Differential Privacy and Neural Networks: A Preliminary Analysis

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Outline

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Differential Privacy

- Releasing (Big) Data sets is fundamental toward analytics tasks (analysis and prediction)
- Releasing or letting analysts to use data raises privacy concerns

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- Releasing (Big) Data sets is fundamental toward analytics tasks (analysis and prediction)
- Releasing or letting analysts to use data raises privacy concerns
- Differential privacy (DP) differs from data anonymization techniques in two main respects:
    - it does not require to modify the data
    - it does not assume any particular background knowledge
- DP ensures the outcomes of queries/calculations to be insensitive to any individual record in a database

Definition

Differential privacy provides information about a group while allowing to learn as little as possible about any individual in it.

Differential Privacy

Definition (Differential Privacy)

A randomized computation M provides ϵ-differential privacy if for any datasets A and B with symmetric difference $A \Delta B = 1$ (A and B differ in one record), and any set of possible outcomes $S \subseteq Range(M)$:

$$\Pr[M(A) \in S] \leq \Pr[M(B) \in S] \times e^{\epsilon} + \delta$$

- ϵ allows to control the level of privacy
- δ allows to introduce an error thus defining (ϵ, δ)-DP
  - lower values of ϵ mean stronger privacy
- The probability that an attacker guesses an individual record is (resp., is not) in the database in at most e^{ϵ}

Differential Privacy for real-valued functions

- DP is achieved by adding noise to the outcome of a query

Definition (Sensitivity of a real-valued function)

Given a function $f\colon D \rightarrow R^d$ the sensitivity of f is:

$$S(\text{f}) = \max_{A,B,A\Delta B=1} ||f(A) - f(B)||_1$$

Theorem (**DP and the Laplace mechanism**)

Given a function $f\colon D \rightarrow R^d$ the computation $M(X) = f(X) + (Laplace(S(f)/\epsilon))^d$ provides ϵ-differential privacy.

Theorem (**Approximate DP and the Gaussian Mechanism**)

Given a function $f\colon D \rightarrow R^d$, the computation $M(X) = f(X) + N(0, \sigma^2)^d + \delta$ provides (ϵ, δ)-differential privacy

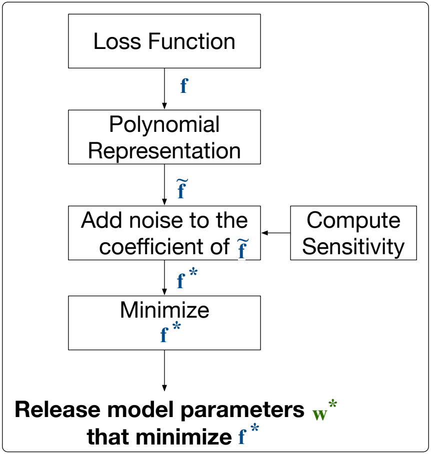
- $\sigma \geq \sqrt{2ln(2/\delta)}/\epsilon \times max_{A\Delta B}||f(A) - f(B)||_2$.

Context

- A dataset $\mathcal{D}$
- Multi Layer Perceptron (MLP) [2].
  - A set of input variables $\mathbf{x_i}$, $i \in [1, d]$
  - A set of output variable $\mathbf{t_l}$, $l \in [1, q]$
  - A a vector $\mathbf{w}$ of adjustable parameters
  - **Network training**: learn the model parameters (weights and biases) that minimize the error function $E(\mathcal{D}, \mathbf{w})$ (e.g., via the back-propagation algorithm [4]).

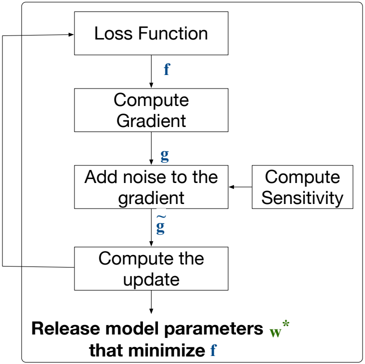
State of the Art

- **Functional Approaches** [6]: *Instead of perturbing the results, one can perturb the objective function and then optimize the perturbed objective function.*



State of the art

- **Non Functional Approaches** [1] are mainly based on *line search methods* such as the (Stochastic) Gradient Descent (SGD).



Our Approach

Our algorithm to learn via Neural Network under Differential Privacy, adopts a Functional Approach. It involves the following four main phases:

1. Find a polynomial representation of the objective function;
2. Compute the sensitivity;
3. Add noise to the polynomial representation;
4. Minimize the perturbed objective function.

Preliminaries: Neural Network Representation

- For simplicity we leave out bias terms
- $w_{i,j}$ corresponds to the weight of the connection between nodes j and i such that $j \prec i$ (j is a predecessor of i).

$$z_{\mathbf{x},i} = \phi_i\left(a_{\mathbf{x},i}\right)$$
$$a_{\mathbf{x},i} = \sum_{j \prec i} w_{i,j} \cdot z_{\mathbf{x},j}$$

- $z_{\mathbf{x},i}$ (resp. $a_{\mathbf{x},i}$) represents the application of a_i to $\mathbf{x}$
- ϕ_i represents the activation function relative to unit i.

