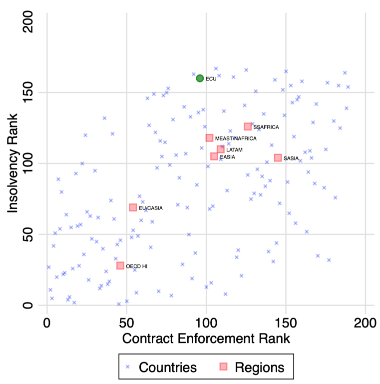
Online Appendix

OA-1 Summary Statistics

Online Appendix Figure OA-1 shows Ecuador's position in terms of contract enforcement and insolvency in the World Bank Doing Business report. Lower numbers represent better institutions to enforce contracts or resolve insolvency cases.

Figure OA-1: **Ranks Insolvency and Enforcement**



Notes: This figure presents Ecuador's rank in the World Bank Doing Business categories of Insolvency (Y-Axis) and Enforcement (X-Axis). The most efficient country in terms of enforcement ranks 1st.

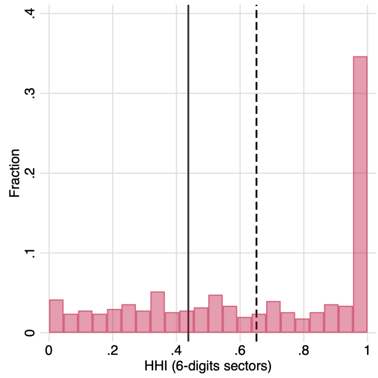
Online Appendix Figure OA-2 shows the distribution of Herfindahl-Hirschman Indices (HHI) for 6-digit manufacturing sectors in 2017. HHI_s for sector s is estimated using the following formula:

$$HHI_s = \sum_{j \in J_s} m_j^2,$$

where m_j is the market share of firm j, and J_s is the set of active firms in sector s. The market share of firm j is obtained by dividing the total revenue of firm j by the total revenue of all firms in sector s.

Online Appendix Figure OA-3 presents the cumulative distribution of quantities, both in relative terms through standardized quantities and in absolute values through residualized log quantities, for different ages of relationships.

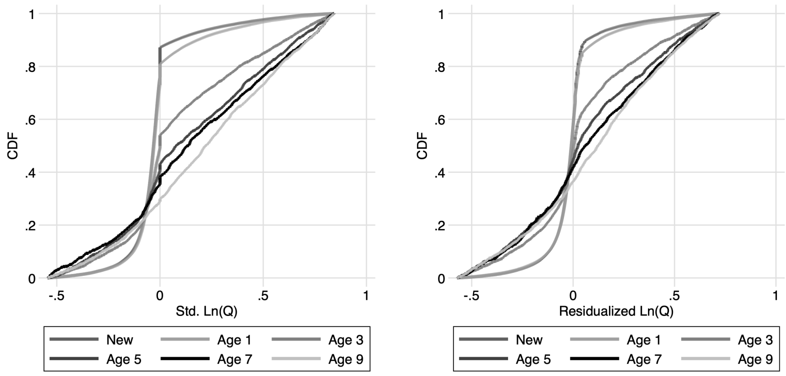
Figure OA-2: **Distribution of Herfindahl-Hirschman Indices for Manufacturing in 2017**



Notes: This figure presents a histogram of estimated Herfindahl-Hirschman Indices (HHI) for 6-digit manufacturing sectors in 2017.

Figure OA-3: **Cumulative Distribution Function of Quantities by Relationship Age**

(a) Product-level Standardized Ln(Q) (b) Seller-level Residualized Ln(Q)



Notes: These figures plot the cumulative distribution functions for standardized log quantities (left) and residualized log quantities (right) by different ages of relationships.

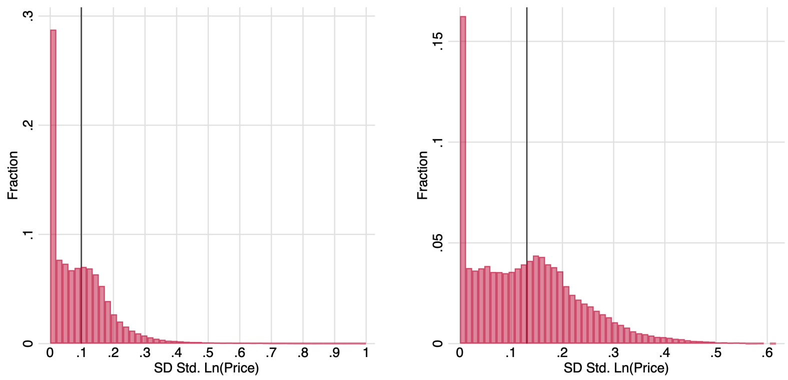
OA-1.1 Price and Quantity Dispersion

Online Appendix Figures OA-4 and OA-5 show the dispersion of standardized log prices and quantities, respectively.

Online Appendix Figure OA-4 shows that the average product has an average standard deviation of prices close to 0.10. This implies that the same product could have prices that are 10% higher or lower than the average price more than 30% of the time. Similarly, Online Appendix Figure OA-5 shows that the average standard deviation of quantities for a given product in a month is close to 0.4.

Figure OA-4: **Product-level Price Dispersion within Month and Year**

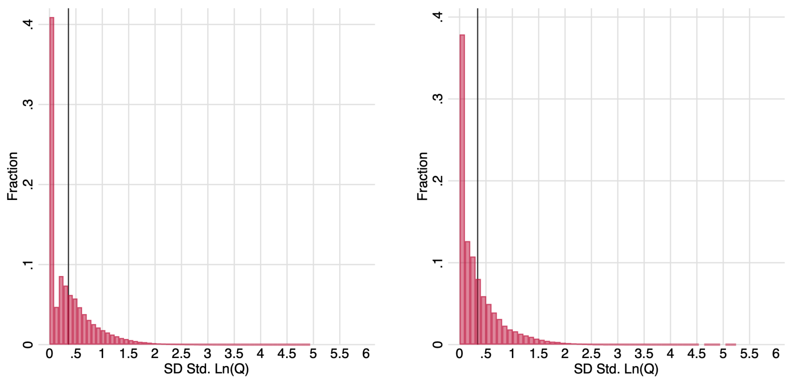
(a) Month (b) Year



Notes: These figures plot histograms of the standard deviation of standardized log prices by month and year, for products that have at least 5 distinct buyers in the time window.

Figure OA-5: **Product-level Quantity Dispersion within Month and Year**

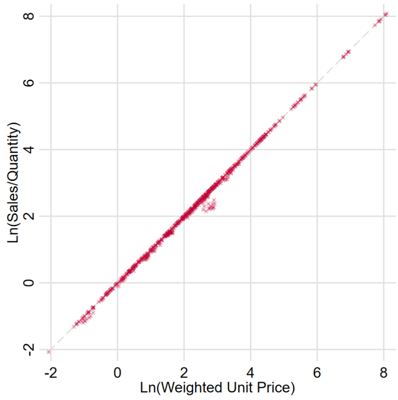
(a) Month (b) Year



Notes: These figures plot histograms of the standard deviation of standardized log quantities by month and year, for products that have at least 5 distinct buyers in the time window.

Online Appendix Figure OA-6 plots the relationship between (log) average unit prices, obtained by dividing the total value of yearly transactions by total quantity purchased, pooled across all different products, against (log) weighted unit prices, summing transaction-product-level unit prices using the total seller-buyer-year share of expenditure as weights.

Figure OA-6: **Average Price vs Weighted Price**



Notes: This figure plots average unit prices (in logs) against weighted prices (in logs) across buyer-seller-year. Average unit prices are calculated by dividing the total value of yearly transactions by total quantity purchased (pooling different products). Weighted prices are calculated by summing transaction-product-level unit prices with total expenditure share as weights.

OA-2 Motivating Evidence - Robustness

In Online Appendix Figure OA-7a, I verify that the differences are not driven by selection, but rather reflect a real increase within pairs. To do so, I run a regression of total quantity q_{ijt} on dummies for the age of the relationship, controlling for pair fixed effects. The figure plots the coefficients for the relationship age dummies and shows that the volume of total quantity purchased grows as relationships age.

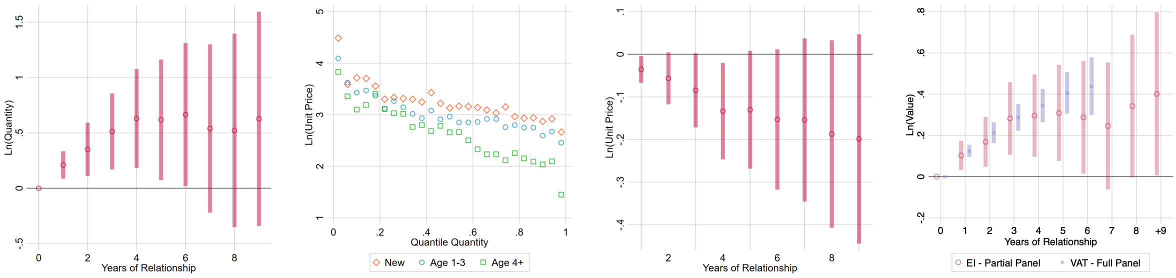
Next, Online Appendix Figure OA-7b plots a binscatter regression of log average unit price on quantiles of quantity, controlling for seller-year fixed effects. The figure documents the presence of quantity discounts within relationship age. Online Appendix Figure OA-7c shows a binscatter plot of log average prices on the age of the relationship, controlling for pair fixed effects. The figure shows that as relationships age, they receive around 1.5% additional discounts per year. Under both formulations, there are price discounts conceded to older clients.

Finally, Online Appendix Figure OA-7d plots regression coefficients for the value of total sales between buyer and supplier on the age of the relationship, controlling for pair fixed effects. The red figures use the electronic invoice database and are constructed using only a partial panel of two observations per pair for the years 2016-2017. The purple marks are constructed using multiple observations of buyer-seller pairs from the VAT B2B database for the years 2007-2015 for the sellers in the electronic invoice database. The figure confirms that relying on only two years of relationship data can properly capture full relationship dynamics observed in longer panel datasets.

Online Appendix Figure OA-8 presents pair-specific changes in prices and quantities between 2016 and 2017, by the age of the relationship in 2016, over quantiles of quantity purchased in 2016. The figures show that prices tend to decrease faster and quantities increase faster for lower quantiles. Over time, backloading in prices and quantities becomes weaker. By age 5, prices and quantities are relatively stable across quantiles.

Online Appendix Table OA-1 shows the results of a regression on log average price on log quantity, controlling for seller-year fixed effects. The table presents a benchmark quantity discount measure of a 2% decrease in price for a 10% increase in total quantity purchased.

Figure OA-7: **Motivating Facts - Robustness (Pair Fixed Effects)**



(a) Q. over Time (b) Q. Discounts (c) Prices over Time (d) Value over Time

Notes: Panel a) plots the coefficients of log total quantity on relationship age dummies, controlling for pair fixed effects. Total quantity is obtained by aggregating all the reported units of sold goods. Standard errors are clustered at the pair level. Panel b) shows the relationship between quantity purchased and average log unit price through binscatters of the measure of unit price against quantile of quantity by age of relationship. Quantiles of quantity are calculated for each seller-relationship age combination. Panel c) plots the regression coefficients of log unit prices on years of relationship, controlling for pair fixed effects. Standard errors are clustered at the pair level. Panel d) plots regression coefficients for the value of total sales between buyer and supplier on the age of the relationship, controlling for pair fixed effects. The red figures use the electronic invoice database and are constructed using only a partial panel of two observations per pair for the years 2016-2017. The purple marks are constructed using multiple observations of buyer-seller pairs from the VAT B2B database for the years 2007-2015 for the sellers in the electronic invoice database.

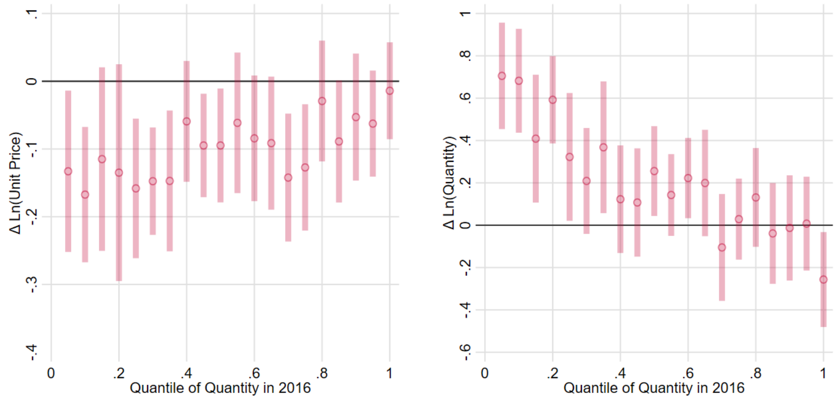
Table OA-1: **Benchmark: Quantity Discounts**

	(1)
VARIABLES	ln(Price)
ln(Quantity)	-0.220***
	(0.0238)
Constant	3.046***
	(0.0718)
Seller-Year FE	Yes
Observations	76,473
R-squared	0.666

Notes: This table presents a regression of log average unit prices on log quantity, controlling for seller-year fixed effects. Standard errors are clustered at the seller-year level. *** p<0.01, ** p<0.05, * p<0.1

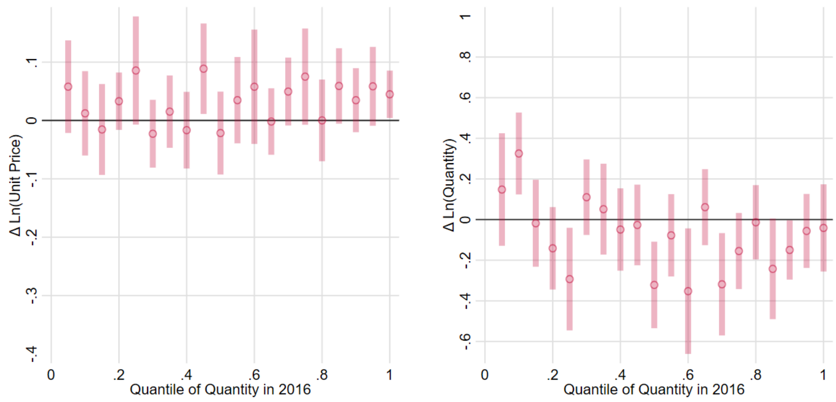
Online Appendix Table OA-2 presents the robustness of relationship discounts to the method of discount allocation (at the bill level vs. at the product level) as well as the weights used to aggregate prices at the seller-buyer-year level (quantities vs. values as weights).

