge through a potential difference is given by $W = qV$. For a proton, with charge $e = 1.602 \times 10^{-19}$ C, accelerated across a potential difference of 1 V, the work done is

$$W = (1.602 \times 10^{-19})(1 \text{ V}) = 1.602 \times 10^{-19} \text{ J}$$

In modern physics calculations, the amount of charge being considered is almost always some multiple of the electron charge. Atoms and nuclei all have an exact multiple of the electron charge (or neutral). For example, some charges are proton $(+e)$, electron $(-e)$, neutron (0), pion $(0, \pm e)$, and a singly ionized carbon atom $(+e)$. The work done to accelerate the proton across a potential difference of 1 V could also be written as

$$W = (1\ e)(1 \text{ V}) = 1 \text{ eV}$$

Use eV for energy

where e stands for the electron charge. Thus eV, pronounced "electron volt," is also a unit of energy. It is related to the SI (*Système International*) unit joule by the two previous equations.

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J} \tag{2.74}$$

The eV unit is used more often in modern physics than the SI unit J. The term eV is often used with the SI prefixes where applicable. For example, in atomic and solid state physics, eV itself is mostly used, whereas in nuclear physics MeV (10^6 eV, *mega*-electron-volt) and GeV (10^9 eV, *giga*-electron-volt) are predominant, and in particle physics GeV and TeV (10^{12} eV, *tera*-electron-volt) are used. When we speak of a particle having a certain amount of energy, the common usage is to refer to the kinetic energy. A 6-GeV proton has a *kinetic* energy of 6 GeV, not a *total* energy of 6 GeV. Because the rest energy of a proton is about 1 GeV, this proton would have a total energy of about 7 GeV.

Like the SI unit for energy, the SI unit for mass, kilogram, is a very large unit of mass in modern physics calculations. For example, the mass of a proton is only 1.6726×10^{-27} kg. Two other mass units are commonly used in modern physics. First, the rest energy E_0 is given by Equation (2.64) as mc^2. The rest energy of the proton is given by

$$\begin{aligned} E_0(\text{proton}) &= (1.67 \times 10^{-27}\ \text{kg})(3 \times 10^8\ \text{m/s})^2 = 1.50 \times 10^{-10}\ \text{J} \\ &= 1.50 \times 10^{-10}\ \text{J}\,\frac{1\ \text{eV}}{1.602 \times 10^{-19}\ \text{J}} = 9.38 \times 10^8\ \text{eV} \end{aligned}$$

The rest energies of the elementary particles are usually quoted in MeV or GeV. (To five significant figures, the rest energy of the proton is 938.27 MeV.) Because $E_0 = mc^2$, the mass is often quoted in units of MeV/c^2; for example, the mass of the proton is given by 938.27 MeV/c^2. We will find that the mass unit of MeV/c^2 is quite useful. The masses of several elementary particles are given on the inside of the front book cover. Although we will not do so, research physicists often quote the mass in units of just eV (or MeV, etc.).

Use MeV/c^2 for mass

The other commonly used mass unit is the (unified) **atomic mass unit**. It is based on the definition that the mass of the neutral carbon-12 (^{12}C) atom is exactly 12 u, where u is one atomic mass unit.* We obtain the conversion between kilogram and atomic mass units u by comparing the mass of one ^{12}C atom.

Atomic mass unit

$$\begin{aligned} \text{Mass}(^{12}\text{C atom}) &= \frac{12\ \text{g/mol}}{6.02 \times 10^{23}\ \text{atoms/mol}} \\ &= 1.99 \times 10^{-23}\ \text{g/atom} \qquad (2.75) \\ \text{Mass}(^{12}\text{C atom}) &= 1.99 \times 10^{-26}\ \text{kg} = 12\ \text{u/atom} \end{aligned}$$

Therefore, the conversion is (when properly done to 6 significant figures)

$$1\ \text{u} = 1.66054 \times 10^{-27}\ \text{kg} \qquad (2.76)$$

$$1\ \text{u} = 931.494\ \text{MeV}/c^2 \qquad (2.77)$$

We have added the conversion from atomic mass units to MeV/c^2 for completeness.

From Equations (2.70) and (2.72) we see that a convenient unit of momentum is energy divided by the speed of light, or eV/c. We will use the unit eV/c for momentum when appropriate. Remember also that we often quote β ($= v/c$) for velocity, so that c itself is an appropriate unit of velocity.

*To avoid confusion between velocity and atomic mass unit, we will henceforth use v for velocity when the possibility exists for confusing the mass unit u with the velocity variable u.

EXAMPLE 2.13

A 2.00-GeV proton hits another 2.00-GeV proton in a head-on collision. (a) Calculate v, β, p, K, and E for each of the initial protons. (b) What happens to the kinetic energy?

Strategy (a) By the convention just discussed, a 2.00-GeV proton has a kinetic energy of 2.00 GeV. We use Equation (2.65) to determine the total energy and Equation (2.70) to determine momentum if we know the total energy. To determine β and v, it helps to first determine the relativistic factor γ, which we can use Equation (2.65) to find. Then we use Equation (2.62) to find β and v. These are all typical calculations that are performed when doing relativistic computations.

Solution (a) We use $K = 2.00$ GeV and the proton rest energy, 938 MeV, to find the total energy from Equation (2.65),

$$E = K + E_0 = 2.00 \text{ GeV} + 938 \text{ MeV} = 2.938 \text{ GeV}$$

The momentum is determined from Equation (2.70).

$$p^2c^2 = E^2 - E_0^2 = (2.938 \text{ GeV})^2 - (0.938 \text{ GeV})^2 = 7.75 \text{ GeV}^2$$

The momentum is calculated to be

$$p = \sqrt{7.75(\text{GeV}/c)^2} = 2.78 \text{ GeV}/c$$

Notice how naturally the unit of GeV/c arises in our calculation.

In order to find β we first find the relativistic factor γ. There are several ways to determine γ; one is to compare the rest energy with the total energy. From Equation (2.65) we have

$$E = \gamma E_0 = \frac{E_0}{\sqrt{1 - u^2/c^2}}$$

$$\gamma = \frac{E}{E_0} = \frac{2.938 \text{ GeV}}{0.938 \text{ GeV}} = 3.13$$

We use Equation (2.62) to determine β.

$$\beta = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} = \sqrt{\frac{3.13^2 - 1}{3.13^2}} = 0.948$$

The speed of a 2.00-GeV proton is $0.95c$ or 2.8×10^8 m/s.

(b) When the two protons collide head-on, the situation is similar to the case when the two blocks of wood collided head-on with one important exception. The time for the two protons to interact is less than 10^{-20} s. If the two protons did momentarily stop at rest, then the two-proton system would have its mass increased by an amount given by Equation (2.68), $2K/c^2$ or 4.00 GeV/c^2. The result would be a highly excited system. In fact, the collision between the protons happens very quickly, and there are several possible outcomes. The two protons may either remain or disappear, and new additional particles may be created. Two of the possibilities are

$$p + p \rightarrow p + p + p + \overline{p} \tag{2.78}$$

$$p + p \rightarrow \pi^+ + d \tag{2.79}$$

where the symbols are p (proton), $\overline{p}$ (antiproton), π (pion), and d (deuteron). We will learn more about the possibilities later when we study nuclear and particle physics. Whatever happens must be consistent with the conservation laws of charge, energy, and momentum, as well as with other conservation laws to be learned. Such experiments are routinely done in particle physics. In the analysis of these experiments, the equivalence of mass and energy is taken for granted.

Binding Energy

The equivalence of mass and energy becomes apparent when we study the binding energy of atoms and nuclei that are formed from individual particles. For example, the hydrogen atom is formed from a proton and electron bound together by the electrical (Coulomb) force. A deuteron is a proton and neutron bound together by the nuclear force. The potential energy associated with the force keeping the system together is called the **binding energy** E_B. The binding energy is the work required to pull the particles out of the bound system into separate, free particles at rest. The conservation of energy is written as

$$M_{\text{bound system}}c^2 + E_B = \sum_i m_i c^2 \tag{2.80}$$

where the m_i values are the masses of the free particles. The binding energy is *the difference between the rest energy of the individual particles and the rest energy of the combined, bound system.*

$$E_B = \sum_i m_i c^2 - M_{\text{bound system}}c^2 \tag{2.81}$$

For the case of two final particles having masses m_1 and m_2, we have

$$E_B = (m_1 + m_2 - M_{\text{bound system}})c^2 = \Delta M c^2 \tag{2.82}$$

where ΔM is the difference between the final and initial masses.

When two particles (for example, a proton and neutron) are bound together to form a composite (like a deuteron), part of the rest energy of the individual particles is lost, resulting in the binding energy of the system. The rest energy of the combined system must be reduced by this amount. The deuteron is a good example. The rest energies of the particles are

$$\text{Proton} \quad E_0 = 1.007276c^2 \text{ u} = 938.27 \text{ MeV}$$

$$\text{Neutron} \quad E_0 = 1.008665c^2 \text{ u} = 939.57 \text{ MeV}$$

$$\text{Deuteron} \quad E_0 = 2.01355c^2 \text{ u} = 1875.61 \text{ MeV}$$

The binding energy E_B is determined from Equation (2.81) to be

$$E_B(\text{deuteron}) = 938.27 \text{ MeV} + 939.57 \text{ MeV} - 1875.61 \text{ MeV} = 2.23 \text{ MeV}$$

CONCEPTUAL EXAMPLE 2.14

Why can we ignore the 13.6 eV binding energy of the proton and electron when making mass determinations for nuclei, but not the binding energy of a proton and neutron?

Solution The binding energy of the proton and electron in the hydrogen atom is only 13.6 eV, which is so much smaller than the 1-GeV rest energy of a neutron and proton that it can be neglected when making mass determinations. The deuteron binding energy of 2.23 MeV, however, represents a much larger fraction of the rest energies and is extremely important. The binding energies of heavy nuclei such as uranium can be more than 1000 MeV, and even that much energy is not large enough to keep uranium from decaying to lighter nuclei. The Coulomb repulsion between the many protons in heavy nuclei is mostly responsible for their instability. Nuclear stability is addressed in Chapter 12.

EXAMPLE 2.15

What is the minimum kinetic energy the protons must have in the head-on collision of Equation (2.79), $p + p \rightarrow \pi^+ + d$, in order to produce the positively charged pion and deuteron? The mass of π^+ is 139.6 MeV/c^2.

Strategy For the minimum kinetic energy K required, we need just enough energy to produce the rest energies of the final particles. We let the final kinetic energies of the pion and deuteron be zero. Because the collision is head-on, the momentum will be zero before and after the collision, so the pion and deuteron will truly be at rest with no kinetic energy. We use the conservation of energy to determine the kinetic energy.

Solution Conservation of energy requires

$$m_p c^2 + K + m_p c^2 + K = m_d c^2 + m_{\pi^+} c^2$$

The rest energies of the proton and deuteron were given in this section, so we solve the previous equation for the kinetic energy.

$$\begin{aligned} K &= \tfrac{1}{2}(m_d c^2 + m_{\pi^+} c^2 - 2m_p c^2) \\ &= \tfrac{1}{2}[1875.6 \text{ MeV} + 139.6 \text{ MeV} - 2(938.3 \text{ MeV})] \\ &= 69 \text{ MeV} \end{aligned}$$

Nuclear experiments like this are normally done with fixed targets, not head-on collisions, and much more energy than 69 MeV is required, because linear momentum must also be conserved.

EXAMPLE 2.16

The atomic mass of the ^{4}He atom is 4.002603 u. Find the binding energy of the ^{4}He nucleus.

Strategy This is a straightforward application of Equation (2.81), and we will need to determine the atomic masses.

Solution Equation (2.81) gives

$$E_B(^4\text{He}) = 2m_pc^2 + 2m_nc^2 - M_{^4\text{He}}\,c^2$$

Later we will learn to deal with atomic masses in cases like this, but for now we will subtract the two electron masses from the atomic mass of ^{4}He to obtain the mass of the ^{4}He nucleus. The mass of the electron is given on the inside of the front cover, along with the masses of the proton and neutron.

$$M_{^4\text{He}}(\text{nucleus}) = 4.002603\text{ u} - 2(0.000549\text{ u})$$
$$= 4.001505\text{ u}$$

We determine the binding energy of the ^{4}He nucleus to be

$$E_B(^4\text{He}) = [2(1.007276\,\text{u}) + 2(1.008665\,\text{u}) - 4.001505\,\text{u}]c^2$$
$$= 0.0304\,c^2\text{u}$$

$$E_B(^4\text{He}) = (0.0304\,c^2\text{u})\frac{931.5\text{ MeV}}{c^2\text{ u}} = 28.3\text{ MeV}$$

The binding energy of the ^{4}He nucleus is large, almost 1% of its rest energy.

EXAMPLE 2.17

The molecular binding energy is called the *dissociation energy*. It is the energy required to separate the atoms in a molecule. The dissociation energy of the NaCl molecule is 4.24 eV. Determine the fractional mass increase of the Na and Cl atoms when they are not bound together in NaCl. What is the mass increase for a mole of NaCl?

Strategy Binding energy is a concept that applies to various kinds of bound objects, including a nucleus, an atom, a molecule, and others. We can use Equation (2.82) in the present case to find ΔM, the change in mass, in terms of the binding energy E_B/c^2. We then divide ΔM by M to find the fractional mass increase.

Solution From Equation (2.82) we have $\Delta M = E_B/c^2$ (the binding energy divided by c^2) as the mass difference between the molecule and separate atoms. The mass of NaCl is 58.44 u. The fractional mass increase is

$$f_r = \frac{\Delta M}{M} = \frac{E_B/c^2}{M} = \frac{4.24\text{ eV}/c^2}{58.44\text{ u}}\frac{c^2\text{ u}}{931\text{ MeV}}\frac{1\text{ MeV}}{10^6\text{ eV}}$$
$$= 7.8 \times 10^{-11}$$

One mole of NaCl has a mass of 58.44 g, so the mass decrease for a mole of NaCl is $f_r \times 58.44$ g or only 4.6×10^{-9} g. Such small masses cannot be directly measured, which is why nonconservation of mass was not observed for chemical reactions—the changes are too small.

EXAMPLE 2.18

A positively charged sigma particle (symbol Σ^+) produced in a particle physics experiment decays very quickly into a neutron and positively charged pion before either its energy or momentum can be measured. The neutron and pion are observed to move in the same direction as the Σ^+ was originally moving, with momenta of 4702 MeV/c and 169 MeV/c, respectively. What was the kinetic energy of the Σ^+ and its mass?

Strategy The decay reaction is

$$\Sigma^+ \rightarrow n + \pi^+$$

where n is a neutron. Obviously the Σ^+ has more mass than the sum of the masses of n and π^+, or the decay would not occur. We have to conserve both momentum and energy for this reaction. We use Equation (2.70) to find the total energy of the neutron and positively charged pion, but in order to determine the rest energy of Σ^+, we need to know the momentum. We can determine the Σ^+ momentum from the conservation of momentum.

Solution The rest energies of n and π^+ are 940 MeV and 140 MeV, respectively. The total energies of E_n and E_{π^-} are, from $E = \sqrt{p^2c^2 + E_0^2}$,

$$E_n = \sqrt{(4702\text{ MeV})^2 + (940\text{ MeV})^2} = 4795\text{ MeV}$$

$$E_{\pi^+} = \sqrt{(169\text{ MeV})^2 + (140\text{ MeV})^2} = 219\text{ MeV}$$

The sum of these energies gives the total energy of the reaction, 4795 MeV + 219 MeV = 5014 MeV, both before and after the decay of Σ^+. Because all the momenta are along the same direction, we must have

$$p_{\Sigma^+} = p_n + p_{\pi^+} = 4702\text{ MeV}/c + 169\text{ MeV}/c$$
$$= 4871\text{ MeV}/c$$

This must be the momentum of the Σ^+ before decaying, so now we can find the rest energy of Σ^+ from Equation (2.70).

$$E_0^2(\Sigma^+) = E^2 - p^2c^2 = (5014\text{ MeV})^2 - (4871\text{ MeV})^2$$
$$= (1189\text{ MeV})^2$$

The rest energy of the Σ^+ is 1189 MeV, and its mass is 1189 MeV/c^2.

We find the kinetic energy of Σ^+ from Equation (2.65).

$$K = E - E_0 = 5014\text{ MeV} - 1189\text{ MeV} = 3825\text{ MeV}$$

2.14 Electromagnetism and Relativity

We have been concerned mostly with the kinematical and dynamical aspects of the special theory of relativity strictly from the mechanics aspects. However, recall that Einstein first approached relativity through electricity and magnetism. He was convinced that Maxwell's equations were invariant (have the same form) in all inertial frames. Einstein wrote in 1952,

> *What led me more or less directly to the special theory of relativity was the conviction that the electromagnetic force acting on a body in motion in a magnetic field was nothing else but an electric field.*

Einstein's conviction about electromagnetism

Einstein was convinced that magnetic fields appeared as electric fields observed in another inertial frame. That conclusion is the key to electromagnetism and relativity.

Maxwell's equations and the Lorentz force law are invariant in different inertial frames. In fact, with the proper Lorentz transformations of the electric and magnetic fields (from relativity theory) together with Coulomb's law (force between stationary charges), Maxwell's equations can be obtained. We will not attempt that fairly difficult mathematical task here, nor do we intend to obtain the Lorentz transformation of the electric and magnetic fields. These subjects are studied in more advanced physics classes. However, we will show qualitatively that the magnetic force that one observer sees is simply an electric force according to an observer in another inertial frame. The electric field arises from charges, whereas the magnetic field arises from *moving* charges.

Magnetism and electricity are relative

Electricity and magnetism were well understood in the late 1800s. Maxwell predicted that all electromagnetic waves travel at the speed of light, and he combined electricity, magnetism, and optics into one successful theory. This classical theory has withstood the onslaught of time and experimental tests.* There were, however, some troubling aspects of the theory when it was observed from different Galilean frames of reference. In 1895 H. A. Lorentz "patched up" the diffi-

*The meshing of electricity and magnetism together with quantum mechanics, called the *theory of quantum electrodynamics* (QED), is one of the most successful theories in physics.

culties with the Galilean transformation by developing a new transformation that now bears his name, the Lorentz transformation. However, Lorentz did not understand the full implication of what he had done. It was left to Einstein, who in 1905 published a paper titled "On the Electrodynamics of Moving Bodies," to fully merge relativity and electromagnetism. Einstein did not even mention the famous Michelson-Morley experiment in this classic 1905 paper, which we take as the origin of the special theory of relativity, and the Michelson-Morley experiment apparently played little role in his thinking. Einstein's belief that *Maxwell's equations describe electromagnetism in any inertial frame* was the key that led Einstein to the Lorentz transformations. Maxwell's assertion that all electromagnetic waves travel at the speed of light and Einstein's postulate that the speed of light is invariant in all inertial frames seem intimately connected.

We now proceed to discuss qualitatively the relative aspects of electric and magnetic fields and their forces. Consider a positive test charge q_0 moving to the right with speed v outside a neutral, conducting wire as shown in Figure 2.34a in the frame of the inertial system K, where the positive charges are at rest and the negative electrons in the wire have speed v to the right. The conducting wire is long and has the same number of positive ions and conducting electrons. For simplicity, we have taken the electrons and the positive charges to have the same speed, but the argument can be generalized.

What is the force on the positive test charge q_0 outside the wire? The total force is given by the Lorentz force

$$\vec{F} = q_0(\vec{E} + \vec{v} \times \vec{B}) \tag{2.83}$$

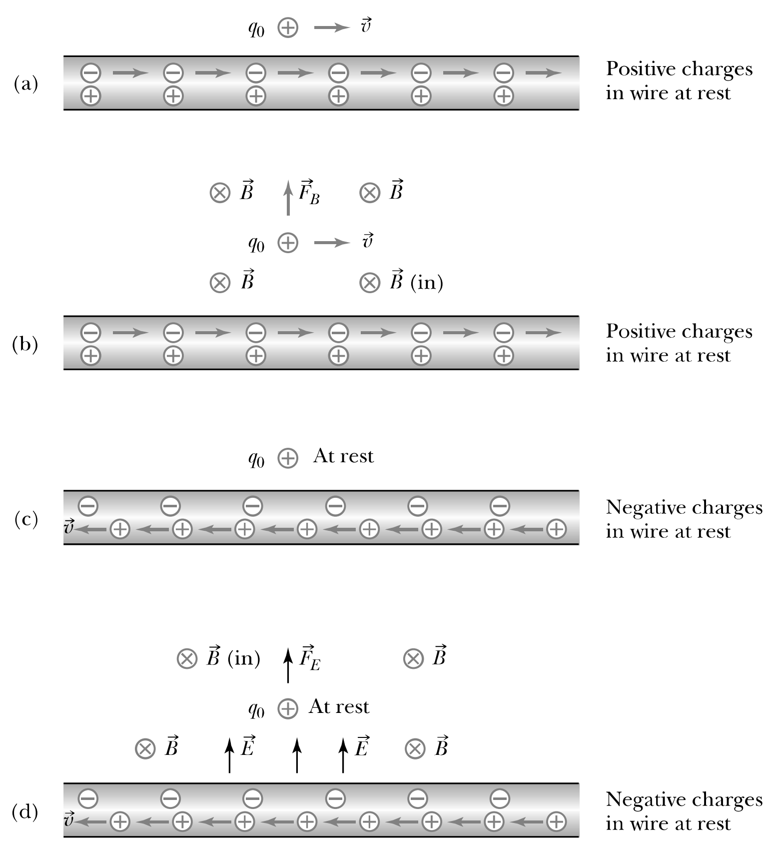


Figure 2.34 (a) A positive charge q_0 is placed outside a neutral, conducting wire. The figure is shown in the system where the positive charges in the wire are at rest. Note that the charge q_0 has the same velocity as the electrons. (b) The moving electrons produce a magnetic field, which causes a force $\vec{F}_B$ on q_0. (c) This is similar to (a), but in this system the electrons are at rest. (d) Now there is an abundance of positive charges due to length contraction, and the resulting electric field repels q_0. There is also a magnetic field, but this causes no force on q_0, which is at rest in this system.

and can be due to an electric field, a magnetic field, or both. Because the total charge inside the wire is zero, the electric force on the test charge q_0 in Figure 2.34a is also zero. But we learned in introductory physics that the moving electrons in the wire (current) produce a magnetic field $\vec{B}$ at the position of q_0 that is into the page (Figure 2.34b). The moving charge q_0 *will be repelled upward by the magnetic force* ($q_0\vec{v} \times \vec{B}$) due to the magnetic field of the wire.

Let's now see what happens in a different inertial frame K′ moving at speed v to the right with the test charge (see Figure 2.34c). Both the test charge q_0 and the negative charges in the conducting wire are at rest in system K′. In this system an observer at the test charge q_0 observes the same density of negative ions in the wire as before. However, in system K′ the positive ions are now moving to the left with speed v. Due to length contraction, the positive ions will appear to be closer together to a stationary observer in K′. Because the positive charges appear to be closer together, there is a higher density of positive charges than of negative charges in the conducting wire. The result is an electric field as shown in Figure 2.34d. The test charge q_0 will now be *repelled* in the presence of the electric field. What about the magnetic field now? The moving charges in Figure 2.34c also produce a magnetic field that is into the page, but this time the charge q_0 is at rest with respect to the magnetic field, so charge q_0 feels no magnetic force.

What appears as a magnetic force in one inertial frame (Figure 2.34b) appears as an electric force in another (Figure 2.34d). Electric and magnetic fields are *relative* to the coordinate system in which they are observed. The Lorentz contraction of the moving charges accounts for the difference. This example can be extended to two conducting wires with electrons moving, and a similar result will be obtained (see Problem 86). It is this experiment, on the force between two parallel, conducting wires, in which current is defined. Because charge is defined using current, the experiment is also the basis of the definition of the electric charge.

We have come full circle in our discussion of the special theory of relativity. The laws of electromagnetism represented by Maxwell's equations have a special place in physics. The equations themselves are invariant in different inertial systems; only the interpretations as electric and magnetic fields are relative.

Summary

Efforts by Michelson and Morley proved in 1887 that either the elusive ether does not exist or there must be significant problems with our understanding of nature.

Albert Einstein solved the problem in 1905 by applying two postulates:

1. The principle of relativity: The laws of physics are the same in all inertial systems.
2. The constancy of the speed of light: Observers in all inertial systems measure the same value for the speed of light in vacuum.

Einstein's two postulates are used to derive the Lorentz transformation relating the space and time coordinates of events viewed from different inertial systems. If system K′ is moving at speed v along the $+x$ axis with respect to system K, the two sets of coordinates are related by

$$\begin{aligned} x' &= \frac{x - vt}{\sqrt{1-\beta^2}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t - (vx/c^2)}{\sqrt{1-\beta^2}} \end{aligned} \tag{2.17}$$

The inverse transformation is obtained by switching the primed and unprimed quantities and changing v to $-v$.

The time interval between two events occurring at the same position in a system as measured by a clock at rest is called the proper time T_0. The time interval T' between the same two events measured by a moving observer is related to the proper time T_0 by the time dilation effect.

$$T' = \frac{T_0}{\sqrt{1 - v^2/c^2}} \tag{2.19}$$

We say that moving clocks run slow, because the shortest time is always measured on clocks at rest.

