0911 Calculo Hayward profesor Milko

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lapsus function

$$F[r] = 1 - \frac{2mr^2}{r^3 + 2a^2m}$$

expansión con $x = \frac{1}{r}$; $F[\frac{1}{x}] = 1 - \frac{2m(1/x)^2}{\frac{1}{x^3} + 2a^2m} = 1 - \frac{1}{x^2}\frac{2mx^3}{1+2a^2mx^3}$

$$F[\tfrac{1}{x}] = 1 - \frac{2mx}{1+2a^2mx^3}$$

$$\frac{dF[1/x]}{dx} = \frac{2m(1+2a^2mx^3) - 2mx(6a^2mx^2)}{(1+2a^2mx^3)^2} = \frac{2m(1+2a^2mx^3 - 6a^2mx^3)}{(1+2a^2mx^3)^2}$$

$$= \frac{2m(1-4a^2mx^3)}{(1+2a^2mx^3)^2} \quad // \qquad \left.\frac{dF[1/x]}{dx}\right|_{x=0} = 2m$$

$$\frac{d^2F[1/x]}{dx^2} = 2m\,\frac{(1-12a^2mx^2)(1+2a^2mx^3)^2 - 2m(1-4a^2mx^3)2(1+2a^2mx^3)(6a^2mx^2)}{(1+2a^2mx^3)^2}$$

$$= -\frac{2m(24a^2mx^2)(a^2mx^3-1)}{(1+2a^2mx^3)^3}$$

$$\left.\frac{d^2F[1/x]}{dx}\right|_{x=0} = \frac{-2m(0)(0-1)}{(1+0)^3} = 0.$$

No se ve una expresión n a simple vista, continuando entonces con mathematica

alrededor de

$$\frac{1}{r} \sim 0$$

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F[r_] := 1 - 2 m r^2/(r^3 + 2 a^2 m)

Series[F[1/x], {x, 0, 15}]
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$1 - 2\,m\,x + 4\,a^2\,m^2\,x^4 - 8\,(a^4\,m^3)\,x^7 + 16\,a^6\,m^4\,x^{10} - 32\,(a^8\,m^5)\,x^{13} + O[x]^{16}$

$$F[r] \simeq 1 - 2m\frac{1}{r} + 2m\,(2a^2m)\frac{1}{r^4} \quad 2m\,(4a^4m^2)\frac{1}{r^7} + 2m\,(8a^6m^3)\frac{1}{r^{10}} + \dots$$

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Table[Simplify[D[F[1/x], {x, n}]/Factorial[n]] /. {x → 0}, {n, 1, 30}]
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$\{-2\,m,\ 0,\ 0,\ 4\,a^2\,m^2,\ 0,\ 0,\ -8\,a^4\,m^3,\ 0,\ 0,\ 16\,a^6\,m^4,$
$0,\ 0,\ -32\,a^8\,m^5,\ 0,\ 0,\ 64\,a^{10}\,m^6,\ 0,\ 0,\ -128\,a^{12}\,m^7,\ 0,\ 0,$
$256\,a^{14}\,m^8,\ 0,\ 0,\ -512\,a^{16}\,m^9,\ 0,\ 0,\ 1024\,a^{18}\,m^{10},\ 0,\ 0\}$

$$F[r] \simeq 1 - \sum_{n=0}^{\infty} 2m\,(2a^2m)^n \frac{(-1)^n}{r^{1+3n}}$$

/ aproximación $r >> 1$

$$F[r] \simeq 1 - \frac{2m}{r}\sum_{n=0}^{\infty} (-1)^n \left(\frac{2a^2m}{r^3}\right)^n$$

$; \quad \frac{2a^2m}{r^3} < 1$

$$\boxed{\sqrt[3]{2a^2m} < r}$$

condición

y para r muy pequeño:

$$F[r] = 1 - \frac{r^2}{a^2} + \frac{r^5}{a^2(2a^2m)} - \frac{r^8}{a^2(4a^6m^2)} + \frac{r^{11}}{a^2(8a^8m^3)} + \dots$$

$$= 1 - \frac{1}{}\sum^{\infty} (-1)^n \frac{r^{2+3n}}{} = 1 - \frac{r^2}{}\sum^{\infty}(-1)^n\left(\frac{r^3}{}\right)^n$$

$$= 1 - \frac{1}{a^2}\sum_{n=0}^{\infty}(-1)^n \frac{r^{2+3n}}{(2a^2m)^n} = 1 - \frac{r^2}{a^2}\sum_{n=0}^{\infty}(-1)^n\left(\frac{r^3}{2a^2m}\right)^n$$