Polynomial representation of the objective function

We consider the following 2nd-order Taylor expansion $\hat{E}(\mathcal{D}, \mathbf{w})$ of the error function $E(\mathcal{D}, \mathbf{w}) = \sum_{\mathbf{x}\in\mathcal{D}} E_{\mathbf{x}}(\mathbf{w})$.

$$\hat{E}(\mathcal{D}, \mathbf{w}) \approx E(\mathcal{D}, \hat{\mathbf{w}}) + (\mathbf{w} - \hat{\mathbf{w}})^T \nabla E(\mathcal{D}, \hat{\mathbf{w}}) + \frac{1}{2}(\mathbf{w} - \hat{\mathbf{w}})^T \nabla^2 E(\mathcal{D}, \hat{\mathbf{w}})(\mathbf{w} - \hat{\mathbf{w}})$$

where: $\nabla E(\mathcal{D}, \hat{\mathbf{w}}) \equiv \frac{\partial E}{\partial w_{i,j}}|_{\mathbf{w}=\hat{\mathbf{w}}}$ and $(\nabla^2 E(\mathcal{D}, \hat{\mathbf{w}}))_{i,j} \equiv \frac{\partial^2 E}{\partial w_i \partial w_j}|_{\mathbf{w}=\hat{\mathbf{w}}}$ are the Jacobian and Hessian matrices, respectively. Denoting by $g_{i,j}$ (resp., $h_{i,j}$) an element of the Jacobian (resp., Hessian) matrix, we obtain:

$$g_{i,j} = \sum_{\mathbf{x}\in\mathcal{D}} \delta_{\mathbf{x},i} z_{\mathbf{x},j} \tag{1}$$

$$\delta_{\mathbf{x},i} = z'_{\mathbf{x},i} \sum_{v:i\prec v} \delta_{\mathbf{x},v} w_{v,i} \tag{2}$$

$$h_{i,j} = \sum_{\mathbf{x}\in\mathcal{D}} b_{\mathbf{x},i} z^2_{\mathbf{x},j} \tag{3}$$

$$b_{\mathbf{x},i} = z''_{\mathbf{x},i} \sum_{v:i\prec v} w_{v,i}\delta_{\mathbf{x},v} + (z'_{\mathbf{x},i})^2 \sum_{v:i\prec v} w^2_{v,i} b_{\mathbf{x},v} \tag{4}$$

Polynomial Represenation of the objective function

To reduce the computational cost we only consider diagonal elements of the Hessian [3]. As for the output units, their output depends from the specific loss function considered. As an example, when considering the Least Squares Error, we obtain:

$$
\begin{aligned}
E(\mathbf{x}, \mathbf{w}) =& \frac{1}{2} \sum_{c=1}^{d} (y_{\mathbf{x},c} - t_{\mathbf{x},c})^2 \\
y_{\mathbf{x},c} =& y_c(\mathbf{x}; \mathbf{w})
\end{aligned}
$$

where $y_{\mathbf{x},c}$ is the c-th component given by the network when giving as input the vector $\mathbf{x}$ and $t_{\mathbf{x},c}$ is the c-th target value. Hence, for output units we have:

$$
\begin{aligned}
\delta_{\mathbf{x},i} =& (\psi(a_{\mathbf{x},i}) - t_{\mathbf{x},i})\psi'(a_{\mathbf{x},i}) && (5) \\
b_{\mathbf{x},i} =& (\psi'(a_{\mathbf{x},i}))^2 + (\psi(a_{\mathbf{x},i}) - t_{\mathbf{x},i})\psi''(a_{\mathbf{x},i})) && (6)
\end{aligned}
$$

where ψ is the activation function for output units.

Compute the sensitivity

We need to estimate its sensitivity to add noise into the polynomial representation of the error function.

Lemma

Let $\mathcal{D}$, $\mathcal{D}'$ be any two databases differing in at most one tuple, and

$$\hat{E}(\mathcal{D}, \mathbf{w}) = \Big(\sum_{\mathbf{x}\in\mathcal{D}}\sum_{j\prec i}\delta_{\mathbf{x},i}z_{\mathbf{x},j}\Big)w_{i,j} + \frac{1}{2}\Big(\sum_{\mathbf{x}\in\mathcal{D}}\sum_{j\prec i}b_{\mathbf{x},i}z^2_{\mathbf{x},j}\Big)w^2_{i,j}$$

$$\hat{E}(\mathcal{D}', \mathbf{w}) = \Big(\sum_{\mathbf{x}'\in\mathcal{D}'}\sum_{j\prec i}\delta_{\mathbf{x}',i}z_{\mathbf{x}',j}\Big)w_{i,j} + \frac{1}{2}\Big(\sum_{\mathbf{x}'\in\mathcal{D}'}\sum_{j\prec i}b_{\mathbf{x}',i}z^2_{\mathbf{x}',j}\Big)w^2_{i,j}$$

the polynomial representation of the error function on $\mathcal{D}$ and $\mathcal{D}'$, respectively. Let $\mathbf{x}$ be an arbitrary tuple.

