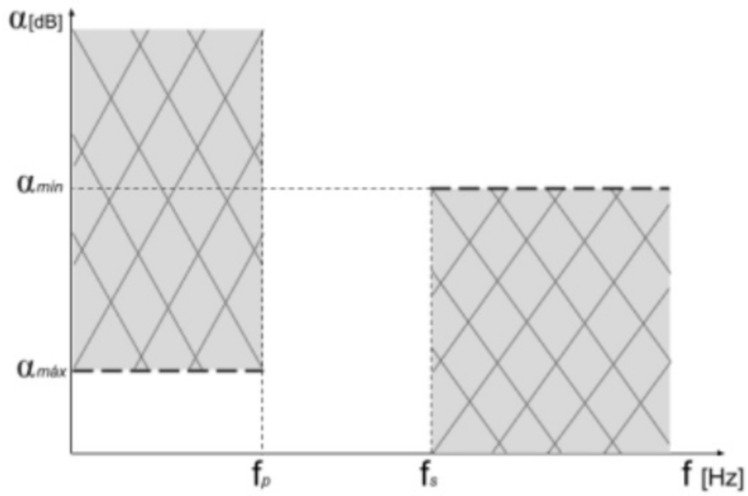
A partir de la siguiente plantilla:



αmáx [dB]	ωp [r/s]	ωs [r/s]
1	1	2,5

1. Obtener la función transferencia T(s) de Bessel para N: 2, 3 y 4 normalizados para $D(\omega = 0) = 1s$ utilizando el método de Storch: ver *Schaumann, R. - Van Valkenburg, Mac E., Design of Analog Filters, Capítulo 10: Delay Filters. Sección 10.2: Bessel-Thomson Response. Página 403.*

2. Elegir la T(s) con el mínimo orden que cumpla con $\alpha_{máx} = 1dB$.

3. Evaluar el retardo de grupo $D(\omega = 2.5)$ y expresar en forma *porcentual [%]* el error o desviamiento respecto a $D(\omega = 0)$.

4. Sintetizar el circuito **NORMALIZADO** con estructuras Sallen-Key *con K=1 (real, negativa y unitaria).*

1)

→ Método de Storch para filtros Bessel-thomson

$$T(s) = \frac{1}{e^{s}} = \frac{1}{\sinh(s) + \cosh(s)}$$

$$\coth(s) = \frac{\cosh(s)}{\operatorname{senh}(s)} = \underbrace{\frac{1}{s}}_{n=1} + \cfrac{1}{\cfrac{3}{s} + \cfrac{1}{\cfrac{5}{s} + \cfrac{1}{\cfrac{7}{s} + \cdots}}}$$

$n=2$ $n=3$ $n=4$

A la suma de $\sinh(s)\big|_n + \cosh(s)\big|_n$ se la llama "B_n" (Polinomio de Bessel)

$$\therefore \; T(s) = \frac{B_n(0)}{B_n(s)}$$

$n=2$

$$\coth(s) = \frac{1}{s} + \cfrac{1}{\cfrac{3}{s}} = \frac{1}{s} + \frac{s}{3} = \frac{3+s^2}{3s}$$

$$B_2(s) = s^2 + 3s + 3$$

$$B_2(0) = 3$$

$$T(s)\Big|_{n=2} = \frac{3}{s^2+3s+3}$$

$n=3$

$$\coth(s) = \frac{1}{s} + \cfrac{1}{\cfrac{3}{s} + \cfrac{1}{\cfrac{5}{s}}} = \frac{1}{s} + \cfrac{1}{\cfrac{3}{s} + \cfrac{s}{5}}$$

$$\coth(s) = \frac{1}{s} + \cfrac{1}{\cfrac{15+s^2}{5s}} = \frac{1}{s} + \frac{5s}{s^2+15}$$

$$\coth(s) = \frac{s^2+15+5s^2}{s^3+15s}$$

$$B_3(s) = s^3 + 6s^2 + 15 + 15$$

$$B_3(0) = 15$$

$$T(s)\Big|_{n=3} = \frac{15}{s^3+6s^2+15s+15}$$

$n=4$

$$\coth(s) = \frac{1}{s} + \cfrac{1}{\cfrac{3}{s} + \cfrac{1}{\cfrac{5}{s} + \cfrac{1}{\cfrac{7}{s}}}} = \frac{1}{s} + \cfrac{1}{\cfrac{3}{s} + \cfrac{7s}{35+s^2}}$$