Figure OA-8: **Backloading in Prices and Quantities by Quantile**



(a) Prices (New Buyers) (b) Quantities (New Buyers)

(c) Prices (Age 1-4) (d) Quantities (Age 1-4)



(e) Prices (Age 5+) (f) Quantities (Age 5+)

Notes: This figure presents pair-specific year-to-year changes in unit prices and quantities from 2016 and 2017 for new buyers, ages 1 to 4, and age 5+, against the quantile of quantity purchased in 2016. The age of relationships is from 2016, and quantiles of quantity are measured in 2016 for each seller-relationship age. Error bars present variation at the 95% level, with standard errors clustered at the seller level.

Table OA-2: **Robustness to Weights and Discount Allocation**

Variable (Weighted Average)	(1) Stdz. Price	(2) Stdz. Price	(3) Stdz. Price	(4) Stdz. Price
Age of Relationship	-0.00690***	-0.00685***	-0.0106***	-0.00884**
	(0.00187)	(0.00177)	(0.00401)	(0.00433)
Weights	Values	Quantity	Values	Quantity
Discount Allocation	Bill	Bill	Product	Product
Observations	76,473	76,473	76,473	76,473
R-Squared	0.018	0.018	0.015	0.009
Pair FE	No	No	No	No
Year FE	Yes	Yes	Yes	Yes
Quantity Control	No	No	No	No

Notes: This table presents regressions of prices on the age of the relationship under different weights for aggregation and methods of allocating discounts. Column (1) is the benchmark and allocates discounts at the bill level, relying on the value share of total yearly transactions as aggregation weights. Column (2) allocates discounts at the bill level and uses total quantity as weights. Column (3) allocates discounts at the product level with values as weights. Column (4) allocates discounts at the product level with quantities as weights. Standard errors are clustered at the seller-year level. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

OA-3 Motivating Facts by Seller Sector

In this section, I present the overall consistency of the motivating facts for each seller sector: namely, textile, cement-products, and pharmaceuticals.

Online Appendix Table OA-3 presents summary statistics by seller's sectors for *Sellers in Sample* (Panel A), *Sellers Not in Sample* (Panel B), which are sellers in the same industry but small enough that they were not covered by the EI seller's database, and *Buyers in Sample* (Panel C). The table demonstrates that the sellers in the sample are significantly larger than their non-sample competitors, with the mean sample seller being 272 times larger in the textile industry, 8 times larger in the pharmaceutical industry, and 32 times larger in the cement-products industry. Furthermore, the sample sellers exhibit a higher exposure to imported materials compared to their non-sample counterparts, with 113 times more reliance in textiles, 4 times more in pharmaceuticals, and 26 times more in cement-products. Additionally, the firms in the sample display a considerably higher capital intensity, with 18 times more capital per dollar in expenditure in textiles, 2 times more in pharmaceuticals, and 8 times more in cement-products. These patterns collectively suggest that (1) the manufacturing firms in the sample are preferred suppliers within their respective industries, (2) there is a degree of product differentiation, likely indicating higher quality given the increased reliance on imported inputs, and (3) the higher capital intensity relative to labor implies a reduced likelihood of production issues.

These statistics also help understand why sellers in the sample have market power in the first place. For textile-products: The average buyer in this industry is smaller than the sellers in the sample, yet significantly larger than the non-sample competitors (59 times larger). Furthermore, the average order in the industry, at 25,000 USD, is substantial relative to the size of the average (40,000 USD) and median (< 9,000 USD) non-sample seller. Therefore, beyond the potential higher quality of goods offered by the sellers in the sample, the relatively large size requirements for the orders imply a scale advantage for in-sample sellers.

In the pharmaceutical-products industry: Products are generally horizontally differentiated, as active components are imperfect substitutes for the final consumer. The size, age, capital intensity, and reliance on imported inputs suggest that sellers in the sample are the preferred suppliers in this differentiated industry.

In the cement-products industry: Manufacturers likely benefit from local market power due to the high transportation costs associated with these types of goods. Additionally, the manufacturers in the sample are likely vertically differentiated due to their capital-intensive production. Similar to the textile industry, a scale argument is valid as well. The average buyer in the industry is 14 times larger than the average non-sample seller, and the orders are relatively large (45,000 USD) compared to the size of the non-sample seller (350,000 USD average; 10,000 USD median).

Decomposing Figure 1 by sector reveals that the vast majority of qualitative results hold individually in each sector, with Online Appendix Figure OA-9 for Textiles, Online Appendix Figure OA-10 for Pharmaceuticals, and Online Appendix Figure OA-11 for Cement-products.

First, a large share of trade is channeled through repeated relationships (Subfigure OA-9a; Subfigure OA-10a; Subfigure OA-11a), though pharmaceutical manufacturers have a lower share of new clients and quantity channeled through new buyers. Still, repeated transactions rather than spot transactions are thus likely important in each industry.

Second, at least 60% of all transactions are financed by trade credit (Subfigure OA-9b; Subfigure OA-10b; Subfigure OA-11b). This implies that opportunism on the buyer side

Table OA-3: **Summary Statistics by Sector - Sellers, Buyers, and Other Competitors**

	Textiles			Pharmaceuticals			Cement-Products		
	Mean	Median	S.D.	Mean	Median	S.D.	Mean	Median	S.D.
Panel A: Sellers in Sample									
Total Sales (million USD)	10.91	3.55	21.26	21.64	12.18	31.78	11.32	7.10	12.89
Expenditures (million USD)	6.48	2.38	12.90	16.13	6.29	27.10	8.73	5.58	8.68
Age	31.47	29.00	18.55	30.89	33.00	20.73	28.25	22.00	19.17
Import Share (%)	20.28	11.67	21.85	35.94	27.65	24.11	13.92	4.99	15.94
Export Share (%)	14.04	0.21	29.20	1.00	0.00	2.14	0.01	0.00	0.05
Capital Share of Expenditures	0.18	0.13	0.18	0.28	0.32	0.15	0.39	0.44	0.16
Observations	19			18			12		
Panel B: Sellers Not in Sample									
Total Sales (million USD)	0.04	0.00	0.69	2.81	0.04	11.54	0.35	0.01	7.51
Expenditures (million USD)	0.03	0.00	0.42	2.44	0.03	12.59	0.22	0.00	3.79
Age	8.94	6.00	8.75	14.32	10.50	14.17	10.83	9.00	8.99
Import Share (%)	0.18	0.00	2.90	9.94	0.00	21.89	0.53	0.00	5.01
Export Share (%)	0.09	0.00	2.67	1.37	0.00	8.90	0.13	0.00	2.89
Capital Share of Expenditures	0.01	0.00	0.08	0.17	0.06	0.24	0.05	0.00	0.17
Observations	24,320			234			3,870		
Panel C: Buyers in Sample									
Total Sales (million USD)	2.37	0.18	25.66	6.21	0.31	58.74	5.01	0.55	44.49
Expenditures (million USD)	1.93	0.13	25.64	5.27	0.28	55.61	4.00	0.50	35.29
Age	15.22	14.00	9.46	16.36	14.00	11.81	15.20	14.00	11.35
Import Share (%)	3.84	0.00	13.62	3.20	0.00	11.14	3.50	0.00	12.55
Export Share (%)	1.14	0.00	9.28	0.48	0.00	4.30	0.63	0.00	6.80
Capital Share of Expenditures	0.17	0.05	0.24	0.12	0.02	0.19	0.16	0.07	0.21
Observations	23,890			2,642			3,053		

Notes: This table reports summary statistics about the size, age, capital intensity, and trade exposure of buyers, sellers in the sample, and sellers not in the sample for the year 2016, separated by seller's sector. Monetary values are in U.S. dollars for 2016.

is relevant for all studied industries.

Third, quantities increase as relationships age, both measured as standardized quantities or total quantity (Subfigure OA-9c and OA-9i; Subfigure OA-10c and OA-10i; Subfigure OA-11c and OA-11i). Quantities grow faster in textiles than in other industries. Moreover, looking at both standardized quantity and total quantity demanded, buyers tend to buy more of the same product over time and in total. In pharmaceutical products, product-specific demand levels off after the first year, while total demand continues to increase; in cement-products, product-specific demand levels off after the second year, while total demand continues to increase. In any case, quantity backloading appears relevant across the board.

Fourth, quantity discounts are observed, both within product and in average prices (Subfigure OA-9d and OA-9g; Subfigure OA-10d and OA-10g; Subfigure OA-11d and OA-11g). Thus, a model with price discrimination in quantities is important.

Fifth, price discounts tend to be offered to older relationships (Subfigure OA-9d and OA-9h; Subfigure OA-10d and OA-10h; Subfigure OA-11d and OA-11h). However, in contrast to the main figure, product-specific discounts are not observed on average in pharmaceuticals, whereas they are present in textiles and cement-products. In terms of average prices, relational discounts are observed across all industries. The contrast between quality-adjusted prices and average prices for pharmaceuticals indicates that product bundles are likely switching over time, allowing buyers to purchase cheaper products either not available or desired at the beginning of the relationship. In any case, a model with dynamic discounts could rationalize observed dynamics for average prices for all industries, as well as for quality-adjusted prices for textiles and cement.

Sixth, relationships that trade more intensively are more likely to survive across all industries (Subfigure OA-9f; Subfigure OA-10f; Subfigure OA-11f), though the heterogeneity across ages is smaller in pharmaceutical products than in textiles and cement.

Online Appendix Table OA-4 shows price dynamics by payment modality, relying on the transaction-level data. Joining all industries together (Column 1), we observe that for transactions conducted via trade-credit, quality-adjusted prices decrease as relationships age, accounting for plausible quantity discounts by controlling for a flexible spline in quantity. Instead, when the transaction's modality is pay-in-advance (Column 2), standardized prices increase as relationships age. The same pattern holds for textiles (Columns 3 and 4), cement-products (Columns 7 and 8), and even so for pharmaceuticals (Columns 5 and 6), where quality-adjusted pair-specific prices do not decrease over time.

Online Appendix Figure OA-12 shows the distribution of trade credit terms offered by industry. Textiles offer on average 40 days of trade credit, with 7, 30, 45, and 60 days as common terms. Pharmaceutical products offer on average 55 days, with 30, 45, and 60 as common terms. Cement-products offer 40 days, with 30, 45, and 60 days as common terms.

Finally, Online Appendix Table OA-5 presents coefficients of variation of sales and expenditures (month-to-month) for sellers and buyers. We can see that sellers have lower variability both in sales and expenditures than buyers, though variability is still present for sellers, as the standard deviation is 25% of the mean sales for pharmaceuticals, 29% for cement-products, and 42% for textiles. Production expenses are also volatile, with the standard deviation representing 30% of mean expenditures, though the difference across industries is much more muted than in sales.

Table OA-4: **Price Dynamics and Payment Method**

	All		Textiles		Pharmaceuticals		Cement-Products	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Payment Method	TC	O	TC	O	TC	O	TC	O
Total Years	-0.00786***	0.00431***	-0.00358***	0.00380***	-0.00641***	0.00770***	-0.0291***	-0.00218
	(0.00214)	(0.00108)	(0.00125)	(0.00110)	(0.00215)	(0.00232)	(0.0106)	(0.00174)
Observations	3,383,399	608,318	2,249,157	305,517	742,940	240,995	391,302	61,806
R-squared	0.954	0.982	0.988	0.977	0.981	0.975	0.758	0.973
Product-Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Quantity Control	Spline	Spline	Spline	Spline	Spline	Spline	Spline	Spline

