

Blackbody Radiation

Equipment

Included:

1	Prism Spectrophotometer Kit	OS-8544
1	Optics Bench (60 cm)	OS-8541
1	Spectrophotometer Accessory Kit	OS-8537
1	Aperture Bracket	OS-8534B
1	Broad Spectrum Light Sensor	PS-2150
1	Rotary Motion Sensor	PS-2120
1	Voltage Sensor	UI-5100
1	Replacement Bulb (10 pk)	SE-8509
1	Banana Plug Cord-Black (5 pack)	SE-9751

Required, but not included:

1	850 Universal Interface	UI-5000
1	PASCO Capstone	

Introduction

This is one of the key experiments that led to the development of Quantum Mechanics.

The spectrum of an incandescent light bulb is scanned by hand using a prism spectrophotometer that measures relative light intensity as a function of angle. A Broad Spectrum Light Sensor is used with a prism so the entire spectrum from approximately 400 nm to 2500 nm can be scanned without the overlapping orders caused by a grating. The wavelengths corresponding to the angles are calculated using the equations for a prism spectrophotometer. The relative light intensity can then be plotted as a function of wavelength as the spectrum is scanned, resulting in the characteristic blackbody curve. The intensity of the light bulb is increased, increasing the temperature, and the scan is repeated to show how the curves nest with a shift in the peak wavelength.

The temperature of the filament of the bulb can be estimated indirectly by determining the resistance of the bulb from the measured voltage and current. From the temperature, the theoretical peak wavelength can be calculated and compared to the measured peak wavelength.

This experiment should be performed in a room with reduced light levels although complete darkness is not required.

Theory

The intensity per wavelength, $I_\lambda(\lambda, T)$, as a function of wavelength of radiation emitted by an ideal body (a blackbody since an ideal emitter must also be an ideal absorber) is given by Planck's Radiation Law:

$$I_\lambda(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \left(\frac{1}{e^{(hc/\lambda kT)} - 1} \right) \quad (1)$$

where c is the speed of light in a vacuum, h is Planck's constant, k is Boltzmann's constant, T is the absolute temperature of the body, and λ is the wavelength of the radiation. Any real object must emit less at all wavelengths.

The wavelength with the greatest intensity is given by

$$\lambda_{\max} = (\text{constant})/T = (0.002898 \text{ m}\cdot\text{K})/T \quad (2)$$

The temperature of the blackbody light filament can be calculated using the resistance of the filament while it is lit. We find the resistance (R) by measuring the voltage (V) and the current (I) and using $R = V/I$. The resistance of the tungsten filament is a nonlinear function of the temperature. Using the measured resistance to calculate the temperature is discussed in Appendix 1.

The wavelength is determined by measuring the angle at which the light is dispersed by a prism. The relationship between the angle and the wavelength is discussed in Appendix 2.

Setup Part A

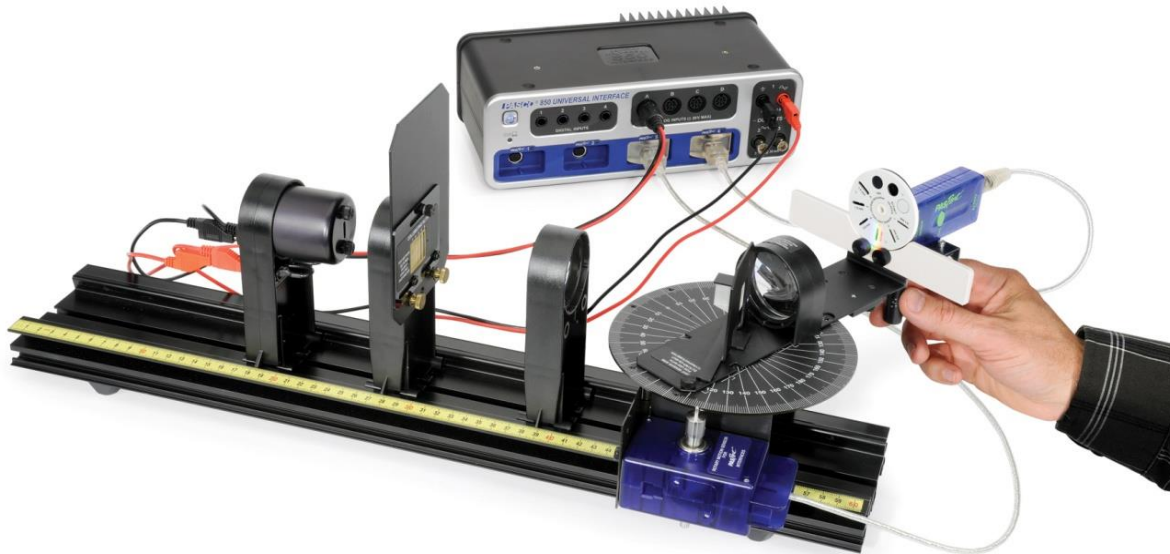


Figure 1: Complete Setup

1. Set up the Prism Spectrophotometer as shown in Figure 1 except place the Blackbody Light Source close to the left end of the track and the Collimating Slit closer to it than is shown in the picture to maximize the intensity. Detailed instructions for mounting the Rotary Motion Sensor and the Degree Plate and Light Sensor Arm to the spectroscopy table may be found in Appendix 3.
2. Attach the Broad Spectrum Light Sensor to the Light Sensor Arm using the $\frac{1}{2}$ inch bolt ($\frac{1}{4} \times 20$) with a large black plastic head. Attach the 2 inch black rod to the bottom of the Light Sensor Arm using one of the vacant holes. This makes a convenient handle for sweeping through the spectrum (see fig. 1 above).
3. Mount the Beveled Stop Piece on the bottom of the Light Sensor Arm with the two supplied bolts (see Figure 2). Position the beveled edge so it will hit against the angle indicator on the spectroscopy table.
4. Note: There is a current sensor built into the 850 output that is used. We do not use the internal voltage sensor since we want the voltage at the lamp and there is some drop across the connecting wires.



