Pontificia Universidad Católica de Chile
Facultad de Matemáticas
**MAT1620 Cálculo II**
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# Ayudantía 7 - Pre I2

## Problema 1

[14.4.46] Dada la función,

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & si \quad (x,y) \neq (0,0) \\ \\ 0 & si \quad (x,y) = (0,0) \end{cases}$$

Demuestre que existen $f_x(0,0)$ y $f_y(0,0)$, pero $f$ no es diferenciable en $(0,0)$. Muestre que $f_x$ y $f_y$ no son continuas en $(0,0)$.

## Problema 2

Explique por qué la función es diferenciable en el punto dado y determine la linealización $L(x,y)$ (plano tangente) en el punto, también encuentre la ecuación vectorial del plano.

$a$)[14.4.11] $f(x,y) = x\sqrt{y},\ (1,4)$ $\qquad$ $b$)[14.4.15] $f(x,y) = e^{-xy}\cos y,\ (\pi,0)$

## Problema 3

a)[14.4.31] Compare los valores de $\Delta z$ y $dz$ si $z = 5x^2 + y^2$ cambia de $(1,2)$ a (1,05, 2,1).

## Problema 4

a) Sea $z = x^3y^3$, $x = r\cos\theta$, $y = r\sin\theta$. Calcular $\frac{\partial z}{\partial r}$ y $\frac{\partial z}{\partial \theta}$

b) Sea $z = f(x,y)$ y $x = r^2\ln t$ e $y = rt^2$, determine todas las segundas derivadas.

c) Si $x - z = \arctan(yz)$. Calcular $\frac{\partial z}{\partial y}$ y $\frac{\partial z}{\partial x}$

## Solución 4.b

Sea $z = f(x, y)$ y $x = r^2 \ln t$ e $y = rt^2$, determine todas las segundas derivadas.

Recordar que $\frac{\partial^2 z}{\partial x \partial y}$ significa derivar $f$ primero con respecto a $y$ y luego con respecto a $x$.
$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ siempre que ambas derivadas cruzadas sean continuas. Y en ese caso el orden de derivación es indiferente. (Esto es lo que ocurre la mayoría de las veces)

$$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r} = \frac{\partial z}{\partial x}(2r \ln t) + \frac{\partial z}{\partial y}(t^2)$$

$$\frac{\partial^2 z}{\partial r^2} = \left(\frac{\partial^2 z}{\partial x^2}\frac{\partial x}{\partial r} + \frac{\partial^2 z}{\partial y \partial x}\frac{\partial y}{\partial r}\right)\frac{\partial x}{\partial r} + \frac{\partial z}{\partial x}\frac{\partial^2 x}{\partial r^2} + \left(\frac{\partial^2 z}{\partial y^2}\frac{\partial y}{\partial r} + \frac{\partial^2 z}{\partial x \partial y}\frac{\partial x}{\partial r}\right)\frac{\partial y}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial^2 y}{\partial r^2}$$

$$= \left(\frac{\partial^2 z}{\partial x^2}(2r \ln t) + \frac{\partial^2 z}{\partial y \partial x}(t^2)\right)(2r \ln t) + \frac{\partial z}{\partial x}(2 \ln t) + \left(\frac{\partial^2 z}{\partial y^2}(t^2) + \frac{\partial^2 z}{\partial x \partial y}(2r \ln t)\right)(t^2) + 0$$

$$\frac{\partial^2 z}{\partial t \partial r} = \left(\frac{\partial^2 z}{\partial x^2}\frac{\partial x}{\partial t} + \frac{\partial^2 z}{\partial y \partial x}\frac{\partial y}{\partial t}\right)\frac{\partial x}{\partial r} + \frac{\partial z}{\partial x}\frac{\partial^2 x}{\partial t \partial r} + \left(\frac{\partial^2 z}{\partial y^2}\frac{\partial y}{\partial t} + \frac{\partial^2 z}{\partial x \partial y}\frac{\partial x}{\partial t}\right)\frac{\partial y}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial^2 y}{\partial t \partial r}$$

$$= \left(\frac{\partial^2 z}{\partial x^2}\left(\frac{r^2}{t}\right) + \frac{\partial^2 z}{\partial y \partial x}(2rt)\right)(2r \ln t) + \frac{\partial z}{\partial x}\left(\frac{2r}{t}\right) + \left(\frac{\partial^2 z}{\partial y^2}(2rt) + \frac{\partial^2 z}{\partial x \partial y}\left(\frac{r^2}{t}\right)\right)(t^2) + \frac{\partial z}{\partial y}(2t)$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = \frac{\partial z}{\partial x}\left(\frac{r^2}{t}\right) + \frac{\partial z}{\partial y}(2rt)$$

$$\frac{\partial^2 z}{\partial t^2} = \left(\frac{\partial^2 z}{\partial x^2}\frac{\partial x}{\partial t} + \frac{\partial^2 z}{\partial y \partial x}\frac{\partial y}{\partial t}\right)\frac{\partial x}{\partial t} + \frac{\partial z}{\partial x}\frac{\partial^2 x}{\partial t^2} + \left(\frac{\partial^2 z}{\partial y^2}\frac{\partial y}{\partial t} + \frac{\partial^2 z}{\partial x \partial y}\frac{\partial x}{\partial t}\right)\frac{\partial y}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial^2 y}{\partial t^2}$$

$$= \left(\frac{\partial^2 z}{\partial x^2}\left(\frac{r^2}{t}\right) + \frac{\partial^2 z}{\partial y \partial x}(2rt)\right)\left(\frac{r^2}{t}\right) + \frac{\partial z}{\partial x}\left(\frac{-r^2}{t^2}\right) + \left(\frac{\partial^2 z}{\partial y^2}(2rt) + \frac{\partial^2 z}{\partial x \partial y}\left(\frac{r^2}{t}\right)\right)(2rt) + \frac{\partial z}{\partial y}(2r)$$

$$\frac{\partial^2 z}{\partial r \partial t} = \left(\frac{\partial^2 z}{\partial x^2}\frac{\partial x}{\partial r} + \frac{\partial^2 z}{\partial y \partial x}\frac{\partial y}{\partial r}\right)\frac{\partial x}{\partial t} + \frac{\partial z}{\partial x}\frac{\partial^2 x}{\partial r \partial t} + \left(\frac{\partial^2 z}{\partial y^2}\frac{\partial y}{\partial r} + \frac{\partial^2 z}{\partial x \partial y}\frac{\partial x}{\partial r}\right)\frac{\partial y}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial^2 y}{\partial r \partial t}$$

$$= \left(\frac{\partial^2 z}{\partial x^2}(2r \ln t) + \frac{\partial^2 z}{\partial y \partial x}(t^2)\right)\left(\frac{r^2}{t}\right) + \frac{\partial z}{\partial x}\left(\frac{2r}{t}\right) + \left(\frac{\partial^2 z}{\partial y^2}(t^2) + \frac{\partial^2 z}{\partial x \partial y}(2r \ln t)\right)(2rt) + \frac{\partial z}{\partial y}(2t)$$