Identification in Differentiated Products Demand Systems

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Abstract

Separability and multi-stage budgeting is some times assumed to estimate parameters of demand systems for differentiated products. This paper shows that if this assumption is valid, then it also provides a set of exclusion restrictions such that the parameters of conditional demand system can be identified if the prices are endogenous.

Key words: Demand, Identification, Multi-stage budgeting
JEL Classification: L10

1. Introduction

Two procedural difficulties in estimating a system of demand equations for a set of differentiated products are (i) estimating a large number of parameters that increase in approximately the square of the number of products and, (ii) finding variables that provide at least as many moment conditions as the number of products with endogenous prices. Depending on the specific context the empirical IO literature deals with these dual problems in a variety of novel methods. To deal with the dimensionality curse, one approach is to assume separability of preferences and multistage budgeting by a representative consumer and estimate a series of conditional demand functions starting at the bottom level (Ellison et al., 1997). Parameters from conditional demand systems are then used to compute an unconditional elasticity matrix. Unless cost shifters for each product are explicitly available for all products, popular approaches to deal with the endogeneity of prices include (i) assuming that the location of products in the characteristics space is exogenous (Bresnahan, 1981), (ii) using counts of observed product characteristics (other than price), counts of same characteristics of other products by the manufacturer as well as similar counts for competitors' products (Berry et al., 1995) and, (iii) invoking error components arguments to use prices in one market as instruments

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for price in another market (Hausman, 1996). In this note I show that the separability and multi-budgeting assumptions once made, also lead to a natural set of instruments for prices and, in principle, can solve the problem of identification. I do not argue that these assumptions are always valid, only that if a researcher is willing to make these assumptions to overcome the problems of dimensionality, then these same assumptions can be used to obtain identification of conditional demand functions.

2. Multi-Budgeting Setup

The main assumption about preferences follows Gorman's multistage budgeting approach. This approach assumes that decisions about consumption are structured so that the allocation of expenditures across the class of products defined by different groups are made independently of the decision about how to allocate the resulting group expenditures across available alternatives. That requires the utility function $U(\cdot)$ to be weakly separable, i.e., the utility function $U = u[f^A(x_1, x_2, x_3), f^B(x_4, x_5, x_6), f^C(x_7, x_8, x_9)]$ be such that if two goods i and j are in different groups then $\frac{\partial}{\partial x_k} \cdot (\frac{u_i}{u_j}) = 0$. A further implication for the demand of good i in group M is that it be a function of only the total expenditures in the group, R_m, and the price of other products in the group but not of prices of goods in other groups, i.e., $x_{i_m} = g(p_{j_m}, R_m)$.

For concreteness, assume a system of I products manufactured by L firms ($L \leq I$) which a researcher has grouped into M groups such that the Mth group contains I_m products. Further, based on the said assumptions, suppose that the researcher specifies a multi-stage budgeting process where the bottom level equations are the flexible LA-AIDS/AIDS share equations within the group, the mid-level are again the share equations of the groups conditional on the total expenditures, and the top level is an aggregate demand equation of total quantity on the price index and exogenous factors. Thus, the bottom level share equations are given by

$$s_{i_m} = \alpha_{i_m} + \beta_{i_m} ln(\frac{R_m}{P_m}) + \sum_{j \in I_m} \gamma_{ij_m} ln P_{j_m}, \tag{1}$$

where s_{i_m} is the share of the ith product in group m, R_m and P_m are the total expenditures and the price index for the group, and P_{j_m} are prices of other goods in the group. The share equation may additionally have other exogenous factors that shift the shares (suppressed for now). The mid-level conditional demand equations would have a similar form (omitted for brevity).

3. Identification via a price competition model

Note that the aggregate *unconditional* demand for product i is given by

$$Q_i = D_i(p_1, \ldots, p_I, Z_i) \tag{2}$$

where Z_i is the vector of exogenous demand shifters. If there are L firms, then the lth firm produces a subset $\mathfrak{L}_l$ products and maximizes its joint profit over these products as

$$\Pi_l = \sum_{i \in \mathfrak{L}_l} (p_i - c_i) D_i(p_1, \ldots, p_I, Z_i)$$

where c_i is the marginal cost. If we assume a Nash-Bertrand price competition among firms, then the price p_i of any product must satisfy the first order conditions

$$Q_i + \sum_{i \in \mathfrak{L}_l} (p_l - c_l) \frac{\partial Q_l(p_1, \ldots, p_I, Z_i)}{\partial p_i} = 0. \tag{3}$$