The length of an object measured by an observer at rest relative to the object is called the proper length L_0. The length of the same object measured by an observer who sees the object moving at speed v is L, where

$$L = L_0\sqrt{1 - v^2/c^2} \tag{2.21}$$

This effect is known as length or space contraction, because moving objects are contracted in the direction of their motion.

If u and u' are the velocities of an object measured in systems K and K′, respectively, and v is the relative velocity between K and K′; the relativistic addition of velocities (Lorentz velocity transformation) is

$$\begin{aligned} u_x &= \frac{dx}{dt} = \frac{u'_x + v}{1 + (v/c^2)u'_x} \\ u_y &= \frac{u'_y}{\gamma[1 + (v/c^2)u'_x]} \\ u_z &= \frac{u'_z}{\gamma[1 + (v/c^2)u'_x]} \end{aligned} \tag{2.23}$$

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{2.8}$$

The Lorentz transformation has been tested for a hundred years, and no violation has yet been detected. Nevertheless, physicists continue to test its validity, because it is one of the most important results in science.

Spacetime diagrams are useful to represent events geometrically. Time may be considered to be a fourth dimension for some purposes. The spacetime interval for two events defined by $\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\,\Delta t^2$ is an invariant between inertial systems.

The relativistic Doppler effect for light frequency f is given by

$$f = \frac{\sqrt{1 + \beta}}{\sqrt{1 - \beta}} f_0 \tag{2.34}$$

where β is positive when source and receiver are approaching one another and negative when they are receding.

The classical form for linear momentum is replaced by the special relativity form:

$$\vec{p} = \gamma m\vec{u} = \frac{m\vec{u}}{\sqrt{1 - u^2/c^2}} \tag{2.48}$$

The relativistic kinetic energy is given by

$$K = \gamma mc^2 - mc^2 = mc^2\left(\frac{1}{\sqrt{1 - u^2/c^2}} - 1\right) \tag{2.58}$$

The *total energy* E is given by

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - u^2/c^2}} = \frac{E_0}{\sqrt{1 - u^2/c^2}} = K + E_0 \tag{2.65}$$

where $E_0 = mc^2$. This equation denotes the equivalence of mass and energy. The laws of the conservation of mass and of energy are combined into one conservation law: the conservation of mass-energy.

Energy and momentum are related by

$$E^2 = p^2c^2 + E_0^2 \tag{2.70}$$

In the case of massless particles (for example, the photon), $E_0 = 0$, so $E = pc$. Massless particles must travel at the speed of light.

The electron volt, denoted by eV, is equal to 1.602×10^{-19} J. The unified atomic mass unit u is based on the mass of the ^{12}C atom.

$$1\text{ u} = 1.66054 \times 10^{-27}\text{ kg} = 931.494\text{ MeV}/c^2 \tag{2.76, 2.77}$$

Momentum is often quoted in units of eV/c, and the velocity is often given in terms of β $(= v/c)$.

The difference between the rest energy of individual particles and the rest energy of the combined, bound system is called the binding energy.

Maxwell's equations are invariant under transformations between any inertial reference frames. What appears as electric and magnetic fields is relative to the reference frame of the observer.

Questions

1. Michelson used the motion of the Earth around the sun to try to determine the effects of the ether. Can you think of a more convenient experiment with a higher speed that Michelson might have used in the 1880s? What about today?
2. If you wanted to set out today to find the effects of the ether, what experimental apparatus would you want to use? Would a laser be included? Why?
3. For what reasons would Michelson and Morley repeat their experiment on top of a mountain? Why would they perform the experiment in summer and winter?
4. Does the fact that Maxwell's equations do not need to be modified because of the special theory of relativity, whereas Newton's laws of motion do, mean that Maxwell's work is somehow greater or more significant than Newton's? Explain.
5. The special theory of relativity has what effect on measurements done today? (a) None whatsoever, because any correction would be negligible. (b) We need to consider the effects of relativity when objects move close to the speed of light. (c) We should always make a correction for relativity because Newton's laws are basically wrong. (d) It doesn't matter, because we can't make measurements where relativity would matter.
6. Why did it take so long to discover the theory of relativity? Why didn't Newton figure it out?
7. Can you think of a way you can make yourself older than those born on your same birthday?
8. Will metersticks manufactured on Earth work correctly on spaceships moving at high speed? Explain.
9. Devise a system for you and three colleagues, at rest with you, to synchronize your clocks if your clocks are too large to move and are separated by hundreds of miles.
10. In the experiment to verify time dilation by flying the cesium clocks around the Earth, what is the order of the speed of the four clocks in a system fixed at the center of the Earth, but not rotating?
11. Can you think of an experiment to verify length contraction directly? Explain.
12. Would it be easier to perform the muon decay experiment in the space station orbiting above Earth and then compare with the number of muons on Earth? Explain.
13. On a spacetime diagram, can events above $t = 0$ but not in the shaded area in Figure 2.25 affect the future? Explain.
14. Why don't we also include the spatial coordinate z when drawing the light cone?
15. What would be a suitable name for events connected by $\Delta s^2 = 0$?
16. Is the relativistic Doppler effect valid only for light waves? Can you think of another situation in which it might be valid?
17. In Figure 2.22, why can a real worldline not have a slope less than one?
18. Explain how in the twin paradox, we might arrange to compare clocks at the beginning and end of Mary's journey and not have to worry about acceleration effects.
19. In each of the following pairs, which is the more massive: a relaxed or compressed spring, a charged or uncharged capacitor, or a piston-cylinder when closed or open?
20. In the fission of ^{235}U, the masses of the final products are less than the mass of ^{235}U. Does this make sense? What happens to the mass?
21. In the fusion of deuterium and tritium nuclei to produce a thermonuclear reaction, where does the kinetic energy that is produced come from?
22. Mary, the astronaut, wants to travel to the star system Alpha Centauri, which is 4.3 lightyears away. She wants to leave on her 30th birthday, travel to Alpha Centauri but not stop, and return in time for her wedding to Vladimir on her 35th birthday. What is most likely to happen? (a) Vladimir is a lucky man, because he will marry Mary after she completes her journey. (b) Mary will have to hustle to get in her wedding gown, and the wedding is likely to be watched by billions of people. (c) It is a certainty that Mary will not reach Alpha Centauri if she wants to marry Vladimir as scheduled. (d) Mary does reach Alpha Centauri before her 35th birthday and sends a radio message to Vladimir from Alpha Centauri that she will be back on time. Vladimir is relieved to receive the message before the wedding date.
23. A salesman driving a very fast car was arrested for driving through a traffic light while it was red, according to a policeman parked near the traffic light. The salesman said that the light was actually green to him, because it was Doppler shifted. Is he likely to be found innocent? Explain.

Problems

Note: The more challenging problems have their problem numbers shaded by a blue box.

2.1 The Need for Ether

1. Show that the form of Newton's second law is invariant under the Galilean transformation.
2. Show that the definition of linear momentum, $p = mv$, has the same form $p' = mv'$ under a Galilean transformation.

2.2 The Michelson-Morley Experiment

3. Show that the equation for t_2 in Section 2.2 expresses the time required for the light to travel to the mirror D and back in Figure 2.2. In this case the light is traveling perpendicular to the supposed direction of the ether. In what direction must the light travel to be reflected by the mirror if the light must pass through the ether?
4. A swimmer wants to swim straight across a river with current flowing at a speed of $v_1 = 0.350$ m/s. If the swimmer swims in still water with speed $v_2 = 1.25$ m/s, at what angle should the swimmer point upstream from the shore, and at what speed will the swimmer swim across the river?
5. Show that the time difference $\Delta t'$ given by Equation (2.4) is correct when the Michelson interferometer is rotated by 90°.
6. In the 1887 experiment by Michelson and Morley, the length of each arm was 11 m. The experimental limit for the fringe shift was 0.005 fringes. If sodium light was used with the interferometer ($\lambda = 589$ nm), what upper limit did the null experiment place on the speed of the Earth through the expected ether?
7. Show that if length is contracted by the factor $\sqrt{1-v^2/c^2}$ in the direction of motion, then the result in Equation (2.3) will have the factor needed to make $\Delta t = 0$ as needed by Michelson and Morley.

2.3 Einstein's Postulates

8. Explain why Einstein argued that the constancy of the speed of light (postulate 2) actually follows from the principle of relativity (postulate 1).
9. Prove that the constancy of the speed of light (postulate 2) is inconsistent with the Galilean transformation.

2.4 The Lorentz Transformation

10. Use the spherical wavefronts of Equations (2.9) to derive the Lorentz transformation given in Equations (2.17). Supply all the steps.
11. Show that both Equations (2.17) and (2.18) reduce to the Galilean transformation when $v \ll c$.
12. Determine the ratio $\beta = v/c$ for the following: (a) A car traveling 95 km/h. (b) A commercial jet airliner traveling 240 m/s. (c) A supersonic airplane traveling at Mach 2.3 (Mach number = v/v_{sound}). (d) The space station, traveling 27,000 km/h. (e) An electron traveling 25 cm in 2 ns. (f) A proton traveling across a nucleus (10^{-14} m) in 0.35×10^{-22} s.
13. Two events occur in an inertial system K as follows:

    Event 1: $x_1 = a$, $t_1 = 2a/c$, $y_1 = 0$, $z_1 = 0$
    Event 2: $x_2 = 2a$, $t_2 = 3a/2c$, $y_2 = 0$, $z_2 = 0$

    In what frame K′ will these events appear to occur at the same time? Describe the motion of system K′.
14. Is there a frame K′ in which the two events described in Problem 13 occur at the same place? Explain.
15. Find the relativistic factor γ for each of the parts of Problem 12.
16. An event occurs in system K′ at $x' = 2$ m, $y' = 3.5$ m, $z' = 3.5$ m, and $t' = 0$. System K′ and K have their axes coincident at $t = t' = 0$, and system K′ travels along the x axis of system K with a speed $0.8c$. What are the coordinates of the event in system K?
17. A light signal is sent from the origin of a system K at $t = 0$ to the point $x = 3$ m, $y = 5$ m, $z = 10$ m. (a) At what time t is the signal received? (b) Find (x', y', z', t') for the receipt of the signal in a frame K′ that is moving along the x axis of K at a speed of $0.8c$. (c) From your results in (b) verify that the light traveled with a speed c as measured in the K′ frame.

2.5 Time Dilation and Length Contraction

18. Show that the experiment depicted in Figure 2.11 and discussed in the text leads directly to the derivation of length contraction.
19. A rocket ship carrying passengers blasts off to go from New York to Los Angeles, a distance of about 5000 km. (a) How fast must the rocket ship go to have its own length shortened by 1%? (b) Ignore effects of general relativity and determine how much time the rocket ship's clock and the ground-based clocks differ when the rocket ship arrives in Los Angeles.
20. Astronomers discover a planet orbiting around a star similar to our sun that is 20 lightyears away. How fast must a rocket ship go if the round trip is to take no longer than 40 years in time for the astronauts aboard? How much time will the trip take as measured on Earth?
21. Particle physicists use particle track detectors to determine the lifetime of short-lived particles. A muon has a mean lifetime of 2.2 μs and makes a track 9.5 cm long before decaying into an electron and two neutrinos. What was the speed of the muon?

22. The Apollo astronauts returned from the moon under the Earth's gravitational force and reached speeds of almost 25,000 mi/h with respect to Earth. Assuming (incorrectly) they had this speed for the entire trip from the moon to Earth, what was the time difference for the trip between their clocks and clocks on Earth?
23. A clock in a spaceship is observed to run at a speed of only 3/5 that of a similar clock at rest on Earth. How fast is the spaceship moving?
24. A spaceship of length 40 m at rest is observed to be 20 m long when in motion. How fast is it moving?
25. The Concorde traveled 8000 km between two places in North America and Europe at an average speed of 375 m/s. What is the total difference in time between two similar atomic clocks, one on the airplane and one at rest on Earth during a one-way trip? Consider only time dilation and ignore other effects such as Earth's rotation.
26. A mechanism on Earth used to shoot down geosynchronous satellites that house laser-based weapons is finally perfected and propels golf balls at $0.94c$. (Geosynchronous satellites are placed 3.58×10^4 km above the surface of the Earth.) (a) What is the distance from the Earth to the satellite, as measured by a detector placed inside the golf ball? (b) How much time will it take the golf ball to make the journey to the satellite in the Earth's frame? How much time will it take in the golf ball's frame?
27. Two events occur in an inertial system K at the same time but 4 km apart. What is the time difference measured in a system K′ moving parallel to these two events when the distance separation of the events is measured to be 5 km in K′?
28. Imagine that in another universe the speed of light is only 100 m/s. (a) A person traveling along an interstate highway at 120 km/h ages at what fraction of the rate of a person at rest? (b) This traveler passes by a meterstick at rest on the highway. How long does the meterstick appear?
29. In another universe where the speed of light is only 100 m/s, an airplane that is 40 m long at rest and flies at 300 km/h will appear to be how long to an observer at rest?
30. Two systems K and K′ synchronize their clocks at $t = t' = 0$ when their origins are aligned as system K′ passes by system K along the x axis at relative speed $0.8c$. At time $t = 3$ ns, Frank (in system K) shoots a proton gun having proton speeds of $0.98c$ along his x axis. The protons leave the gun at $x = 1$ m and arrive at a target 120 m away. Determine the event coordinates (x, t) of the gun firing and of the protons arriving as measured by observers in both systems K and K′.

2.6 Addition of Velocities

31. A spaceship is moving at a speed of $0.84c$ away from an observer at rest. A boy in the spaceship shoots a proton gun with protons having a speed of $0.62c$. What is the speed of the protons measured by the observer at rest when the gun is shot (a) away from the observer and (b) toward the observer?
32. A proton and an antiproton are moving toward each other in a head-on collision. If each has a speed of $0.8c$ with respect to the collision point, how fast are they moving with respect to each other?
33. Imagine the speed of light in another universe to be only 100 m/s. Two cars are traveling along an interstate highway in opposite directions. Person 1 is traveling 110 km/h, and person 2 is traveling 140 km/h. How fast does person 1 measure person 2 to be traveling? How fast does person 2 measure person 1 to be traveling?
34. In the Fizeau experiment described in Example 2.5, suppose that the water is flowing at a speed of 5 m/s. Find the difference in the speeds of two beams of light, one traveling in the same direction as the water and the other in the opposite direction. Use $n = 1.33$ for water.
35. Three galaxies are aligned along an axis in the order A, B, C. An observer in galaxy B is in the middle and observes that galaxies A and C are moving in opposite directions away from him, both with speeds $0.60c$. What is the speed of galaxies B and C as observed by someone in galaxy A?
36. Consider the *gedanken* experiment discussed in Section 2.6 in which a giant floodlight stationed 400 km above the Earth's surface shines its light across the moon's surface. How fast does the light flash across the moon?

2.7 Experimental Verification

37. A group of scientists decide to repeat the muon decay experiment at the Mauna Kea telescope site in Hawaii, which is 4205 m above sea level. They count 10^4 muons during a certain time period. Repeat the calculation of Section 2.7 and find the classical and relativistic number of muons expected at sea level. Why did they decide to count as many as 10^4 muons instead of only 10^3?
38. Consider a reference system placed at the U.S. Naval Observatory in Washington, D.C. Two planes take off from Washington Dulles Airport, one going eastward and one going westward, both carrying a cesium atomic clock. The distance around the Earth at 39° latitude (Washington, D.C.) is 31,000 km, and Washington rotates about the Earth's axis at a speed of 360 m/s. Calculate the predicted differences between the clock left at the observatory and the two clocks in the airplanes (each traveling at 300 m/s) when the airplanes return to Washington. Include the rotation of the Earth but no general relativistic effects. Compare with the predictions given in the text.

2.8 Twin Paradox

39. Derive the results in Table 2.1 for the frequencies f' and f''. During what time period do Frank and Mary receive these frequencies?

40. Derive the results in Table 2.1 for the time of the total trip and the total number of signals sent in the frame of both twins. Show your work.

2.9 Spacetime

41. Use the Lorentz transformation to prove that $s^2 = s'^2$.

42. Prove that for a timelike interval, two events can never be considered to occur simultaneously.

43. Prove that for a spacelike interval, two events cannot occur at the same place in space.

44. Given two events, (x_1, t_1) and (x_2, t_2), use a spacetime diagram to find the speed of a frame of reference in which the two events occur simultaneously. What values may Δs^2 have in this case?

45. (a) Draw on a spacetime diagram in the fixed system a line expressing all the events in the moving system that occur at $t' = 0$ if the clocks are synchronized at $t = t' = 0$. (b) What is the slope of this line? (c) Draw lines expressing events occurring for the four times t'_4, t'_3, t'_2, and t'_1 where $t'_4 < t'_3 < 0 < t'_2 < t'_1$. (d) How are these four lines related geometrically?

46. Consider a fixed and a moving system with their clocks synchronized and their origins aligned at $t = t' = 0$. (a) Draw on a spacetime diagram in the fixed system a line expressing all the events occurring at $t' = 0$. (b) Draw on this diagram a line expressing all the events occurring at $x' = 0$. (c) Draw all the worldlines for light that pass through $t = t' = 0$. (d) Are the x' and ct' axes perpendicular? Explain.

47. Use the results of the two previous problems to show that events simultaneous in one system are not simultaneous in another system moving with respect to the first. Consider a spacetime diagram with x, ct and x', ct' axes drawn such that the origins coincide and the clocks were synchronized at $t = t' = 0$. Then consider events 1 and 2 that occur simultaneously in the fixed system. Are they simultaneous in the moving system?

2.10 Doppler Effect

48. An astronaut is said to have tried to get out of a traffic violation for running a red light ($\lambda = 650$ nm) by telling the judge that the light appeared green ($\lambda = 540$ nm) to her as she passed by in her high-powered transport. If this is true, how fast was the astronaut going?

49. Derive Equation (2.32) for the case where the source is fixed but the receiver approaches it with velocity v.

50. Do the complete derivation for Equation (2.33) when the source and receiver are receding with relative velocity v.

51. A spacecraft traveling out of the solar system at a speed of $0.95c$ sends back information at a rate of 1400 kHz. At what rate do we receive the information?

52. Three radio-equipped plumbing vans are broadcasting on the same frequency f_0. Van 1 is moving east of van 2 with speed v, van 2 is fixed, and van 3 is moving west of van 2 with speed v. What is the frequency of each van as received by the others?

53. Three radio-equipped plumbing vans are broadcasting on the same frequency f_0. Van 1 is moving north of van 2 with speed v, van 2 is fixed, and van 3 is moving west of van 2 with speed v. What frequency does van 3 hear from van 2; from van 1?

54. A spaceship moves radially away from Earth with acceleration 29.4 m/s^2 (about $3g$). How much time does it take for the sodium streetlamps ($\lambda = 589$ nm) on Earth to be invisible (with a powerful telescope) to the human eye of the astronauts? The range of visible wavelengths is about 400 to 700 nm.

2.11 Relativistic Momentum

55. Newton's second law is given by $\vec{F} = d\vec{p}/dt$. If the force is always perpendicular to the velocity, show that $\vec{F} = m\gamma\vec{a}$, where $\vec{a}$ is the acceleration.

56. Use the result of the previous problem to show that the radius of a particle's circular path having charge q traveling with speed v in a magnetic field perpendicular to the particle's path is $r = p/qB$. What happens to the radius as the speed increases as in a cyclotron?

57. Newton's second law is given by $\vec{F} = d\vec{p}/dt$. If the force is always parallel to the velocity, show that $\vec{F} = \gamma^3 m\vec{a}$.

58. Find the force necessary to give a proton an acceleration of 10^{19} m/s^2 when the proton has a velocity (along the same direction as the force) of (a) $0.01c$, (b) $0.1c$, (c) $0.9c$, and (d) $0.99c$.

59. A particle having a speed of $0.92c$ has a momentum of 10^{-16} kg · m/s. What is its mass?

60. A particle initially has a speed of $0.5c$. At what speed does its momentum increase by (a) 1%, (b) 10%, (c) 100%?

61. The Bevatron accelerator at the Lawrence Berkeley Laboratory accelerated protons to a kinetic energy of 6.3 GeV. What magnetic field was necessary to keep the protons traveling in a circle of 15.2 m? (See Problem 56.)

62. Show that linear momentum is conserved in Example 2.9 as measured by Mary.

2.12 Relativistic Energy

63. Show that $\frac{1}{2}m\gamma v^2$ does not give the correct kinetic energy.

64. How much ice must melt at 0°C in order to gain 2 g of mass? Where does this mass come from? The heat of fusion for water is 334 J/g.

65. Physicists at the Stanford Linear Accelerator Center (SLAC) bombarded 9-GeV electrons head-on with 3.1-GeV positrons to create B mesons and anti-B mesons. What speeds did the electron and positron have when they collided?

66. The Tevatron accelerator at the Fermi National Accelerator Laboratory (Fermilab) outside Chicago boosts protons to 1 TeV (1000 GeV) in five stages (the numbers given in parentheses represent the total kinetic energy at the end of each stage): Cockcroft-Walton (750 keV), Linac (400 MeV), Booster (8 GeV), Main ring or injector (150 GeV), and finally the Tevatron itself (1 TeV). What is the speed of the proton at the end of each stage?

67. Calculate the momentum, kinetic energy, and total energy of an electron traveling at a speed of (a) $0.020c$, (b) $0.20c$, and (c) $0.90c$.

68. The total energy of a body is found to be twice its rest energy. How fast is it moving with respect to the observer?

69. A system is devised to exert a constant force of 8 N on an 80-kg body of mass initially at rest. The force pushes the mass horizontally on a frictionless table. How far does the body have to be pushed to increase its mass-energy by 25%?

70. What is the speed of a proton when its kinetic energy is equal to twice its rest energy?

71. What is the speed of an electron when its kinetic energy is (a) 10% of its rest energy, (b) equal to the rest energy, and (c) 10 times the rest energy?

72. Derive the following equation:

$$\beta = \frac{v}{c} = \sqrt{1 - \left(\frac{E_0}{E_0 + K}\right)^2}$$

73. Prove that $\beta = pc/E$. This is a useful relation to find the velocity of a highly energetic particle.

74. A good rule of thumb is to use relativistic equations whenever the kinetic energies determined classically and relativistically differ by more than 1%. Find the speeds when this occurs for (a) electrons and (b) protons.

75. How much mass-energy (in joules) is contained in a peanut weighing 0.1 ounce? How much mass-energy do you gain by eating 10 ounces of peanuts? Compare this with the food energy content of peanuts, about 100 kcal per ounce.

76. Calculate the energy needed to accelerate a spaceship of mass 10,000 kg to a speed of $0.3c$ for intergalactic space exploration. Compare this with a projected annual energy usage on Earth of 10^{21} J.

77. Derive Equation (2.58) for the relativistic kinetic energy and show all the steps, especially the integration by parts.

78. A test automobile of mass 1000 kg moving at high speed crashes into a wall. The average temperature of the car is measured to rise by 0.5°C after the wreck. What is the change in mass of the car? Where does this change in mass come from? (Assume the average specific heat of the automobile is close to that of steel, $0.11\ \text{cal} \cdot \text{g}^{-1} \cdot {}^\circ\text{C}^{-1}$.)

2.13 Computations in Modern Physics

79. A helium nucleus has a mass of 4.001505 u. What is its binding energy?

80. A free neutron is an unstable particle and beta decays into a proton with the emission of an electron. How much kinetic energy is available in the decay?

81. The Large Hadron Collider at Europe's CERN facility is designed to produce 7.0 TeV (that is, 7.0×10^{12} eV) protons. Calculate the speed, momentum, and total energy of the protons.

82. What is the kinetic energy of (a) an electron having a momentum of 40 GeV/c? (b) a proton having a momentum of 40 GeV/c?

83. A muon has a mass of 106 MeV/c^2. Calculate the speed, momentum, and total energy of a 200-MeV muon.

84. The reaction $^2\text{H} + {}^2\text{H} \rightarrow n + {}^3\text{He}$ (where n is a neutron) is one of the reactions useful for producing energy through nuclear fusion. (a) Assume the deuterium nuclei (^{2}H) are at rest and use the atomic mass units of the masses in Appendix 8 to calculate the mass-energy imbalance in this reaction. (Note: You can use atomic masses for this calculation, because the electron masses cancel out.) This amount of energy is given up when this nuclear reaction occurs. (b) What percentage of the initial rest energy is given up?

85. The reaction $^2\text{H} + {}^3\text{H} \rightarrow n + {}^4\text{He}$ is one of the reactions useful for producing energy through nuclear fusion. (a) Assume the deuterium (^{2}H) and tritium (^{3}H) nuclei are at rest and use the atomic mass units of the masses in Appendix 8 to calculate the mass-energy imbalance in this reaction. This amount of energy is given up when this nuclear reaction occurs. (b) What percentage of the initial rest energy is given up?

2.14 Electromagnetism and Relativity

86. Instead of one positive charge outside a conducting wire, as was discussed in Section 2.14 and shown in Figure 2.34, consider a second conducting wire parallel to the first one. Both wires have positive and negative charges, and the wires are electrically neutral. Assume that in both wires the positive charges travel to the right and negative charges to the left. (a) Consider an inertial frame moving with the negative charges of wire 1. Show that the second wire is attracted to the first wire in this frame. (b) Now consider an inertial frame moving with the positive charges of the second wire. Show that the first wire is attracted to the second. (c) Use this argument to show that electrical and magnetic forces are relative.