$$\boxed{\sqrt[3]{2a^2m} > r}$$

Por tanto ahora la integración de 0 a infinito, puede separarse en terminos de la series

$$\int_0^R r\sqrt{F[r]}\,dr = \int_0^{\sqrt[3]{2a^2m}} r\sqrt{1-\frac{r^2}{a^2}\sum_{n=0}^{\infty}(-1)^n\left(\frac{r^3}{2a^2m}\right)^n}\,dr + \int_{\sqrt[3]{2a^2m}}^{R} r\sqrt{1-\frac{2m}{r}\sum_{n=0}^{\infty}(-1)^n\left(\frac{2a^2m}{r^3}\right)^n}\,dr$$

$$= \int_0^{\sqrt[3]{2a^2m}} r\left(1-\frac{r^2}{2a^2}\sum_{n=0}^{\infty}(-1)^n\left(\frac{r^3}{2a^2m}\right)^n\right)dr + \int_{\sqrt[3]{2a^2m}}^{R} r\left(1-\frac{m}{r}\sum_{n=0}^{\infty}(-1)^n\left(\frac{2a^2m}{r^3}\right)^n\right)dr$$

$$= \int_0^{\sqrt[3]{2a^2m}} r - \frac{r^3}{2a^2}\sum_{n=0}^{\infty}(-1)^n\left(\frac{r^3}{2a^2m}\right)^n dr + \int_{\sqrt[3]{2a^2m}}^{R} r - m\sum_{n=0}^{\infty}(-1)^n\left(\frac{2a^2m}{r^3}\right)^n dr$$

$$= \int_0^R r\,dr - \sum_{n\geq 0}\frac{(-1)^n}{2a^2}\int_0^{\sqrt[3]{2a^2m}} r^3\left(\frac{r^3}{2a^2m}\right)^n dr - \sum_{n\geq 0} m(-1)^n\int_{\sqrt[3]{2a^2m}}^{R}\left(\frac{2a^2m}{r^3}\right)^n dr$$

$$I_1 = \int_0^{\sqrt[3]{2a^2m}} r^3\left(\frac{r^3}{2a^2m}\right)^n dr \quad \left| \quad \begin{array}{l} u = r/\sqrt[3]{2a^2m} \;; \quad \sqrt[3]{2a^2m}\,du = dr \\ u(0) = 0 \qquad\qquad r^3 = (2a^2m)u^3 \\ u(\sqrt[3]{2a^2m}) = 1 \end{array}\right.$$

$$I_1 = (2a^2m)^{4/3} \int_0^1 u^3 u^{3n} du = (2a^2m)^{4/3} \int_0^1 u^{3n+3} du = (2a^2m)^{4/3} \frac{1}{3n+4}$$

$$I_2 = \int_{\sqrt[3]{2a^2m}}^{R} \left(\frac{2a^2m}{r^3}\right)^n dr \quad \left| \begin{array}{l} w = \frac{r}{\sqrt[3]{2a^2m}} \ ; \ dr = \sqrt[3]{2a^2m}\, dw \\ w(\sqrt[3]{2a^2m}) = 1 \\ w(R) = \frac{R}{\sqrt[3]{2a^2m}} \end{array} \right\} = \int_1^{w(R)} \frac{(2a^2m)^{1/3}}{w^{3n}} dw$$

$$= (2a^2m)^{1/3} \left.\frac{w^{1-3n}}{1-3n}\right|_1^{w(R)} = (2a^2m)^{1/3} \left(\frac{R}{\sqrt[3]{2a^2m}}\right) \frac{(2a^2m)^n}{R^{3n}} \frac{1}{1-3n} - \frac{(2a^2m)^{1/3}}{1-3n}$$

$$I_2 = \frac{R}{1-3n} \left(\frac{2a^2m}{R^3}\right)^n - \frac{(2a^2m)^{1/3}}{1-3n}$$

$$\int_0^R r\sqrt{F[r]}\, dr = \int_0^R r\, dr - \sum_{n\geq 0} \frac{(-1)^n}{2a^2} (2a^2m)^{4/3} \frac{1}{3n+4} - \sum_{n\geq 0} m(-1)^n \left(\frac{R}{1-3n}\left(\frac{2a^2m}{R^3}\right)^n - \frac{(2a^2m)^{1/3}}{1-3n}\right)$$

