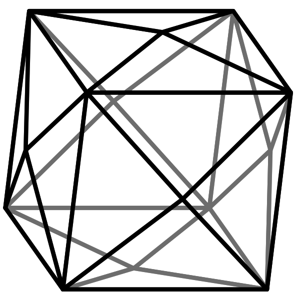
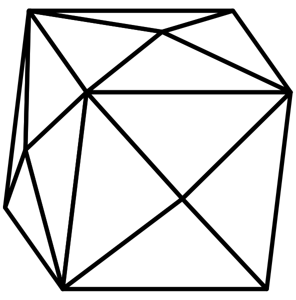
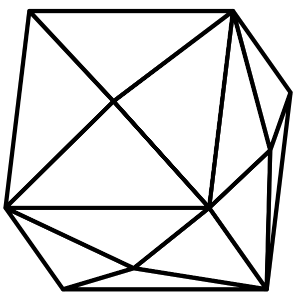
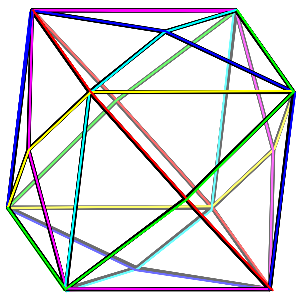


A different realization of the Coexter complex on 4 variables. To get the Ehrhart picture we have to cone over this subdivision of a sphere twice.

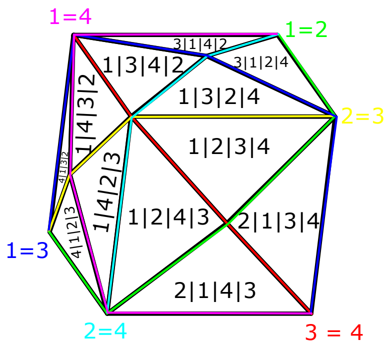


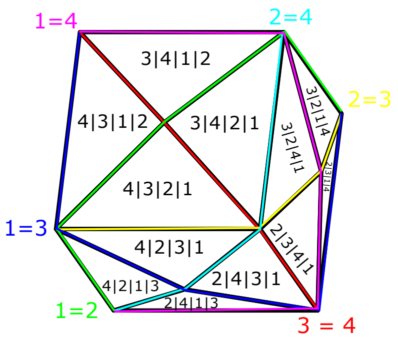


Front and back faces.

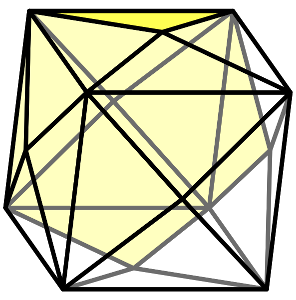


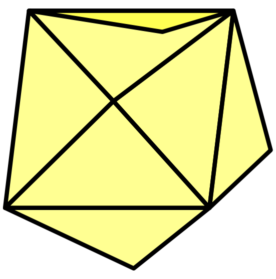
Edges highlighted according to which of the six Braid hyperplanes they lie on.



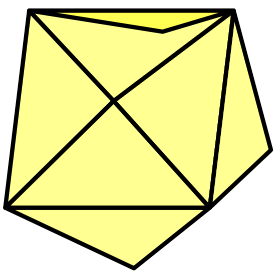


Faces labled with ordered set partitions. Note that we label a two dimensional face in this picture with the label of the corresponding four dimensional cell in the Ehrhart picture, i.e., with a permutation.



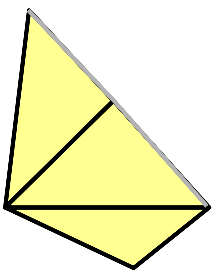


A subcomplex we will represent as a decision tree.
Open edges will be marked in gray.



cut using x3 = x4

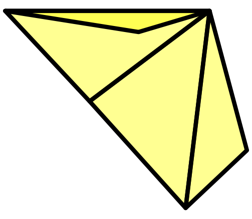
x4 < x3



As we are coning over the main diagonal of the cube, this region is actually convex. It can be described using the following addtional constraints:

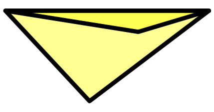
x1 >= x3
x2 >= x4

x4 >= x3

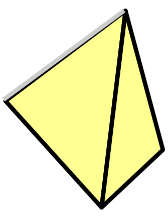


cut using x1 = x2

x2 >= x1



x2 < x1



these regions are now convex and can resp. be defined using the following addition constraints.

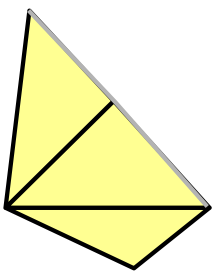
x1 >= x3
x2 >= x4

x1 >= x4
x2 >= x3

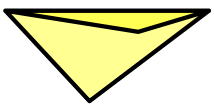
The whole boolean formula:

```
if x4 >= x3:
    if x2 >= x1:
        x1 >= x3 and x2 >= x4
    else:
        x1 >= x4 and x2 >= x3
else:
    x1 >= x3 an x2 >= x4
```

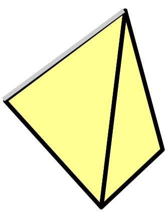
In total the inequality descriptions of the individual pieces are:



x4 < x3
x3 <= x1
x4 <= x2



x3 <= x4
x1 <= x2
x3 <= x1
x4 <= x2



x3 <= x4
x2 < x1
x4 <= x1
x3 <= x2