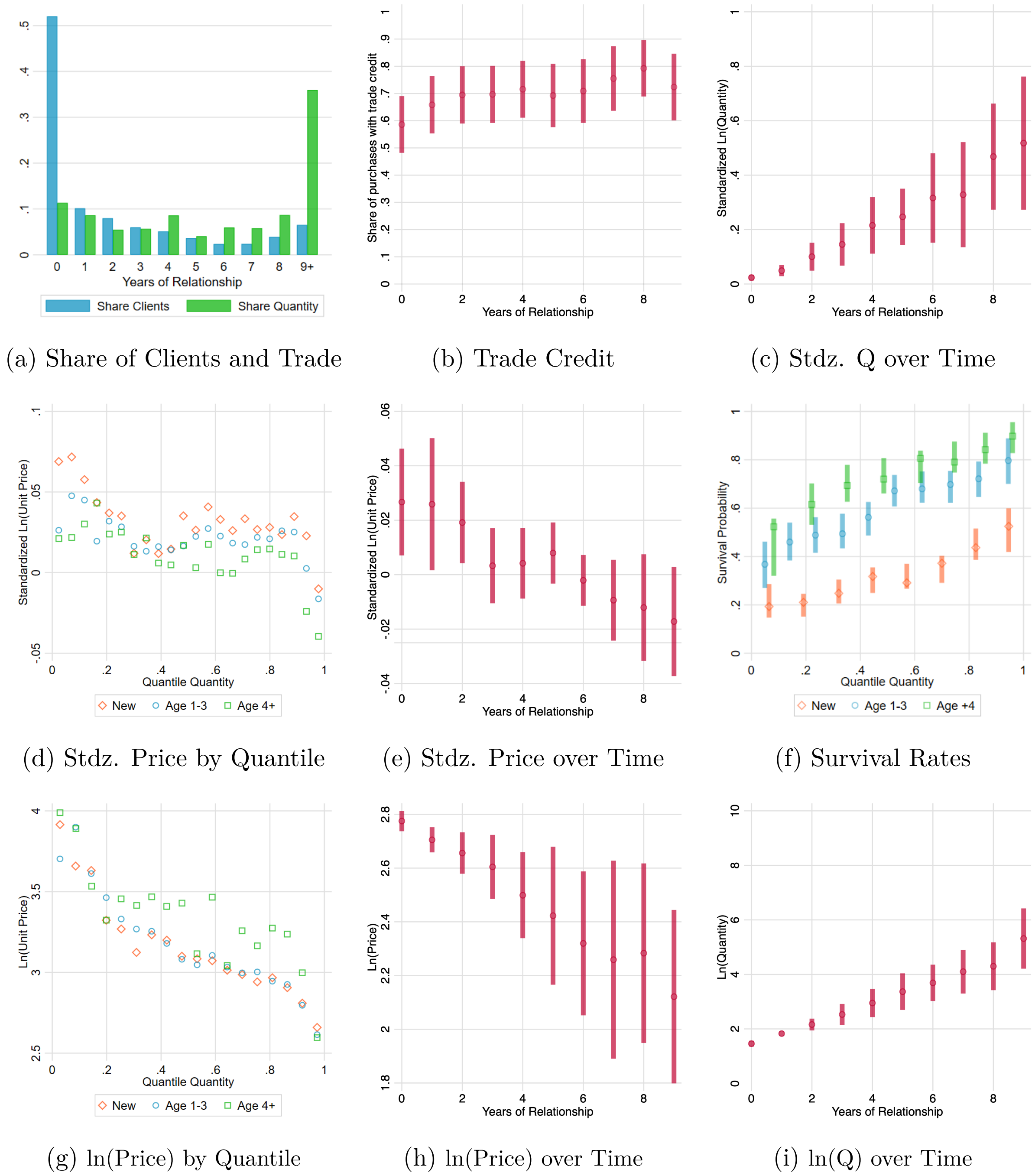
Notes: This table presents transactions-level regression of log unit prices on the age of relationship, controlling for a flexible spline of quantity and product-year fixed effects, by payment modality and sector. Columns (1) and (2) present results for all sectors, for trade-credit transactions and all others, respectively. Columns (3) and (4) report results for textiles, Columns (5) and (6) for pharmaceuticals, and Columns (7) and (8) for cement-products. Standard errors are clustered at the pair-year level. *** p<0.01, ** p<0.05, * p<0.1

Table OA-5: **Coefficient of Variation of Sales and Expenditures**

	CV Sales Seller	CV Expenditures Seller	CV Sales Buyer	CV Expenditures Buyer
Textiles	0.42	0.28	0.65	0.65
Pharmaceuticals	0.25	0.34	0.77	0.55
Cement-Products	0.29	0.31	0.86	0.68

Notes: This table presents coefficients of variation (CV) in monthly sales and expenditures for sellers and buyers between 2016 and 2017.

Figure OA-9: **Motivating Facts: Textile-Products**



(a) Share of Clients and Trade

(b) Trade Credit

(c) Stdz. Q over Time

(d) Stdz. Price by Quantile

(e) Stdz. Price over Time

(f) Survival Rates

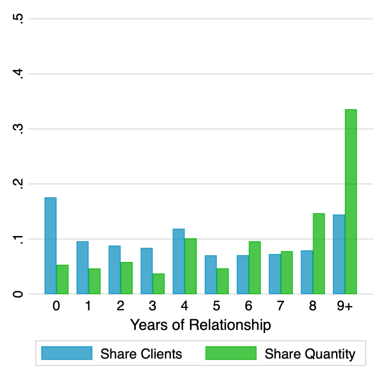
(g) ln(Price) by Quantile

(h) ln(Price) over Time

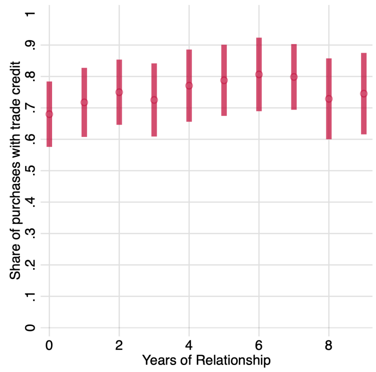
(i) ln(Q) over Time

Notes: This figure replicates Figure 1 for Textile-Products only. Subfigure a) displays the distribution of the average of the share of clients and quantity sold by relationship age, calculated across all sellers in 2016. Subfigure b) displays the average of the share of purchases channeled through trade credit, along with a 90% confidence interval, calculated across all sellers. Subfigure c) displays the evolution of standardized log quantities, with their corresponding 90% confidence intervals, calculated across all sellers. The standardized log quantity is obtained by taking the average quantity sold in a given year for each seller-product and subtracting the log average quantity for that year. The standard errors are calculated at the seller-year level. Subfigure d) shows the relationship between quantity purchased and standardized log unit price through a binscatter plot that displays the measure of unit price against the quantity sold, based on relationship age. The standardized log unit price is obtained by netting out the average log unit price for that year for each seller-product. The quantiles of quantity are calculated for each seller-relationship age combination. Subfigure e) presents a binscatter plot of standardized log unit prices against years of relationship, controlling for a flexible spline of standardized log quantities. The standard errors are calculated at the seller-year level. Subfigure f) displays a binscatter plot of the average survival rate of pairs at different ages and quantiles of quantity. Subfigure g) presents a binscatter of (log) average price on the quantile of quantity by relationship age, controlling for seller-year fixed effects. Subfigure h) presents a binscatter of (log) average price on years of relationship controlling for a flexible spline of quantity and seller-year fixed effects. Subfigure i) presents a binscatter of (log) total quantity on years of relationship controlling for seller-year fixed effects. The quantiles of quantities are calculated for each seller-age combination, and the error bars represent a 90% level of variation across all sellers.

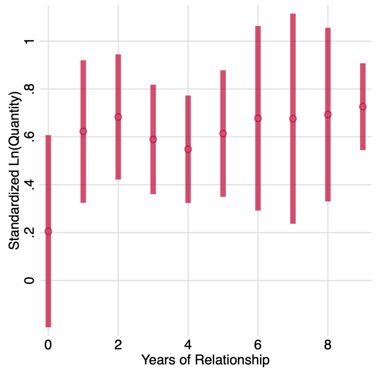
Figure OA-10: **Motivating Facts: Pharmaceutical-Products**



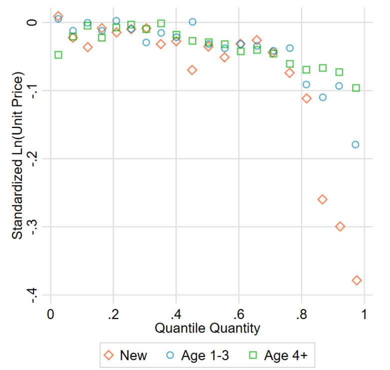
(a) Share of Clients and Trade



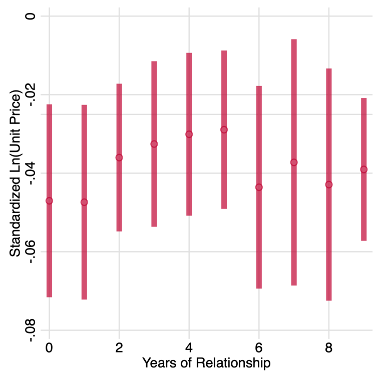
(b) Trade Credit



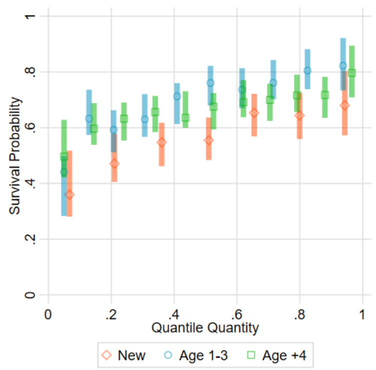
(c) Stdz. Q over Time



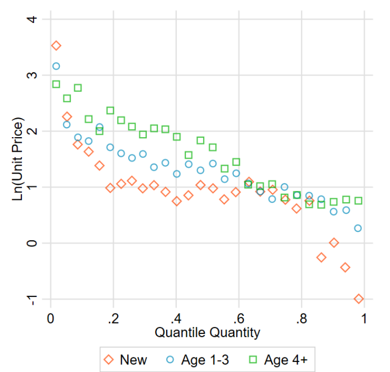
(d) Stdz. Price by Quantile



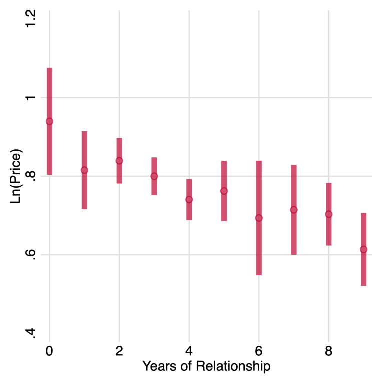
(e) Stdz. Price over Time



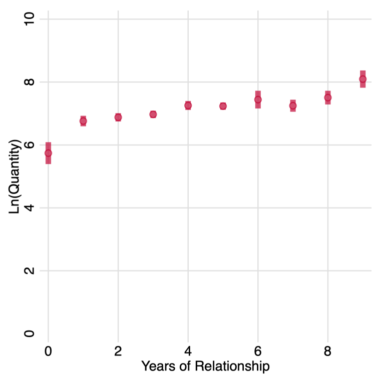
(f) Survival Rates



(g) ln(Price) by Quantile



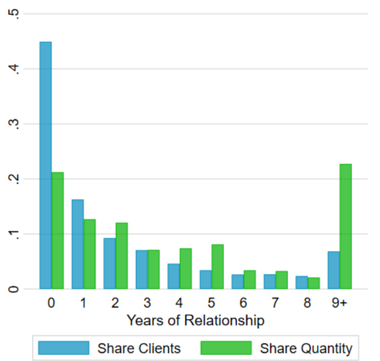
(h) ln(Price) over Time



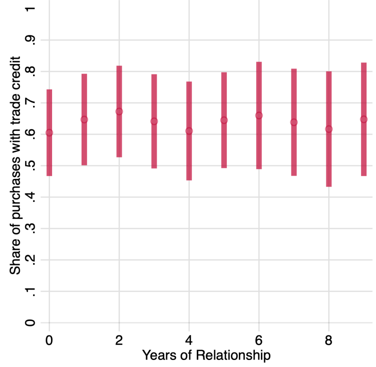
(i) ln(Q) over Time

Notes: This figure replicates Figure 1 for Pharmaceutical-Products only. Subfigure a) displays the distribution of the average of the share of clients and quantity sold by relationship age, calculated across all sellers in 2016. Subfigure b) displays the average of the share of purchases channeled through trade credit, along with a 90% confidence interval, calculated across all sellers. Subfigure c) displays the evolution of standardized log quantities, with their corresponding 90% confidence intervals, calculated across all sellers. The standardized log quantity is obtained by taking the average quantity sold in a given year for each seller-product and subtracting the log average quantity for that year. The standard errors are calculated at the seller-year level. Subfigure d) shows the relationship between quantity purchased and standardized log unit price through a binscatter plot that displays the measure of unit price against the quantity sold, based on relationship age. The standardized log unit price is obtained by netting out the average log unit price for that year for each seller-product. The quantiles of quantity are calculated for each seller-relationship age combination. Subfigure e) presents a binscatter plot of standardized log unit prices against years of relationship, controlling for a flexible spline of standardized log quantities. The standard errors are calculated at the seller-year level. Subfigure f) displays a binscatter plot of the average survival rate of pairs at different ages and quantiles of quantity. Subfigure g) presents a binscatter of (log) average price on the quantile of quantity by relationship age, controlling for seller-year fixed effects. Subfigure h) presents a binscatter of (log) average price on years of relationship controlling for a flexible spline of quantity and seller-year fixed effects. Subfigure i) presents a binscatter of (log) total quantity on years of relationship controlling for seller-year fixed effects. The quantiles of quantities are calculated for each seller-age combination, and the error bars represent a 90% level of variation across all sellers.

