DeMorgan's Laws:

- $\overline{A \cap B} = \overline{A} \cup \overline{B}$
- $\overline{A \cup B} = \overline{A} \cap \overline{B}$

Countable if elements can be put in one-to-one with $\mathbb{N} = \{1, 2, 3, \dots\}$

Uncountable sets:

- $\mathbb{R} = (-\infty, \infty)$
- $[0, 1]$ and $(0, 1)$
- $(a, b), \forall a, b \in \mathbb{R}$ such that $a < b$

A family of sets $\{\mathcal{A}_i, i \in I\}$ is a **partition of** $\mathcal{S}$ if it is disjoint and collectively exhaustive over $\mathcal{S}$.

Axioms of Probability

1. $\mathcal{P}(A) \geq 0, \forall A \in \mathcal{F}(\mathcal{S}))$
2. $\mathcal{P}(\mathcal{S}) = 1$
3. $\mathcal{P}(\cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} \mathcal{P}(A_i)$

Probability Equations

1. $\mathcal{P}(A \cup B) = \mathcal{P}(A) + \mathcal{P}(B) - \mathcal{P}(A \cap B)$
2. $\mathcal{P}(A|B) = \frac{\mathcal{P}(A \cap B)}{\mathcal{P}(B)} = \frac{\mathcal{P}(B|A)\mathcal{P}(A)}{\mathcal{P}(B)}$

Given $\mathcal{S}$ and a family of subsets $\mathcal{G} = \{\mathcal{A}_i, i \in I\}$ of $\mathcal{S}$, the **σ-field generated by** $\mathcal{G}$, denoted $\sigma(\mathcal{G})$, is the smallest σ field containing all the subsets in $\mathcal{G}$.
By "smallest" σ-field, we mean that for any σ-field $\mathcal{F}_0$ containing all the sets in $\mathcal{G}$:

$$\sigma(\mathcal{G}) \subset \mathcal{F}_0$$

Given $\mathbb{R}$, the **Borel field of** $\mathbb{R}$ is defined as the σ-field generated by the family of all open intervals

$$\mathcal{G} = \{(a, b) : \forall (a, b) \in \mathbb{R} \text{ such that } a < b\}.$$

We denote the Borel field of $\mathbb{R}$ by $\mathcal{B}(\mathbb{R})$.

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pmfs
uniform
$p(\omega) = \frac{1}{n}, \forall \omega \in \mathcal{S}$
binomial
$p(k) = \binom{n}{k} a^k (1-a)^{n-k}$
geometric
$p(k) = (1-a)a^k, a \in (0, 1)$
poisson
$p(k) = \frac{\lambda^k e^{-k}}{k!}; k = 0, 1, 2, \dots; \lambda > 0$

Properties of the pdf:

1. $f(r) \geq 0, \forall r \in \mathbb{R}$
2. $\int_{-\infty}^{\infty} f(r) dr = 1$

Given a valid pdf, we can get a valid probability measure $\mathcal{P}(\cdot)$:

$$\mathcal{P}(A) = \int_A f(r) dr = \int_{-\infty}^{\infty} \cdot 1_A(r) dr$$

where

$$1_A(r) = \begin{cases} 1, r \in A \\ 0, r \notin A \end{cases}$$

is called the **indicator function** of the set A.

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pdfs
uniform
$f(r) = \frac{1}{b-a} 1_{[a,b]}(r)$
exponential
$f(r) = \lambda e^{-\lambda r} \cdot 1_{[0,\infty]}(r), \lambda > 0$
$\hookrightarrow = \begin{cases} \lambda e^{-\lambda r}, r \geq 0 \\ 0, r < 0 \end{cases}$
gaussian
$f(r) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-(r-\mu)^2}{2\sigma^2}\right), r \in \mathbb{R}, \mu \in \mathbb{R}, \sigma > 0$

Conditional Probability

$$\mathcal{P}(A|B) = \frac{\mathcal{P}(A \cap B)}{\mathcal{P}(B)}$$

Bayes Formula

$$\mathcal{P}(A|B) = \frac{\mathcal{P}(B|A)\mathcal{P}(A)}{\mathcal{P}(B)}$$

Total Probability Law

$$\mathcal{P}(B) = \sum_{i=1}^{n} \mathcal{P}(B|A_i)\mathcal{P}(A_i)$$

Bayes Theorem

$$\mathcal{P}(A_m|B) = \frac{\mathcal{P}(B|A_m)\mathcal{P}(A_m)}{\sum_{i=1}^{n} \mathcal{P}(B|A_i)\mathcal{P}(A_i)}$$

Events A and B are **statistically independent** if and only if

$$\mathcal{P}(A \cap B) = \mathcal{P}(A)\mathcal{P}(B)$$

Bernoulli Trials

$$p_n(k) = \mathcal{P}(\mathcal{B}_k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where $p = \mathcal{P}_0(A)$.