General Problems

87. An Ω^- particle has rest energy 1672 MeV and mean lifetime 8.2×10^{-11} s. It is created and decays in a particle track detector and leaves a track 24 mm long. What is the total energy of the Ω^- particle?

88. Show that the following form of Newton's second law satisfies the Lorentz transformation. Assume the force is parallel to the velocity.

$$F = m\frac{dv}{dt}\frac{1}{[1 - (v^2/c^2)]^{3/2}}$$

89. Use the results listed in Table 2.1 to find (a) the number of signals Frank receives at the rate f' and the time at which Frank detects Mary's turnaround, and (b) the number of signals Mary receives at the rate f' and her clock reading when she turns around. (c) From Frank's perspective, find the time for the remainder of the trip (after he detects Mary's turnaround), the number of signals he receives at the rate f'', the total number of signals he receives, and Mary's age, based on that total number of signals. (d) From Mary's perspective, find the time for the remainder of the trip (after her turnaround), the number of signals she receives at the rate f'', the total number of signals she receives, and Frank's age, based on that total number of signals.

90. For the twins Frank and Mary described in Section 2.8, consider Mary's one-way trip at a speed of $0.8c$ to the star system 8 lightyears from Earth. Compute the spacetime interval s in the fixed frame and s' in the moving frame, and compare the results.

91. Frank and Mary are twins. Mary jumps on a spaceship and goes to the star system Alpha Centauri (4.30 lightyears away) and returns. She travels at a speed of $0.8c$ with respect to Earth and emits a radio signal every week. Frank also sends out a radio signal to Mary once a week. (a) How many signals does Mary receive from Frank before she turns around? (b) At what time does the frequency of signals Frank receives suddenly change? How many signals has he received at this time? (c) How many signals do Frank and Mary receive for the entire trip? (d) How much time does the trip take according to Frank and to Mary? (e) How much time does each twin say the other twin will measure for the trip? Do the answers agree with those for (d)?

92. A police radar gun operates at a frequency of 10.5 GHz. The officer, sitting in a patrol car at rest by the highway, directs the radar beam toward a speeding car traveling 80 mph directly away from the patrol car. What is the frequency shift of the reflected beam, relative to the original radar beam?

93. A spaceship moving $0.80c$ direction away from Earth fires a missile that the spaceship measures to be moving at $0.80c$ perpendicular to the ship's direction of travel. Find the velocity components and speed of the missile as measured by Earth.

94. An electron has a total energy that is 250 times its rest energy. Determine its (a) kinetic energy, (b) speed, and (c) momentum.

95. A proton moves with a speed of $0.90c$. Find the speed of an electron that has (a) the same momentum as the proton, and (b) the same kinetic energy.

96. A high-speed K^0 meson is traveling at a speed of $0.90c$ when it decays into a π^+ and a π^- meson. What are the greatest and least speeds that the mesons may have?

97. Frank and Mary are twins, and Mary wants to travel to our nearest star system, Alpha Centauri (4.30 lightyears away). Mary leaves on her 30th birthday and intends to return to Earth on her 52nd birthday. (a) Assuming her spaceship returns from Alpha Centauri without stopping, how fast must her spaceship travel? (b) How old will Frank be when she returns?

98. The International Space Federation constructs a new spaceship that can travel at a speed of $0.995c$. Mary, the astronaut, boards the spaceship to travel to Barnard's star, which is the second nearest star to our solar system after Alpha Centauri and is 5.98 lightyears away. After reaching Barnard's star, the spaceship travels slowly around the star system for three years doing research before returning back to Earth. (a) How much time does her journey take? (b) How much older is her twin Frank than Mary when she returns?

99. A powerful laser on Earth rotates its laser beam in a circle at a frequency of 0.030 Hz. (a) How fast does the spot that the laser makes on the moon move across the moon's landscape? (b) With what rotation frequency should the laser rotate if the laser spot moves across the moon's landscape at speed c?

100. The Lockheed SR-71 Blackbird may be the fastest nonresearch airplane ever built; it traveled at 2200 miles/hour (983 m/s) and was in operation from 1966 to 1990. Its length is 32.74 m. (a) By what percentage would it appear to be length contracted while in flight? (b) How much time difference would occur on an atomic clock in the plane compared to a similar clock on Earth during a flight of the Blackbird over its range of 3200 km?

101. A spaceship is coming directly toward you while you are in the International Space Station. You are told that the spaceship is shining sodium light (with an intense yellow doublet of wavelengths 588.9950 and 589.5924 nm). You have an apparatus that can resolve two closely spaced wavelengths if the difference is $\Delta\lambda < 0.55$ nm. If you find that you can just resolve the doublet, how fast is the spaceship traveling with respect to you?

102. Quasars are among the most distant objects in the universe and are moving away from us at very high speeds, as discussed in Chapter 16. Astrophysicists use the redshift parameter z to determine the redshift of such rapidly moving objects. The parameter z is determined by observing a wavelength λ' of a known spectral line of wavelength λ_{source} on Earth; $z = \Delta\lambda/\lambda_{\text{source}} = (\lambda' - \lambda_{\text{source}})/\lambda_{\text{source}}$. Find the speed of two quasars having z values of 1.9 and 4.9.

103. One possible decay mode of the neutral kaon is $K^0 \rightarrow \pi^0 + \pi^0$. The rest energies of the K^0 and π^0 are 498 MeV and 135 MeV, respectively. The kaon is initially at rest when it decays. (a) How much energy is released in the decay? (b) What are the momentum and relative directions of the two neutral pions (π^0)?

104. The sun radiates energy at a rate of 3.9×10^{26} W. (a) At what rate is the sun losing mass? (b) At that rate, how much time would it take to exhaust the sun's fuel supply? The sun's mass is 2.0×10^{30} kg, and you may assume that the reaction producing the energy is about 0.7% efficient. Compare your answer with the sun's expected remaining lifetime, about 5 Gy.

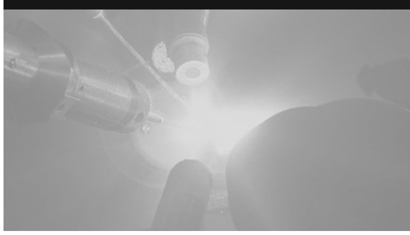
105. One way astrophysicists have identified "extrasolar" planets orbiting distant stars is by observing redshifts or blueshifts in the star's spectrum due to the fact that the star and planet each revolve around their common center of mass. (See *Scientific American*, August 2010, p. 41.) Consider a star the size of our sun (mass $= 1.99 \times 10^{30}$ kg), with a planet the size of Jupiter (1.90×10^{27} kg) in a circular orbit of radius 7.79×10^{11} m and a period of 11.9 years. (a) Find the speed of the star revolving around the system's center of mass. (b) Assume that Earth is in the planet's orbital plane, so that at one point in its orbit the star is moving directly toward Earth, and at the opposite point it moves directly away from Earth. How much is 550-nm light redshifted and blueshifted at those two extreme points?

106. Small differences in the wavelengths in the sun's spectrum are detected when measurements are taken from different parts of the sun's disk. Specifically, measurements of the 656-nm line in hydrogen taken from opposite sides on the sun's equator—one side approaching Earth and the other receding—differ from each other by 0.0090 nm. Use this information to find the rotational period of the sun's equator. Express your answer in days. (The sun's equatorial radius is 6.96×10^8 m.)

3

CHAPTER

The Experimental Basis of Quantum Physics



As far as I can see, our ideas are not in contradiction to the properties of the photoelectric effect observed by Mr. Lenard.

Max Planck, 1905

As discussed in Chapter 1, during the final decades of the 1800s scientists discovered phenomena that could not be explained by what we now call classical physics. Despite the prevalent confidence in the laws of classical physics, the few exceptions to these laws discovered during the latter part of the nineteenth century led to the fabulous 30-year period of 1900–1930, when our understanding of the laws of physics was dramatically changed. One of these exceptions led to the special theory of relativity, which was introduced by Einstein in 1905 and successfully explained the null result of the Michelson-Morley experiment. The other great conceptual advance of twentieth-century physics, the quantum theory that is the subject of this chapter, began in 1900 when Max Planck introduced his explanation of blackbody radiation.

We begin this chapter with Wilhelm Röntgen's discovery of the x ray and J. J. Thomson's discovery of the electron. Robert Millikan later determined the electron's charge. We shall see that, although it was necessary to assume that certain physical quantities may be quantized, scientists found this idea hard to accept. We discuss the difficulties of explaining blackbody radiation with classical physics and how Planck's proposal solved the problem. Finally, we will see that Einstein's explanation of the photoelectric effect and Arthur Compton's understanding of data on x-ray scattering made the quantum hypothesis difficult to refute. After many difficult and painstaking experiments, it became clear that quantization was not only necessary, it was also the correct description of nature.

3.1 Discovery of the X Ray and the Electron

In the 1890s scientists and engineers were familiar with the "cathode rays" that were generated from one of the metal plates in an evacuated tube across which a large electric potential had been established. The origin and constitution of cathode rays were not known. The concept of an atomic substructure of matter was widely accepted because of its use in explaining the results of chemical experiments. Therefore, it was surmised that cathode rays had something to do with atoms. It was known, for example, that cathode rays could penetrate matter, and their properties were of great interest and under intense investigation in the 1890s.

In 1895 Wilhelm Röntgen was studying the effects of cathode rays passing through various materials and noticed a nearby phosphorescent screen glowing vividly in the darkened room. Röntgen soon realized he was observing a new kind of ray, one that, unlike cathode rays, was unaffected by magnetic fields and was far more penetrating than cathode rays. These **x rays**, as he called them, were apparently produced by the cathode rays bombarding the glass walls of his vacuum tube. Röntgen studied their transmission through many materials and even showed that he could obtain an image of the bones in a hand when the x rays were allowed to pass through as shown in Figure 3.1. This experiment created tremendous excitement, and medical applications of x rays were quickly developed. For this discovery, Röntgen received the first Nobel Prize for Physics in 1901.

New penetrating ray: x ray

For several years before the discovery of x rays, J. J. Thomson (1856–1940), professor of experimental physics at Cambridge University, had been studying the properties of electrical discharges in gases. Thomson's apparatus was similar to that used by Röntgen and many other scientists because of its simplicity (Figure 3.2). Thomson believed that cathode rays were particles, whereas several respected German scientists (such as Heinrich Hertz) believed they were wave phenomena.

Bettmann/Corbis

Wilhelm Röntgen (1845–1923), born in Germany but raised in the Netherlands, studied mechanical engineering at the University of Zurich. After holding several university appointments, he went to the University of Munich as Chair of Physics in 1900, where he remained for the rest of his life. As a professor at the University of Würzburg in 1895, he discovered x rays while investigating the passage of electric current through low-pressure gases. He preferred working alone and built most of his own apparatus. He refused to benefit from his many discoveries and died nearly bankrupt after World War I.

Thomson was able to prove in 1897 that the charged particles emitted from a heated electrical cathode were in fact the same as cathode rays. The main features of Thomson's experiment are shown in the schematic apparatus of Figure 3.2. The rays from the cathode are attracted to the positive potential on aperture A (anode) and are further collimated by aperture B to travel in a straight line and strike a fluorescent screen in the rear of the tube, where they can be visually detected by a flash of light. A voltage across the deflection plates sets up an electric field that deflects charged particles. Previously, in a similar experiment, Hertz had observed no effect on the cathode rays due to the deflecting voltage. Thomson at first found the same result, but on further evacuating the glass tube, he observed the deflection and proved that cathode rays had a negative charge. The previous experiment, in a poorer vacuum, had failed because the cathode rays had interacted with and ionized the residual gas. Thomson also studied the effects of a magnetic field upon the cathode rays and proved convincingly that the cathode rays acted as negatively charged particles (electrons) in both electric and magnetic fields, for which he received the Nobel Prize for Physics in 1906.

Thomson's method of measuring the ratio of the electron's charge to mass, e/m, is now a standard technique and generally studied as an example of charged particles passing through perpendicular electric and magnetic fields as shown schematically in Figure 3.3. With the magnetic field turned off, the electron entering the region between the plates is accelerated upward by the electric field

$$F_y = ma_y = qE \tag{3.1}$$

where m and q are, respectively, the mass and charge of the electron, and a_y is its resulting acceleration. The time for the electron to traverse the deflecting plates (length $= \ell$) is $t \approx \ell / v_0$. The exit angle θ of the electron is then given by

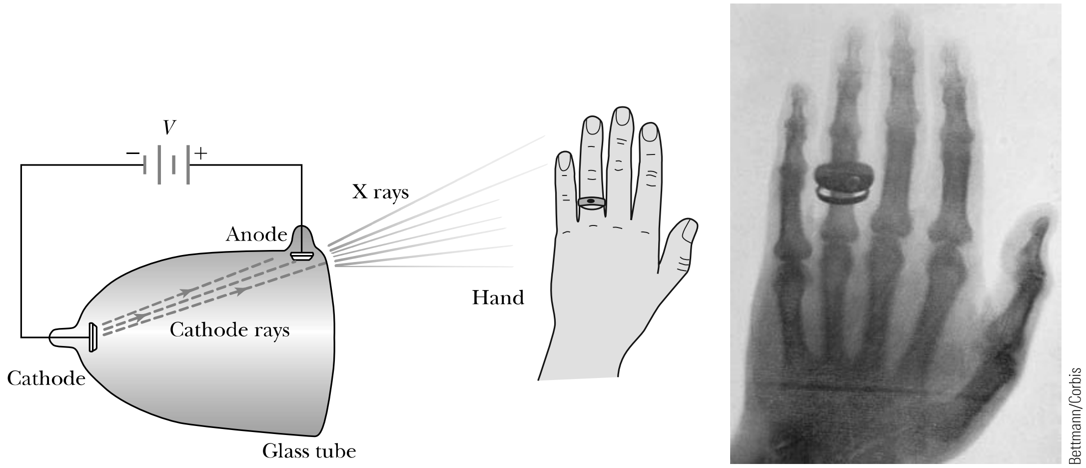


Figure 3.1 In Röntgen's experiment, "x rays" were produced by cathode rays (electrons) hitting the glass near the anode. He studied the penetration of the x rays through several substances and even noted that if the hand was held between the glass tube and a screen, the darker shadow of the bones could be discriminated from the shadow of the hand.

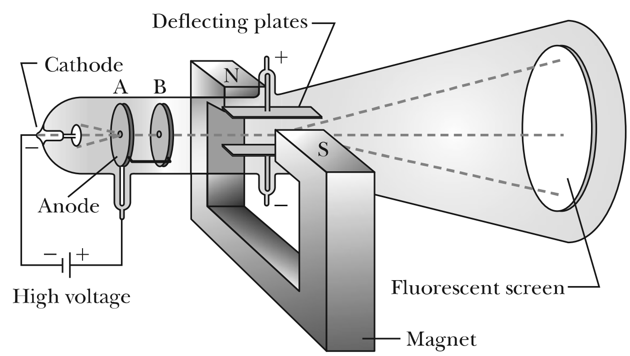


Figure 3.2 Apparatus of Thomson's cathode-ray experiment. Thomson proved that the rays emitted from the cathode were negatively charged particles (electrons) by deflecting them in electric and magnetic fields. The key to the experiment was to evacuate the glass tube.

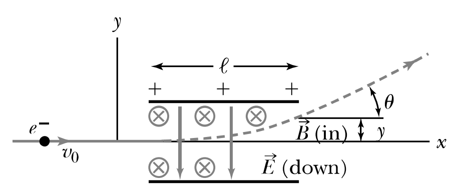


Figure 3.3 Thomson's method of measuring the ratio of the electron's charge to mass was to send electrons through a region containing a magnetic field ($\vec{B}$ into page) perpendicular to an electric field ($\vec{E}$ down). The electrons having $v = E/B$ go through undeflected. Then, using electrons of the same energy, the magnetic field is turned off and the electric field deflects the electrons, which exit at angle θ. The ratio of e/m can be determined from $\vec{B}$, $\vec{E}$, θ, and ℓ, where ℓ is the length of the field distance and θ is the emerging angle. See Equation (3.5).

$$\tan\theta = \frac{v_y}{v_x} = \frac{a_y t}{v_0} = \frac{qE}{m}\frac{\ell}{v^2_0} \tag{3.2}$$

The ratio q/m can be determined if the velocity is known. By turning on the magnetic field and adjusting the strength of $\vec{B}$ so that no deflection of the electron occurs, the velocity can be determined. The condition for zero deflection is that the net force on the electron is exactly zero:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} = 0 \tag{3.3}$$

Hence,

$$\vec{E} = -\vec{v} \times \vec{B}$$

Because $\vec{v}$ and $\vec{B}$ are perpendicular, the electric and magnetic field strengths are related by

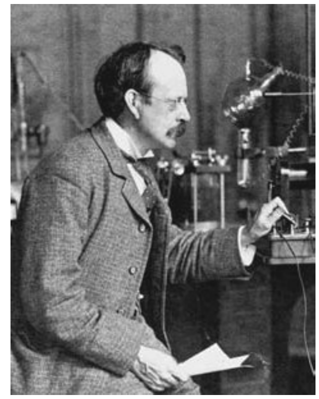
$$|\vec{E}| = |v_x|\,|\vec{B}|$$

so that

$$v_x = \frac{E}{B} = v_0 \tag{3.4}$$

where we have used $E = |\vec{E}|$ for the magnitude of the electric field and similarly B for the magnitude of $\vec{B}$. If we insert this value for v_0 into Equation (3.2), we extract the ratio q/m.

$$\frac{q}{m} = \frac{v^2_0 \tan\theta}{E\ell} = \frac{E\tan\theta}{B^2\ell} \tag{3.5}$$



Sir Joseph John Thomson (1856–1940), universally known as "J.J.," went to Cambridge University at age 20 and remained there for the rest of his life. Thomson's career with the Cavendish Laboratory spanned a period of over 50 years, and he served as director from 1884 until 1918 when he stepped down in favor of Ernest Rutherford. Thomson was exceptional in designing apparatus and diagnosing problems, although he was not a particularly gifted experimentalist with his hands. His guidance at the Cavendish Laboratory was partly instrumental in the award of seven Nobel Prizes in Physics to him and his peers during his 50 years at the lab.

EXAMPLE 3.1

In an experiment similar to Thomson's, we use deflecting plates 5.0 cm in length with an electric field of 1.2×10^4 V/m. Without the magnetic field we find an angular deflection of 30°, and with a magnetic field of 8.8×10^{-4} T we find no deflection. What is the initial velocity of the electron and its q/m?

Strategy Because we know the values of E and B for which there is no deflection, we use Equation (3.4) to determine the electron's velocity v_0. Then we can use Equation (3.5) to determine q/m for the situation with no magnetic field.

Solution We insert the values of E and B into Equation (3.4) to find

$$v_0 = \frac{E}{B} = \frac{1.2 \times 10^4\ \text{V/m}}{8.8 \times 10^{-4}\ \text{T}} = 1.4 \times 10^7\ \text{m/s}$$

Because all our units for E and B are in the international system (SI), the value for v_0 is in meters/second. Equation (3.5) gives the following result for q/m:

$$\frac{q}{m} = \frac{E\tan\theta}{B^2\ell} = \frac{(1.2 \times 10^4\ \text{V/m})(\tan 30°)}{(8.8 \times 10^{-4}\ \text{T})^2(0.050\ \text{m})}$$

$$= 1.8 \times 10^{11}\ \text{C/kg}$$

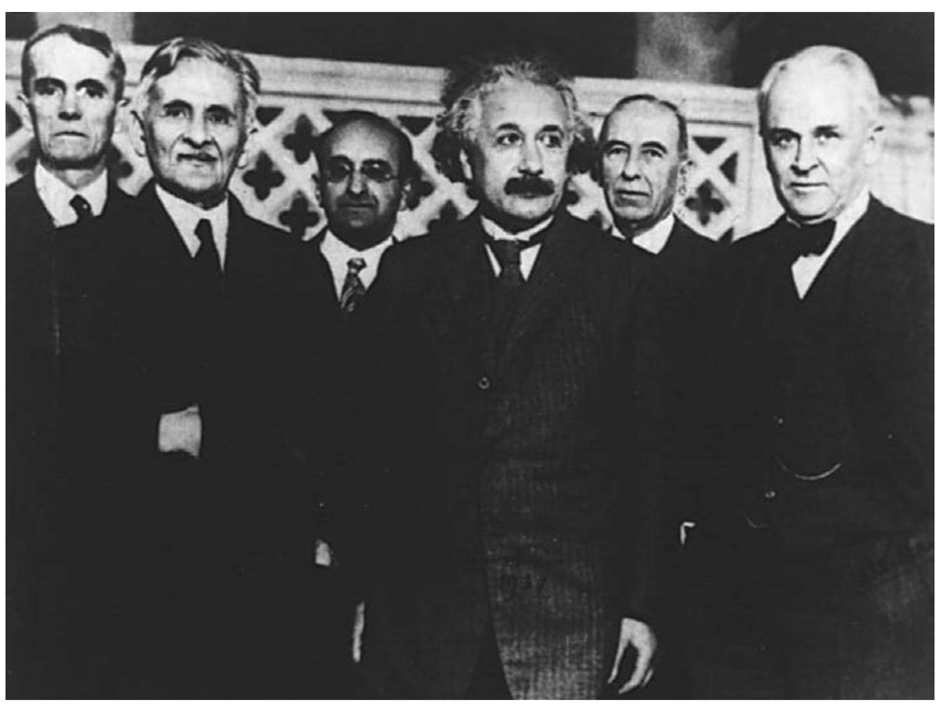
Thomson's actual experiment, done in the manner of the previous example, obtained a result about 35% lower than the presently accepted value of 1.76×10^{11} C/kg for e/m. Thomson realized that the value of e/m (e = absolute value of electron charge) for an electron was much larger than had been anticipated and a factor of 1000 larger than any value of q/m that had been previously measured (for the hydrogen atom). He concluded that either m was small or e was large (or both), and the "carriers of the electricity" were quite penetrating compared with atoms or molecules, which must be much larger in size.

3.2 Determination of Electron Charge

After Thomson's measurement of e/m and the confirmation of the cathode ray as a charge carrier (called *electron*), several investigators attempted to determine the actual magnitude of the electron's charge. In 1911 the American physicist Robert A. Millikan (1868–1953) reported convincing evidence for an accurate determination of the electron's charge. Millikan's classic experiment began in 1907 at the University of Chicago. The experiment consisted of visual observation of the motion of uncharged and both positively and negatively charged oil drops moving under the influence of electrical and gravitational forces. The essential parts of the apparatus are shown in Figure 3.4. As the drops emerge from the nozzle, frictional forces sometimes cause them to be charged. Millikan's method consisted of balancing the upward force of the electric field between the plates against the downward force of the gravitational field.

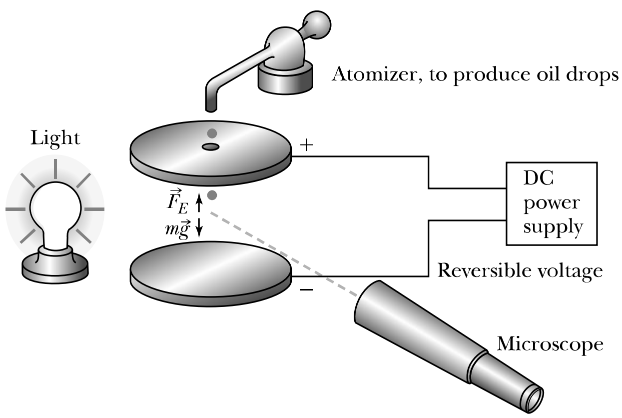
When an oil drop falls downward through the air, it experiences a frictional force $\vec{F}_f$ proportional to its velocity due to the air's viscosity:

$$\vec{F}_f = -b\vec{v} \tag{3.6}$$



Courtesy of California Institute of Technology

Three great physicists (foreground), 1931: Michelson, Einstein, and Robert A. Millikan (1868–1953). Millikan received his degrees from Oberlin College and Columbia University and was at the University of Chicago from 1896 to 1921 before leaving to join the California Institute of Technology, where he was chair of the Executive Council from 1921 to 1945 (de facto president) and helped Caltech become a leading research institution. His important work included the famous oil-drop experiment to determine the electron charge, a confirmation of Einstein's photoelectric theory in which Millikan measured Planck's constant h, and Brownian motion. He received the Nobel Prize in Physics in 1923 for the first two experiments. He also did important work in cosmic ray physics and is given credit for the name *cosmic rays*. In later life he became particularly interested in teaching and was a prolific textbook author.