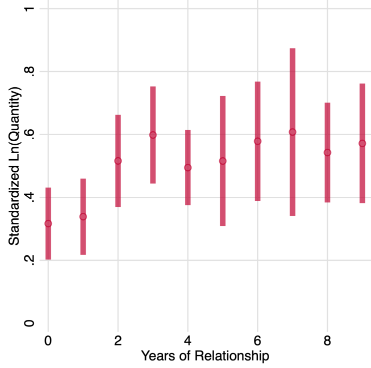
Figure OA-11: **Motivating Facts: Cement-Products**



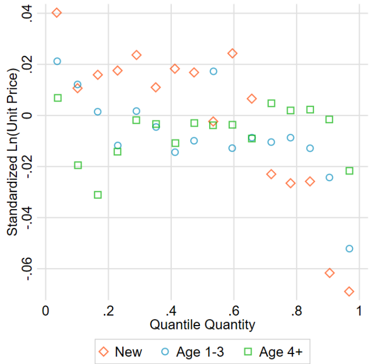
(a) Share of Clients and Trade



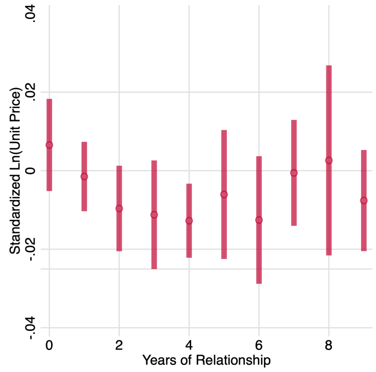
(b) Trade Credit



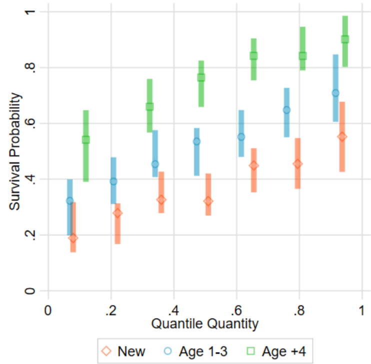
(c) Stdz. Q over Time



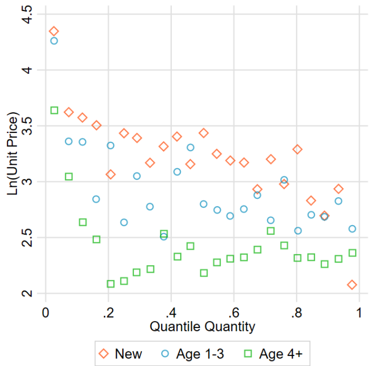
(d) Stdz. Price by Quantile



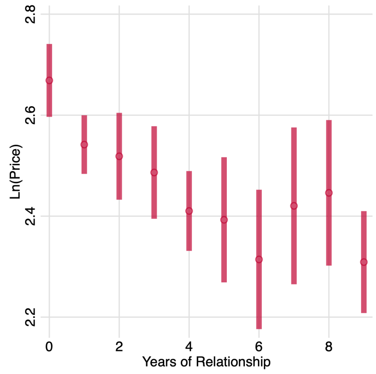
(e) Stdz. Price over Time



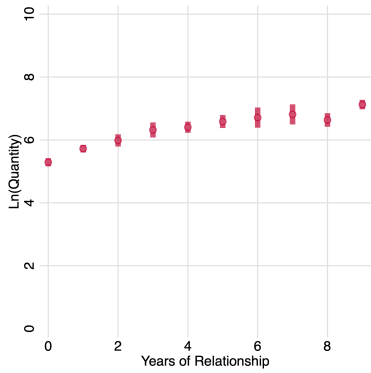
(f) Survival Rates



(g) ln(Price) by Quantile



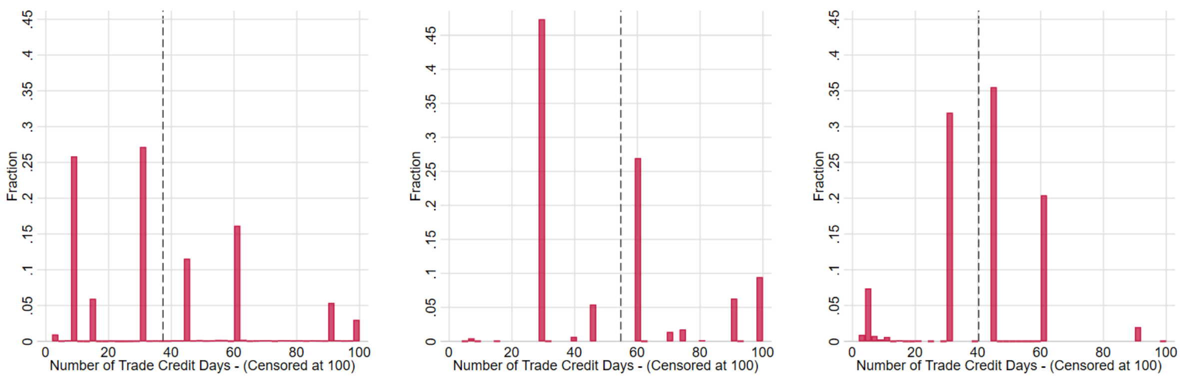
(h) ln(Price) over Time



(i) ln(Q) over Time

Notes: This figure replicates Figure 1 for Cement-Products only. Subfigure a) displays the distribution of the average of the share of clients and quantity sold by relationship age, calculated across all sellers in 2016. Subfigure b) displays the average of the share of purchases channeled through trade credit, along with a 90% confidence interval, calculated across all sellers. Subfigure c) displays the evolution of standardized log quantities, with their corresponding 90% confidence intervals, calculated across all sellers. The standardized log quantity is obtained by taking the average quantity sold in a given year for each seller-product and subtracting the log average quantity for that year. The standard errors are calculated at the seller-year level. Subfigure d) shows the relationship between quantity purchased and standardized log unit price through a binscatter plot that displays the measure of unit price against the quantity sold, based on relationship age. The standardized log unit price is obtained by netting out the average log unit price for that year for each seller-product. The quantiles of quantity are calculated for each seller-relationship age combination. Subfigure e) presents a binscatter plot of standardized log unit prices against years of relationship, controlling for a flexible spline of standardized log quantities. The standard errors are calculated at the seller-year level. Subfigure f) displays a binscatter plot of the average survival rate of pairs at different ages and quantiles of quantity. Subfigure g) presents a binscatter of (log) average price on the quantile of quantity by relationship age, controlling for seller-year fixed effects. Subfigure h) presents a binscatter of (log) average price on years of relationship controlling for a flexible spline of quantity and seller-year fixed effects. Subfigure i) presents a binscatter of (log) total quantity on years of relationship controlling for seller-year fixed effects. The quantiles of quantities are calculated for each seller-age combination, and the error bars represent a 90% level of variation across all sellers.

Figure OA-12: **Trade Credit Terms by Sector**



(a) Textiles (b) Pharmaceutical (c) Cement-Products

Notes: This figure plots the distribution of trade-credit days offered by the seller's sector.

OA-4 Existence and Non-Stationarity

To prove existence, I build on two key results from the literature. First, I utilize the result of non-linear pricing from Jullien (2000) to demonstrate the existence of a stationary optimal contract in the presence of heterogeneous participation constraints. This is achieved by showing the equivalence between the stationary contract with limited enforcement and a non-linear pricing problem with heterogeneous outside options. Subsequently, similar to the argument in Martimort et al. (2017), I present a simple non-stationary deviation that outperforms the stationary optimal contract.

It is important to note that I will show existence results under the assumption of no exit, i.e., $X(\theta) = 0$ for all θ. To prove existence with exit, one must replace the discount factor δ with $\tilde{\delta} \equiv \min\{\delta(\theta)\}$, where $\delta(\theta) = \delta(1 - X(\theta))$ accounts for heterogeneous breakups. This adjustment only affects one of the assumptions discussed below and sets an upper bound on the worst-case exit rate.

OA-4.1 Existence of Stationary Contract

The model in Jullien (2000) solves the following problem:

$$\max_{\{t(\theta),q(\theta)\}} \int_{\underline{\theta}}^{\bar{\theta}} [t(\theta) - cq(\theta)]f(\theta)d\theta \quad \text{s.t.} \tag{IR Problem}$$

$$v(\theta, q(\theta)) - t(\theta) \geq v(\theta, q(\hat{\theta})) - t(\hat{\theta}) \quad \forall \theta, \hat{\theta} \tag{IC}$$

$$v(\theta, q(\theta)) - t(\theta) \geq \bar{u}(\theta) \quad \forall \theta. \tag{IR}$$

Under a modified first-order approach, the seller's first-order condition is given by:

$$v_q(\theta, q(\theta)) - c = \frac{\gamma(\theta) - F(\theta)}{f(\theta)} v_{\theta q}(\theta, q(\theta), \tag{38}$$

for each type θ, and the complementary slackless condition on the IR constraints:

$$\int_{\underline{\theta}}^{\bar{\theta}} [u(\theta) - \bar{u}(\theta)]d\gamma(\theta) = 0. \tag{39}$$

Jullien (2000) shows that under three assumptions there exists a unique optimal solution in which all consumers participate. This solution is characterized by the first-order conditions 38 and complementary slackless condition 39 with $q(\theta)$ increasing.

The first assumption is potential separation (PS), which requires that the optimal solution is non-decreasing in θ, and satisfied under weak assumptions on the distribution of θ and the curvature of the surplus relative to the return of the buyer. In particular, it requires that

$$\frac{d}{d\theta}\left(\frac{S_q(\theta, q)}{v_{\theta q}(\theta, q)}\right) \geq 0$$
$$\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right) \geq 0 \geq \frac{d}{d\theta}\left(\frac{1 - F(\theta)}{f(\theta)}\right).$$

The second and *key* assumption is homogeneity (H), requiring that there exists a quantity profile $\{\bar{q}(\theta)\}$ such that the allocation with full participation $\{\bar{u}(\theta), \bar{q}(\theta)\}$ is implementable in that $\bar{u}'(\theta) = v_\theta(\theta, \bar{q}(\theta))$ and $\bar{q}(\theta)$ is weakly increasing. This assumption implies that the reservation return can be implemented as a contract without excluding any type, ensuring that incentive compatibility is not an issue when the individual rationality constraint is binding.

Lastly, the assumption of full participation (FP) posits all types participate, and is satisfied when (H) holds and the surplus generated in the reservation return framework is greater than the private return to the buyer, i.e. $s(\theta, \bar{q}(\theta)) \geq \bar{u}(\theta)$.

I show that my setting can be rewritten in terms of Jullien (2000), implying that an optimal separating stationary contract exists. The seller chooses the optimal stationary contract $\{t(\theta), q(\theta)\}$ that satisfy incentive-compatibility and the limited enforcement constraint. Formally, the seller solves the problem:

$$\max_{\{t(\theta),q(\theta)\}} \frac{1}{1-\delta} \int_{\underline{\theta}}^{\bar{\theta}} [t(\theta) - cq(\theta)] f(\theta) d\theta \quad \text{s.t.} \tag{LE Problem}$$

$$v(\theta, q(\theta)) - t(\theta) \geq v(\theta, q(\hat{\theta})) - t(\hat{\theta}) \quad \forall \theta, \hat{\theta} \tag{IC}$$

$$\frac{\delta}{1-\delta}\Big(v(\theta, q(\theta)) - t(\theta)\Big) \geq t(\theta) = v(\theta, q(\theta)) - u(\theta), \quad \forall \theta, \tag{LC}$$

where $u(\theta)$ is the return obtained by type θ. The limited enforcement constraint can be easily written as the IR constraint in Jullien (2000):

$$u(\theta) \geq (1-\delta) v(\theta, q(\theta)) \equiv \bar{u}(\theta) \quad \forall \theta. \tag{LE'}$$

In my model, with $v(\theta, q) = \theta v(q)$, the first condition of assumption PS is always satisfied as

$$\frac{d}{d\theta}\left(\frac{S_q(\theta, q)}{v_{\theta q}(\theta, q)}\right) = \frac{d}{d\theta}\left(\theta - \frac{c}{v'(q)}\right) \geq 0 \iff 1 \geq 0 \tag{A1}$$

As stated earlier, the second condition of assumption PS is satisfied for a wide-range of distributions for θ. Therefore, assumption PS is satisfied for any of those distributions.

Then, consider Assumption H. It requires that an allocation $\{\bar{q}(\theta)\}$ exists such that $\bar{u}'(\theta) = v_\theta(\theta, \bar{q}(\theta))$ and $\bar{q}(\theta)$ is weakly increasing. Notice that under LE', we can define $\bar{q}(\theta)$ as $\bar{u}'(\theta) = (1-\delta)[\theta v'(q(\theta)) q'(\theta) + v(q(\theta)] = v(\bar{q}(\theta))$. Define $G(\bar{q}, \theta) = v(\bar{q}) - (1-\delta)[\theta v'(q(\theta)) q'(\theta) + v(q(\theta))] = 0$. By the implicit function theorem, $\bar{q}(\theta)$ is weakly increasing if

$$\begin{aligned}
\bar{q}'(\theta) &= -\frac{dG/d\theta}{dG/d\bar{q}} \\
&= \frac{(1-\delta)[v'(q(\theta))q'(\theta) + \theta v''(q(\theta))(q'(\theta))^2 + \theta v'(q(\theta))q''(\theta) + v'(q(\theta))]}{v'(\bar{q})} \geq 0 \\
&\iff v'(q(\theta))[1 + q'(\theta) + \theta q''(\theta))] + \theta v''(q(\theta))(q'(\theta))^2 \geq 0 \\
&\iff \frac{q'(\theta) + \theta q''(\theta) + 1}{\theta (q'(\theta))^2} \geq A(q) \\
&\iff \left(\frac{T''(q)}{T'(q)} + A(q)\right)\Big(1 + \theta(q)\theta'(q)r(q) + \theta'(q)\Big) \geq A(q) \\
&\iff \frac{T''(q)}{T'(q)} \frac{M(q)}{M(q)-1} \geq A(q),
\end{aligned}$$

where $M(q) \equiv 1 + \theta(q)\theta'(q)r(q) + \theta'(q)$ and $r(q) = g^{-1}(q)$ for $g(\theta) \equiv q''(\theta)$. As we expect $T''(q) < 0$ and $T'(q) > 0$, it is necessary that $M(q)/(M(q)-1) < 0$. Such condition will be satisfied if $M(q) < 1$ and $M(q) > 0$, which imply that

$$r(q)\theta(q) < -1$$
$$\text{and} \tag{A2}$$
$$\theta'(q) < \frac{1}{\theta(q)|r(q)| - 1}.$$

The first condition sets restrictions on the rate of change of quantities, which requires $q''(\theta)$ to be negative, restricting how convex $u(\theta)$ can be. The second condition requires that quantities increase at a minimum rate. Moreover, the condition sets bounds on the price discounts offered relative to the buyers' return curvature at a given quantity.

Lastly, full participation requires H to hold as well as $s(\theta, \bar{q}(\theta)) \geq (1-\delta)\theta v(\bar{q}(\theta))$. The condition becomes:

$$\delta \geq \frac{c\bar{q}(\theta)}{\theta v(\bar{q}(\theta))}, \tag{A3}$$

which requires that agents value the future high enough, such that discount factor be greater than the ratio of average cost to average return.

Let $\{t^{st}(\theta), q^{st}(\theta)\}$ be the solution to the to the problem characterized by equations 38 and 39. Assuming that the primitives $v(\cdot)$, $F(\theta)$, and δ are such that conditions A1, A2, and A3 hold for $\{t^{st}(\theta), q^{st}(\theta)\}$, then $\{t^{st}(\theta), q^{st}(\theta)\}$ is uniquely optimal.

