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# Introductory Microeconomics ECO/1A1Y

## Games – Thinking Strategically

# Outline
# Week 3 Spring 2013

- Topics
  - Price competition revisited
  - Thinking strategically: some game theory
  - How to solve games
  - Price competition as a two players' game

  - A new equilibrium solution (iterated dominance)
  - Nash equilibrium (mutual best responses)
  - Deterministic versus probabilistic (mixed) strategies

# Price Competition
## A simple example of strategic interaction

- Duopoly: two firms (A and B) sell an identical good
- $TC = Q$ and $AC = MC = 1$
  - Firms are price setters (P can be as low as 0, as high as 5, or infinite)
  - $\Pi = TR - TC = pQ - Q = (p-1)Q$

- Demand is fixed $Q = 20$
- Two cases:
  - If $\mathrm{P_A} = \mathrm{P_B}$ then $\mathrm{Q_A} = \mathrm{Q_B} = 10$
  - If $P_A < P_B$ then $Q_A = 20$ and $Q_B = 0$ (symmetric for B)

# Price Competition
## A simple example of strategic interaction

- Price competition in a simple (payoff) matrix

Firm B

| Firm A | | High price=5 (collusion) | Lower price=4 (undercut) |
|---|---|---|---|
| | High price=5 (collusion) | 40 for A<br>40 for B | 0 for A<br>60 for B |
| | Lower price=4 (undercut) | 60 for A<br>0 for B | 30 for A<br>30 for B |

# Duopoly And Collusion

- Collusion is bad for consumers, good for firms
- In the real world open collusion is illegal
  - Exceptions: OPEC
  - If firms could, they would run a group monopoly and get highest profit

- Competition Commission monitors such activities
  - But, should we worry about collusion?
  - Can we make a prediction about this game?

# Economic Games

- Two firms engaged in patent race or innovation (Blue ray versus DVD)
- Supermarkets contesting for a prime location
- Firms bidding for a government contract
- Competing for a job
- Boeing and Airbus Industries are competing for a lucrative deal with an airline

# Game = Players + Actions + Outcomes

- Players: firms, or any other decision maker
- Actions: Quantities or prices or … move your piece in chess
- Outcomes: Firm’s profit, or prize resulting from win, loss or draw

# Strategy And Actions

- Strategy is a decision rule over actions
- In games where decisions are one-shot and simultaneous, strategy and action are the same
- In games where decisions are taken in sequence, strategy and actions are different
- There are different types of strategies (see the additional slides: pure and mixed)

# How to solve a game?

- There are several methods, depending on the game
- In all cases, a solution is always a **stable** outcome
- In other words: once the solution is reached, it is stable because none can do any better
- Unilateral deviations do not pay: ***rational thinking***
- [How do we get to the solution? That's a different story]

# Methods: some intuitions

- Dominant strategy solution
  - There is only one best action (for every action of the other player)

- Nash equilibrium
  - We simultaneously do our best
  - Every dominant strategy solution is a Nash equilibrium, but not the other way

- Iterated dominance
  - We discard bad decisions and reach a solution

- Subgame perfect equilibrium (sequential games)
  - Doing our best over time

# Game 1: Prisoners' dilemma

- A simple story
- Two people are arrested on charges of bank robbery (major crime)
- Bad news: Evidence links them together as collaborators and proves them to be at least offenders (minor crime)
- Good news: it cannot be proved that they are the real robbers, unless one of them confesses
- If someone confesses, she benefits from collaborating with the police…

# The rules of the game

- The police keep them in two separate cells
- They give them the following choices:
- Confess (***defection***)
- Keep silent (***cooperation***)
- They are also told that both are given the same choices
- Both will face the prison terms based on their actions

# A Normal Form Game

| | | B: Confess (Defect) | B: Remain silent (Cooperate) |
|---|---|---|---|
| A | Confess (Defect) | **5** years for A<br>**5** years for B | **1** year for A<br>**8** years for B |
| | Remain silent (Cooperate) | **8** years for A<br>**1** year for B | **3** years for A<br>**3** years for B |

# Player A thinks strategically

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| | | If B Confess... (Defect) | Remain silent (Cooperate) |
|---|---|---|---|
| A | Confess (Defect) | 5 years for A<br>5 years for B | **1** year for A<br>**8** years for B |
| | Remain silent (Cooperate) | 8 years for A<br>1 year for B | **3** years for A<br>**3** years for B |

# Player A keeps thinking strategically

| | | Confess (Defect) | If B remains silent… (Cooperate) |
|---|---|---|---|
| A | Confess (Defect) | **5** years for A<br>**5** years for B | 1 year for A<br>**8** years for B |
| | Remain silent (Cooperate) | **8** years for A<br>**1** year for B | 3 years for A<br>**3** years for B |

# A knows what it is best for him

- Regardless of B's action she is always better off confessing:
  - If B confesses, 5 years is better than 8
  - If B does not confess, 1 year is better than 3
- Confess is a dominant strategy for player A

- ***Dominant strategy:*** the strategy in a game that produces better results irrespective of the strategy chosen by one's opponent.