This in turn implies a price equation for each product as

$$p_i = c_i + \Omega^{-1} Q_i(p_1, \ldots, p_I, Z_i)) \tag{4}$$

where $\Omega_{li} = -\Theta_{li} \frac{\partial Q_l(p_1, \ldots, p_I, Z_i)}{\partial p_i}$ and Θ_{li} is a 1/0 matrix with ones in the leading diagonal and in locations where a firm jointly produces products l and i (see Nevo (1998)). An ideal set of instruments would then be the marginal costs c_i by each product (or factors that shift the marginal costs) – which most researchers may not have. An additional issue arises if the researcher is using retail level scanner data since then she would need access to cost shifters by each product and market. However, if the retailers in market a set the retail level price as a *fixed* markup α_a over the corresponding wholesale price, then the manufactures would take this market specific markup into account when setting the price of their product. Hausman and Leonard (2002) show this fixed markup is equivalent to a manufacturer redefining the marginal cost as $(1 + \alpha_a)c_i$. This in turn implies that the profit maximizing price for product i in market a would be given by

$$p_{ai} = (1 + \alpha_a)c_i + \Omega^{-1} Q_i(p_{a1}, \ldots, p_{aI}, Z_{ai})). \tag{5}$$

Thus, the price of product i in any one market depends on the price of *all* other products in the market since these prices enter the unconditional demand function. Note, however, that in the multi-budgeting system the researcher is *not* estimating the parameters of the unconditional demand system, but rather of the more narrowly defined segment demand systems conditional on expenditures on that particular segment. For instance, if the first bottom level segment ($m = 1$) has just three products in the segment, then the conditional demand for each of the three products is a function of total expenditures, exogenous demand shifters and the prices of *only* these three products (see equation 1). Thus, the conditional demand of product i in market a and in segment m is given by

$$Q_{ai_m} = D(p_{a1_m}, \ldots, p_{aI_m}, Z_{ai}; R_m). \tag{6}$$

The fact that the price of the other products, those not in segment m are excluded from the conditional demand functions, provides us with an identifying strategy: in the estimation of the conditional demand for products in segment m, use the price of related products not in the segment as instruments for the price of products in the segment. The prices of the products not in the segment are relevant instruments per equation (5) due to the assumed form of price competition.[1] These prices are also valid instruments due to the exclusion restrictions per equation (6) and arise specifically because of the multistage budgeting process. Thus, this strategy spares the researcher from trying to find cost shifters by each product and market i.e., identification via the usual $(1+\alpha_a)c_i$ variables.

Just to be sure, suppose there are three segments ($M = 3$), the first segment has three products and the remaining two segments have three and four products respectively (for a total of 10 products). Then, in the estimation of the first bottom level segment we use the price of goods 4 through 10 as the instruments for the prices of goods 1 through 3. A similar method is used in the estimation of the other two bottom level segments. At the next level up, we have three demand (or share) equations, one for each segment and each equation is a function of three price indexes. The first is a price index of all products in the first segment, the second is a price index of all products in the second segment and so on. More care is needed in estimating these mid-level demand equations via a 2SLS (or equivalent method). In the 'first-stage', the first price index must be regressed on prices of products 4 through 10, the second on the prices of products 1 through 3 and 8 through 10, and the third on prices 1 through 7 (as well as on the vector Z). These projections of the price indexes can then be used in the 'second stage' estimation of the mid-level demand functions. Note that if a canned routine for 2SLS/IV is used, it will typically result in obtaining projections of these price indexes on all of the prices which is not valid in this situation.

Finally, note that the top equation cannot be estimated via the method outlined above. This equation involves an overall price index for all products and is a share weighted average of the three price indexes at the mid-level equations. Unless the researcher has reason to believe that the top-level price index is not correlated with the error term, the alternative is to find a valid instrument

[1] Note that this methodology allows identification even if firms are colluding. For instance, in the case of a cartel or a single producer, all the elements of Ω in equation 5 would be 1, whereas Ω would be an identity matrix in the case where each product corresponds to a unique non-colluding firm.

for the overall price index. However, this may not be so difficult since only a single instrument that effects the overall price index is needed, rather than 10 variables that provide 10 non-redundant moment restrictions.

4. Conclusion

The search for valid and relevant instruments in demand analysis is almost always a challenging task. The BLP instruments, which require the assumption that observed characteristics be uncorrelated with the unobserved characteristics, may or may not be valid in a specific situation. Similarly, the Hausman instruments rely on the assumption that prices in two different markets be correlated via common cost shocks and not via common demand side shocks. Similarly, the methodology outlined here is not universal and has its own limitations. First, and most obviously, the order condition can fail if the separability assumption is not valid (in principle this assumption can be tested via the usual separability tests). Second, even if separability holds, there may not be enough moment conditions in all segments. For instance, if there are three segments and products are divided as 7, 2 and 1 in the three segments respectively, then the conditional demand parameters of the first seven products cannot be identified (though the later three are over identified). Finally, the rank condition can also fail if the above assumed Nash-Bertrand price competition model is not appropriate for the industry under study. One example would be if the price setting strategy of a firm involves only the prices of other products in the same segment and not prices of products in other segments. In such a case the instruments would not be relevant. In summary, if the separability condition holds, the instruments suggested here may be applicable in a wide range of cases and may even provide over identification if there are several segments and few products in each segment. Alternatively, if the assumption is not valid, it calls into question the appropriateness of the multi-budgeting process.

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