THE LINEAR REGRESSION PROCESS

From Generation to Interpretation

Presented By

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Photo credit - Vito Palmisano

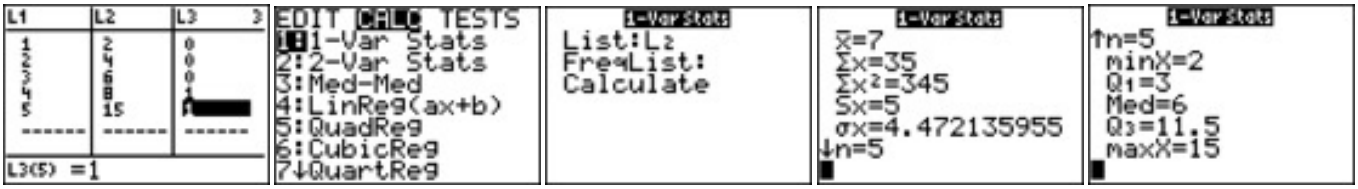
Contents

The Story

A statistics teacher gives a quiz to a class. The scores were 2, 4, 6, 8, and 15 with one student being absent. Absent student returns the next day...
Student: How am I going to do on the quiz?
Teacher: Well, the class average was...

1 Standard Deviation: The Non-Resistant Measure of Spread

How much variability, on average, is there around the mean?



Score x	Deviation	Squared Deviation
2		
4		
6		
8		
15		

The Story Part 2

A statistics teacher gives a quiz to a class. The scores were 2, 4, 6, 8, and 15 with one student being absent. After surveying the class, the teacher knows the hours studied were 1, 2, 3, 4 and 5, respectively. Absent student returns the next day...
Student: How am I going to do on the quiz?
Teacher: That depends - how long did you study?

2 Getting Least Squares Line of Best Fit

Hours (x)	1	2	3	4	5
Score (y)	2	4	6	8	15

2.1 From Summary Statistics

Formulas (given): $\hat{y} = a + bx \qquad b = r\frac{s_y}{s_x} \qquad a = \bar{y} - b\bar{x}$

Descriptive Statistics: x, y

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
x	5	0	3.000	0.707	1.581	1.000	1.500	3.000	4.500	5.000
y	5	0	7.00	2.24	5.00	2.00	3.00	6.00	11.50	15.00

Correlations: x, y

```
Pearson correlation of x and y = 0.949
P-Value = 0.014
```

2.2 From Output

Regression Analysis: y versus x

```
The regression equation is
y = - 2.00 + 3.00 x

Predictor     Coef   SE Coef          T       P
Constant    -2.000     1.915     -1.04   0.373
x           3.0000    0.5774      5.20   0.014

s = 1.82574    R-sq = 90.0%    R-Sq(adj) = 86.7%
```

Analysis of Variance

```
Source           DF         SS        MS        F        P
Regression        1     90.000    90.000    27.00    0.014
Residual Error    3     10.000     3.333
Total             4    100.000
```

2.3 Interpretation

2.3.1 Slope

Slope represents the **predicted** change in response associated with each unit increase in the explanatory variable, **on average**.

2.3.2 Y-Intercept

Y-intercept is the predicted value when the explanatory (x) is 0. [Often the y-intercept is useless.]

2.4 Predicted Values and Residuals

$\hat{y} = -2 + 3x$

Hours	Score	Predicted	Residual
x	y	$\hat{y}$	$y - \hat{y}$
1	2		
2	4		
3	6		
4	8		
5	15		

- Predicted $\hat{y}$: substitute explanatory (x) values into regression equation.
- Residual $y - \hat{y}$ (also called *regression error*); actual minus predicted.

The Story Part 3

A statistics teacher gives a quiz to a class. The scores were 2, 4, 6, 8, and 15 with one student being absent. After surveying the class, the teacher knows the hours studied were 1, 2, 3, 4 and 5, respectively. Absent student returns the next day...
Student: How am I going to do on the quiz?
Teacher: That depends - how long did you study?
Student: Does how long I studied really make a difference?

2.5 The Coefficient of Determination r^2 - Comparing Models

Regression Analysis: y versus x

```
The regression equation is
y = - 2.00 + 3.00 x

Predictor      Coef   SE Coef         T       P
Constant     -2.000     1.915     -1.04   0.373
x            3.0000    0.5774      5.20   0.014

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Analysis of Variance

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Source           DF         SS        MS        F        P
Regression        1     90.000    90.000    27.00    0.014
Residual Error    3     10.000     3.333
Total             4    100.000
```
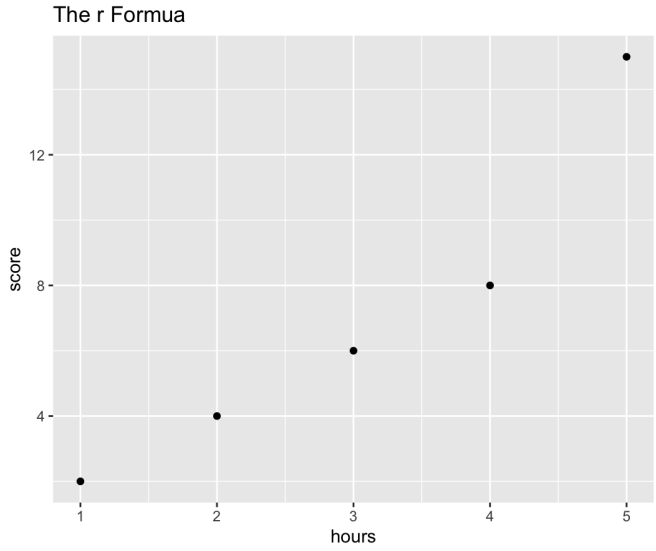
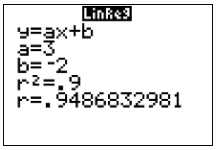

Hours	Score	Predicted	Error	$(\text{Error})^2$	Residual	$(\text{Residual})^2$
x	y	$\hat{y}$	$y-\bar{y}$	$(y-\bar{y})^2$	$y-\hat{y}$	$(y-\hat{y})^2$
1	2					
2	4					
3	6					
4	8					
5	15					

3 Summary & Examples

3.1 Getting r

- $r = \frac{\Sigma z_x \cdot z_y}{n-1}$
- Never calculate by hand; use calculator or computer output.
- Know formula *properties*.





Correlations: x, y

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3.2 Five R Properties

- Examples