

## Homework 6

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### Part A.i

Compute the covariance matrix of  $\mathbf{Z}$

$$\text{Var}(Z_1) = \frac{1}{2} \text{Var}(X_1 + X_2) = \frac{1}{2} (\sigma_1^2 + \sigma_2^2)$$

$$\text{Var}(Z_2) = \frac{1}{2} \text{Var}(X_1 - X_2) = \frac{1}{2} (\sigma_1^2 + \sigma_2^2)$$

$$\text{Cov}(Z_1, Z_2) = \mathbb{E}(Z_1 Z_2) - \mathbb{E}Z_1 \mathbb{E}Z_2 = \mathbb{E}(Z_1 Z_2)$$

and

$$\mathbb{E}(Z_1 Z_2) = \frac{1}{2} \mathbb{E}(X_1^2 - X_2^2) = \frac{1}{2} (\sigma_1^2 - \sigma_2^2)$$

Thus,

$$\Sigma = \frac{1}{2} \begin{pmatrix} \sigma_1^2 + \sigma_2^2 & \sigma_1^2 - \sigma_2^2 \\ \sigma_1^2 - \sigma_2^2 & \sigma_1^2 + \sigma_2^2 \end{pmatrix}$$

Compute  $\Lambda$

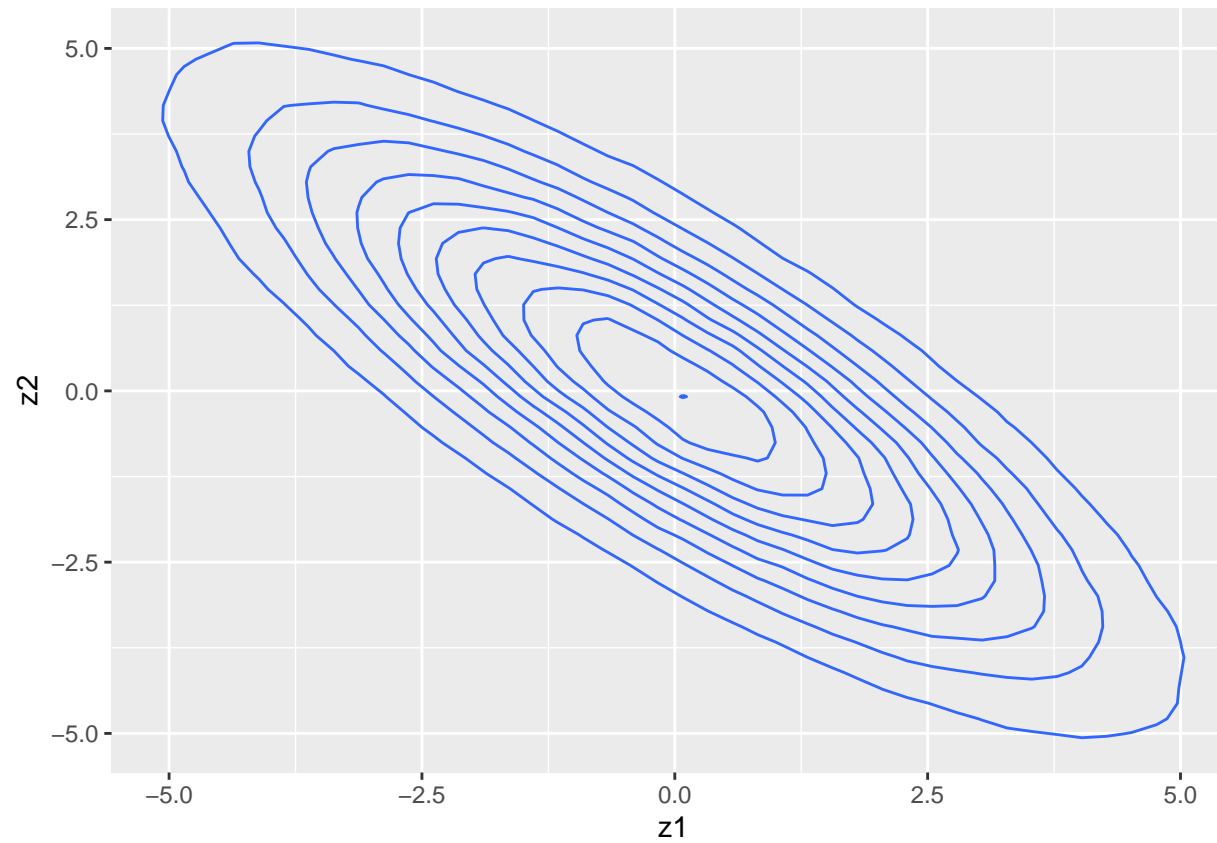
Compute eigenvalues and eigenvectors of  $\Lambda$

Level curves of the pdf

```
s1 <- 1  
s2 <- 10
```

```
x1 <- rnorm(n = 1000000, mean = 0, sd = sqrt(s1))
x2 <- rnorm(n = 1000000, mean = 0, sd = sqrt(s2))
z1 <- (x1 + x2) / sqrt(2)
z2 <- (x1 - x2) / sqrt(2)
```

```
library(ggplot2)
library(magrittr)
tibble::tibble(z1 = z1, z2 = z2) %>%
  ggplot() + geom_density_2d(aes(x = z1, y = z2))
```



**Part A.ii**

Find the minimizer

Level curves

**Part A.iii**

Find the minimizer

Level curves