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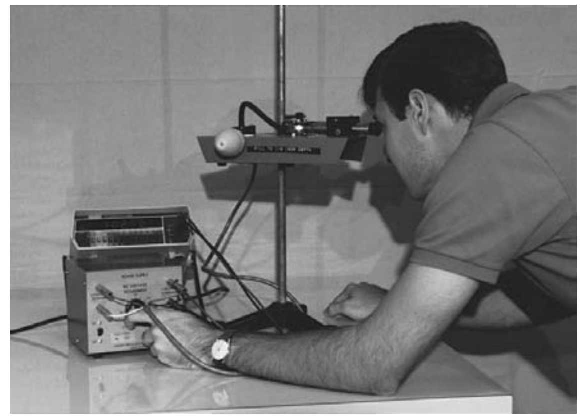


Stephen T. Thornton

(b)

Figure 3.4 (a) Diagram of the Millikan oil-drop experiment to measure the charge of the electron. Some of the oil drops from the atomizer emerge charged, and the electric field (voltage) is varied to slow down or reverse the direction of the oil drops, which can have positive or negative charges. (b) A student looking through the microscope is adjusting the voltage between the plates to slow down a tiny plastic ball that serves as the oil drop.

This force has a minus sign because a drag force always opposes the velocity. The constant b is determined by Stokes's law and is proportional to the oil drop's radius. Millikan showed that Stokes's law for the motion of a small sphere through a resisting medium was incorrect for small-diameter spheres because of the atomic nature of the medium, and he found the appropriate correction. The buoyancy of the air produces an upward force on the drop, but we can neglect this effect for a first-order calculation.

To suspend the oil drop at rest between the plates, the upward electric force must equal the downward gravitational force. The frictional force is then zero because the velocity of the oil drop is zero.

$$\vec{F}_E = q\vec{E} = -m\vec{g} \quad (\text{when } v = 0) \tag{3.7}$$

The magnitude of the electric field is $E = V/d$, and V is the voltage across large, flat plates separated by a small distance d. The magnitude of the electron charge q may then be extracted as

$$q = \frac{mgd}{V} \tag{3.8}$$

To calculate q we have to know the mass m of the oil drops. Millikan found he could determine m by turning off the electric field and measuring the terminal velocity of the oil drop. The radius of the oil drop is related to the terminal velocity by Stokes's law (see Problem 7). The mass of the drop can then be determined by knowing the radius r and density ρ of the type of oil used in the experiment:

$$m = \tfrac{4}{3}\pi r^3\rho \tag{3.9}$$

If the power supply has a switch to reverse the polarity of the voltage and an adjustment for the voltage magnitude, the oil drop can be moved up and down in the apparatus at will. Millikan reported that in some cases he was able to observe a given oil drop for up to six hours and that the drop changed its charge several times during this time period.

Measurement of electron charge

Millikan made thousands of measurements using different oils and showed that there is a basic quantized electron charge. Millikan's value of e was very close to our presently accepted value of 1.602×10^{-19} C. Notice that we always quote a positive number for the charge e. The charge on an electron is then $-e$.

EXAMPLE 3.2

For an undergraduate physics laboratory experiment we often make two changes in Millikan's procedure. First, we use plastic balls of about 1 micrometer (μm or micron) diameter, for which we can measure the mass easily and accurately. This avoids the measurement of the oil drop's terminal velocity and the dependence on Stokes's law. The small plastic balls are sprayed through an atomizer in liquid solution, but the liquid soon evaporates in air. The plastic balls are observed by looking through a microscope. One other improvement is to occasionally bombard the region between the plates with ionizing radiation, such as an electron (beta particle) from a radioactive source. This radiation ionizes the air and makes it easier for the charge on a ball to change. By making many measurements we can determine whether the charges determined from Equation (3.8) are multiples of some basic charge unit.

In an actual undergraduate laboratory experiment the mass of the balls was $m = 5.7 \times 10^{-16}$ kg and the spacing between the plates was $d = 4.0$ mm. Therefore q can be found from Equation (3.8):

$$q = \frac{mgd}{V} = \frac{(5.7 \times 10^{-16}\ \text{kg})(9.8\ \text{m/s}^2)(4.0 \times 10^{-3}\ \text{m})}{V}$$

$$q = \frac{(2.23 \times 10^{-17}\ \text{V})}{V}\ \text{C}$$

where V is the voltage between plates when the observed ball is stationary. Two students observed 30 balls and found the values of V shown in Table 3.1 for a stationary ball. In this experiment the voltage polarity can be easily changed, and a positive voltage represents a ball with a positive charge. Notice that charges of both signs are observed.

Table 3.1 Student Measurements in Millikan Experiment

Particle	Voltage (V)	q ($\times 10^{-19}$ C)	Particle	Voltage (V)	q	Particle	Voltage (V)	q
1	−30.0	−7.43	11	−126.3	−1.77	21	−31.5	−7.08
2	+28.8	+7.74	12	−83.9	−2.66	22	−66.8	−3.34
3	−28.4	−7.85	13	−44.6	−5.00	23	+41.5	+5.37
4	+30.6	+7.29	14	−65.5	−3.40	24	−34.8	−6.41
5	−136.2	−1.64	15	−139.1	−1.60	25	−44.3	−5.03
6	−134.3	−1.66	16	−64.5	−3.46	26	−143.6	−1.55
7	+82.2	+2.71	17	−28.7	−7.77	27	+77.2	+2.89
8	+28.7	+7.77	18	−30.7	−7.26	28	−39.9	−5.59
9	−39.9	−5.59	19	+32.8	+6.80	29	−57.9	−3.85
10	+54.3	+4.11	20	−140.8	+1.58	30	+42.3	+5.27

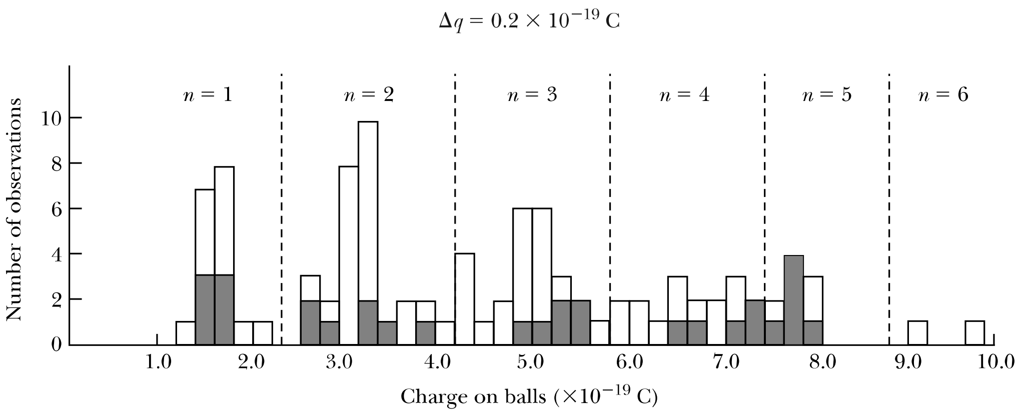


Figure 3.5 A histogram of the number of observations for the charge on a ball in a student Millikan experiment. The histogram is plotted for $\Delta q = 0.2 \times 10^{-19}$ C. The solid area refers to the first group's 30 measurements, and the open area to another 70 measurements. Notice the peaks, especially for the first three ($n = 1, 2, 3$) groups, indicating the electron charge quantization. When the basic charge q_0 is found from $q = nq_0$ (n = integer), $q_0 = 1.6 \times 10^{-19}$ C was determined in this experiment from all 100 observations.

The values of $|q|$ are plotted on a histogram in units of $\Delta q = 0.2 \times 10^{-19}$ C in Figure 3.5 (solid area). When 70 additional measurements from other students are added, a clear pattern of quantization develops with a charge $q = nq_0$, especially for the first three groups. The areas of the histogram can be separated for the various n values, and the value of q_0 found for each measurement is then averaged. For the histogram shown we find $q_0 = 1.7 \times 10^{-19}$ C for the first 30 measurements and $q_0 = 1.6 \times 10^{-19}$ C for all 100 observations.

3.3 Line Spectra

In contrast to the smooth, continuous radiation spectrum obtained from thermal bodies, chemical elements produce unique wavelengths (colors) when burned in a flame or when excited in an electrical discharge, a fact already known in the early 1800s. Prisms had been used to investigate these early sources of spectra, and optical spectroscopy became an important area of experimental physics, primarily because of the modern development of diffraction gratings by Henry Rowland* (1848–1901) of Johns Hopkins University in the 1880s.

An example of a spectrometer used to observe optical spectra is shown in Figure 3.6. An electrical discharge excites atoms of a low-pressure gas contained in the tube. The collimated light passes through a diffraction grating with thousands of ruling lines per centimeter, and the diffracted light is separated at angle θ according to its wavelength. The equation expressing diffraction maxima is

$$d \sin \theta = n\lambda \tag{3.10}$$

Diffraction maxima

where d is the distance between rulings, and n (an integer) is called the order number ($n = 1$ has the strongest scattered intensity). The resulting pattern of light

*Rowland was one of the first six professors chosen in 1875 for the founding of Johns Hopkins University and, together with Albert Michelson, was one of the foremost American physicists of the last part of the nineteenth century. He was a founder and was elected the first president of the American Physical Society in 1899. Albert Michelson was the vice president; neither had formally earned a Ph.D. degree.

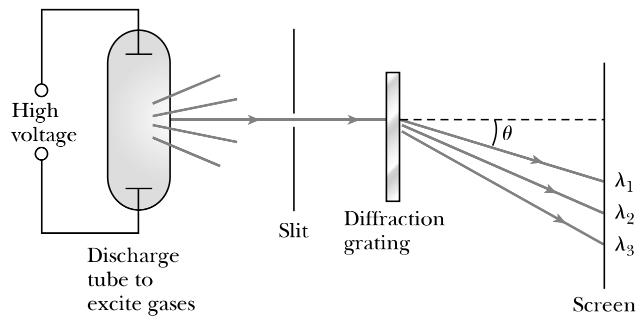


Figure 3.6 Schematic of an optical spectrometer. Light produced by a high-voltage discharge in the glass tube is collimated and passed through a diffraction grating, where it is deflected according to its wavelength. See Equation (3.10).

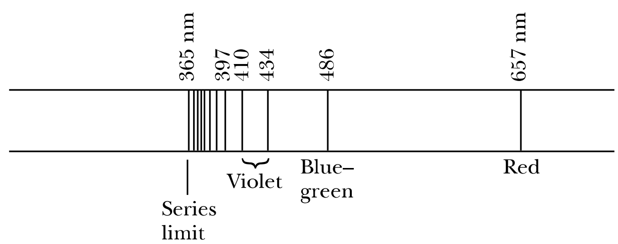


Figure 3.7 The Balmer series of line spectra of the hydrogen atom with wavelengths indicated in nanometers. The four visible lines are noted as well as the lower limit of the series.

bands and dark areas on the screen is called a *line spectrum*. By 1860 Bunsen and Kirchhoff realized that the wavelengths of these line spectra would allow identification of the chemical elements and the composition of materials. It was discovered that each element had its own characteristic wavelengths (see examples shown on the inside back cover). The field of spectroscopy flourished because finer and more evenly ruled gratings became available, and improved experimental techniques allowed more spectral lines to be observed and catalogued. Particular attention was paid to the sun's spectrum in hopes of understanding the origin of sunlight. The helium atom was actually "discovered" by its line spectra from the sun before it was recognized on Earth (see Special Topic, "The Discovery of Helium").

Characteristic line spectra of elements

Many scientists believed that the increasing number of spectral lines suggested a complicated internal structure of the atom, and that by carefully investigating the wavelengths for many elements, the structure of atoms and matter could be understood. That belief was eventually partially realized.

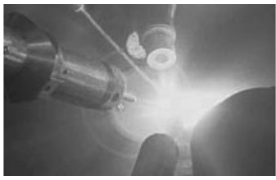
For much of the nineteenth century, scientists attempted to find some simple underlying order for the characteristic wavelengths of line spectra. Hydrogen appeared to have an especially simple-looking spectrum, and because some chemists thought hydrogen atoms might be the constituents of heavier atoms, hydrogen was singled out for intensive study. Finally, in 1885, Johann Balmer, a Swiss schoolteacher, succeeded in obtaining a simple empirical formula that fit the wavelengths of the four lines then known in the hydrogen spectrum and several ultraviolet lines that had been identified in the spectra of white stars. This series of lines, called the *Balmer series*, is shown in Figure 3.7. Balmer found that the expression

Balmer's empirical result

$$\lambda = 364.56 \frac{k^2}{k^2 - 4} \text{ nm} \tag{3.11}$$

(where $k = 3, 4, 5, \ldots; k > 2$) fit all the visible hydrogen lines. Wavelengths are normally given in units of nanometers* (nm).

*Wavelengths were formerly listed in units of angstroms [one angstrom (Å) = 10^{-10} m], named after Anders Ångstrom (1817–1874), one of the first persons to observe and measure the wavelengths of the four visible lines of hydrogen.



Special Topic

The Discovery of Helium

It might seem that the discovery of helium, the second simplest of all elements, would have occurred centuries ago. In fact the discovery occurred over a period of several years in the latter part of the nineteenth century as scientists were scrambling to understand unexpected results. The account here is taken from *Helium*, by William H. Keesom.*

A schematic diagram of an optical spectroscope is shown in Figure 3.11. Its first use in a solar eclipse was on August 18, 1868, to investigate the sun's atmosphere. Several people (including P. J. C. Janssen, G. Rayet, C. T. Haig, and J. Herschel) at the solar eclipse regions in India and Malaysia reported observing, either directly or indirectly, an unusual yellow line in the spectra that would later prove to be due to helium. It occurred to Janssen the day of the eclipse that it should be possible to see the sun's spectrum directly without the benefit of the eclipse, and he did so with a spectroscope in the next few days. The same idea had occurred to J. N. Lockyer earlier, but he did not succeed in measuring the sun's spectrum until October 1868, a month or so after Janssen. This method of observing the sun's atmosphere at any time was considered to be an important discovery, and Janssen and Lockyer are prominently recognized not only for their role in the evolution of helium's discovery, but also for their method of studying the sun's atmosphere.

The actual discovery of helium was delayed by the fact that the new yellow line seen in the sun's atmosphere was very close in wavelength to two well-known yellow lines of sodium. This is apparent in the atomic line spectra of both helium and sodium seen on the inside back cover of this text. By December 1868, Lockyer, A. Secchi, and Janssen each independently recognized that the yellow line was different from that of sodium.

Another difficulty was to prove that the new yellow line, called D3, was not due to some other known element, especially hydrogen. For many years Lockyer thought that D3 was related to hydrogen, and he and E. Frankland performed several experiments in an unsuccessful attempt to prove his thesis. Lockyer wrote as late as 1887 that D3 was a form of hydrogen. Despite Lockyer's convictions, Lord Kelvin reported in 1871 during his presidential address to the British Association that Frankland and Lockyer could not find the D3 line to be related to any terrestrial (from Earth) flame. Kelvin reported that it seemed to represent a new substance, which Frankland and Lockyer proposed to call helium (from the Greek word *helios* for "sun").

It was not until 1895 that helium was finally clearly observed on Earth by Sir William Ramsay, who had received a letter reporting that W. F. Hillebrand had produced nitrogen gas by boiling uranium ores (*pitchblende*) in dilute sulfuric acid. Ramsay was skeptical of the report and proceeded to reproduce it. He was astounded, after finding a small amount of nitrogen and the expected argon gas, to see a brilliant yellow line that he compared with those from sodium, finding the wavelengths to be slightly different. Sir William Crookes measured the wavelength and reported the following day that it was the D3 line, proving the terrestrial existence of helium. Later in 1895 H. Kayser found the helium line in spectra taken from a gas that had evolved from a spring in Germany's Black Forest. Eventually, in 1898, helium was confirmed in the Earth's atmosphere by E. C. Baly. No one person can be credited with the discovery of helium.

The remarkable properties of *liquid* helium are discussed in Section 9.7.

*W. H. Keesom, *Helium,* Amsterdam, London, and New York: Elsevier (1942).

It is more convenient to take the reciprocal of Equation (3.11) and write Balmer's formula in the form

$$\frac{1}{\lambda} = \frac{1}{364.56\ \text{nm}}\frac{k^2 - 4}{k^2} = \frac{4}{364.56\ \text{nm}}\left(\frac{1}{2^2} - \frac{1}{k^2}\right) = R_H\left(\frac{1}{2^2} - \frac{1}{k^2}\right) \tag{3.12}$$

where R_H is called the *Rydberg constant* (for hydrogen) and has the more accurate value $1.096776 \times 10^7\ \text{m}^{-1}$, and k is an integer greater than two ($k > 2$).

By 1890, efforts by Johannes Rydberg and particularly Walther Ritz resulted in a more general empirical equation for calculating the wavelengths, called the *Rydberg equation*.

Rydberg equation

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{k^2}\right) \tag{3.13}$$

where $n = 2$ corresponds to the Balmer series and $k > n$ always. In the next 20 years after Balmer's contribution, other series of the hydrogen atom's spectral lines were discovered, and by 1925 five series had been discovered, each having a different integer n (Table 3.2). The understanding of the Rydberg equation (3.13) and the discrete spectrum of hydrogen were important research topics early in the twentieth century.

Table 3.2 Hydrogen Series of Spectral Lines

Discoverer (year)	Wavelength	n	k
Lyman (1916)	Ultraviolet	1	>1
Balmer (1885)	Visible, ultraviolet	2	>2
Paschen (1908)	Infrared	3	>3
Brackett (1922)	Infrared	4	>4
Pfund (1924)	Infrared	5	>5

EXAMPLE 3.3

The visible lines of the Balmer series were observed first because they are most easily seen. Show that the wavelengths of spectral lines in the *Lyman* ($n = 1$) and *Paschen* ($n = 3$) series are not in the visible region. Find the wavelengths of the four visible atomic hydrogen lines. Assume the visible wavelength region is $\lambda = 400\text{–}700$ nm.

Strategy We use Equation (3.13) to determine the various wavelengths for $n = 1$, 2, and 3. If the wavelengths are between 400 and 700 nm, we conclude they are in the visible region. Otherwise, they are not visible.

Solution We use Equation (3.13) first to examine the Lyman series ($n = 1$):

$$\frac{1}{\lambda} = R_H\left(1 - \frac{1}{k^2}\right)$$

$$= 1.0968 \times 10^7\left(1 - \frac{1}{k^2}\right)\ \text{m}^{-1}$$

$$k = 2:\ \frac{1}{\lambda} = 1.0968 \times 10^7\left(1 - \frac{1}{4}\right)\ \text{m}^{-1}$$

$$\lambda = 1.216 \times 10^{-7}\ \text{m} = 121.6\ \text{nm}\ \ \text{(Ultraviolet)}$$

$$k = 3:\ \frac{1}{\lambda} = 1.0968 \times 10^7\left(1 - \frac{1}{9}\right)\ \text{m}^{-1}$$

$$\lambda = 1.026 \times 10^{-7}\ \text{m} = 102.6\ \text{nm}\ \ \text{(Ultraviolet)}$$

Because the wavelengths are decreasing for higher k values, all the wavelengths in the Lyman series are in the ultraviolet region and not visible by eye.

For the Balmer series ($n = 2$) we find

$$k = 3:\ \frac{1}{\lambda} = 1.0968 \times 10^7\left(\frac{1}{4} - \frac{1}{9}\right)\ \text{m}^{-1}$$

$$\lambda = 6.565 \times 10^{-7}\ \text{m} = 656.5\ \text{nm}\ \ (\text{Red})$$

$$k = 4:\ \frac{1}{\lambda} = 1.0968 \times 10^7\left(\frac{1}{4} - \frac{1}{16}\right)\ \text{m}^{-1}$$

$$\lambda = 4.863 \times 10^{-7}\ \text{m} = 486.3\ \text{nm}\ \ (\text{Blue-green})$$

$$k = 5:\ \frac{1}{\lambda} = 1.0968 \times 10^7\left(\frac{1}{4} - \frac{1}{25}\right)\ \text{m}^{-1}$$

$$\lambda = 4.342 \times 10^{-7}\ \text{m} = 434.2\ \text{nm}\ \ (\text{Violet})$$

$$k = 6:\ \frac{1}{\lambda} = 1.0968 \times 10^7\left(\frac{1}{4} - \frac{1}{36}\right)\ \text{m}^{-1}$$

$$\lambda = 4.103 \times 10^{-7}\ \text{m} = 410.3\ \text{nm}\ \ (\text{Violet})$$

$$k = 7:\ \frac{1}{\lambda} = 1.0968 \times 10^7\left(\frac{1}{4} - \frac{1}{49}\right)\ \text{m}^{-1}$$

$$\lambda = 3.971 \times 10^{-7}\ \text{m} = 397.1\ \text{nm}\ \ (\text{Ultraviolet})$$

Therefore $k = 7$ and higher k values will be in the ultraviolet region. The four lines $k = 3$, 4, 5, and 6 of the Balmer series are visible, although the 410-nm ($k = 6$) line is difficult to see because it is barely in the visible region and is weak in intensity.

The next series, $n = 3$, named after Paschen, has wavelengths of

$$k = 4:\quad \frac{1}{\lambda} = 1.0968 \times 10^7\left(\frac{1}{9} - \frac{1}{16}\right)\ \text{m}^{-1}$$

$$\lambda = 1.876 \times 10^{-6}\ \text{m} = 1876\ \text{nm}\ \ (\text{Infrared})$$

$$k = 5:\quad \frac{1}{\lambda} = 1.0968 \times 10^7\left(\frac{1}{9} - \frac{1}{25}\right)\ \text{m}^{-1}$$

$$\lambda = 1.282 \times 10^{-6}\ \text{m} = 1282\ \text{nm}\ \ (\text{Infrared})$$

$$k = \infty:\quad \frac{1}{\lambda} = 1.0968 \times 10^7\left(\frac{1}{9} - \frac{1}{\infty}\right)\ \text{m}^{-1}$$

$$\lambda = 8.206 \times 10^{-7}\ \text{m} = 820.6\ \text{nm}\ \ (\text{Infrared})$$

Thus the Paschen series has wavelengths entirely in the infrared region. The *series limit* is the smallest wavelength that can occur for each series (see Problem 9). Notice that the series limit is found for $k = \infty$ and is equal to 820.6 nm for the Paschen series. The higher series, $n \geq 4$, will all have wavelengths above the visible region.

3.4 Quantization

As we discussed in Chapter 1, some early Greek philosophers believed that matter must be composed of fundamental units that could not be further divided. The word *atom* means "not further divisible." Today some scientists believe, as these ancient philosophers did, that matter must eventually be indivisible. However, as we have encountered new experimental facts, our ideas about the fundamental, indivisible "building blocks" of matter have changed. "Elementary" particles are discussed further in Chapter 14.

Is matter indivisible?

Whatever the elementary units of matter may turn out to be, we suppose there are some basic units of mass-energy of which matter is composed. This idea is hardly foreign to us: we have already seen that Millikan's oil-drop experiment showed the quantization of electric charge. Modern theories predict that charges are quantized in units (called **quarks**) of $\pm e/3$ and $\pm 2e/3$, but quarks are not directly observed experimentally. The charges of particles that have been directly observed are quantized in units of $\pm e$.

Electric charge is quantized

In nature we see other examples of quantization. The measured atomic weights are not continuous—they have only discrete values, which are close to integral multiples of a unit mass. Molecules are formed from an integral number of atoms. The water molecule is made up of exactly two atoms of hydrogen and one of oxygen. The fact that an organ pipe produces one fundamental musical note with overtones is a form of quantization arising from fitting a precise number (or fractions) of sound waves into the pipe.

Quantization occurs often in nature

Line spectra provide a prime example of quantization. We have learned that the hydrogen line spectra have precise wavelengths that can be described empirically by simple equations. We will see in the next chapter that Niels Bohr used some simple assumptions based on the new quantum theory to model the atom and successfully predict these wavelengths. By the end of the nineteenth century radiation spectra had been well studied. There certainly didn't appear to be any quantization effects observed in blackbody radiation spectra emitted by hot bodies. However, the explanation of blackbody radiation spectra was to have a tremendous influence on the discovery of quantum physics.

3.5 Blackbody Radiation

It has been known for many centuries that when matter is heated, it emits radiation. We can feel heat radiation emitted by the heating element of an electric stove as it warms up. As the heating element reaches 550°C, its color becomes dark red, turning to bright red around 700°C. If the temperature were increased still further, the color would progress through orange, yellow, and finally white. We can determine experimentally that a broad spectrum of wavelengths is emitted when matter is heated. This process was of great interest to physicists of the nineteenth century. They measured the intensity of radiation being emitted as a function of material, temperature, and wavelength.

All bodies simultaneously emit and absorb radiation. When a body's temperature is constant in time, the body is said to be in *thermal equilibrium* with its surroundings. In order for the temperature to be constant, the body must absorb thermal energy at the same rate as it emits it. This implies that a good thermal emitter is also a good absorber.

Radiation emission and absorption

Blackbody radiation is unique

Physicists generally try to study first the simplest or most idealized case of a problem to gain the insight needed to analyze more complex situations. For thermal radiation the simplest case is a **blackbody**, which has the ideal property that it absorbs all the radiation falling on it and reflects none. The simplest way to construct a blackbody is to drill a small hole in the wall of a hollow container as shown in Figure 3.8. Radiation entering the hole will be reflected around inside the container and then eventually absorbed. Only a small fraction of the entering rays will be reemitted through the hole. If the blackbody is in thermal equilibrium, then it must also be an excellent emitter of radiation.