Figure 2: Beveled Stop Piece

Setup Part B

1. Attach the Mounted Prism to the spectroscopy table by screwing it into the hole in the center of the table. Screw it down until it almost touches the table. *It is critical that the Mounted Prism does not touch the table so the table is free to move without moving the prism.* Orient the prism with its apex toward the light source as shown in Figure 4. The prism base must be perpendicular to the incoming light beam. To do this set, turn the table until the index mark is on 0° , and then set the base of the Mounted Prism so it lies along the 0° - 180° line on the table. Secure the prism in place using the wing nut and lock washer on the bottom of the bolt sticking through the spectroscopy table.
2. Ground the Spectrometer by attaching an alligator jumper cable from the ground post on the bottom of the spectroscopy table (on the side opposite the Rotary Motion Sensor), and attaching the other end to a ground. A convenient ground is the silver outside connector for the #2 or #3 Outputs at the lower right on the 850 Universal Interface.
3. Plug the Blackbody Light Source into the #1 Output on the top right of the 850 Universal Interface. Polarity does not matter.
4. Plug the Broad Spectrum Light Sensor and the Rotary Motion Sensor into PASPORT inputs on the 850 Universal Interface. Set the sample rate of the Light Sensor to 20 Hz and the sample rate of the Rotary Motion Sensor to 100 Hz.
5. Plug the Voltage Sensor into the Analog A input on the 850 (see Figure 1). Attach the red lead to the red banana lead on the Blackbody Light Source. Attach the black lead to the black banana on the Blackbody Light Source. **Do Not** attach the leads to the output jacks on the 850. The current is large enough that there is a voltage drop along the wires connecting the 850 output to the Blackbody Light Source and we want to measure the voltage at the light source. Set the Voltage Sensor sample rate to 20 Hz.

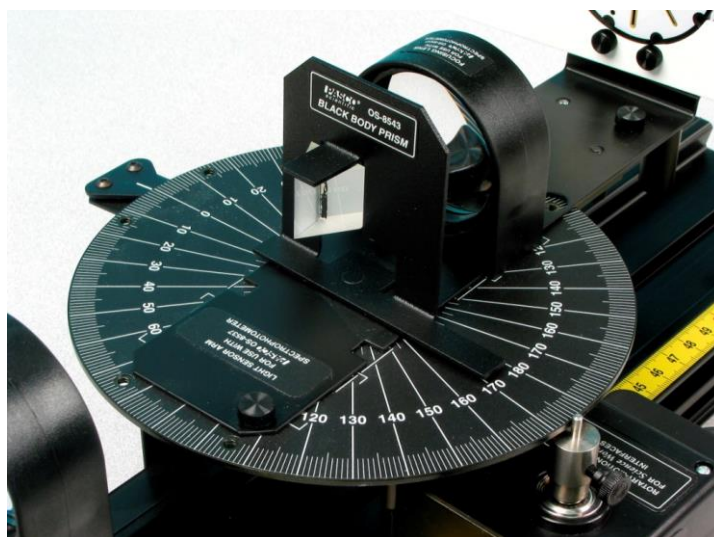


Figure 4: The prism apex should be oriented toward the light source.

be

Software Setup

1. Create a graph of Relative Intensity vs. Angle in radians.
2. Create a table and, in the first column, create a new User-Entered Data Set called “Table Angle” and make the units degrees.
3. In the second column, create a new User-Entered Data Set called “Shaft Angle” and make the units radians.
4. Insert a third column and create a calculation:

$$\text{AngCorr} = [\text{Table Angle } (^{\circ}), \nabla] / [\text{Shaft Angle (rad)}, \nabla]$$
 and make the units deg/rad.
5. Click the Signal Generator at the left of the screen. Set the waveform for DC and the Voltage for 7.0 V either by typing it in or by using the up/down keys to the right of the DC Voltage bar. Don’t turn on the signal generator until you are instructed to do so in the procedure (this lengthens the bulb life).

Procedure

1. Set the collimating slits on Slit #4. Set the Aperture Disk in front of the Light Sensor on Slit #4.

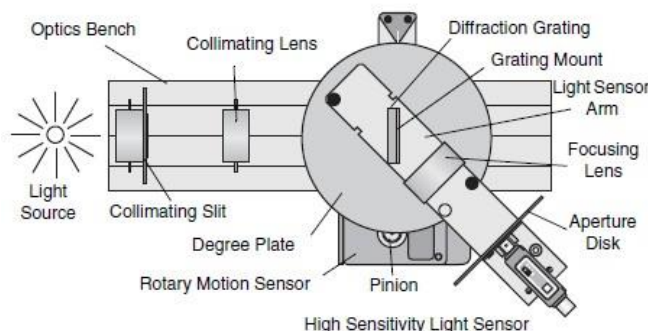


Figure 3: Spectrophotometer System (top view)

2. Collimating the system: the Collimating Slit must be at the focal point of the first lens and the Sensor Mask and Aperture Disk must be at the focal point of the second lens. Move the spectroscopy table back to the end of the track it is out of the way. Place the Blackbody Light Source near the end of the track and the Collimating Slit near the blackbody light source. Move the Collimating Lens (see figure 3 above) at least 12 cm from the slit. Have someone with 20/20 vision (corrected by glasses is fine) look through the lens at the slit. Move the lens toward the slit until it first comes into sharp focus. The slit should be about 10 cm from the lens. Now move the spectroscopy table as close to the Collimating Lens as possible. Set the Focusing lens 10 cm from the Sensor Mask. We will adjust this more exactly later.

