CT Graphs Scheduling Problem

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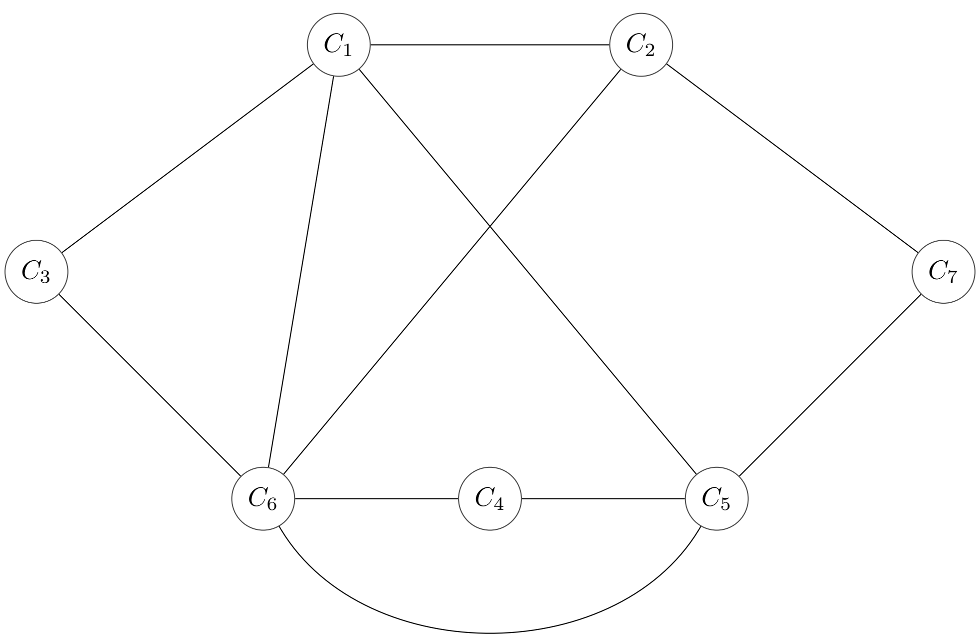
A.3.1

To represent this as a graph I have let each node represent a class and each edge represent at least one person is shared between the classes. If the graph corresponding to the problem can be coloured with ≤ 4 colours then the University would be able to timetable the classes in the four available time-slots. The graph is represented as the following adjacency matrix:

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

where row i and column j represent C_i and C_j respectively.



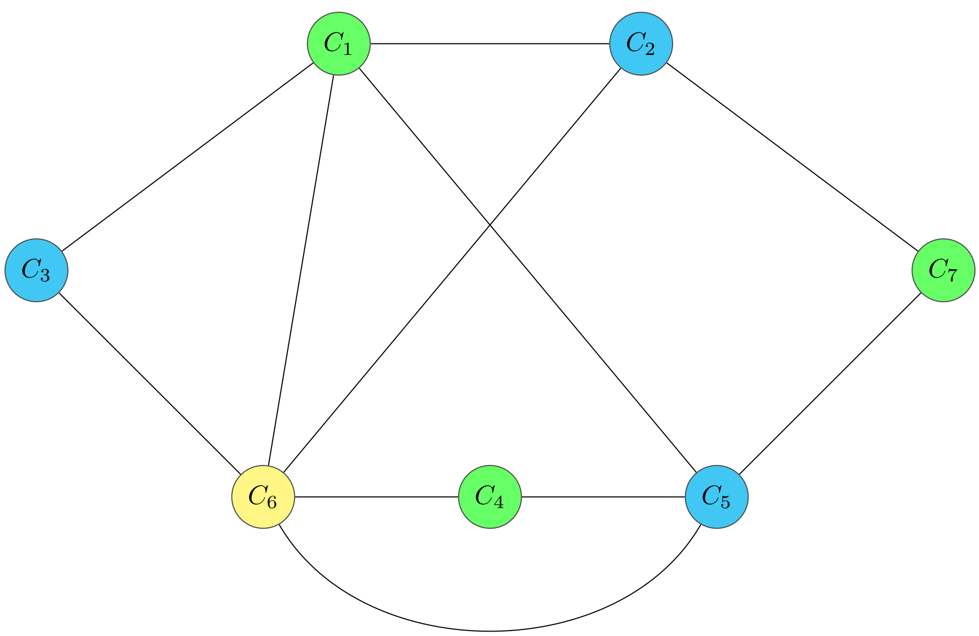
A.3.2

Starting from C_1, the order of visited vertices is $C_1, C_2, C_3, C_5, C_4, C_6, C_7$.

A.3.3

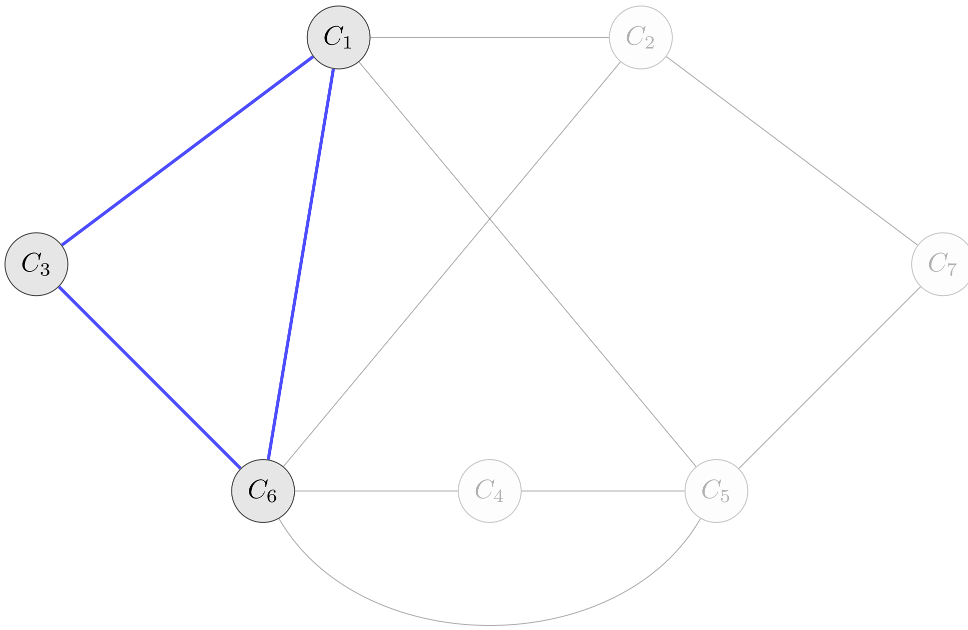
The colour of each vertex is as follows:

- C_1 has colour 1
- C_2 has colour 2
- C_3 has colour 2
- C_4 has colour 1
- C_5 has colour 2
- C_6 has colour 3
- C_7 has colour 1



A.3.4

The chromatic number of G, $\chi(G) = 3$. This is because the graph certainly cannot be 1-coloured as it is a non-trivial graph and it certainly cannot be 2-coloured because a bipartite graph requires no cycles of odd length to exist in the graph but at least one does exist in this graph:



Hence, because we have found a 3-colouring this must be the minimum amount of colours and thus $\chi(G) = 3$.