Take the Goods and Run: Contracting Frictions and Market Power in Supply Chains*

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Abstract

This paper studies the efficiency of self-enforced relational agreements, a common solution to contracting frictions, when sellers have market power and contracts cannot be externally enforced. To this end, I develop a dynamic contracting model with limited enforcement in which buyers can default on their trade-credit debt and estimate it using a novel dataset from the Ecuadorian manufacturing supply-chain. The key empirical finding is that bilateral trade is inefficiently low in early periods of the relationship, but converges toward efficiency over time, despite sellers' market power. Counterfactual simulations imply that both market power and enforcement contribute to inefficiencies in trade.

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1 Introduction

When courts cannot enforce contracts, trading partners often resort to long-term relational contracts, sustained through repeated interactions, to ease frictions and constrain opportunistic behavior (Johnson et al., 2002). As weak contract enforcement is a common feature of developing economies, relational agreements are highly relevant inter-firm organizational structures. Understanding the efficiency of these informal agreements is essential for policy-makers in developing countries, as they frequently have to make trade-offs regarding where to focus their reform efforts.

The traditional view sees contracting frictions as a hindrance that distorts productive decisions (La Porta et al., 1997; Nunn, 2007), implying that, as a standard solution, relational contracts may be inefficient. However, it is noteworthy that the same economies where enforcement constraints are likely to be a significant factor may also encounter additional frictions, such as high market concentration, making them second-best environments (Rodrik, 2008). In the presence of seller market power, weak enforcement may increase the buyer's relative bargaining power, thereby limiting downstream distortions while improving the efficiency of a relationship as opposed to a perfect enforcement world (Genicot and Ray, 2006). Thus, the efficiency of relational agreements remains unclear.

This paper uses theory and data to quantify the static (period-by-period) efficiency of self-enforced long-term relationships in the presence of seller market power and limited external enforcement of contracts. I develop a novel long-term contracting model where 1) the seller can price discriminate across buyers and time, and 2) the buyer can act opportunistically and simply *take the goods and run* whenever the delivery of the goods occurs before payment. Without access to external enforcement, the seller uses the value of the relationship itself to discipline the buyer's behavior. The modeling framework is applied to examine self-enforced relationships in the manufacturing supply chain in Ecuador, a middle-income country with slow commercial courts and concentrated sectors.

The paper has two novel empirical contributions. Firstly, by utilizing a structural econometric model, it provides the first empirical evidence regarding the efficiency evolution of long-term trade relationships. The findings demonstrate that relationships tend to be highly inefficient at the early stages, but over time, such inefficiencies diminish, indicating the crucial role of repeated informal agreements in creating surplus. Secondly, the study examines the counterfactual scenario of implementing best-practice institutions (e.g., eliminating contracting frictions) and finds an intertemporal trade-off. In the short term, the implementation of best-practice institutions leads to an increase in welfare. However, in the medium and long term, such institutional changes are found to result in welfare losses when compared to the second-best equilibrium. In contrast, efficiency improves when all modeled frictions are addressed simultaneously.

I start by documenting six fundamental patterns that provide the basis for the key elements of the model. First, it is observed that most trade takes place through repeated relationships. Second, the vendor finances a majority of transactions using trade-credit, even in new relationships. Third, relationships exhibit growth in both quantity and value as they age. Fourth, sellers offer considerable quantity discounts, such that a 10% increase in quantity is associated with a 2% unit price decrease. Fifth, older buyers receive up to 3% discounts relative to new buyers, conditional on the quantity purchased, as the relationship evolves. Importantly, these discounts are observed only in cases where buyers use trade-credit as opposed to paying the full order amount upfront. Moreover, given accounting markups of 20%, these discounts are economically significant with respect to quantities and the age of relationships. Finally, the survival probability of relationships is observed to increase in quantity and as relationships mature. These patterns provide valuable insights into the nature of long-term relationships in the manufacturing supply chain in Ecuador, which are used to build the theoretical model and to inform the empirical analysis.

Standard models in the literature (such as efficiency gains, learning, demand assurance, or supply-side enforcement issues) are not able to capture all of these patterns under realistic assumptions. For that reason, to account for these patterns and assess the efficiency of relationships over time, I develop a dynamic contracting model by embedding a non-linear pricing model with heterogeneous participation constraints (Jullien, 2000; Attanasio and Pastorino, 2020) into an infinitely repeated game with limited enforcement (Martimort et al., 2017; Pavoni et al., 2018; Marcet and Marimon, 2019). In the model, sellers and buyers with private heterogeneous demand meet randomly and have the opportunity to engage in repeated trade. The seller has all the bargaining power and proposes a dynamic contract of prices and quantities, for which they have commitment. Consistent with the data, the seller in the model finances all the transactions using trade-credit. Buyer heterogeneity provides incentives to price discriminate, so the seller offers menus of quantities and prices that satisfy *incentive compatibility* and induce revelation of the buyer asymmetric information.

Crucially, the buyer cannot commit to paying their debts and is subject to forward-looking *limited enforcement* constraints. The future stream of benefits created by the relationship must be large enough to secure the payment. To prevent a *take the goods and run* scenario, the seller must share a greater amount of surplus than otherwise. Thus, enforcement constraints could act against the seller's profit maximizing incentives to distort trade downward through inefficiently low quantities. Matching the empirical picture described above, the optimal dynamic menu of quantities and prices in a setting with limited enforcement features *backloading*: both the total surplus generated by the relationship and the surplus captured by the buyer increase over time.

To determine the optimal quantity allocations in this setting, I use a recursive Lagrangian approach (Pavoni et al., 2018; Marcet and Marimon, 2019), which characterizes the optimal dynamic contract in terms of *past* and *present* limited enforcement Lagrange multipliers (LE multipliers). The present LE multipliers capture the current limited enforcement constraints, while past LE multipliers account for promises made in the past to prevent default and serve as promise-keeping constraints. In equilibrium, the optimal quantity allocations are then determined by a *modified virtual surplus*, which takes into account the standard informational rents due to incentive compatibility, as well as the shadow costs of binding enforcement constraints.

The paper proposes an econometric model that is directly derived from the theoretical model and shows that the parameters of the model can be identified using cross-sectional data on prices, quantities, age of relationships, and marginal costs for one seller. The model relies on the seller's optimality conditions and the buyer's dynamic first-order conditions for incentive compatibility (as in the static results of Luo et al., 2018 and Attanasio and Pastorino, 2020) to identify the dynamic effects of limited enforcement on trade. The identification intuition is twofold. First, the seller offers prices that induce the revelation of information about buyers' types and discriminate across them. This implies that price variation across buyers is a signal of their underlying types. Second, the degree of trade distortion relative to the efficient outcome provides information on whether current or past enforcement concerns are constraining the trade relationship. By examining the difference between marginal prices and marginal costs, which indicates the presence of downward and upward distortions, we can identify the extent of additional distortions due to limited enforcement.

I estimate the model using three administrative databases collected by the Ecuadorian government for tax purposes that match the objects in the theoretical model. I obtain pair-specific unit prices and quantities using a new electronic invoice database that contains all domestic sales for 49 manufacturing firms in the textile, pharmaceutical, and cement-product sectors for 2016-2017, each with a large number of buyers each year (median of 600). The age of relationships is inferred through the universe of firm-to-firm VAT database, which tracks the total volume of bilateral trade from 2008-2015. Lastly, a measure of seller's costs comes from information on total variable costs (i.e., intermediate inputs expenditure and labor wages) contained in usual financial statements reported to the tax authority.

The model fits the data well, and the estimation reveals that enforcement concerns are relevant throughout the life-cycle of a relationship. Specifically, almost all new relationships have binding enforcement constraints. As relationships age, these constraints are relaxed, reflecting the increase in quantities coming from past promises made by the seller. Given the large number of trading partners, I explore the heterogeneity of enforcement constraints and find they differ significantly by buyers' and sellers' characteristics. For example, they are more likely to bind when the seller and buyer's headquarters are far away.

Using the estimated model parameters, I evaluate the efficiency of transactions at any given point and examine the division of surplus. My findings indicate that new relationships operate at approximately 30% of the optimal (i.e., frictionless) level, but efficiency increases as relationships age. Relationships lasting five years or more can achieve efficiencies upwards of 80%. In the aggregate, my analysis reveals that sellers heavily distort quantities early on. Specifically, only 5% of suppliers achieve levels of aggregate output that are indistinguishable from efficient output when dealing with new buyers, whereas 84% of sellers achieve long-term aggregate output levels that cannot be distinguished from efficient levels. Remarkably, these patterns hold for each industry studied, talking to the generality of the result. As for the division of surplus, I find that sellers capture the majority (around 70%) of the generated surplus, although the largest buyers may capture up to 50% of the total surplus. These findings suggest that market power favors the seller.

The paper proceeds to investigate counterfactual scenarios that have surprising implications. Firstly, the analysis shows that addressing enforcement constraints alone, without addressing market power, can lead to higher surplus in the short term, but result in a lower total surplus in the medium and long term. Similarly, only addressing market power leads to substantial welfare losses across different types and time periods. These findings are consistent with the *theory of second-best* (Lipsey and Lancaster, 1956), which suggests that in the presence of one friction, the effect on welfare of removing one friction alone is uncertain. In this particular case, each friction serves to counterbalance the other. Secondly, the paper explores the effects of addressing both frictions simultaneously. The results indicate that most relationships achieve a higher total surplus and lower surplus for the seller when both frictions are addressed together. Overall, these counterfactual analyses underscore the significance of recognizing the interplay between various frictions in markets. Simply addressing one friction in isolation may not produce the desired outcome and could result in unintended consequences.

This paper contributes to several strands of the theoretical and empirical literatures. First, I contribute to a vast and diverse theoretical literature on imperfect lending and contracting (Bull, 1987; MacLeod and Malcomson, 1989; Thomas and Worrall, 1994; Watson, 2002; Ray, 2002; Levin, 2003; Albuquerque and Hopenhayn, 2004; Board, 2011; Halac, 2012; Andrews and Barron, 2016; Martimort et al., 2017; Troya-Martinez, 2017). The closest theoretical paper to mine is Martimort et al. (2017), which provides a theory of a two-sided limited enforcement problem in which buyers can default on debts and sellers can cheat on quality. In their setting, the buyer is the principal and increasingly shares a greater amount of surplus with the seller, implying dynamics where quantities *and* prices both increase. These dynamics do not match those observed in the setting I study, with frictions that are common in many parts of the developing world. In contrast, I consider a model where, besides the incentives to default, the buyer has private information about the value of the relationship and the seller has the bargaining power.

Second, I contribute to the empirical literature on imperfect lending and contracting (McMillan and Woodruff, 1999; Banerjee and Duflo, 2000; Karaivanov and Townsend, 2014; Antras and Foley, 2015; Macchiavello and Morjaria, 2015; Boehm and Oberfield, 2020; Startz, 2021; Blouin and Macchiavello, 2019; Heise, 2019; Ghani and Reed, 2020; Ryan, 2020). Several papers, including Blouin and Macchiavello (2019), Ryan (2020), and Startz (2021), have previously estimated the efficiency losses arising from imperfect contracting. For instance, Blouin and Macchiavello (2019) analyze strategic default on forward-contracts by sellers in the international coffee market, Ryan (2020) focuses on contract renegotiation in public procurement, and Startz (2021) studies weak contract enforcement in relation to seller opportunism and the presence of search frictions. My contribution is to quantify the inefficiencies from buyer opportunism in trade-credit agreements, in conjunction with seller market power. Additionally, to my knowledge, my paper is the first empirical study to quantify the evolution of efficiency in relationships over time.

This work also follows the theoretical and empirical literature related to price discrimination (Maskin and Riley, 1984; Jullien, 2000; Villas-Boas, 2004; Grennan, 2013; Luo et al., 2018; Attanasio and Pastorino, 2020; Marshall, 2020).[1] The works by Luo et al. (2018) and Attanasio and Pastorino (2020) provide estimation methodology and identification results for static non-linear pricing problems, with and without binding participation constraints, respectively. This paper generalizes their models and estimation methods to a multi-period setting by the relying on the recursive Lagrangian approach, a tool typically used in sovereign-debt macroeconomic models (Aguiar and Amador, 2014). Furthermore, while Attanasio and Pastorino (2020) provide identification results for non-linear pricing models with participation constraints under constant participation multipliers, I extend their findings by showing that, for non-constant multipliers, these models are only set identified unless a parametric assumption is made.

More generally, this paper relates to works in finance and development studying manifestations of the *theory of second-best* (e.g., Petersen and Rajan, 1995; Genicot and Ray, 2006; Macchiavello and Morjaria, 2021). For example, Macchiavello and Morjaria (2021) examine the impact of competition on welfare in the coffee supply chain in Rwanda, finding that increased competition can reduce parties' ability to sustain self-enforced agreements, resulting in adverse effects on welfare. Similarly, Petersen and Rajan (1995) demonstrate that increasing competition in bank lending can actually harm buyers by decreasing overall volumes of lending when buyers have limited commitment to paying their debts. This paper contributes to this literature by empirically showing that fixing only one market friction can result in welfare losses and that addressing enforcement and seller market power together could increase welfare. Additionally, my counterfactual results are consistent

[1] This paper is related to the literature studying the durable/storable-goods monopolist (e.g. Coase, 1972; Bulow, 1982; Dudine et al., 2006; Hendel and Nevo, 2013; Hendel et al., 2014). However, it differs from it, as this paper treats inputs as non-durable and non-storable by assuming the buyer's production opportunity is time-specific.

with the theoretical results of Genicot and Ray (2006), who show that improving enforcement reduces the buyer's expected payoff when the seller has the bargaining power, and of Troya-Martinez (2017), who finds that total welfare decreases as enforcement quality increases beyond a certain level.

The empirical facts presented in Section 3 have been individually documented by previous works. For instance, Heise (2019) and, partially, Monarch and Schmidt-Eisenlohr (2017) have previously documented the fact of relationship dynamics in quantities and prices for international trade. The persistence of intra-national links has been documented by Huneeus (2018) for Chile. Price discrimination in the context of medical devices and wholesale food has been documented by Grennan (2013) and Marshall (2020), respectively. Similarly, Antras and Foley (2015), Garcia-Marin et al. (2019), and Amberg et al. (2020) have documented similar patterns of trade-credit issuance. However, to the best of my knowledge, this paper is the first to document relationship dynamics regarding prices and quantities intra-nationally and to present all of these facts in the same setting.

The paper is organized as follows. Section 2 provides a description of the context and presents summary statistics of the data. Section 3 offers the motivating facts that the model needs to match. Section 4 presents the model. Section 5 discusses identification and the estimation procedure. Section 6 offers the estimated results, model fit, and discusses the performance of alternative models. Section 7 discusses welfare and presents three counterfactual exercises. Finally, Section 8 concludes the paper.

2 Context, Interviews, and Data

Ecuador is an upper middle income country with weak enforcement of contracts and concentrated manufacturing markets. According to the World Bank Doing Business survey, Ecuador ranks as a median country in terms of Contract Enforcement, measuring the efficiency of courts in resolving commercial disputes, and one of the worst in terms of Insolvency measures, reflecting the inefficiency of courts in dealing with debt defaults due to bankruptcy. Additionally, the country's manufacturing sectors exhibit high levels of concentration, with average Herfindahl-Hirschman Indices of 0.6 for 6-digit economic codes, which are significantly higher than the concentration threshold of 0.25 used by the US Justice Department to identify highly concentrated markets.

2.1 Interviews

To gain a deeper understanding of the relationship management practices of manufacturing firms in Ecuador, I conducted hour-long interviews with high-ranking managers from 10 manufacturing firms in the spring of 2019. The following are the key findings from these interviews:

- Relationships among firms are not primarily based on written contracts but rather on informal agreements. Although transactions are documented, they are usually managed without the involvement of third-party enforcement, as formal enforcement is seen as costly and inefficient.[2]

- Quality issues from suppliers are not a major concern, as the inputs used are highly standardized.
- Enforcing payment for trade-credit transactions requires some investment in terms of time and personnel to pressure buyers to pay their debts.
- Most firms are aware that cash transactions offer discounts compared to trade-credit, but they often resort to trade-credit due to a lack of short-term liquidity.

This paper will not attempt to explain the underlying causes of these features, but instead will focus on how they shape ongoing relationships.

2.2 Administrative Data

The data used in this paper come from various administrative databases collected by Ecuador's Servicio of Rentas Internas (IRS) for tax purposes.

2.2.1 *VAT database*

By law, since 2008, firms are required to report all of their firm-to-firm inputs and purchases with information on the identity of the buyer and seller through the B2B VAT system. I use the universe of business-to-business (B2B) VAT database for 2008-2015 to measure the lengths of relationships. In particular, I define *age of relationship* as the total number of years that the seller has sold some positive value to the buyer in the past. Given the first year of observation is 2008, age of relationship is censored at +9.

2.2.2 *Electronic Invoicing*

The primary data source for the analysis is the electronic invoicing (EI) system. In 2014, Ecuador started rolling out a new EI system to collect VAT information more consistently, requiring large firms to implement this new technology. By 2015, the largest 5000 firms were required to use the EI system for all sales. This system would send a copy of the transaction information to the buyer and government immediately after the transaction occurs. For each sale done by a firm in the system, the EI collects product-level information, including a bar-code identifier, product description, unit price, quantities, discounts, as well as transaction-level information, such as buyer unique national identifier and method of payment. Method of payment can be: cash, check, credit card, trade-credit offered by seller with trade-credit payment terms, amongst others.

[2]The Judicial Magazine of the Ecuadorian Government, available here, provides further evidence of the inefficiency of the court system. Two recent cases of buyer default were found, one taking 6 years to resolve and the other 4 years. A 2016 reform was made to the *Código Orgánico General de Procesos* to speed up debt collection, but in practice, this route is used as a last resort and takes around 2 years to enforce payment, according to personal estimates from 7,000 cases in the Civil Court in Quito in 2017.

The EI system records transactional information in real-time and sends a copy of the transaction to both the buyer and the government. The system captures both product-level and transaction-level information, such as the bar-code identifier, product description, unit price, quantities, discounts, buyer's unique national identifier, and the method of payment (which could be cash, check, credit card, trade-credit offered by the seller, among others).

The data collected for this study is drawn from the EI system and pertains to 49 manufacturing firms operating in the textiles, pharmaceuticals, and cement sectors for the years 2016-2017. These firms are large, with an average of 8000 buyers and a market share of 24% in their 6-digit sector at the national level and 50% in their sector at the provincial level. The database coverage is deemed to be good, with the average selling firm in the sample having more than 90% of its reported sales captured by the EI system. Managerial interviews have revealed that most of these firms use the invoices received and sent for internal accounting purposes.

The study defines a *product* as a bar-code identifier and description combination. The discounts offered in a transaction are allocated equally to all the products purchased in that transaction by adjusting the product unit prices. For example, if a 5% discount is offered on a transaction, the reported unit prices of all the products are adjusted by 5%. Let p_{ijgry} be the discount adjusted unit price and q_{ijgry} be the reported quantity for buyer i from seller j for good g in transaction r during year y.

I define standardized unit prices at the transaction-product level $\tilde{p}_{ijgry}$ as

$$\tilde{p}_{ijgry} = ln(p_{ijgry}) - \overline{ln(p_{jgy})}, \tag{1}$$

where $\overline{ln(p_{jgy})}$ is the average log discount adjusted price for the good g of seller j in year y. I define standardized quantity at the transaction-product level $\tilde{q}_{ijgry}$ in an analogous manner.

To obtain pair-year-level values of the standardized prices and quantities, I aggregate them by the respective share of total expenditures. Define V_{ijy} as the total value of transactions between buyer i and seller j in year y. Let $s_{ijgry} = v_{ijgry}/V_{ijy}$ be the share of expenditure that good g in transaction r represents for the pair and $v_{ijgry} = p_{ijgry} * q_{ijgry}$ be the transfer value. Then, define pair-year level equivalents for the standardized prices and quantities as:

$$\tilde{p}_{ijy} = \sum_{r \in R_{ijy}} \sum_{g \in G_{ijry}} s_{ijgry} * \tilde{p}_{ijgry}, \tag{2}$$

where R_{ijy} is the set of all the transactions between i and j in year y and G_{ijry} is the set of all goods in transaction r.

In order to address the potential concern that cross-sectional differences in prices and quantities could be driven by variations in the bundles of goods purchased by buyers and over time, I present statistics on the patterns and dynamics of prices and quantities using standardized measures. The use of these measures indicates that differences in the products purchased by buyers do not influence the results.

For estimation purposes, I use the following definitions of prices and quantities, as they are better suited to the structure of the model. For total quantity q_{ijy}, I sum over all reported quantities over all goods and all transactions:

$$q_{ijy} = \sum_{r \in R_{ijry}} \sum_{g \in G_{ijry}} q_{ijgry}. \tag{3}$$

For prices, I obtain average unit price by dividing total value of transactions by total quantity:

$$p_{ijy} = V_{ijy}/q_{ijy}. \tag{4}$$

This definition of prices is consistent with the weighted average of product-level discount inclusive prices, as demonstrated in Online Appendix Figure OA-1.

The total quantity produced by seller j in year y is given by $Q_{jy} = \sum_{i \in I_{jy}} q_{ijy}$, where I_{jy} is the set of all buyers that transacted with the seller in the year. While the measures of quantities differ between the model and the motivating evidence, all motivating facts hold when using aggregate total quantities, both in the cross-section and in the short panel structure (controlling for buyer-seller pair fixed effects).

2.2.3 *Financial Statements*

I complement this information with yearly data on expenditures and wage bill from financial statements for all sellers for 2016-2017.[3]

2.3 Overview of the data

Online Appendix OA-1 provides descriptive statistics of the data. Table OA-1 shows that the sellers are typically large and well-established, with median sales of 8 million USD in 2016 and a median age of 29 years. These sellers have accounting markups of 20 percent and use directly imported goods in their production (approximately 21 percent of all inputs). They primarily channel their sales through the local market rather than exporting. On the other hand, buyers are smaller and younger, with a median sales of 200K USD in 2016 and a median age of 14 years. They have smaller accounting markups of 10 percent and have limited direct contact with international trade.

Table OA-2 shows the industrial composition of buyers by selling sector.The market segments of buyers for Textile products are 40% from Wholesale and Retail, 15% from

[3]In robustness exercises, I also use sales, exports, imports, total assets, total debt, total receivables, and total uncollectibles for all buyers and sellers in the data for 2008-2017. This data is obtained from the financial statements. I also add information on 6-digit sector code, GPS location of headquarters' neighborhood, year founded, type of ownership (multinational, local, part of a business group), and whether the buyer and seller are vertically integrated.

Manufacturing, 8% from Professional Services, 5% from Agriculture, 3% from Transportation and Storage, and the remaining 28% are spread across various other sectors. The majority of Pharmaceutical product buyers, 46%, come from Wholesale and Retail, 17% from the Human Health sector (such as hospitals and doctors), 10% from Manufacturing, 4% from Construction, 3% from Professional Services, and the rest, 20%, are distributed among other sectors. Finally, the market segments of buyers for Cement-Products are 25% from Wholesale and Retail, 20% from Construction, 16% from Professional Services, 8% from Manufacturing, 5% from Real Estate, and 26% from other sectors. This composition of buyers from different sectors is beneficial as it indicates that buyers, with the exception of those from Construction and Professional Services for Cement-Products, are likely to have linear input needs.

Online Appendix Table OA-3 presents summary statistics about quantities, values, and the number of buyers per seller obtained through the EI dataset.[4] Notice that the reporting threshold is smaller than in previous work (Bernard et al., 2019; Alfaro-Urena et al., 2022), implying a larger number of buyers. Despite the large number of buyers, the yearly bills are not small for the country, with median (average) bill of 9K USD (44K USD). At the same time, due to the staggered rollout of the policy, data is sourced from the largest firms in the economy. Indeed, the size in number of buyers and total sales of the median manufacturing firm in my sample corresponds to size of manufacturing firms between the top 5 and 10 percent in Costa Rica (Alfaro-Urena et al., 2022) and between the top 25 and 10 percent in Belgium (Bernard et al., 2019).

The median (average) buyer purchases around 1.5K (12.5K) units of product. What are these products? Table OA-4 provides information on a random sample of products, including their prices and inferred marginal costs. The prices are obtained directly from invoices, while the marginal costs are imputed by dividing the total variable costs, including wages and intermediate inputs, by the total output in units for each firm.

In the textiles industry, products may include shirts, skirts, hats, or jackets, with different patterns or sizes also considered separate products. Thus, the aggregation over quantities corresponds to individual clothing items. Although prices are aggregated over all types of products, the variation in the sample suggests a reasonable approximation. For example, a Panama Hat is sold for 34 USD, but it costs only 12 USD to manufacture, while a shirt is sold for 19 USD and costs 9.85 USD to manufacture.

In the pharmaceutical sector, products are typically packages of tablets or bottles, with the aggregation taking place over packages that would be sold to the final consumer. For example, Vitamin B Syrup is sold for 2.3 USD, but it costs only 80 cents to manufacture. In the example, we see two pairs of firms that produce multiple products, but also have different marginal costs for each product. This is because of sampling from two different

[4] Product-level variation of standardized prices and quantities is available in Supplemental Material, available on my website http://www.felipebrugues.com.

years for each firm.

In the cement-products industry, products may include stones, mortar, concrete, and the like. While aggregating over these types of products can be more challenging, it should be noted that firms producing products such as mortar do not typically produce tiles, poles, or stones. Two other annotations are in order. First, there are three different firms selling mortar at similar prices, despite being headquartered in different and distant cities. This suggests that despite the products being substitutes, sellers may still have local market power due to transportation costs. Second, one firm produces two types of pole products, sold at different prices but with the same cost of production. Another firm produces two types of concrete products, sold at different prices but with the same cost of production. As costs will enter into the dependent variable in my main estimation process, these mistakes would enter as measurement error in the econometric model.

To provide further confidence that aggregating over products does not result in significant information distortion, Figure OA-1 presents the fit between inferred prices and observed weighted prices. Inferred prices are constructed as the ratio of total sales to total quantity in the period, while observed weighted prices are obtained by aggregating unit prices using the share of total quantity of the good sold as weights. The figure shows a strong fit between the two measures, with a correlation of 0.58 at the buyer-seller-year level.

3 Motivating Evidence

This section presents evidence on how buyer-seller relationships work in the Ecuadorian supply chain. Based on the data analyzed, there are three key findings: i) Trade heavily relies on past relationships and trade-credit arrangements. ii) As relationships mature, the quantity of goods exchanged increases, while prices decrease. iii) At any given time, larger purchases are associated with lower prices. In Section 4, a long-term contract model is proposed to capture these dynamics. The model allows the seller to use price discrimination across buyers and time, and enables buyers to default on trade-credit debts without facing legal consequences.

Fact 1: Large amount of trade occurs via repeated relationships

Figure 1a demonstrates the significance of repeated relationships for the sellers included in this study. The blue bars represent the average proportion of clients by length of relationship, while the green bars indicate the average proportion of the total quantity sold. The results reveal that although roughly 35% of all buyer-seller pairs consist of new buyers, only about 10% of the total trade is conducted through these fresh relationships. In contrast, relationships that have endured for at least nine years constitute less than 10% of all pairs but contribute to over 30% of the total trade.

Fact 2: Most transactions occur via trade-credit

The EI database includes payment method information, which specifies if the seller financed the transaction and the credit terms in days. For this analysis, I only consider whether the buyer was offered trade-credit, irrespective of the terms of the agreement.[5] Figure 1b displays the average proportion, across sellers, of relationships of a certain age that involved trade-credit at some point during the year. The data reveals that the use of trade-credit is widespread, with approximately 85% of relationships receiving trade-credit in the first year of contact. By the eighth year, almost all relationships involve trade-credit at some point during the year.

