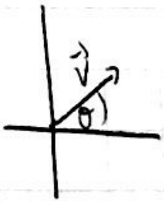
P1

$$D_v f = \frac{\vec{\nabla} f \cdot \vec{v}}{|\vec{v}|} \qquad \text{(Derivad. direccional)}$$



$$\vec{v} = \langle \cos(\theta), \sin(\theta) \rangle$$

1) $f(x,y) = x^3y^4 + x^4y^3$, $(1,1)$ $\theta = \pi/6 = 30°$

$$\nabla f = \begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{bmatrix} 3x^2y^4 + 4x^3y^3 \\ 4x^3y^3 + 3y^2x^3 \end{bmatrix}$$

$$\nabla f(1,1) = \begin{bmatrix} 7 \\ 7 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix} \Rightarrow D_v f(1,1) = \begin{bmatrix} 7 \\ 7 \end{bmatrix} \cdot \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix} = 7\left(\frac{\sqrt{3}+1}{2}\right)_{//}$$

2) $f(x,y) = e^x \cos(y)$, $(0,0)$ $\theta = \pi/6$

$$\nabla f = \begin{matrix} e^x \cos(y) \\ -e^x \sin(y) \end{matrix}$$

$$\nabla f(0,0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix} \Rightarrow D_v f(0,0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix} = \sqrt{3}/2$$

P2)

a) $f(x,y) = \dfrac{x}{x^2+y^2}$, $(1,2)$ $\quad V = \langle 3,5 \rangle$

$|V| = \sqrt{3^2+5^2} = \sqrt{9+25}$
$= \sqrt{34}$

$f_x = \dfrac{x^2+y^2 - 2x^2}{(x^2+y^2)^2} = \dfrac{y^2-x^2}{(x^2+y^2)^2}$

$f_y = -\dfrac{2y}{(x^2+y^2)^2}$

$\nabla f(1,2) = \begin{bmatrix} 3/25 \\ -4/25 \end{bmatrix} \Rightarrow D_v f(1,2) = \begin{bmatrix} 3/25 \\ -4/25 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \end{bmatrix} \cdot \dfrac{1}{\sqrt{34}}$

$= \dfrac{9-20}{25\sqrt{34}} = \dfrac{-11}{25\sqrt{34}}$

b) $f(x,y,z) = \sqrt{xyz}$ $\quad (3,2,6)$ $\quad V = \langle -1, 2, 2 \rangle$

$|V| = \sqrt{(-1)^2+2^2+2^2} = 3$

$f_x = \dfrac{1}{2} \dfrac{yz}{\sqrt{xyz}}$

$\nabla f(3,2,6) = \dfrac{1}{2} \begin{bmatrix} \frac{12}{6} \\ \frac{18}{6} \\ \frac{6}{6} \end{bmatrix} = \begin{bmatrix} 1 \\ 3/2 \\ 1/2 \end{bmatrix}$

$f_y = \dfrac{1}{2} \dfrac{xz}{\sqrt{xyz}}$

$f_z = \dfrac{1}{2} \dfrac{xy}{\sqrt{xyz}}$

$D_v f(3,2,6) = \begin{bmatrix} 1 \\ 3/2 \\ 1/2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} \cdot \dfrac{1}{3} = (-1+3+1)\dfrac{1}{3} = 1$

P3

$V_1 = \langle 1,1 \rangle \quad |V_1| = \sqrt{2}$

$V_2 = \langle 1,3 \rangle \quad |V_2| = \sqrt{10}$

$D_{v_1} f(1,2) = \sqrt{2}$

$\begin{bmatrix} f_x \\ f_y \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} = \sqrt{2}$

(1) $\boxed{f_x + f_y = 2}$

$D_{v_2} f(1,3) = \sqrt{10}$

$\begin{bmatrix} f_x \\ f_y \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \end{bmatrix} \cdot \frac{1}{\sqrt{10}} = \sqrt{10}$