Blackbody radiation is theoretically interesting because of its universal character: the radiation properties of the blackbody (that is, the cavity) are independent of the particular material of which the container is made. Physicists can study the previously mentioned properties of intensity versus wavelength (called *spectral distribution*) at fixed temperatures without having to understand the details of emission or absorption by a particular kind of atom. The question of precisely what the thermal radiation actually consisted of was also of interest, although it was assumed, for lack of evidence to the contrary (and correctly, it turned out!), to be electromagnetic radiation.

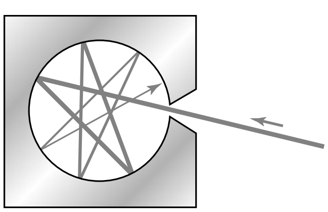


Figure 3.8 Blackbody radiation. Electromagnetic radiation (for example, light) entering a small hole reflects around inside the container before eventually being absorbed.

The intensity $\mathcal{I}(\lambda, T)$ is the total power radiated per unit area per unit wavelength at a given temperature. Measurements of $\mathcal{I}(\lambda, T)$ for a blackbody are displayed in Figure 3.9. Two important observations should be noted:

1. The maximum of the distribution shifts to smaller wavelengths as the temperature is increased.
2. The total power radiated increases with the temperature.

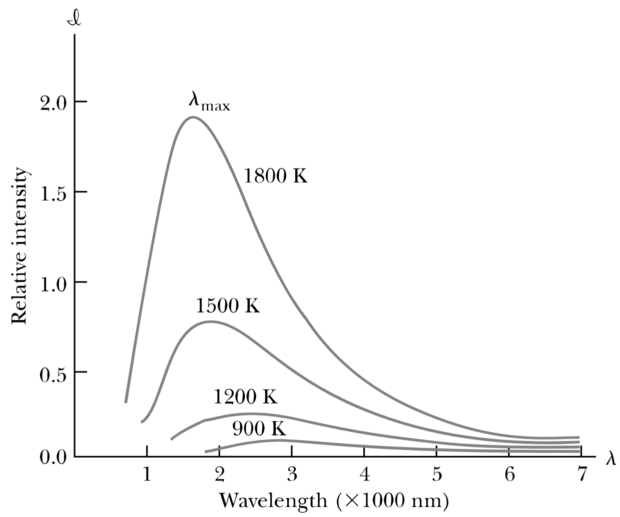


Figure 3.9 Spectral distribution of radiation emitted from a blackbody for different blackbody temperatures.

The first observation is expressed in **Wien's displacement law**:

$$\boxed{\lambda_{max}T = 2.898 \times 10^{-3}\ \mathrm{m \cdot K}} \tag{3.14}$$

Wien's displacement law

where λ_{max} is the wavelength of the peak of the spectral distribution at a given temperature. We can see in Figure 3.9 that the position of λ_{max} varies with temperature as prescribed by Equation (3.14). Wilhelm Wien received the Nobel Prize in 1911 for his discoveries concerning radiation. We can quantify the second observation by integrating the quantity $\mathcal{I}(\lambda, T)$ over all wavelengths to find the power per unit area at temperature T.

$$R(T) = \int_0^\infty \mathcal{I}(\lambda, T)\, d\lambda \tag{3.15}$$

Josef Stefan found empirically in 1879, and Boltzmann demonstrated theoretically several years later, that $R(T)$ is related to the temperature by

$$\boxed{R(T) = \epsilon\sigma T^4} \tag{3.16}$$

Stefan-Boltzmann law

This is known as the **Stefan-Boltzmann law**, with the constant σ experimentally measured to be $5.6705 \times 10^{-8}\ \mathrm{W/(m^2 \cdot K^4)}$. The Stefan-Boltzmann law equation can be applied to any material for which the emissivity is known. The **emissivity** ϵ ($\epsilon = 1$ for an idealized blackbody) is simply the ratio of the emissive power of an object to that of an ideal blackbody and is always less than 1. Thus, Equation (3.16) is a useful and valuable relation for practical scientific and engineering work.

EXAMPLE 3.4

A furnace has walls of temperature 1600°C. What is the wavelength of maximum intensity emitted when a small door is opened?

Strategy We assume the furnace with a small door open is a blackbody so that we can determine $\lambda_{\max}$ from Equation (3.14).

Solution We first convert the temperature to kelvin.

$$T = (1600 + 273)\ \text{K} = 1873\ \text{K}$$

Equation (3.14) gives

$$\lambda_{\max}(1873\ \text{K}) = 2.898 \times 10^{-3}\ \text{m} \cdot \text{K}$$

$$\lambda_{\max} = 1.55 \times 10^{-6}\ \text{m} = 1550\ \text{nm}$$

The peak wavelength is in the infrared region.

EXAMPLE 3.5

The wavelength of maximum intensity of the sun's radiation is observed to be near 500 nm. Assume the sun to be a blackbody and calculate (a) the sun's surface temperature, (b) the power per unit area $R(T)$ emitted from the sun's surface, and (c) the energy received by the Earth each day from the sun's radiation.

Strategy (a) We use Equation (3.14) with $\lambda_{\max}$ to determine the sun's surface temperature. (b) We assume the sun is a blackbody. We use the temperature T with Equation (3.16) to determine the power per unit area $R(T)$. (c) Because we know $R(T)$, we can determine the amount of the sun's energy intercepted by the Earth each day.

Solution (a) From Equation (3.14) we calculate the sun's surface temperature with $\lambda_{\max} = 500$ nm.

$$(500\ \text{nm})T_{\text{sun}} = 2.898 \times 10^{-3}\ \text{m} \cdot \text{K}\frac{10^9\ \text{nm}}{\text{m}}$$

$$T_{\text{sun}} = \frac{2.898 \times 10^6}{500}\ \text{K} = 5800\ \text{K} \tag{3.17}$$

(b) The power per unit area $R(T)$ radiated by the sun is

$$R(T) = \sigma T^4 = 5.67 \times 10^{-8}\frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}(5800\ \text{K})^4$$

$$= 6.42 \times 10^7\ \text{W/m}^2 \tag{3.18}$$

(c) Because this is the power per unit surface area, we need to multiply it by $4\pi r^2$, the surface area of the sun. The radius of the sun is 6.96×10^5 km.

$$\text{Surface area (sun)} = 4\pi(6.96 \times 10^8\ \text{m})^2 = 6.09 \times 10^{18}\ \text{m}^2$$

Thus the total power, P_{sun}, radiated from the sun's surface is

$$P_{\text{sun}} = 6.42 \times 10^7\frac{\text{W}}{\text{m}^2}(6.09 \times 10^{18}\ \text{m}^2)$$

$$= 3.91 \times 10^{26}\ \text{W} \tag{3.19}$$

The fraction F of the sun's radiation received by Earth is given by the fraction of the total area over which the radiation is spread.

$$F = \frac{\pi r_E^2}{4\pi R_{Es}^2}$$

where r_E = radius of Earth = 6.37×10^6 m, and R_{Es} = mean Earth-sun distance = 1.49×10^{11} m. Then

$$F = \frac{\pi r_E^2}{4\pi R_{Es}^2} = \frac{(6.37 \times 10^6\ \text{m})^2}{4(1.49 \times 10^{11}\ \text{m})^2} = 4.57 \times 10^{-10}$$

Thus the radiation received by the Earth from the sun is

$$P_{\text{Earth}}(\text{received}) = (4.57 \times 10^{-10})(3.91 \times 10^{26}\ \text{W})$$

$$= 1.79 \times 10^{17}\ \text{W}$$

and in one day the Earth receives

$$U_{\text{Earth}} = 1.79 \times 10^{17}\ \frac{\text{J}}{\text{s}}\ \frac{60\ \text{s}}{\text{min}}\ \frac{60\ \text{min}}{\text{h}}\ \frac{24\ \text{h}}{\text{day}}$$

$$= 1.55 \times 10^{22}\ \text{J} \tag{3.20}$$

The power received by the Earth per unit of exposed area is

$$R_{\text{Earth}} = \frac{1.79 \times 10^{17}\ \text{W}}{\pi(6.37 \times 10^6\ \text{m})^2} = 1400\ \text{W/m}^2 \tag{3.21}$$

This is the source of most of our energy on Earth. Measurements of the sun's radiation outside the Earth's atmosphere give a value near 1400 W/m^2, so our calculation is fairly accurate. Apparently the sun does act as a blackbody, and the energy received by the Earth comes primarily from the surface of the sun.

Attempts to understand and derive from basic principles the shape of the blackbody spectral distribution (Figure 3.9) were unsuccessful throughout the 1890s and presented a serious dilemma to the best scientists of the day. The nature of the dilemma can be understood from classical electromagnetic theory, together with statistical thermodynamics. The radiation emitted from a blackbody can be expressed as a superposition of electromagnetic waves of different frequencies within the cavity. That is, radiation of a given frequency is represented by a standing wave inside the cavity. The equipartition theorem of thermodynamics (Chapter 9) assigns equal average energy kT to each possible wave configuration.

Lord Rayleigh used the classical theories of electromagnetism and thermodynamics to show in June 1900 that the blackbody spectral distribution should have a $1/\lambda^4$ dependence, which is completely inconsistent with the experimental result at low wavelength shown in Figure 3.9. Later, in 1905, after Sir James Jeans helped Rayleigh determine the factor in front of this distribution, they presented their complete result to be

$$\mathcal{I}(\lambda, T) = \frac{2\pi ckT}{\lambda^4} \tag{3.22}$$

Rayleigh-Jeans formula

This result is known as the **Rayleigh-Jeans formula**, and it is the best formulation that classical theory can provide to describe blackbody radiation. For long wavelengths there are few configurations through which a standing wave can form inside the cavity. However, as the wavelength becomes shorter the number of standing wave possibilities increases, and as $\lambda \to 0$, the number of possible configurations increases without limit. This means the total energy of all configurations is infinite, because each standing wave configuration has the nonzero energy kT. We show a graph of the Rayleigh-Jeans result compared with experimental data in Figure 3.10, and although the prediction approaches the data at long wavelengths, it deviates badly at short wavelengths. In 1911 Paul Ehrenfest dubbed this situation the "ultraviolet catastrophe," and it was one of the outstanding exceptions that classical physics could not explain.

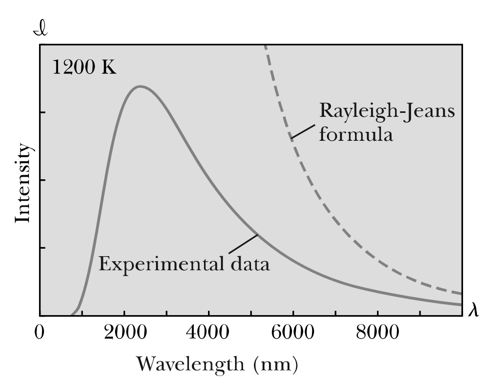
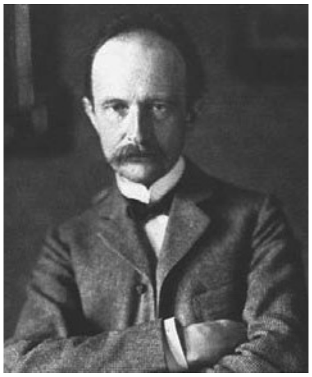


Figure 3.10 The spectral distribution calculated by the Rayleigh-Jeans formula is compared with blackbody radiation experimental data at 1200 K. The formula approaches the data at large wavelengths but disagrees badly at low wavelengths.



AIP/Emilio Segrè Visual Archives

Max Planck (1858–1947) spent most of his productive years as a professor at the University of Berlin (1889–1928). Planck was one of the early theoretical physicists and did work in optics, thermodynamics, and statistical mechanics. His theory of the *quantum of action* was slow to be accepted. Finally, after Einstein's photoelectric effect explanation and Rutherford and Bohr's atomic model, Planck's contribution became widely acclaimed. He received many honors, among them the Nobel Prize in Physics in 1918.

In the 1880s the German Max Planck, who was an expert on the second law of thermodynamics, rejected Boltzmann's statistical version of thermodynamics and even doubted the atomic theory of matter or "atomism." Planck was appointed Professor of Physics at the University of Berlin in 1889, and his views began to change. He was not quite ready to accept atomism, but he set out in 1895 to examine the irreversibility of radiation processes. He thought he had shown that laws of electromagnetism distinguished between past and present, but Boltzmann showed in 1897 that there could be no difference. Planck then began to consider blackbody radiation. Planck tried various functions of wavelength and temperature until he found a single formula that fit the measurements of $\mathcal{I}(\lambda, T)$ over the entire wavelength range. It is not clear that Planck was even aware of Lord Rayleigh's result. Planck was simply looking for a formula that fit the known blackbody spectral distribution. Planck reported his formula in October 1900, but he realized a month later it was nothing but an inspired guess. By then Planck had accepted Boltzmann's view. Planck followed Hertz's work using oscillators to confirm the existence of Maxwell's electromagnetic waves, and lacking detailed information about the atomic composition of the cavity walls, Planck assumed that the radiation in the cavity was emitted (and absorbed) by some sort of "oscillators" that were contained in the walls. When adding up the energies of the oscillators, he assumed (for convenience) that each one had an energy that was an integral multiple of hf, where f is the frequency of the oscillating wave and h is a constant. He was applying a technique invented by Boltzmann, and Planck ultimately expected to take the limit $h \to 0$, to include all the possibilities. However, he noticed that by keeping h nonzero, he arrived at the equation needed for $\mathcal{I}(\lambda, T)$:

Planck's radiation law

$$\mathcal{I}(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \tag{3.23}$$

Equation (3.23) is **Planck's radiation law,** which he reported in December 1900. The derivation of Equation (3.23) is sufficiently complicated that we have omitted it here, but we revisit it in Chapter 9. No matter what Planck tried, he could arrive at agreement with the experimental data only by making two important modifications of classical theory:

Planck's constant h

1. The oscillators (of electromagnetic origin) can only have certain discrete energies determined by $E_n = nhf$, where n is an integer, f is the frequency, and h is called **Planck's constant** and has the value

$$h = 6.6261 \times 10^{-34}\ \mathrm{J \cdot s} \tag{3.24}$$

2. The oscillators can absorb or emit energy in discrete multiples of the fundamental quantum of energy given by

$$\Delta E = hf \tag{3.25}$$

Planck found these results quite disturbing and spent several years trying to find a way to keep the agreement with experiment while letting $h \to 0$. Each attempt failed, and Planck's quantum result became one of the cornerstones of modern science.

EXAMPLE 3.6

Show that Wien's displacement law follows from Planck's radiation law.

Strategy Wien's law, Equation (3.14), refers to the wavelength for which $\mathcal{I}(\lambda, T)$ is a maximum for a given temperature. From calculus we know we can find the maximum value of a function for a certain parameter by taking the derivative of the function with respect to the parameter, set the derivative to zero, and solve for the parameter.

Solution Therefore, to find the value of the Planck radiation law for a given wavelength we set $d\mathcal{I}/d\lambda = 0$ and solve for λ.

$$\frac{d\mathcal{I}(\lambda, T)}{d\lambda} = 0 \quad \text{for } \lambda = \lambda_{\text{max}}$$

$$2\pi c^2 h \frac{d}{d\lambda}\left[\lambda^{-5}(e^{hc/\lambda kT} - 1)^{-1}\right]\Bigg|_{\lambda_{\text{max}}} = 0$$

$$-5\lambda_{\text{max}}^{-6}(e^{hc/\lambda_{\text{max}}kT} - 1)^{-1} - \lambda_{\text{max}}^{-5}(e^{hc/\lambda_{\text{max}}kT} - 1)^{-2} \times \left(\frac{-hc}{kT\lambda_{\text{max}}^2}\right)e^{hc/\lambda_{\text{max}}kT} = 0$$

Multiplying by $\lambda_{\text{max}}^6(e^{hc/\lambda_{\text{max}}kT} - 1)$ results in

$$-5 + \frac{hc}{\lambda_{\text{max}}kT}\left(\frac{e^{hc/\lambda_{\text{max}}kT}}{e^{hc/\lambda_{\text{max}}kT} - 1}\right) = 0$$

Let

$$x = \frac{hc}{\lambda_{\text{max}}kT}$$

Then

$$-5 + \frac{xe^x}{e^x - 1} = 0$$

and

$$xe^x = 5(e^x - 1)$$

This is a transcendental equation and can be solved numerically (try it!) with the result $x \approx 4.966$, and therefore

$$\frac{hc}{\lambda_{\text{max}}kT} = 4.966$$

$$\lambda_{\text{max}}T = \frac{hc}{4.966\, k} = \frac{1240 \text{ eV}\cdot\text{nm}}{4.966\left(8.617 \times 10^{-5}\dfrac{\text{eV}}{\text{K}}\right)}\frac{10^{-9}\text{ m}}{\text{nm}}$$

and finally,

$$\lambda_{\text{max}}T = 2.898 \times 10^{-3} \text{ m}\cdot\text{K}$$

which is the empirically determined Wien's displacement law.

EXAMPLE 3.7

Use Planck's radiation law to find the Stefan-Boltzmann law.

Strategy We determine $R(T)$ by integrating $\mathcal{I}(\lambda, T)$ over all wavelengths.

Solution

$$R(T) = \int_0^\infty \mathcal{I}(\lambda, T)\, d\lambda$$

$$= 2\pi c^2 h \int_0^\infty \frac{1}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}\, d\lambda$$

Let

$$x = \frac{hc}{\lambda kT}$$

Then

$$dx = -\frac{hc}{kT}\frac{d\lambda}{\lambda^2}$$

Now we have

$$R(T) = -2\pi c^2 h \int_\infty^0 \left(\frac{kT}{hc}\right)^6 x^5 \frac{1}{e^x - 1}\frac{1}{x^2}\left(\frac{hc}{kT}\right)^2 dx$$

$$= +2\pi c^2 h\left(\frac{kT}{hc}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1}\, dx$$

We look up this integral in Appendix 7 and find it to be $\pi^4/15$.

$$R(T) = 2\pi c^2 h\left(\frac{kT}{hc}\right)^4 \frac{\pi^4}{15}$$

$$R(T) = \frac{2\pi^5 k^4}{15h^3c^2}T^4$$

Putting in the values for the constants k, h, and c results in

$$R(T) = 5.67 \times 10^{-8}\, T^4 \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}$$

EXAMPLE 3.8

Show that the Planck radiation law agrees with the Rayleigh-Jeans formula for large wavelengths.

Strategy We use Equation (3.23) for the Planck radiation law, let $\lambda \to \infty$ for the term involving the exponential, and see whether the result agrees with Equation (3.22).

Solution We follow the strategy and find the result for the term involving the exponential.

$$\frac{1}{e^{hc/\lambda kT} - 1} = \frac{1}{\left[1 + \frac{hc}{\lambda kT} + \left(\frac{hc}{\lambda kT}\right)^2 \frac{1}{2} + \cdots\right] - 1} \to \frac{\lambda kT}{hc}$$

for large λ

Equation (3.23) now becomes

$$\mathcal{I}(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{\lambda kT}{hc} = \frac{2\pi ckT}{\lambda^4}$$

which is the same as the Rayleigh-Jeans result in Equation (3.22).

EXAMPLE 3.9

Show that Planck's radiation law resolves the *ultraviolet catastrophe*.

Strategy The ultraviolet catastrophe occurs because the number of configurations through which a standing wave can form inside the cavity becomes infinite as $\lambda \to 0$. We want to find out what happens to $\mathcal{I}(\lambda, T)$ if we let $\lambda \to 0$. We also need to investigate the total energy of the system, especially for the large number of small-wavelength oscillators.

Solution If we let $\lambda \to 0$ in Equation (3.23), the value of $e^{hc/\lambda kT} \to \infty$. The exponential term dominates the λ^5 term as $\lambda \to 0$, so the denominator in Equation (3.23) is infinite, and the value of $\mathcal{I}(\lambda, T) \to 0$. Note that as the wavelength decreases, the frequency increases ($f = c/\lambda$), and $hf \gg kT$. Few oscillators will be able to obtain such large energies, partly because of the large energy necessary to take the energy step from 0 to hf. The probability of occupying the states with small wavelengths (large frequency and high energy) is vanishingly small, so the total energy of the system remains finite. The ultraviolet catastrophe is avoided.

3.6 Photoelectric Effect

Perhaps the most compelling, and certainly the simplest, evidence for the quantization of radiation energy comes from the only acceptable explanation of the **photoelectric effect**. While Heinrich Hertz was performing his famous experiment in 1887 that confirmed Maxwell's electromagnetic wave theory of light, he noticed that when ultraviolet light fell on a metal electrode, a charge was produced that separated the leaves of his electroscope. Although Hertz recognized this discovery of what would become known as the photoelectric effect, it was of little use to him at the time, and he left the exploitation of the effect to others, particularly Philipp Lenard. The photoelectric effect is one of several ways in which electrons can be emitted by materials. By the early 1900s it was known that electrons are bound to matter. The valence electrons in metals are "free"—they are able to move easily from atom to atom but are not able to leave the surface of the material. The methods known now by which electrons can be made to completely leave the material include

Table 3.3 Work Functions

Element	ϕ (eV)	Element	ϕ (eV)	Element	ϕ (eV)
Ag	4.64	K	2.29	Pd	5.22
Al	4.20	Li	2.93	Pt	5.64
C	5.0	Na	2.36	W	4.63
Cs	1.95	Nd	3.2	Zr	4.05
Cu	4.48	Ni	5.22		
Fe	4.67	Pb	4.25		

From *Handbook of Chemistry and Physics*, 90th ed. Boca Raton, Fla.: CRC Press (2009–10), pp. 12-114.

Methods of electron emission

1. Thermionic emission: Application of heat allows electrons to gain enough energy to escape.
2. Secondary emission: The electron gains enough energy by transfer from a high-speed particle that strikes the material from outside.
3. Field emission: A strong external electric field pulls the electron out of the material.
4. Photoelectric effect: Incident light (electromagnetic radiation) shining on the material transfers energy to the electrons, allowing them to escape.

It is not surprising that electromagnetic radiation interacts with electrons within metals and gives the electrons increased kinetic energy. Because electrons in metals are weakly bound, we expect that light can give electrons enough extra kinetic energy to allow them to escape. We call the ejected electrons **photoelectrons**. The minimum extra kinetic energy that allows electrons to escape the material is called the **work function** ϕ. The work function is the minimum binding energy of the electron to the material (see Table 3.3 for work function values for several elements).

Photoelectrons

Work function

Experimental Results of Photoelectric Effect

Experiments carried out around 1900 showed that photoelectrons are produced when visible and/or ultraviolet light falls on clean metal surfaces. Photoelectricity was studied using an experimental apparatus shown schematically in Figure 3.11. Incident light falling on the **emitter** (also called the **photocathode** or **cathode**)

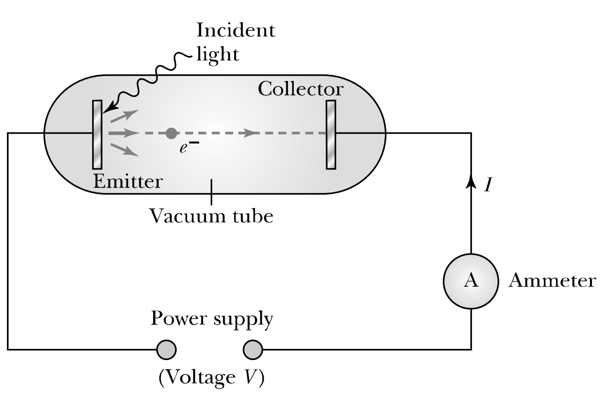


Figure 3.11 Photoelectric effect. Electrons emitted when light shines on a surface are collected, and the photocurrent I is measured. A negative voltage, relative to that of the emitter, can be applied to the collector. When this retarding voltage is sufficiently large, the emitted electrons are repelled, and the current to the collector drops to zero.

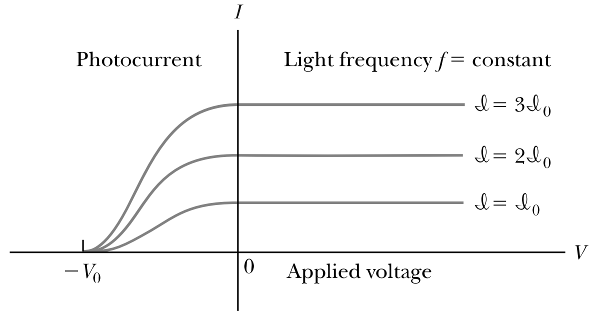


Figure 3.12 The photoelectric current I is shown as a function of the voltage V applied between the emitter and collector for a given frequency f of light for three different light intensities. Notice that no current flows for a retarding potential more negative than $-V_0$ and that the photocurrent is constant for potentials near or above zero (this assumes that the emitter and collector are closely spaced or in spherical geometry to avoid loss of photoelectrons).

ejects electrons. Some of the electrons travel toward the **collector** (also called the **anode**), where either a negative (retarding) or positive (accelerating) applied voltage V is imposed by the power supply. The current I measured in the ammeter (photocurrent) arises from the flow of photoelectrons from emitter to collector.

