Dipolo Electrico con Potenciales Retardados

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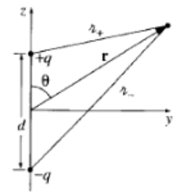

$$V[\vec{r},t) = \frac{1}{4\pi\epsilon_0}\left[\frac{q_0\cos[\omega t_r)}{\mathscr{r}_+} - \frac{q_0\cos[\omega t_r)}{\mathscr{r}_-}\right]$$

$$t_r = t - \mathscr{r}/c$$

$$\mathscr{r}_\pm = \sqrt{r^2 \mp dr\cos\theta + \left(\frac{d}{2}\right)^2} \qquad \boxed{d << r}$$

$$\mathscr{r}_\pm = r\sqrt{1 \mp \frac{d}{r}\cos\theta + \left(\frac{d}{2}\right)^2\frac{1}{r^2}} \qquad \frac{d^2}{r^2} \to 0$$

$$r\left(1 \mp \frac{d}{r}\cos\theta + \left(\frac{d}{2r}\right)^2\right)^{1/2} = r\left(1 \mp 2x\cos\theta + x^2\right)^{1/2} = f[x]$$

Taylor Expansion $\quad f[x) = \sum \frac{f^{(n)}[x)}{n!}\Big|_{x=0} x^n$

$$f_\pm[0] = r(1) = r$$

$$f'_\pm[x] = r\,\frac{1}{2}\left(1 \mp 2x\cos\theta + x^2\right)^{-1/2}\left(\mp 2\cos\theta + 2x\right)$$

$$= \frac{r(x \mp \cos\theta)}{\left(1 \mp 2x\cos\theta + x^2\right)^{1/2}}$$

$$f'_\pm[0] = \mp r\cos\theta$$

$$f''[x] = r(1)\left(1 \mp 2x\cos\theta + x^2\right)^{1/2} - r(x \mp \cos\theta)\frac{(x \mp \cos\theta)}{(1 \mp 2x\cos\theta + x^2)^{1/2}}$$

$$f_\pm[\ldots] \; \ldots \; \frac{\ldots (1 \mp 2x\cos\theta + x^{\ldots})}{(1 \mp 2x\cos\theta + x^2)}$$

$$f''_-[0] = r - r(-\cos\theta)^2 = r(1-\cos^2\theta) = r\sin^2\theta$$

$$f''_+[0] = r - r(\cos\theta)^2 = r\sin^2\theta$$

$\longrightarrow$

$$f_\pm[x] = f[0] + f'[0]\,x + f''[0]\,x^2 + O(x^3)$$

$$r_\pm = f_\pm\left[x = \frac{d}{2r}\right] = r \mp r\cos\theta\left(\frac{d}{2r}\right) + r\sin^2\theta\left(\frac{d}{2r}\right)^2$$

$$r_\pm \simeq r\left(1 \mp \frac{d}{2r}\cos\theta + \sin^2\theta\left(\frac{d}{2r}\right)^2\right)$$

Usando $r_\pm \simeq r\left(1 \mp \frac{d}{2r}\cos\theta\right)$

$$\cos\left(w\left(t - \frac{r_\pm}{c}\right)\right) = \cos\left[wt - \frac{w}{c}r\left(1 \mp \frac{d}{2r}\cos\theta\right)\right]$$

$$I_\pm = \cos\left[w\left(t - \frac{r}{c}\right) \pm \frac{wd}{2c}\cos\theta\right]$$

$$\cos(a \pm b) = \cos a\cos b \mp \sin a \sin b$$

$$I_\pm = \cos\left[w\left(t-\frac{r}{c}\right)\right]\cos\left[\frac{wd}{2c}\cos\theta\right] \mp \sin\left[w\left(t-\frac{r}{c}\right)\right]\sin\left[\frac{wd}{2c}\cos\theta\right]$$

$$I_\pm = \cos\left[w\left(t-\frac{r}{c}\right)\right]\cos\left[\frac{wd}{2c}\cos\theta\right] \mp \sin\left[w\left(t-\frac{r}{c}\right)\right]\sin\left[\frac{wd}{2c}\cos\theta\right]$$

aproximación $d \ll \frac{c}{w} \longrightarrow \frac{wd}{c} \ll 1$

$$\cos\left[\frac{wd}{2c}\cos\theta\right] \simeq 1$$

$$\sin\left[\frac{wd}{2c}\cos\theta\right] \simeq \frac{wd}{2c}\cos\theta$$

$$\therefore \cos\left[w\left(t-\frac{r_\pm}{c}\right)\right] \cong \cos\left[w\left(t-\frac{r}{c}\right)\right] \mp \frac{wd}{2c}\cos\theta \, \sin\left[w\left(t-\frac{r}{c}\right)\right]$$