(2) $\boxed{f_x + 3f_y = 10}$

(2)-(1) $= 2f_y = 8 \Rightarrow \boxed{f_y = 4}$

$\Rightarrow \boxed{f_x = -2}$

~~P4~~

P4 $\underbrace{x^2 + 2y^2 + 3z^2}_{f(x,y,z)} = 21$ $= C$

~~paralelo~~ $x + 4y + 6z = 0$

$\Downarrow$

$\vec{n}_2 = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix}$

$\vec{n}_1 = \vec{\nabla} f = \begin{bmatrix} 2x \\ 4y \\ 6z \end{bmatrix}$

$\downarrow$ plano Tangente

$n_1 = \lambda \overset{\text{ponderado}}{\vec{n}_2} \Rightarrow$

$2x = \lambda \rightarrow x = \lambda/2$

$4y = 4\lambda \rightarrow y = \lambda$

$6z = 6\lambda \rightarrow z = \lambda$

$\lambda \in \mathbb{R}$

$f(\lambda/2, \lambda, \lambda) = 21$

$\frac{\lambda^2}{4} + 2\lambda^2 + 3\lambda^2 = 21 \; / \cdot 4$

$\lambda^2 + 8\lambda^2 + 12\lambda^2 = 84$

$21\lambda^2 = 84$

$\lambda^2 = 4$

$\boxed{\lambda = \pm 2}$

Pts que sirve

$(1,2,2)$ y $(-1,-2,-2)$

$\downarrow$ plano Tangente

(1) $\vec{\nabla} f(1,2,2) \cdot \left(\begin{smallmatrix} x \\ y \\ z \end{smallmatrix} - \begin{smallmatrix} 1 \\ 2 \\ 2 \end{smallmatrix} \right) = 0$

$\begin{bmatrix} 2 \\ 8 \\ 12 \end{bmatrix} \cdot \begin{bmatrix} x-1 \\ y-2 \\ z-2 \end{bmatrix} = 0$ //

(2) $\vec{\nabla} f(-1,-2,-2) \cdot \begin{pmatrix} x+1 \\ y+2 \\ z+2 \end{pmatrix} = 0$ //

P5)

elipsoide $\underbrace{3x^2+2y^2+z^2=9}_{f(x,y,z)}$ esfera $\underbrace{x^2+y^2+z^2+8x-6y-8z+24=0}_{g(x,y,z)}$ ⊖ → Error

¿error e) $\boxed{33}$

Tangentes en $(1,1,2)$

$$f(1,1,2) = g(1,1,2)$$

$$3+2+4 \qquad 1+1+4-8-6-16+17$$

$$\cancel{16}\ 6-30+\cancel{8}+33$$

$$9 = 9$$

$$\vec{\nabla} f = \begin{bmatrix} 6x \\ 4y \\ 2z \end{bmatrix} \xrightarrow{(1,1,2)} \begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix}$$

$$\vec{\nabla} g = \begin{bmatrix} 2x-8 \\ 2y-6 \\ 2z-8 \end{bmatrix} \xrightarrow{(1,1,2)} \boxed{\begin{matrix} -6 \\ -4 \\ -4 \end{matrix}}$$

$$\vec{\nabla} f \ // \ \vec{\nabla} g$$

Entonces son tangentes porque comparten el plano tangente en $(1,1,2)$

P6)

1) $f(x,y) = x^2+y^2-2x$ $\qquad D =$

$$\nabla f = \begin{bmatrix} 2x-2 \\ 2y \end{bmatrix}$$

$f_x = 0 \to x=1$ $\quad \to (1,0)$

$f_y = 0 \to y = 0$

$f_{xx} = 2 \qquad f_{xy} = 0 = f_{yx}$

$f_{yy} = 2$

$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix} > 0$ y $f_{xx}(1,0) > 0 \Rightarrow (1,0)$ es mínimo

máximo debe estar en el borde superior derecho, debido a la forma del gradiente, borde superior derecho es

$$y = 2 - x$$

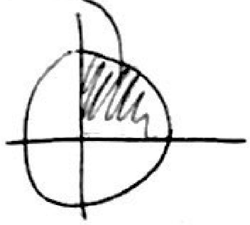
$$f(x, y=2-x) = x^2 + (2-x)^2 - 2x = \cancel{x^2+4-4x+x^2-2x} = 2x-4$$

$$= 2x^2 - 4x + 4 - 2x = 2x^2 - 6x + 4$$

$\cancel{f}$ $f = 2x-4$ → máximo cuando $x = 0$ ~~...~~ máximo

Entonces $Y=2$ el máximo $(0,2)$

P6) 2) $f(x,y) = xy^2$ $D = \{(x,y) \mid 0 \leq x, 0 \leq y \quad x^2+y^2 \leq 1\}$



$\nabla f = \begin{bmatrix} y^2 \\ 2xy \end{bmatrix}$

$\nabla f = 0 \Rightarrow (x,y) = (0,0) \rightarrow$ es claro que es un mínimo

El máximo está en el borde por la forma del gradiente

borde $y = \sqrt{1-x^2}$

$f(x, y=\sqrt{1-x^2}) = x(1-x^2) = x - x^3$

$f' = 1-3x^2 = 0 \rightarrow x = \pm 1/3$ como $x \geq 0$

$\rightarrow x = 1/3$ máximo

Entonces $y = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$

Max máximo $\left(1/3, \frac{2\sqrt{2}}{3}\right)$