3. Click the Signal Generator at the left of the screen. Turn on the Signal Generator by clicking ON.
4. Set the moveable arm at the center of the track so that the un-deviated light that passes above the prism from the slit strikes the Sensor Mask. Adjust the Focusing Lens so the image on the Sensor Mask is as sharp as possible. The system is now well collimated. Look at the light coming from the Blackbody Light Source. Observe the color. (Yes, white is a color.)

5. Rotate the movable arm until you see the spectrum. Look at the spectrum on the Light Sensor screen. Are all the colors (from red to violet) present? What does this show about white light?

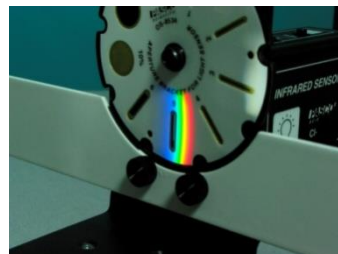






Figure 5: Spectrum on Light Sensor Mask

6. Rotate the scanning arm until it touches the stop. This will be the starting position for all the scans.
7. Before starting to record, review the steps you will perform during recording: The Broad Spectrum Light Sensor tends to drift so the following steps need to be performed as written. With the sensor are pressed against the stop, press the Tare button on the Broad Spectrum Light Sensor (the button is illuminated) to zero it. Observe the *width of the visible pattern*. Using the handle below the light sensor, sweep rapidly from the stop to a position about one *visible spectrum width* to the left (ultraviolet side) of the pattern. Slowly rotate the scanning arm through the spectrum to point about two *visible spectrum widths* to the right (infrared side) of the visible pattern. You should try to complete this operation in less than 30 seconds from when you press the Tare button, but try to sweep at a uniform rate. Now continue rapidly all the way past zero degrees (the position where the light sensor is directly opposite the light source), slowing as you sweep across the white light peak at zero degrees. It is important that you only sweep in one direction! If you attempt to go back, the Rotary Motion sensor will lose track of where you are!
8. You must be holding the scanning arm against the stop when you press RECORD!!! If it is not against the stop, each run will have a different zero position and you will not see the position of the peak correctly. Now click RECORD and perform the scan as described in part 7.
9. Click Stop. On the Signal Generator, click Off. Click on the Signal Generator Button to close the Signal Generator panel.
10. If the curve does not fill the graph, click on the Re-size Tool () at the upper left of the graph toolbar. *The angles may all be negative, depending on how you set up the spectrometer. If so, place the hand icon over where it says "angle" at the bottom of the screen and when the blue box appears, left click, select QuickCalc at the top of the pop-*

up and then select $-\theta$ in the pop-up that appears to the side. On both sides of the spectrum peak the Relative Intensity should be approximately zero. It does not matter if the intensity zero drifts as you sweep to the central white-light peak as long as you can see the peak. If either of these is not true, click the Delete Last Run button at the lower right and repeat the run. When you have a good run, click the Data Summary button at the left edge of the page, double click on the good run (probably Run #1) and re-label it 7V Run. Click the Data Summary button to close the Data Summary panel.

11. Examine the Relative Intensity vs. Angle graph. Note that it says that the unit for the angle is radians. Since we have only rotated the table by about 80° this is clearly not correct. The reason is that the Rotary Motion Sensor measures the angle that its own shaft turns through, but we need the angle that the table turns through. The diameter of the table is approximately 60 times the diameter of the Rotary Motion Sensor shaft. It turns out that the number of radians for the shaft is approximately equal to the number of degrees for the table. We measure the true correction in step 12.
12. Press the TARE button on the Broad Spectrum Sensor. With the table set so the index mark is on 50° click RECORD. Rotate the table 100° to the other 50° mark, let it sit on the 50° degree mark for about ten seconds (drift in the reading will then make the stop point obvious) and click STOP. Click Data Summary (top left of screen) and label this run Calibrate. Close the Data Summary panel. On the graph, select the Calibrate Run. Find the angle (rad) that the shaft turned through. The initial angle should be exactly 0.
To find the final angle, click on the Data Selection icon () on the graph toolbar. A highlighted region with handles should appear on the graph. Drag the handles to highlight the region between 100° and 106° . Click the Scale-to-Fit tool () . You should now be able to read the angle to the nearest tenth of a degree. Record this value in the table in the Shaft Angle column. Your value should be close the one already in the table, but change the value to the one you measured since the instruments may vary slightly.
13. Click on the Calculator on the left side of the screen and create a calculation:

$$\text{true angle} = 0.9569 * \text{abs}([\text{angle (rad)}, \blacktriangledown])$$
and give it units of $^\circ$ (degrees).
If your value differs from the sample data, replace the sample value (0.9569) with yours in the calculation for the true angle. ($100^\circ/\text{shaft angle} = 0.9569$) On the graph, delete the selection and then click on the Scale-to-Fit tool to return the graph to normal.
14. Create a new page and create two digits displays with the Voltage Ch.A and Output Current and create a graph of Relative Intensity vs. true angle. There will be a peak on the intensity vs. angle graph where the light sensor is aligned with the light source because some light passes by the prism instead of going through the prism. This peak enables the initial angle to be exactly determined. Select the 7V Run if it isn't already displayed. Click the Scale-to-Fit tool. Click on the Coordinates Tool () from the graph toolbar. Click on the box and drag it until it is directly above the central peak (above it, not on the line – you should see an arrow pointing down to the peak. Release the Smart Cursor and it will snap to the peak. The two numbers in the info box are [angle, relative intensity]. Click the Calculator (left side of screen) and enter this

equation:

$$\text{CorrTrueAng} = \text{Initial Angle} - \text{True Angle}$$

and define Initial Angle to be the angle read on the graph. This amount will be subtracted from all the angles so we are measuring the angle that the light bent from the straight through path.