$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\left[\frac{q_0\cos[wt_r)}{r_+} - \frac{q_0\cos[wt_r)}{r_-}\right]$$

mientras que $\dfrac{1}{r_\pm} \simeq \dfrac{1}{r}\left(1 \pm \dfrac{d}{2r}\cos\theta\right)$

$$V(r,\theta,t) = \frac{q_0}{4\pi\epsilon_0}\left[\frac{1}{r}\left(1+\frac{d}{2r}\cos\theta\right)\left\{\cos\left[w\left(t-\frac{r}{c}\right)\right] - \frac{wd}{2c}\cos\theta\,\sin\left[w\left(t-\frac{r}{c}\right)\right]\right\}\right.$$
$$\left. - \frac{1}{r}\left(1-\frac{d}{2r}\cos\theta\right)\left\{\cos\left[w\left(t-\frac{r}{c}\right)\right] + \frac{wd}{2c}\cos\theta\,\sin\left[w\left(t-\frac{r}{c}\right)\right]\right\}\right]$$

$$\cos\left[w\left(t-\frac{r}{c}\right)\right] = \Omega$$
$$\sin\left[w\left(t-\frac{r}{c}\right)\right] = \xi$$

$$V = \frac{q}{4\pi\epsilon_0}\frac{1}{r}\left[\Omega - \frac{wd}{2c}\cos\theta\,\xi + \frac{d}{2r}\Omega - \frac{wd^2}{4rc}\cos\theta\,\xi\right.$$
$$\left. - \Omega - \frac{wd}{2c}\cos\theta\,\xi + \frac{d}{2r}\Omega + \frac{wd^2}{4rc}\cos\theta\,\xi\right]$$

$$V = \frac{q}{4\pi\epsilon_0}\frac{1}{r}\left[\cancel{\Omega} - \frac{wd}{2c}\cos\theta\,\xi + \frac{d}{2r}\Omega - \frac{wd^2}{\ } \cancel{\cos\theta\,\xi}\right.$$

$$V = \frac{q}{4\pi\epsilon_0}\frac{1}{r}\left[\cancel{\Omega} - \frac{wd}{2c}\cos\theta\ \xi + \frac{d}{2r}\Omega - \frac{wd^2}{4rc}\cancel{\cos\theta\ \xi} \right.$$
$$\left. - \cancel{\Omega} - \frac{wd}{2c}\cos\theta\ \xi + \frac{d}{2r}\Omega + \frac{wd^2}{4rc}\cancel{\cos\theta\ \xi} \right]$$

$$V = \frac{q}{4\pi\epsilon_0 r}\left[\frac{d}{r}\cos\left[w\left(t-\frac{r}{c}\right)\right] - \frac{wd}{c}\cos\theta\ \sin\left[w\left(t-\frac{r}{c}\right)\right]\right]$$

$$|p = qd$$

$$V = \frac{p}{4\pi\epsilon_0 r}\left[\frac{\cos[w(t-\frac{r}{c})]}{r} - \frac{w}{c}\cos\theta\ \sin\left[w\left(t-\frac{r}{c}\right)\right]\right]$$

Sin asumir la aproximación 3 : $r \gg \frac{c}{w}$

$$\begin{cases} V = \frac{p}{4\pi\epsilon_0}\frac{1}{r^2}\left(\Omega - \frac{rw}{c}\cos\theta\ \xi\right) \\ V = \frac{p}{4\pi\epsilon_0 r}\left[\frac{\cos[w(t-\frac{r}{c})]}{r} - \frac{w}{c}\cos\theta\ \sin\left[w\left(t-\frac{r}{c}\right)\right]\right] \end{cases}$$

$$\nabla V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{\phi}$$

$$\partial_r \Omega = \frac{\partial}{\partial r}\cos\left[w\left(t-\frac{r}{c}\right)\right] = -\sin\left[w\left(t-\frac{r}{c}\right)\right]\left(-\frac{w}{c}\right) = \frac{w}{c}\sin\left[w\left(t-\frac{r}{c}\right)\right]$$

$$\partial_r \xi = \frac{\partial}{\partial r}\sin\left[w\left(t-\frac{r}{c}\right)\right] = -\frac{w}{c}\cos\left[w\left(t-\frac{r}{c}\right)\right]$$