OA-4.1.1 *Solution to Stationary* $\Gamma^{st}(\theta)$

The seller's first-order condition defines the following differential equation in the stationary equilibrium

$$\theta u'(q^{st}(\theta)) - c = \frac{\Gamma^{st}(\theta) - F(\theta) + (1-\delta)\theta\gamma^{st}(\theta)}{f(\theta)} u'(q^{st}(\theta)). \tag{40}$$

The solution $\Gamma^{st}(\theta)$ to the equation above is given by:

$$\Gamma^{st}(\theta) = \frac{\int_{\underline{\theta}}^{\theta} x^{\delta/(1-\delta)}[xf(x) - c(u'(q^{st}(x))^{-1}f(x) + F(x)]dx + K}{\theta^{1/(1-\delta)}(1-\delta)}, \tag{41}$$

which by integration by parts reduces to:

$$\Gamma^{st}(\theta) = \frac{F(\theta)}{1-\delta} - \frac{\delta \int^{\theta} x^{\delta/(1-\delta)} F(x)dx}{(1-\delta)\theta^{1/(1-\delta)}} - \frac{cE[x^{\delta/(1-\delta)}u'({}^{st}q(x))^{-1}|x \leq \theta]}{(1-\delta)\theta^{1/(1-\delta)}} + \frac{K}{(1-\delta)\theta^{1/(1-\delta)}} \tag{42}$$

The constant is obtained by using the boundary condition $\Gamma^{st}(\bar{\theta}) = 1$. Therefore,

$$K = cE[x^{\delta/(1-\delta)}u'(q^{st}(x))^{-1})] - \delta\bar{\theta}^{1/(1-\delta)} + \delta \int x^{\delta/(1-\delta)} F(x)dx. \tag{43}$$

OA-4.2 Optimality of Non-Stationary Contracts

Having established the existence of an optimal stationary contract, I now show that a non-stationary contract exists which dominates the stationary contract. A similar argument was briefly discussed in the working paper version of Martimort et al. (2017).

Consider the following deviation from the stationary contract, in which at tenure 0, the return obtained by the buyer is given by:

$$u_0(\theta) = u^{st}(\theta) - \varepsilon,$$

for some $\varepsilon > 0$ sufficiently small, where $u^{st}(\theta) = \theta v(q^{st}(\theta)) - t^{st}(\theta)$ and $t_0(\theta) = t^{st}(\theta)$. Define $q_0(\theta)$ to satisfy this deviation. Under this deviation, the enforcement constraint at $\tau = 0$ is:

$$t^{st}(\theta) \leq \frac{\delta}{1-\delta}\left[\theta v(q^{st}(\theta)) - t^{st}(\theta)\right],$$

which is identical to the one in the stationary contract, which we know $\{t^{st}(\theta), q^{st}(\theta)\}$ satisfy. Moreover, the incentive compatibility constraint is still satisfied as $\hat{\theta}$ maximizes

$$u_0(\theta, \hat{\theta}) + \frac{\delta}{1-\delta} u^{st}(\theta, \hat{\theta}) = \frac{\delta}{1-\delta} u^{st}(\theta, \hat{\theta}) - \varepsilon,$$

where $u_\tau(\theta, \hat{\theta}) \equiv \theta v(q_\tau(\hat{\theta})) - t_\tau(\hat{\theta})$.

Under this alternative scheme, the seller obtains an additional payoff ε per buyer while still satisfying both the incentive compatibility and limited enforcement constraints. Therefore, the optimal contract is non-stationary.

OA-5 Proof of Lemma 1: $\Gamma_\tau(\overline{\theta}) = 1$

I prove that $\Gamma_\tau(\overline{\theta}) = 1$ for all τ. To begin, recall we assumed the outside option $\overline{u}_\tau(\theta)$ was equal to zero for all τ and all θ. Suppose instead that at some period k, the outside option is uniformly shifted downward by $\varepsilon > 0$ for all θ, that is, $\overline{u}_k(\theta) = -\varepsilon$. The enforcement constraint at k is now given by:

$$\delta[\sum_{s=1}^{\infty} \delta^{s-1} u_{k+s}(\theta)] - \overline{u}_k(\theta) = \sum_{s=1}^{\infty} \delta^s u_{k+s}(\theta) + \varepsilon \geq t_k(\theta) = \theta v(q_k(\theta)) - u_k(\theta). \tag{44}$$

The seller's problem in the Lagrangian-form is

$$W(\varepsilon) = \max_{\{q_\tau(\theta), u_\tau(\theta)\}} \sum_{\tau=0}^{\infty} \delta^\tau \Big\{ \int_{\underline{\theta}}^{\overline{\theta}} [\theta v(q_\tau(\theta)) - c q_\tau(\theta) - u_\tau(\theta)] f(\theta) d\theta + \tag{45}$$

$$\int_{\underline{\theta}}^{\overline{\theta}} [\sum_{s=1}^{\infty} \delta^s u_{\tau+s} + \varepsilon \times 1\{\tau = k\} - t_\tau(\theta)] d\Gamma_\tau(\theta) \Big\} \tag{46}$$

such that $u'_\tau(\theta) = \theta v'(q_\tau(\theta))$ for all τ, θ. The change in the value of the seller's problem given the uniform change in outside options is:

$$\frac{dW(\varepsilon)}{d\varepsilon} = \delta^k \int_{\underline{\theta}}^{\overline{\theta}} d\Gamma_k(\theta), \tag{47}$$

where the integral is the cumulative multiplier.

I argue that the quantities that solve the original problem still maximize the current one but that the transfers are all shifted upward by the constant ε. That is, if $q_\tau(\theta)$ is the solution for the problem with $\overline{u}_\tau(\theta) = 0$ for all θ and all τ with associated $t_\tau(\theta)$, $q_\tau(\theta)$ is also the solution for the problem with outside options $\overline{u}_\tau(\theta) = -\varepsilon \times 1\{\tau = k\}$ for all θ and all τ with associated transfers equal to $t_\tau(\theta) + \varepsilon \times 1\{\tau = k\}$. The value of the problem for the seller is:

$$W(\varepsilon) = \sum_{\tau=0}^{\infty} \delta^{\tau} \Big\{ \int_{\underline{\theta}}^{\overline{\theta}} [t_{\tau}(\theta) + \varepsilon \times 1\{\tau = k\} - cq_{\tau}(\theta)] f(\theta) d\theta \Big\} \tag{48}$$

$$= \sum_{\tau=0}^{\infty} \delta^{\tau} \Big\{ \int_{\underline{\theta}}^{\overline{\theta}} [t_{\tau}(\theta) - cq_{\tau}(\theta)] f(\theta) d\theta \Big\} + \delta^{k}\varepsilon. \tag{49}$$

So

$$\frac{dW(\varepsilon)}{d\varepsilon} = \delta^{k}. \tag{50}$$

Therefore, from equations 47 and 50, the cumulative multiplier for any k will satisfy the following property:

$$\Gamma_k(\overline{\theta}) \equiv \int_{\underline{\theta}}^{\overline{\theta}} d\Gamma_k(\theta) = \frac{dW(\varepsilon)}{d\varepsilon} \frac{1}{\delta^k} = 1. \tag{51}$$

OA-6 A Two-Type Illustrative Example

The purpose of this example is four-fold. First, I illustrate how the introduction of the limited enforcement constraint may distort quantities relative to perfect enforcement. Second, I show that lower types unambiguously reap higher net returns due to the enforcement constraint. The introduction of the enforcement constraints effectively raises their reservation return to participate in trade, forcing the seller to offer larger net return values to lower types. Third, I demonstrate that the optimal contract must be non-stationary. Fourth, I show through a solved example that the optimal stationary contract features *backloading*: unit prices decrease while quantities increase as relationships age.

OA-6.1 Buyer's Types

A buyer type-θ gains a gross return θq^{β} from q units of the product sold by the seller. Assume there are positive, yet diminishing marginal returns, i.e., $\beta \in (0, 1)$. The buyer types can take values $\{\theta_L, \theta_H\}$, such that $\theta_L < \theta_H$. Let f_L (resp. f_H) be the probability that buyer is type L (resp. type H) and assume no exit, i.e., $X(\theta) = 0$.

OA-6.2 A Stationary Contract

For now, consider the optimal *stationary* contract. The optimal choice gives the buyer the net return $R(\theta_i) = \theta_i q_i^{\beta} - T(q_i)$. The seller designs the scheme to maximize:

$$max_{\{T_i, q_i\}} f_L(T_L - cq_L) + (1 - f_L)(T_H - cq_H)$$

where $T_i \equiv T(q_i)$, subject to incentive-compatibility constraints:

$$R(\theta_H) \equiv \theta_H q_H^{\beta} - T_H \geq \theta_H q_L^{\beta} - T_L, \tag{IC-H}$$

$$R(\theta_L) \equiv \theta_L q_L^{\beta} - T_L \geq \theta_L q_H^{\beta} - T_H. \tag{IC-L}$$

as well as the limited enforcement constraint:

$$\frac{\delta}{1-\delta}(R(\theta_i)) \geq T_i \quad i = L, H. \tag{LE-i}$$

This last constraint effectively (weakly) raises the minimum net rent that each buyer needs to obtain to participate in trade. The usual nonlinear pricing problem only requires that $R(\theta_i) \geq 0$. Instead, the limited enforcement case requires that $R(\theta_i) \geq (1-\delta)/\delta T_i >$

0, where the minimum return is endogenously determined. Notice that as $\delta \to 1$, the limiting case becomes the standard nonlinear pricing problem.[63]

To simplify the problem, assume that the IC-L and LE-H are slack while IC-H and LE-L are binding.[64] By using these assumptions on the constraints, one can obtain the optimal quantity allocations:

$$q_H^* = \left(\frac{\beta}{c}\theta_H\right)^{\frac{1}{1-\beta}},$$
$$q_L^* = \left(\frac{\beta}{c}\left[\theta_L - \frac{(1-\delta)\theta_L}{f_L} - \frac{(1-f_L)(\theta_H-\theta_L)}{f_L}\right]\right)^{\frac{1}{1-\beta}},$$

and optimal tariffs:

$$T_H^* = \theta_H q_H^\beta + (\delta\theta_L - \theta_H)q_L^\beta,$$
$$T_L^* = \delta\theta_L q_L^\beta.$$

The tariffs are similar to that in the standard case, with the exception that the discount factor now enters the terms multiplying θ_L. Therefore, for a given quantity, tariffs are lower for both types.

The program's solution implies there is no distortion in quantities for type-H, as they purchase at the first-best level. However, type-L's purchases are shifted downwards. First, as is common in adverse selection problems, their purchases are distorted downwards to incentivize the revelation of type-H.

Second, contrary to the standard problem, extracting all rents from type-L is no longer feasible, as type-L would default. This generates a second downward pressure for quantities, as the standard quantity allocation for θ_L (i.e., when $\delta = 1$), together with the optimal tariffs for L under limited enforcement do not satisfy IC-H. To see this, notice that as IC-H was binding in the standard problem, type-H was on the margin between their standard bundle and the standard bundle for type-L. Thus, if the limited enforcement bundle for type-L keeps quantities fixed (relative to the standard menu) and at the same time asks for lower tariffs, type-H buyers would now prefer the menu intended for type-L. As a result, the seller needs to reduce type-L's allocation, even further than would be required under the standard adverse selection problem.

OA-6.3 Non-Stationarity

Relative to the standard problem, the seller now needs to offer positive net returns to all buyers, in order to prevent default. Contrary to the results in Baron and Besanko (1984), the stationary contract is no longer the optimal contract. Instead, the seller could offer a dynamic contract with intertemporal incentives that use the promise of future returns to the buyer to discipline their behavior now. Through this approach, the seller can extract higher shares of surplus early on than would be feasible under a stationary contract, increasing their present-value lifetime profits.

The exact dynamic path depends on the return function and distribution of types of the buyer, as well as the marginal cost of the seller and the common discount factor. For that reason, I consider next a solved numerical example.

[63] The theoretical result that the buyer benefits from a deterioration of enforcement was previously discussed by Genicot and Ray (2006). In their model, they find that if better enforcement brings with it the deterioration of outside options and the seller has the bargaining power, the buyer will see their expected payoff increase. The opposite holds when the buyer has the bargaining power.

[64] All slack constraints are verified for the numerical example discussed below.

OA-6.4 A Visual Example

To visualize the problem, I consider a numerical example with the following values for the parameters: $\beta = 0.5$, $c = 1$, $f_L = 0.95$, $\theta_L = 1$, $\theta_H = 3$, $\delta = 0.9$.[65] Besides the incentive compatibility constraint and the limited enforcement constraint, I have also included the interim individual rationality constraint.

Online Appendix Figure OA-13 shows the levels of quantities, prices, profits per buyer, and buyer's net return for the example discussed above for different regimes: stationary with perfect enforcement (Baron-Myerson), stationary with limited enforcement, and dynamic with limited enforcement.

In solid green, the figure shows the allocation for type-H. As mentioned above, limited enforcement of contracts does not distort their consumption relative to perfect enforcement. In solid blue, the figure shows the allocation for type-L under perfect enforcement. Type-L receives lower quantities and higher prices than type-H and receives zero net return. In dashed-dot blue, the figure shows the stationary allocation for type-L under limited enforcement. Relative to perfect enforcement, type-L sees a reduction in quantities and an increase in net return, in line with the logic explained above. Importantly, as the buyer's return function features diminishing returns in q, lower levels of quantity for lower values of δ also imply the seller can charge *higher* unit prices to type-L.