The pertinent experimental facts about the photoelectric effect are these:

Photoelectric experimental results

1. The kinetic energies of the photoelectrons are independent of the light intensity. In other words, a stopping potential (applied voltage) of $-V_0$ is sufficient to stop all photoelectrons, *no matter what the light intensity*, as shown in Figure 3.12. For a given light intensity there is a maximum photocurrent, which is reached as the applied voltage increases from negative to positive values.
2. The maximum kinetic energy of the photoelectrons, for a given emitting material, depends only on the frequency of the light. In other words, for light of different frequency (Figure 3.13) a different retarding potential $-V_0$ is required to stop the most energetic photoelectrons. The value of V_0 depends on the frequency f but not on the intensity (see Figure 3.12).
3. The smaller the work function ϕ of the emitter material, the lower is the threshold frequency of the light that can eject photoelectrons. No photoelectrons are produced for frequencies below this threshold frequency, no matter what the intensity. Data similar to Millikan's results (discussed later)

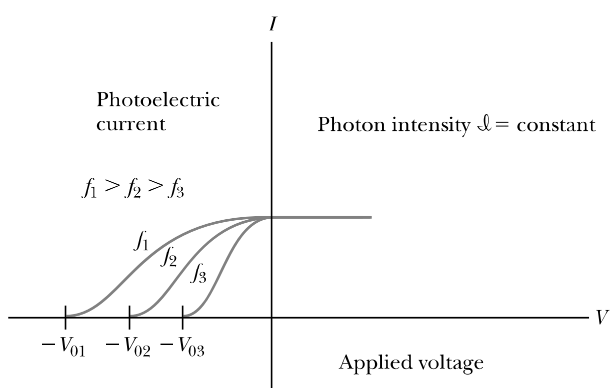


Figure 3.13 The photoelectric current I is shown as a function of applied voltage for three different light frequencies. The retarding potential $-V_0$ is different for each f and is more negative for larger f.

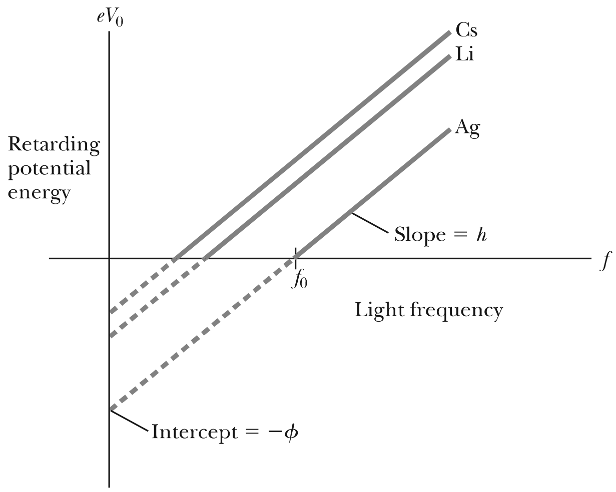


Figure 3.14 The retarding potential energy eV_0 (maximum electron kinetic energy) is plotted versus light frequency for three emitter materials.

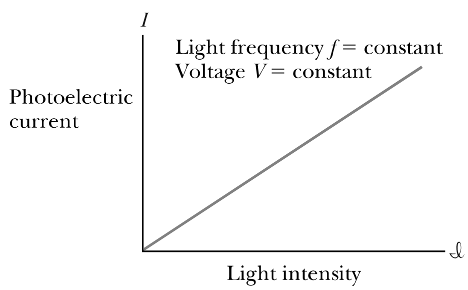


Figure 3.15 The photoelectric current I is a linear function of the light intensity for a constant f and V.

are shown in Figure 3.14, where the threshold frequencies f_0 are measured for three metals.

4. When the photoelectrons are produced, however, their number is proportional to the intensity of light as shown in Figure 3.15. That is, the maximum photocurrent is proportional to the light intensity.
5. The photoelectrons are emitted almost instantly ($\leq 3 \times 10^{-9}$ s) following illumination of the photocathode, independent of the intensity of the light.

Except for result 5, these experimental facts were known in rudimentary form by 1902, primarily due to the work of Philipp Lenard, who had been an assistant to Hertz in 1892 after Hertz had moved from Karlsruhe to Bonn. Lenard, who extensively studied the photoelectric effect, received the Nobel Prize in Physics in 1905 for this and other research on the identification and behavior of electrons.

Classical Interpretation

As stated previously, classical theory allows electromagnetic radiation to eject photoelectrons from matter. However, classical theory predicts that the total amount of energy in a light wave increases as the light intensity increases. Therefore, according to classical theory, the electrons should have more kinetic energy if the light intensity is increased. However, according to experimental result 1 and Figure 3.12, a characteristic retarding potential $-V_0$ is sufficient to stop all photoelectrons for a given light frequency f, no matter what the intensity. Classical electromagnetic theory is unable to explain this result. Similarly, classical theory cannot explain result 2, because the maximum kinetic energy of the photoelectrons depends on the value of the light frequency f and not on the intensity.

The existence of a threshold frequency, shown in experimental result 3, is completely inexplicable in classical theory. Classical theory cannot predict the results shown in Figure 3.14. Classical theory does predict that the number of photoelectrons ejected will increase with intensity in agreement with experimental result 4.

Finally, classical theory would predict that for extremely low light intensities, a long time would elapse before any one electron could obtain sufficient energy to escape. We observe, however, that the photoelectrons are ejected almost immediately. For example, experiments have shown that a light intensity equivalent to the illumination produced over a 1-cm^2 area by a 100-watt incandescent bulb at a distance of 1000 km is sufficient to produce photoelectrons within a second.

EXAMPLE 3.10

Photoelectrons may be emitted from sodium ($\phi = 2.36$ eV) even for light intensities as low as 10^{-8} W/m^2. Calculate classically how much time the light must shine to produce a photoelectron of kinetic energy 1.00 eV.

Strategy We will assume that all of the light is absorbed in the first layer of atoms in the surface. Then we calculate the number of sodium atoms per unit area in a layer one atom thick. We assume that each atom in a single atomic layer absorbs equal energy, but a single electron in each of these atoms receives all the energy. We then calculate how long it takes these electrons to attain the energy (2.36 eV + 1.00 eV = 3.36 eV) needed for the electron to escape.

Solution We first find the number of Na atoms/volume:

$$\frac{\text{Avogadro's number}}{\text{Na gram molecular weight}} \times \text{density} = \frac{\text{number of Na atoms}}{\text{volume}}$$

$$\frac{6.02 \times 10^{23}\ \text{atoms/mol}}{23\ \text{g/mol}} \times 0.97 \frac{\text{g}}{\text{cm}^3} = 2.54 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3} = 2.54 \times 10^{28} \frac{\text{atoms}}{\text{m}^3} \tag{3.26}$$

To estimate the thickness of one layer of atoms, we assume a cubic structure.

$$\frac{1\ \text{atom}}{d^3} = 2.54 \times 10^{28} \frac{\text{atoms}}{\text{m}^3}$$

$$d = 3.40 \times 10^{-10}\ \text{m}$$
$$= \text{thickness of one layer of sodium atoms}$$

If all the light is absorbed in the first layer of atoms, the number of exposed atoms per m^2 is

$$2.54 \times 10^{28} \frac{\text{atoms}}{\text{m}^3} \times 3.40 \times 10^{-10}\ \text{m} = 8.64 \times 10^{18} \frac{\text{atoms}}{\text{m}^2}$$

With the intensity of 10^{-8} W/m^2, each atom will receive energy at the rate of

$$1.00 \times 10^{-8} \frac{\text{W}}{\text{m}^2} \times \frac{1}{8.64 \times 10^{18}\ \text{atoms/m}^2}$$
$$= 1.16 \times 10^{-27}\ \text{W}$$
$$= 1.16 \times 10^{-27} \frac{\text{J}}{\text{s}} \times \frac{1}{1.6 \times 10^{-19}\ \text{J/eV}}$$
$$= 7.25 \times 10^{-9}\ \text{eV/s}$$

If energy is absorbed at the rate of 7.25×10^{-9} eV/s for a single electron, we can calculate the time t needed to absorb 3.36 eV:

$$t = \frac{3.36\ \text{eV}}{7.25 \times 10^{-9}\ \text{eV/s}} = 4.63 \times 10^{8}\ \text{s} = 14.7\ \text{years}$$

Based on classical calculations, the time required to eject a photoelectron should be 15 years!

Einstein's Theory

Albert Einstein was intrigued by Planck's hypothesis that the electromagnetic radiation field must be absorbed and emitted in quantized amounts. Einstein took Planck's idea one step further and suggested that the *electromagnetic radiation field itself is quantized* and that "the energy of a light ray spreading out from a point source is not continuously distributed over an increasing space but consists of a finite number of energy quanta which are localized at points in space, which move without dividing, and which can only be produced and absorbed as complete units."* We now call these energy quanta of light **photons**. According to Einstein each photon has the energy quantum

Photons

$$E = hf \tag{3.27}$$

Energy quantum

where f is the frequency of the electromagnetic wave associated with the light, and h is Planck's constant. Notice that Equation (3.27) is consistent with Planck's relation for quantum of energy presented in Equation (3.25). The photon travels at the speed of light c in a vacuum, and its wavelength is given by

$$\lambda f = c \tag{3.28}$$

In other words, Einstein proposed that in addition to its well-known wavelike aspect, amply exhibited in interference phenomena, *light should also be considered to have a particle-like aspect.* Einstein suggested that the photon (quantum of light) delivers its entire energy hf to a single electron in the material. To leave the material, the struck electron must give up an amount of energy ϕ to overcome its binding in the material. The electron may lose some additional energy by interacting with other electrons on its way to the surface. Whatever energy remains will then appear as kinetic energy of the electron as it leaves the emitter. The conservation of energy requires that

$$hf = \phi + \text{K.E. (electron)} \tag{3.29}$$

Because the energies involved here are on the order of electron volts, we are safe in using the nonrelativistic form of the electron's kinetic energy, $\frac{1}{2}mv^2$. The electron's kinetic energy will be degraded as it passes through the emitter material, so, strictly speaking, we want to experimentally detect the maximum value of the kinetic energy.

$$hf = \phi + \tfrac{1}{2}mv^2_{\text{max}} \tag{3.30}$$

The retarding potentials measured in the photoelectric effect are thus the opposing potentials needed to stop the most energetic electrons.

$$eV_0 = \tfrac{1}{2}mv^2_{\text{max}} \tag{3.31}$$

Quantum Interpretation

We should now reexamine the experimental results of the photoelectric effect to see whether Einstein's quantum interpretation can explain all the data. The first and second experimental results (which indicate that the kinetic energies of

*For an English translation of A. Einstein, *Annalen der Physik* **17**, 132 (1905), see A. B. Arons and M. B. Peppard, *American Journal of Physics* **33**, 367 (1965).

the photoelectrons depend on the light frequency, but not the light intensity) can be explained. The kinetic energy of the electrons, K.E. (electron) $= hf - \phi$ [see Equation (3.29)], does not depend on the light intensity at all, but only on the light frequency and the work function of the material.

$$\frac{1}{2} mv_{\text{max}}^2 = eV_0 = hf - \phi \tag{3.32}$$

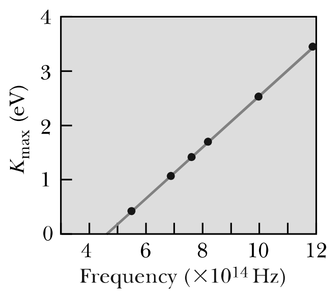


Figure 3.16 Millikan published data in 1916 for the photoelectric effect in which he shone light of varying frequency on a sodium electrode and measured the maximum kinetic energies of the photoelectrons. He found that no photoelectrons were emitted below a frequency of 4.39×10^{14} Hz (or longer than a wavelength of 683 nm). The results were independent of light intensity, and the slope of a straight line drawn through the data produced a value of Planck's constant in excellent agreement with Planck's theory. Even though Millikan admitted his own data were sufficient proof of Einstein's photoelectric effect equation, Millikan was not convinced of the photon concept for light and its role in quantum theory.

A potential slightly more positive than $-V_0$ will not be able to repel all the electrons, and, for a close geometry of the emitter and collector, practically all the electrons will be collected when the retarding voltage is near zero. For very large positive potentials all the electrons will be collected, and the photocurrent levels off as shown in Figure 3.12. If the light intensity increases, there will be more photons per unit area, more electrons ejected, and therefore a higher photocurrent, as displayed in Figure 3.12.

If a different light frequency is used, say f_2, then a different stopping potential is required to stop the most energetic electrons [see Equation (3.32)], $eV_{02} = hf_2 - \phi$. For a constant light intensity (more precisely, a constant number of photons/area/time), a different stopping potential V_0 is required for each f, but the maximum photocurrent will not change, because the number of photoelectrons ejected is constant (see Figure 3.13). The quantum theory easily explains Figure 3.15, because the number of photons increases linearly with the light intensity, producing more photoelectrons and hence more photocurrent.

Equation (3.32), proposed by Einstein in 1905, predicts that the stopping potential will be linearly proportional to the light frequency, with a slope h/e, where *h is the same constant found by Planck*. The slope is independent of the metal used to construct the photocathode. Equation (3.32) can be rewritten

$$eV_0 = \frac{1}{2} mv_{\text{max}}^2 = hf - hf_0 = h(f - f_0) \tag{3.33}$$

where $\phi = hf_0$ represents the negative of the y intercept. The frequency f_0 represents the threshold frequency for the photoelectric effect (when the kinetic energy of the electron is precisely zero). The data available in 1905 were not sufficiently accurate either to prove or disprove Einstein's theory, and even Planck himself, among others, viewed the theory with skepticism. R. A. Millikan, then at the University of Chicago, tried to show that Einstein was wrong by undertaking a series of elegant experiments that required almost 10 years to complete. In 1916 Millikan reported data shown in Figure 3.16 that confirmed Einstein's prediction. Millikan found the value of h from the slope of the line in Figure 3.16 to be 4.1×10^{-15} eV · s, in good agreement with the value of h determined for blackbody radiation by Planck. Einstein's theory of the photoelectric effect was gradually accepted after 1916; finally in 1922 he received the Nobel Prize for the year 1921, primarily for his explanation of the photoelectric effect.*

Quantization of electromagnetic radiation field

We should summarize what we have learned about the quantization of the electromagnetic radiation field. First, electromagnetic radiation consists of photons, which are particle-like (or corpuscular), each consisting of energy

$$E = hf = \frac{hc}{\lambda} \tag{3.34}$$

*R. A. Millikan also received the Nobel Prize in Physics in 1923, partly for his precise study of the photoelectric effect and partly for measuring the charge of the electron. Millikan's award was the last in a series of Nobel Prizes spanning 18 years that honored the fundamental efforts to measure and understand the photoelectric effect: Lenard, Einstein, and Millikan.

where f and λ are the frequency and wavelength of the light, respectively. The total energy of a beam of light is the sum total of the energy of all the photons and for monochromatic light is an integral multiple of hf (generally the integer is very large).

This representation of the photon picture must be true over the entire electromagnetic spectrum from radio waves to visible light, x rays, and even high-energy gamma rays. This must be true because, as we saw in Chapter 2, a photon of given frequency, observed from a moving system, can be redshifted or blueshifted by an arbitrarily large amount, depending on the system's speed and direction of motion. We examine these possibilities later. During emission or absorption of any form of electromagnetic radiation (light, x rays, gamma rays, and so on), photons must be created or absorbed. The photons have only one speed: the speed of light ($= c$ in vacuum).

EXAMPLE 3.11

Light of wavelength 400 nm is incident upon lithium (ϕ = 2.93 eV). Calculate (a) the photon energy and (b) the stopping potential V_0.

Strategy (a) Light is normally described by wavelengths in nm, so it is useful to have an equation to calculate the energy in terms of λ.

$$E = hf = \frac{hc}{\lambda}$$

$$= \frac{(6.626 \times 10^{-34}\,\text{J}\cdot\text{s})(2.998 \times 10^{8}\,\text{m/s})}{\lambda(1.602 \times 10^{-19}\,\text{J/eV})(10^{-9}\,\text{m/nm})}$$

$$\boxed{E = \frac{1.240 \times 10^{3}\,\text{eV}\cdot\text{nm}}{\lambda}} \tag{3.35}$$

(b) We use Equation (3.32) to determine the stopping potential once we know the frequency f and work function ϕ.

Solution (a) For a wavelength of λ = 400 nm we use Equation (3.35) to determine the photon's energy

$$E = \frac{1.240 \times 10^{3}\,\text{eV}\cdot\text{nm}}{400\,\text{nm}} = 3.10\,\text{eV}$$

(b) For the stopping potential, Equation (3.32) gives

$$eV_0 = hf - \phi = E - \phi = 3.10\,\text{eV} - 2.93\,\text{eV} = 0.17\,\text{eV}$$

$$V_0 = 0.17\,\text{V}$$

A retarding potential of 0.17 V will stop all photoelectrons.

EXAMPLE 3.12

(a) What frequency of light is needed to produce electrons of kinetic energy 3.00 eV from illumination of lithium?
(b) Find the wavelength of this light and discuss where it is in the electromagnetic spectrum.

Strategy We have enough information to determine the photon energy needed from Equation (3.30), and we can determine the frequency from $E = hf$.

Solution From Equation (3.30), we have

$$hf = \phi + \tfrac{1}{2} m v_{\max}^2$$

$$= 2.93\,\text{eV} + 3.00\,\text{eV} = 5.93\,\text{eV}$$

The photon frequency is now found to be

$$f = \frac{E}{h} = \frac{(5.93\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV})}{(6.626 \times 10^{-34}\,\text{J}\cdot\text{s})}$$

$$= 1.43 \times 10^{15}\,\text{s}^{-1} = 1.43 \times 10^{15}\,\text{Hz}$$

(b) The wavelength of the light can be found from $c = \lambda f$.

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8}\,\text{m/s}}{1.43 \times 10^{15}\,\text{Hz}} = 2.10 \times 10^{-7}\,\text{m} = 210\,\text{nm}$$

This is ultraviolet light, because the wavelength 210 nm is below the range of visible wavelengths 400 to 700 nm.

EXAMPLE 3.13

For the light intensity of Example 3.10, $\mathscr{I} = 10^{-8}$ W/m^2, a wavelength of 350 nm is used. What is the number of photons/(m$^2 \cdot$ s) in the light beam?

Strategy We first find the photon energy, and because we know the intensity, we will be able to determine the photon flux.

Solution From Equation (3.35) we find the photon energy E:

$$E = \frac{1.240 \times 10^3 \text{ eV}\cdot\text{nm}}{350 \text{ nm}} = 3.5 \text{ eV}$$

The intensity $\mathscr{I}$ is the product of the photon flux N and photon energy E:

$$\text{Intensity } \mathscr{I} = \left[N\left(\frac{\text{photons}}{\text{m}^2\cdot\text{s}}\right)\right]\left[E\left(\frac{\text{energy}}{\text{photon}}\right)\right]$$
$$= NE\left(\frac{\text{energy}}{\text{m}^2\cdot\text{s}}\right)$$

where we have put the units of N and E in parentheses. We solve this for N:

$$N = \frac{\mathscr{I}}{E} = \frac{1.0 \times 10^{-8}\,\text{J}\cdot\text{s}^{-1}\,\text{m}^{-2}}{(1.6 \times 10^{-19}\,\text{J/eV})(3.5\text{ eV/photon})}$$
$$= 1.8 \times 10^{10}\,\frac{\text{photons}}{\text{m}^2\cdot\text{s}}$$

Thus even a low-intensity light beam has a large flux of photons, and even a few photons can produce a photocurrent (albeit a very small one!).

The photoelectric effect is responsible for many applications in the detection of light. These include the photomultiplier tube for counting individual light pulses, photoelectric cells for light-activated devices (such as door openers and intrusion alarms), and solar panels.

3.7 X-Ray Production

In the photoelectric effect, a photon gives up all of its energy to an electron, which may then escape from the material in which it was bound. Can the inverse process occur? Can an electron (or any charged particle) give up its energy and create a photon? The answer is yes, but the process must be consistent with the laws of physics. Recall that photons must be created or absorbed as whole units. A photon cannot give up half its energy; it must give up all its energy. If in some physical process only part of the photon's energy were required, then a *new* photon would be created to carry away the remaining energy.

Unlike a photon, an electron may give up part or all of its kinetic energy and still be the same electron. When an electron interacts with the strong electric field of the atomic nucleus and is consequently accelerated, the electron radiates electromagnetic energy. According to classical electromagnetic theory, it should do so continuously. In the quantum picture we must think of the electron as emitting a series of photons with varying energies; this is the only way that the inverse photoelectric effect can occur. An energetic electron passing through matter will radiate photons and lose kinetic energy. The process by which photons are emitted by an electron slowing down is called **bremsstrahlung**, from the German word for "braking radiation." The process is shown schematically in Figure 3.17 where an electron of energy E_i passing through the electric field of

Bremsstrahlung process

a nucleus is accelerated and produces a photon of energy $E = hf$. The final energy of the electron is determined from the conservation of energy to be

$$E_f = E_i - hf \tag{3.36}$$

Because linear momentum must be conserved, the nucleus absorbs very little energy, and it is ignored. One or more photons may be created in this way as electrons pass through matter.

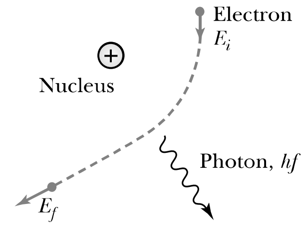


Figure 3.17 Bremsstrahlung is a process through which an electron is accelerated while under the influence of the nucleus. The accelerated electron emits a photon.

In Section 3.1 we mentioned Röntgen's discovery of x rays. The x rays are produced by the bremsstrahlung effect in an apparatus shown schematically in Figure 3.18. Current passing through a filament produces copious numbers of electrons by thermionic emission. These electrons are focused by the cathode structure into a beam and are accelerated by potential differences of thousands of volts until they impinge on a metal anode surface, producing x rays by bremsstrahlung (and other processes) as they stop in the anode material. Much of the electron's kinetic energy is lost by heating the anode material and not by bremsstrahlung. The x-ray tube is evacuated so that the air between the filament and anode will not scatter the electrons. The x rays produced pass through the sides of the tube and can be used for a large number of applications, including medical diagnosis and therapy, fundamental research in crystal and liquid structure, and engineering diagnoses of flaws in large welds and castings.

X rays from a standard tube include photons of many wavelengths. By scattering x rays from crystals we can produce strongly collimated monochromatic (single-wavelength) x-ray beams. Early x-ray spectra produced by x-ray tubes of accelerating potential 35 kV are shown in Figure 3.19. These particular tubes had targets of tungsten, molybdenum, and chromium. The smooth, continuous x-ray spectra are those produced by bremsstrahlung, and the sharp "characteristic x rays" are produced by atomic excitations and are explained in Section 4.6. X-ray wavelengths typically range from 0.01 to 1 nm. However, high-energy accelerators can produce x rays with wavelengths as short as 10^{-6} nm.

Notice that in Figure 3.19 the minimum wavelength $\lambda_{\min}$ for all three targets is the same. The minimum wavelength $\lambda_{\min}$ corresponds to the maximum frequency $f_{\max}$. If the electrons are accelerated through a voltage V_0, then their kinetic energy is eV_0. The maximum photon energy therefore occurs when the electron gives up all of its kinetic energy and creates one photon (this is

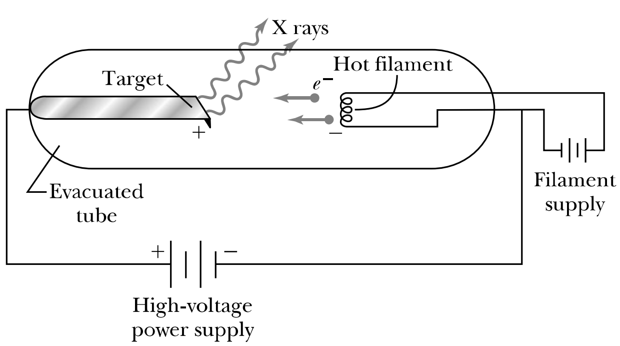


Figure 3.18 Schematic of x-ray tube where x rays are produced by the bremsstrahlung process of energetic electrons.