This fact has two important implications. Firstly, the seller assumes a substantial portion of the risks associated with the transaction. In the absence of a strong legal enforcement framework, any opportunistic action taken by the buyer would result in the direct costs being absorbed by the seller. Secondly, the seller's opportunistic actions, such as cheating in quality or quantity, are likely to be limited (Smith, 1987). Post-delivery, the buyer may retain the value of the transaction as a guarantee of quality. Therefore, when the seller finances transactions, the terms of trade tend to favor the buyer.[6]

Fact 3: Quantities increase as relationships age

I now present empirical evidence on the life-cycle of quantities in buyer-seller relationships, which is depicted in Figure 1c. The figure shows a binscatter regression of standardized log quantities, $\tilde{q}_{ijgry}$, on dummies for different ages of relationships in the cross-section. I find that older relationships tend to purchase more of a given product within a given year than younger relationships. To ensure the robustness of these findings, I also verify the dynamics using pair-fixed effects in Online Appendix Figure OA-2a.[7]

Fact 4: Quantity discounts for a given age of relationship

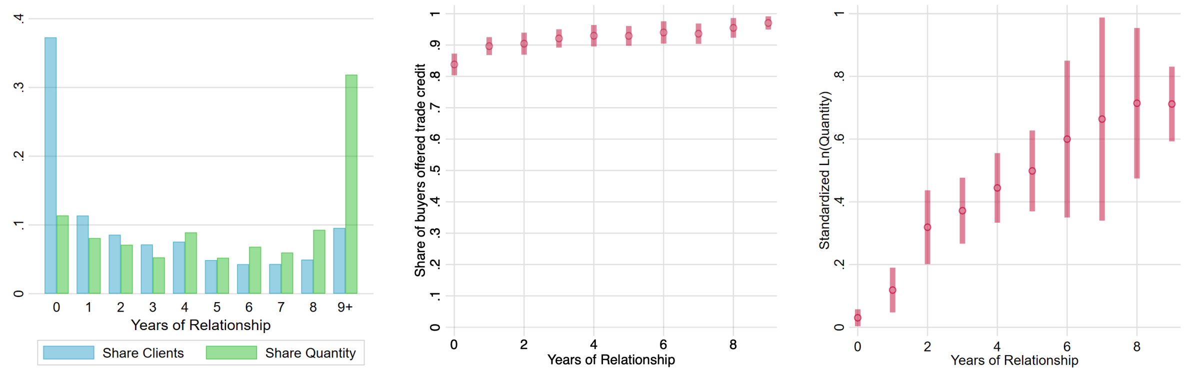
Next, I examine the link between prices and quantities. In light of the differences in the quantities sold by different manufacturers, I present quantities as quantiles, calculated

[5] On average, trade-credit agreements have a maturity of 29 days.

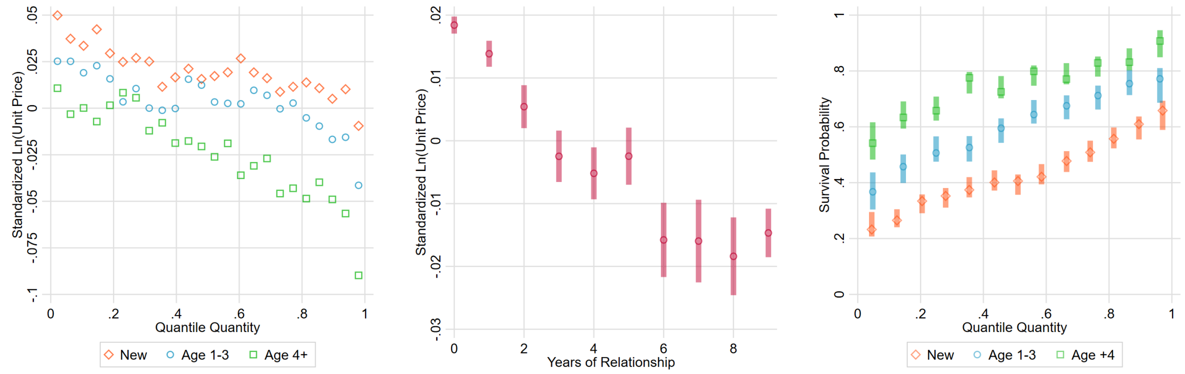
[6] It is possible that this empirical pattern is valid for the sample of large manufacturing firms in my sector, but may not hold for smaller or informal firms. To evaluate the representativeness of my type of selling firms in the entire economy, I analyze the 2010 Economic Census from Ecuador, which represents the universe of firms, including micro-enterprises. According to the census, formal firms (similar to those captured in my data) account for 45% (42%) of sales (employment) in textiles, 99% (74%) in pharmaceutical, and 82% (60%) in cement-products.

[7] Note, however, that I only observe at most two years per pair. For that reason, within-pair growth uses partial information to reconstruct the whole path of quantities. To verify that the partial panel of quantities is capturing correctly the growth of relationships, in Online Appendix Figure OA-2d, I plot the path of total value transacted in relationships using both the partial panel captured in the EI database as well as a longer panel using VAT data for years 2008-2015. To correctly measure the age of a relationship in the VAT data, I drop relationships that start during the first year that a seller appeared in the data. Moreover, to correct for partial-year effects in exit (Bernard et al., 2017), I drop the last observation available for each pair. The figure shows that under both databases, the value transacted within pairs increases as they age. Moreover, the EI database's partial panel accurately captures the full growth path observed in the VAT data.

Figure 1: **Motivating Facts**



(a) Share of Clients and Trade (b) trade-credit (c) Quantity over Time



(d) Quantity Discounts (e) Prices over Time (f) Survival Rates

Notes: Subfigure a) displays the distribution of the average of the share of clients and quantity sold by relationship age, calculated across all sellers in 2016. Sub-figure b) displays the average of the proportion of relationships that used trade-credit at some point during the year, along with a 90% confidence interval, calculated across all sellers. Subfigure c) displays the evolution of standardized log quantities, with their corresponding 90% confidence intervals, calculated across all sellers. The standardized log quantity is obtained by taking the average quantity sold in a given year for each seller-product and subtracting the log average quantity for that year. The standard errors are calculated at the seller-year level. Subfigure d) shows the relationship between quantity purchased and standardized log unit price through a binscatter plot that displays the measure of unit price against the quantity sold, based on relationship age. The standardized log unit price is obtained netting out average log unit price for that year for each seller-product. The quantiles of quantity are calculated for each seller-relationship age combination. Subfigure e) presents a binscatter plot of standardized log unit prices against years of relationship, controlling for a flexible spline of standardized log quantities. The standard errors are calculated at the seller-year level. Subfigure f) displays a bisscatter plot of the average survival rate of pairs at different ages and quantiles of quantity. The quantiles of quantities are calculated for each seller-age combination, and the error bars represent a 90% level of variation across all sellers.

within each seller and across the following relationship categories: i) new relationships, ii) relationships aged 1-3, iii) relationships aged 4 or more. To compare quality-adjusted prices, the standardized unit price by quantiles of quantity is displayed as a binscatterplot in Figure 1d. The standardization allows comparing quality-adjusted prices, as the variation is at the product-level. The results demonstrate that, regardless of the relationship's age, larger quantities attract lower quality-adjusted prices. This finding holds true when considering average unit prices and total quantities, as shown in Online Appendix Figure OA-2b. In terms of magnitude, a 10% increase in total quantity purchased is linked with an average price decrease of 2% (Online Appendix Table OA-5). Besides the evidence on concentrated market and positive markups, the existence of price discrimination is prima facie evidence of the existence of market power varian1989price.

Fact 5: For a given quantity, older relationships pay lower unit prices

Figure 1e presents the relationship between unit prices and the age of the relationship. By using a binscatter regression of standardized log prices on age of relationship dummies, controlling for a flexible spline of standardized quantities, the figure reveals that older relationships receive up to 3% more quality-adjusted additional discounts than new relationships. These discounts are economically significant as well, given accounting markups of 20%. I

Importantly, these dynamic discounts over time remain robust even after controlling for pair fixed effects (Online Appendix Figure OA-2c). In addition, I performed a replication of Figure 1e in Online Appendix Table OA-6 and discovered that the impact of relationship age on discounts remains stable even after controlling for possible omitted variables. Specifically, I controlled for various buyer characteristics such as their age, distance between headquarters, size (in sales, number of employees, assets), and whether they are multinational, exporter, importer, or part of a business group. Additionally, I accounted for the importance of the relationship for both the buyer (in terms of the supply share) and seller (in terms of demand share) to capture any potential asymmetries in bilateral market power (Dhyne et al., 2022). Moreover, the observed effect was consistent across all three industries analyzed, as evidenced in Online Appendix Table OA-7.

I interpret these findings, coupled with the phenomenon of backloading of quantities, as supporting a model with limited contract enforcement. Such a model can explain the observed price and quantity dynamics if the seller has a profit-maximizing incentive. By delaying the buyer's portion of the surplus, the seller can motivate the buyer to act in a disciplined manner, leading to the maximization of expected profits.

To further support the interpretation of limited contract enforcement as the underlying mechanism for the observed price and quantity dynamics (over alternatives such as efficiency gains or demand assurance), I explore the effect of payment modality on these patterns. This analysis, presented in Online Appendix Table OA-8, considers both cross-sectional and panel data. Specifically, I examine whether pay-in-advance buyers, who face strong incentives not to default, receive any price discounts over time. As expected, the results show that pay-in-advance buyers do not experience any discounts, as revealed in Column (1) and (3) for quality-adjusted and average prices, respectively. Although the estimates are positive, they are statistically insignificant. In contrast, trade-credit buyers (Columns 2 and 4) continue to receive discounts over time, consistent with the notion that limited contract enforcement enables the seller to discipline the buyer's behavior and maximize profits by postponing the buyer's share of the surplus.

Fact 6: Relationships that trade more are more likely to survive

Lastly, relationships are persistent. Figure 1f plots the share of relationships that survive from 2016 until 2017 by quantile of quantity in 2016 and age of relationship. The figure reports the share of new links that survive in red, in blue for links age 1-3, and in green for links age 4 or older. I find that around 40 percent of new relationships survive at least one more year, 60 percent of relationships age 1-3 survive, and more than 75 percent of relationships of 4 years or more survive. Moreover, within a relationship age, pairs that trade more volume are also more likely to survive from year to year.

4 Model of a Dynamic Contract

This section introduces the dynamic model, which serves three primary purposes. Firstly, it allows for dispersion in quantity. Secondly, it captures quantity discounts at any point in time. Thirdly, it obtains the backloading of prices and quantities. To accomplish the first two goals, I incorporate heterogeneous private information on the buyer's side into the model. To capture the backloading of prices and quantities, I include a limited enforcement constraint that prevents the buyer from defaulting on their trade-credit debts.

Preliminaries

Consider an infinitely repeated relationship between a seller (the principal) and a buyer (the agent). Time is indexed by $\tau \geq 0$ and we denote by $\delta < 1$ the common discount factor. Buyers' preferences depend on a private information match attribute (or type) θ, continuously distributed with support $[\underline{\theta}, \bar{\theta}]$, $\underline{\theta} > 0$, cumulative distribution function $F(\theta)$ and probability density function $f(\theta)$. This match attribute is drawn at the beginning of the relationship and is kept constant over time. Although the parameter is private information, the distribution $F(\cdot)$ is common knowledge.

Relationships end due to exogenous shocks that happen at every period τ with probability $X(\theta)$.[8] The exit probability $X(\cdot)$ is also common knowledge. Due to this, the type's distribution evolves over time. Define $f_\tau(\theta) = f(\theta)(1 - X(\theta))^\tau / \int (f(m)(1 - X(m))^\tau)dm$ as the probability density function for time τ and $F_\tau(\theta)$ as its corresponding density function.

A trade profile stipulates an infinite array of transfers t_τ and quantities q_τ for each time period τ, $\{t_\tau, q_\tau\}_{\tau=0}^{\infty}$.[9] The trade profile gives the following discounted payoff to the

[8]The model can accommodate for dynamic hazard rates $X_\tau(\theta)$.

[9]Throughout the next sections, I will use the terms transfers and tariffs interchangeably.

principal

$$\sum_{\tau=0}^{\infty} \delta^{\tau}(t_{\tau} - cq_{\tau}) \tag{5}$$

and to the buyer

$$\sum_{\tau=0}^{\infty} \delta(\theta)^{\tau}(\theta v(q_{\tau}) - t_{\tau}), \tag{6}$$

where $v(\cdot)$ is the base return function and $\delta(\theta) \equiv \delta(1 - X(\theta))$. I consider $v(\cdot)$ strictly increasing and strictly concave.[10]

4.1 Full Enforcement

As a benchmark, consider the case of full enforcement, both with symmetric and asymmetric information.

4.1.1 *Complete Information*

Under complete information and full enforcement, the seller acts as a monopolist practicing first-degree price discrimination implementing a stationary contract $(t^{1d}(\theta), q^{1d}(\theta))$, which is defined as

$$\theta v'(q^{1d}(\theta)) = c \quad \text{and} \quad t^{1d}(\theta) = \theta v(q^{1d}(\theta)).$$

The seller offers first-best quantities but extracts all the rents from the buyer. This allocation is infinitely repeated over time.

4.1.2 *Asymmetric Information*

The principal has commitment and wants to design a dynamic tariff scheme $t_{\tau}(\cdot)$ that maximizes their lifetime expected profit. The revelation principle applies to single-agent dynamic setups (Baron and Besanko, 1984; Sugaya and Wolitzky, Forthcoming), so there is no loss of generality in restricting the study to an infinite sequence menu $\{t_{\tau}(\theta), q_{\tau}(\theta)\}_{\underline{\theta},\overline{\theta}}$ that induces the agent to report their true type.

The theoretical insights from Baron and Besanko (1984) apply in this setup.[11] The optimal dynamic contract with full enforcement is equal to repeated Baron-Myerson static contracts with quantities determined by:

$$\theta v'(q_{\tau}^{fe}) = c - \frac{1 - F_{\tau}(\theta)}{f_{\tau}(\theta)} v(q_{\tau}^{fe}(\theta)), \tag{PE}$$

[10]This property of the buyer's return function can be micro-founded by using diminishing returns in production for one input, keeping at least one other input fixed. This assumption is common in the literature. For instance, standard production function estimation generally assumes that capital is set one year in advance (e.g., Levinsohn and Petrin, 2003).

[11]Theorem 4' offers the results for fully persistent types in an infinite horizon model.

and tariffs such that

$$t_\tau^{fe}(\theta) = \theta v(q_\tau^{fe}(\theta)) - \int_{\underline{\theta}}^{\theta} v(q_\tau^{fe}(x))dx.$$

It is possible to show that under positive selection (i.e., $X'(\theta) < 0, \forall\theta$), average and type-specific quantities *decrease* over time. Similarly, average and type-specific unit prices can be shown to increase.[12] Instead, without selection patterns (i.e., $X'(\theta) = 0, \forall\theta$), the optimal full enforcement contract with asymmetric information is stationary. Hence, with realistic selection patterns, a quantity discounting model will not be able to capture dynamic discounts over time.

4.2 Limited Enforcement

While the seller can commit fully to the long-term contract, the buyer can act opportunistically. I assume that, in each period, the seller first delivers the goods and has to wait for the buyer to transfer the promised amount before the end of the period, effectively offering trade-credit to the buyer in every transaction. While this assumption is strong, it reduces the complexity of the problem and data shown in Section 3 shows trade-credit is extremely common, if not the norm.

The direct mechanism $C(\theta) = \{q_\tau(\theta), t_\tau(\theta)\}_{\tau=0}^{\infty}$ stipulates quantities and post-delivery transfers in each period for agent reporting type θ. The seller offers the menu of $\{\theta, C(\theta)\}_{\underline{\theta},\bar{\theta}}$, with combinations of available reporting types and corresponding allocations.

4.2.1 *Timing*

The contracting game takes places in the following order:

1. Prior to trade, at $\tau = 0$, the buyer observes their private type θ. The seller offers the mechanisms menu $\{C(\theta)\}$. The buyer either accepts or rejects the offer. If they accept, they report type $\widehat{\theta}$. If they reject, both the seller and buyer receive their outside options, normalized to 0.[13]

2. In each trading period $\tau \geq 0$:
   - The seller produces and delivers $q_\tau(\widehat{\theta})$.
   - The post-delivery payment $t_\tau(\widehat{\theta})$ is paid by the buyer, or they breach the contract.
   - Following a breach on the buyer's side, the contract is terminated.

[12] With positive selection, informational rents given to middle-types decrease, as the distribution is shifting towards higher-types $F_\tau(\theta) > F_{\tau+1}(\theta)$. In order to incentivize the highest types still active, middle-types will be distorted downwards in the future. Marginal unit prices are given by $p(q(\theta)) = c + (1 - F_\tau(\theta)/f_\tau(\theta)$ (Armstrong, 2016), which will be generally larger for each θ, and as such, average price will be larger at each q.

[13] For the buyer, this normalization is not restrictive if they use a production function that mixes the inputs linearly or if the supplier is a true monopolist. For the seller, the normalization is not restrictive if there are no scale economies.

As it will made clear below, the contract considered is default-free, through the use of enforcement constraints, and features no renegotiation. Since default never occurs in equilibrium, there is no loss in assuming that the seller terminates trade following a breach (Abreu, 1988; Levin, 2003).

4.2.2 *Constraints*

Let us now characterize the set of constraints in the main problem. The set of constraints contain the usual individual rationality and incentive compatibility constraints of adverse selection problems. This setting's novelty is to include an additional enforcement constraint in each trading period, which acts as an endogenously determined participation constraint. Each of the enforcement constraints will ensure the buyer will not default in the specific time period.

Buyer's Incentive Compatibility

Under the assumption of perfectly persistent types, as in Martimort et al. (2017), incentive compatibility requires that the agent evaluates their lifetime return:

$$\sum_{\tau=0}^{\infty} \delta(\theta)^{\tau} u_{\tau}(\theta) \geq \sum_{\tau=0}^{\infty} \delta(\theta)^{\tau} [\theta v(q_{\tau}(\widehat{\theta})) - t_{\tau}(\widehat{\theta})] \quad \forall \theta, \widehat{\theta}, \tag{IC-B}$$

where $u_{\tau}(\theta) = \theta v(q_{\tau}(\theta)) - t_{\tau}(\theta)$.

Buyer's Limited Enforcement Constraint

The key friction in the model is the limited enforcement of the trade-credit contracts, which allows for the possibility of buyer's default. Under the assumption of contracting termination following a breach, a *default-free* menu satisfies the limited enforcement constraint of the buyer:

$$t_{\tau}(\theta) \leq \sum_{s=1}^{\infty} \delta(\theta)^{s} u_{s}(\theta) \quad \forall \theta, \tau. \tag{LE-B}$$

The condition requires that the costs of breaking the relationship, in terms of the forgone opportunities of trade, have to be greater than the benefits from breaching the contract.

The buyer's LE-B constraint at $\tau = 0$ implies the individual rationality constraint required for buyer participation in trade.[14] For that reason, only LE-B and IC-B are considered. From this, it follows that ex-ante trade under limited enforcement should leave participating buyers weakly better than under perfect enforcement whenever the seller has the bargaining power.

[14] A mechanism C is individually rational if the participation constraint at $\tau = 0$ holds: $\sum_{\tau=0}^{\infty} \delta(\theta)^{\tau}(u_{\tau}(\theta)) \geq 0 \quad \forall \theta$. To see how LE-B implies this, add $u_0(\theta)$ on both sides and note that $u(\theta) + t_{\tau}(\theta) = \theta v(q_{\tau}(\theta)) > 0$.

4.2.3 *Optimal Contract with Limited Enforcement*

Denote total surplus as $s(\theta, q, c) = \theta v(q) - cq$. The principal's problem becomes

$$\max_{\{u_\tau(\theta), q_\tau(\theta)\}} \sum_{\tau=0}^{\infty} \delta^\tau \int_{\underline{\theta}}^{\overline{\theta}} [s(\theta, q_\tau(\theta), c) - u_\tau(\theta)] f_\tau(\theta) d\theta, \tag{SP}$$

such that IC-B and LE-B are satisfied. That is, the objective of the seller is to maximize total surplus while reducing the share of surplus given to the buyer as much as possible without violating the constraints.

The solution in a static setting (e.g., in Jullien (2000)) follows the first-order approach of Mirrlees (1971), which substitutes the global incentive compatibility constraint with a local one. Recent results in dynamic mechanism design (Pavan et al., 2014; Battaglini and Lamba, 2019) show that a dynamic envelope theorem for the relaxed problem can be used to characterize under certain conditions the global solution to the full contract. In particular, Battaglini and Lamba (2019) argue that if types are fully persistent, strictly monotonic contracts (i.e., those with $q'_\tau(\theta) > 0$ for all θ and τ) will be globally incentive compatible. Throughout this section, I will assume that allocated quantities satisfy this monotonicity property.

Following Pavan et al. (2014), an implementable menu satisfies dynamic incentive-compatibility if it satisfies the dynamic envelope formula:

$$\sum_{\tau=0}^{\infty} \delta(\theta)^\tau u'_\tau(\theta) = \sum_{\tau=0}^{\infty} \delta(\theta)^\tau v(q_\tau(\theta)), \tag{7}$$

for any arbitrary $0 < \delta(\theta) < 1$ function and $u'_\tau(\theta) \equiv du_\tau(\theta)/d\theta$. Substituting the envelope condition 7 with $\delta(\theta) = \delta$ into the seller's problem SP yields

$$\sum_{\tau=0}^{\infty} \delta^\tau \int_{\underline{\theta}}^{\overline{\theta}} [s(\theta, q_\tau(\theta), c) - \int_{\underline{\theta}}^{\theta} v(q_\tau(x)) dx] f_\tau(\theta) d\theta - \sum_{\tau=0}^{\infty} \delta^\tau u_\tau(\underline{\theta}). \tag{8}$$

The return term of the buyer acknowledges the rents that have to be given to higher types in order to preserve incentive compatibility.

I follow Jullien (2000) and write the problem in Lagrangian-type form. For this formulation, the dynamic LE-B constraint at time τ is given by:

$$\int_{\underline{\theta}}^{\overline{\theta}} \{\sum_{s=1}^{\infty} \delta^s (1 - X(\theta))^s u_{\tau+s}(\theta) - [\theta v(q_\tau(\theta)) - u_\tau(\theta)]\} d\Gamma_\tau(\theta) = 0, \tag{Langrangian-D-LE}$$

where $\Gamma_\tau(\theta) = \int_{\underline{\theta}}^{\theta} \gamma_\tau(x) dx$ is the cumulative LE multiplier with derivative $\gamma_\tau(\theta)$. The derivative $\gamma_\tau(\theta) > 0$ whenever the limited enforcement constraint binds and it captures the shadow value of the enforcement constraint at θ. The cumulative multiplier $\Gamma_\tau(\theta)$ captures the extent by which trade is distorted by limited enforcement. It represents the shadow value of relaxing the enforcement constraints uniformly from $\underline{\theta}$ to θ, capturing the amount of profits lost by the seller due to enforcement incentives. As extending θ

increases the set on which the enforcement constrained is relaxed, Γ_τ is nonnegative and nondecreasing. Moreover, by relaxing uniformly the constraints, the seller can reduce the buyers' net returns by keeping quantities unchanged, so $\Gamma_\tau(\overline{\theta}) = 1$.[15] Thus, the cumulative multiplier has the properties of a cumulative distribution function.

After manipulating the limited enforcement constraints,[16] one can obtain the full Lagrangian maximand:

$$\sum_{\tau=0}^{\infty} \delta^\tau \int_{\underline{\theta}}^{\overline{\theta}} [s_\tau(\theta, q_\tau(\theta)) - v(q_\tau(\theta)) \frac{\Gamma_\tau(\theta) - F_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s^\tau(\theta))\tilde{\Gamma}_s^\tau(\overline{\theta}) + \theta\gamma_\tau(\theta)}{f_\tau(\theta)}] f_\tau(\theta) d\theta, \tag{10}$$

with the corresponding slackness condition Langrangian-D-LE and where $\Gamma_s^\tau(\theta)$ is the conditional cumulative LE multiplier constraint defined by

$$\Gamma_s^\tau(\theta) = \frac{\int_{\underline{\theta}}^{\theta} (1 - X(x))^{\tau-s} \gamma_s(x) dx}{\tilde{\Gamma}_s^\tau(\overline{\theta})}, \tag{11}$$

for $\tilde{\Gamma}_s^\tau(\overline{\theta}) = \int (1 - X(\theta))^{\tau-s}\gamma_s(\theta) d\theta$. The conditional cumulative multiplier constraint adjusts for the likelihood that a given θ has survived $\tau - s$ periods, assigning lower weights to θ's that are less likely to survive.

The corresponding seller's first order condition determining the allocation rule at any relationship tenure τ is:

$$\theta v'(q_\tau(\theta)) - c = \frac{\Gamma_\tau(\theta) - F_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s^\tau(\theta))\tilde{\Gamma}_s^\tau(\overline{\theta}) + \theta\gamma_\tau(\theta)}{f_\tau(\theta)} v'(q_\tau(\theta)). \tag{SFOC}$$

The allocation equation responds to intuitive forces. For expositional purposes, assume that the breakup probability is zero for all types, $X(\theta) = 0$ for all θ. Then, $\Gamma_s^\tau(\theta) = \Gamma_s(\theta)$, $\tilde{\Gamma}_s^\tau(\overline{\theta}) = 1$, $F_\tau(\theta) = F(\theta)$, $f_\tau(\theta) = f(\theta)$. Assume as well that $v(q) = kq^\beta$. The equation can be written as:

$$q_\tau(\theta)^{1-\beta} = \overbrace{\frac{k\beta}{c}}^{\text{Inv. } \mu} \Big[\overbrace{\theta - \frac{1-F(\theta)}{f(\theta)}}^{\text{Virtual Surplus}} - \overbrace{\frac{\theta\gamma_\tau(\theta)}{f(\theta)}}^{\text{LE}} + \overbrace{\frac{(1-\Gamma_\tau(\theta))}{f(\theta)}}^{\text{LE+IC}} + \overbrace{\frac{\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))}{f(\theta)}}^{\text{Past LE + IC}} \Big] \tag{Q-CES}$$

which resembles the usual solution to an adverse selection problem in which the allocation is determined by an inverse markup (μ) rule adjusted by the *modified virtual surplus*,

[15] In Online Appendix Section OA-4, I show formally that $\Gamma_\tau(\overline{\theta}) = 1$.

[16] Pre-multiply each constraint by δ^τ and sum over τ. Reorder internal summations, substitute in the dynamic envelope condition, and eliminate constant terms to obtain:

$$\sum_{\tau=0}^{\infty} \delta^\tau \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} v(q_\tau(x)) dx \sum_{s=0}^{\tau-1} (1 - X(\theta))^{\tau-s} d\Gamma_s(\theta)$$
$$- \sum_{\tau=0}^{\infty} \delta^\tau \int_{\underline{\theta}}^{\overline{\theta}} [\theta v(q_\tau(\theta)) - \int_{\underline{\theta}}^{\theta} v(q_\tau(x)) dx] d\Gamma_\tau(\theta). \tag{9}$$

Then integrate by parts.

which accounts for necessary rents due to incentive compatibility and due to the limited enforcement constraint.