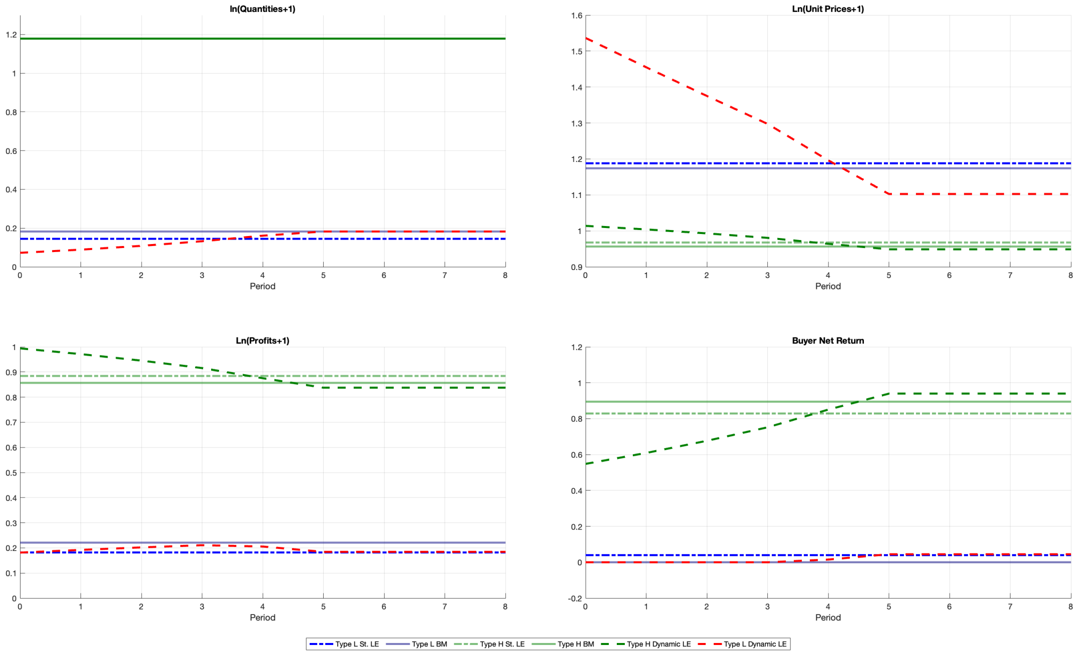
Lastly, the figure shows the optimal non-stationary path of prices and quantities in the dashed lines (red for type-L and green for type-H). The optimal path features *backloading* as quantities (weakly) increase and unit prices (weakly) decrease over time. As shown in the figure, this path of prices and quantities increases short-term expected profits from *each* buyer relative to the optimal stationary contract. Thus, the dynamics allow the seller to extract higher short-term profits for the high type as well. Indeed, in this example, the lifetime total profit in the dynamic case is 91% the level of the Baron-Myerson profit levels, whereas the stationary equilibrium reaches 88%. The seller can effectively prevent default now and increase present-value lifetime profits by offering higher surplus levels to the buyers in the future.

Interestingly, the optimal path in the solved example features consumption for type-L in the long run that is greater than the stationary contracts with limited enforcement, as it converges to the Baron-Myerson allocation. Thus, in this case, the dynamics increase the long-term efficiency of the contracts.

In any case, the example shows that through the interaction market power on the seller side (which is reflected in the ability to offer incentive-compatible profit-maximizing menus) and the limited enforcement constraint, long-term contracts may display dynamics in which average quantities increase and unit prices decrease over time. Moreover, at any point in time, types consuming higher levels of quantities also enjoy lower unit prices. That is, this model of price discrimination with limited enforcement of contracts features i) *backloading* of prices and quantities, and ii) *quantity discounts* at any point in time.

[65] The higher the difference between types, the higher the discount factor, the higher the elasticity β, or the bigger the share of high types, the longer the path to convergence.

Figure OA-13: **Example - Nonlinear Pricing and Limited Enforcement**



Notes: This figure shows Quantities, Prices, Profits, and Buyer Net Return for different enforcement and contract regimes. In dash-dot green, the optimal stationary contract for type-H under limited enforcement. In dashed green, the optimal dynamic contract for type-H under limited enforcement. In solid green, the optimal stationary contract for type-H under perfect enforcement. In solid blue, the optimal stationary contract for type-L under perfect enforcement. In dash-dot blue, the optimal stationary contract for type-L under limited enforcement. In dashed red, the optimal dynamic contract for type-H under limited enforcement. The parameters used in the example are: $\{\beta = 0.5,\ c = 1,\ f_L = 0.95,\ \theta_L = 1,\ \theta_H = 3,\ \delta = 0.9\}$.

OA-7 Additional Theoretical Results

OA-7.1 Model Dynamics

Quantity Discounts

Define $T_\tau(q_\tau(\theta)) \equiv t_\tau(\theta_\tau(q))$, $\Lambda_\tau(\theta) \equiv \Gamma_\tau(\theta) - \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\theta)) + \theta\gamma_\tau(\theta)$, and $\lambda_\tau(\theta) \equiv d\Lambda_\tau/d\theta$. The price schedule is said to feature quantity discounts if $T''_\tau(q) < 0$.

Proposition 3. *Assume strict monotonicity of quantity $q'_\tau(\theta) > 0$ and that $\lambda_\tau(\theta) < f_\tau(\theta)$. If the densities $f_\tau(\theta)$ satisfy log-concavity and $d(F_\tau(\theta)/f_\tau(\theta))/d\theta \geq F_\tau(\theta)/[(\theta - 1)f_\tau(\theta)]$, then the tariff schedule exhibits quantity discounts, $T''_\tau(q) \leq 0$ for each $q = q_\tau(\theta)$, $\theta \in (\underline{\theta}, \overline{\theta})$ and τ.*

Proof of Proposition 3. Recall the quantity function $q_\tau(\theta)$ and its inverse function $\theta_\tau(q)$. Further differentiating the derivative of the incentive-compatible tariff schedule $T'_\tau(q_\tau(\theta)) = \theta v'(q_\tau(\theta))$ gives:

$$T''_\tau(q) = \theta'_\tau(q)v'(q) + \theta_\tau(q)v''(q) = \theta(q)v'(q)\Big[\frac{\theta'_\tau(q)}{\theta_\tau(q)} + \frac{v''(q)}{v'(q)}\Big] \tag{52}$$

$$= T'(q)\Big[\frac{1}{\theta_\tau(q)q'_\tau(\theta)} - A(q)\Big], \tag{53}$$

for $A(q) = -v''(q)/v'(q)$ and $\theta'_\tau(q) = 1/q'_\tau(\theta)$.

By implicit differentiation on the seller's first-order condition, we obtain an expression for $q'_\tau(\theta)$:

$$q'_\tau(\theta) = -\frac{\frac{d}{d\theta}\Big[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\Big]v'(q_\tau(\theta))}{\Big[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\Big]v''(q_\tau(\theta))}$$

$$= \frac{1}{A(q_\tau(\theta))}\frac{\frac{d}{d\theta}\Big[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\Big]}{\Big[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\Big]}$$

From SFOC, the denominator of the equation above is positive as $v'(q_\tau(\theta)) > 0$ and $c > 0$. By assumption, strict monotonicity holds ($q'_\tau(\theta) > 0$), which implies that the numerator is also positive. Substituting into (52) and using the fact that $T'_\tau(q) > 0$ and $A(q_\tau) > 0$, quantity discounts $T''_\tau(q) \leq 0$ hold if and only if

$$\frac{\Big[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\Big]}{\theta\frac{d}{d\theta}\Big[\theta - \frac{\Gamma_\tau(\theta)-F_\tau(\theta)-\sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta))+\theta\gamma_\tau(\theta)}{f_\tau(\theta)}\Big]} \leq 1 \tag{54}$$

Inequality 54 holds if

$$\theta - \frac{\Lambda_\tau(\theta) - F_\tau(\theta)}{f_\tau(\theta)} \leq \theta - \theta\frac{(\lambda_\tau(\theta) - f_\tau(\theta))f_\tau(\theta) - (\Lambda_\tau(\theta) - F_\tau(\theta))f'_\tau(\theta)}{f_\tau(\theta)^2}.$$

Rearranging, one obtains

$$[\Lambda_\tau(\theta) - F_\tau(\theta)][f_\tau(\theta) + f'_\tau(\theta)\theta] \geq \theta f(\theta)[\lambda_\tau(\theta) - f_\tau(\theta)]. \tag{55}$$

From the positive denominator above, one can obtain that $\theta f_\tau(\theta) \geq \Lambda_\tau(\theta) - F_\tau(\theta)$. Moreover, note that the log-concavity of the density $F_\tau(\theta)$ is sufficient to satisfy the standard assumption of the monotone hazard condition. So concentrating on log-concave densities, the following inequality holds: $f_\tau(\theta) \geq f'_\tau(\theta)\theta$. Therefore, if $\Lambda_\tau(\theta) > F_\tau(\theta)$, then a sufficient condition for quantity discounts is $\lambda_\tau(\theta) < f_\tau(\theta)$.

Instead if $\Lambda_\tau(\theta) < F_\tau(\theta)$, one can write 55 as

$$(\theta - 1)f_\tau(\theta) + f_\tau(\theta) \geq [F_\tau(\theta) - \Lambda_\tau(\theta)]\Big(1 + \frac{f'_\tau(\theta)\theta}{f_\tau(\theta)}\Big) + \lambda_\tau(\theta). \tag{56}$$

If $f'_\tau(\theta) < 0$, then a sufficient condition is $(\theta - 1)f_\tau(\theta) \geq F_\tau(\theta)$. If $f'_\tau(\theta) > 0$, then a sufficient condition is that $(\theta - 1)f(\theta) \geq F_\tau(\theta)(1 + \theta f'_\tau(\theta)/f_\tau(\theta))$. Both conditions can be expressed as:

$$\frac{d}{d\theta}\Big(\frac{F_\tau(\theta)}{f_\tau(\theta)}\Big) = \frac{f_\tau(\theta)^2 - F_\tau(\theta)f'_\tau(\theta)}{f_\tau(\theta)^2} \geq \frac{F_\tau(\theta)}{(\theta - 1)f_\tau(\theta)}. \tag{57}$$

$\square$

Intuitively, the condition states that for a general class of distributions, as long as the incentive-compatibility marginal effects dominate those of the limited enforcement, the seller finds it optimal to offer quantity discounts at any relationship age. This condition is likely to be satisfied if the limited enforcement constraint is slack for some buyers even at their first interaction. Moreover, it also requires the enforcement constraint to be slack for all buyers in the long run. This last requirement aligns with the model of Martimort et al. (2017), where buyers reach a *mature* phase in which the constraints no longer bind. This is also consistent with Proposition 4 below, which finds that trade reaches a mature phase.

In terms of generality, the usual monopolist screening problem requires (or uses) log-concavity of $f(\theta)$.[66] I am strengthening the requirement that the evolution of the distribution also satisfies log-concavity, implicitly placing bounds on the distribution of exit rates over types.

The second condition strengthens the requirements on the dynamic distribution of types to ensure that the seller desires to price discriminate across types.

An alternative way to consider this property is to use (t-RULE) to obtain that the tariff schedule is concave if and only if $q_\tau'(\theta) > \frac{v'(q_\tau(\theta))}{-v''(q_\tau(\theta))\theta}$. As long as quantities increase by types fast enough, the seller will offer quantity discounts. The rate at which the quantities have to increase is determined by the level of the type and the curvature of the return function.

Evolution of Quantities

Next, I discuss how quantities evolve in Proposition 4.

Proposition 4. *For each θ, quantity increases monotonically in τ (i.e., $q_\tau(\theta) \leq q_{\tau+1}(\theta)$) if and only if the limited enforcement constraint is relaxed over time ($\gamma_\tau(\theta) \geq \gamma_{\tau+1}(\theta)$). Moreover, there is a time τ^* such that $\forall \tau \geq \tau^*$, $\gamma_{\tau^*}(\theta) = 0$ for all $\theta > \underline{\theta}$ and $q_{\tau^*}(\theta) \geq q_\tau(\theta)$ for all $\tau < \tau^*$ and all θ.*

Proof of Proposition 4. Notice that by the seller's first-order condition and $v'(\cdot) > 0$, $q_\tau(\theta) \leq q_{\tau+1}(\theta)$ holds if and only if

$$
\begin{aligned}
V_\tau(\theta) &\equiv \frac{\Gamma_\tau(\theta) - F_\tau(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_\tau(\theta)}{f_\tau(\theta)} \\
&\geq \frac{f_\tau(\theta)}{f_{\tau+1}(\theta)} \frac{\Gamma_\tau(\theta) - F_{\tau+1}(\theta) - \sum_{s=0}^{\tau-1}(1-\Gamma_s(\theta)) + \theta\gamma_{\tau+1}(\theta)}{f_\tau(\theta)} + \frac{\Gamma_{\tau+1}(\theta) - 1}{f_{\tau+1}(\theta)} \equiv V_{\tau+1}(\theta)v,
\end{aligned}
$$

which can be written as

$$
V_\tau(\theta) \geq \frac{f_\tau(\theta)}{f_{\tau+1}(\theta)} V_\tau(\theta) + \frac{\Gamma_{\tau+1}(\theta) - 1}{f_{\tau+1}(\theta)} + \frac{\theta[\gamma_{\tau+1}(\theta) - \gamma_\tau(\theta)]}{f_{\tau+1}(\theta)} - \frac{F_{\tau+1}(\theta) - F_\tau(\theta)}{f_{\tau+1}(\theta)}.
$$

With no selection pattern, i.e. $f_\tau(\theta) = f_{\tau+1}(\theta)$, the condition reduces to

$$
\frac{1 - \Gamma_{\tau+1}(\theta)}{f_\tau(\theta)} \geq \frac{\theta[\gamma_{\tau+1}(\theta) - \gamma_\tau(\theta)]}{f_\tau(\theta)}.
$$

[66] Log-concavity of a density function $g(x)$ is equivalent to $g'(x)/g(x)$ being monotone decreasing. Families of density functions satisfying log-concavity include: uniform, normal, extreme value, exponential, amongst others.

