SPHERE PACKINGS V. PENTAHEDRAL PRISMS

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Abstract. This paper is the fifth in a series of papers devoted to the proof of the Kepler conjecture, which asserts that no packing of congruent balls in three dimensions has density greater than the face-centered cubic packing.

In this paper, we prove that decomposition stars associated with the plane graph of arrangements we term pentahedral prisms do not contravene. Recall that a contravening decomposition star is a potential counterexample to the Kepler conjecture. We use interval arithmetic methods to prove particular linear relations on components of any such contravening decomposition star. These relations are then combined to prove that no such contravening stars exist.

Introduction

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Pentahedral prisms come remarkably close to achieving the optimal score of 8 pt, that achieved by the decomposition stars of the face-centered cubic lattice packing. In this sense, we consider pentahedral prisms to be "worst case" decomposition stars.

Pentahedral prisms constituted a counterexample to an early version of Hales's approach to a proof of the Kepler conjecture, and have always been a somewhat thorny obstacle to the proof of the conjecture. Relations required to treat pentahedral prisms are delicate in contrast to the more general bounds which suffice to treat other decomposition stars.

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References

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