15. Repeat Steps 6 through 9 for voltages of 4 V and 10 V. **Caution:** If 10 volts is applied to the blackbody light for an extended amount of time, the life of the bulb will be reduced. Only turn on the bulb when taking measurements. Notice (with your eyes) how the spectrum changes. On a new page in Capstone, create a graph of Light Intensity vs. True Angle and select the 4V Run. Use the Smart Cursor to verify that the central peak agrees with the 7V Run within 0.1 degrees. Then check the 10V Run. *If the central peaks disagree by more than 0.1 degrees, you did not start from the same position (against the stop) and should redo one or more runs.*

Analysis Part 1

The wavelengths (λ) for the Tungsten Curves are calculated from the measured dispersion angles. See Appendix 1 (A&B) for details of this calculation. The larger the angle, the shorter the wavelength. The wavelengths are limited to the range 400 nm to 2500 nm since the relationship relating angle to wavelength is only valid for this range and because the glass of the prism does not pass wavelengths above 2500 nm.


In the calculator, create the following equations (see Appendix 3 for explanations of these equations):

$$\lambda_{cal} = 320 + 1 / ([\text{Index of Refraction}] - 1.635) + 0.2 / ([\text{Index of Refraction}] - 1.635)^2 + 0.19 / ([\text{Index of Refraction}] - 1.635)^3$$

$$n = \sqrt{((2 * \sin([\text{CorrTrueAng} (^{\circ})]) / \sqrt{3}) + 0.5)^2 + 3/4)}$$

$$\lambda = \text{filter}(400, 2500, \lambda_{cal})$$

On a new page in Capstone, create a graph of Light Intensity vs. Wavelength, λ .

If the 4V, 7V, and 10V curves don't all show on the graph, click the Data Selection Tool () so that it is pushed in and then click the small black triangle and select all three runs.

1. Does the peak shift toward shorter or longer wavelengths as the temperature is increased?
2. How does the intensity change as the temperature is increased? Does this agree with what Equation 2 (see Theory) predicts?
3. How did the color of the bulb change with temperature? How did the color composition

of the spectrum change with temperature? Considering the peak wavelengths, why is a bulb's filament red at low temperatures and white at high temperatures?

4. At about what wavelength is the peak wavelength of our Sun? What color is our Sun? Why?
5. For the highest temperature, is more of the intensity (area of the intensity vs. wavelength graph) in the visible part of the spectrum or in the infrared part of the spectrum? How could a light bulb be made more efficient so it puts out a greater percentage of its light in the visible?




Temperature

1. On a new page, create a table with a new User-Entered Data set called "R holder" with units of Ω in the first column. Then create the following equations in the calculator:

$$\rho = 5.65 * ([\text{Voltage Ch A (V)}] / [\text{Output Current Ch H (A)}] - [\text{R holder } (\Omega)]) / 0.93$$

$$\text{Abs Temp} = 103 + 38.1\rho - 0.095\rho^2 + 0.000248\rho^3$$

Then select Abs Temp in the second column of the table and choose the 4V run. Then add two more columns and select Abs Temp in each one, selecting the 7V run in one and the 10 V run in the other.

2. The Filament Temperature for the run in the table is calculated using the measured voltage across the lamp holder and current through the filament. For details of the calculation see Appendix 2.
3. Put in the values 1.0 Ω , 2.0 Ω and 4.0 Ω for the resistance of the Lamp Holder (R_{holder}) in the first column of the table. These values are given to show how sensitive the Temperature value is to this measurement.
4. Create a graph of Light Intensity vs. wavelength, λ . Click on the Data Display icon () on the graph toolbar and select the 4V Run. Click the Re-size Tool (). Use the Smart Tool () on the Tungsten Intensity vs. Wavelength graph to find the wavelength where the intensity is maximum. It is probably best to just eyeball the center of the peak rather than using the Snap to the peak function of the Smart Cursor which tends to be confused by the rather flat peak. Calculate the temperature using Equation (2) from the Theory section. Repeat for the 7V Run and the 10V Run.

Analysis 2: Planck's Equation

1. Calculated temperatures (from max wavelength) :
 - a. 10V Run:
 - b. 7V Run:
 - c. 4V Run:
2. Does your calculated temperature agree with that inferred from the filament resistance?
3. Create the following calculations (explained in Appendix 3):

$$I = \left(\frac{C1}{[\lambda \text{ (nm)}] / 10^9} \right)^5 \left(\frac{1}{\exp\left(\frac{C2}{([\lambda \text{ (nm)}] / 10^9) * [\text{Temp}]} \right) - 1} \right)$$



$$I_{\text{theory}} = [I] * (1 * 10^{-11}) * [\text{scale}]$$

$$C1 = 2 * \pi * [\text{Planck's constant (J}\cdot\text{s)}] * [\text{Speed of light (m/s)}]^2$$

$$C2 = [\text{Planck's constant (J}\cdot\text{s)}] * [\text{Speed of light (m/s)}] / [\text{Boltzmann's constant (J/K)}]$$