$$\coth(s) = \frac{1}{s} + \frac{s^3 + 35s}{3(35+s^2) + 7s^2}$$

$$\coth(s) = \frac{105 + 3s^2 + 7s^2 + s^4 + 35s^2}{105s + 3s^3 + 7s^3}$$

$$\coth(s) = \frac{s^4 + 45s^2 + 105}{10s^3 + 105s}$$

$$B_4(s) = s^4 + 10s^3 + 45s^2 + 105s + 105$$

$$B_4(0) = 105$$

$$H(s)\Big|_{n=4} = \frac{105}{s^4 + 10s^3 + 45s^2 + 105s + 105}$$

2) $\alpha_{max} = 1\,dB$ ¿ obtener el mínimo orden

$$\alpha(w) = -20 \log |H(w)|$$

$$|T(w)| = 10^{-\alpha(w)/20}$$

→ se debe evaluar en $w = w_p = 1$

Para $n=2$

$$H(s)\Big|_{n=2} = \frac{3}{s^2 + 3s + 3}$$

$$|H(jw)| = \frac{|3|}{|(jw)^2 + 3jw + 3|} = \frac{3}{\sqrt{9w^2 + (3-w^2)^2}}$$

Si $w=1 \rightarrow |H(jw)| = \dfrac{3}{\sqrt{9+4}} = \dfrac{3}{\sqrt{13}}$

$$|H(1)| = 0.832$$

$$\alpha(1) = -20\log|H(1)| = 1.597$$

Para $n=3$

$$H(s)\Big|_{n=3} = \frac{15}{s^3+6s^2+15s+15}$$

$$|H(j\omega)| = \frac{\|15\|}{\|(j\omega)^3+6(j\omega)^2+15j\omega+15\|}$$

$$|H(j\omega)| = \frac{15}{\|-j\omega^3-6\omega^2+15j\omega+15\|}$$

$$|H(j\omega)| = \frac{15}{\sqrt{(15\omega-\omega^3)^2+(15-6\omega^2)^2}}$$

Si $\omega=1 \rightarrow \dfrac{15}{\sqrt{(15-1)^2+(15-6)^2}} = \dfrac{15}{\sqrt{14^2+9^2}}$

$$|H(1)| = 0.90126$$

$$\alpha(1) = -20\log|H(1)| = 0.90297$$

n=4

$$H(s)\Big|_{n=4} = \frac{105}{s^4+10s^3+45s^2+105s+105}$$

$$|H(j\omega)| = \frac{|105|}{\left|\omega^4 - 10j\omega^3 - 45\omega^2 + 105j\omega + 105\right|}$$

$$|H(j\omega)| = \frac{105}{\sqrt{(105\omega-10\omega^3)^2+(\omega^4-45\omega^2+105)^2}}$$

$$|H(1)| = \frac{105}{\sqrt{(95)^2+(61)^2}} = 0.930$$

$$\alpha(1) = -20\log|H(1)| = 0.630$$

3) $$H(s)\Big|_{n=3} = \frac{15}{s^3+6s^2+15s+15}$$

$$H(jw)\Big|_{n=3} = \frac{15}{-jw^3-6w^2+jw15+15}$$

$$\measuredangle H = 0 - \operatorname{Arctg}\left(\frac{15w-w^3}{15-6w^2}\right)$$

$$D = -\frac{d\measuredangle H}{dw} = \frac{6w^4+45w^2+225}{w^6+6w^4+45w^2+225}$$