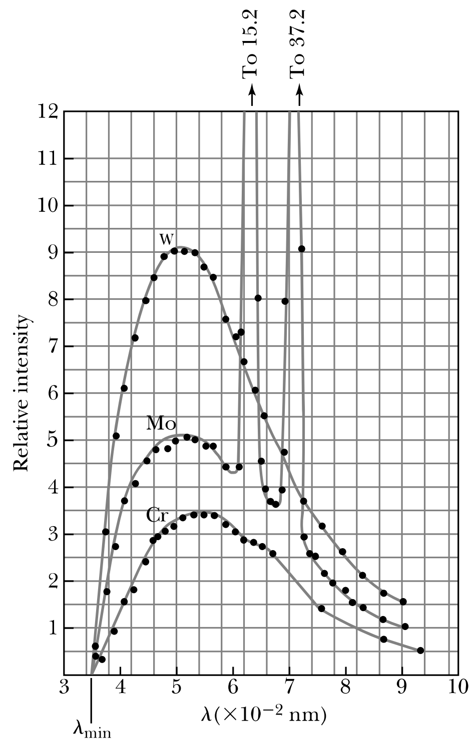


Figure 3.19 The relative intensity of x rays produced in an x-ray tube is shown for an accelerating voltage of 35 kV. Notice that $\lambda_{\min}$ is the same for all three targets. *From C. T. Ulrey,* Physical Review ***11,*** *405 (1918).*

relatively unlikely, however). This process is the **inverse photoelectric effect**. The conservation of energy requires that the electron kinetic energy equal the maximum photon energy (where we neglect the work function ϕ because it is normally so small compared with eV_0).

$$eV_0 = hf_{\max} = \frac{hc}{\lambda_{\min}}$$

or

Duane-Hunt rule

$$\lambda_{\min} = \frac{hc}{e}\frac{1}{V_0} = \frac{1.240 \times 10^{-6}\ \mathrm{V \cdot m}}{V_0} \tag{3.37}$$

The relation Equation (3.37) was first found experimentally and is known as the **Duane-Hunt rule** (or **limit**). Its explanation in 1915 by the quantum theory is now considered further evidence of Einstein's photon concept. The value $\lambda_{\min}$ depends only on the accelerating voltage and is the same for all targets.

Only the quantum hypothesis explains all of these data. Because the heavier elements have stronger nuclear electric fields, they are more effective in accelerating electrons and making them radiate. The intensity of the x rays increases with the square of the atomic number of the target. The intensity is also approximately proportional to the square of the voltage used to accelerate the electrons. This is why high voltages and tungsten anodes are so often used in x-ray machines. Tungsten also has a very high melting temperature and can withstand high electron-beam currents.

CONCEPTUAL EXAMPLE 3.14

Explain how the Duane-Hunt rule can be used to determine the electron bombarding energy in a device such as a scanning electron microscope.

Solution If we look closely at Equation (3.37), we can see that any reduction in the acceleration voltage V_0 will lead to an increase in the value of λ_{min}. A careful analysis of the minimum value of the wavelength should be in agreement with the expected voltage V_0. If the value of λ_{min} varies over time, for example depending on the electron beam current, it may be due to anomalous charging effects in the beam acceleration/transport system. Solutions for problems like this require painstaking efforts and may dictate the experimental conditions, such as using lower beam currents to avoid problems.

EXAMPLE 3.15

If we have a tungsten anode (work function $\phi = 4.63$ eV) and electron acceleration voltage of 35 kV, why do we ignore in Equation (3.36) the initial kinetic energy of the electrons from the filament and the work functions of the filaments and anodes? What is the minimum wavelength of the x rays?

Strategy We can ignore the initial electron kinetic energies and the work functions, because they are on the order of a few electron volts (eV), whereas the kinetic energy of the electrons due to the accelerating voltage is 35,000 eV. The error in neglecting everything but eV_0 is small. We will use Equation (3.37) to determine the minimum wavelength.

Solution We use the Duane-Hunt rule of Equation (3.37) to determine

$$\lambda_{\text{min}} = \frac{1.240 \times 10^{-6}\ \text{V}\cdot\text{m}}{35.0 \times 10^{3}\ \text{V}} = 3.54 \times 10^{-11}\ \text{m}$$

which is in good agreement with the data of Figure 3.19.

3.8 Compton Effect

When a photon enters matter, it is likely to interact with one of the atomic electrons. According to classical theory, the electrons will oscillate at the photon frequency because of the interaction of the electron with the electric and magnetic field of the photon and will reradiate electromagnetic radiation (photons) at this same frequency. This is called **Thomson scattering**. However, in the early 1920s Arthur Compton experimentally confirmed an earlier observation by J. A. Gray that, especially at backward-scattering angles, there appeared to be a component of the emitted radiation (called a **modified** wave) that had a longer wavelength than the original primary (**unmodified**) wave. Classical electromagnetic theory cannot explain this modified wave. Compton then attempted to understand theoretically such a process and could find only one explanation: *Einstein's photon particle concept must be correct.* The scattering process is shown in Figure 3.20.

Thomson scattering

Compton proposed in 1923 that the photon is scattered from only one electron, rather than from all the electrons in the material, and that the laws of the conservation of energy and momentum apply as in any elastic collision between two particles. We recall from Chapter 2 that the momentum of a particle moving at the speed of light (photon) is given by

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \tag{3.38}$$

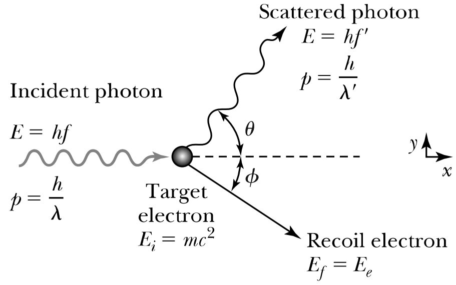


Figure 3.20 Compton scattering of a photon by an electron essentially at rest.

We treat the photon as a particle with a definite energy and momentum. Scattering takes place in a plane, which we take to be the xy plane in Figure 3.20. Both x and y components of momentum must be conserved, because of the vector nature of the linear momentum. The energy and momentum before and after the collision (treated relativistically) are given in Table 3.4. The incident and scattered photons have frequencies f and f', respectively. The recoil electron has energy E_e and momentum p_e.

In the final system the electron's total energy is related to its momentum by Equation (2.70):

$$E_e^2 = (mc^2)^2 + p_e^2c^2 \tag{3.39}$$

We can write the conservation laws now, initial = final, as

$$\text{Energy} \quad hf + mc^2 = hf' + E_e \tag{3.40a}$$

$$p_x \quad \frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + p_e\cos\phi \tag{3.40b}$$

$$p_y \quad \frac{h}{\lambda'}\sin\theta = p_e\sin\phi \tag{3.40c}$$

We will relate the change in wavelength $\Delta\lambda = \lambda' - \lambda$ to the scattering angle θ of the photon. We first eliminate the recoil angle ϕ by squaring Equations (3.40b) and (3.40c) and adding them, resulting in

$$p_e^2 = \left(\frac{h}{\lambda}\right)^2 + \left(\frac{h}{\lambda'}\right)^2 - 2\left(\frac{h}{\lambda}\right)\left(\frac{h}{\lambda'}\right)\cos\theta \tag{3.41}$$

Arthur Compton (1892–1962) is shown here in 1931 looking into an ionization chamber that he designed to study cosmic rays in the atmosphere. Compton received his degrees from the College of Wooster and Princeton University. He spent most of his career at the University of Chicago and Washington University, St. Louis. After his early work with x rays for which he received the Nobel Prize in 1927, he was a pioneer in high-energy physics through his cosmic ray studies. Compton was also a leader in the establishment of the Manhattan Project to produce the atomic bomb during World War II and, afterwards, for nuclear power generation.

Table 3.4 Results of Compton Scattering

Energy or Momentum	Initial System	Final System
Photon energy	hf	hf'
Photon momentum in x direction (p_x)	$\frac{h}{\lambda}$	$\frac{h}{\lambda'}\cos\theta$
Photon momentum in y direction (p_y)	0	$\frac{h}{\lambda'}\sin\theta$
Electron energy	mc^2	$E_e = mc^2 + \text{K.E.}$
Electron momentum in x direction (p_x)	0	$p_e\cos\phi$
Electron momentum in y direction (p_y)	0	$-p_e\sin\phi$

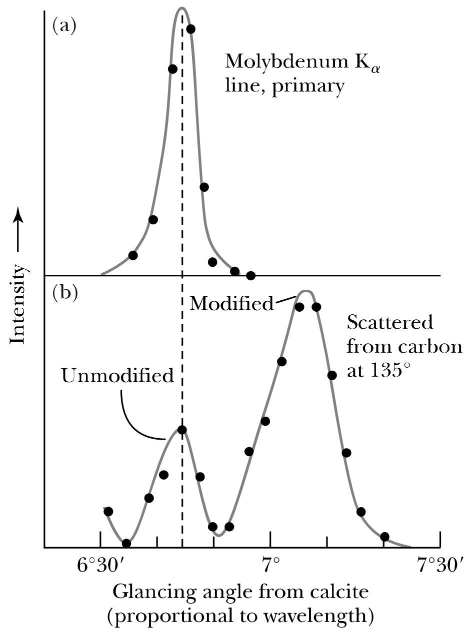


Figure 3.21 Compton's original data showing (a) the primary x-ray beam from Mo unscattered and (b) the scattered spectrum from carbon at 135° showing both the modified and unmodified wave. *Adapted from Arthur H. Compton,* Physical Review *22, 409-413 (1923).*

Then we substitute E_e from Equation (3.40a) and p_e from Equation (3.41) into Equation (3.39) (setting $\lambda = c/f$).

$$[h(f - f') + mc^2]^2 = m^2c^4 + (hf)^2 + (hf')^2 - 2(hf)(hf')\cos\theta$$

Squaring the left-hand side and canceling terms leaves

$$mc^2(f - f') = hff'(1 - \cos\theta)$$

Rearranging terms gives

$$\frac{h}{mc^2}(1 - \cos\theta) = \frac{f - f'}{ff'} = \frac{\dfrac{c}{\lambda} - \dfrac{c}{\lambda'}}{\dfrac{c^2}{\lambda\lambda'}} = \frac{1}{c}(\lambda' - \lambda)$$

or

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta) \tag{3.42}$$

Compton effect

which is the result Compton found in 1923 for the increase in wavelength of the scattered photon.

Compton then proceeded to check the validity of his theoretical result by performing a careful experiment in which he scattered x rays of wavelength 0.071 nm from carbon at several angles. He showed that the modified wavelength was in good agreement with his prediction.* A part of his data is shown in Figure 3.21, where both the modified (λ') and unmodified (λ) scattered waves are identified.

*An interesting personal account of Compton's discovery can be found in A. H. Compton, *American Journal of Physics* **29**, 817–820 (1961).

The kinetic energy and scattering angle of the recoiling electron can also be calculated. Experiments in which the recoiling electrons were detected were soon carried out, thus completely confirming Compton's theory. The process of elastic photon scattering from electrons is now called the **Compton effect**. Note that the difference in wavelength, $\Delta\lambda = \lambda' - \lambda$, depends only on the constants h, c, and m_e in addition to the scattering angle θ. The quantity $\lambda_C = h/m_e c = 2.426 \times 10^{-3}$ nm is called the **Compton wavelength** of the electron. Only for wavelengths on the same order as λ_C (or shorter) will the fractional shift $\Delta\lambda/\lambda$ be large. For visible light, for example with $\lambda = 500$ nm, the maximum $\Delta\lambda/\lambda$ is on the order of 10^{-5} and $\Delta\lambda$ would be difficult to detect. The probability of the occurrence of the Compton effect for visible light is also quite small. However, for x rays of wavelength 0.071 nm used by Compton, the ratio of $\Delta\lambda/\lambda$ is $\sim$0.03 and could easily be observed. Thus, the Compton effect is important only for x rays or γ-ray photons and is small for visible light.

Compton wavelength

The physical process of the Compton effect can be described as follows. The photon elastically scatters from an essentially free electron in the material. (The photon's energy is so much larger than the binding energy of the almost free electron that the atomic binding energy can be neglected.) The newly created scattered photon then has a modified, longer wavelength. What happens if the photon scatters from one of the tightly bound inner electrons? Then the binding energy is not negligible, and the electron might not be dislodged. The scattering in this case is effectively from the entire atom (nucleus + electrons). Then the mass in Equation (3.42) is several thousand times larger than m_e, and $\Delta\lambda$ is correspondingly smaller. Scattering from tightly bound electrons results in the unmodified photon scattering ($\lambda \approx \lambda'$), which is also observed in Figure 3.21. Thus, the quantum picture also explains the existence of the unmodified wavelength predicted by the classical theory (Thomson scattering) alluded to earlier.

The success of the Compton theory convincingly demonstrated the correctness of both the quantum concept and the particle nature of the photon. The use of the laws of the conservation of energy and momentum applied relativistically to pointlike scattering of the photon from the electron finally convinced the great majority of scientists of the validity of the new modern physics. Compton received the Nobel Prize in Physics for this discovery in 1927.

EXAMPLE 3.16

An x ray of wavelength 0.050 nm scatters from a gold target. (a) Can the x ray be Compton-scattered from an electron bound by as much as 62 keV? (b) What is the largest wavelength of scattered photon that can be observed? (c) What is the kinetic energy of the most energetic recoil electron and at what angle does it occur?

Strategy We first determine the x-ray energy to see if it has enough energy to dislodge the electron. We use Equation (3.42) with both the atomic and electron mass to determine the scattered photon wavelength. We then use the conservation of energy to determine the recoil electron kinetic energy.

Solution From Equation (3.35) the x-ray energy is

$$E_{\text{x ray}} = \frac{1.240 \times 10^3 \text{ eV} \cdot \text{nm}}{0.050 \text{ nm}} = 24{,}800 \text{ eV} = 24.8 \text{ keV}$$

Therefore, the x ray does not have enough energy to dislodge the inner electron, which is bound by 62 keV. In this case we have to use the atomic mass in Equation (3.42), which results in little change in the wavelength (Thomson scattering).

Scattering may still occur from outer electrons, so we examine Equation (3.42) with the electron mass. The longest wavelength $\lambda' = \lambda + \Delta\lambda$ occurs when $\Delta\lambda$ is a maximum or when $\theta = 180°$.

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos 180°) = \lambda + \frac{2h}{m_e c}$$

$$= 0.050 \text{ nm} + 2(0.00243 \text{ nm}) = 0.055 \text{ nm}$$

The energy of the scattered photon is then a minimum and has the value

$$E'_{\text{x ray}} = \frac{1.240 \times 10^3 \text{ eV} \cdot \text{nm}}{0.055 \text{ nm}} = 2.25 \times 10^4 \text{ eV} = 22.5 \text{ keV}$$

The difference in energy of the initial and final photon must equal the kinetic energy of the electron (neglecting binding energies). The recoil electron must scatter in the forward direction at $\phi = 0°$ when the final photon is in the backward direction ($\theta = 180°$) to conserve momentum. The kinetic energy of the electron is then a maximum.

$$E_{\text{x ray}} = E'_{\text{x ray}} + \text{K.E. (electron)}$$

$$\text{K.E. (electron)} = E_{\text{x ray}} - E'_{\text{x ray}}$$

$$= 24.8 \text{ keV} - 22.5 \text{ keV} = 2.3 \text{ keV}$$

Because $\Delta\lambda$ does not depend on λ or λ', we can determine the wavelength (and energy) of the incident photon by merely observing the kinetic energy of the electron at forward angles (see Problem 60).

3.9 Pair Production and Annihilation

A guiding principle of scientific investigation, if not a general rule of nature, is that if some process is not absolutely forbidden (by some law such as conservation of energy, momentum, or charge), then we might expect that it will eventually occur. In the photoelectric effect, bremsstrahlung, and the Compton effect, we have studied exchanges of energy between photons and electrons. Have we covered all possible mechanisms? For example, can the kinetic energy of a photon be converted into particle mass and vice versa? It would appear that if none of the conservation laws are violated, then such a process should be possible.

First, let us consider the conversion of photon energy into mass. The electron, which has a mass ($m = 0.511 \text{ MeV}/c^2$), is the lightest particle within an atom. If a photon can create an electron, it must also create a positive charge to balance charge conservation. In 1932, C. D. Anderson (Nobel Prize in Physics, 1936) observed a positively charged electron (e^+) in cosmic radiation. This particle, called a **positron**, had been predicted to exist several years earlier by P. A. M. Dirac (Nobel Prize in Physics, 1933). It has the same mass as the electron but an opposite charge. Positrons are also observed when high-energy gamma rays (photons) pass through matter. Experiments show that a photon's energy can be converted entirely into an electron and a positron in a process called **pair production**. The reaction is

Positron

$$\gamma \rightarrow e^+ + e^- \tag{3.43}$$

Pair production

However, this process occurs only when the photon passes through matter, because energy and momentum would not be conserved if the reaction took place in isolation. The missing momentum must be supplied by interaction with a nearby massive object such as a nucleus.

EXAMPLE 3.17

Show that a photon cannot produce an electron-positron pair in free space as shown in Figure 3.22a.

Strategy We need to look carefully at the conservation of momentum and energy to see whether pair production can occur in free space.

Solution Let the total energy and momentum of the electron and the positron be E_-, p_- and E_+, p_+, respectively. The conservation laws are then

Energy $$hf = E_+ + E_- \tag{3.44a}$$

Momentum, p_x $$\frac{hf}{c} = p_- \cos\theta_- + p_+ \cos\theta_+ \tag{3.44b}$$

Momentum, p_y $$0 = p_- \sin\theta_- - p_+ \sin\theta_+ \tag{3.44c}$$

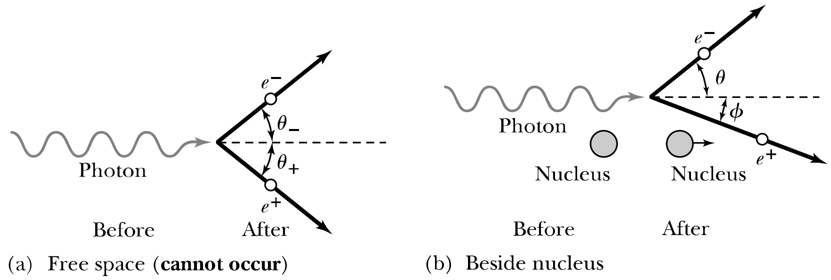


Figure 3.22 (a) A photon cannot decay into an electron-positron pair in free space, but (b) if a nucleus is nearby, the nucleus can absorb sufficient linear momentum to allow the process to proceed.

Equation (3.44b) can be written as

$$hf = p_- c \cos\theta_- + p_+ c \cos\theta_+ \tag{3.45}$$

If we insert $E_\pm^2 = p_\pm^2 c^2 + m^2 c^4$ into Equation (3.44a), we have

$$hf = \sqrt{p_+^2 c^2 + m^2 c^4} + \sqrt{p_-^2 c^2 + m^2 c^4} \tag{3.46}$$

The maximum value of hf is, from Equation (3.45),

$$hf_{\max} = p_- c + p_+ c$$

However, from Equation (3.46), we also have

$$hf > p_- c + p_+ c$$

Equations (3.45) and (3.46) are inconsistent and cannot simultaneously be valid. Equations (3.44), therefore, do not describe a possible reaction. The reaction displayed in Figure 3.22a is not possible, because energy and momentum are not simultaneously conserved.

Consider the conversion of a photon into an electron and a positron that takes place inside an atom where the electric field of a nucleus is large. The nucleus recoils and takes away a negligible amount of energy but a considerable amount of momentum. The conservation of energy will now be

$$hf = E_+ + E_- + \text{K.E. (nucleus)} \tag{3.47}$$

A diagram of the process is shown in Figure 3.22b. The photon energy must be at least equal to $2m_e c^2$ in order to create the masses of the electron and positron.

$$hf > 2m_e c^2 = 1.022 \text{ MeV} \qquad \text{(for pair production)} \tag{3.48}$$

The probability of pair production increases dramatically both with higher photon energy and with higher atomic number Z of the atom's nucleus because of the correspondingly higher electric field that mediates the process.

The next question concerns the new particle, the positron. Why is it not commonly found in nature? We also need to answer the question posed earlier: can mass be converted to energy?

Positrons are found in nature. They are detected in cosmic radiation and as products of radioactivity from several radioactive nuclei. However, their existences are doomed because of their interaction with electrons. When positrons and electrons are in proximity for even a short time, they annihilate each other, producing photons. A positron passing through matter will quickly lose its kinetic energy through atomic collisions and will likely **annihilate** with an electron. After a positron slows down, it is drawn to an electron by their mutual electric attraction, and the electron and positron may then form an atomlike configuration called **positronium**, in which they orbit around their common center of mass.

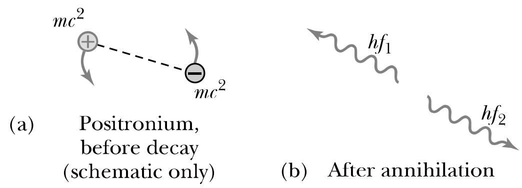


Figure 3.23 Annihilation of positronium atom (consisting of an electron and positron), producing two photons.

Eventually the electron and positron annihilate each other (typically in 10^{-10} s), producing electromagnetic radiation (photons). The process $e^+ + e^- \rightarrow \gamma + \gamma$ is called **pair annihilation**.

Pair annihilation

Consider a positronium "atom" at rest in free space. It must emit at least two photons to conserve energy and momentum. If the positronium annihilation takes place near a nucleus, it is possible that only one photon will be created, because the missing momentum can be supplied by nucleus recoil as in pair production. Under certain conditions three photons may be produced. Because the emission of two photons is by far the most likely annihilation mode, let us consider this mode, displayed in Figure 3.23. The conservation laws for the process $(e^+e^-)_{\text{atom}} \rightarrow \gamma + \gamma$ will be (we neglect the atomic binding energy of about 6.8 eV)

$$\text{Energy} \qquad 2m_ec^2 \approx hf_1 + hf_2 \tag{3.49a}$$

$$\text{Momentum} \qquad 0 = \frac{hf_1}{c} - \frac{hf_2}{c} \tag{3.49b}$$

By Equation (3.49b), the frequencies are identical, so we left $f_1 = f_2 = f$. Thus Equation (3.49a) becomes

$$2m_ec^2 = 2hf$$

or

$$hf = m_ec^2 = 0.511 \text{ MeV} \tag{3.50}$$

In other words, the two photons from positronium annihilation will move in opposite directions, each with energy 0.511 MeV. This is exactly what is observed experimentally.

The production of two photons in opposite directions with energies just over 0.5 MeV is so characteristic a signal of the presence of a positron that it has useful applications. **Positron emission tomography (PET)** scanning has become a standard diagnostic technique in medicine. A positron-emitting radioactive chemical (containing a nucleus such as ^{15}O, ^{11}C, ^{13}N, or ^{18}F) injected into the body causes two characteristic annihilation photons to be emitted from the points where the chemical has been concentrated by physiological processes. The location in the body where the photons originate is identified by measuring the directions of two gamma-ray photons of the correct energy that are detected in coincidence, as shown in Figure 3.24. Measurement of blood flow in the brain is an example of a diagnostic tool used in the evaluation of strokes, brain tumors, and other brain lesions.

PET scan

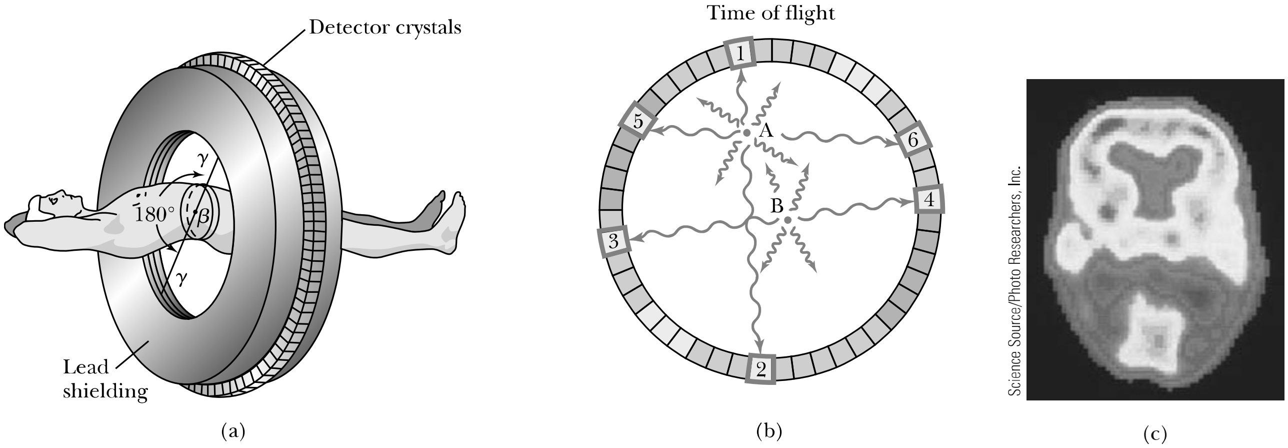


Figure 3.24 Positron emission tomography (PET) is a useful medical diagnostic tool to study the path and location of a positron-emitting radiopharmaceutical in the human body. (a) Appropriate radiopharmaceuticals are chosen to concentrate by physiological processes in the region to be examined. (b) The positron travels only a few millimeters before annihilation, which produces two photons that can be detected to give the positron position. (c) PET scan of a normal brain. *(a) and (b) are after G. L. Brownell et al.,* Science ***215**, 619 (1982)*

CONCEPTUAL EXAMPLE 3.18

Fluorine-18 is a radioactive nuclide that is a e^+ emitter and is used with blood flow within the brain to study brain disorders. Positron emission tomography (PET) scans rely on gamma rays (photons) being emitted in opposite directions for detection in scans. How is it possible for gamma rays to be simultaneously emitted in opposite directions?

Solution When fluorine-18 emits a e^+, the e^+ soon binds with an electron to form a positronium atom, which has a relatively low energy and linear momentum. When the annihilation occurs, 1.022 MeV is available, and both the conservation of energy and linear momentum must be obeyed. Conservation of momentum requires the photons (gamma rays) to emerge in precisely opposite directions with equal energies if the initial linear momentum is zero.