$$\text{Temp} = 2500$$

$$\text{scale} = 1.02$$

4. On the graph of Light Intensity vs. wavelength, create a second vertical axis and select the theoretical Intensity (I_{theory}). Click on the Data Display icon () and select the 10V Run. Click the Re-size tool () . Change the temperature (Temp) in the calculator to match your calculated temperature above. Be sure that the scales on the left and right sides of the graph are the same! If they are not, change one of the scales by moving the cursor above a number on that vertical scale and when the hand icon changes to a parallel plate icon, click and drag to stretch the scale until both are the same.
5. The “scale” is included in the calculation because the Broad Spectrum Sensor is not calibrated. Also, we need a scale factor of roughly 10^{-11} that is built into line 11 in the Calculator to adjust for the fact that the reading from the light sensor is in units of % but the units of the theory are in W/m^3 . The “scale” is currently 1.02. Change this until the Blackbody curve (I_{meas}) roughly matches the Tungsten curve but is above it everywhere.
6. Does the shape of the curve match the theoretical curve? Can the bulb really be considered a blackbody?
7. Real objects radiate less than a Blackbody. For a real object $I_{\text{real}} = e(\lambda) I_{\text{Plank}}$ where $e(\lambda)$ is called the emissivity and is a function of wavelength and is always less than 1. For unoxidized tungsten at 2000 K, the value of $e(\lambda)$ averaged over all wavelength is 0.260 and at 3000 K it is 0.334 (CRC Handbook, 63rd edition, page E-387). To show the true blackbody curve, triple the “scale” value, so the Blackbody curve is about 3x as high as

the Tungsten curve. You will need to adjust the vertical scales. First adjust the I_{theory} scale on right until the I_{theory} plot all shows on the graph. Then adjust the left hand scale until both scales are the same. Tungsten actually peaks at a wavelength about 20 nm shorter than the Blackbody peak. This is too small to see in your data. You now have the true and ideal curves for tungsten.

Appendix 1: Finding the wavelength as a function of angle.

Wavelength Calculation: The index of refraction of the prism glass varies with the wavelength of the light. To determine the wavelength as a function of the angle, the relationship between the index of refraction and the angle is determined using Snell's Law at each face of the prism and some geometry and basic trigonometry.

$$\sin 60^\circ = n \sin \theta_2 \quad (\text{A1})$$

$$\text{and} \quad \sin \theta = n \sin \theta_3 \quad (\text{A2})$$

where n is the index of refraction of the prism.

$$n \sin \theta_3 = n \sin (60^\circ - \theta_2) = n(\sin 60^\circ \cos \theta_2 - \cos 60^\circ \sin \theta_2)$$

$$= n \sin 60^\circ \cos \theta_2 - \cos 60^\circ \sin 60^\circ \text{ (using Equation A1)}$$

Rearranging this and using Equation A2 yields

$$n \cos \theta_2 = (\sin \theta / \sin 60^\circ) + \sin 60^\circ \quad (\text{A3})$$

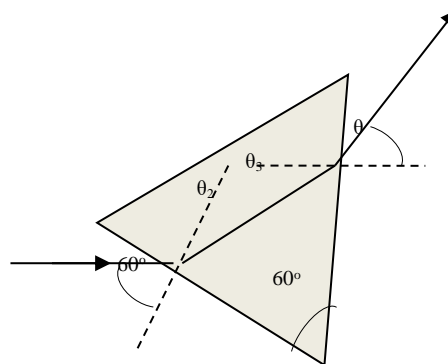
Squaring Equations A1 and A3 and adding them to gives

$$n^2 (\sin^2 \theta_2 + \cos^2 \theta_2) = n^2 = [(\sin \theta / \sin 60^\circ) + \sin 60^\circ]^2 + \sin^2 60^\circ$$

Putting in values for $\sin 60^\circ$ and $\cos 60^\circ$ yields

$$n = \sqrt{\left(\frac{2}{\sqrt{3}} \sin \theta + \frac{1}{2}\right)^2 + \frac{3}{4}} \quad (\text{A4})$$

We use this equation to calculate index of refraction (n) values for our measured angles. We then use the n values to calculate the wavelength using values relating the index of refraction to wavelength for the prism (provided by the supplier of the prism)(see table under tab Append 1B).

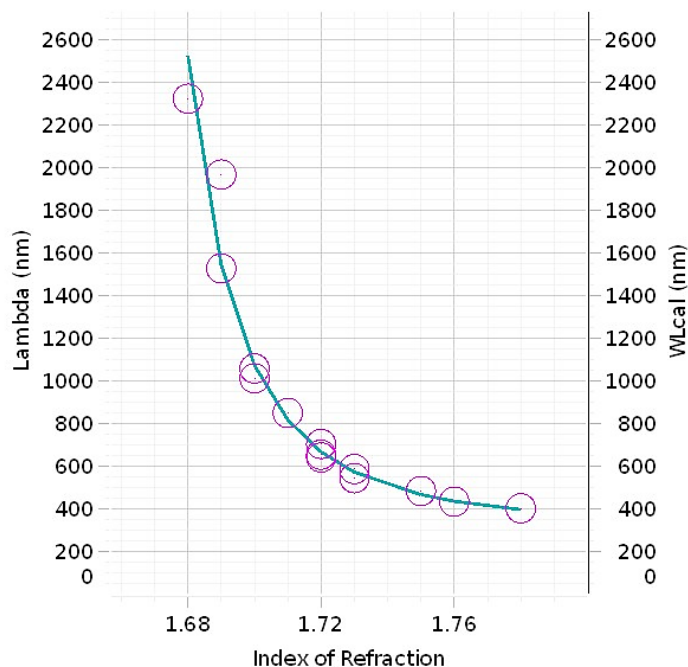


We need an equation based on the data in the table to use in our calculations. We use a polynomial and choose the values of the constants to fit the prism data. The results are not unique but fit the data within the uncertainty in the index implicit in the data table of at least 0.005. The equation is

$$\lambda = A + B(n-E)^{-1} + C(n-E)^{-2} + D(n-E)^{-3} \quad (A5)$$

where λ is the wavelength in nm, and the constants have values: $A = 320$ nm, $B = 1$ nm, $C = 0.2$ nm, $D = 0.19$ nm, and $E = 1.635$. On the graph, the open circles represent the prism supplier's data with the size of the circles showing the uncertainty, and the curved line is from the above equation.