# B thinks the same way

| | B: Confess (Defect) | B: Remain silent (Cooperate) |
| --- | --- | --- |
| If A confesses (Defect) | 5 years for B | 8 years for B |
| If A remains silent (Cooperate) | 8 years for A<br>1 year for B | 3 years for A<br>3 years for B |

# B keeps thinking…

| | B | |
|---|---|---|
| | Confess (Defect) | Remain silent (Cooperate) |
| If A confesses (Defect) | **5** years for A<br>**5** years for B | **1** year for A<br>**8** years for B |
| If A remains silent (Cooperate) | 1 year for B | 3 years for B |

# B also knows what it is best for him

- Regardless of A's action she is always better off confessing:
  - If A confesses, 5 years is better than 8
  - If A does not confess, 1 year is better than 3
- Confess is a dominant strategy also for player B

- ***Dominant strategy:*** the strategy in a game that produces better results irrespective of the strategy chosen by one's opponent.

# A solution

- We can make a prediction for this game
  - As long as each player's outcome is truly represented, both players are rational, and know the other player is rational

- Both players are better off if both remain silent
  - This is optimal from the collective point of view
  - Is this a (stable) solution of the game?
  - NO

# A solution

- No!

- Each player has an **incentive to confess**
  - Regardless of the other player's action, a player is always better off confessing

- Cooperation (remaining silent) is NOT a stable outcome

- Defection (confessing) is the only equilibrium of the game: it is **dominant strategy** for both players

# Dominant Strategy Solutions

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- Some games luckily have dominant strategy solution: easy to solve
- But this is typically not the case in most strategic environments
- In the PD, both players are better off confessing
- In such games, the prediction is clear cut

# Back to price competition

A game with only two players

# Duopoly/Oligopoly

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- Firms in the real world face the prisoners' dilemma games everyday
- Dilemma: whether to compete, or be nice to each other (secretly collude)
- Competition erodes everybody's profit. Why not avoid competition?
- Every firm wants to avoid competition, but … cannot resist the temptation of making a quick hit-and-run profit!!

# Price Competition
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- Two cases:
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# Back to price competition

| | P=5 | P=4 | P=3 | P=2 | P=1 | P=0 |
|---|---|---|---|---|---|---|
| P=5 | 40 / 40 | 0 / 60 | | | | |
| P=4 | 60 / 0 | 30 / 30 | | | | |
| P=3 | | | | | | |
| P=2 | | | | | | |
| P=1 | | | | | | |
| P=0 | | | | | | |

# Is (4,4) an equilibrium?

| | P=5 | P=4 | P=3 | P=2 | P=1 | P=0 |
|---|---|---|---|---|---|---|
| P=5 | 40 / 40 | 0 / 60 | 0 / 40 | 0 / 20 | 0 / 0 | 0 / -20 |
| P=4 | 60 / 0 | 30 / 30 | 0 / 40 | 0 / 20 | 0 / 0 | 0 / -20 |
| P=3 | 40 / 0 | 40 / 0 | 20 / 20 | 0 / 20 | 0 / 0 | 0 / -20 |
| P=2 | 20 / 0 | 20 / 0 | 20 / 0 | 10 / 10 | 0 / 0 | 0 / -20 |
| P=1 | 0 / 0 | 0 / 0 | 0 / 0 | 0 / 0 | 0 / 0 | 0 / -20 |
| P=0 | -20 / 0 | -20 / 0 | -20 / 0 | -20 / 0 | -20 / 0 | -20 / -20 |

# Back to price competition

| | P=5 | P=4 | P=3 | P=2 | P=1 | P=0 |
|---|---|---|---|---|---|---|
| P=5 | 40, 40 | 0, 60 | 0, 40 | 0, 20 | 0, 0 | 0, -20 |
| P=4 | 60, 0 | 30, 30 | 0, 40 | 0, 20 | 0, 0 | 0, -20 |
| P=3 | 40, 0 | 40, 0 | 20, 20 | 0, 20 | 0, 0 | 0, -20 |
| P=2 | 20, 0 | 20, 0 | 20, 0 | 10, 10 | 0, 0 | 0, -20 |
| P=1 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 0, -20 |
| P=0 | -20, 0 | -20, 0 | -20, 0 | -20, 0 | -20, 0 | -20, -20 |

# Undercutting the rival

- Is $\mathrm{P_A} = \mathrm{P_B} = 3$ an equilibrium of the game?