In particular, the incentive compatibility constraint forces the seller to give higher quantities to higher types through $F(\theta)$ as informational rents. Moreover, when the current limited enforcement constraint is binding ($\gamma_\tau(\theta) > 0$), it limits the volume of trade. Keeping the future stream of quantities constant, if the buyer is on the verge of defaulting, the seller needs to reduce tariffs now. But, given profit maximizing incentives, the seller must also decrease quantities. Hence, enforcement concerns decrease contemporaneous quantities. Yet, at the same time, a countervailing force exists: to preserve incentive compatibility and prevent low-types from pretending to be higher-types, quantities are uniformly shifted upwards by $1 - \Gamma_\tau(\theta)$.

One key difference from Jullien (2000) is the inclusion of past cumulative multipliers ($\sum_{s=0}^{\tau-1}(1 - \Gamma_s(\theta))$), which generate the backloading of quantities. This multiplier acts as a promise-keeping constraint, where types whose limited enforcement constraint was binding in the past receive higher quantities in the present. A similar result is presented in Martimort et al. (2017) for a discrete number of types. With exogenous exit ($X > 0$), promises made in the distant past have less weight now. However, if relationships never end, promises made in the past will affect trade levels forever, as in Marcet and Marimon (1992).

The combination of $\Gamma_\tau(\theta)$, $\Gamma_s(\theta)$, and $\theta\gamma_\tau(\theta)$ at equilibrium determines whether the allocated quantity is greater or lower than what it would be under full enforcement. Additionally, as is typical, the amount of allocated quantities decreases as the markup that a seller would charge under linear monopolist pricing increases.

The results of Pavan et al. (2014) allow us to construct the transfers $t_\tau(\theta)$ satisfying the necessary first-order conditions with the corresponding allocation rule specified in SFOC. Specifically, if the contract allocation satisfies a strict monotonicity assumption,[17] the following transfer rule satisfies the buyer's dynamic envelope formula:

$$t_\tau(\theta) = \theta v(q_\tau(\theta)) - \int_{\underline{\theta}}^{\theta} v(q(x))dx - u_\tau(\underline{\theta}), \tag{t-RULE}$$

and so the derivative of the transfer rule with respect to type is

$$t_\tau'(\theta) = \theta v'(q_\tau(\theta))q_\tau'(\theta). \tag{t-RULEp}$$

4.2.4 *Non-Stationary Equilibrium*

This paper aims to learn about the primitives of the model by using data, rather than characterizing the full optimal dynamic contract. However, it's worth noting that I prove the optimal contract cannot be stationary in Online Appendix OA-3 through two steps. First, I prove that a unique stationary equilibrium exists. Second, I demonstrate that a

[17] Pavan et al. (2014) use a weaker assumption, integral monotonicity, which is implied by the strict monotonicity assumption.

non-stationary deviation exists that dominates the stationary equilibrium. As a result, if an optimal contract exists, it must be non-stationary.

4.2.5 *Model Dynamics and Static Efficiency*

In Online Appendix Section OA-5, I discuss the model dynamics and efficiency in more detail. Specifically, I provide sufficient conditions for observing backloading of quantities and prices, as well as cross-sectional quantity discounts. Additionally, I demonstrate that the model eventually reaches a stationary long-term equilibrium that may be either efficient or inefficient, potentially resulting in under-consumption or over-consumption. This means that the model does not necessarily imply long-term efficiency. Next, I compare the efficiency of the limited enforcement contract to that under perfect enforcement. Despite the model's theoretical ambiguity, long-term relational contracts could generate higher levels of efficiency than allocations under perfect enforcement.[18]

5 Identification and Estimation of Dynamic Contracts

In this section, I will discuss the identification of the model primitives θ, $v(\cdot)$, and $\Gamma_\tau(\cdot)$. To derive a key identifying equation that maps data into primitives, I will use the first-order condition of the seller and the rate of change of the tariff function captured by the variation in prices. I will present the necessary assumptions to derive the equation and demonstrate how to use it to identify all the required elements for welfare analysis. Additionally, I will provide a detailed discussion regarding the identification assumptions and offer guidance on how misspecification of the model may affect the results. Finally, I will describe an estimation procedure that relies on the main identification equation to recover the model primitives from the available data.

5.1 Identification

For each seller in a given year, the observables are unit prices $p_\tau(q)$ (or transfers $t_\tau(q)$) and quantities q_τ for different buyers with relationship age τ, as well as marginal costs c. Throughout this section, I abstract away from the possibility of exogenous breakups. The possibility of breakups will be reintroduced in estimation.[19]

As shown in Section 4, the dynamic contract is a complex object. Rather than deriving the full equilibrium contract by forward-iteration, I rely on the following assumption.

Identification Assumption 1. *Each seller offers a unique menu of dynamic contracts to all buyers, and such menu satisfies equations SFOC and t-RULEp for all θ and τ.*

[18] To see an example, please refer to Supplemental Material Section SM4, which presents a solved two-type example illustrating these features.

[19] As exogenous breakups can be directly estimated in the data, they would be treated as known during identification. Their inclusion would only add complexity in the notation without adding substantial insights regarding identification.

Under assumption 1, I can collapse all information about future unobserved quantities and transfers into the limited enforcement multipliers. The seller is aware of the solution and the future promises —the first order conditions are consistent with them— and I exploit this knowledge to learn about enforcement distortions. Although the assumption is strong, it is often used in the identification of dynamic games, as these types of games may have multiple equilibria (Aguirregabiria and Nevo, 2013).

Identification Assumption 2. *Within each period, quantity increases strictly monotonically with type θ: $q'_\tau(\theta) > 0$.*

Assumption 2 directly links observed quantities with underlying unobserved types, i.e., one can infer that if a buyer purchases higher quantities, they must have better match-quality with the seller. Moreover, we can deduce the distribution of types from the distribution of quantities, $F_\tau(\theta(\alpha)) = \alpha$, for quantile α. This also implies $f_\tau(\theta(\alpha)) = 1/\theta'_\tau(\alpha)$ through the chain-rule. Lastly, we can derive the following relationship, $\Gamma_\tau(\theta_\tau(\alpha)) = \Gamma_\tau(\alpha)$ and $\gamma_\tau(\theta(\alpha))\theta'_\tau(\alpha) = \gamma_\tau(\alpha)$.

Identification Assumption 3. *Types θ are fully persistent.*

Assumption 3 allows us to link past estimated multipliers $\Gamma_s(\theta)$ for $s < \tau$ over time. Specifically, the assumption, in conjunction with Assumption 2, implies that buyers at the quantity quantile α at time τ correspond to multipliers of buyers at the quantity quantile α at time s, $\Gamma_s(\alpha)$. Access to a long-term panel would enable the relaxation of this assumption, as the econometrician could track, for each buyer b, past estimated multipliers $\Gamma_s(\theta_{s,b})$, even when the types change over time ($\theta_{s,b} \neq \theta_{\tau,b}$). Unfortunately, my short panel of two years only permits the tracking of multipliers for one year in the past, and only for buyers who have just begun trading, rather than buyers in long-term relationships.

Deriving the Key Identification Equation

Exploiting the fact that the mapping from agent type θ to quantity q_τ is strictly monotone (Assum. 2), one can write the first-order condition of the seller SFOC and the derivative of the transfer rule of the buyer t-RULE in terms of quantiles α (Luo et al., 2018; Luo, 2018):

$$\theta_\tau(\alpha)v'(q_\tau(\alpha)) - c =$$

$$\Big[\Gamma_\tau(\alpha) - \alpha - \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\alpha)) + \frac{\theta_\tau(\alpha)}{\theta'_\tau(\alpha)}\gamma_\tau(\alpha)\Big]\theta_\tau(\alpha)v'(q_\tau(\alpha))\frac{\theta'_\tau(\alpha)}{\theta_\tau(\alpha)},$$

$$T'_\tau(q_\tau(\alpha)) = \theta_\tau(\alpha)v'(q_\tau(\alpha)),$$

where $\alpha \in [0,1]$ and I used the fact that observed price schedule can be mapped to the model transfer schedule by $T_\tau(q_\tau(\theta(\alpha))) = t_\tau(\theta(\alpha))$. Moreover, $\theta_\tau(\alpha)$ and $q_\tau(\alpha)$ are the

α-quantiles of the agent's type and quantity at tenure τ, respectively. Notice as well that I have used $f_\tau(\theta(\alpha)) = 1/\theta'_\tau(\alpha)$ and $\gamma_\tau(\theta_\tau(\alpha)) = \gamma_\tau(\alpha)/\theta'_\tau(\alpha)$ (Assum. 2) and linked multipliers $\Gamma_s(\alpha)$ with the buyer's current quantitle α (Assum. 3).

Together, the key identification equation becomes:

$$\frac{T'_\tau(q_\tau(\alpha)) - c}{T'_\tau(q_\tau(\alpha))} = \frac{\theta'_\tau(\alpha)}{\theta_\tau(\alpha)}\Big[\Gamma_\tau(\alpha) - \alpha - \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\alpha))\Big] + \gamma_\tau(\alpha), \tag{I:EQ}$$

where $\theta_\tau(\cdot)$, $\theta'_\tau(\cdot)$, $\Gamma_\tau(\cdot)$, and $\gamma_\tau(\cdot)$ are unknown. This equation will be the base for the identification of all unknown functions. Assumption 1 implies I can use cross-sectional variation across cohorts to learn about past enforcement multipliers, as well as stating that variation in prices and quantities across buyers reflect variation in buyer type θ rather than differences in contracts.

Note that the price schedule $T_\tau(\cdot)$ and its derivatives are nonparametrically identified from information on prices and quantities alone, so in this section, I treat them as known. Moreover, I treat c as known, as I can back-out average cost (across all product varieties) by using information on total variable costs and total seller output.

Set Identification of the Limited Enforcement Multiplier $\Gamma_\tau(\theta)$

The identification argument is recursive and takes the primitives at time $s < \tau$, and in particular, $\Gamma_s(\alpha)$, as known (mapped over time using assumptions 2 and 3).

Define $\Xi_\tau(\alpha) = \Gamma_\tau(\alpha) + \theta_\tau(\alpha)/\theta'_\tau(\alpha)\gamma_\tau(\alpha)$. Substituting in and reordering, equation I:EQ becomes:

$$\Xi_\tau(\alpha) = \alpha + \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\alpha)) + \frac{T'_\tau(q_\tau(\alpha)) - c}{T'_\tau(q_\tau(\alpha))}\frac{\theta_\tau(\alpha)}{\theta'_\tau(\alpha)}. \tag{12}$$

As $\theta_\tau(\alpha) > 0$ and $\theta'_\tau(\alpha) > 0$ (Assum. 2), then $\Xi_\tau(\alpha)$ is set identified. In particular,

$$\Xi_\tau(\alpha) \in \begin{cases} [0, \alpha - \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\alpha))) \text{ if } T'_\tau(q_\tau(\alpha)) < c, \\ [\alpha - \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\alpha)), 1] \text{ if } T'_\tau(q_\tau(\alpha)) \geq c, \end{cases}$$

where $\Gamma_s(\alpha)$ is assumed to be known. The set identification condition presented has an intuitive interpretation. Suppose a new buyer enters at time period $\tau = 0$. If we observe that the seller's marginal revenue is lower than the marginal cost, $T'_\tau(q_\tau(\alpha) < c$, this implies overconsumption relative to the first-best. In order for this to occur, the enforcement constraints embedded in $\Xi_0(\alpha)$ must put greater upward pressure relative to the downward distortions from the asymmetric information incentives, and thus $\Xi_0(\alpha) < \alpha$. Therefore, information on marginal revenues vis-a-vis marginal costs allows us to identify whether enforcement constraints distort consumption upward or downward relative to the traditional informational constraint.

Given set identification of $\Xi_\tau(\alpha)$, $\Gamma_\tau(\alpha)$ is also set identified after defining a boundary condition.

Identification Assumption 4. *For quantile $\alpha = 1$, $\Gamma_\tau(1)$ is known for all τ.*

Given knowledge of $\theta_\tau(\alpha)/\theta'_\tau(\alpha)$, this boundary condition implies there is a solution to the differential equations defined in $\Xi_\tau(\alpha) = \Gamma_\tau(\alpha) + \theta_\tau(\alpha)/\theta'_\tau(\alpha)\gamma_\tau(\alpha)$. In Online Appendix OA-4, I show this boundary condition is $\Gamma_\tau(1) = 1$ for all τ.

Then note that for every value $\xi_\tau(\alpha) \in \Xi_\tau(\alpha)$, there is a *unique* known value for $\theta_\tau(\alpha)/\theta'_\tau(\alpha)$ that is consistent with equation 12. Therefore, for quantile α and for each value $\xi(\alpha)$ of the set $\Xi_\tau(\alpha)$, $\Gamma^\xi_\tau(\alpha)$ is identified from the solution to the differential equation:

$$\gamma^\xi_\tau(\alpha) + \Gamma^\xi_\tau(\alpha)\frac{\theta_\tau(\alpha)}{\theta'_\tau(\alpha)} = \xi(\alpha)\frac{\theta_\tau(\alpha)}{\theta'_\tau(\alpha)}, \tag{13}$$

with the boundary condition $\Gamma_\tau(1) = 1$ for all τ and the known value $\theta_\tau(\alpha)/\theta'_\tau(\alpha)$. For that reason, $\Gamma_\tau(\cdot)$ is set identified from the union of $\Gamma^\xi_\tau(\cdot)$ for all $\xi \in \Xi_\tau(\cdot)$.[20]

Point Identification of the Limited Enforcement Multiplier $\Gamma_\tau(\theta)$

I now show stronger identification results by making a parametric assumption on the return function.[21]

Identification Assumption 5. *The return function is of the form $v(q) = kq^\beta$, for $k > 0$ and $\beta \in (0,1)$.*

Appendix A provides the details on how Assum. 5 provides point identification of $\Gamma_\tau(\theta)$.

Generally speaking, the set identification issue arises in equation I:EQ as both the ratio $\theta'(\alpha)/\theta(\alpha)$ and the multipliers are unknown. Although there are unique mappings between those two elements, they are also many such joint mappings. Yet, without parametrizing, one can show that the ratio $\theta'(\alpha)/\theta(\alpha)$ will be equal to $T''(q(\alpha))/T'(q(\alpha)) - v''(q(\alpha))/v'(q(\alpha))$. Hence from equation 12 above, $\Xi_\tau(\alpha)$ and consequently $\Gamma_\tau(\alpha)$ are point identified up to $-v''(q(\alpha))/v'(q(\alpha))$. Thus, providing further structure on the curvature

[20] If $\Gamma_s(\alpha)$ is taken to be a set, then the identification set for $\Xi_\tau(\alpha)$ should be defined as:

$$\Xi_\tau(\alpha) \in \begin{cases} [0, \alpha - \sum_{s=0}^{\tau-1}(1 - \Gamma^{SUP}_s(\alpha))) \text{ if } T'_\tau(q_\tau(\alpha)) < c, \\ [\alpha - \sum_{s=0}^{\tau-1}(1 - \Gamma^{INF}_s(\alpha)), 1] \text{ if } T'_\tau(q_\tau(\alpha)) \geq c, \end{cases}$$

where $\Gamma^{SUP}_s(\alpha)$ is the supremum and $\Gamma^{INF}_s(\alpha)$ is the infimum in identified set for $\Gamma_s(\alpha)$. Although the bounds for $\Xi_\tau(\alpha)$ are wider, the identification argument for $\Gamma_\tau(\cdot)$ remains unchanged. For every value $\Xi_\tau(\alpha)$ and $\sum_{s=0}^{\tau-1}(1 - \Gamma_s(\alpha))$, there is a unique value for $\theta_\tau(\alpha)/\theta'_\tau(\alpha)$. Therefore, for each combination of $\{\Xi_\tau(\alpha), \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\alpha))\}_{\alpha\in[0,1]}\}$, $\Gamma_\tau(\theta)$ is identified from the solution to the differential equation 13.

[21] Luo (2018) studies the nonparametric identification of this model and of $\Gamma_\tau(\alpha)$ in particular with observations on prices and quantities alone. They find that this model is nonparametrically identified if one can find an alternative efficient market, for which $\Gamma_\tau(\alpha) = 1$ for all α, in order to learn about $\theta'_\tau(\alpha)/\theta_\tau(\alpha)$. With information on $\theta'_\tau(\alpha)/\theta_\tau(\alpha)$ in hand, $\Gamma_\tau(\alpha)$ is nonparametrically identified from information on transfers and prices alone. This approach is not feasible in my setting as each seller is considered a market, and it is impossible to find something that could be regarded as an alternative efficient market for each seller.

of the return function ($v''(q(\alpha))/v'(q(\alpha))$) will also provide direct information on which ratios $\theta'(\alpha)/\theta(\alpha)$ are consistent with prices ($T''(q(\alpha))/T'(q(\alpha))$).

By parametrizing $v(q)$ under Assum. 5, the ratio $-v''(q(\alpha))/v'(q(\alpha))$ becomes $(1-\beta)/q$, which depends only on one parameter. he intuition behind this identification is twofold, and it rests on the fact that the multipliers are identified for the highest type without additional assumptions. Firstly, any distortions to the consumption of the highest type reflect only enforcement constraints. If such constraints were absent, the consumption of the highest type would be efficient. Therefore, any gap in marginal revenue and marginal cost for the highest type *directly* identifies the enforcement multiplier $\gamma_0(1)$ for $\tau = 0$ and $\alpha = 1$. Secondly, the knowledge of all enforcement multipliers at $\alpha = 1$ allows for the approximation of the identification equation for an arbitrary quantile $1-\varepsilon$, for a small enough ε, through a Taylor expansion. As a result, the identification equation for such quantile $1-\varepsilon$ will have only one unknown β, which is now identified on observations of quantities, prices, and marginal cost.[22] Thus, the difference in marginal prices for $1-\varepsilon$ relative to the highest type will inform about the degree of dispersion required to satisfy incentive compatibility due to changes in the marginal return for the buyer, and thus informative about the curvature of the return function (β).

I build on the identification approach of Attanasio and Pastorino (2020) while also making two novel contributions. Firstly, I establish clear set identification bounds for enforcement multipliers without any reliance on parametric assumptions. Secondly, I offer point identification results though a parametrization, even in cases where the multipliers may vary. In contrast, Attanasio and Pastorino (2020) only provide point identification results for the case of constant multipliers, and address identification concerns by using a parametrized multiplier function.

For estimation, nonetheless, I follow the approach of Attanasio and Pastorino (2020), which consider a parametrization of $\Gamma_\tau(\cdot)$ as a flexible function of q_τ rather than parametrizing $v(\cdot)$. Throughout the remainder of the section, I consider $\Gamma_\tau(\cdot)$ as point identified.

Identification of Types θ

Using the allocation equation at τ and the fact that $\partial \ln(\theta_\tau(\alpha))/\partial\alpha = \theta'_\tau(\alpha)/\theta_\tau(\alpha)$, I obtain the following expression for $\theta_\tau(\alpha)$:

$$\ln(\theta_\tau(\alpha)) = \ln(\underline{\theta}_\tau) + \int_\tau^\alpha \frac{\partial \ln(\theta_\tau(x))}{\partial x} dx \tag{14}$$

$$= \ln(\underline{\theta}_\tau) + \int_0^\alpha \frac{1}{\Gamma_\tau(x) - x - \sum_{s=0}^{\tau-1}(1-\Gamma_s(x))} \Big[\frac{T'_\tau(q_\tau(x)) - c}{T'_\tau(q_\tau(x))} - \gamma_\tau(x)\Big] dx, \tag{15}$$

which identifies the quantile function of type $\theta_\tau(\cdot)$ up to $\underline{\theta}_\tau$ and $\Gamma_\tau(\alpha)$. Thus, it necessary to make a scale normalization.

[22] Notice this argument can be extended to show that without any additional assumptions, the enforcement multipliers at $\alpha = 0$ are also identified. Thus, permitting the identification of an additional higher-order parameter on q.

Identification Assumption 6. $\underline{\theta}_\tau = 1$.

Making the scale normalization on types $\underline{\theta}_\tau \equiv \underline{\theta} = 1$, the quantile function for types becomes:

$$\theta_\tau(\alpha) = \exp\Big(\int_0^\alpha \frac{1}{\Gamma_\tau(x) - x - \sum_{s=0}^{\tau-1}(1-\Gamma_s(x))} \Big[\frac{T'_\tau(q_\tau(x)) - c}{T'_\tau(q_\tau(x))} - \gamma_\tau(x) \Big] dx \Big). \quad (16)$$

As Luo et al. (2018) show, $\underline{\theta} = 1$ is a normalization for a general function $v(\cdot)$. Under a parametrization $v(q) = kq^\beta$, which provides point identification for $\Gamma_\tau(\cdot)$, $\underline{\theta} = 1$ is also a normalization as it suffices to multiply k by the normalization constant to obtain an observationally equivalent structure. Hence, the distribution of θ is semi-parametrically identified.

The distribution $f_\tau(\theta)$ is identified from $\theta'_\tau(\alpha)$ since $f_\tau(\theta) = 1/\theta'_\tau(\alpha)$ and $\theta'_\tau(\alpha)$ is obtained from

$$\theta'_\tau(\alpha) = \frac{\theta_\tau(\alpha)}{\Gamma_\tau(\alpha) - \alpha - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\alpha))} \Big[\frac{T'_\tau(q_\tau(\alpha)) - c}{T'_\tau(q_\tau(\alpha)} - \gamma_\tau(\alpha) \Big]. \quad (17)$$

Identification of the Base Return Function $v(\cdot)$

The base return function $v(q_\tau(\alpha))$ is identified in two steps under the assumed parametrization. First, the elasticity β is identified from observations of quantities, prices, and marginal costs, as detailed in Appendix A. The level shifter k is identified using the derivative of the transfer rule $T'_\tau(q_\tau(\alpha)) = \theta_\tau(\alpha)v'(q_\tau(\alpha)) = \theta_\tau(\alpha)k\beta q_\tau(\alpha)^{\beta-1}$, as $\theta_\tau(\alpha)$ and β are identified, while $q_\tau(\alpha)$ and $T'_\tau(q_\tau(\alpha))$ are known.

5.2 Discussion

In my methodology, I leverage the fact that the seller knows the optimal contract solution, which must satisfy the first-order conditions of both the seller and the buyer. Observable quantity distortions relative to an unconstrained world are then used to infer informational and enforcement rents. This approach offers several benefits, as it allows me to identify and estimate the model without needing to solve it fully. A full-solution approach would require repetitive forward iteration of the model for each specific type, which could become computationally burdensome and limit the dimension of the type-space.

This assumption also proves useful in cases of model mispecification. Specifically, it permits the buyer to have outside options that the econometrician cannot observe, as long as the seller is aware of them, and these outside options will be accounted for in the enforcement constraints. Although the econometrician may not be able to distinguish the effects of outside options from those of future promises, this does not create any identification issues for the welfare analysis primitives.[23]

[23] Mispecification of the model will impact the equilibrium transfer solution. For example, if buyers have a constant outside option, equilibrium transfers will be lower by the value of the outside option.

One important limitation of this approach is that it cannot address counterfactuals involving dynamic quantities, since solving for such quantities would require iterative solution methods. Nevertheless, this methodology provides valuable insights for understanding the efficiency of *actual* trade, which is the focus of this paper.[24]

The methodology also implicitly relies on the commitment of the seller to the mechanism. Given full persistence of types, starting in the second period, the seller knows the asymmetric information of the buyer, but does not change the mechanism to improve their profits by reducing the informational rents given to the buyers. This assumption is arguably strong in a setting where one of the parties can defect.

Despite this disadvantage, I consider the commitment solution as relevant for three reasons. First, the commitment solution is a natural benchmark. Indeed, it is often used in model of sovereign default (Dovis, 2019), firm dynamics (Roldan-Blanco and Gilbukh, 2021), and risk-sharing in developing countries (Ábrahám and Laczó, 2018). As such, the new identification results for dynamic contracts have broad potential applications. Second, solving dynamic mechanism problems with limited commitment on the principal's side is complex. While recent advances have been made in addressing this issue Doval and Skreta (2020), introducing limited commitment would require the addition of new sequential rationality constraints, which could generate identification issues in both constraints. Lastly, the concerns associated with the commitment problem can be mitigated by obtaining longer panel data that tracks the evolution of quantities for each pair over time. With such data, the assumption of fully persistent types can be relaxed, and a more flexible Markov approach can be used. If the seller is unable to fully learn the buyer's type, the interest in updating the allocation rule will be reduced, thus alleviating the concerns of the commitment problem.

5.3 Estimation

To estimate the key parameters of the model, I use identification equations I:EQ and 16. These equations are estimated separately for each seller j and tenure τ, using $N_{j\tau}$ observations for each year.

The estimation procedure consists of several steps. First, I estimate tariff functions from payments and quantities for each tenure, using ordinary least squares. Then, I estimate constant marginal costs using information on total quantities and variable costs. Next, I estimate heterogeneous hazard rates at the percentile-tenure level and obtain percentile-to-percentile transition matrices over time using pair-wise information.

However, this will not affect the marginal prices and thus the primitives identified from them, including the base marginal return and the type, remain unaffected as well.

[24]Model mispecification in terms of outside options also affects the counterfactuals of different enforcement or pricing regimes. If the outside options are constant, the counterfactuals will be correct in terms of efficiency. Surplus division will be biased towards the seller. If outside options are heterogeneous, the counterfactual efficiency will also be affected. Yet, the direction of the bias is uncertain ex-ante, as it depends on the distribution of types and the curvature of the return function.

After completing these steps, I begin the iterative process. Starting at $\tau = 0$, I estimate enforcement multipliers $\widehat{\Gamma}_\tau(\cdot)$ using the empirical analog of equation I:EQ. To do this, I parametrize the multipliers as a logistic distribution and use maximum-likelihood estimation. Next, using the estimated multipliers and the empirical analog of equation 16, I obtain estimated types $\widehat{\theta}$ for each τ. I then obtain the base marginal return functions $\widehat{v}'(\cdot)$ from prices and estimated types. Finally, I use the transition matrix to link estimated multipliers for $s < \tau$ to quantiles of quantity at τ, and repeat the process for $\tau + 1$.

For more information on the estimation approach and corresponding parametric assumptions, please see Online Appendix Section OA-6. Additionally, Online Appendix Section OA-7 provides Monte Carlo simulations demonstrating the accuracy of the estimation method for a two-period dynamic contract.

6 Empirical Results

In this section, I first explain the definition of relationship tenure, then discuss the estimates of primitives of the model and show the data fit (both quantitatively and qualitatively). I present the results pooling all sellers together but conduct estimation at the seller-year level.

6.1 Definitions of Relationship Tenure and Estimation Sample

I make two restrictions to facilitate estimation and reduce measurement error in relationship ages. First, I require that buyers have at least one previous relationship with some seller (not necessarily those in my sample) prior to 2016.[25] Second, I pool relationships ages using the following classification method and define *relationship tenure* between seller i and buyer j at year t as:

$$tenure_{ijt} = \begin{cases} \text{pair-age}_{ijt} & \text{if pair-age}_{ijt} < 5 \\ 5 & \text{if pair-age}_{ijt} \geq 5. \end{cases}$$

The final sample with estimated structural model is of 24 sellers with information for 2016 and 2017 as well as 25 sellers with information for either 2016 or 2017. I consider these 73 seller-year observations on their own, but use sellers that appear on multiple years to validate fit over time.

6.2 Estimation Results

My model relies on the following seller-dependent ingredients: initial distribution of private types θ, the base return function $v(\cdot)$, and the limited enforcement multipliers $\Gamma_\tau(\cdot)$ for tenure $\tau \in \{0, 1, .., 4, 5\}$.

[25]I verify that this restriction is not driving the results by estimating the model with *all* available buyers, despite the possible measurement error in age of relationship. Overall, results are very consistent with those presented here. Results of this robustness check are available upon request.