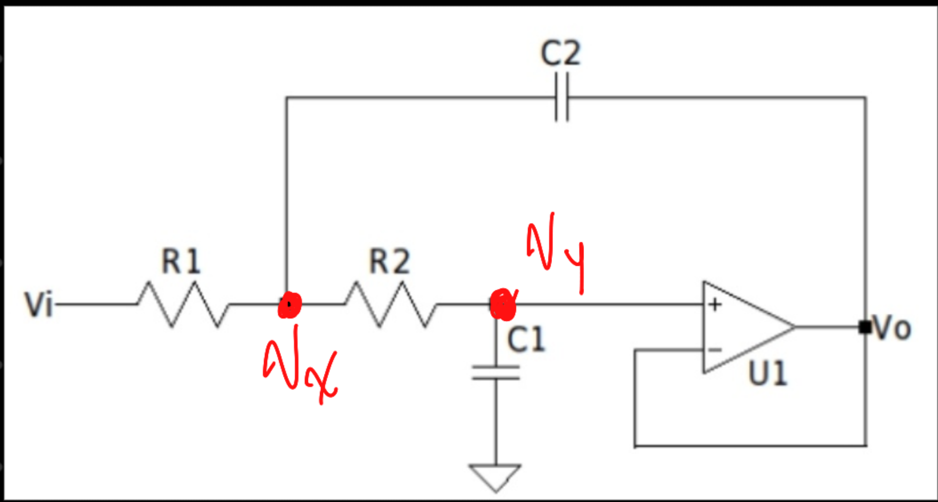
$$D(0) = \frac{225}{225} = 1$$

$$D(2.5) = 0.752$$

$$\frac{D(2.5)}{D(0)} = 0.752 \rightarrow 75.2\ \%\ \text{de}\ D(0)$$

desviamiento de $-24.8\ \%$ con respecto a $D(0)$

4) Sintetizar con Sallen-Key $K=1$



$$\begin{cases} V_y = V_o \\ V_x\left(G_1 + G_2 + sC_2\right) + V_{in} G_1 + V_o sC_2 + V_y G_2 = 0 \\ V_y\left(G_2 + sC_1\right) + V_x G_2 = 0 \end{cases}$$

$$V_x G_2 = -V_y\left(G_2 + sC_1\right)$$

① $$V_x = -V_y \frac{\left(G_2 + sC_1\right)}{G_2}$$

$$-\frac{(G_2+sC_1)}{G_2}(G_1+G_2+sC_2)V_0 + V_{in}G_1 + V_0(sC_2+G_2) = 0$$

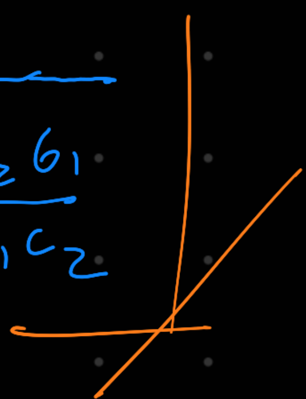
$$V_{in}G_1 = \frac{V_0}{G_2}\left[\frac{(G_2+sC_1)(G_1+G_2+sC_2)}{\cancel{G_2}} - G_2(sC_2+G_2)\right]$$

$$\frac{V_{in}}{V_0}G_1G_2 = G_2G_1 + \cancel{G_2^2} + \cancel{G_2 sC_2} + sC_1G_1 + sC_1G_2 + s^2C_1C_2 - \cancel{G_2 sC_2} - \cancel{G_2^2}$$

$$\frac{V_{in}}{V_0}G_1G_2 = G_2G_1 + sC_1(G_1+G_2) + s^2C_1C_2$$

$$\frac{V_0}{V_{in}}\frac{1}{G_1G_2} = \frac{\frac{1}{C_1C_2}}{s^2 + s\frac{(G_1+G_2)}{C_2} + \frac{G_2G_1}{C_1C_2}}$$

$$\frac{V_0}{V_{in}} = \frac{\frac{G_2G_1}{C_1C_2}}{s^2+s\left(\frac{G_1+G_2}{C_2}\right)+\frac{G_2G_1}{C_1C_2}}$$



$$\frac{G_2 G_1}{C_1 C_2} = \omega_0^2$$

$$\frac{G_1 + G_2}{C_2} = \frac{\omega_0}{Q}$$

$$\frac{\sqrt{G_1 G_2}}{\sqrt{C_1 C_2}} = \boxed{\omega_0}$$

$$Q = \omega_0 \boxed{\frac{C_2}{G_1 + G_2}}$$

$$Q = \frac{\sqrt{G_1 G_2}}{\sqrt{C_1}} \sqrt{C_2} \; \boxed{\frac{1}{G_1 + G_2}}$$

(Sigue en el Notebook)