Index of Refraction	Wavelength (nm)
1.68	2325.40
1.69	1970.10
1.69	1529.60
1.70	1060.00
1.70	1014.00
1.71	852.10
1.72	706.50
1.72	656.30
1.72	643.00
1.72	632.80
1.73	589.30
1.73	546.10
1.75	486.10
1.76	435.80
1.78	404.70



Prism Wavelength versus Index of Refraction

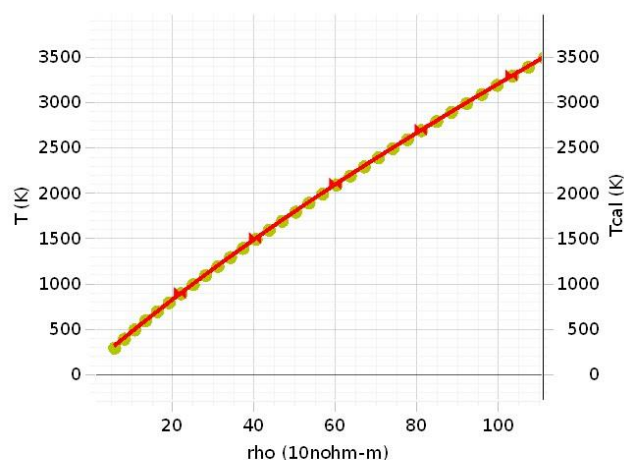
Appendix 2

Finding the Temperature: the resistivity of Tungsten over a broad range of temperature is shown in the table (CRC Handbook, 63rd edition, page E-387). A function that approximates the data is also given

$$T(K) = 103 + 38.1\rho - 0.095\rho^2 + 0.000248\rho^3 \quad (A6)$$

where ρ is the resistivity in units of $10^{-8} \Omega\cdot m$. The graph shows the fit between the measured values of $T(K)$ from the table (circles) and the values from equation A6 (line).

resistivity ($\times 10^{-8} \Omega\cdot m$)	Temperature (K)	resistivity ($\times 10^{-8} \Omega\cdot m$)	Temperature (K)
5.65	300	60.06	2100
8.06	400	63.48	2200
10.56	500	66.91	2300
13.23	600	70.39	2400
16.09	700	73.91	2500
19.00	800	77.49	2600
21.94	900	81.04	2700
24.93	1000	84.70	2800
27.94	1100	88.33	2900
30.98	1200	92.04	3000
34.08	1300	95.76	3100
37.19	1400	99.54	3200
40.36	1500	103.3	3300
43.55	1600	107.2	3400
46.78	1700	111.1	3500
50.05	1800	115.0	3600
53.35	1900	115.0	3600
56.67	2000	115.0	3600



Tungsten Temperature vs Resistivity

We may ignore the expansion of the filament to good approximation, so the resistivity is directly proportional to the resistance and we write

$$\rho/\rho_o = R_{fil}/R_o = (R_{meas} - R_{holder})/R_o = (V/I - R_{holder})/R_o$$

$$\rho = \rho_o(V/I - R_{holder})/R_o \quad (A7)$$

where ρ_o is the resistivity at room temperature ($\rho_o = 5.65 \times 10^{-8} \Omega\cdot m$), R_o is the resistance of the filament at room temperature ($R_o = 0.93 \Omega$), R_{fil} is the resistance of the filament at some temperature, R_{holder} is the resistance of the lamp holder, $R_{meas} = V/I$ is the measured resistance of the bulb holder + lamp, I is the current through the filament, and V is the voltage measured directly across the lamp holder. By measuring V , I , and R_{holder} and using Equations A6 & A7 we determine the temperature of the filament.

Appendix 3: Explanation of Equations in Capstone File

Note that the down arrow (▼) indicates that you can click on it to choose a specific run of that measurement rather than having the calculation apply to all runs. Normally calculations are for all runs except when you may want to subtract a particular run from all the other runs, as you may do for calibration.

1. $\text{AngCorr} = [\text{Table Angle } (^{\circ}), \text{▼}] / [\text{Shaft Angle (rad)}, \text{▼}]$

$$\text{AngCorr} = \frac{\text{Table Angle (degrees)}}{\text{Shaft Angle (rad)}}$$

This equation adjusts for the fact that as the shaft of the Rotary Motion Sensor rotates through one revolution, the table only rotates through 1/60th of one revolution.

2. $\lambda_{\text{cal}} = 320 + 1/([\text{Index of Refraction}, \text{▼}] - 1.635) + 0.2/([\text{Index of Refraction}, \text{▼}] - 1.635)^2 + 0.19/([\text{Index of Refraction}, \text{▼}] - 1.635)^3$

$$\lambda_{\text{cal}} = 320 + \frac{1}{(n - 1.635)} + \frac{0.2}{(n - 1.635)^2} + \frac{0.19}{(n - 1.635)^3}$$

This equation uses the information from the manufacturer of the prism to determine the wavelength as it depends on the index of refraction. The numbers are given to us by the manufacturer. The manufacturer determines these numbers empirically.