$$||\hat{E}(\mathcal{D}, \mathbf{w}) - \hat{E}(\mathcal{D}', \mathbf{w})||_1 \leq 2\sum_{j\prec i}\max_{\mathbf{x}}\left(|\delta_{\mathbf{x},i}z_{\mathbf{x},j}| + |b_{\mathbf{x},i}z^2_{\mathbf{x},j}|\right)$$

An overview of the Algorithm

Algorithm 1 `FunctionalNetDP` (Privacy budget ϵ)

1: Initialize $\mathbf{w}$
2: $g, h \leftarrow$ Polynomial Representation of the loss function
3: Find $\widetilde{g}$ and $\widetilde{h}$ via `addNoise`($\mathbf{w}$,ϵ) /* Algorithm 2 */
4: Compute $\tilde{\mathbf{w}}=\underset{\mathbf{w}}{argmin}\ \widetilde{E}(\mathcal{D}, \mathbf{w})$
5: **return** $\tilde{\mathbf{w}}$ / set of weights that minimizes $\widetilde{E}(\mathcal{D}, \mathbf{w})$ /

Algorithm 2 `addNoise`(Weights $\mathbf{w}$, privacy budget ϵ)

1: Let $\mathcal{D}$ be the dataset
2: Set $S(\widetilde{E})=2\sum_{j\prec i} \max_{\mathbf{x}} \left(|\delta_{\mathbf{x},i} z_{\mathbf{x},j}|+|b_{\mathbf{x},i} z^2_{\mathbf{x},j}|\right)$
3: Find g via eq. (1) and h via eq. (3) using $\mathcal{D}$
4: g$\leftarrow g/min(1, ||g||^2/C)$ /* clip Gradient */
5: h$\leftarrow h/min(1, ||h||^2/C)$ /* clip Hessian */
6: $\widetilde{g} \leftarrow g$+$Lap(S(\widetilde{E}) \mid \epsilon)$
7: $\widetilde{h} \leftarrow h$+$Lap(S(\widetilde{E}) \mid \epsilon)$
8: **return** $\widetilde{g}$ and $\widetilde{h}$

Theorem

Algorithm 1 satisfies ϵ-differential privacy.

A fine-grained analysis of the Laplace noise (I)

Assuming that for both ϕ and ψ the logistic function $\sigma(x) = \frac{1}{1+e^{-x}}$ is used, then we have that:

- $\sigma(x) \in [0, 1]$
- $\sigma'(x) \in [0, 0.25]$,
- $\sigma(x)'' \in [-0.1, 0.1]$

Moreover, we assume that:

- $w_{i,j} \in [0, 1]$
- for each dimension d of the input tuple $\mathbf{x}^{\mathbf{d}} \in [0, 1]$

To bound the amount of Laplace noise needed, we need to bound the components $\sum_{j \prec i} |\delta_{\mathbf{x},i} z_{\mathbf{x},j}|$ and $\sum_{j \prec i} |b_{\mathbf{x},i} z^2_{\mathbf{x},j}|$ in the previous Lemma.

Let m be the depth of the network. For each layer $j \in \{1, ..., m\}$ we denote by δ^j (resp., b^j) the coefficient of a generic unit in the layer j. Moreover, s_j represents the number of units at layer j. We now analyze the amount of noise as per our previous sensitivity analysis.

Lemma

The noise to be introduced by the $\sum_{j \prec i} |\delta_{\mathbf{x},i} z_{\mathbf{x},j}|$ *component is* $\leq 0.25^m \times \prod_{j=1}^{j=m} s_j$.

Lemma

The noise to be introduced by the $\sum_{j \prec i} |b_{\mathbf{x},i} z^2_{\mathbf{x},j}|$ *component is:*

$$\leq \prod_{j=1}^{j=m} s_j \left(0.1^m + 0.001625 \times m\right)$$

A fine-grained analysis (II)

Theorem

For a network of m layers with s_j units in each layer, where $j \in [1, n]$, the amount of Laplacian noise to ensure differential privacy is:

$$\leq 2\left(0.25^m \prod_{q=1}^{q=m} s_q + \prod_{j=1}^{j=m} s_j \times \left(0.1^m + 0.001625 \times m\right)\right)$$

The above analysis shows that the amount of noise to ensure differential privacy is dominated by the number of units in the network.

Concluding Remarks

- We have reported on the usage of differential privacy in neural networks and discussed our preliminary findings in adding Laplacian noise to a low-polynomial representation of the error function.
- We want to underline the following aspects emerging from our preliminary analysis :
  1. The sensitivity basically depends on the topology of the network.
     - Finding a better bound is in our research agenda
  2. Algorithm 1 works by approximating the Hessian to its diagonal components as done by other approaches .
     - We are considering the usage of 2nd order methods (e.g., Hessian Free optimization [5])

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