Pension model

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1 The model

In each period, each mother with a child of age t, chooses consumption c_t, assets A_{t+1}, and hours h_t (that determines the pension earning points EP_{t+1}) by maximizing her lifetime utility:

$$V\left(A_t, EP_t, t\right) = \max_{c_t, h_t, A_{t+1} \geq 0, EP_{t+1}} \left\{ u\left(c_t, h_t\right) + \frac{1}{1+\delta} V\left(A_{t+1}, EP_{t+1}, t+1\right) \right\}$$

where the period period utility is

$$u\left(c_t, h_t\right) = \frac{c_t^{1-\gamma_c}}{1-\gamma_c} + \beta \frac{\left(H - h_t\right)^{1-\gamma_h}}{1-\gamma_h}.$$

The budget constraint prior to retirement, i.e. $t \leq T_{ret}$, utility is maximized subject to the intertemporal budget constraint:

$$A_{t+1} = (1+r)A_t + w_t\left(1-\tau_t\right)h_t - c_t + Y_t$$

while the budget constraint under retirement is:

$$A_{t+1} = (1+r)A_t + \rho EP_t - c_t + Y_t.$$

Note that Y_t is unearned income (mostly coming from husband/father earnings?) and ρ is the monetary value of pension points.

Law of motion for pension earning points is:

$$EP_{t+1} = EP_t + f_t\left(E_t\right)$$
$$f_t(x) = \begin{cases} \frac{x}{\bar{E}} & \text{if } a_t \notin [3, 10] \\ \max\{\min\{1.5\frac{x}{\bar{E}}, 1\}, \frac{x}{\bar{E}}\} & \text{if } a_t \in [3, 10] \end{cases}$$

where $\bar{E}$ are average earnings and $E_t = h_t w_t$.

2 Solving the model

The model can be solved by backward induction combined with the endogenous gridpoints method. To do so, we need the FOCs before and after retirement. The Lagrangian constrain on pension points can be computed recursively using the FOCs. The Lagraingian before retirement is

$$\mathcal{L} = \max_{c_t,h_t,A_{t+1}\geq 0,EP_{t+1}} \left\{ u\left(c_t,h_t\right) + \frac{1}{1+\delta} V\left(A_{t+1},EP_{t+1},t+1\right) \right\} \tag{1}$$

$$+\lambda_t[(1+r)A_t + w_t\left(1-\tau_t\right)h_t - c_t + Y_t - A_{t+1}] \tag{2}$$

$$+\chi_t[-\mathrm{EP}_{t+1} + \mathrm{EP}_t + Mh_t w_t/\bar{E}] \tag{3}$$

$$+\mu_t A_{t+1}; \tag{4}$$

$$\mu_t A_{t+1} \geq 0, \mu_t \geq 0, A_{t+1} \geq 0 \tag{5}$$

where M is the pension point "multiplier." The focs are:

$$\frac{\partial\mathcal{L}}{\partial c_t} = 0 \implies c_t^{-\gamma_c} = \lambda_t$$

$$\frac{\partial\mathcal{L}}{\partial h_t} = 0 \implies \beta(H-h_t)^{-\gamma_h} = \lambda_t w_t(1-\tau_t) + \chi_t M w_t/\bar{E}$$

$$\frac{\partial\mathcal{L}}{\partial a_{t+1}} = 0 \implies \lambda_{t+1}\frac{(1+r)}{(1+\delta)} + \mu_t = \lambda_t$$

$$\frac{\partial\mathcal{L}}{\partial \mathrm{PEN}_{t+1}} = 0 \implies \chi_t = \chi_{t+1}\frac{1}{1+\delta}$$

After retirement:

$$\mathcal{L} = \max_{c_t,h_t,A_{t+1}\geq 0,EP_{t+1}} \left\{ u\left(c_t,h_t\right) + \frac{1}{1+\delta} V\left(A_{t+1},EP_{t+1},t+1\right) \right\} \tag{6}$$

$$+\lambda_t[(1+r)A_t + \mathrm{EP}_t\rho - c_t + Y_t - A_{t+1}] \tag{7}$$

$$+\chi_t[-\mathrm{EP}_{t+1} + \mathrm{EP}_t] \tag{8}$$

$$+\mu_t A_{t+1}; \tag{9}$$

$$\mu_t A_{t+1} \geq 0, \mu_t \geq 0, A_{t+1} \geq 0 \tag{10}$$

where M is the pension point "multiplier." The focs are:

$$\frac{\partial\mathcal{L}}{\partial c_t} = 0 \implies c_t^{-\gamma_c} = \lambda_t$$

$$\frac{\partial\mathcal{L}}{\partial a_{t+1}} = 0 \implies \lambda_{t+1}\frac{(1+r)}{(1+\delta)} + \mu_t = \lambda_t$$

$$\frac{\partial\mathcal{L}}{\partial \mathrm{PEN}_{t+1}} = 0 \implies \chi_t = \frac{1}{1+\delta}(\chi_{t+1} + \lambda_t\rho)$$