$$\frac{\partial V}{\partial r} = \frac{p}{4\pi\epsilon_0}\frac{\partial}{\partial r}\left[\frac{1}{r^2}\left(\Omega - \frac{w}{c}\cos\theta\ r\ \xi\right)\right]$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right) = -2\frac{1}{r^3}$$

$$\left\{ \partial_r\Omega = \frac{w}{c}\xi \right. \qquad \partial_r\xi = -\frac{w}{c}\Omega$$

$$\psi = \partial_r\left(\frac{1}{r^2}\right)\left(\Omega - \frac{w}{c}\cos\theta\, r\, \xi\right) + \frac{1}{r^2}\left[\partial_r \Omega - \frac{w}{c}\cos\theta\left(\partial_r(r)\xi + r\partial_r\xi\right)\right]$$

$$= -\frac{2}{r^3}\left(\Omega - \frac{w}{c}r\cos\theta\, \xi\right) + \frac{1}{r^2}\left[\frac{w}{c}\xi - \frac{w}{c}\cos\theta\, \xi + \left(\frac{w}{c}\right)^2 r\, \Omega\right]$$

$$= -\frac{2\Omega}{r^3} + \frac{2}{r^2}\frac{w}{c}\cos\theta\, \xi + \frac{1}{r^2}\frac{w}{c}\xi - \frac{1}{r^2}\frac{w}{c}\cos\theta\, \xi + \frac{1}{r}^2\left(\frac{w}{c}\right)\Omega$$

$$= -\frac{2\Omega}{r^3} + \frac{1}{r^2}\frac{w}{c}\xi\left(1+\cos\theta\right) + \frac{1}{r}\left(\frac{w}{c}\right)^2\Omega \quad //$$

$$(\nabla\psi)_r = \left(\frac{1}{r}\left(\frac{w}{c}\right)^2 - \frac{2}{r^3}\right)\cos\left[w\left(t-\frac{r}{c}\right)\right] + \frac{1+\cos\theta}{r^2}\left(\frac{w}{c}\right)\sin\left[w\left(t-\frac{r}{c}\right)\right]$$

donde $V = \frac{p_0}{4\pi\epsilon_0}\psi$

luego $\frac{\partial\psi}{\partial\theta} = \frac{\partial}{\partial\theta}\frac{1}{r^2}\left(\Omega - \frac{rw}{c}\cos\theta\, \xi\right) = -\frac{rw}{r^2 c}\xi\frac{\partial\cos\theta}{\partial\theta}$; $(\nabla\psi)_\theta = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$

$$(\nabla\psi)_\theta = \frac{1}{r}\frac{w}{cr}\sin\theta\, \sin\left[w\left(t-\frac{r}{c}\right)\right] = \frac{w}{c}\frac{1}{r^2}\sin\theta\, \xi$$

$\therefore\ \vec{E} = -\nabla V = -\frac{p_0}{4\pi\epsilon_0}\nabla\psi$

$$\vec{E} = -\frac{p_0}{4\pi\epsilon_0}\left\{\left(\frac{1}{r}\left(\frac{w}{c}\right)^2 - \frac{2}{r^3}\right)\Omega + \frac{1+\cos\theta}{r^2}\left(\frac{w}{c}\right)\xi\right\}\hat{r}$$

$$+ \frac{p_0}{4\pi\epsilon_0}\frac{1}{r^2}\frac{w}{c}\sin\theta\, \xi\, \hat{\theta}$$

Computar y Simular
en coordenadas cartesianas

unidad de campo $\vec{E}$

$$\hat{r} = \begin{bmatrix} \cos\phi \sin\theta \\ \sin\phi \sin\theta \\ \cos\theta \end{bmatrix} \quad ; \quad \hat{\theta} = \begin{bmatrix} \cos\phi \cos\theta \\ \sin\phi \cos\theta \\ -\sin\theta \end{bmatrix} \quad ; \quad \frac{P_0}{4\pi\epsilon_0} \Rightarrow 1\,[u]$$

$$\vec{E}[u] = -\left\{ \left(\frac{1}{r} \left(\frac{\omega}{c} \right)^2 - \frac{2}{r^3} \right) \Omega + \frac{1+\cos\theta}{r^2} \left(\frac{\omega}{c} \right) \xi \right\} \begin{bmatrix} \cos\phi \sin\theta \\ \sin\phi \sin\theta \\ \cos\theta \end{bmatrix}$$

$$+ \quad \frac{1}{r^2} \frac{\omega}{c} \sin\theta \, \xi \begin{bmatrix} \cos\phi \cos\theta \\ \sin\phi \cos\theta \\ -\sin\theta \end{bmatrix}$$