2015 State competition, Question 18

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Problem statement

The quadratic equation

$$x^2 + px + 2p = 0$$

has solutions $x = a$ and $x = b$. If the

quadratic equation

$$x^2 + cx + d = 0$$

has solutions $x = a + 2$ and $x = b + 2$, what is the value of d?

Approach

We'll rely on factorization of quadratics to solve this.

First, note that we're given the two solutions to the first equation, a and b. This tells us that:

$$(x - a)(x - b) = 0 = x^2 + px + 2p$$

We can expand the left-hand side to get:

$$(x - a)(x - b) = x^2 - (a + b)x + ab = x^2 + px + 2p$$

For the two sides of the equation to be equal, their coefficients for the x terms must be equal.

That is, we must have

$$-(a + b) = p$$

and

$$ab = 2p$$

From these two equations, we see that:

$$-2(a + b) = 2p = ab$$

Using "guess and check", notice that $a = 2$ and $b = -1$ is one solution to the above equation.

We're also given the two solutions to the second equation, $a + 2$ and $b + 2$.

$$(x - (a + 2))(x - (b + 2)) = 0 = x^2 + cx + d$$

Using the same approach, ie, matching the coefficients of the powers of x, we see that

$$(a + 2)(b + 2) = d$$

Plug in the values of a and b that solved the first quadratic equation to get:

$$d = (2 + 2)(-1 + 2) = 4$$

Resources

Git info

```
git2r::last_commit()
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## [87e662d] 2021-11-08: added solution to 2015 state question 18
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