First, Figure 2a shows the average estimated log type θ by quantile of quantity for tenure 0. Recall that for identification, I normalized the lowest type $\underline{\theta}$ to 1. The figure illustrates that, on average across sellers, types tend to increase with the quantity purchased, with a more significant increase in the top quantiles of quantities. Error bars show the dispersion across sellers for a given quantile.[26]

Next, Figure 2b plots the average estimated base marginal return $v'(\cdot)$ by quantity quantile and relationship tenure. Consistent with the model, the base marginal return function $v'(\cdot)$ decreases as quantity increases for all. In addition, the figure reveals that the functions $v'(\cdot)$ for older tenures shift downwards for many quantiles, reflecting the greater consumption levels as time goes by. The estimated values have a clear economic interpretation, as $v'(\cdot)$ represents the marginal revenue for the buyer of an extra unit of the good for a given type. For the median new buyer (respectively, tenure 5), spending one dollar on manufacturing the good generates 2.5(1.25) dollars of revenue for the buyer. These values suggest that inefficiencies are more prevalent in new relationships than in older ones. For more information, refer to Online Appendix Figure OA-6.

Since the buyer is purchasing inputs using trade-credit, it is possible to translate the figures into the marginal product of capital (MPK) per dollar price of credit (interest rate). The MPK measures the return the buyer would receive if given an extra unit of the input at their transaction price. I find a wedge of 40% between MPK and the transaction price for the median new relationship and 34% for the median tenure 5 relationship. Although these wedges are smaller than the gaps of 80% estimated for Indian firms by Banerjee and Duflo (2014), they are larger than the average gaps of 6% calculated by Blouin and Macchiavello (2019).[27]

Finally, Figure 2c presents the average estimated limited enforcement multiplier $\Gamma_\tau(\cdot)$. The figure indicates that almost all new pairs are constrained, as the average multiplier $\Gamma_0(\cdot)$ equals only 1 for the top 1% of pairs, on average across sellers. However, as time goes by, the average multiplier approaches 1 for lower quantiles of trade, suggesting that the limited enforcement constraint is less restrictive.

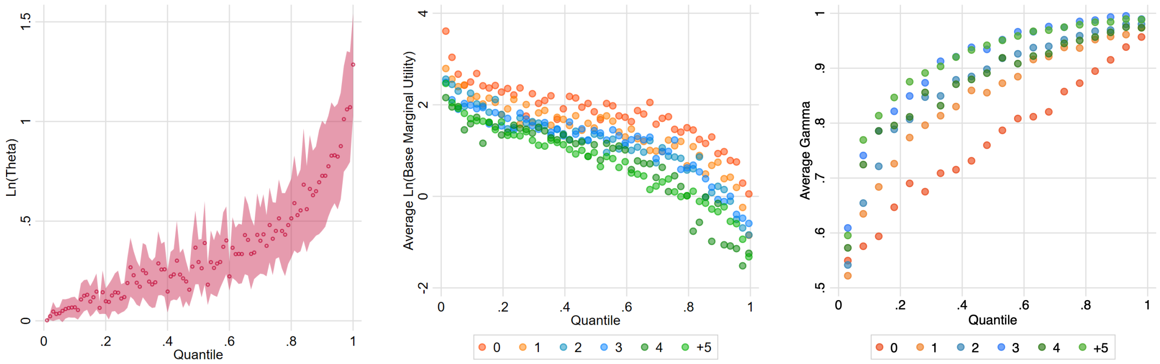
Online Appendix Table OA-8.1 displays the distribution of t-statistics for the LE multiplier at tenure 0 (Γ_0) to test against the null hypothesis of a standard model. Based on the significance of the parameters of estimated $\widehat{\Gamma}_0(\underline{\theta})$, I reject the null that the standard nonlinear pricing model applies in my setup for 86% of the markets (seller-years).

Online Appendix Table OA-10 provides the estimated values for k and β of the parametrization of $v(\cdot)$. These values will be used to obtain quantities in counterfactual simulations.

[26] For more information on the distribution of types per seller-year, with confidence intervals constructed via bootstrap, refer to the Supplemental Material.

[27] It is important to note that the estimated gaps for micro-enterprises are even greater, ranging from 300% to 500% in Mexico (McKenzie and Woodruff, 2008). However, since the median buyer in my sample has total yearly sales of USD 200,000, they cannot be directly compared to micro-enterprises.

Figure 2: **Estimated Primitives**



(a) Type (b) Base Marginal Return (c) Average Multiplier Γ_τ

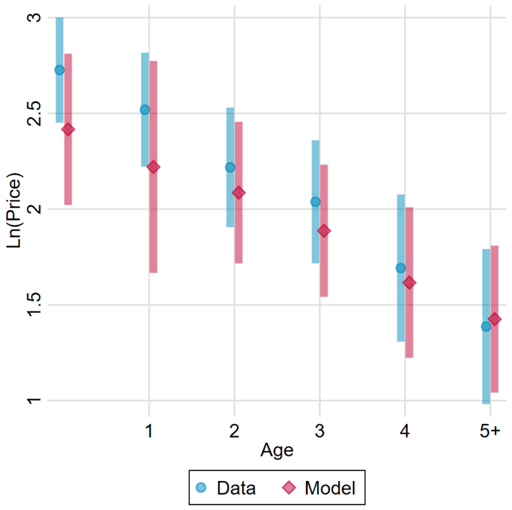
Notes: Sub-figure a) shows the average log type $ln(\theta)$ by quantile of quantity, across-sellers, with error bars representing the dispersion of ± 1.96 standard errors for each quantile across sellers. Sub-figure b) displays the average base marginal returns, across-sellers, for different estimation tenure groups, by quantile of quantity. Sub-figure c) presents the average estimated limited enforcement multiplier by tenure and quantile of quantity, across-sellers.

6.3 Model Fit

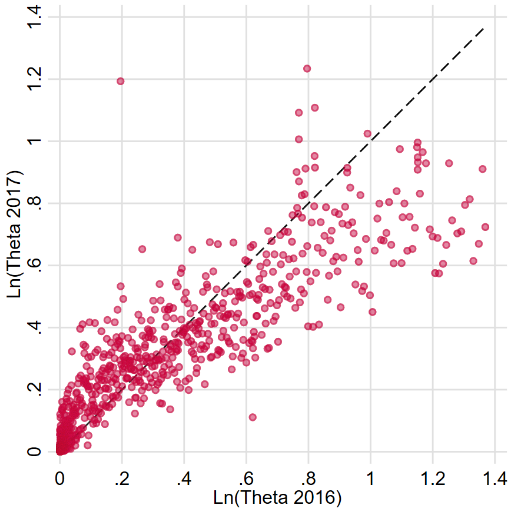
I use four different measures to assess the fit of the model. First, Online Appendix Figure OA-7 demonstrates that the model has good statistical fit across tenures. Second, I compare the observed quantities with model-predicted quantities. The predicted quantities, obtained using the closed form solution of the seller's first-order condition under the parametrization of $v(\cdot)$, match well with the observed quantities in all tenures, as shown in Online Appendix Figure OA-8. Third, using predicted quantities and the incentive-compatible tariff function t-RULE, I generate predicted tariffs. Online Appendix Figure OA-9 demonstrates that the model-generated tariffs match the observed tariffs well across tenures. Fourth, I compare the non-targeted observed cross-sectional unit price discounts by tenure to those generated by the model in Figure 3a, and the model replicates the observed discounts quite well.

To validate the model's within-pair dynamics, I consider a fifth validation exercise. I can use the panel structure to verify that the primitives of the model are similar over time within pairs. Given that the model is estimated using cross-sectional information for each seller separately in 2016 and 2017, Figure 3b shows the value of estimated $\widehat{\theta}$ in 2017 against the value of estimated $\widehat{\theta}$ in 2016 for pairs that are active on both years. The figure illustrates a good correspondence between both estimated values, with the markers overlaying the diagonal in the graph. This result helps validate both the estimation procedure, as similar results are obtained via two independent estimation processes, and the persistency assumption for the types.

Figure 3: **Non-targeted Moments**



(a) Price Discounts



(b) Type in 2016 v. 2017

Notes: Sub-figure a) presents a plot of unit prices by tenure over time using a binscatter plot, comparing prices in the data with model-generated prices. Model-generated unit prices are calculated by dividing model-generated tariffs by model-generated quantities. The error bars represent 95% confidence intervals, with standard errors clustered at the seller-year level.Sub-figure b) shows the estimated types θ in 2017 plotted against those estimated in 2016, for buyer-seller pairs that appear in both years. These estimates were obtained through separate seller-specific estimations for each year using cross-sectional variation only. The dashed line represents the 45 degree line.

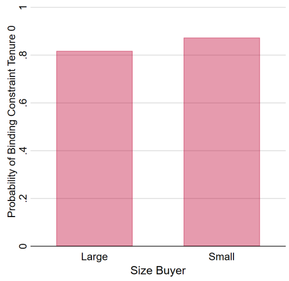
6.4 Qualitative Results

To further explore the implications of the estimated model, I investigate heterogeneity in limited enforcement constraints by seller and buyer characteristics. Although these exercises are outside the scope of the model, they serve to provide interpretation for the enforcement parameter.

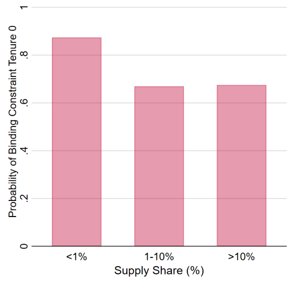
Recall that $\gamma_0(\cdot) > 0$ implies that the buyer's limited enforcement constraint is binding. Figure 4 shows the probability that the constraint is binding at tenure 0 by different buyer characteristics, which offers qualitative differences consistent with previous literature on enforcement constraints. For example, larger firms, exporters, importers, or firms in business groups are less likely to have a binding constraint. All of these would generally be more reliable buyers. Moreover, buyers that might find it hard to locate an alternative supplier, such as those that depend heavily on the seller as measured by their supply share, are also less likely to have a binding constraint (McMillan and Woodruff, 1999). Lastly, distant buyers, who plausibly impose higher enforcement costs for the seller, are more likely to experience binding enforcement constraints (Antras and Foley, 2015).

To better understand the factors affecting limited enforcement constraints from the sellers' perspective, Figure 5 displays the coefficients from regressions of the share of constrained buyers at tenure 0 on various sellers' characteristics. The results show that larger sellers, measured by total sales, total assets, or cash holdings, tend to have a

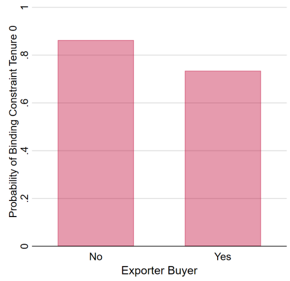
Figure 4: **Enforcement Constraints and Buyer Characteristics**



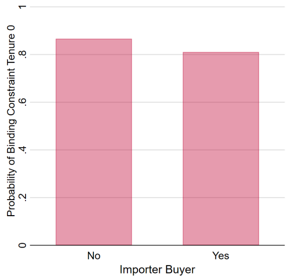
(a) Size



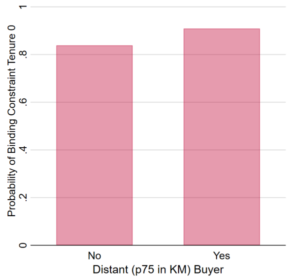
(b) Supply Share



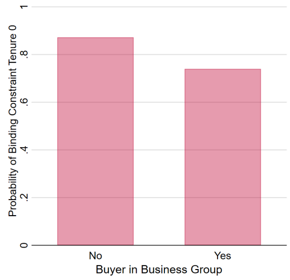
(c) Exporter



(d) Importer



(e) Distance

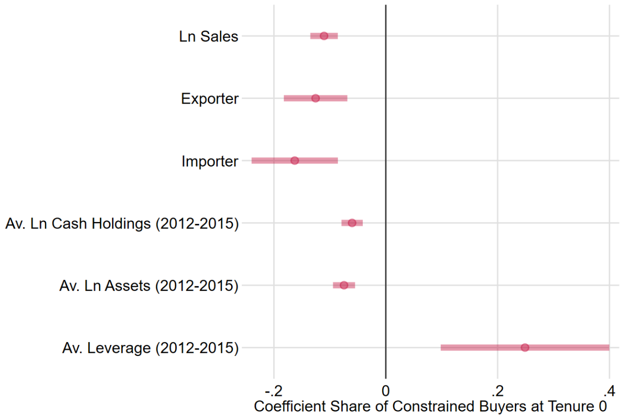


(f) Business Group

Notes: These figures present heterogeneity of estimated limited enforcement multipliers by buyer's characteristics. The figures shows the share of buyers in each group with positive enforcement constraint $\gamma_0(\cdot)$ in tenure 0. A firm is large if they are in the top 25 of sales from the set of buyers. Buyer is classified as exporter if they report at least \$5,000 USD of exports and importer if they report at least \$5,000 USD of imports. Distance between headquarters is calculated as KM. between neighborhoods as the crow flies. I classify a buyer as part of a business group if they have at least link with another firm in the economy through an shareholder that owns at least 1% shares in each firm.

lower share of constrained buyers. Similarly, sellers that export or import also have a lower share of constrained buyers. The finds suggest that these sellers may be of higher quality and thus generate larger surpluses for their clients at any level of quantity, making buyers less likely to cheat. On the other hand, higher levels of seller leverage, which may signal financial distress or uncertainty, are associated with a higher share of constrained buyers, indicating that buyers may be more cautious and less willing to consider long-term future relationships with these sellers. Overall, these results suggest that the quality and financial stability of sellers can play an important role in determining the level of limited enforcement constraints in the market.

Figure 5: **Correlation Seller Characteristics and Share of Constrained Buyers**



Notes: This figure plots the estimated coefficients of a regression of the share of constrained buyers for each seller-year on different seller's characteristics. Sales refer to total sales. I classify a seller as an exporter if they report exports of at least $5,000 USD and as an importer if they report imports of at least $5,000 USD. Cash holdings, total debt, and total assets are obtained through the financial statements. Leverage is estimated as total debt over total assets.

7 Welfare and Counterfactuals

In this section, I analyze the efficiency of the relationships over time using the estimated model. Additionally, I evaluate the welfare performance of different pricing and enforcement schemes. I focus on three margins: i) perfect enforcement with full price discrimination, ii) limited enforcement with uniform pricing, and iii) perfect enforcement with uniform pricing.

7.1 Efficiency Relative to First-Best

Under the parametrization $v(q) = kq^{\beta}$, first-best quantities for each pair is given by:

$$q^{fb}(\theta) = \left(\frac{k\beta\theta}{c}\right)^{1/(1-\beta)}. \tag{18}$$

Moreover, total surplus is a function of buyer's type θ, quantity q and seller's marginal cost c: $Surplus(\theta, q, c) = \theta k q^{\beta} - cq$. Hence, static efficiency of allocation q for buyer type θ is defined as:

$$\text{Efficiency}(\theta, q, c) = \frac{Surplus(\theta, q, c)}{\max_q Surplus(\theta, q, c)}.$$

Figure 6a plots the average efficiency for each tenure across quantity deciles, averaging over all pairs, excluding tenure 1 and 3 for clearer visualization. The figure shows that new relationships are severely constrained, with the median buyer trading only at around 30% their optimal level. However, as relationships age, efficiency increases. The median buyer now trades at 60% of optimal levels at tenure 2, 75% at tenure 4, and over 80% at tenure 5. Additionally, the figure demonstrates a significant heterogeneity in traded efficiency within relationship age: partners trading little have greater distortions than partners trading more intensively.

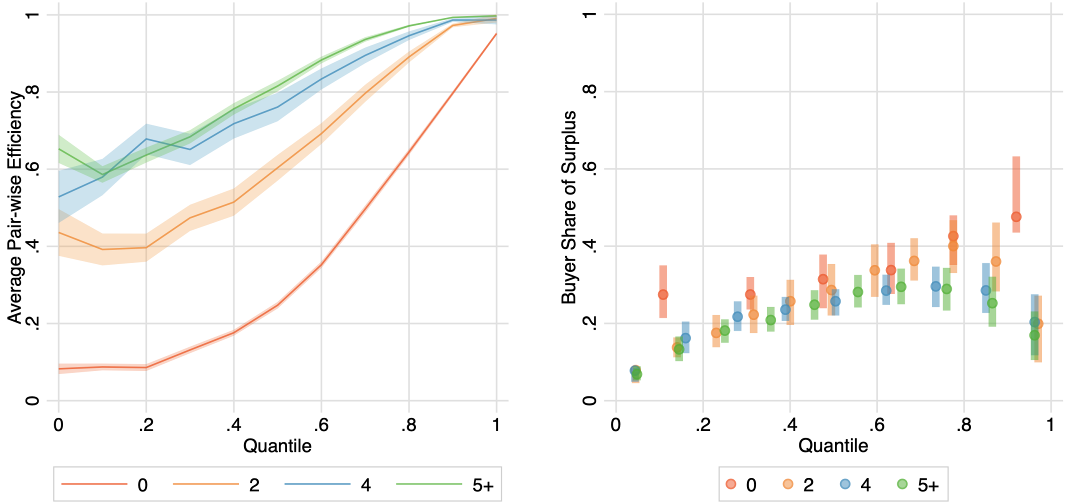
Of course, this characterization of efficiency might be too strict if the majority of trade is channelled through large buyers. To account for the intensity-inclusive efficiency, I study the weighted average efficiency of all transactions per seller. This approach considers the potential efficiency losses and constructs weights using the share of total efficient quantities at a given tenure. Under this measure, the total output is inefficient early on but converges towards efficiency in the medium and long term. In Panel A of Table 1, I report the share of sellers trading at efficient levels, both in average total output and with the average buyer.[28] The results indicate that only 5% of sellers are trading efficiently with new buyers, but efficiency increases quickly, with 70% of sellers trading efficiently by tenure 2. In the long term, 84% of sellers transact with their buyers at efficient levels.

To better understand the long-term efficiency of relationships across different selling sectors, I present the share of sellers trading at aggregate efficient levels in Panel B of Table 1. While at the beginning of relationships almost no seller is trading efficiently, efficiency levels start to diverge at tenure 2. Starting at this point, Textiles shows slower growth in efficiency, while Pharmaceutical and Cement-Products continue to improve. By tenure 5, almost all Pharmaceutical and Cement-Products sellers are trading at aggregate efficient levels, while 70% of Textiles sellers do so. Despite this heterogeneity across sectors, the general takeaway is clear: even in different sectors, aggregate trade efficiency is high in the medium and long-term.

To provide a benchmark for the estimated inefficiencies due to imperfect contracting, it is helpful to compare these results to previous estimates. While the specific settings and frictions may vary, this comparison offers valuable insights. For instance, previous studies such as Blouin and Macchiavello (2019) find that strategic default reduces output by 16% for the mean relationship, with only 26% of relationships operating at first-best. Similarly, Ryan (2020) finds that weak contract enforcement reduces efficiency by 10% on average, while Startz (2021) finds that jointly contracting and search frictions reduce welfare by 14%. In contrast, the results presented in this paper offer a dire look at the relationship-level, with average output at only 38% of first-best. However, when weighting for the size of relationships, the estimated inefficiencies are more moderate and in line with the literature, with a weighted average loss of 15%. It is worth highlighting that the previous studies only estimate efficiency for stationary relationships, whereas this paper offers efficiency estimates over the lifespan of a relationship. Additionally, these magnitudes of relationship-level inefficiencies may not be specific to developing countries, as contemporaneous work by Harris and Nguyen (2022) finds that the median relationship in the US trucking industry achieves only 44% of first-best output.

[28] I test for seller-level efficiency via 30 bootstrap simulations and call seller's output efficient if the 95th percentile of weighted surplus is within 1% of efficiency.

Figure 6: **Efficiency and Buyer Surplus**



(a) Efficiency

(b) Buyer Surplus

Notes: Sub-figure a) presents average efficiency by quantile of quantity and tenure over all sellers. Error bars show dispersion of ± 1.96 standard errors for each quantile across sellers. Sub-figure b) shows buyer share of surplus for quantile of quantity and tenure. Error bars show ± 1.96 standard errors, clustered at the seller-year level.

To analyze surplus division, I present Figure 6b. This figure displays the average share of surplus captured by buyers, across sellers, by bins over quantiles of quantity purchased at different tenures. The results show that sellers capture the majority of the surplus, with the median buyer in any tenure capturing around 30 percent of the generated surplus. The figure also reveals that, consistent with the nonlinear pricing scheme, buyers who trade more intensively capture a larger share of the surplus, up to 50 percent. However, the smallest buyers may capture less than 10 percent of the total surplus. The results show that sellers have significant market power in this setting.

Table 1: **% Share of Sellers with Efficient Trade**

	Tenure 0	Tenure 1	Tenure 2	Tenure 3	Tenure 4	Tenure 5
Panel A: All Sectors						
Weighted	5.48	41.10	69.86	79.45	75.00	84.29
Unweighted	5.48	21.92	32.88	36.99	37.50	30.00
Panel B: Weighted, By Sector						
Textiles	4.55	45.45	59.09	63.64	63.64	68.18
Pharmaceutical	0.00	30.77	73.08	88.46	73.08	88.46
Cement-Products	12.50	50.00	75.00	87.50	86.96	95.24

Notes: This table reports the share of sellers that trade efficiently. Panel A presents results across all sectors. The first measure (Weighted) computes the share of sellers whose weighted average output cannot be rejected to be different from the efficient output at the 10% level. The weights are constructed over potential output for each seller-tenure. The second measures (Unweighted) computes the share of sellers for which the surplus created by the average buyer cannot be rejected to be different from efficient at the 10% level. Panel B presents results using the Weighted measure for each selling sector.

7.2 Counterfactuals

In the following section, I will use the estimated model to explore the implications of improving enforcement of trade-credit contracts and enforcing current Ecuadorian legislation that forbids price discrimination on identical transactions. I will consider three counterfactual scenarios: 1) Maintaining price discrimination but improving contract enforcement; 2) Maintaining limited enforcement but eliminating price discrimination; 3) Eliminating both limited enforcement and price discrimination

Counterfactual 1: *Perfect Enforcement + Price Discrimination*

One natural question is to consider what the surplus and corresponding surplus shares would be in a world of perfect enforcement of contracts. To answer this question, I use the distribution of types at different tenures and equation Q-CES with $\Gamma_\tau(\cdot)$ set to 1 and $\gamma_\tau(\cdot)$ set to 0. I also set $\Gamma_s(\cdot)$ to 1 for $s < \tau$.

Counterfactual 2: *Limited Enforcement + Uniform Pricing*

Written law in Ecuador, the European Union, and the US forbid price discrimination that applies differential treatment to customers performing an otherwise equivalent transaction, including possibly preferential treatment due to tenure.[29] This counterfactual

[29] In Ecuador, Art. 9 of *Ley Orgánica de Regulación y Control del Poder de Mercado*. In the EU, Art. 102(c) of *Treaty on the Functioning of the European Union* (ex of Art. 82(c) of. *EC Treaty*). In the US, Section 2(a) of the *Robinson-Patman Act*. In practice, only the EU has enforced such a law in court. See, for instance, the cases *Hoffmann-La Roche v. Commission* and *Manufacture française des pneumatiques Michelin v Commission*. In the US, *some* variants of preferential pricing (such as loyalty discounts in multiproduct markets) have been upheld in court. See, for instance, cases *LePage's v 3M* and *SmithKline v Eli Lilly*. Moreover, in the US, discounts below cost are seen as anticompetitive (see *Eisai Inc. v. Sanofi-Aventis U.S., LLC*). In Ecuador, no cases have been brought to court regarding the specific Art 9.

studies the welfare effects of a policy that enforces uniform pricing but keeps the limited enforcement regime active.

Under the assumed base return function, the optimal uniform price is $p^l = c/\beta$ for any quantity. The corresponding type θ's demand is given by $q^l(\theta) = (k\beta\theta/p^l)^{1/(1-\beta)}$. This stationary menu will be insufficient for some enforcement constraints. Given exogenous hazard rates $X(\theta)$, the stationary enforcement constraint will be given by:

$$\delta(1 - X(\theta)) \geq \beta, \tag{L-LE}$$

which indicates that the rate of return captured by β has to be smaller than the buyer-specific discount rate. Notice that this limited enforcement constraint will hold for any other uniform price, so buyers who are willing to default at the optimal uniform price p^l will also be willing to default at any other alternative uniform price p^l_a.

Under a monotonicity assumption on $X(\theta)$,[30] the seller will set a minimum quantity $\underline{q}^l$ that the buyer needs to announce in order to be served. In particular, it will only serve $q(\theta) \geq \underline{q}^l$, where $\underline{q}^l = min\{q^l(\theta)|\delta(1 - X(\theta)) \geq \beta\}$. In the counterfactual exercise, I set their quantities to zero to those θ with $q^l(\theta) < \underline{q}^l$.[31]

Counterfactual 3: *Perfect Enforcement + Uniform Pricing*

Lastly, I consider optimal uniform pricing under perfect enforcement. I use quantities and prices as in counterfactual 2 above. However, as buyers are precluded from the possibility of default, the seller serves all buyers. Thus, no quantity is set to zero.

7.2.1 *Discussion of Counterfactual Results*

Table 2 displays the results of the counterfactual exercises. The table shows the average surplus as a percentage of the baseline for each percentile group in quantity and tenure. Additional results for the three counterfactual exercises related to buyer net return, profits, and prices are presented in Online Appendix Section OA-9.

Panel A shows the results for counterfactual 1, which considers nonlinear pricing with perfect enforcement. The policy exercise generates an inter-temporal trade-off. Fixing enforcement generates massive gains for middle and lower types in the early stages of the relationship. That is, weak enforcement forces the seller to generate further downward distortions when buyers can default on trade. Fixing enforcement alone would increase surplus for 75% of the buyers in tenure 0 and 1. However, as relationships age, contract enforcement distortions become of second order. By tenure 3 and onward, limited enforcement contracts actually help discipline the market power of the seller. Fixing enforcement would decrease the generated surplus in old relationships for essentially all buyers, as the seller increases quantities over time to incentivize debt repayment from the buyer side. In the long-term, the threat of default is sufficient to overcome sellers' market power.

[30] The monotonicity on the hazard rate $X'(\theta) < 0$ is observed in the data.

[31] In this counterfactual exercise, I use an additional assumption: buyers demand the optimal level of quantity that is consistent with prices and full enforcement.

Panel B presents the results for counterfactual 2, which considers uniform pricing with limited enforcement. The surplus is between 0 to 40 percent of the baseline surplus across time and types. The surprisingly low performance of this alternative regime is explained by the large share of buyers that would be excluded from trade, as some buyers cannot credibly commit to repaying their debts and the seller cannot use dynamic incentives to discipline their behavior. Thus, in the presence of limited enforcement, the seller's ability to price discriminate actually improves the situation for both buyers and sellers by increasing the share of buyers that can be credibly incentivized not to default.[32]

Panel C reports the results for counterfactual 3 (uniform pricing with perfect enforcement). The table shows that surplus increases relative to baseline, except for the highest types. Welfare gains are concentrated in the lowest types (who see gains of up to 46,000%), although even median types also see large increases (from 12% up to 8,000%). Given that this counterfactual allows the seller to choose the profit maximizing uniform price, performance would improve the better the efforts to reduce seller market power, while contemporaneously addressing enforcement.

The counterintuitive results that solving only one friction at once may lead to welfare losses is a direct manifestation of the *theory of second best* (Lipsey and Lancaster, 1956). In the presence of multiple market frictions, eliminating one friction will not necessarily lead to higher welfare. In fact, in the presence of one market friction, an additional friction might be necessary to reach second-best.