3. $T_{\text{cal}} = 103 + 38.1 * [\rho (10^{-8} \Omega\text{-m}), \text{▼}] - 0.095 * [\rho (10^{-8} \Omega\text{-m}), \text{▼}]^2 + 0.000248 * [\rho (10^{-8} \Omega\text{-m}), \text{▼}]^3$

$$T_{\text{cal}} = 103 + 38.1\rho - 0.095\rho^2 + 0.000248\rho^3$$

This equation calculates the temperature of the filament from the resistivity, which is determined by the voltage across the filament and the current through the filament. The equation comes from the CRC Handbook, relating how the resistivity of Tungsten changes with temperature.

4. $\text{true angle} = 0.9569 * \text{abs}([\text{angle (rad)}, \text{▼}])$

$$\text{true angle} = 0.9569 \text{abs}(\text{angle})$$

The true angle is calibrated by moving the spectrometer arm through a known angle read from the spectrometer table and reading the angle from the Rotary Motion Sensor. The number is entered by the user and is calculated by taking the ratio of the Table Angle in degrees over the Rotary Motion Sensor Angle in radians. Therefore, the true angle is in degrees. The absolute value of the Rotary Motion Sensor is taken to ensure a positive angle even if the Rotary Motion Sensor happens to be set up so that it reads negative.

5. $\text{CorrTrueAng} = [\text{Initial angle}] - [\text{true angle}, \nabla]$

$$\text{CorrTrueAng} = \text{Initial Angle} - \text{True Angle}$$

The CorrTrueAng is measured from the central maximum. The Initial Angle is the angle of the central maximum measured on an Intensity vs. True Angle graph.

6. $n = \sqrt{(2 \sin([\text{CorrTrueAng} (^\circ), \nabla]) / \sqrt{3} + 0.5)^2 + 3/4}$

$$n = \sqrt{\left(\frac{2 \sin(\text{CorrTrueAng})}{\sqrt{3}} + 0.5\right)^2 + \frac{3}{4}}$$

This equation is derived in Appendix 1. The index of refraction, n , of the prism glass varies with the wavelength of the light. To determine the wavelength that corresponds to the angle the light is bent, the index of refraction must be known.

7. $\lambda = \text{filter}(400, 2500, 320 + 1/([n, \nabla] - 1.635) + 0.2/([n, \nabla] - 1.635)^2 + 0.19/([n, \nabla] - 1.635)^3)$

$$\lambda = \text{filter}(400, 2500, \lambda_{\text{cal}})$$

The wavelength, λ_{cal} , determined from the angle is given by Equation (A5) in Appendix 1. It is found from the values given by the prism manufacturer. This equation is a filter that only gives a value for the wavelength if the wavelength is between 400 nm and 2500 nm, which is the useful range of the prism glass.

8. $\text{resist} = 5.65 * ([\text{Voltage Ch A (V), } \nabla] / [\text{Output Current Ch H (A), } \nabla] - [\text{R holder (ohm), } \nabla]) / 0.93$

From Appendix 2, the equation for the resistivity is:

$$\rho = \rho_o (V/I - R_{\text{holder}}) / R_o$$

where ρ_o is the resistivity at room temperature ($\rho_o = 5.65 \times 10^{-8} \Omega\text{-m}$), R_o is the resistance of the filament at room temperature ($R_o = 0.93 \Omega$),

$$\rho = 5.65 \left(\frac{V/I - R_{\text{holder}}}{0.93} \right)$$

9. $\text{Abs Temp} = 103 + 38.1 * [\text{resist} (10^{-8} \Omega\text{-m}), \nabla] - 0.095 * [\text{resist} (10^{-8} \Omega\text{-m}), \nabla]^2 + 0.000248 * [\text{resist} (10^{-8} \Omega\text{-m}), \nabla]^3$

$$\text{Abs Temp} = 103 + 38.1\rho - 0.095\rho^2 + 0.000248\rho^3$$

$$10. \quad I = \frac{C1}{([\lambda \text{ (nm)}, \nabla]/10^9)^5} \left(\frac{1}{\exp([C2]/([\lambda \text{ (nm)}, \nabla]/10^9) * [\text{Temp}]) - 1} \right)$$

$$I = \frac{C1}{\lambda^5} \left(\frac{1}{e^{\frac{C2}{\lambda T}} - 1} \right)$$

This is the theoretical equation from the theory Equation (1). The temperature is entered by the user in the constant equation below (14) to affect the shape of the curve.

$$11. \quad I_{\text{theory}} = [I, \nabla] * (1 * 10^{(-11)}) * [\text{scale}]$$

$$I_{\text{theory}} = (I \times 10^{-11})(\text{scale})$$

This is the theoretical equation multiplied by a factor of 10^{-11} to make the values in the same range as the measured light intensity values that are in percent of maximum. It is also multiplied a scale factor that you can adjust to make the height of the theoretical curve match more closely to the data. This calculation does not affect the shape of the curve.

$$12. \quad C1 = 2 * \pi * [\text{Planck's constant (J}\cdot\text{s)}] * [\text{Speed of light (m/s)}]^2$$

$$C1 = 2\pi hc^2$$

$$13. \quad C2 = [\text{Planck's constant (J}\cdot\text{s)}] * [\text{Speed of light (m/s)}] / [\text{Boltzmann's constant (J/K)}]$$

$$C2 = \frac{hc}{k}$$

$$14. \quad \text{Temp} = 2500$$

$$15. \quad \text{Initial angle} = 68.9$$

$$16. \quad \text{scale} = 1.02$$

Appendix 4

Spectrophotometer Set Up

This part of the manual describes how to set up the Spectrophotometer System (see Fig. 3).