Antiparticles

Before leaving the subject of positrons we should pursue briefly the idea of **antiparticles**. The positron is the antiparticle of the electron, having the opposite charge but the same mass.* In 1955 the antiproton was discovered by E. G. Segrè and O. Chamberlain (Nobel Prize, 1959), and today, many antiparticles are known. We now believe that every particle has an antiparticle. In some cases, as for photons or neutral pi-mesons, the particle and antiparticle are the same, but for most other particles, the particle and antiparticle are distinct. For example, both the neutron and proton have anitparticles called the antineutron and antiproton.

We know that matter and antimatter cannot exist together in our world, because their ultimate fate will be annihilation. However, we may let our speculation run rampant! If we believe in symmetry, might there not be another world, perhaps in a distant galaxy, that is made of antimatter? Because galaxies are so far apart in space, annihilation would be infrequent. However, if a large chunk of antimatter ever struck the Earth, it would tend to restore the picture of a symmetric universe. As we see from Problem 59, however, in such an event there would be no one left to receive the appropriate Nobel Prize.

*Other particle properties (for example, spin) are described later (particularly in Chapters 7 and 14) and also need to be considered.

Summary

In 1895 Röntgen discovered x rays, and in 1897 Thomson proved the existence of electrons and measured their charge-to-mass ratio. Finally, in 1911 Millikan reported an accurate determination of the electron's charge. Experimental studies resulted in the empirical Rydberg equation to calculate the wavelengths of the hydrogen atom's spectrum:

$$\frac{1}{\lambda} = R_{\text{H}}\left(\frac{1}{n^2} - \frac{1}{k^2}\right) \quad k > n \tag{3.13}$$

where $R_H = 1.096776 \times 10^7\ \text{m}^{-1}$.

In order to explain blackbody radiation Planck proposed his quantum theory of radiation in 1900, which signaled the era of modern physics. From Planck's theory we can derive Wien's displacement law:

$$\lambda_{\max} T = 2.898 \times 10^{-3}\ \text{m} \cdot \text{K} \tag{3.14}$$

and the Stefan-Boltzmann law:

$$R(T) = \epsilon \sigma T^4 \tag{3.16}$$

Planck's radiation law gives the power radiated per unit area per unit wavelength from a blackbody.

$$\mathcal{I}(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \tag{3.23}$$

The oscillators of the electromagnetic radiation field can change energy only by quantized amounts given by $\Delta E = hf$, where $h = 6.6261 \times 10^{-34}\ \text{J} \cdot \text{s}$ is called *Planck's constant.*

Classical theory could not explain the photoelectric effect, but in 1905 Einstein proposed that the electromagnetic radiation field itself is quantized. We call these particle-like quanta of light *photons*, and they each have energy $E = hf$ and momentum $p = h/\lambda$. The photoelectric effect is easily explained by the photons each interacting with only one electron. The conservation of energy gives

$$hf = \phi + \tfrac{1}{2} mv_{\max}^2 \tag{3.30}$$

where ϕ is the work function of the emitter. The retarding potential required to stop all electrons depends only on the photon's frequency

$$eV_0 = \tfrac{1}{2} mv_{\max}^2 = hf - hf_0 \tag{3.33}$$

where $\phi = hf_0$. Millikan showed experimentally in 1916 that Einstein's theory was correct.

Bremsstrahlung radiation (x rays) is emitted when charged particles (for example, electrons) pass through matter and are accelerated by the nuclear field. These x rays have a minimum wavelength

$$\lambda_{\min} = \frac{hc}{eV_0} \tag{3.37}$$

where electrons accelerated by a voltage of V_0 impinge on a target.

In the Compton effect a photon scatters from an electron with a new photon created, and the electron recoils. For an incident and an exit photon of wavelength λ and λ', respectively, the change in wavelength is

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta) \tag{3.42}$$

when the exit photon emerges at angle θ to the original photon direction. The Compton wavelength of the electron is $\lambda_C = h/m_e c = 2.426 \times 10^{-3}$ nm. The success of the Compton theory in 1923 convincingly demonstrated the particle-like nature of the photon.

Finally, photon energy can be converted into mass in pair production:

$$\gamma \rightarrow e^+ + e^- \tag{3.43}$$

where e^+ is the positron, the antiparticle of the electron. Similarly, a particle and antiparticle annihilate catastrophically in the process

$$e^+ + e^- \rightarrow \gamma + \gamma$$

called pair annihilation.

Questions

1. How did the ionization of gas by cathode rays prevent H. Hertz from discovering the true character of electrons?
2. In Thomson's e/m experiment, does it matter whether the electron passing through interacts first with the electric field or with the magnetic field or both simultaneously? Explain.
3. Women in the late 1890s were terrified about the possible misuse of the new Röntgen x rays. What use do you think they envisioned?

4. In the late 1890s many people had x rays taken of their body. X-ray machines were common in shoe stores in the late 1940s and early 1950s for people to examine how their shoes fit; customers enjoyed seeing pictures of their bones. Discuss the safety of these undertakings.
5. Parents tell their children not to sit close to the television screen. Can x rays be produced in old, cathode-ray-type televisions? Explain.
6. In Example 3.2, why would you be concerned about observing a cluster of several balls in the Millikan electron charge experiment?
7. In Figure 3.5, why are the histogram peaks more difficult to identify as the charge increases?
8. How is it possible for the plastic balls in Example 3.2 to have both positive and negative charges? What is happening?
9. Why do you suppose Millikan tried several kinds of oil, as well as H_2O and Hg, for his oil-drop experiment?
10. In the experiment of Example 3.2, how could you explain an experimental value of $q = 0.8 \times 10^{-19}$ C?
11. Why do you suppose scientists worked so hard to develop better diffraction gratings?
12. Why was helium discovered in the sun's spectrum before being observed on Earth? Why was hydrogen observed on Earth first?
13. Do you believe there is any relation between the wavelengths of the Paschen (1908) and Pfund (1924) series and the respective dates they were discovered? Explain.
14. It is said that no two snowflakes look exactly alike, but we know that snowflakes have a quite regular, although complex, crystal structure. Discuss how this could be due to quantized behavior.
15. Why do we say that the elementary units of matter or "building blocks" must be some basic unit of mass-energy rather than only of mass?
16. Why is a red-hot object cooler than a white-hot one of the same material?
17. Why did scientists choose to study blackbody radiation from something as complicated as a hollow container rather than the radiation from something simple, such as a thin, solid cylinder (such as a dime)?
18. Why does the sun's radiation output match that of a blackbody?
19. Astronomers determine the surface temperature of a star by measuring its brightness at different frequencies. Explain how they can then use the Planck radiation law to obtain the surface temperature.
20. In a typical photoelectric effect experiment, consider replacing the metal photocathode with a gas. What difference would you expect?
21. What do the work functions of Table 3.3 tell us about the properties of particular metals? Which have the most tightly and least tightly bound electrons?
22. Why is it important to produce x-ray tubes with high accelerating voltages that are also able to withstand electron currents?
23. For a given beam current and target thickness, why would you expect a tungsten target to produce a higher x-ray intensity than targets of molybdenum or chromium?
24. List all possible known interactions between photons and electrons discussed in this chapter. Can you think of any more?
25. Discuss why it is difficult to see the Compton effect using visible light.
26. What do you believe to be an optimum lifetime for a positron-emitting radioactive nuclide used in brain tumor diagnostics? Explain.

Problems

Note: The more challenging problems have their problem numbers shaded by a blue box.

3.1 Discovery of the X Ray and the Electron

1. Describe the design features of an apparatus that will produce the correct magnetic field needed in Figure 3.2.
2. For an electric field of 2.5×10^5 V/m, what is the strength of the magnetic field needed to pass an electron of speed 2.2×10^6 m/s with no deflection? Draw the mutually perpendicular $\vec{v}$, $\vec{E}$, and $\vec{B}$ directions that allow this to occur.
3. Across what potential difference does an electron have to be accelerated to reach the speed $v = 1.8 \times 10^7$ m/s? Work the problem both nonrelativistically and relativistically and compare the results.
4. An electron entering Thomson's e/m apparatus (Figures 3.2 and 3.3) has an initial velocity (in horizontal direction only) of 4.0×10^6 m/s. In the lab is a permanent horseshoe magnet of strength 12 mT, which you would like to use. (a) What electric field will you need in order to produce zero deflection of the electrons as they travel through the apparatus? (b) The length of nonzero $\vec{E}$ and $\vec{B}$ fields is 2.0 cm. When the

magnetic field is turned off, but the same electric field remains, how far in the vertical direction will the electron beam be deflected over this length?

3.2 Determination of Electron Charge

5. Consider the following possible forces on an oil drop in Millikan's experiment: gravitational, electrical, frictional, and buoyant. Draw a diagram indicating the forces on the oil drop (a) when the electric field is turned off and the droplet is falling freely and (b) when the electric field causes the droplet to rise.

6. Neglect the buoyancy force on an oil droplet and show that the terminal speed of the droplet is $v_t = mg/b$, where b is the coefficient of friction when the droplet is in free fall. (Remember that the frictional force $\vec{F}_f$ is given by $\vec{F}_f = -b\vec{v}$ where velocity is a vector.)

7. Stokes's law relates the coefficient of friction b to the radius r of the oil drop and the viscosity η of the medium the droplet is passing through: $b = 6\pi\eta r$. Show that the radius of the oil drop is given in terms of the terminal velocity v_t (see Problem 6), η, g, and the density of the oil by $r = 3\sqrt{\eta v_t/2g\rho}$.

8. In a Millikan oil-drop experiment the terminal velocity of the droplet is observed to be 1.3 mm/s. The density of the oil is $\rho = 900\ \text{kg/m}^3$, and the viscosity of air is $\eta = 1.82 \times 10^{-5}\ \text{kg/m} \cdot \text{s}$. Use the results of the two previous problems to calculate (a) the droplet radius, (b) the mass of the droplet, and (c) the coefficient of friction.

3.3 Line Spectra

9. What is the series limit (that is, the smallest wavelength) for (a) the Lyman series and (b) the Balmer series?

10. Light from a slit passes through a transmission diffraction grating of 400 lines/mm, which is located 3.0 m from a screen. What are the distances on the screen (from the unscattered slit image) of the three brightest visible (first-order) hydrogen lines?

11. A transmission diffraction grating with 420 lines/mm is used to study the light intensity of different orders (n). A screen is located 2.8 m from the grating. What are the positions on the screen of the three brightest red lines for a hydrogen source?

12. Calculate the four largest wavelengths for the Brackett and Pfund series for hydrogen.

13. Josef von Fraunhofer made the first diffraction grating in 1821 and used it to measure the wavelengths of specific colors as well as the dark lines in the solar spectrum. His first diffraction grating consisted of 262 parallel wires. Assume that the wires were 0.20 mm apart and that Fraunhofer could resolve two spectral lines that were deflected at angles 0.50 min of arc apart. Using this grating, what is the minimum separation (in wavelength) that can be resolved of two first-order spectral lines near a wavelength of 400 nm?

14. Suppose that a detector in the Hubble Space Telescope was capable of detecting visible light in the wavelength range of 400 to 700 nm. (a) List all the wavelengths for the hydrogen atom that are in this range and their series name. (b) The detector measures visible wavelengths of 537.5 nm, 480.1 nm, and 453.4 nm that researchers believe are due to the hydrogen atom. Why are all the known visible hydrogen lines not detected? (c) Use these data to calculate the speed of the stellar object that emitted the spectra. Assume that the object is not rotating. Why might rotation be an issue?

15. The Spitzer Space Telescope was launched in 2003 to detect infrared radiation. Suppose a particular detector on the telescope is sensitive over part of the near-infrared region of wavelengths 980 to 1920 nm. Astronomers want to detect the radiation being emitted from a red giant star and decide to concentrate on wavelengths from the Paschen series of the hydrogen atom. (a) What are the known wavelengths in this wavelength region? (b) The detector measures wavelengths of 1334.5, 1138.9, and 1046.1 nm believed to be from the Paschen series. Why are these wavelengths different from those found in part (a)? (c) How fast is the star moving with respect to us?

3.4 Quantization

16. Quarks have charges $\pm e/3$ and $\pm 2e/3$. What combination of three quarks could yield (a) a proton, (b) a neutron?

3.5 Blackbody Radiation

17. Calculate λ_{max} for blackbody radiation for (a) liquid helium (4.2 K), (b) room temperature (293 K), (c) a steel furnace (2500 K), and (d) a blue star (9000 K).

18. Calculate the temperature of a blackbody if the spectral distribution peaks at (a) gamma rays, $\lambda = 1.50 \times 10^{-14}$ m; (b) x rays, 1.50 nm; (c) red light, 640 nm; (d) broadcast television waves, $\lambda = 1.00$ m; and (e) AM radio waves, $\lambda = 204$ m.

19. (a) A blackbody's temperature is increased from 900 K to 2300 K. By what factor does the total power radiated per unit area increase? (b) If the original temperature is again 900 K, what final temperature is required to double the power output?

20. (a) At what wavelength will the human body radiate the maximum radiation? (b) Estimate the total power radiated by a person of medium build (assume an area given by a cylinder of 175-cm height and 13-cm radius). (c) Using your answer to (b), compare the energy radiated by a person in one day with the energy intake of a 2000-kcal diet.

21. White dwarf stars have been observed with a surface temperature as hot as 200,000°C. What is the wavelength of the maximum intensity produced by this star?

22. For a temperature of 5800 K (the sun's surface temperature), find the wavelength for which the spectral distribution calculated by the Planck and Rayleigh-Jeans results differ by 5%.

23. A tungsten filament of a typical incandescent lightbulb operates at a temperature near 3000 K. At what wavelength is the intensity at its maximum?

24. Use a computer to calculate Planck's radiation law for a temperature of 3000 K, which is the temperature of a typical tungsten filament in an incandescent lightbulb. Plot the intensity versus wavelength. (a) How much of the power is in the visible region (400–700 nm) compared with the ultraviolet and infrared? (b) What is the ratio of the intensity at 400 nm and 700 nm to the wavelength with maximum intensity?

25. Show that the ultraviolet catastrophe is avoided for short wavelengths ($\lambda \to 0$) with Planck's radiation law by calculating the limiting intensity $\mathcal{I}(\lambda, T)$ as $\lambda \to 0$.

26. Estimate the power radiated by (a) a basketball at 20°C and (b) the human body (assume a temperature of 37°C).

27. At what wavelength is the radiation emitted by the human body at its maximum? Assume a temperature of 37°C.

28. If we have waves in a one-dimensional box, such that the wave displacement $\Psi(x, t) = 0$ for $x = 0$ and $x = L$, where L is the length of the box, and

$$\frac{1}{c}\frac{\partial^2 \Psi}{\partial t^2} - \frac{\partial^2 \Psi}{\partial x^2} = 0 \quad \text{(wave equation)}$$

show that the solutions are of the from

$$\Psi(x, t) = a(t) sin\left(\frac{n\pi x}{L}\right) \quad (n = 1,2,3,...)$$

and $a(t)$ satisfies the (harmonic-oscillator) equation

$$\frac{d^2 a(t)}{dt^2} + \omega_n^2 a(t) = 0$$

where $\omega_n = n\pi c/L$ is the angular frequency $2\pi f$.

29. If the angular frequencies of waves in a three-dimensional box of sides L generalize to

$$\omega = \frac{\pi c}{L}(n_x^2 + n_y^2 + n_z^2)^{1/2}$$

where all n are integers, show that the number of distinct states in the frequency interval $f(= \omega/2\pi)$ to $f + \Delta f$ is given by (where f is large)

$$dN = 4\pi\frac{L^3}{c^3}f^2 df$$

30. Let the energy density in the frequency interval f to $f + df$ within a blackbody at temperature T be $dU(f, T)$. Show that the power emitted through a small hole of area ΔA in the container is

$$\frac{c}{4}dU(f,T)\Delta A$$

31. Derive the Planck radiation law emitted by a blackbody. Remember that light has two directions of polarization and treat the waves as an ensemble of harmonic oscillators.

3.6 Photoelectric Effect

32. An FM radio station of frequency 98.1 MHz puts out a signal of 50,000 W. How many photons/s are emitted?

33. How many photons/s are contained in a beam of electromagnetic radiation of total power 180 W if the source is (a) an AM radio station of 1100 kHz, (b) 8.0-nm x rays, and (c) 4.0-MeV gamma rays?

34. What is the threshold frequency for the photoelectric effect on lithium ($\phi = 2.93$ eV)? What is the stopping potential if the wavelength of the incident light is 380 nm?

35. What is the maximum wavelength of incident light that can produce photoelectrons from silver ($\phi = 4.64$ eV)? What will be the maximum kinetic energy of the photoelectrons if the wavelength is halved?

36. A 2.0-mW green laser ($\lambda = 532$ nm) shines on a cesium photocathode ($\phi = 1.95$ eV). Assume an efficiency of 10^{-5} for producing photoelectrons (that is, one photoelectron produced for every 10^5 incident photons) and determine the photoelectric current.

37. An experimenter finds that no photoelectrons are emitted from tungsten unless the wavelength of light is less than 270 nm. Her experiment will require photoelectrons of maximum kinetic energy 2.0 eV. What frequency of light should be used to illuminate the tungsten?

38. The human eye is sensitive to a pulse of light containing as few as 100 photons. For orange light of wavelength 610 nm, how much energy is contained in the pulse?

39. In a photoelectric experiment it is found that a stopping potential of 1.00 V is needed to stop all the electrons when incident light of wavelength 260 nm is used and 2.30 V is needed for light of wavelength 207 nm. From these data determine Planck's constant and the work function of the metal.

40. Find the wavelength of light incident on a tungsten target that will release electrons with a maximum speed of 1.4×10^6 m/s.

3.7 X-Ray Production

41. What is the minimum x-ray wavelength produced for a dental x-ray machine operated at 30 kV?

42. The Stanford Linear Accelerator can accelerate electrons to 50 GeV (50×10^9 eV). What is the minimum wavelength of photon it can produce by bremsstrahlung? Is this photon still called an x ray?

43. A cathode-ray tube in a scanning electron microscope operates at 25 keV. What is λ_{min} for the continuous x-ray spectrum produced when the electrons hit the target?

44. Calculate λ_{min} for all three elements shown in Figure 3.19. Use the value of the work function for tungsten in Table 3.3 and calculate the percentage error in neglecting the work function for the Duane-Hunt rule using the data of Figure 3.19.
45. The two peaks for the molybdenum spectra of Figure 3.19 are *characteristic* spectral lines for the molybdenum element. What is the minimum potential difference needed to accelerate electrons in an x-ray tube to produce both of these lines?

3.8 Compton Effect

46. Calculate the maximum $\Delta\lambda/\lambda$ of Compton scattering for blue light ($\lambda = 480$ nm). Could this be easily observed?
47. A photon having 40 keV scatters from a free electron at rest. What is the maximum energy that the electron can obtain?
48. If a 7.0-keV photon scatters from a free proton at rest, what is the change in the photon's wavelength if the photon recoils at 90°?
49. Is it possible to have a scattering similar to Compton scattering from a proton in H_2 gas? What would be the Compton wavelength for a proton? What energy photon would have this wavelength?
50. An instrument has resolution $\Delta\lambda/\lambda = 0.40\%$. What wavelength of incident photons should be used in order to resolve the modified and unmodified scattered photons for scattering angles of (a) 30°, (b) 90°, and (c) 170°?
51. Derive the relation for the recoil kinetic energy of the electron and its recoil angle ϕ in Compton scattering. Show that

$$\text{K.E. (electron)} = \frac{\Delta\lambda/\lambda}{1 + (\Delta\lambda/\lambda)} hf$$

$$\cot\phi = \left(1 + \frac{hf}{mc^2}\right)\tan\frac{\theta}{2}$$

52. A 650-keV gamma ray Compton-scatters from an electron. Find the energy of the photon scattered at 110°, the kinetic energy of the scattered electron, and the recoil angle of the electron.
53. A photon of wavelength 2.0 nm Compton-scatters from an electron at an angle of 90°. What is the modified wavelength and the fractional change, $\Delta\lambda/\lambda$?

3.9 Pair Production and Annihilation

54. How much photon energy is required to produce a proton-antiproton pair? Where could such a high-energy photon come from?
55. What is the minimum photon energy needed to create an e^--e^+ pair when a photon collides (a) with a free electron at rest and (b) with a free proton at rest?

General Problems

56. What wavelength photons are needed to produce 30.0-keV electrons in Compton scattering?
57. A typical person can detect light with a minimum intensity of 4.0×10^{-11} W/m^2. For light of this intensity and $\lambda = 550$ nm, how many photons enter the eye each second if the pupil is open wide with a diameter of 9.0 mm?
58. A copper wire carrying a high current glows "red hot" just before the wire melts at a temperature of 1085°C. (a) What is the peak wavelength of the emitted radiation? (b) Given your answer to part (a), how can the wire be "red hot"?
59. The gravitational energy of Earth is approximately $0.5(GM_E^2/R_E)$ where M_E is the mass of Earth. This is approximately the energy needed to blow the planet into small fragments (the size of asteroids). How large would an antimatter meteorite the density of nickel-iron ($\rho \approx 5 \times 10^3$ kg/m^3) have to be in order to blow up Earth when it strikes? Compute the energy involved in the particle-antiparticle annihilation and compare it with the total energy in all the nuclear arsenals of the world [~5000 megatons (MT), where 1 MT = 4.2×10^{15} J].
60. Show that the maximum kinetic energy of the recoil electron in Compton scattering is given by

$$\text{K.E.}_{\text{max}}\ \text{(electron)} = hf\frac{\dfrac{2hf}{mc^2}}{1 + \dfrac{2hf}{mc^2}}$$

At what angles θ and ϕ does this occur? If we detect a scattered electron at $\phi = 0°$ of 100 keV, what energy photon was scattered?
61. Use the Wien displacement law to make a log-log plot of λ_{max} (from 10^{-8} m to 10^{-2} m) versus temperature (from 10^0 K to 10^5 K). Mark on the plot the regions of visible, ultraviolet, infrared, and microwave wavelengths. Put the following points on the line: sun (5800 K), furnace (1900 K), room temperature (300 K), and the background radiation of the universe (2.7 K). Discuss the electromagnetic radiation that is emitted from each of these sources. Does it make sense?
62. (a) What is the maximum possible energy for a Compton-backscattered x ray ($\theta = 180°$)? Express your answer in terms of λ, the wavelength of the incoming photon. (b) Evaluate numerically when the incoming photon's energy is 100 keV.
63. The naked eye can detect a stellar object of sixth magnitude in the night sky. With binoculars, we can see an object of the ninth magnitude. The sun's brightness at Earth is 1400 W/m^2. The Hubble Space Telescope can detect an object of the 30th magnitude, which amounts to a brightness of about 2×10^{-20} W/m^2. (a) Consider a detector in the Hubble Space Tele-

scope with a collection area of 0.30 m^2. If you assume hydrogen light of frequency 486 nm (blue-green), how many photons/s enter the telescope from a 30th-magnitude star? (b) An increase of magnitude one represents a decrease in brightness by a factor of $100^{1/5}$. Estimate how many photons/s from a sixth-magnitude star would enter your eye if the diameter of your pupil is 6.5 mm.

64. Original data from Millikan's pivotal photoelectric experiment that confirmed Einstein's quantum explanation is shown in Figure 3.16 [from R. A. Millikan, *Physical Review* **7**, 362 (1916)]. Sodium was the photocathode. Use the data to find the work function for sodium and Planck's constant.

65. A prototype laser weapon tested in 2010 used a laser with an infrared wavelength of 1.06 μm, because the atmosphere is fairly transparent at that wavelength. The laser's continuous output was 25 kW. How many photons per second were produced?

66. A typical chemical reaction such as an explosive combustion releases about 5 MJ of energy per kg fuel used. At the sun's current rate of energy production, how much time would the sun last at that rate? Compare your answer with the sun's estimated lifetime of 10 billion years.

67. The bright star Sirius A has a diameter 1.6 times the sun's and surface temperature 9600 K. (a) What is the peak wavelength of radiation emitted from the surface? (Note: Sirius has a distinctive blue tint when viewed with the naked eye.) (b) Find the net power output from the surface of Sirius A and compare with that from the sun.

68. In developing Equation (3.36), we argued that the recoiling nucleus could be ignored. Consider again the x-ray tube described in Example 3.15 with 35-keV electrons striking a tungsten target. Suppose an electron is deflected through a negligible angle and its kinetic energy drops to 30 keV in a scattering event with a nucleus. Assuming that the nucleus was initially at rest, use conservation of momentum to find the kinetic energy of the recoiling nucleus and comment on the result.

69. The Fermi Gamma-ray Space Telescope, launched in 2008, can detect gamma rays with energies ranging from 10 keV to 300 MeV. For each of those energy extremes, find the resulting kinetic energy and speed of an electron created by the gamma ray as part of an electron-positron pair. Assume that the electron has half of the gamma ray's energy.

70. Gamma-ray detectors like the one described in the preceding problem often use calorimetry to determine gamma-ray energies. Suppose a beam of 100-MeV gamma rays strikes a target with a mass of 2.5 kg and specific heat 430 J/(kg · K). How many gamma rays are needed to raise the target's temperature by 10 mK?