[32] In results not presented here, I consider as well an alternative counterfactual regarding uniform pricing. I consider a pooling contract that offers only a unique price and quantity mix for all buyers. The seller picks the mix to prevent default of all buyers above a targeted threshold. Hence, the seller picks the profit margin and the share of defaults. In such counterfactual, I obtain again much lower surplus than in baseline.

Table 2: **Average Surplus as % of Baseline**

	10%	25%	50%	75%	100%	10%	25%	50%	75%	100%
	Panel A: Nonlinear + Perfect Enforcement					*Panel C: Uniform + Perfect Enforcement*				
Tenure 0	1,508.4	1,419.0	628.0	150.3	56.5	46,633.7	42,233.1	8,487.5	1,083.0	64.0
Tenure 1	430.3	430.6	256.0	112.0	49.8	13,887.9	12,003.0	8,472.0	649.7	49.4
Tenure 2	164.8	139.9	102.6	59.7	44.2	5,399.0	4,161.9	1,531.8	97.7	35.9
Tenure 3	80.5	82.7	68.6	53.4	43.2	1,816.5	1,198.1	417.5	63.3	33.5
Tenure 4	72.4	72.7	67.9	54.0	45.2	745.0	624.2	294.0	60.8	35.1
Tenure 5	60.7	66.4	60.2	53.9	47.0	224.6	195.7	112.2	49.9	36.8
	Panel B: Uniform + Limited Enforcement					*% Excluded*				
Tenure 0	1.0	1.3	1.5	2.1	3.2	97.3	96.4	95.8	94.1	90.4
Tenure 1	2.8	4.0	5.8	5.7	5.3	93.4	91.9	88.6	87.3	85.8
Tenure 2	12.2	14.1	18.4	16.7	15.4	79.8	77.4	70.0	65.6	61.3
Tenure 3	16.9	19.4	26.6	23.0	19.4	74.4	69.2	58.0	51.4	50.0
Tenure 4	17.7	25.3	33.4	28.9	24.6	71.3	60.1	44.7	38.5	37.4
Tenure 5	28.6	37.9	43.5	34.0	29.2	62.3	50.7	36.4	30.3	25.8

Notes: This table presents average efficiency measures as % of baseline (nonlinear price with limited enforcement) of different pricing and enforcement regimes by percentile groups of quantity and tenure. For instance, 10% collects all buyers between percentiles 0 and 10%. Panel A reports results for nonlinear pricing with perfect enforcement. Panel B reports optimal monopolistic uniform price with limited enforcement. Subpanel reports the share of excluded buyers in this counterfactual. Panel C reports results for optimal monopolistic uniform price with perfect enforcement. No buyer is excluded in Panel A and C.

8 Conclusion

This paper examines how frictions in the manufacturing supply chain impact long-term relationships. Using a novel theoretical model, the study shows that allocating bargaining power to the seller while allowing the buyer to *take the goods and run* has significant implications for surplus division, price and quantity dynamics. The paper demonstrates that limited enforcement constraints, which prevent buyer defaults, require the seller to offer larger amounts of surplus than perfect enforcement would require. This creates an incentive for the seller to distort trade inter-temporally by promising larger quantities and lower prices in the future to reap larger profits now.

The study employs a unique intra-national database from Ecuador to estimate the structural model of relational contracting with seller market power, with the main contribution being the quantification of the efficiency of dynamic trade. The results suggest that trade is highly inefficient at the start of relationships, but transacted quantities approach full efficiency in the long-term despite the seller's market power. These findings highlight the significant value created by informal agreements between buyers and sellers, but also demonstrate the fragility of these agreements. Unilateral reforms aimed at improving enforcement or applying Ecuadorian antitrust policy may undermine the long-term efficiency of relational contracts. However, addressing multiple frictions at once could lead to welfare gains.

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Appendix

A Point Identification of Gamma

In this section, I detail how $\Gamma_\tau(\cdot)$ is point identified with observations of prices, quantities, and marginal cost for one seller under the parametrization of $v(q) = kq^\beta$ for $k > 0$ and $\beta \in (0, 1)$.

A.1 Step 1: Show β is identified

We first show that β is identified from observations on prices, quantities and marginal cost for $\tau = 0$. In this step, we omit subscripts $\tau = 0$.

Consider $\rho(\alpha) = \partial \ln(\theta(\alpha))/\partial\alpha = \theta'(\alpha)/\theta(\alpha)$. Substituting in, the key identification equation I:EQ becomes

$$\frac{T'(q(\alpha)) - c}{T'(q(\alpha))} = \rho(\alpha)\Big[\Gamma(\alpha) - \alpha\Big] + \gamma(\alpha). \tag{19}$$

Evaluating at $\alpha = 1$ and using the fact that $\Gamma(1) = 1$, yields

$$\gamma(1) = \frac{T'(q(1)) - c}{T'(q(1))}. \tag{20}$$

Therefore, all parameters, except $\rho(\alpha)$ are known at the boundary $\alpha = 1$.

As an auxiliary result, note that:

$$\gamma'(1) = \frac{cT''(q(1))}{T'(q(1))}, \tag{21}$$

which is known.

Then consider the first-order condition at $\alpha = 1-\varepsilon$ using Taylor approximations for the enforcement multipliers:

$$\frac{T'(q(1-\varepsilon))-c}{T'(q(1-\varepsilon))} \approx \rho(1-\varepsilon)\Big[\Gamma(1)-\gamma(1)\varepsilon-1+\varepsilon\Big]+\gamma(1)-\gamma'(1)\varepsilon, \tag{22}$$

under the assumption that Γ is regular and second-order differentiable as it approaches $\alpha = 1$. From this equation, the value for $\rho(1-\varepsilon)$ is identified.

Use the derivative of the transfer rule to obtain $\rho(\alpha) = \theta'(\alpha)/\theta(\alpha) = T''(q(\alpha))/T'(q(\alpha)) + A(q(\alpha))$, where $A(q(\alpha)) = -v''(q(\alpha))/v'(q(\alpha))$. The assumed parametrization implies $A(q) = (1-\beta)/q$. As $T'_\tau(\cdot)$, $T''_\tau(\cdot)$, and $q_\tau(\cdot)$ are known, $\rho(\cdot)$ depends on only one unknown parameter β, which is identified from the value of $\rho(1-\varepsilon)$ above.

A.2 Step 2: Show Γ_0 is identified from β

Consider equation 12 and use the parametrized version of $\rho_0(\alpha)$:

$$\Xi_0(\alpha) = \alpha + \frac{T_0'(q_0(\alpha))-c}{T_0'(q_0(\alpha))}\left[\frac{T_0''(q_0(\alpha))}{T_0'(q_0(\alpha))}+\frac{1-\beta}{q_0(\alpha)}\right]^{-1}. \tag{23}$$

As c, $T_0'(\cdot)$, $T_0''(\cdot)$, $q_0(\cdot)$ are known, $\Xi_0(\alpha)$ is identified up to β. As β is identified from observations of prices, quantities and marginal cost, then $\Xi_0(\alpha)$ is identified.

Then, $\Gamma_0(\alpha)$ is identified from the solution to the differential equation

$$\gamma_0(\alpha)+\Gamma_0(\alpha)\left[\frac{T_0''(q_0(\alpha))}{T_0'(q_0(\alpha))}+\frac{1-\beta}{q_0(\alpha)}\right]^{-1} = \Xi_0(\alpha)\left[\frac{T_0''(q_0(\alpha))}{T_0'(q_0(\alpha))}+\frac{1-\beta}{q_0(\alpha)}\right]^{-1}, \tag{24}$$

using the boundary condition $\Gamma_0(1) = 1$, and the fact that $T_0''(\cdot)$, $T_0'(\cdot)$, $q_0(\cdot)$, and β are known or identified.

A.3 Step 3: Show Γ_τ is identified from β and Γ_s for $s < \tau$

Start recursively from $\tau = 1$. With knowledge of $\Gamma_s(\cdot)$ for $s < \tau$ and β, note that

$$\Xi_\tau(\alpha) = \alpha + \sum_{s=0}^{\tau-1}(1-\Gamma_s(\alpha)) + \frac{T_\tau'(q_\tau(\alpha))-c}{T_\tau'(q_\tau(\alpha))}\left[\frac{T_\tau''(q_\tau(\alpha))}{T_\tau'(q_\tau(\alpha))}+\frac{1-\beta}{q_\tau(\alpha)}\right]^{-1} \tag{25}$$

is identified as $\Gamma_s(\cdot)$, c, $T_\tau'(\cdot)$, $T_\tau''(\cdot)$, $q_\tau(\cdot)$, and β are known or identified.

Then, $\Gamma_\tau(\alpha)$ is identified from the solution to the differential equation

$$\gamma_\tau(\alpha)+\Gamma_\tau(\alpha)\left[\frac{T_\tau''(q_\tau(\alpha))}{T_\tau'(q_\tau(\alpha))}+\frac{1-\beta}{q_\tau(\alpha)}\right]^{-1} = \Xi_\tau(\alpha)\left[\frac{T_\tau''(q_\tau(\alpha))}{T_\tau'(q_\tau(\alpha))}+\frac{1-\beta}{q_\tau(\alpha)}\right]^{-1}, \tag{26}$$

using the boundary condition $\Gamma_\tau(1) = 1$, and the fact that $T_\tau''(\cdot)$, $T_\tau'(\cdot)$, $q_\tau(\cdot)$, and β are known or identified.

Online Appendix

OA-1 Summary Statistics

Table OA-1: **Summary Statistics - Sellers and Buyers in 2016**

	Sellers			***Buyers***		
	Mean	Median	SD	Mean	Median	SD
Total Sales (million USD)	14.95	8.26	24.33	2.35	0.20	24.33
Total Inputs (millon USD)	10.58	5.31	18.94	1.92	0.15	24.13
Accounting Markup	1.21	1.20	0.20	1.21	1.10	0.61
Age	30.47	29.00	19.16	15.18	14.00	9.75
Import Share (%)	24.47	21.38	22.96	3.82	0.00	13.49
Export Share (%)	5.81	0.00	19.11	1.06	0.00	8.87
Observations	49			28,138		

Notes: This table reports summary statistics about the size, age, and trade exposure of buyers and sellers in the sample for the year 2016. Monetary values are in U.S. dollars for 2016.

Table OA-2: **Industrial Composition of Buyers by Selling Sector**

Seller Industry	Ranking	Buyer Industry	Average % Share Pairs
Textiles	1	Wholesale & Retail	40
Textiles	2	Manufacturing	15
Textiles	3	Professional Activities	8
Textiles	4	Agriculture	5
Textiles	5	Transporation & Storage	3
Textiles	6	Other	28
Pharmaceutical	1	Wholesale & Retail	46
Pharmaceutical	2	Human Health	17
Pharmaceutical	3	Manufacturing	10
Pharmaceutical	4	Construction	4
Pharmaceutical	5	Professional Activities	3
Pharmaceutical	6	Other	20
Cement-Products	1	Wholesale & Retail	25
Cement-Products	2	Construction	20
Cement-Products	3	Professional Activities	16
Cement-Products	4	Manufacturing	8
Cement-Products	5	Real Estate	5
Cement-Products	6	Other	26

Notes: This table provides a breakdown of the industrial composition of buyers for each selling sector.

Table OA-3: **Summary Statistics - Electronic Invoice Database**

	Mean	Median	SD
N. Buyers	8,028.41	613.50	25,078.11
N. Buyers (> USD250)	801.86	281.50	1,269.02
N. Buyers (> USD4,800)	160.49	73.00	265.19
Total Sales (million USD)	16.58	7.23	29.44
Total Q (million)	5.42	1.20	9.01
Q per Buyer	12,455.39	1,495.22	25,823.40
Bill per Buyer (USD)	43,490.37	9,067.65	105,840.28
Bill per Buyer (USD) (> USD250)	61,756.13	14,898.73	143,297.12
Bill per Buyer (USD) (> USD4,800)	176,632.43	46,899.44	444,389.15
Observations	49		

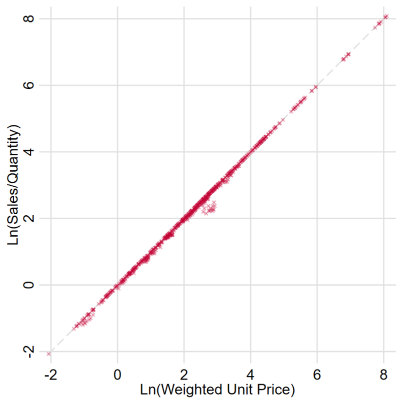
Notes: This table reports summary statistics of the electronic invoice database. N. buyers refers to the number of unique buyers each seller in the sample has on average over 2016 and 2017. Quantity is the sum of all quantities across products. Bill per buyer is the total value of the transactions between buyer and seller. The thresholds > 250 and $> 4,800$ correspond to the data selection thresholds in Bernard et al. (2019) and Alfaro-Urena et al. (2022), respectively.

Table OA-4: **Example - Product Information, Prices, and Marginal Costs**

Industry	Firm-ID	Product Description	Observed Unit Price	Imputed Marginal Cost
Textiles	1	Teddy King, Size 55, Brim 7CM, Color-B02 [Panama Hat]	33.9	11.96397
Textiles	2	Shirt, R:1931, Squares	19.34	9.851618
Textiles	3	Tank Undershirt, Male, Size M, White	10.26758	6.716691
Textiles	4	Betty K246	19.44	16.9426
Textiles	5	Bikini, Woman, 500306, Black, L	13.5	16.77684
Textiles	6	Ribbon, Black, 30 mm X 700	26.62	1.859854
Textiles	7	Skirt, Tropical Squares, Scottish	46.01	17.76808
Textiles	8	Boots, LLN NG AM, Size 39	7.091094	2.168639
Textiles	9	Elastic Socks, Nylon and Cotton	16.55658	8.476095
Textiles	10	Jacket, Kids, Spiderman Print, Hoodie	18.3	7.112451
Pharmaceutical	1	Nitazoxanida, 500mg X 6 tablets	5.27	4.831101
Pharmaceutical	1	Clopidogrel Tarbis 75 mg film-coated tablets	12.9	6.566845
Pharmaceutical	2	Losartan/Hydrochlorothiazide, 100mg X 28 tablets	5.04	.7821692
Pharmaceutical	3	B Complex, Syrup 120 ml	2.32	.8053265
Pharmaceutical	4	Sodium perborate, mint oil, saccharin	4.688695	1.814284
Pharmaceutical	5	Boldenone 50, Injectable, Bottle X 500 ml	123.12	3.014165
Pharmaceutical	6	Pinaver, Film-coated, 100 mg X 20 tables	10.32	2.623771
Pharmaceutical	7	Endobion X 60 tablets	14.83333	5.48487
Pharmaceutical	7	Prostageron X 60 capsules	14.75	7.036339
Pharmaceutical	8	Oral rehydration solution, cherry, 500ml	2.67	1.796762
Cement-Products	1	Gray French Pedestrian Paving Stone	11.27652	18.10799
Cement-Products	2	Corrugated Plate	23.73	9.56013
Cement-Products	3	Polymer-modified adhesive mortar for ceramics, 25kg	6.310047	2.988913
Cement-Products	4	Polymer-modified adhesive mortar for ceramics, 25kg	6.944	12.36252
Cement-Products	5	Polymer-modified adhesive mortar for ceramics, 25kg	6.650971	3.450306
Cement-Products	6	Straight Pole 21m x 1400kg, Reinforced Concrete	882	73.94785
Cement-Products	6	Straight Pole 21m x 2400kg, Reinforced Concrete	1362.73	73.94785
Cement-Products	7	Tile 50x50x2 cm (Color)	32	6.622508
Cement-Products	8	MFC Concrete, 300, XXXXX XXXX-XXXX	94	50.34137
Cement-Products	8	CFC Concrete, 240, XXXXX XXXX-XXXX	79.428	50.34137

Notes: This table presents a sample of ten random products from each of the studied sectors (textiles, pharmaceutical, and cement-products), with product descriptions translated into English and sensitive information, such as brand names, removed to ensure confidentiality. The observed average unit prices reflect the listed prices reported by the firms, while the imputed marginal costs are estimated using the firms' average cost as a proxy.

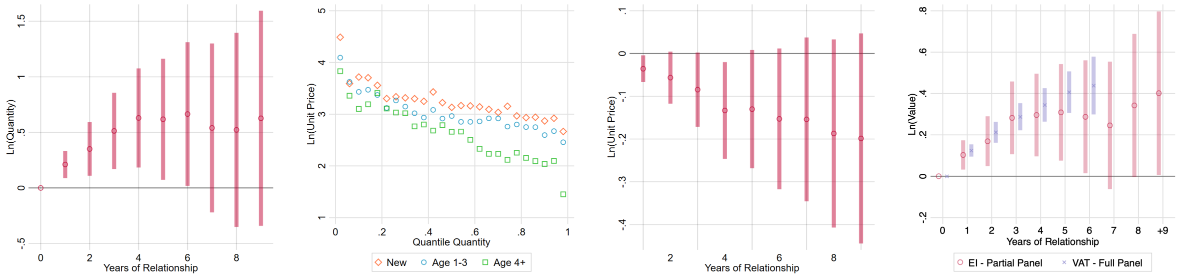
Figure OA-1: **Average Price vs Weighted Price**



Notes: This figure plots average unit prices (in logs) against weighted prices (in logs) across buyer-seller-year. Average unit prices are calculated by dividing total value of yearly transactions by total quantity purchased (pooling different products). Weighted prices are calculated by summing transaction-product-level unit prices by total expenditure share.

OA-2 Motivating Evidence - Robustness

Figure OA-2: **Motivating Facts - Robustness**



(a) Quantity over Time (b) Quantity Discounts (c) Prices over Time (d) Prices over Time

Notes: Panel a) plots the coefficients of log total quantity on relationship age dummies controlling for pair fixed effects. Total quantity is obtained by aggregating all the reported units of sold goods. Standard errors are clustered at the pair-level. Panel b) shows the relationship between quantity purchased and average log unit price (right-panel) through binscatters of the measure of unit price against quantile of quantity by age of relationship. Quantiles of quantity are calculated for each seller-relationship age combination. Panel c) plots the regression coefficients of log unit prices on years of relationship, controlling for pair fixed effects. Standard errors are clustered at the pair-level. Panel d) plots regression coefficients for the the value of total sales between buyer and supplier on age of relationship, controlling for pair fixed effects. The red figures use the electronic invoice database and are constructed using only a partial panel of two observations per pair for years 2016-2017. The purple marks are constructed using multiple observations of buyer-seller pairs from the VAT B2B database for years 2007-2015 for the sellers in the electronic invoice database.

Table OA-5: **Benchmark: Quantity Discounts**

	(1)
VARIABLES	ln(Price)
ln(Quantity)	-0.220***
	(0.0238)
Constant	3.046***
	(0.0718)
Seller-Year FE	Yes
Observations	76,473
R-squared	0.666

Notes: This table presents a regression of log average unit prices on log quantity, controlling for seller-year fixed effects. Standard errors are clustered at the seller-year level. *** p<0.01, ** p<0.05, * p<0.1

Table OA-6: **Robustness - Standardized Log Price**

VARIABLES	(1) Stdz. ln(Price)	(13) Stdz. ln(Price)
Stdz. ln(Quantity)	-0.0463***	-0.0454***
	(0.00722)	(0.00631)
Age of Relationship	-0.00552***	-0.00395***
	(0.00146)	(0.00134)
ln(Assets Buyer)	0.00131***	0.000832***
	(0.000318)	(0.000230)
Supply Share	0.0262*	0.0143
	(0.0157)	(0.0145)
Demand Share	0.0119	0.00402
	(0.0486)	(0.0454)
Observations	73,633	73,626
R-squared	0.082	0.091
Controls	Yes	Yes
Year FE	Yes	Yes
Buyer Sector FE	No	Yes

Notes: This table presents regressions regressions of standardized unit prices on age of relationship, standardized quantity, and different buyer characteristics. Controls include Exporter, Importer, Business Group, Multinational Dummies, as well as Distance between HQs, Sales, Age, and Number of Employees of Buyer. Standard errors are clustered at the seller-year level. *** p<0.01, ** p<0.05, * p<0.1

Table OA-7: **Robustness - Seller's Sector**

VARIABLES	(1) Stdz. ln(Price)	(2) ln(Price)
Textiles		
Age of Relationship	-0.00319*	-0.0973***
	(0.00180)	(0.0305)
Pharmaceuticals		
Age of Relationship	-0.00691***	-0.0308**
	(0.00185)	(0.0139)
Cements		
Age of Relationship	-0.00447***	-0.0387***
	(0.00141)	(0.00728)
Seller-Year FE	No	Yes
Controls	Yes	Yes
Observations	73,633	73,633
R-squared	0.083	0.608

Notes: This table presents regression of prices on age of relationship by sector of the seller. Column (1) presents results for the standardized log prices. Column (2) presents results for log average price, controlling for seller-year fixed effects. Both columns control for standardized quantities as well as all variables in Table OA-6. Standard errors are clustered at the seller-year level. *** p<0.01, ** p<0.05, * p<0.1

Table OA-8: **Price Dynamics by Payment Method**

VARIABLES	(1) Stdz. ln(Price)	(2) Stdz. ln(Price)	(3) ln(Price)	(4) ln(Price)
Payment Method	Pay-in-advance	Trade-Credit	Pay-in-advance	Trade-Credit
Age of Relationship	0.000360	-0.00248***	0.0314	-0.0240***
	(0.00215)	(0.000258)	(0.0266)	(0.00516)
Quantity Control	Yes	Yes	Yes	Yes
Seller-Year FE	Yes	Yes	No	No
Pair FE	No	No	Yes	Yes
Observations	8,777	68,730	1,548	33,680
R-squared	0.148	0.134	0.947	0.937

Notes: This table presents a regression of unit prices on age of relationship, controlling for quantity, by payment modality. The sample excludes buyers that switch between modalities. Columns (1) and (2) uses standardized prices and controls for seller-year fixed effects, while Columns (3) and (4) relies instead on average unit prices while controlling for seller-buyer fixed effects. Standard errors are clustered at the seller-buyer level. *** p<0.01, ** p<0.05, * p<0.1

OA-3 Existence and Non-Stationarity

To prove existence, I build on two results of the literature. First, I use the result of non-linear pricing of Jullien (2000) to prove the existence of a stationary optimal contract in the presence of heterogeneous participation constraints. I do so by showing the equivalence between the stationary contract with limited enforcement and a non-linear pricing problem with heterogeneous outside options. Then, similar to the argument in Martimort et al. (2017), I offer an simple non-stationary deviation that dominates the stationary optimal contract.

Note that I will show existence results under the assumption of no exit, i.e., $X(\theta) = 0$ for all θ. To prove existence with exit, one must simply replace the discount factor δ for $\tilde{\delta} \equiv min\{\delta(\theta)\}$, where $\delta(\theta) = \delta(1 - X(\theta))$ is the discount factor that accounts for heterogeneous breakups. This change will only affect one of the assumptions discussed below and set an upper bound in the worse-case exit rate.

OA-3.1 Existence of Stationary Contract

The model in Jullien (2000) solves the following problem:

$$\max_{\{t(\theta),q(\theta)\}} \int_{\underline{\theta}}^{\overline{\theta}} [t(\theta) - cq(\theta)]f(\theta)d\theta \quad \text{s.t.} \tag{IR Problem}$$

$$v(\theta, q(\theta)) - t(\theta) \geq v(\theta, q(\hat{\theta})) - t(\hat{\theta}) \quad \forall \theta, \hat{\theta} \tag{IC}$$

$$v(\theta, q(\theta)) - t(\theta) \geq \bar{u}(\theta) \quad \forall \theta. \tag{IR}$$

Under a modified first-order approach, the seller's first-order condition is given by:

$$v_q(\theta, q(\theta)) - c = \frac{\gamma(\theta) - F(\theta)}{f(\theta)} v_{\theta q}(\theta, q(\theta), \tag{27}$$

for each type θ, and the complementary slackless condition on the IR constraints:

$$\int_{\underline{\theta}}^{\overline{\theta}} [u(\theta) - \bar{u}(\theta)]d\gamma(\theta) = 0. \tag{28}$$

Jullien (2000) shows that under three assumptions there exists a unique optimal solution in which all consumers participates, which is characterized by the first-order conditions 27 and complementary slackless condition 28 with $q(\theta)$ increasing. The first-assumption is potential separation (PS), which requires that the optimal solution is non-decreasing in θ, and satisfied under weak assumptions on the distribution of θ and the curvature of the surplus relative to the return of the buyer. In particular, it requires that

$$\frac{d}{d\theta}\Big(\frac{S_q(\theta, q)}{v_{\theta q}(\theta, q)}\Big) \geq 0$$
$$\frac{d}{d\theta}\Big(\frac{F(\theta)}{f(\theta)}\Big) \geq 0 \geq \frac{d}{d\theta}\Big(\frac{1 - F(\theta)}{f(\theta)}\Big).$$

The second and *key* assumption is homogeneity (H), requiring that there exists a quantity profile $\{\bar{q}(\theta)\}$ such that the allocation with full participation $\{\bar{u}(\theta), \bar{q}(\theta)\}$ is implementable in that $\bar{u}'(\theta) = v_\theta(\theta, \bar{q}(\theta))$ and $\bar{q}(\theta)$ is weakly increasing. This assumption implies that the reservation return can be implemented as a contract without excluding any type, ensuring that incentive compatibility is not an issue when the individual rationality constraint is binding. Lastly, the assumption of full participation (FP) assumes all types participate, and is satisfied when (H) holds and the surplus generated in the reservation return framework is greater than the private return to the buyer, i.e. $s(\theta, \bar{q}(\theta)) \geq \bar{u}(\theta)$.

I show that my setting can be rewritten in terms of Jullien (2000), implying that an optimal separating stationary contract exists. The seller chooses the optimal stationary contract $\{t(\theta), q(\theta)\}$ that satisfy incentive-compatibility and the limited enforcement constraint. Formally, the seller solves the problem:

$$\max_{\{t(\theta),q(\theta)\}} \frac{1}{1-\delta}\int_{\underline{\theta}}^{\overline{\theta}} [t(\theta)-cq(\theta)]f(\theta)d\theta \;\; \text{s.t.} \tag{LE Problem}$$

$$v(\theta,q(\theta))-t(\theta) \geq v(\theta,q(\hat{\theta}))-t(\hat{\theta}) \;\; \forall\theta,\hat{\theta} \tag{IC}$$

$$\frac{\delta}{1-\delta}\Big(v(\theta,q(\theta))-t(\theta)\Big) \geq t(\theta) = v(\theta,q(\theta))-u(\theta), \quad \forall\theta, \tag{LC}$$

where $u(\theta)$ is the return obtained by type θ. The limited enforcement constraint can be easily written as the IR constraint in Jullien (2000):

$$u(\theta) \geq (1-\delta)v(\theta,q(\theta)) \equiv \bar{u}(\theta) \;\; \forall\theta. \tag{LE'}$$

In my model, with $v(\theta,q) = \theta v(q)$, the first condition of assumption PS is always satisfied as

$$\frac{d}{d\theta}\left(\frac{S_q(\theta,q)}{v_{\theta q}(\theta,q)}\right) = \frac{d}{d\theta}\left(\theta - \frac{c}{v'(q)}\right) \geq 0 \iff 1 \geq 0 \tag{A1}$$

As stated earlier, the second condition of assumption PS is satisfied for a wide-range of distributions for θ. Therefore, assumption PS is satisfied for any of those distributions.