As $\gamma_\tau(\theta) > 0$ by assumption and the left-hand side is (weakly) positive due to $\Gamma_{\tau+1}(\theta) \leq 1$, a sufficient condition is that $\gamma_{\tau+1}(\theta) < \gamma_\tau(\theta)$. To obtain necessity, consider the Lagrangian keeping future return U^+ constant. The seller chooses $q(\theta)$ maximizing the following program:

$$L(\theta, U, q, \lambda, \gamma) = (\theta v(q(\theta)) - cq(\theta) - U)f(\theta) + \lambda v(q(\theta)) + \gamma(U + \delta U^+ - \theta v(q(\theta))), \quad (58)$$

where λ is the co-state variable for the incentive-compabilitity constraint and γ is the multiplier for the limited enforcement constraint. Noting that the necessary conditions are also sufficient (Seierstad and Sydsaeter, 1986) (pg. 276), the relevant optimality conditions are:

$$f(\theta)[\theta v'(q(\theta)) - c] + \lambda(\theta)v'(q(\theta)) = \gamma(\theta)\theta v'(q(\theta))$$

and

$$\dot{\lambda}(\theta) = f(\theta) - \gamma(\theta)$$

which imply

$$\gamma(\theta) = f(\theta) - \frac{cf(\theta)}{\theta v'(q(\theta))} + \frac{F(\theta) - \Gamma(\theta)}{\theta}.$$

Therefore, a higher level of quantity $q(\theta)$ is implied by a lower $\gamma(\theta)$.

Next, to obtain that $\gamma_\tau(\theta) = 0$ for some finite $\tau > \tau^*$ for all $\theta > \underline{\theta}$. Suppose otherwise, such that $\gamma_\tau(\tilde{\theta}) > 0$ for some $\tilde{\theta}$ and all τ. Then, $\Gamma_\tau(\theta) < 1$ for all $\theta \leq \tilde{\theta}$. Therefore, $1 - \Gamma_\tau(\theta) > 0$ for all $\theta \leq \tilde{\theta}$. Thus, as $\tau \to \infty$, $\sum_{s=0}^{\tau}(1 - \Gamma_s(\theta)) \to \infty$ for all $\theta \leq \tilde{\theta}$. Thus, as long as $q_\tau(\theta) < \infty$ for all θ, τ, it must be the case that some finite τ^* exists such that $\gamma_\tau(\theta) = 0$ for all $\tau > \tau^*$ and for all θ. It is possible however for enforcement constraints to bind for $\underline{\theta}$, as in that case $\Gamma_\tau(\underline{\theta}) = 1$ and quantities would be finite.

Finally, to obtain $q_{\tau^*}(\theta) \geq q_\tau(\theta)$ for all $\tau < \tau^*$ and all θ. Notice that $q_{\tau^*}(\theta) \geq q_\tau(\theta)$ if and only if

$$\theta\gamma_\tau(\theta) + \sum_{s=\tau+1}^{\tau^*-1}(1 - \Gamma_s(\theta)) \geq 0,$$

which always holds. It holds with strict inequality whenever the enforcement constraint binds at period τ, or when it binds in some period between τ and τ^* for some θ between $\underline{\theta}$ and θ.

$\square$

In the model, quantities go hand-in-hand with enforcement constraints. Although the exact path depends on further assumptions on the return function and the distribution of types, the model predicts that quantities will reach a mature phase in which constraints no longer bind, except perhaps for the lowest type. At this mature phase, quantities will be at their highest level in the relationship.

Discounts over time

The model also offers conditions under which discounts over time are observed.

Proposition 5. *If $M_{\tau+1}(\theta) \equiv q_{\tau+1}(\theta) - q_\tau(\theta) \geq 0$ for all θ and with strict inequality for $\underline{\theta}$, then $p_{\tau+1}(q) \equiv T_{\tau+1}(q)/q < T_\tau(q)/q \equiv p_\tau(q)$.*

Proof of Proposition 5. Use the marginal price function $T'_\tau(q) = \theta_\tau(q)v'(q)$. Average unit prices $p_\tau(q)$ for $q > 0$ are given by:

$$p_\tau(q) = \frac{T_\tau(q)}{q} = \frac{\int_0^q \theta_\tau(x)v'(x)dx}{q},$$

where I have used the normalization $T_\tau(0) = 0$ and the inverse function $\theta_\tau(q)$. Average prices decrease over time if and only if

$$\int_0^q \theta_\tau(x)v'(x)dx > \int_0^q \theta_{\tau+1}(x)v'(x)dx$$
$$\iff$$
$$\int_0^q [\theta_\tau(x) - \theta_{\tau+1}]v'(x)dx > 0.$$

By assumption, $q_\tau(\theta) \geq q_{\tau+1}(\theta)$ (and strictly so for $\underline{\theta}$). Thus, $\theta_\tau(q) > \theta_{\tau+1}(q)$ for all q and the inequality holds.

□

As long as quantities (weakly) increase from τ to $\tau + 1$, unit prices at any given q decrease. The intuition behind this result is that marginal prices match marginal returns. A right-ward shift in quantities for (some) buyers further lowers marginal returns, requiring a decrease in marginal prices as well. As such, average prices will be lower at each q as well.

To further understand the dynamics in the model, I present a solved two-type example in Online Appendix Section OA-6. The example illustrates the backloading of prices and quantities together with quantity discounts as a way to maximize lifetime profits for the seller while preventing opportunistic behavior from the buyer.

OA-7.2 Static Efficiency of Limited Enforcement

We now turn to analyzing the efficiency of contracts with limited enforcement. Relationship-specfic total surplus (and thus efficiency) is determined by the total quantity transacted at a point in time. I concentrate on static (period-by-period) efficiency, as it is common in the relational contracting literature (e.g., as in Fong and Li, 2017; Kostadinov and Kuvalekar, 2022), rather than total lifetime efficiency.

For simplicity, suppose that $\theta\gamma_\tau(\theta)$ is small enough so the quantities allocated in the limited enforcement contract with no exit ($X(\theta) = 0$) and the assumed parametrization of $v(\cdot)$ are given by:

$$q_\tau^{LEm}(\theta)^{1-\beta} \approx \frac{k\beta}{c}\left[\theta - \frac{\Gamma_\tau(\theta) - F_\tau(\theta) - \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\theta))}{f(\theta)}\right].$$

With some abuse of notation, define the modified value of the cumulative multiplier at time τ as $\tilde{\Gamma}_\tau(\theta) = \Gamma_\tau(\theta) - \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\theta))$, so the allocation is given by:

$$q_\tau^{LEm}(\theta)^{1-\beta} \approx \frac{k\beta}{c}\left[\theta - \frac{\tilde{\Gamma}_\tau(\theta) - F(\theta)}{f(\theta)}\right].$$

Moreover, recall that the first-best outcome is given by:

$$q_\tau^{FB}(\theta)^{1-\beta} = \frac{k\beta}{c}\theta.$$

If $\tilde{\Gamma}_\tau(\theta) < F(\theta)$, there is overconsumption relative to first best. If $\tilde{\Gamma}_\tau(\theta) > F(\theta)$, there is underconsumption. If $\tilde{\Gamma}_\tau(\theta) = F(\theta)$, trade is fully efficient. Therefore, this limited enforcement model allows for the possibility of efficient trade, as well as inefficient trade either through underconsumption or overconsumption.

For the case with underconsumption, i.e., $\tilde{\Gamma}_\tau(\theta) > F(\theta)$, efficiency increases over time if $\tilde{\Gamma}_\tau(\theta) < \tilde{\Gamma}_{\tau-1}(\theta)$. By reordering and eliminating repeated terms, the condition becomes $\Gamma_\tau(\theta) < 1$. Thus, under the case with no exit and underconsumption, we expect efficiency to increase until pair-wise trade becomes unconstrained. Note, however, that quantities may converge at inefficient levels.

OA-7.3 Static Efficiency Relative to Perfect Enforcement

Comparing equations SFOC and PE-Q, in the case with no exit $X(\theta) = 0$ for all θ, the total quantity transacted is greater under full enforcement than under limited enforcement if:

$$(1 - \Gamma_\tau(\theta)) + \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\theta)) - \theta\gamma_\tau(\theta) < 0. \tag{59}$$

For the types for which the limited enforcement constraint is not binding (so $\gamma_\tau(\theta) = 0$), except for the highest type, the inequality does not hold, and pair-wise welfare decreases under full enforcement. This will likely matter for middle/high types early on. Moreover, it might apply too for lower types in the long-term that started with binding constraints at the beginning for the contract but that grew over time to become unconstrained. Therefore, welfare can be greater under a long-term relational contract with limited enforcement than under perfect enforcement.

For types with $\gamma_\tau(\theta) > 0$, the inequality can be written as:

$$\theta - \frac{1 - \Gamma_\tau(\theta)}{\gamma_\tau(\theta)} > \frac{\sum_{s=0}^{\tau-1}(1 - \Gamma_s(\theta))}{\gamma_\tau(\theta)}.$$

The inequality above reminds us of a modified virtual surplus, where instead of the distribution of types we use the distribution of enforcement constraints. For perfect enforcement to be welfare increasing, the virtual surplus accounting for contemporaneous information rents of limited enforcement has to be greater than the information rents (promises to increase quantity) stemming from past enforcement constraints. Of course, early on, perfect enforcement could be more efficient, yet, as relationships age this might be more difficult to sustain.

In contrast with the arguments set forward in past literature, I have shown that in the interaction of market power and enforcement constraints could imply that weak legal enforcement is actually efficiency *increasing* at some points in time, and particularly so in the long-run. Intuitively, absent enforcement constraints, the seller is able to offer *the* profit-maximizing menu of quantities and prices. The buyer's ability to act opportunistically restricts how much the seller can extract and changes the surplus in favor of the buyer.

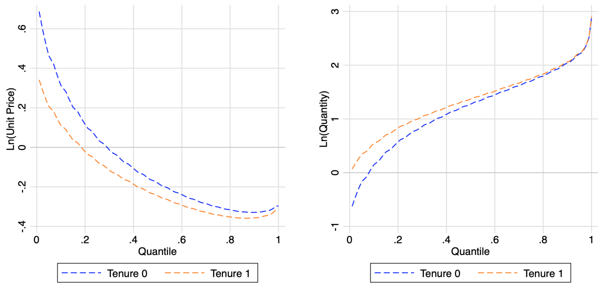
OA-8 Monte Carlo Study

The Monte Carlo studies the behavior of my estimators for two periods of a dynamic contract without breakups. I use the following design. The return function is $v(\theta, q) = \theta q^{1/2}$. The type distribution is Weibull with scale parameter equal to 1 and shape parameter equal to 2, $F(\theta) = 1 - exp(-(\theta - 1)^k)$, normalized so $\underline{\theta} = 1$.[67] Marginal

[67] Recall that the model requires the type distribution to verify the monotone hazard rate condition, $\frac{d}{d\theta}\frac{F(\theta)}{f(\theta)} \geq 0 \geq \frac{d}{d\theta}\frac{1-F(\theta)}{f(\theta)}$. Distributions that satisfy the monotone hazard rate condition include: Uniform,

Figure OA-14: **Prices and Quantities by Quantile**

(a) **Unit Prices** (b) **Quantity**



Notes: These figures show the level of prices and quantities by quantile of quantity for tenure 0 and tenure 1 in the Monte Carlo simulation.

cost is 0.45. Although the multiplier function $\Gamma_\tau(\theta)$ is the solution to a differential equation linking the type distribution $F(\theta)$, the marginal cost, and the average base marginal return of types $\tilde{\theta} \leq \theta$, I parametrize it as a logistic distribution. In tenure 0, $\Gamma_0(\theta)$ has location parameter equal to 1 and scale parameter equal to 0.5. Instead, in tenure 1, $\Gamma_1(\theta)$ has location parameter 1 and scale 0.35. The lower scale parameter at tenure 1 reflects the idea that over time, the limited enforcement constraint is less binding. I construct the tariffs following Pavan et al. (2014): $t_\tau(\theta) = \theta q_\tau(\theta)^{1/2} - \int_{\underline{\theta}}^{\theta} q_\tau(x)^{1/2} dx$.

I randomly draw 1000 values of θ using $F(\theta)$ and obtain corresponding quantities $q_0(\theta)$ and $q_1(\theta)$ using the first-order condition of the seller and the assumed parametrizations of the return function, marginal cost, and multiplier at tenure 0 and 1. Then, I obtain the corresponding tariffs and I apply my estimator as defined in the previous sections to estimate $\{\theta, U(\cdot), \Gamma_\tau(\cdot)\}$. I repeat this 300 times to construct the dispersion for my estimates.

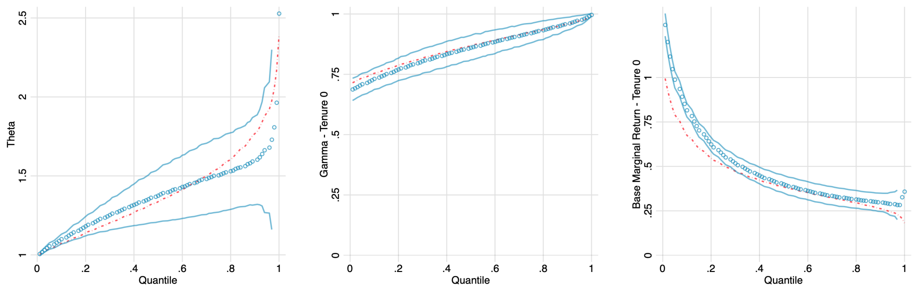
Online Appendix Figure OA-14 shows the (log) average prices and average quantities generated by the model for the two types of tenure. The model delivers quantity discounts (decreasing unit prices in θ), strict mononoticity of quantity (increasing quantities in θ), and backloading in the dynamic model, namely, further discounts and larger quantities in tenure 1 for each θ.

Online Appendix Figure OA-15 shows the results of the estimated Gamma distribution and the base marginal return, again in blue the estimated results and in red the true values. Both cases indicate good fit. Subfigure (a) shows the estimated $\hat{\theta}$ in blue and true θ in red by quantile. Dispersion at the 95 percent level are included for all except the top 2 quantiles, as they start to diverge. Overall, the figure shows a good fit, with most sections of including the true θ within their dispersion.