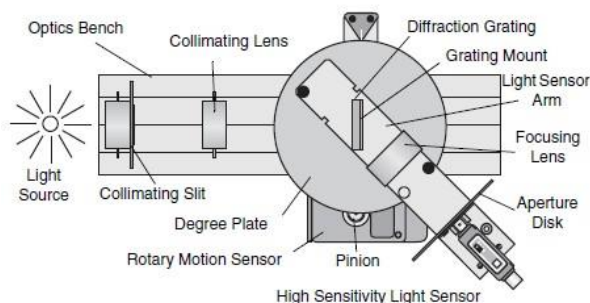


Figure 3: Spectrophotometer System (top view)

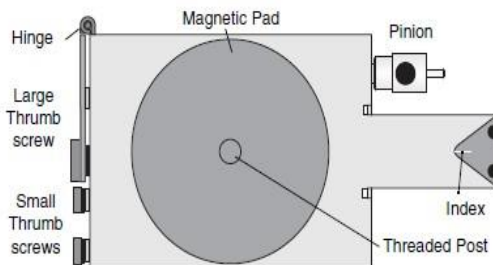


Figure 4: Spectrophotometer Base (top view)

Mounting the Rotary Motion Sensor

This describes how to mount the Rotary Motion Sensor to the hinge on the Spectrophotometer Base. The top of the Spectrophotometer Base has a short threaded post for centering the circular Degree Plate and for holding the Grating Mount. It also has a magnetic pad for holding the Degree Plate, and a triangular shaped index marker. One side of the base has a post upon which the Pinion can be stored when it is not in use. The other side has a spring-loaded hinge and two small thumbscrews for mounting the Rotary Motion Sensor (included in the Spectrophotometer System). On both sides of the base are large thumbscrews and square nuts used for mounting the Spectrophotometer Base on the Optics Bench (see Fig. 4).

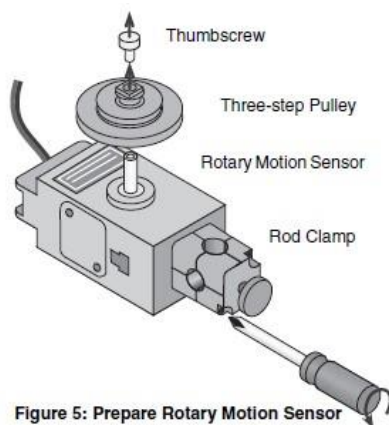


Figure 5: Prepare Rotary Motion Sensor

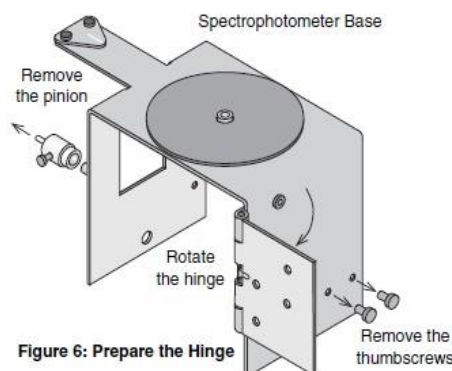
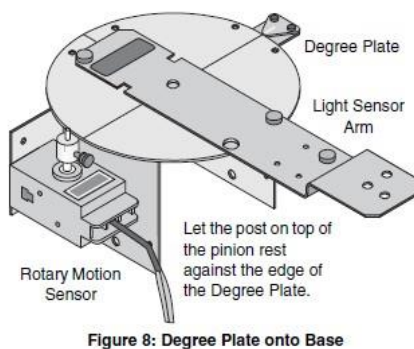
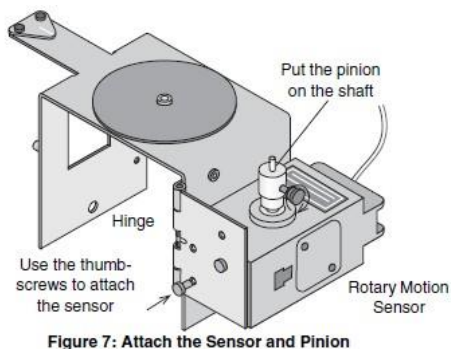


Figure 6: Prepare the Hinge

The Rotary Motion Sensor has a three step pulley attached to its shaft with a small thumbscrew. The sensor also has a rod clamp attached at one end. First, remove the small thumbscrew and three step pulley from the Rotary Motion Sensor shaft. Then, remove the rod clamp from the Rotary Motion Sensor (see Fig. 5). Remove the two small thumbscrews from the threaded storage holes on the side of the Spectrophotometer Base and set them aside for the

moment. Remove the Pinion from the storage post on the opposite side of the Spectrophotometer Base and set the Pinion aside for a moment (see Fig. 6). Rotate the hinge away from the side of the base until the hinge is almost perpendicular to the base. Use the two small thumbscrews to fasten the Rotary Motion Sensor to the *lower* set of holes on the inside of the hinge. Place the Pinion all the way onto the Rotary Motion Sensor shaft and tighten the Pinion on the shaft by turning the small thumbscrew on the side of the Pinion (see Fig. 7). Connect the Rotary Motion Sensor to the PASCO interface.



Mounting the Degree Plate and Light Sensor Arm

The Degree Plate and Light Sensor Arm are shipped as a unit. The Light Sensor Arm is attached to the circular Degree Plate with two small thumbscrews. The hole in the center of the Degree Plate fits over the short threaded post on the top of the Spectrophotometer Base. Hold the Rotary Motion Sensor slightly away from the base so the small diameter post on top of the Pinion is not in the way of the edge of the Degree Plate. Position the hole in the plate over the short threaded post on the top of the base. Place the Degree Plate onto the Spectrophotometer Base. Let the small diameter post on the top of the Pinion rest against the *edge* of the Degree Plate (see Fig. 8).