CT Graphs: B.2 Traversing Problem

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February 2020

B.2.1

The adjacency matrix is:

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

where row i and column j represent node i and node j respectively.
The adjacency linked lists are:

$$1 \rightarrow [2, 3, 4, 5]$$
$$2 \rightarrow [1, 6, 7]$$
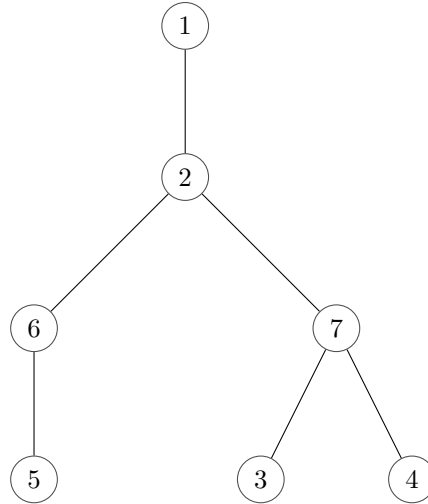$$3 \rightarrow [1, 7]$$
$$4 \rightarrow [1, 7]$$
$$5 \rightarrow [1, 6]$$
$$6 \rightarrow [2, 5]$$
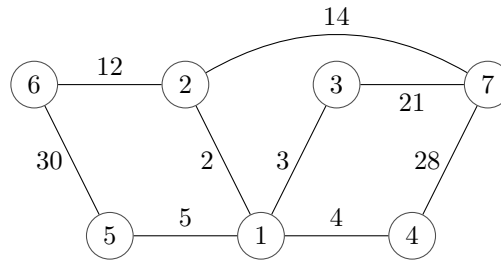$$7 \rightarrow [2, 3, 4]$$

B.2.2

The DFS tree for the graph is: (left paths were followed before the right paths)

The order in which the vertices were visited was $1, 2, 6, 5, 7, 3, 4$.
The order in which the vertices became dead ends was $5, 6, 3, 4, 7, 2, 1$.
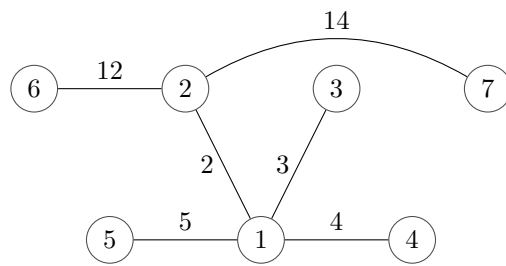
B.2.3

The edges sorted by their weights are as follows:

$$edges = \{(1,2) : 2, (1,3) : 3, (1,4) : 4, (1,5) : 5, (2,6) : 12, (2,7) : 14, (3,7) : 21, (4,7) : 28, (5,6) : 30\}$$

The order they are considered in and whether they were used is:

1. $(1, 2)$ does not cause a cycle therefore it is added to the tree.

2. $(1, 3)$ does not cause a cycle therefore it is added to the tree.

3. $(1, 4)$ does not cause a cycle therefore it is added to the tree.

4. $(1, 5)$ does not cause a cycle therefore it is added to the tree.

5. $(2, 6)$ does not cause a cycle therefore it is added to the tree.

6. $(2, 7)$ does not cause a cycle therefore it is added to the tree.

7. $(3, 7)$ does causes a cycle therefore it is rejected.

8. $(4, 7)$ does causes a cycle therefore it is rejected.

9. $(5, 6)$ does causes a cycle therefore it is rejected.

This produces the minimum spanning tree:

It has total weight 40.