- Both firms make 20

- If $\mathrm{P_A} = 2$ (while $\mathrm{P_B} = 3$), the profit is still $20$ $[= (2 - 1) * 20]$

- But, A may undercut its rival's price a bit:
- $\mathrm{P_A} = 2.9$ (and $\mathrm{P_B} = 3$)
- A gets the full market and a profit of $38$ $[= (2.9 - 1) * 20]$
- Better than 20!!

- Until… $P = MC = 1$, with no profit at all… bad news for firms!

# Takeaway messages

- Price competition generates no profits for firms, even when collusion is a tempting option
- Game theory helps us to understand the logic of strategic interaction
- We may try to find a stable outcome of every interaction: an equilibrium in which no firm has an incentive to unilaterally deviate
- Some games have dominant strategies, so finding the equilibrium of the game is easy (but these cases are the exception, not the rule)

# Collective versus individual gains

- In most games, as the PD, a single player would make a different decision: collective rationality is at odds with individual rationality

- Some games are **zero-sum** games (as in many sports, one's win means other's loss), but most economic situations are not **zero-sum** games

- We will work on this in the workshop and seminars

# Pure and mixed strategies

- **Pure strategy**: When a player prefers to play one action over the other, it is called a pure strategy (paying This or That with certainty)

- **Mixed strategy**: When a player is uncertain between two (or more) actions, he can take a particular action only with some probability. It is called mixed strategy

# Duopoly: Collusion over time

- The outcome of the prisoners' dilemma game is different, if the players play this game again and again over LONG period of time

- Over time: The prisoners would talk beforehand to avoid confessions. Each could reciprocate by being nice in all future occasions. This is called a Tit for Tat strategy.

- Firms could do the same (playing Tit for Tat) in their long term interests.

- They could limit PRICE WAR to a tolerable degree, and if possible avoid it altogether, or choose a softer form of competition.

# Iterated dominance

Finding a stable solution

# Iterated dominance

- Most games do not have a dominant strategies
- We may still identify **dominated** strategies
  - A rational player is better off by playing differently
- We then can eliminate this strategy on the ground that rational (self interested and capable of reasoning based on the available information) players do not play strictly dominated strategies!
- By eliminating such dominated strategies one by one we can arrive at a unique pair of strategies (sometimes)

# An example

| | L | C | R |
|---|---|---|---|
| U | 2, 3 | 6, 4 | 6, 5 |
| M | 4, 3 | 8, 0 | 4, 1 |
| D | 10, 3 | 3, 4 | 5, 5 |

# Thinking rationally (dominated strategies)

| | L | C | R |
|---|---|---|---|
| U | 2, 3 | 6, 4 | 6, 5 |
| M | 4, 3 | 8, 0 | 4, 1 |
| D | 10, 3 | 3, 4 | 5, 5 |

- Compare C to R

# Rational players don't play dominated strategies!

| | L | | R |
|---|---|---|---|
| U | 2, 3 | | 6, 5 |
| M | 4, 3 | | 4, 1 |
| D | 10, 3 | | 5, 5 |

- Compare C to R
  - C would not be played

# Iterated dominance

| | L | | R |
|---|---|---|---|
| U | 2, 3 | | 6, 5 |
| M | 4, 3 | | 4, 1 |
| D | **10**, 3 | | **5**, 5 |

- ~~Compare C to R~~
  - ~~C would not be played~~
- Now compare M to D

# Iterated dominance

| | L | | R |
|---|---|---|---|
| U | 2, 3 | | 6, 5 |
| | | | |
| D | 10, 3 | | 5, 5 |

- ~~Compare C to R~~
  - ~~C would not be played~~
- Now compare M to D
  - M would not be played

# Iterated dominance

| | L | | R |
|---|---|---|---|
| U | 2, 3 | | 6, **5** |
| | | | |
| D | 10, 3 | | 5, **5** |

- ~~Compare C to R~~
  - ~~C would not be played~~
- ~~Now compare M to D~~
  - ~~M would not be played~~
- Compare L to R

# Iterated dominance

| | | | R |
|---|---|---|---|
| U | | | 6, 5 |
| | | | |
| D | | | 5, 5 |

- ~~Compare C to R~~
  - ~~C would not be played~~
- ~~Now compare M to D~~
  - ~~M would not be played~~
- Compare L to R
  - L would not be played