Then, consider Assumption H. It requires that an allocation $\{\bar{q}(\theta)\}$ exists such that $\bar{u}'(\theta) = v_\theta(\theta,\bar{q}(\theta))$ and $\bar{q}(\theta)$ is weakly increasing. Notice that under LE', we can define $\bar{q}(\theta)$ as $\bar{u}'(\theta) = (1-\delta)[\theta v'(q(\theta))q'(\theta)+v(q(\theta)] = v(\bar{q}(\theta))$. Define $G(\bar{q},\theta) = v(\bar{q})-(1-\delta)[\theta v'(q(\theta))q'(\theta)+v(q(\theta))] = 0$. By the implicit function theorem, $\bar{q}(\theta)$ is weakly increasing if

$$\begin{aligned}
\bar{q}'(\theta) &= -\frac{dG/d\theta}{dG/d\bar{q}} \\
&= \frac{(1-\delta)[v'(q(\theta))q'(\theta)+\theta v''(q(\theta))(q'(\theta))^2+\theta v'(q(\theta))q''(\theta)+v'(q(\theta))]}{v'(\bar{q})} \geq 0 \\
&\iff v'(q(\theta))[1+q'(\theta)+\theta q''(\theta))]+\theta v''(q(\theta))(q'(\theta))^2 \geq 0 \\
&\iff \frac{q'(\theta)+\theta q''(\theta)+1}{\theta(q'(\theta))^2} \geq A(q) \\
&\iff \left(\frac{T''(q)}{T'(q)}+A(q)\right)\Big(1+\theta(q)\theta'(q)r(q)+\theta'(q)\Big) \geq A(q) \\
&\iff \frac{T''(q)}{T'(q)}\frac{M(q)}{M(q)-1} \geq A(q),
\end{aligned}$$

where $M(q) \equiv 1+\theta(q)\theta'(q)r(q)+\theta'(q)$ and $r(q) = g^{-1}(q)$ for $g(\theta) \equiv q''(\theta)$. As we expect $T''(q) < 0$ and $T'(q) > 0$, it is necessary that $M(q)/(M(q)-1) < 0$. Such condition will be satisfied if $M(q) < 1$ and $M(q) > 0$, which imply that

$$\begin{gathered}
r(q)\theta(q) < -1 \\
\text{and} \\
\theta'(q) < \frac{1}{\theta(q)|r(q)|-1}.
\end{gathered} \tag{A2}$$

The first condition sets restrictions on the rate of change of quantities, which requires $q''(\theta)$ to be negative, restricting how convex $u(\theta)$ can be. The second condition requires that quantities increase at a minimum rate. Moreover, the condition sets bounds on the price discounts offered relative to the buyers' return curvature at a given quantity.

Lastly, full participation requires H to hold as well as $s(\theta, \bar{q}(\theta)) \geq (1-\delta)\theta v(\bar{q}(\theta))$. The condition becomes:

$$\delta \geq \frac{c\bar{q}(\theta)}{\theta v(\bar{q}(\theta))}, \tag{A3}$$

which requires that agents value the future high enough, such that discount factor be greater than the ratio of average cost to average return.

Let $\{t^{st}(\theta), q^{st}(\theta)\}$ be the solution to the to the problem characterized by equations 27 and 28. Assuming that the $v(\cdot)$, $F(\theta)$, and δ are such that A1, A2, and A3 hold for $\{t^{st}(\theta), q^{st}(\theta)\}$, then $\{t^{st}(\theta), q^{st}(\theta)\}$ is uniquely optimal.

OA-3.2 Optimality of Non-Stationary Contracts

Having established the existence of an optimal stationary contract, I now show that a non-stationary contract exists, which dominates the stationary contract. A similar argument was briefly discussed in the working paper version of Martimort et al. (2017).

Consider the following deviation from the stationary contract, in which at tenure 0, the return obtained by the buyer is given by:

$$u_0(\theta) = u^{st}(\theta) - \varepsilon,$$

for some $\varepsilon > 0$ sufficiently small, $u_{st=\theta v(q^{st}(\theta))-t^{st}(\theta)}$ and $t_0(\theta) = t^{st}(\theta)$. Define $q_0(\theta)$ to so it satisfies that the deviation defined above.. Under this deviation, the enforcement constraint at $\tau = 0$ is:

$$t^{st}(\theta) \leq \frac{\delta}{1-\delta}\Big[\theta v(q^{st}(\theta)) - t^{st}(\theta)\Big],$$

which is identical to the one in the stationary contract, which we know $\{t^{st}(\theta), q^{st}(\theta)\}$ satisfy. Moreover, the incentive compatibility constraint is still satisfied as $\hat{\theta}$ maximizes

$$u_0(\theta, \hat{\theta}) + \frac{\delta}{1-\delta} u^{st}(\theta, \hat{\theta}) = \frac{\delta}{1-\delta} u^{st}(\theta, \hat{\theta}) - \varepsilon,$$

where $u_\tau(\theta, \hat{\theta}) \equiv \theta v(q_\tau(\hat{\theta})) - t_\tau(\hat{\theta})$.

Under this alternative scheme, the seller obtains additional payoff ε while still satisfying both the incentive compatibility and limited enforcement constraints. Therefore, the optimal contract is non-stationary.

OA-4 Proof that Gamma Equals One for Highest Type

I prove that $\Gamma_\tau(\overline{\theta}) = 1$ for all τ. To begin, recall we assumed the outside option $\overline{u}_\tau(\theta)$ was equal to zero for all τ and all θ. Suppose instead that at some k, the outside option is uniformly shifted downward by > 0 for all θ, that is, $\overline{u}_k(\theta) = -\varepsilon$. The enforcement constraint at k is now given by:

$$\delta[\sum_{s=1}^{\infty} \delta^{s-1} u_{k+s}(\theta)] - \overline{u}_k(\theta) = \sum_{s=1}^{\infty} \delta^s u_{k+s}(\theta) + \varepsilon \geq t_k(\theta) = \theta v(q_k(\theta)) - u_k(\theta). \tag{29}$$

The seller's problem in the Lagrangian-form is

$$W(\varepsilon) = \max_{\{q_\tau(\theta), u_\tau(\theta)\}} \sum_{\tau=0}^{\infty} \delta^\tau \Big\{ \int_{\underline{\theta}}^{\overline{\theta}} [\theta v(q_\tau(\theta)) - cq_\tau - u_\tau(\theta)] f(\theta) d\theta + \tag{30}$$

$$\int_{\underline{\theta}}^{\overline{\theta}} [\sum_{s=1}^{\infty} \delta^s u_{\tau+s} + \varepsilon * 1\{\tau = k\} - t_\tau(\theta)] d\Gamma_\tau(\theta) \Big\} \tag{31}$$

such that $u'_\tau(\theta) = \theta v'(q_\tau(\theta))$ for all τ, θ. The change in the value of the problem of the seller given the uniform change in outside options is:

$$\frac{dW(\varepsilon)}{d\varepsilon} = \delta^k \int_{\underline{\theta}}^{\overline{\theta}} d\Gamma_k(\theta), \tag{32}$$

where the integral is the cumulative multiplier.

I argue that the quantities that solve the original problem still maximize the current one but that the transfers are all shifted upward by the constant . That is, if $q_\tau(\theta)$ is the solution for the problem with $\overline{u}_\tau(\theta) = 0$ for all θ and all τ with associated $t_\tau(\theta)$, $q_\tau(\theta)$ is also the solution for the problem with outside options $\overline{u}_\tau(\theta) = -\varepsilon 1\{\tau = k\}$ for all θ and all τ with associated transfers equal to $t_\tau(\theta) + \varepsilon 1\{\tau = k\}$. The value of the problem for the seller is:

$$W(\varepsilon) = \sum_{\tau=0}^{\infty} \delta^\tau \Big\{ \int_{\underline{\theta}}^{\overline{\theta}} [t_\tau(\theta) + \varepsilon 1\{\tau = k\} - cq_\tau] f(\theta) d\theta \Big\} \tag{33}$$

$$= \sum_{\tau=0}^{\infty} \delta^\tau \Big\{ \int_{\underline{\theta}}^{\overline{\theta}} [t_\tau(\theta - cq_\tau] f(\theta) d\theta \Big\} + \delta^k \varepsilon. \tag{34}$$

So

$$\frac{dW(\varepsilon)}{d\varepsilon} = \delta^k. \tag{35}$$

Therefore, the cumulative multiplier for any k will satisfy the following property:

$$\Gamma_k(\overline{\theta}) \equiv \int_{\underline{\theta}}^{\overline{\theta}} d\Gamma_k(\theta) = \frac{dW(\varepsilon)}{d\varepsilon} \frac{1}{\delta^k} = 1. \tag{36}$$

OA-5 Additional Theoretical Results

OA-5.1 Model Dynamics

Proofs are available in Supplemental Material Section SM3.

Quantity Discounts

Define $T_\tau(q_\tau(\theta)) \equiv t_\tau(\theta_\tau(q))$, $\Lambda_\tau(\theta) \equiv \Gamma_\tau(\theta) - \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\theta)) + \theta\gamma_\tau(\theta)$, and $\lambda_\tau(\theta) \equiv d\Lambda_\tau/d\theta$. The price schedule is said to feature quantity discounts if $T''_\tau(q) < 0$.

Proposition 1. *Assume strict monotonicity of quantity $q'_\tau(\theta) > 0$ and that $\lambda_\tau(\theta) < f_\tau(\theta)$. If the densities $f_\tau(\theta)$ satisfy log-concavity and $d(F_\tau(\theta)/f_\tau(\theta))/d\theta \geq F_\tau(\theta)/[(\theta - 1)f_\tau(\theta)]$, then the tariff schedule exhibits quantity discounts, $T''_\tau(q) \leq 0$ for each $q = q_\tau(\theta)$, $\theta \in (\underline{\theta}, \overline{\theta})$ and τ.*

Intuitively, the condition states that for a general class of distributions, as long as the incentive-compatibility marginal effects dominate those of the limited enforcement, the seller finds it optimal to offer quantity discounts at any relationship age. This is likely to be satisfied if the limited enforcement constraint is slack for some buyers already at their first interaction. Moreover, it also requires the enforcement constraint is slack for all buyers in the long run. This last requirement is in line with the model of Martimort et al. (2017), where buyers reach a *mature* phase in which the constraints no longer bind, as well as Proposition 2 below, which also finds that trade reaches a mature phase.

In terms of generality, the usual monopolist screening problem requires (or uses) log-concavity of $f(\theta)$.[33] I am strengthening the requirement that the evolution of the distribution also satisfies log-concavity, implicitly placing bounds on the distribution of exit rates over types.

The second condition strengthens the conditions on the dynamic distribution of types, in order to guarantee that the seller has the desire of price discriminating across types.

An alternative way to consider this property is to use t-RULE to obtain that the tariff schedule is concave if and only if $q'_\tau(\theta) > v'(q_\tau(\theta))/[-v''(q_\tau(\theta)\theta]$. As long as quantities increase by types fast enough, then the seller will offer quantity discounts. The rate at which the quantities have to increase is determined by the level of the type and the curvature of the return function.

Evolution of Quantities

Next, I discuss how quantities evolve in Proposition 2.

Proposition 2. *For each θ, quantity increases monotonically in τ (i.e., $q_\tau(\theta) \leq q_{\tau+1}(\theta)$) if and only if the limited enforcement constraint is relaxed over time ($\gamma_\tau(\theta) \geq \gamma_{\tau+1}(\theta)$). Moreover, there is a time τ^* such that $\forall\tau \geq \tau^*$, $\gamma_{\tau^*}(\theta) = 0$ for all θ and $q_{\tau^*}(\theta) \geq q_\tau(\theta)$ for all $\tau < \tau^*$ and all θ.*

In the model, quantities go hand-in-hand with enforcement constraints. Although the exact path depends on further assumptions on the return function and the distribution of types, the model predicts that quantities will reach a mature phase in which constraints no longer bind. At this mature phase, quantities will be at their highest level in the relationship.

Discounts over time

The model also offers conditions under which discounts over time are observed.

Proposition 3. *If $M_{\tau+1}(\theta) \equiv q_{\tau+1}(\theta) - q_\tau(\theta) \geq 0$ for all θ and with strict inequality for $\underline{\theta}$, then $p_{\tau+1}(q) \equiv T_{\tau+1}(q)/q < T_\tau(q)/q \equiv p_\tau(q)$.*

As long as quantities (weakly) increase from τ to $\tau + 1$, unit prices at any given q decrease. The intuition behind this result is that marginal prices match marginal returns. A right-ward shift in quantities for (some) buyers further lowers marginal returns, requiring a decrease in marginal prices as well. As such, average prices will be lower at each q as well.

To further understand the dynamics in the model, I present a solved two-type example in Supplemental Material Section SM4. The example illustrates the backloading of prices and quantities together with quantity discounts as a way to maximize lifetime profits for the seller while preventing opportunistic behavior from the buyer.

[33]Log-concavity of a density function $g(x)$ is equivalent to $g'(x)/g(x)$ being monotone decreasing. Families of density functions satisfying log-concavity include: uniform, normal, extreme value, exponential, amongst others.

OA-5.2 Static Efficiency of Limited Enforcement

We now turn to analyzing the efficiency of contracts with limited enforcement. Relationship-specfic total surplus (and thus efficiency) is determined by the total quantity transacted at a point in time. I concentrate on static (period-by-period) efficiency, as it is common in the relational contracting literature (e.g., as in Fong and Li, 2017; Kostadinov and Kuvalekar, Forthcoming), rather than total lifetime efficiency.

For simplicity, suppose that $\theta\gamma_\tau(\theta)$ is small enough so the quantities allocated in the limited enforcement contract with no exit ($X(\theta) = 0$) and the assumed parametrization of $v(\cdot)$ are given by:

$$q_\tau^{LEm}(\theta)^{1-\beta} = \frac{k\beta}{c}\Big[\theta - \frac{\Gamma_\tau(\theta) - F_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))}{f(\theta)}\Big].$$

With some abuse of notation, define the modified value of the cumulative multiplier at time τ as $\tilde{\Gamma}_\tau(\theta) = \Gamma_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))$, so the allocation is given by:

$$q_\tau^{LEm}(\theta)^{1-\beta} = \frac{k\beta}{c}\Big[\theta - \frac{\tilde{\Gamma}_\tau(\theta) - F(\theta)}{f(\theta)}\Big].$$

Moreover, recall that the first-best outcome is given by:

$$q_\tau^{FB}(\theta)^{1-\beta} = \frac{k\beta}{c}\theta.$$

If $\tilde{\Gamma}_\tau(\theta) < F(\theta)$, there is overconsumption relative to first best. If $\tilde{\Gamma}_\tau(\theta) > F(\theta)$, there is underconsumption. If $\tilde{\Gamma}_\tau(\theta) = F(\theta)$, trade is fully efficient. Therefore, this limited enforcement model allows for the possibility of efficient trade, as well as inefficient trade either through underconsumption or overconsumption.

For the case with underconsumption, i.e., $\tilde{\Gamma}_\tau(\theta) > F(\theta)$, efficiency increases over time if $\tilde{\Gamma}_\tau(\theta) < \tilde{\Gamma}_{\tau-1}(\theta)$. By reordering and eliminating repeated terms, the condition becomes $\Gamma_\tau(\theta) < 1$. Thus, under the case with no exit and underconsumption, we expect efficiency to increase until pair-wise trade becomes unconstrained. Note, however, that quantities may converge at inefficient levels.

OA-5.3 Static Efficiency Relative to Perfect Enforcement

Comparing equations SFOC and PE, in the case with no exit $X(\theta) = 0$ for all θ, the total quantity transacted is greater under full enforcement than under limited enforcement if:

$$(1-\Gamma_\tau(\theta)) + \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) - \theta\gamma_\tau(\theta) < 0. \tag{37}$$

For the types for which the limited enforcement constraint is not binding (so $\gamma_\tau(\theta) = 0$), except for the highest type, the inequality does not hold, and pair-wise welfare decreases under full enforcement. This will likely matter for middle/high types early on. Moreover, it might apply too for lower types in the long-term that started with binding constraints at the beginning for the contract but that grew over time to become unconstrained. Therefore, welfare can be greater under a long-term relational contract with limited enforcement than under perfect enforcement.

For types with $\gamma_\tau(\theta) > 0$, the inequality can be written as:

$$\theta - \frac{1-\Gamma_\tau(\theta)}{\gamma_\tau(\theta)} > \frac{\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))}{\gamma_\tau(\theta)}.$$

The inequality above reminds us of a modified virtual surplus, where instead of the distribution of types we use the distribution of enforcement constraints. For perfect enforcement to be welfare increasing, the virtual surplus accounting for contemporaneous information rents of limited enforcement has to be greater than the information rents (promises to increase quantity) stemming from past enforcement constraints. Of course, early on, perfect enforcement could be more efficient, yet, as relationships age this might be more difficult to sustain.

In contrast with the arguments set forward in past literature, I have shown that in the interaction of market power and enforcement constraints could imply that weak legal enforcement is actually efficiency *increasing* at some points in time, and particularly so in the long-run. Intuitively, absent enforcement constraints, the seller is able to offer *the* profit-maximizing menu of quantities and prices. The buyer's ability to act opportunistically restricts how much the seller can extract and changes the surplus in favor of the buyer.

OA-6 Estimation

This section discusses the details of the estimation procedure.

OA-6.1 Tariff Function

In identification, I treated the tariff function $T_\tau(\cdot)$ as given. However, I observe only pairs of payments and quantities $(t_{i\tau}, q_{i\tau})$ for $i = 1, 2, ..., N_\tau$ for each tenure. The pricing model discussed in section 4 implies that observed transfers lie on the curve $t = T(q)$, as they are both functions of the type $\theta_{i\tau}$ in a given tenure. As noted by Luo et al. (2018), observed prices and quantities may not lie on the curve, if there is measurement error or unobserved heterogeneity, introducing additional randomness beyond $\theta_{i\tau}$.

To deal with this additional randomness, I follow Perrigne and Vuong (2011), which show that the tariff function is nonparametrically identified under the assumption that observed tariffs differ from optimal tariffs due to random measurement error. In particular, observed tariffs are a function of optimal tariffs $t_{i\tau} = T(q_{i\tau})e^{\upsilon_{i\tau}}$, such that $\upsilon_{i\tau}$ is independent of $q_{i\tau}$.

I consider a parametric version of the model, in which $T_\tau(q) = e^{\beta_{0\tau}} q^{\beta_{1\tau}}$. This leads to the estimation model with measurement error:

$$ln(t_{i\tau}) = \beta_{0\tau} + \beta_{1\tau} ln(q_{i\tau}) + \upsilon_{i\tau}, \tag{38}$$

where $t_{i\tau}$ is the observed tariff and $q_{i\tau}$ is the observed quantities for buyer i with tenure τ. Under the given assumption of independence, the tariff schedule can be estimated via ordinary least squares. The estimated tariff schedule linking observed quantities is $\widehat{T}_\tau(q_{i\tau}) = e^{\hat{\beta}_{0\tau}} q_{i\tau}^{\hat{\beta}_{1\tau}}$, while the marginal tariff is $\widehat{T}'_\tau(q_{i\tau}) = \hat{\beta}_{1\tau} t_{i\tau}/q_{i\tau}$. Note that I allow for differences in tariff schedules across τ, responding to the dynamic treatment of the problem, i.e. the same level of quantity q may have different associated tariffs if the buyer-seller relationship is new or have been sustained for some years.

OA-6.1.1 *Heterogenous Hazard Rates*

I estimate heterogenous hazard rates at the percentile-tenure level. In particular, I rank buyers in percentiles of quantity for each tenure in 2016. I then calculate the share of buyers in each percentile that survived until 2017. To reduce the noise and preserve a monotonicity of hazard rate, I then approximate the estimated nonparametric hazard rates as a logistic function of percentiles:

$$S_\tau(r) = \frac{exp(a_\tau + b_\tau r)}{1 + exp(a_\tau + b_\tau r)} + \varepsilon^s_\tau(r), \tag{39}$$

where $S_\tau(r)$ is the share of buyers surviving from 2016 until 2017 in percentile rank r for tenure τ and $\varepsilon^s_\tau(r)$ is Gaussian noise orthogonal to r.

OA-6.2 Marginal Cost

Marginal cost is estimated directly from the data under the assumption that marginal cost is equal to average variable cost. As defined in Section 2, average variable cost is defined as total expenditures and total wages divided by total quantity sold.

OA-6.3 LE Multipliers

Recall that the LE multiplier $\Gamma_\tau(\alpha)$ has the properties of a cumulative distribution function. Following Attanasio and Pastorino (2020), I parametrize the multiplier as a logistic distribution:[34]

$$\Gamma_\tau(\alpha) = \frac{exp(\phi_\tau(q_\tau(\alpha)))}{1 + exp(\phi_\tau(q_\tau(\alpha)))}, \tag{40}$$

where $\phi_\tau(q_\tau(\alpha))$ is a polynomial up to the second degree. Under this parametrization, the derivative of the multiplier is $\gamma_\tau(\alpha) = \phi'_\tau(q_\tau(\alpha))\Gamma_\tau(\alpha)(1 - \Gamma_\tau(\alpha))$.

Moreover, I parametrize $\theta'(\alpha)/\theta(\alpha)$ as a inverse quadratic function of quantity:

$$\frac{\theta'(\alpha)}{\theta(\alpha)} = \frac{1}{d_0 + d_1 q_\tau(\alpha) + d_2 q_\tau(\alpha)^2}. \tag{41}$$

The key identification equation I:EQ provides the following estimating equation:

$$\begin{aligned}\frac{\hat\beta_{1\tau} p_\tau(\alpha) - \widehat{c}}{\hat\beta_{1\tau} p_\tau(\alpha)} = & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(Main Est. Eq.)}\\ & \frac{1}{d_0 + d_1 q_\tau(\alpha) + d_2 q_\tau(\alpha)^2}\Big[\frac{exp(\phi_\tau(q_\tau(\alpha)))}{1 + exp(\phi_\tau(q_\tau(\alpha)))} - \alpha - \widehat{M}_\tau(\alpha)\Big]\\ & + \phi'_\tau(q_\tau(\alpha))\frac{exp(\phi_\tau(q_\tau(\alpha)))}{1 + exp(\phi_\tau(q_\tau(\alpha)))}\Big(1 - \frac{exp(\phi_\tau(q_\tau(\alpha)))}{1 + exp(\phi_\tau(q_\tau(\alpha)))}\Big) + \varepsilon^g_\tau(\alpha),\end{aligned}$$

where I have used $p_{i\tau} = t_{i\tau}/q_{i\tau}$ and where ε^g is measurement error coming from the mispecification of Γ, the tariff function, or the marginal cost. Moreover, past multipliers are captured by $\widehat{M}_\tau(\alpha) \equiv \sum_{s=0}^{\tau-1} \widehat{S}_s^{\tau-s}(1 - \widehat{\Gamma}_s(\alpha))$, with $\widehat{S}_s$ and $\widehat{\Gamma}_s(\alpha)$ for $s < \tau$ estimated in earlier stages and taken in τ as given. The equation is estimated via maximum likelihood under the assumption that ε^g is drawn from a Gaussian with parameters $(0, \sigma^{\varepsilon^g})$. This step in the estimation process recovers the parameters $\{\phi_\tau, d_0, d_1, d_2, \sigma^{\varepsilon^g}\}$.

To match previously estimated LE multipliers $\Gamma_s(\theta)$ to $\theta(\alpha)$ at tenure τ, I use the estimated hazard rates to generate a percentile-percentile transition matrix. Then, I can match percentiles matching α_s for $s < \tau$ to percentiles matching α_τ. Moreover, I use the estimated hazard rates for τ corresponding to α to properly discount past promises captured in past multipliers.

[34]The multiplier function is the solution to a differential equation. As shown in Supplemental Material Section SM2, it is a function of the cumulative distribution of types θ, the marginal cost, and the expected base marginal return (i.e., depends on the curvature of the return function).

OA-6.4 Buyer Types and Type Distribution

Once Γ_τ and γ_τ are estimated, the consumer type $\theta_\tau(\alpha)$ is obtained from

$$ln(\widehat{\theta}_\tau(\alpha)) = \tag{42}$$

$$\frac{1}{N_\tau}\sum_{k=1}^{N_\tau}\frac{1\{\alpha \geq k/N_\tau\}}{\widehat{\Gamma}_\tau(k/N_\tau) - k/N_\tau - \widehat{M}_\tau(k/N_\tau)}\Big[1 - \frac{\widehat{c}}{\hat{\beta}_{1\tau}p_\tau(k/N_\tau)} - \widehat{\gamma}_\tau(k/N_\tau)\Big], \tag{43}$$

for $\alpha \in [0, (N_\tau - 1)/N_\tau]$ and where N_τ is the total count of buyers of tenure τ. The estimator for $\theta'_\tau(\alpha)$ is

$$\widehat{\theta'}_\tau(\alpha) = \frac{\widehat{\theta}_\tau(\alpha)}{\widehat{\Gamma}_\tau(\alpha) - \alpha - \widehat{M}_\tau(k/N_\tau)}\Big[1 - \frac{\widehat{c}}{\hat{\beta}_{1\tau}p_\tau(\alpha)} - \widehat{\gamma}_\tau(\alpha)\Big]. \tag{44}$$

Finally, the density function $\widehat{f}_\tau(\theta(\alpha))$ is $1/\widehat{\theta'}_\tau(\alpha)$.

OA-6.4.1 *Base Marginal Return and Return Function*

The derivative of the transfer rule links the base marginal return with the marginal tariff and the consumer type: $v'(q_\tau(\alpha)) = T'_\tau(q_\tau(\alpha))/\theta_\tau(\alpha)$. Therefore, an estimator for the base marginal return is

$$\widehat{v'(q_\tau(\alpha))} = \frac{\hat{\beta}_{1\tau}p_\tau(\alpha)}{\widehat{\theta}_\tau(\alpha)}. \tag{45}$$

Following the discussion in the identification section, $v(\cdot)$ is estimated by

$$v(q_\tau(\alpha)) = \widehat{T}_\tau(q_\tau(0)) + \frac{1}{N_\tau}\sum_{k=1}^{N_\tau}\widehat{v'(q_\tau(k/N_\tau))}1\{\alpha \geq k/N_\tau\}. \tag{46}$$

OA-6.5 Parametrization of $v(\cdot)$ for Counterfactual Analysis

To calculate pair-specific efficient (first-best) quantities, I require estimated buyer types θ, base marginal returns $v'(\cdot)$ and seller marginal costs c. The range of optimal quantities may not be covered by the range of realized quantities, and thus, base marginal returns may be undefined for some quantities. For that reason, during counterfactual analysis, I parametrize the seller-specific marginal return functions $v(\cdot)$ as $v(q) = kq^\beta$, for $k > 0$ and $\beta \in (0, 1)$ and estimate these functions for each seller using linear least squares and the values of estimated marginal returns $\widehat{v'(\cdot)}$.

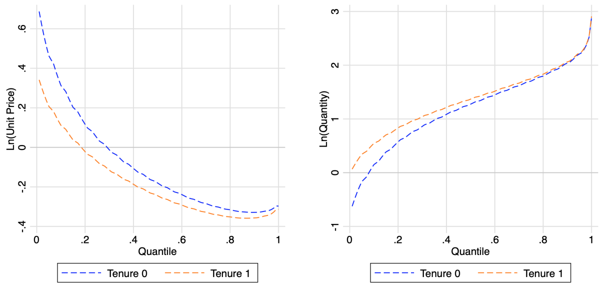
OA-7 Monte Carlo Study

The Monte Carlo studies the behavior of my estimators for two periods of a dynamic contract without breakups. I use the following design. The return function is $v(\theta, q) = \theta q^{1/2}$. The type distribution is Weibull with scale parameter equal to 1 and shape parameter equal to 2, $F(\theta) = 1 - exp(-(\theta - 1)^k)$, normalized so $\underline{\theta} = 1$.[35] Marginal cost is 0.45. Although the multiplier function $\Gamma_\tau(\theta)$ is the solution to a differential equation linking the type distribution $F(\theta)$, the marginal cost, and the average base marginal return of types $\tilde{\theta} \leq \theta$, I parametrize it as a logistic distribution. In tenure 0, $\Gamma_0(\theta)$ has

[35] Recall that the model requires the type distribution to verify the monotone hazard rate condition, $\frac{d}{d\theta}\frac{F(\theta)}{f(\theta)} \geq 0 \geq \frac{d}{d\theta}\frac{1-F(\theta)}{f(\theta)}$. Distributions that satisfy the monotone hazard rate condition include: Uniform, Normal, Logistic, Extreme Value (including Frechet), Weibull (shape parameter ≥ 1), Exponential, and Power functions.

