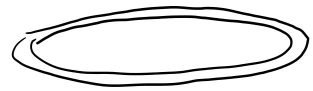
Integral de electro

Tuesday, March 28, 2023 7:14 PM

$$d\vec{E} = \frac{\hat{k}}{4\pi\varepsilon_0} \frac{z}{(z^2+s^2)^{3/2}} \, dq$$

$dq = 2\pi s \, ds \, \sigma$



$$\vec{E} = \frac{z\hat{k}}{4\pi\varepsilon_0} 2\pi\sigma \int_0^\infty \frac{s \, ds}{(z^2+s^2)^{3/2}}$$

\# Lo primero es expandir funciones como series de potencias

$$(z^2+s^2)^{-\frac{3}{2}} = |z|^{-3}\left(1+\frac{s^2}{z^2}\right)^{-3/2}$$

$$ {}_1F_0\left(\begin{matrix} 3/2 \\ \cdot \end{matrix} \middle| -\frac{s^2}{z^2}\right) = \sum_n \frac{(3/2)_n}{n!} \left(\frac{s}{z}\right)^{2n} (-1)^n$$

$$ {}_1F_0\left(\begin{matrix} 3/2 \\ \cdot \end{matrix} \middle| \left(\frac{s}{z}\right)^2\right) = \left(1+\left(\frac{s}{z}\right)^2\right)^{-3/2}$$

$$\therefore \int_0^\infty \frac{s \, ds}{(z^2+s^2)^{3/2}} = \int_0^\infty \frac{s}{|z|^3} \, {}_1F_0\left(\begin{matrix} 3/2 \\ \cdot \end{matrix} \middle| \frac{s^2}{z^2}\right) ds = \int_0^\infty \frac{s}{|z|^3} \sum_n \frac{(3/2)_n}{n!} \left(\frac{s}{z}\right)^{2n} (-1)^n ds$$

$$I = \sum_{n=0}^\infty \frac{(3/2)_n}{|z|^{3+2n}} \frac{(-1)^n}{n!} \int_0^\infty s^{1+2n} ds$$

def. braket

$$\left\{ \int_0^\infty s^{\alpha-1} ds = \langle \alpha \rangle \right.$$

$$\therefore \int_0^\infty s^{1+2n+1-1} ds = \langle 2+2n \rangle$$

// definición indicador $\frac{(-1)^n}{n!} = \phi_n$

$$I = \sum_{n=0}^\infty \frac{(3/2)_n}{|z|^{3+2n}} \phi_n \langle 2+2n \rangle$$

regla de sumación

$$\sum_{n=0}^\infty f(n) \phi_n \langle n+\beta \rangle = f(n)\Gamma[-n] \Big|_{n=-\beta}$$

// ... $\langle \alpha n+\beta \rangle = \frac{1}{|\alpha|} \left\langle n + \frac{\beta}{\alpha} \right\rangle$ $\quad \left\{ \langle 2n+2 \rangle = \frac{1}{2} \langle n+1 \rangle \right.$

// regla de escalamiento $\langle \alpha n + \beta \rangle = \frac{1}{|\alpha|} \langle n + \frac{\beta}{\alpha} \rangle$ $\quad \{ \langle 2n+2 \rangle = \frac{1}{2} \langle n+1 \rangle$

$$I = \sum_{n=0}^{\infty} \frac{(3/2)_n}{|z|^{3+2n}} \phi_n \frac{1}{2} \langle n+1 \rangle = \frac{\left(\frac{3}{2}\right)_n}{2 z^{3+2n}} \Gamma[-n] \Big|_{n=-1}$$

$$I = \frac{\left(\frac{3}{2}\right)_{(-1)}}{2|z|^{3-2}} \Gamma[1]$$

$\Gamma[1] = 0! = 1$ $\quad \Gamma[\alpha+1] = \alpha \Gamma[\alpha]$

$\left(\frac{3}{2}\right)_n = \frac{\Gamma[\frac{3}{2}+n]}{\Gamma[\frac{3}{2}]}$; $\Gamma[\frac{3}{2}] = \Gamma[1+\frac{1}{2}] = \frac{1}{2}\Gamma[\frac{1}{2}]$

$\left(\frac{3}{2}\right)_{(-1)} = \frac{\Gamma[\frac{3}{2}-1]}{\Gamma[\frac{3}{2}]} = \frac{\Gamma[\frac{1}{2}]}{\Gamma[\frac{3}{2}]} = \frac{\cancel{\Gamma[\frac{1}{2}]}}{\frac{1}{2}\cancel{\Gamma[\frac{1}{2}]}} = 2$

$$I = \frac{2\,\Gamma[1]}{2|z|} = \frac{2}{2|z|} = \frac{1}{|z|} \quad //$$

$$\int_0^{\infty} \frac{s\, ds}{(z^2+s^2)^{3/2}} = \frac{1}{|z|}$$

$\text{signo}(z) = \frac{z}{|z|}$

$$\vec{E} = \frac{z\hat{k}}{4\pi\varepsilon_0} 2\pi\sigma \int_0^{\infty} \frac{s\, ds}{(z^2+s^2)^{3/2}} = \frac{z\hat{k}}{4\pi\varepsilon_0} 2\pi\sigma \frac{1}{|z|} = \frac{\sigma}{2\varepsilon_0}\hat{k}\ \text{sign}(z)$$