# Iterated dominance

| | | | R |
|---|---|---|---|
| U | | | 6, 5 |
| | | | |
| D | | | 5, 5 |

Solution

- ~~Compare C to R~~
  - ~~C would not be played~~
- ~~Now compare M to D~~
  - ~~M would not be played~~
- ~~Compare L to R~~
  - ~~L would not be played~~

# A solution

- Rational players prefer more to less: they never choose dominated strategies

- Rationality is common knowledge: when both players know they are rational, they can eliminate strategies that will never be played

- *Sometimes* we arrive at a stable solution: no incentives to do something different (*no unilateral incentives to deviate*)

# Nash equilibrium

Mutual best responses

# Real life games

- Two firms think about a simultaneous investment (in a new technology)
- It is better if they share the cost (and the benefits)

| | L | R |
|---|---|---|
| U | 1, 1 | 1, 0 |
| D | 0, 1 | 2, 2 |

# Thinking strategically

- Do they have a dominant (or dominated strategy)?

| | L | R |
|---|---|---|
| U | 1, 1 | 1, 0 |
| D | 0, 1 | 2, 2 |

# Thinking strategically

| | L | R |
|---|---|---|
| **U** | **1**, 1 | 1, 0 |
| D | 0, 1 | 2, 2 |

# Thinking strategically

| | L | R |
|---|---|---|
| U | 1, 1 | 1, 0 |
| D | 0, 1 | 2, 2 |

# Thinking strategically

| | L | R |
|---|---|---|
| U | 1, 1 | 1, 0 |
| **D** | 0, 1 | **2**, 2 |

# Do we have a prediction?

| | L | R |
|---|---|---|
| U | **1, 1** | 1, 0 |
| D | 0, 1 | **2, 2** |

- Dominant strategies and iterated dominance are not always useful
- Best response is not unique…
- *Homework: what about column player?*

# Multiple solutions

| | L | R |
|---|---|---|
| U | **1, 1** | 1, 0 |
| D | 0, 1 | **2, 2** |

- U,L and D,R are *stable* solutions of the game (mutual best responses)
- No unilateral incentives to deviate: equilibria
- ***Nash equilibria***

# Nash equilibrium

- There are games with no dominant strategies that are not solvable by iterated dominance
- Then we use the concept of Nash equilibrium, which specifies a combination of the mutually best response strategies
- If player 1 does his best by playing strategy A, when player 2 plays strategy a, and similarly if player 2 does his best by playing strategy a against player 1's play of strategy A, then the pair of strategies (A,a) is a Nash equilibrium.

- ***Nash equilibrium:*** the combination of strategies in a game such that neither player has any incentive to change strategies given the strategy of his opponent.
  - A Nash equilibrium does not require both players to have a dominant strategy!

# Multiple solutions? I need a prediction

- D, R is better than U, L if we care about efficiency (social welfare)
- ***Pareto*** efficiency: 2,2 > 1,1 (next lecture…)
- U, L is safer than D, R if we try to maximize the minimum payoff (***maximin***): 1>0

| | L | R |
|---|---|---|
| U | **1, 1** | 1, 0 |
| D | 0, 1 | **2, 2** |

- ***Maximin strategy:*** choosing the option that makes the lowest payoff one can receive as large as possible.

# Extra Slides …
# Mixed strategies

Playing a probabilistic game

# Real life games

The penalty dilemma

Dominant strategy? Dominated strategies? Nash?

| | L | R |
|---|---|---|
| U (L) | 0, 1 | 1, 0 |
| D (R) | 1, 0 | 0, 1 |

# Real life games

I don´t want the other player to anticipate…
What if I make him indifferent?

| | L | R |
|---|---|---|
| U **(p)** | 0, 1 | 1, 0 |
| D **(1-p)** | 1, 0 | 0, 1 |

$$\pi^{e}_{COL}(L) = p \cdot 1 + (1-p) \cdot 0 = p$$

$$\pi^{e}_{COL}(R) = p \cdot 0 + (1-p) \cdot 1 = (1-p)$$

$$\pi^{e}_{COL}(L) = p = \pi^{e}_{COL}(R) = (1-p) \qquad p = \frac{1}{2}$$

# Real life games

The ROW player does the same

| | L **(q)** | R **(1-q)** |
|---|---|---|
| U **(p)** | 0, 1 | 1, 0 |
| D **(1-p)** | 1, 0 | 0, 1 |

$$\pi^e_{ROW}(U) = q \cdot 0 + (1-q) \cdot 1 = (1-q)$$

$$\pi^e_{ROW}(D) = q \cdot 1 + (1-q) \cdot 0 = q$$

$$\pi^e_{ROW}(U) = (1-q) = \pi^e_{ROW}(L) = q \qquad q = \frac{1}{2}$$

# A probabilistic solution

This is the only equilibrium of the game

*Can you prove that? (Does a unilateral deviation pay?)*

| | L (q) | R (1-q) |
|---|---|---|
| U **(p)** | 0, 1 | 1, 0 |
| D **(1-p)** | 1, 0 | 0, 1 |