$$= \frac{R^2}{2} - \sum_{n\geq 0} m(2a^2m)^{1/3} \frac{(-1)^n}{3n+4} - \sum_{n\geq 0} mR \frac{(-1)}{1-3n}\left(\frac{2a^2m}{R^3}\right)^n + \sum_{n\geq 0} m(2a^2m)^{1/3} \frac{(-1)^n}{1-3n}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{3n+4} = 1 - \frac{1}{3}\left(\frac{\pi}{\sqrt{3}} + \ln(2)\right)$$

$$\boxed{\begin{array}{l} \frac{\sqrt{3}}{9} = \frac{3^{1/2}}{3^2} = 3^{\frac{1}{2}-\frac{4}{2}} = 3^{-\frac{3}{2}} = \frac{1}{\sqrt{27}} \\ \frac{1}{3\sqrt{3}} = \frac{1}{\sqrt{27}} \end{array}}$$

$$\sum_{n\geq 0} m(2a^2m)^{1/3} \frac{(-1)^n}{1-3n} = m\sqrt[3]{2a^2m}\left(1 - \frac{\pi}{\sqrt{27}} - \frac{1}{3}\ln(2)\right)$$

$n \geq 0$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{1-3n} = \frac{1}{9}\left(9+\sqrt{3}\,\pi - 3\ln(2)\right)$$

$$\sum_{n\geq 0} m(-1)^n \frac{\sqrt[3]{2a^2m}}{1-3n} = m\sqrt[3]{2a^2m}\left(1+\frac{\pi}{\sqrt{27}}-\frac{1}{3}\ln(2)\right)$$

$$\int_0^R r\sqrt{F[r]}\,dr = \frac{R^2}{2} + \sum_{n\geq 0} m\left(2a^2m\right)^{1/3}\frac{(-1)^n}{1-3n} - \sum_{n\geq 0} m\left(2a^2m\right)^{1/3}\frac{(-1)^n}{3n+4} - \sum_{n\geq 0} mR\,\frac{(-1)^n}{1-3n}\left(\frac{2a^2m}{R^3}\right)^n$$

$$= \frac{R^2}{2} + m\sqrt[3]{2a^2m}\left(1+\frac{\pi}{\sqrt{27}}-\frac{1}{3}\ln(2)\right)$$

$$- m\sqrt[3]{2a^2m}\left(1-\frac{\pi}{\sqrt{27}}-\frac{1}{3}\ln(2)\right) - mR\sum_{n\geq 0}\frac{(-1)^n}{1-3n}\left(\frac{2a^2m}{R^3}\right)^n$$

$$= \frac{R^2}{2} + m\sqrt[3]{2a^2m}\left(\frac{2\pi}{\sqrt{27}}\right) - mR\sum_{n\geq 0}\frac{(-1)^n}{1-3n}\left(\frac{2a^2m}{R^3}\right)^n$$

$$mR\sum_{n\geq 0}\frac{(-1)^n}{1-3n}\left(\frac{2a^2m}{R^3}\right)^n = mR\;{}_2F_1\left[\begin{matrix}-\frac{1}{3},\; 1\\ \frac{2}{3}\end{matrix}\;\middle|\; -\left(\frac{2a^2m}{R^3}\right)^n\right]$$

Hypergeometrica

Si nos ponemos a listar los n

$$\int_0^R r\sqrt{F[r]}\,dr = \frac{R^2}{2} + m\sqrt[3]{2a^2m}\left(\frac{2\pi}{\sqrt{27}}\right) - mR\sum_{n\geq 0}\frac{(-1)^n}{1-3n}\left(\frac{2a^2m}{R^3}\right)^n$$

$$= \frac{R^2}{2} + m\sqrt[3]{2a^2m}\left(\frac{2\pi}{\sqrt{27}}\right) - mR - \frac{a^2m^2}{R^2} + \frac{4}{5}\frac{a^4m^3}{R^5} - \frac{a^6m^4}{R^8} + \dots$$

$$= \frac{R^2}{2}\left(1 - \frac{2m}{R} + \frac{m\sqrt[3]{2a^2m}}{R^2}\left(\frac{4\pi}{\sqrt{27}}\right) - 2\frac{a^2m^2}{R^4} + \dots\right)$$

$$> \int_0^R = \frac{R^2}{2} + 2mR + \frac{2a^2m^2}{R^2} = \frac{R^2}{2}\left(1 + \frac{4m}{R} + \frac{4a^2m^2}{R^4}\right)$$