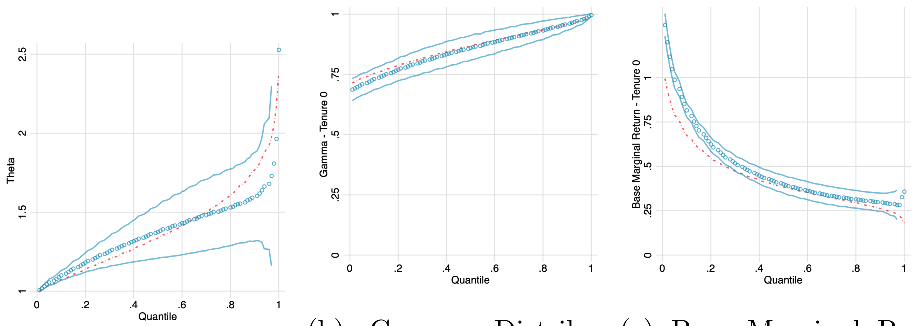
Figure OA-3: **Prices and Quantities by Quantile**

(a) **Unit Prices** (b) **Quantity**



Notes: These figures show the level of prices and quantities by quantile of quantity for tenure 0 and tenure 1 in the Monte Carlo simulation.

Figure OA-4: **Monte Carlo Results for Tenure 0**



(a) Types θ (b) Gamma Distribution (c) Base Marginal Return

Notes: Panel (a) plots the true (red) and estimated distribution of types (in blue) by quantile of quantity. Panel (b) plots the true (red) and estimated value (blue) of the LE multiplier for tenure 0 by quantile of quantity. Panel (c) plots the true (red) and estimated value (blue) of the base marginal return for tenure 0 by quantile of quantity. Error margins indicate ± 1.96 variation around estimated mean from 300 simulations.

location parameter equal to 1 and scale parameter equal to 0.5. Instead, in tenure 1, $\Gamma_1(\theta)$ has location parameter 1 and scale 0.35. The lower scale parameter at tenure 1 reflects the idea that over time, the limited enforcement constraint is less binding. I construct the tariffs following Pavan et al. (2014): $t_\tau(\theta) = \theta q_\tau(\theta)^{1/2} - \int_{\underline{\theta}}^{\theta} q_\tau(x)^{1/2} dx$.

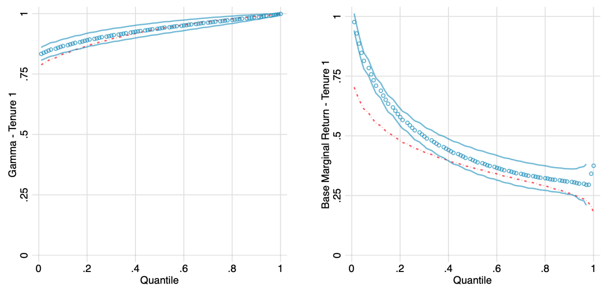
I randomly draw 1000 values of θ using $F(\theta)$ and obtain corresponding quantities $q_0(\theta)$ and $q_1(\theta)$ using the first-order condition of the seller and the assumed parametrizations of the return function, marginal cost, and multiplier at tenure 0 and 1. Then, I obtain the corresponding tariffs and I apply my estimator as defined in the previous sections to estimate $\{\theta, U(\cdot), \Gamma_\tau(\cdot)\}$. I repeat this 300 times to construct the dispersion for my estimates.

Online Appendix Figure OA-3 shows the (log) average prices and average quantities generated by the model for the two types of tenure. The model delivers quantity discounts (decreasing unit prices in θ), strict mononoticity of quantity (increasing quantities in θ), and backloading in the dynamic model, namely, further discounts and larger quantities in tenure 1 for each θ.

Online Appendix Figure OA-4 shows the results of the estimated Gamma distribution

Figure OA-5: **Monte Carlo Results for Tenure 1**

(a) Gamma Distribution (b) Base Marginal Return



Notes: Panel (a) plots the true (red) and estimated value (blue) of the LE multiplier for tenure 1 by quantile of quantity, with error margins indicating ±1.96 variation around the estimated mean. Panel (b) plots the true (red) and estimated value (blue) of the base marginal return for tenure 1 by quantile of quantity, with error margins indicating ±1.96 variation around the estimated mean from 300 simulations.

and the base marginal return, again in blue the estimated results and in red the true values. Both cases indicate good fit. Subfigure (a) shows the estimated $\hat{\theta}$ in blue and true θ in red by quantile. Dispersion at the 95 percent level are included for all except the top 2 quantiles, as they start to diverge. Overall, the figure shows a good fit, with most sections of including the true θ within their dispersion.

Next, I show the tenure 1's results estimates. Recall that the first-order condition of the seller now includes a backward-looking variable $1 - \Gamma_0(\theta)$ that keeps track of whether the limited commitment constraint was binding in the past. This variable is used by seller as a promise-keeping constraint that guarantees the seller delivers higher quantities and return in the future to prevent buyers from defaulting in the past. In my estimation, I use the tenure 0's predicted $\widehat{\Gamma}_0(\theta(\alpha))$ for each quantile α. Online Appendix Figure OA-5 shows the estimated Gamma distribution and the base marginal return. Although the fit is worse than in tenure 0, the dispersion of both gamma and the base marginal return include tend to include their true values.

With respect to the differences between true and estimated functions, I find that the slight upward bias in the Gamma function for tenure 1 disappears if I use the true $\Gamma_0(\theta)$ function instead of the estimated $\widehat{\Gamma}_0$, suggesting that the bias is generated by sampling error in the tenure 0 estimates. Moreover, differences in the base marginal return for both tenure 0 and tenure 1 come from approximating the tariff function as log-linear. In the Monte-Carlo, the change in unit price is very steep for low-types, and this generates some approximation error for low-types in terms of the base marginal return function. Despite this error, the coefficient of the base return function is correctly estimated when using the assumed parametrization, observations of quantity, and the nonparametric estimates of $v'(\cdot)$ as target. In particular, the estimated coefficient cannot be rejected to be different from 0.5 (the assumed value in simulation).

OA-8 Additional Estimation Results and Model Fit

OA-8.1 Distribution of t-Statistics against Standard Model Null

Online Appendix Table OA-8.1 show the distribution of t-statistics for tests against a standard model null.

Table OA-9: **Distribution of t-Statistics**

	p10	p25	p50	p75	p90
Tenure 0	0.31	4.64	11.55	30.08	109.27

Notes: This table reports distribution of t-statistics for tests against a standard model null (e.g., $\Gamma_0(\cdot) = 1$).

OA-8.2 Parametrization of the Base Return Function

To conduct counterfactual experiments that consider quantities beyond those observed in the data, I parametrize the seller-specific buyer's return function $v(q) = kq^{\beta}$ for $k > 0$ and $\beta \in (0, 1)$. This return function satisfies modelling assumptions $v'(\cdot) > 0$ and $v''(\cdot) < 0$. To estimate parameters, I consider tenure 0 transactions between buyer i and seller j at year t and perform the following uniform least squares regression:

$$ln(\widehat{v'}_{ijt}) = ln(k) + ln(\beta) + (\beta - 1)ln(q_{ijt}) + \varepsilon_{ijt},$$

using $v'(q) = k\beta q^{\beta-1}$, the estimated base marginal returns $\widehat{v'}_{ijt}$ and under the assumption that ε_{ijt} is Gaussian error. Online Appendix Table OA-10 present the distribution of k and β.

Table OA-10: **Parameters of Return Function**

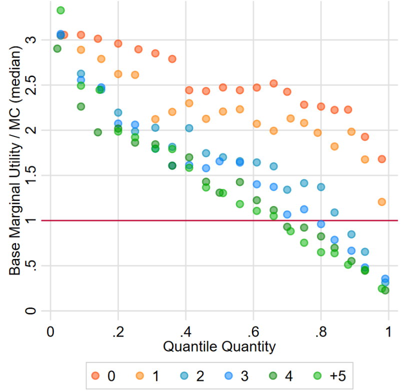
	mean	p10	p25	p50	p75	p90
β	0.56	0.30	0.48	0.61	0.76	0.82
k	171.23	9.00	17.24	39.64	86.61	282.40

Notes: This table reports distribution of estimated values for the ex-post parametrization of the return function.

OA-8.3 Economic Magnitudes: Base Marginal Return

Online Appendix Figure OA-6 presents a binscatter of the ratio marginal revenue product (base marginal return) over marginal costs aginst the quantile of quantity, across sellers for tenure 0. It shows that the return of the input for the buyer is greater than the private marginal cost of providing it for the seller, for a majority of the buyers. For instance, the median buyer obtains 1.5 dollars of revenue for each dollar spent by the seller to produce the product.

Figure OA-6: **Base Marginal Return over Marginal Costs**

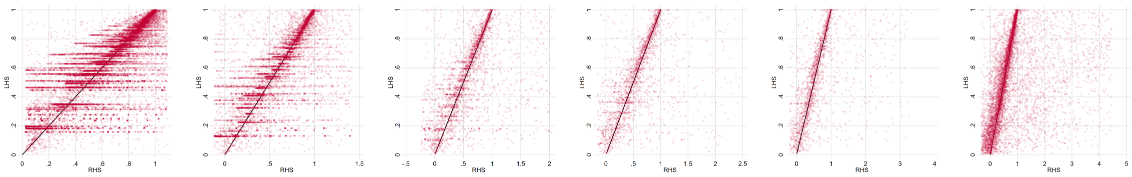


Notes: This figure plots the median of the ratio of base marginal return by marginal costs across sellers by quantile of quantity for each tenure.

OA-8.4 Model Fit

Online Appendix Figure OA-7 presents the statistical fit of the model across tenures. It plots a reordered equation I:EQ's left-hand side on the X-axis and the model's prediction using estimated coefficients of the right-hand side on the Y-axis.[36] Fit generally worsens for higher tenures; the results from Monte Carlo studies in Online Appendix OA-7 suggest that the decrease in statistical fit is driven by noise from using estimates for limited enforcement multipliers $\Gamma_s(\cdot)$ for earlier tenures s.

Figure OA-7: **Model Fit - Statistical**



(a) T0 (b) T1 (c) T2 (d) T3 (e) T4 (f) T5

Notes: These figures show binscatters of statistical fit of the model across tenures as implied by identification equation I:EQ. On the X-axis, it shows the predicted cumulative distribution function for the observation while on the Y-axis it plots the observed value.

Online Appendix Figure OA-8 shows the fit in terms of quantities. To obtain quantities, I use the parametrization $v(q) = kq^{\beta}$, for $k > 0$ and $\beta \in (0, 1)$ and the closed-form formula in Q-CES.

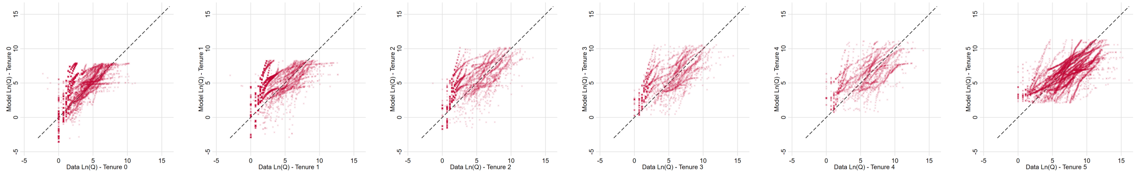
Online Appendix Figure OA-9 shows the fit of tariffs. To generate tariffs in the model, I use the empirical equivalent of equation t-RULE.

[36] Reorder equation I:EQ to obtain:

$$\alpha = \Gamma_{\tau}(\alpha) - \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\alpha)) - \left[\frac{T'_{\tau}(q_{\tau}(\alpha)) - c_{\tau}}{T'_{\tau}(q_{\tau}(\alpha))} - \gamma_{\tau}(\alpha)\right]\frac{\theta_{\tau}(\alpha)}{\theta'_{\tau}(\alpha)},$$

and use the estimated analogues of the right-hand side to make the predictions.

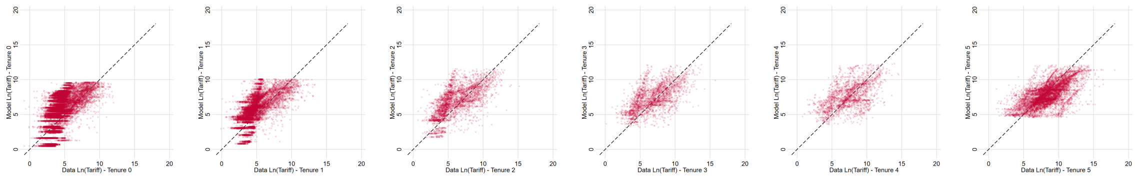
Figure OA-8: **Model Fit - Quantities**



(a) T0 (b) T1 (c) T2 (d) T3 (e) T4 (f) T5

Notes: These figures display binscatters of model fit according to quantities. Estimated quantities use the close-form formula under the CES parametrization of the return function, as discussed in Appendix OA-10. The X-axis plots the observed (log) quantities and Y-axis model predicted (log) quantities.

Figure OA-9: **Model Fit - Tariffs**



(a) T0 (b) T1 (c) T2 (d) T3 (e) T4 (f) T5

Notes: These figures display binscatters of model fit according to tariffs. Estimated tariffs are generated by using the empirical analogue of the transfer rule *t*-RULE, taking as inputs estimated parameters θ, the parametrized return function $v(\cdot)$, and model generated quantities. The X-axis plots the observed (log) tariffs and Y-axis model predicted (log) tariffs.

OA-9 Additional Counterfactual Results

This subsection presents comparisons of different counterfactual models relative to baseline nonlinear pricing regime with limited enforcement. The tables present the share of observations in each percentile group for which each reported category (e.g., buyer's net return) is greater under the baseline than under the alternative. Online Appendix Table OA-11 shows all the results.

The main takeaways are the following.

Buyers. Small-quantity buyers tend to prefer limited enforcement of contracts over perfect enforcement. They can effectively use the threat of default to reap higher returns. Instead, the median and top buyers prefer perfect enforcement in the short-term but limited enforcement in the long-term.

Under weak enforcement of contract, buyers prefer price discrimination than uniform pricing, as otherwise they would be excluded from trade (only median and top buyers prefer uniform pricing in the long-term). However, if exclusion and default are restricted, most buyers prefer uniform pricing.

Sellers. They prefer limited enforcement in the short-term but perfect enforcement in the long-term. Under weak enforcement of contract, they enjoy the ability to price discriminate, as it allows them to sell to buyers that would otherwise be excluded from trade. Instead, if enforcement is strong, they prefer uniform pricing in the short-term but price discrimination in the long-term. This is driven by the rapid increase in quantities, despite the decrease in unit prices offer to most buyers as incentive not to default.

Table OA-11: **Counterfactual Policies**

		Nonlinear + Perfect					**Uniform + Limited**					**Uniform + Perfect**				
		10%	25%	50%	75%	100%	10%	25%	50%	75%	100%	10%	25%	50%	75%	100%
Buyer Return	*Tenure 0*	24.2	24.2	10.9	4.8	6.8	97.3	96.5	96.0	94.3	91.6	0.1	0.2	0.6	6.9	40.6
	Tenure 1	68.3	55.3	23.0	9.4	11.8	94.6	92.2	88.6	88.1	87.4	0.1	0.1	0.2	14.0	55.9
	Tenure 2	62.5	45.3	30.6	25.9	28.4	82.1	79.2	70.2	66.5	63.2	0.9	0.4	0.9	10.0	31.8
	Tenure 3	65.3	59.9	40.2	32.3	38.0	76.9	71.6	58.3	54.4	55.9	3.1	0.8	1.6	11.2	28.3
	Tenure 4	63.5	43.4	36.9	33.9	44.2	74.6	62.1	44.8	41.7	39.8	5.4	1.2	4.9	8.7	17.4
	Tenure 5	52.7	59.6	65.7	60.8	68.9	65.5	53.7	37.3	33.2	30.9	0.7	1.6	2.8	7.0	19.9
Seller Profit	*Tenure 0*	34.1	41.6	88.3	95.0	93.1	92.7	92.6	96.4	98.0	98.8	7.1	7.4	11.1	35.0	47.9
	Tenure 1	53.8	55.0	83.3	90.6	88.2	99.1	96.7	94.8	97.1	90.5	28.4	19.1	29.8	45.9	51.3
	Tenure 2	49.0	50.4	72.0	74.1	71.6	95.0	97.0	98.2	99.5	97.6	34.3	35.1	50.9	70.2	85.7
	Tenure 3	45.2	48.1	61.7	68.1	62.0	96.5	99.0	97.0	99.3	93.7	50.2	50.0	61.9	76.9	86.6
	Tenure 4	48.6	53.2	65.3	66.1	55.8	92.0	98.5	95.1	95.2	94.5	54.0	64.2	71.5	86.5	93.5
	Tenure 5	61.1	44.5	37.7	39.3	31.1	94.0	87.8	95.1	97.4	95.8	63.2	66.3	81.9	92.8	94.6
Surplus	*Tenure 0*	18.6	18.9	9.0	3.8	2.6	98.4	98.1	98.8	98.5	99.5	3.8	4.1	5.2	12.0	65.0
	Tenure 1	40.5	41.7	30.3	12.6	29.6	97.5	96.2	97.3	99.2	100.0	6.0	7.4	11.0	31.9	76.1
	Tenure 2	47.2	50.7	48.3	63.2	72.6	90.0	92.2	91.7	98.6	99.7	17.0	18.8	27.6	57.0	95.0
	Tenure 3	60.9	57.6	69.7	76.6	70.1	90.6	92.5	88.1	98.4	99.6	24.3	26.0	37.4	69.1	98.4
	Tenure 4	66.7	71.9	74.5	77.1	67.3	86.2	89.6	85.9	98.3	99.5	28.1	40.1	53.8	79.2	97.9
	Tenure 5	74.2	79.6	88.6	91.4	84.8	82.0	78.3	80.3	96.9	100.0	30.5	34.2	53.7	86.1	99.9
Unit Prices	*Tenure 0*	76.3	75.7	89.1	94.6	93.1	93.6	93.1	95.4	90.3	43.6	93.6	93.1	95.4	90.3	43.6
	Tenure 1	56.0	55.7	77.8	90.5	88.1	98.6	96.8	87.9	68.2	25.0	98.6	96.8	87.9	68.2	25.0
	Tenure 2	44.0	56.0	68.3	74.0	71.7	92.6	94.8	90.9	64.0	18.3	92.3	94.7	90.9	64.0	18.3
	Tenure 3	37.3	41.5	58.8	67.9	61.8	91.2	96.7	89.2	55.1	13.3	90.8	96.7	89.2	55.1	13.3
	Tenure 4	38.2	56.4	63.1	67.6	55.6	88.3	95.5	88.0	65.3	20.0	87.7	95.5	88.0	65.2	20.0
	Tenure 5	43.5	38.2	35.9	38.2	31.1	90.6	91.3	87.7	53.5	10.5	89.8	91.1	87.6	53.2	10.0
% Excluded	*Tenure 0*	-	-	-	-	-	97.3	96.4	95.8	94.1	90.4	-	-	-	-	-
	Tenure 1	-	-	-	-	-	93.4	91.9	88.6	87.3	85.8	-	-	-	-	-
	Tenure 2	-	-	-	-	-	79.8	77.4	70.0	65.6	61.3	-	-	-	-	-
	Tenure 3	-	-	-	-	-	74.4	69.2	58.0	51.4	50.0	-	-	-	-	-
	Tenure 4	-	-	-	-	-	71.3	60.1	44.7	38.5	37.4	-	-	-	-	-
	Tenure 5	-	-	-	-	-	62.3	50.7	36.4	30.3	25.8	-	-	-	-	-

Notes: This table reports the % share of observations for which the reported category (e.g., Buyer's Net Return) is greater under the observed nonlinear pricing regime than under the alternative policy. The values are reported across different tenures and percentiles in the distribution of types. The policies considered are 1) Nonlinear pricing with perfect enforcement, 2) Uniform monopolist pricing with limited enforcement, and 3) Uniform monopolist pricing with perfect enforcement. The reported categories are Buyer's Net Return, Seller's Profits, Total Surplus, Unit Prices, and percentage of Excluded Buyers.

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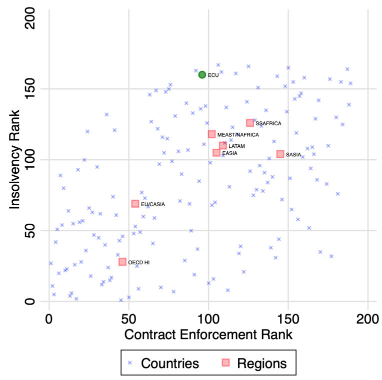
Supplemental Material (Not for Publication)

SM1 Additional Descriptive Statistics

Supplemental Material Figure SM1 shows the position of Ecuador in terms of contract enforcement and insolvency in the World Bank Doing Business report. Lower numbers represent better institutions to enforce contracts or solve insolvency cases.

Figure SM1: **Ranks Insolvency and Enforcement**



Notes: This figure presents the location of Ecuador in the World Bank Doing Business ranks in the categories of Insolvency (Y-Axis) and Enforcement (X-Axis). Most efficient country in terms of enforcement ranks 1st.

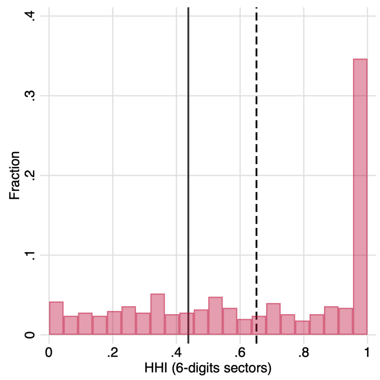
Supplemental Material Figure SM2 shows the distribution of Herfindahl-Hirschman Indices (HHI) for manufacturing 6-digit sectors in 2017. HHI_s for sector s is estimated using the following formula:

$$HHI_s = \sum_{j \in J_s} m_j^2,$$

where m_j is the market share of firm j, J_s is the set of active firms in sector s. The market share of firm j is obtained by dividing total revenue of firm j by the sum total revenue of all firms in sector s.

Supplemental Material Figure SM3 presents the cumulative distribution of quantities, both in relative terms through standardized quantities and in absolute values through residualized log quantities, for different age of relationships.

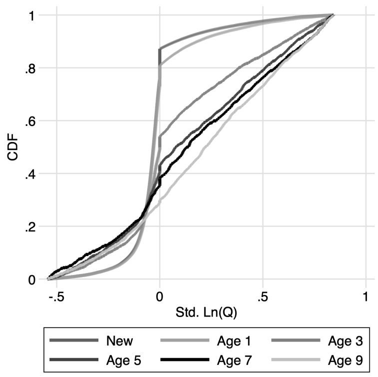
Figure SM2: **Distribution of Herfindahl-Hirschman Indices for Manufacturing in 2017**



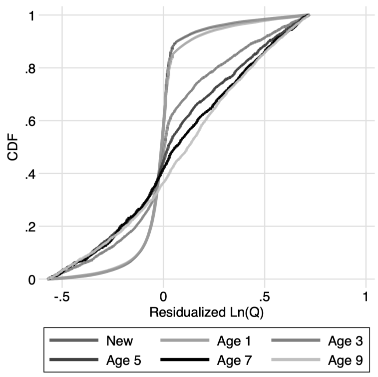
Notes: This figure presents a histogram of estimated Herfindahl-Hirschman Indices (HHI) for 6-digit manufacturing sectors in 2017.

Figure SM3: **Cumulative Distribution Function of Quantities by Relationship Age**

(a) Product-level Standardized Ln(Q)



(b) Seller-level Residualized Ln(Q)



Notes: These figures plot the cumulative distribution functions for standardized log quantities (left) and residualized log quantities (right) by different ages of relationship.

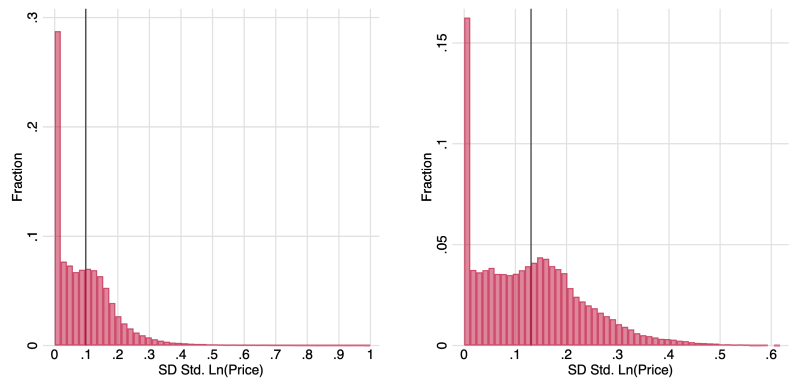
SM1.1 Price and Quantity Dispersion

Supplemental Material Figures SM4 and SM5 show the dispersion of standardized log prices and quantities, respectively.

Supplemental Material Figure SM4 shows that the average product has an average standard deviation of prices close to 0.10. This implies that in a given, the same product could have prices that are 10% higher or lower than the average price more than 30% of the time. Similarly, Supplemental Material Figure SM5 shows that the average standard deviation of quantities for a given product in a month is close to 0.4.

Figure SM4: **Product-level Price Dispersion within Month and Year**

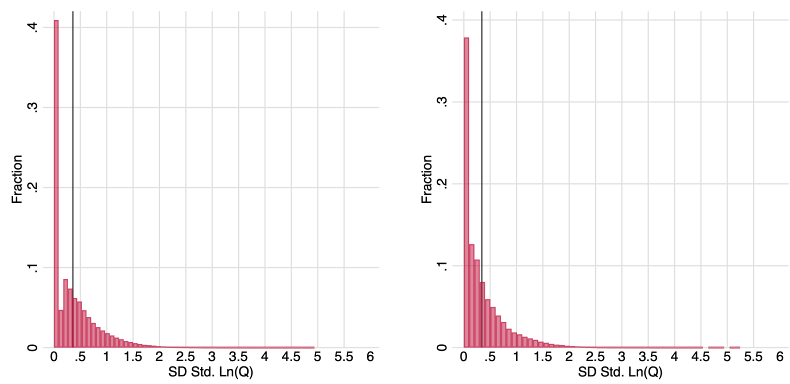
(a) Month (b) Year



Notes: These figures plot histograms of the standard deviation of standardized log prices by month and year, for products that have at least 5 distinct buyers in time window.

Figure SM5: **Product-level Quantity Dispersion within Month and Year**

(a) Month (b) Year



Notes: These figures plot histograms of the standard deviation of standardized log quantity by month and year, for products that have at least 5 distinct buyers in time window.