# Rock, Paper, Scissors

- A two-player game with *three* strategies – how to find the mixed strategy equilibrium in that case?
- Rules are: Two players simultaneously choose Rock, Paper or Scissors with the understanding that Rock beats Scissors; Paper beats Rock; Scissors beats paper. All other outcomes, e.g. Rock, Rock result in ties.
- Denote a win by a 1, a loss by -1 and a tie by 0

# Rock, Paper, Scissors

- The game is symmetric, so it suffices to find one player's mixed strategy, say the Column player.
- Let $r = \Pr(Rock)\,, p = \Pr(Paper)\; and\; 1 - r - p = \Pr(Scissors)$

| | Rock | Paper | Scissors | Expected |
|---|---|---|---|---|
| **Rock** | 0,0 | -1, 1 | 1, -1 | $\mathbf{1 - r - 2p}$ |
| **Paper** | 1, -1 | 0, 0 | -1, 1 | $\mathbf{2r + p - 1}$ |
| **Scissors** | -1, 1 | 1, -1 | 0, 0 | $\mathbf{p - r}$ |

- Column chooses r and p the make row indifferent:
  - $2r + p - 1 = -r + p \rightarrow 3r = 1, r = 1/3;$
  - $1 - r - 2p = 2r + p - 1 \quad 1 - 1/3\; - 2p = 2/3 + p - 1 \quad 1 = 3p, p = 1/3;$
- It follows that the unique, mixed strategy equilibrium is $p = r = 1/3$, i.e., play all three strategies with equal probability

# Market Entry Game

- Two fast food chains, A and B, decide if they should open a branch in a mall which already has several other food stores
- Strategies: Enter, Don't
- If a firm does not enter, it earns 0 profit
- If a firm enters and the competitor does not, it earns £300K per year
- If both enter, each makes a loss of £100K

# Market Entry Game

| Payoffs in 1000s | B: Enter | B: Don't |
|---|---|---|
| A: Enter | -1,-1 | 3,0 |
| A: Don't | 0,3 | 0,0 |

# Market Entry Game

| Payoffs in 1000s | B: Enter | B: Don't |
|---|---|---|
| A: Enter | -1,-1 | 3,0 |
| A: Don't | 0,3 | 0,0 |

**2 Pure Strategy Equilibria**

**Are there any mixed strategy equilibria?**

# Market Entry Game
## Are there any mixed strategy equilibria?

| Payoffs in 1000s | B: Enter (q) | B: Don't (1-q) |
|---|---|---|
| A: Enter (p) | -1,-1 | 3,0 |
| A: Don't (1-p) | 0,3 | 0,0 |

A plays Enter with probability p and Don't enter with 1-p
B plays Enter with probability q and Don't enter with 1-q

# Market Entry Game

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| Payoffs in 1000s | B: Enter (q) | B: Don't (1-q) |
|---|---|---|
| A: Enter (p) | -1,-1 | 3,0 |
| A: Don't (1-p) | 0,3 | 0,0 |

- A wants to choose p so as to make B indifferent between entering or not entering
    - B's payoff from enter: (p)(-1) + (1-p)(3)
    - B's payoff from don't:  (p)(0) + (1-p)(0)
    - Indifferent → (p)(-1) + (1-p)(3)  = (p)(0) + (1-p)(0) → p=3/4

- B wants to choose q so as to make A indifferent between entering or not entering
    - A's payoff from enter: (q)(-1) + (1-q)(3)
    - A's payoff from don't:  (q)(0) + (1-q)(0)
    - Indifferent → (q)(-1) + (1-q)(3)  = (q)(0) + (1-q)(0) → p=3/4

- In the mixed strategy Nash equilibrium both firms choose enter with probability ¾, and Don't Enter with probability ¼

# Takeaway messages

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- Dominant and dominated strategies play a limited role in many games. But, they make your life easier
- Nash equilibrium is a very general concept based on mutual best responses (no way to do better), even with more than one equilibria
- Games sometimes have no equilibrium in pure strategies but still have an equilibrium in mixed (probabilistic strategies)