CT Graphs: A.3 Scheduling Problem

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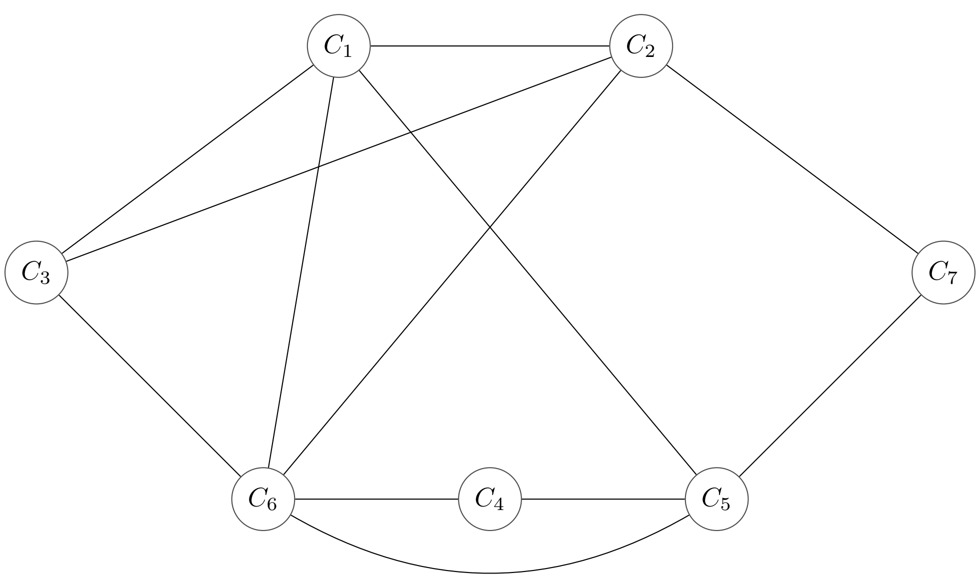
A.3.1

To represent this as a graph I have let each node represent a class and each edge represent at least one person is shared between the classes. If the graph corresponding to the problem can be coloured with ≤ 4 colours then the University would be able to timetable the classes in the four available time-slots. The graph is represented as the following adjacency matrix:

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

where row i and column j represent C_i and C_j respectively.



A.3.2

The order that the algorithm would visit the vertices of G is as follows:

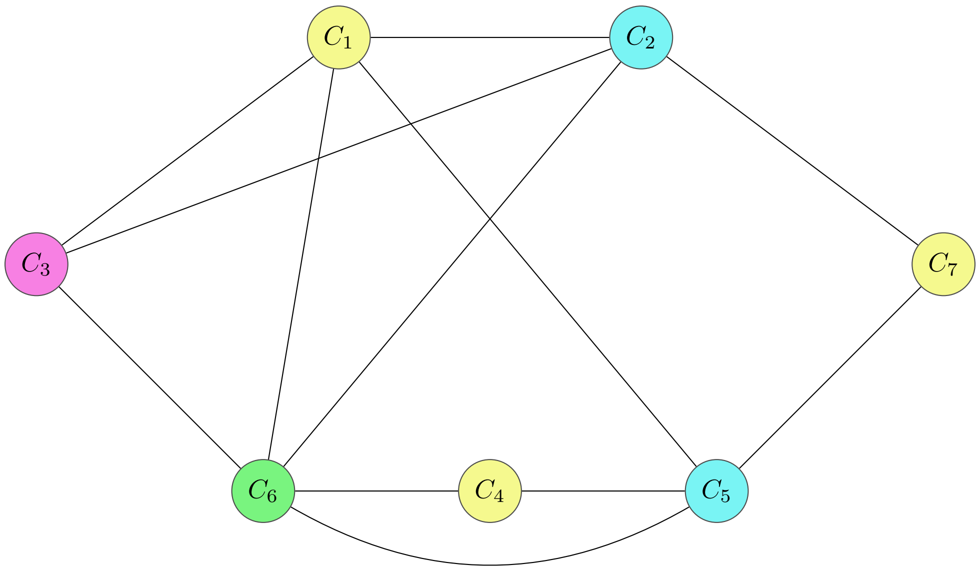
1. Start at C_1.
2. Available vertices to visit: $\{C_2, C_3, C_5, C_6\}$. Visit C_2.
3. Available vertices to visit: $\{C_3, C_5, C_6, C_7\}$. Visit C_3.
4. Available vertices to visit: $\{C_5, C_6, C_7\}$. Visit C_5.
5. Available vertices to visit: $\{C_4, C_6, C_7\}$. Visit C_4.
6. Available vertices to visit: $\{C_6, C_7\}$. Visit C_6.
7. Available vertices to visit: $\{C_7\}$. Visit C_7.
8. All vertices visited.

So the order that the vertices were visited in was $\{C_1, C_2, C_3, C_5, C_4, C_6, C_7\}$

A.3.3

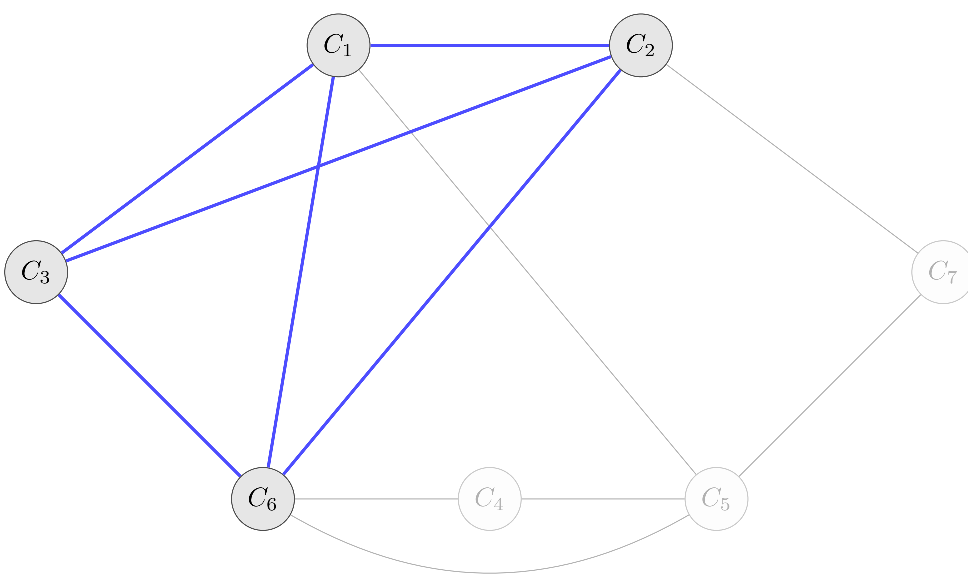
The colour of each vertex is as follows:

- C_1 has colour 1
- C_2 has colour 2
- C_3 has colour 3
- C_4 has colour 1
- C_5 has colour 2
- C_6 has colour 4
- C_7 has colour 1



A.3.4

The chromatic number of G, $\chi(G) = 4$. This is because of the sub-graph below which can only be coloured in 4 distinct colours because each vertex connects to the other 3 vertices:



Hence, because I have found a 4-colouring for the whole graph this must be the absolute minimum number of colours required so $\chi(G) = 4$.