SM2 Solution of Gamma Function for Stationary Equilibrium

The seller's first order condition defines the following differential equation in the stationary equilibrium

$$\theta u'(q(\theta)) - c = \frac{\Gamma(\theta) - F(\theta) + (1-\delta)\theta\gamma(\theta)}{f(\theta)} u'(q(\theta)). \tag{47}$$

The solution $\Gamma(\theta)$ to the equation above is given by:

$$\Gamma(\theta) = \frac{\int_{\underline{\theta}}^{\theta} x^{\delta/(1-\delta)}[xf(x) - c(u'(q(x))^{-1}f(x) + F(x)]dx + K}{\theta^{1/(1-\delta)}(1-\delta)}, \tag{48}$$

which by integration by parts reduces to:

$$\Gamma(\theta) = \frac{F(\theta)}{1-\delta} - \frac{\delta \int^{\theta} x^{\delta/(1-\delta)} F(x)dx}{(1-\delta)\theta^{1/(1-\delta)}} - \frac{cE[x^{\delta/(1-\delta)}u'(q(x))^{-1}|x \leq \theta]}{(1-\delta)\theta^{1/(1-\delta)}} + \frac{K}{(1-\delta)\theta^{1/(1-\delta)}} \tag{49}$$

The constant is obtained by using the boundary condition $\Gamma(\overline{\theta}) = 1$. Therefore,

$$K = cE[x^{\delta/(1-\delta)}u'(q(x))^{-1})] - \delta\overline{\theta}^{1/(1-\delta)} + \delta \int x^{\delta/(1-\delta)}F(x)dx. \tag{50}$$

SM3 Proofs - Model Dynamics

Proof of Proposition 1. Recall the quantity function $q_\tau(\theta)$ and its inverse function $\theta_\tau(q)$. Further differentiating the derivative of the incentive-compatible tariff schedule $T'_\tau(q_\tau(\theta)) = \theta v'(q_\tau(\theta))$ gives:

$$T''_\tau(q) = \theta'_\tau(q)v'(q) + \theta_\tau(q)v''(q) = \theta(q)v'(q)\left[\frac{\theta'_\tau(q)}{\theta_\tau(q)} + \frac{v''(q)}{v'(q)}\right] \tag{51}$$

$$= T'(q)\left[\frac{1}{\theta_\tau(q)q'_\tau(\theta)} - A(q)\right], \tag{52}$$

for $A(q) = -v''(q)/v'(q)$ and $\theta'_\tau(q) = 1/q'_\tau(\theta)$.

By implicit differentiation on the seller's first-order condition Number we obtain an expression for $q'_\tau(\theta)$:

$$q'_\tau(\theta) = -\frac{\frac{d}{d\theta}\left[\theta - \frac{\Gamma_\tau(\theta) - F_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]v'(q_\tau(\theta))}{\left[\theta - \frac{\Gamma_\tau(\theta) - F_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]v''(q_\tau(\theta))}$$

$$= \frac{1}{A(q_\tau(\theta))} \frac{\frac{d}{d\theta}\left[\theta - \frac{\Gamma_\tau(\theta) - F_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]}{\left[\theta - \frac{\Gamma_\tau(\theta) - F_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]}$$

The denominator of the equation above is positive as $v'(q_\tau(\theta)) > 0$ and $c > 0$. As by assumption, strict monotonicity holds $q'_\tau(\theta) > 0$, then the numerator is also positive. Substituting in 51 and using the fact that $T'_\tau(q) > 0$ and $A(q_\tau) > 0$, quantity discounts $T''_\tau(q) \leq 0$ hold if and only if

$$\frac{\left[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]}{\theta\frac{d}{d\theta}\left[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\right]} \leq 1 \tag{53}$$

Define the $\Lambda_\tau(\theta) \equiv \Gamma_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_\tau(\theta)$ and $\lambda_\tau(\theta) \equiv d\Lambda_\tau(\theta)/d\theta$. Inequality 53 holds if

$$\theta - \frac{\Lambda_\tau(\theta) - F_\tau(\theta)}{f_\tau(\theta)} \leq \theta - \theta\frac{(\lambda_\tau(\theta) - f_\tau(\theta))f_\tau(\theta) - (\Lambda_\tau(\theta) - F_\tau(\theta))f_\tau'(\theta)}{f_\tau(\theta)^2}.$$

Rearranging, one obtains

$$[\Lambda_\tau(\theta) - F_\tau(\theta)][f_\tau(\theta) + f_\tau'(\theta)\theta] \geq \theta f(\theta)[\lambda_\tau(\theta) - f_\tau(\theta)]. \tag{54}$$

As noted above, $\theta f_\tau(\theta) \geq \Lambda_\tau(\theta) - F_\tau(\theta)$. Note that log-concavity of the density $F_\tau(\theta)$ is sufficient to satisfy the assumption of monotone hazard condition. For log-concave densities, the following inequality holds $f_\tau(\theta) \geq f_\tau'(\theta)\theta$. Therefore, if $\Lambda_\tau(\theta) > F_\tau(\theta)$, then a sufficient condition for quantity discounts is $\lambda_\tau(\theta) < f_\tau(\theta)$.

Instead if $\Lambda_\tau(\theta) < F_\tau(\theta)$, one can write 53 as

$$(\theta-1)f_\tau(\theta) + f_\tau(\theta) \geq [F_\tau(\theta) - \Lambda_\tau(\theta)]\Big(1 + \frac{f_\tau'(\theta)\theta}{f_\tau(\theta)}\Big) + \lambda_\tau(\theta). \tag{55}$$

If $f_\tau'(\theta) < 0$, then a sufficient condition is $(\theta-1)f_\tau(\theta) \geq F_\tau(\theta)$. If $f_\tau'(\theta) > 0$, then a sufficient condition is that $(\theta-1)f(\theta) \geq F_\tau(\theta)(1+\theta f_\tau'(\theta)/f_\tau(\theta))$, which can be expressed as:

$$\frac{d}{d\theta}\Big(\frac{F_\tau(\theta)}{f_\tau(\theta)}\Big) = \frac{f_\tau(\theta)^2 - F_\tau(\theta)f_\tau'(\theta)}{f_\tau(\theta)^2} \geq \frac{F_\tau(\theta)}{(\theta-1)f_\tau(\theta)}. \tag{56}$$

□

Proof of Proposition 2. Notice that by the seller's first-order condition and $v'(\cdot) > 0$, $q_\tau(\theta) \leq q_{\tau+1}(\theta)$ holds if and only if

$$\begin{aligned} V_\tau(\theta) &\equiv \frac{\Gamma_\tau(\theta) - F_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_\tau(\theta)}{f_\tau(\theta)} \\ &\geq \frac{f_\tau(\theta)}{f_{\tau+1}(\theta)}\frac{\Gamma_\tau(\theta) - F_{\tau+1}(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_{\tau+1}(\theta)}{f_\tau(\theta)} + \frac{\Gamma_{\tau+1}(\theta)-1}{f_{\tau+1}(\theta)}, \end{aligned}$$

which can be written as

$$V_\tau(\theta) \geq \frac{f_\tau(\theta)}{f_{\tau+1}(\theta)}V_\tau(\theta) + \frac{\Gamma_{\tau+1}(\theta)-1}{f_{\tau+1}(\theta)} + \frac{\theta[\gamma_{\tau+1}(\theta) - \gamma_\tau(\theta)]}{f_{\tau+1}(\theta)} - \frac{F_{\tau+1}(\theta) - F_\tau(\theta)}{f_{\tau+1}(\theta)}.$$

With no selection pattern, i.e. $f_\tau(\theta) = f_{\tau+1}(\theta)$, the condition reduces to

$$\frac{1-\Gamma_{\tau+1}(\theta)}{f_\tau(\theta)} \geq \frac{\theta[\gamma_{\tau+1}(\theta) - \gamma_\tau(\theta)]}{f_\tau(\theta)}.$$

As $\gamma_\tau(\theta) > 0$ by assumption and the left-hand side is (weakly) positive due to $\Gamma_{\tau+1}(\theta) \leq 1$, a sufficient condition is that $\gamma_{\tau+1}(\theta) < \gamma_\tau(\theta)$. To obtain necessity, consider the Lagrangian keeping future return U^+ constant. The seller chooses $q(\theta)$ maximizing the following program:

$$L(\theta, U, q, \lambda, \gamma) = (\theta v(q(\theta)) - cq(\theta) - U)f(\theta) + \lambda v(q(\theta)) + \gamma(U + \delta U^+ - \theta v(q(\theta))), \quad (57)$$

where λ is the co-state variable for the incentive-compabilitity constraint and γ is the multiplier for the limited enforcement constraint. Noting that the necessary conditions are also sufficient (Seierstad and Sydsaeter, 1986) (pg. 276). The relevant optimality conditions are:

$$f(\theta)[\theta v'(q(\theta)) - c] + \lambda(\theta)v'(q(\theta)) = \gamma(\theta)\theta v'(q(\theta))$$

and

$$\dot{\lambda}(\theta) = f(\theta) - \gamma(\theta)$$

which imply

$$\gamma(\theta) = f(\theta) - \frac{cf(\theta)}{\theta v'(q(\theta))} + \frac{F(\theta) - \Gamma(\theta)}{\theta}.$$

Therefore, a higher level of quantity $q(\theta)$ is implies with a lower $\gamma(\theta)$.

For $\gamma_\tau(\theta) = 0$ for some finite $\tau > \tau^*$ for all θ. Suppose otherwise, such that $\gamma_\tau(\tilde{\theta}) > 0$ for some $\tilde{\theta}$ and all τ. Then, $\Gamma_\tau(\theta) < 1$ for all $\theta \leq \tilde{\theta}$. Therefore, $1 - \Gamma_\tau(\theta) > 0$ for all $\theta \leq \tilde{\theta}$. Thus, as $\tau \to \infty$, $\sum_{s=0}^{\tau}(1 - \Gamma_s(\theta)) \to \infty$ for all $\theta \leq \tilde{\theta}$. Thus, as long as $q_\tau(\theta) < \infty$ for all θ, τ, it must be the case that some finite τ^* exists such that $\gamma_\tau(\theta) = 0$ for all $\tau > \tau^*$ and for all θ.

For $q_{\tau^*}(\theta) > q_\tau(\theta)$ for all $\tau < \tau^*$ and all θ. Notice that $q_{\tau^*}(\theta) \geq q_\tau(\theta)$ if and only if

$$\theta\gamma_\tau(\theta) + \sum_{s=\tau+1}^{\tau^*-1}(1 - \Gamma_s(\theta)) \geq 0,$$

which always holds. It holds with strict inequality whenever the enforcement constraint binds, or when it binds in some period between τ and τ^* for some θ between θ and $\overline{\theta}$.

□

Proof of Proposition 3. Use the marginal price function $T'_\tau(q) = \theta_\tau(q)v'(q)$. Average unit prices $p_\tau(q)$ for $q > 0$ are given by:

$$p_\tau(q) = \frac{T_\tau(q)}{q} = \frac{\int_0^q \theta_\tau(x)v'(x)dx}{q},$$

where I have used the normalization $T_\tau(0) = 0$ and the inverse function $\theta_\tau(q)$. Average prices decrease over time if and only if

$$\int_0^q \theta_\tau(x)v'(x)dx > \int_0^q \theta_{\tau+1}(x)v'(x)dx$$

$$\iff$$

$$\int_0^q [\theta_\tau(x) - \theta_{\tau+1}]v'(x)dx > 0.$$

By assumption, $q_\tau(\theta) \geq q_{\tau+1}(\theta)$ (and strictly so for $\underline{\theta}$). Thus, $\theta_\tau(q) > \theta_{\tau+1}(q)$ for all q and the inequality holds. □

SM4 A Two-Type Illustrative Example

The purpose of this example is four-fold. First, I illustrate how the introduction of the limited enforcement constraint may distort quantities relative to perfect enforcement. Second, I show that lower types unambiguously reap higher net returns due to the enforcement constraint. The introduction of the enforcement constraints effectively raises their reservation return to participate in trade, forcing the seller to offer larger shares of surplus to lower types. Third, I demonstrate that the optimal contract must be non-stationary. Fourth, I show through a solved example that the optimal stationary contract features *backloading*: unit prices decrease while quantities increase as relationships age.

SM4.1 Buyer's Types

A buyer type-θ gains a gross return θq^β from q units of the product sold by the seller. Assume there are positive, yet diminishing marginal returns, i.e., $\beta \in (0, 1)$. The buyer types can take values $\{\theta_L, \theta_H\}$, such that $\theta_L < \theta_H$. Let f_L (resp. f_H) be the probability that buyer is type L (resp. type H) and assume no exit, i.e., $X(\theta) = 0$.

SM4.2 A Stationary Contract

For now, consider the optimal *stationary* contract. The optimal choice gives the buyer the net return $R(\theta_i) = \theta_i q_i^\beta - T(q_i)$. The seller designs the scheme to maximize:

$$max_{\{T_i, q_i\}} f_L(T_L - cq_L) + (1 - f_L)(T_H - cq_H)$$

where $T_i \equiv T(q_i)$, subject to incentive-compatibility constraints:

$$R(\theta_H) \equiv \theta_H q_H^\beta - T_H \geq \theta_H q_L^\beta - T_L, \tag{IC-H}$$

$$R(\theta_L) \equiv \theta_L q_L^\beta - T_L \geq \theta_L q_H^\beta - T_H. \tag{IC-L}$$

as well as the limited enforcement constraint:

$$\frac{\delta}{1 - \delta}(R(\theta_i)) \geq T_i \quad i = L, H. \tag{LE-i}$$

This last constraint effectively (weakly) raises the minimum net rent that each buyer needs to obtain to participate in trade. The usual nonlinear pricing problem only requires that $R(\theta_i) \geq 0$. Instead, the limited enforcement case requires that $R(\theta_i) \geq (1 - \delta)/\delta T_i > 0$, where the minimum return is endogenously determined. Notice that as $\delta \to 1$, the limiting case becomes the standard nonlinear pricing problem.[37]

To simplify the problem, assume that the IC-L and LE-H are slack while IC-H and LE-L are binding.[38] By using these assumptions on the constraints, one can obtain the optimal quantity allocations:

$$q_H^* = \left(\frac{\beta}{c}\theta_H\right)^{\frac{1}{1-\beta}},$$
$$q_L^* = \left(\frac{\beta}{c}\left[\theta_L - \frac{(1-\delta)\theta_L}{f_L} - \frac{(1-f_L)(\theta_H - \theta_L)}{f_L}\right]\right)^{\frac{1}{1-\beta}},$$

[37] The theoretical result that the buyer benefits from a deterioration of enforcement was previously discussed by Genicot and Ray (2006). In their model, they find that if better enforcement brings with it the deterioration of outside options and the seller has the bargaining power, the buyer will see their expected payoff increase. The opposite holds when the buyer has the bargaining power.

[38] All slack constraints are verified for the numerical example discussed below.

and optimal transfers:

$$T_H^* = \theta_H q_H^\beta + (\delta\theta_L - \theta_H) q_L^\beta,$$
$$T_L^* = \delta\theta_L q_L^\beta.$$

The program's solution implies there is no distortion in quantities for type-H, as they purchase at the first-best level. However, type-L's purchases are shifted downwards. First, as is common in adverse selection problems, their purchases are distorted downwards to incentivize the revelation of type-H.

Second, contrary to the standard problem, extracting all rents from type-L is no longer feasible. As such, the standard quantity allocation for θ_L (i.e., when $\delta = 1$), together with the optimal transfers for L under limited enforcement do not satisfy IC-H. To see this, notice that as IC-H was binding in the standard problem, type-H was on the margin between their standard bundle and the standard bundle for type-L. Thus, if the limited enforcement bundle for type-L keeps quantities fixed (relative to the standard menu) and at the same time asks or lower transfers, type-H buyers would now prefer the menu intended for type-L. As a result, the seller needs to reduce type-L's allocation, even further than would be required under the standard adverse selection problem.

SM4.3 Non-Stationarity

Relative to the standard problem, the seller now needs to offer positive net returns to all buyers, in order to prevent default. Contrary to the results in Baron and Besanko (1984), the stationary contract is no longer the optimal contract. Instead, the seller could offer a dynamic contract with intertemporal incentives that uses the promise of future returns to the buyer to discipline their behavior now. Through this approach, the seller can extract higher shares of surplus early on than would be feasible under a stationary contract, increasing their present-value lifetime profits.

The exact dynamic path depends on the return function and distribution of types of the buyer, as well as the marginal cost of the seller and the common discount factor. For that reason, I consider next a solved numerical example.

SM4.4 A Visual Example

To visualize the problem, I consider a numerical example with the following values for the parameters: $\beta = 0.25$, $c = 1$, $f_L = 0.95$, $\theta_L = 10$, $\theta_H = 20$, $\delta = 0.9$.

Supplemental Material Figure SM6 shows the levels of quantities, prices, profits per buyer, and buyer's net return for the example discussed above for different regimes: stationary with perfect enforcement, stationary with limited enforcement, and dynamic with limited enforcement.

With the solid lines, the figure shows the stationary solution both under weak enforcement and perfect enforcement. In solid green, the figure shows the allocation for type-H. As mentioned above, limited enforcement of contracts does not distort their consumption relative to perfect enforcement. In solid blue, the figure shows the allocation for type-L under perfect enforcement. Type-L receives lower quantities and higher prices than type-H and receives zero net return.

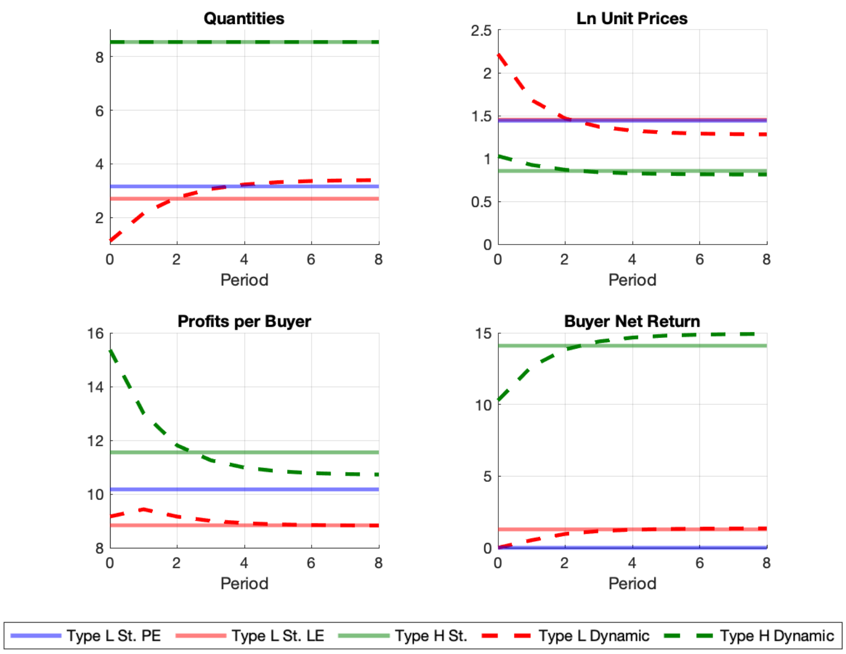
In solid red, the figure shows the allocation for type-L under limited enforcement. Relative to perfect enforcement, type-L sees a reduction in quantities and an increase in net return, in line with the logic explained above. Importantly, as the buyer's return function features diminishing returns in q, lower levels of quantity for lower values of δ also imply the seller can charge *higher* unit prices to type-L.

Lastly, the figure shows the optimal non-stationary path of prices and quantities in the dashed lines. The optimal path features *backloading* as quantities (weakly) increase and unit prices (weakly) decrease over time. As shown in the figure, this path of prices and quantities increases expected present-value lifetime profits from each buyer relative to the optimal stationary contract. The seller can effectively prevent default now and increase present-value lifetime profits by offering higher surplus levels to the buyers in the future.

Interestingly, the optimal path in the solved example features consumption for type-L in the long-run that is greater than the stationary contracts with and without limited enforcement. That is, through dynamic contracts, long-term allocations could potentially be more efficient than contracts under perfect enforcement.

In any case, the example shows that through the interaction market power on the seller side (which is reflected in the ability to offer incentive-compatible profit-maximizing menus) and the limited enforcement constraint, long-term contracts may display dynamics in which average quantities increase and unit prices decrease over time. Moreover, at any point in time, types consuming higher levels of quantities also enjoy lower unit prices. That is, this model of price discrimination with limited enforcement of contracts features i) *backloading* of prices and quantities, and ii) *quantity discounts* at any point in time.

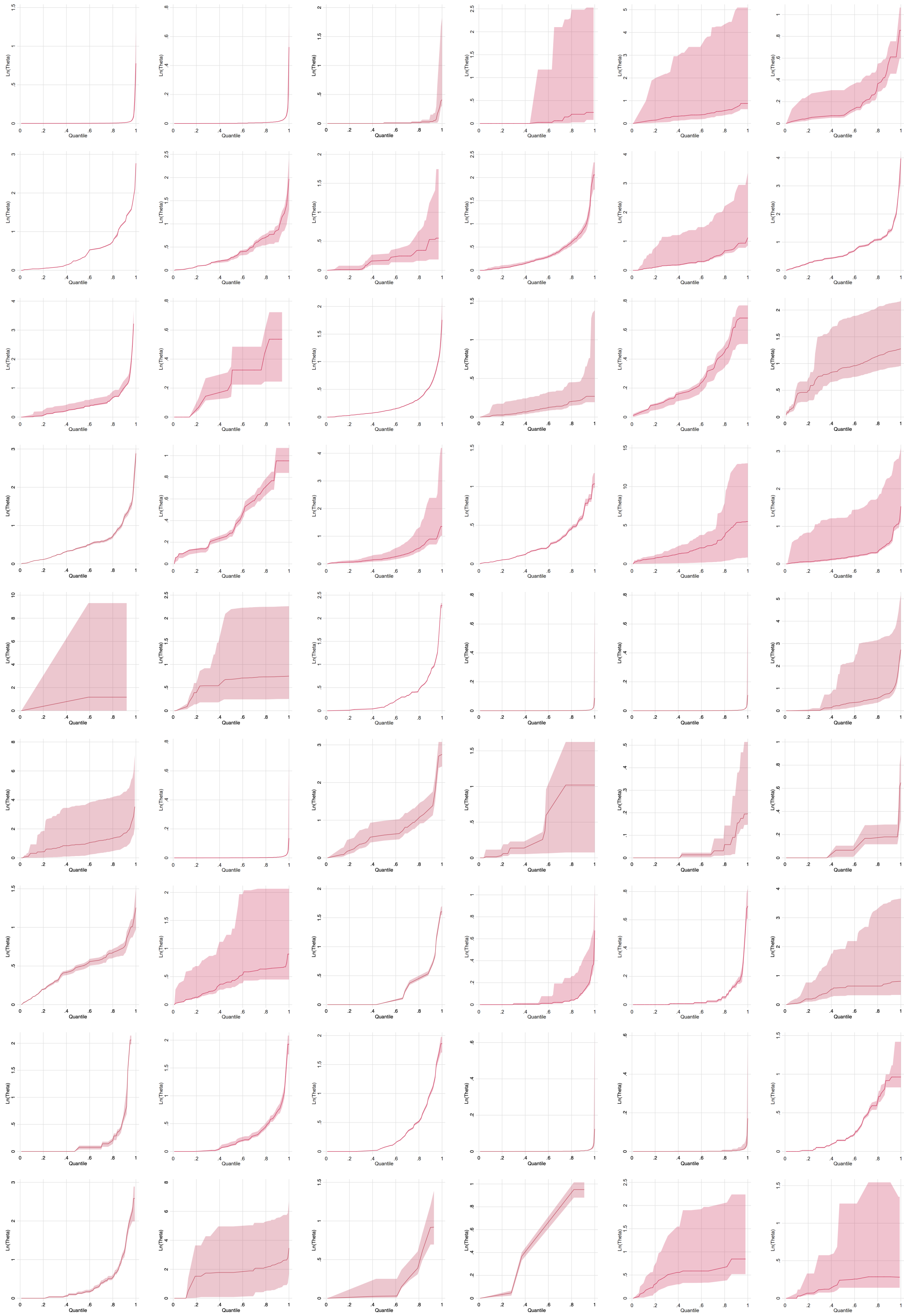
Figure SM6: **Example - Nonlinear Pricing and Limited Enforcement**

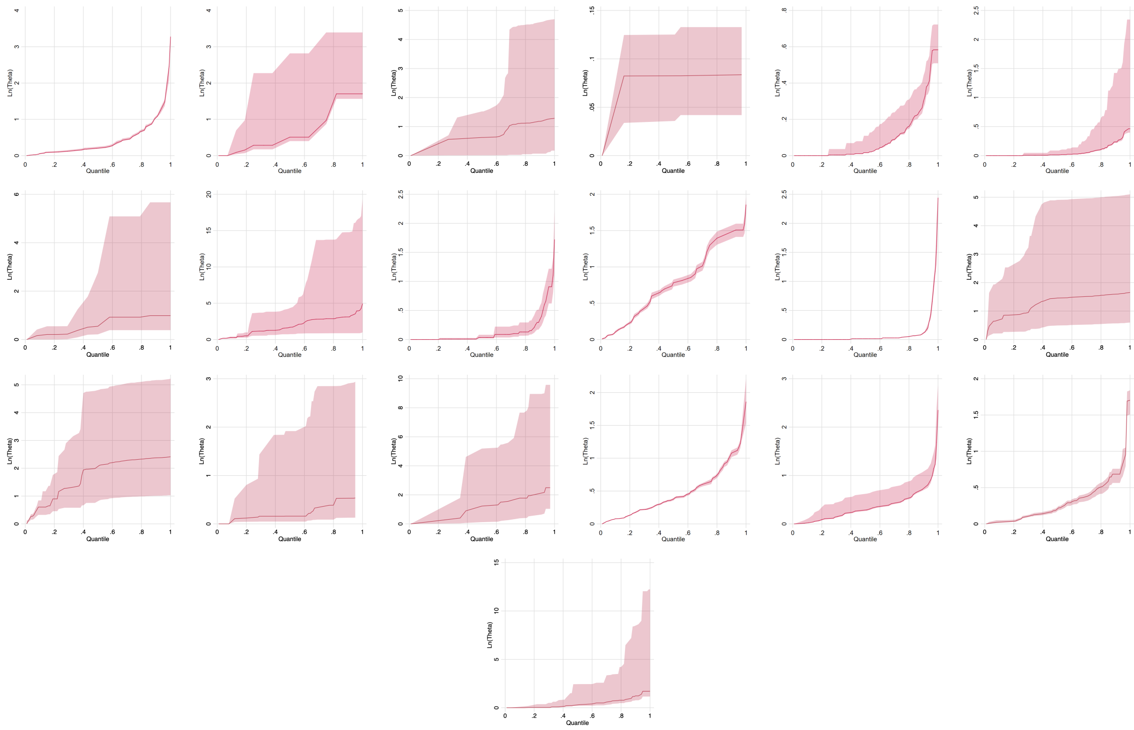


Notes: This figure shows Quantities, Prices, Profits, and Buyer Net Return for different enforcement and contract regimes. In solid green, the optimal stationary contract for type-H under perfect enforcement and limited enforcement. In dashed green, the optimal dynamic contract for type-H under limited enforcement. In solid blue, the optimal stationary contract for type-L under perfect enforcement. In solid red, the optimal stationary contract for type-L under limited enforcement. In dashed red, the optimal dynamic contract for type-H under limited enforcement. The parameters used in the example are: $\{\beta = 0.25,\ c = 1,\ f_L = 0.95,\ \theta_L = 10,\ \theta_H = 20,\ \delta = 0.9\}$.

SM5 Bootstrapped Distribution of Types

In this section, I present the boostrapped distribution for types of each seller-year.




Ln(Theta)
Quantile

Supplemental Material References

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