Next, I show the tenure 1's results estimates. Recall that the first-order condition of the seller now includes a backward-looking variable $1 - \Gamma_0(\theta)$ that keeps track of whether the limited commitment constraint was binding in the past. This variable is used by seller as a promise-keeping constraint that guarantees the seller delivers higher quantities and return in the future to prevent buyers from defaulting in the past. In my estimation, I use the tenure 0's predicted $\widehat{\Gamma}_0(\theta(\alpha))$ for each quantile α. Online Appendix Figure OA-16 shows the estimated Gamma distribution and the base marginal return. Although the fit

Normal, Logistic, Extreme Value (including Frechet), Weibull (shape parameter ≥ 1), Exponential, and Power functions.

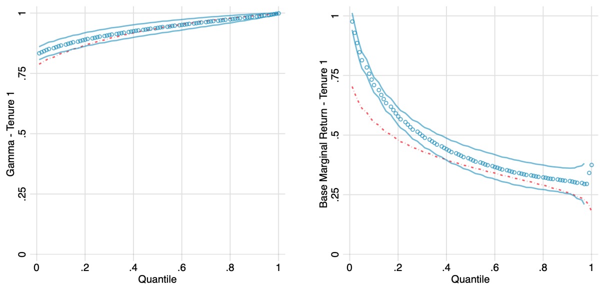
Figure OA-15: **Monte Carlo Results for Tenure 0**



(a) Types θ (b) Γ_0 Distribution (c) $v'(\cdot)$

Notes: Panel (a) plots the true (red) and estimated distribution of types (in blue) by quantile of quantity. Panel (b) plots the true (red) and estimated value (blue) of the LE multiplier for tenure 0 by quantile of quantity. Panel (c) plots the true (red) and estimated value (blue) of the base marginal return for tenure 0 by quantile of quantity. Error margins indicate ± 1.96 variation around estimated mean from 300 simulations.

Figure OA-16: **Monte Carlo Results for Tenure 1**



(a) Γ_1 Distribution (b) $v'(\cdot)$

Notes: Panel (a) plots the true (red) and estimated value (blue) of the LE multiplier for tenure 1 by quantile of quantity, with error margins indicating ± 1.96 variation around the estimated mean. Panel (b) plots the true (red) and estimated value (blue) of the base marginal return for tenure 1 by quantile of quantity, with error margins indicating ± 1.96 variation around the estimated mean from 300 simulations.

is worse than in tenure 0, the dispersion of both gamma and the base marginal return include tend to include their true values.

With respect to the differences between true and estimated functions, I find that the slight upward bias in the Gamma function for tenure 1 disappears if I use the true $\Gamma_0(\theta)$ function instead of the estimated $\widehat{\Gamma}_0$, suggesting that the bias is generated by sampling error in the tenure 0 estimates. Moreover, differences in the base marginal return for both tenure 0 and tenure 1 come from approximating the tariff function as log-linear. In the Monte-Carlo, the change in unit price is very steep for low-types, and this generates some approximation error for low-types in terms of the base marginal return function. Despite this error, the coefficient of the base return function is correctly estimated when using the assumed parametrization, observations of quantity, and the nonparametric estimates of $v'(\cdot)$ as target. In particular, the estimated coefficient cannot be rejected to be different from 0.5 (the assumed value in simulation).

OA-9 Evidence for Marginal Costs Constancy Assumption

I provide empirical support for the assumption of constant marginal cost in three ways.

First, I present evidence that average variable cost (AVC) is relatively constant over time. For each seller i at time t, I construct *quarterly* measures of average cost by dividing total variable cost (intermediate inputs plus labor) in the quarter by total quantity sold in the quarter:

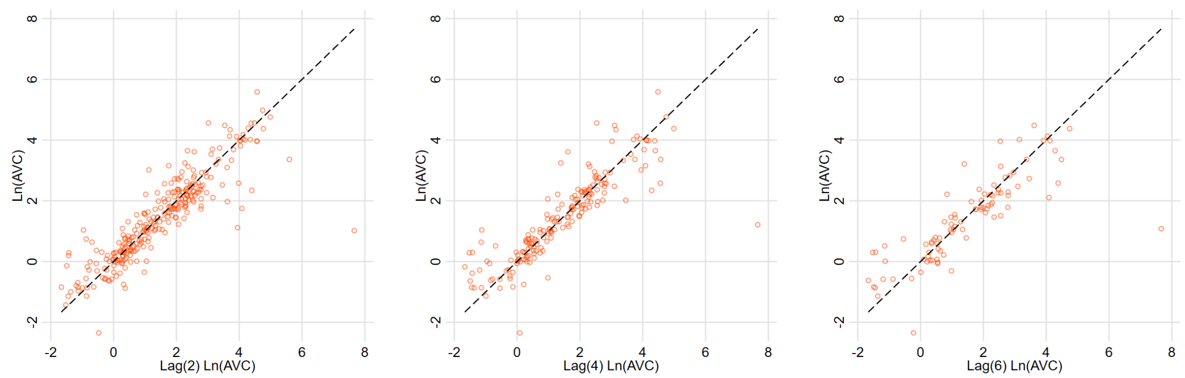
$$AVC_{it} = \frac{VC_i(Q_{it})}{Q_{it}},$$

where $VC_i(\cdot)$ is the variable cost function. Marginal cost is related to the previous equation via the derivative of the variable cost function: $MC_i(Q_{it}) = VC_i'(Q_{it})$. If marginal cost is constant, then $VC_i'(Q_{it}) = c_i Q_{it}$ and $AVC_{it} = c_i$. Therefore, strong serial correlation in AVC across periods indicates the following relationship:

$$AVC_{it} = c_i + \varepsilon_{it}.$$

Appendix Figure OA-17 presents a scatter plot of the (log) average variable cost on two, four, and six lags, with the dashed diagonal presenting a 1-to-1 fit. The figure shows that even after one and a half years apart, the average variable cost traces the diagonal fairly well.[68] This type of test is meaningful as sellers do experience variation in sales across months (Online Appendix Table OA-5), and therefore Q_{it} is non-constant.

Figure OA-17: **Serial Correlation**



(a) 2 Lags (b) 4 Lags (c) 6 Lags

Notes: These figures present the scatter plots of firm-level quarterly measures of average variable costs against 2 quarter lags (a), 4 quarters (b) and 6 quarters (c).

Second, I verify the constancy of average variable costs using a regression framework by regressing (log) average variable costs on seller fixed effects. I find that seller effects explain 87% of all variation using quarterly data and 84% using monthly data.

Third, under the assumption of constant marginal costs, we obtain the following accounting relationship for total variable costs: $VC_{it} = c_i Q_{it}$. Taking logs yields:

$$\ln(VC_{it}) = \ln(c_i) + \ln(Q_{it}).$$

This equation creates a testable framework for regression:

$$\ln(VC_{it}) = \beta_Q^c \ln(Q_{it}) + \ln(c_i) + \varepsilon_{it},$$

[68] A similar relationship exists if we focus only on monthly variation.

Table OA-6: Test for constancy of marginal cost

VARIABLES	(1) ln(VC)	(2) ln(VC)
ln(Q)	0.163**	0.757**
	(0.0723)	(0.302)
P-Value ($\beta^c_Q = 1$)	0.000	0.415
Observations	384	384
Seller FE	Yes	Yes
Time	Quarterly	Quarterly
Method	OLS	IV

Notes: This table presents the results of the test for constancy of marginal costs, of (log) total variables costs on (log) quantity. Column(1) reports OLS and Column (2) reports the instrumental variable results. Unit of observation is at the seller-quarter-level. Standard errors are clustered at the seller level. ***p<0.01, **p<0.05, *p<0.1

where $\beta^c_Q = 1$ under constant marginal costs, $\ln(c_i)$ is captured by a seller fixed effect, and ε_{it} is noise, possibly stemming from model specification (i.e., true costs are non-constant and thus c_{it} is time-varying).

Notice that an OLS regression would not serve to test this equation if true marginal costs are time-varying, even if they are constant at the output level within the time period. An increase in true time-varying marginal cost is likely associated with a total decrease in quantity sold (as the seller increases prices to buyers). Thus, as quantity increases total variable costs, $\beta^c_Q > 0$, the negative relationship between costs and observed quantities implies downward bias in OLS due to omitted variable bias.

For that reason, I test this equation using an instrumental variable approach that exogenously shifts Q_{it} from changes in marginal costs captured by ε_{it}. Specifically, I use downstream demand shift-share style shocks in the spirit of Acemoglu et al. (2016) and Huneeus (2018). For a given selling firm i, I consider their 2015 demand share s^{2015}_{ij} over buyers j. Then, for each buyer, I regress their quarterly volume of log sales on buyer fixed effects and quarter-year fixed effects and collect the residuals as demand shocks $shock^d_{jt}$. For each seller, I obtain the weighted average of their exposure to potential demand shocks IV^d_{it} as follows:

$$IV^d_{it} = \sum_j s^{2015}_{ij} \times shock^d_{jt}.$$

I then run a regression for the testing equation using quarterly data at the seller level, using IV^d_{it} as an instrument for quarterly quantity Q_{it}.

Internet Appendix Table OA-6 shows the results. First, OLS (Column 1) shows a downward bias relative to the IV (Column 2), indicating some degree of model misspecification or measurement error in total quantity. Second, in the instrumental variable approach, we fail to reject that β^c_Q is equal to 1 (although, the point estimate is not precisely estimated at 1). Therefore, the test is again consistent with a constant marginal cost assumption.

Thus, all in all, the constant marginal cost assumption is not incredibly restrictive in this setting.

OA-10 Additional Estimation Results and Model Fit

OA-10.0.1 *Tariff Function*

Despite the simple approximation of the tariff function in equation 9, the within-tenure seller-specific tariff functions show a good fit. The average R-squared is close to 0.80, and the distribution of R-squared estimates for each seller-tenure (Figure OA-18) shows a good fit across the board.

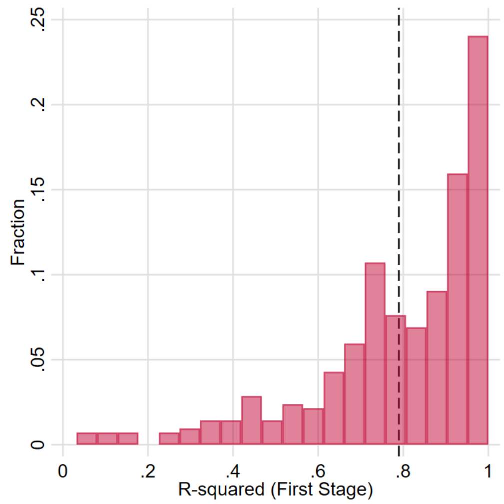


Figure OA-18: R-squared Distribution in the Estimation of the Tariff Function

Notes: This figure presents the distribution of R-squared values from seller-tenure-year regressions (equation 9).

Of course, as the fit is not perfect, it is worth highlighting some sources of measurement error in the tariff function. First, it is possible that the firm price schedule has higher-order terms, which would generate measurement error. However, this concern is small, as estimating a quadratic model only improves the R-squared on average by 0.008. Second, it is possible that, besides pricing on tenure and quantity, the firm is also pricing based on other unobservable characteristics (to the econometrician), which creates misspecification error, translating into measurement error. This would be particularly worrisome if the price schedule over quantities and tenure is not linearly separable from the other pricing characteristics. However, as shown in Table 5, the coefficients for prices on quantities and tenure are unaffected by the inclusion of a large set of buyer characteristics, supporting the assertion that pricing on other (plausibly unobserved) characteristics might enter as orthogonal measurement error.

OA-10.1 Survival Function Probability

Online Appendix Figure OA-19 presents estimated survival probabilities by age of relationship and quantile of quantity, with variation representing differences across seller-years.

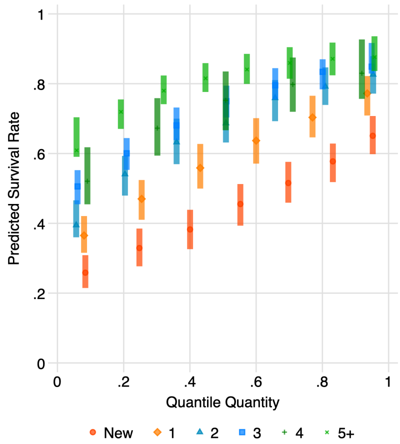


Figure OA-19: Survival Probability Function

Notes: This figure presents the estimated survival probability by quantile of quantity and age of relationship across seller-years. Confidence intervals represent the 90% level of variation across sellers, with standard errors clustered at the seller-year level.

OA-10.2 Distribution of t-Statistics against Standard Model Null

Online Appendix Table OA-7 shows the distribution of t-statistics for tests against a standard model null.

Table OA-7: **Distribution of t-Statistics**

	p10	p25	p50	p75	p90
Tenure 0	0.31	4.64	11.55	30.08	109.27

Notes: This table reports distribution of t-statistics for tests against a standard model null (e.g., $\Gamma_0(\cdot) = 1$).

OA-10.3 Parametrization of the Base Return Function

To conduct counterfactual experiments that consider quantities beyond those observed in the data, I parametrize the seller-specific buyer's return function $v(q) = kq^\beta$ for $k > 0$ and $\beta \in (0, 1)$. This return function satisfies the modeling assumptions $v'(\cdot) > 0$ and $v''(\cdot) < 0$.

To estimate the parameters, I consider tenure 0 transactions between buyer i and seller j at year t and perform the following least squares regression:

$$\ln(\widehat{v'}_{ijt}) = \ln(k) + \ln(\beta) + (\beta - 1)\ln(q_{ijt}) + \varepsilon_{ijt},$$

using $v'(q) = k\beta q^{\beta-1}$, the estimated base marginal returns $\widehat{v'}_{ijt}$, and under the assumption that ε_{ijt} is Gaussian error.

Online Appendix Table OA-8 presents the distribution of k and β.

Table OA-8: **Parameters of Return Function**

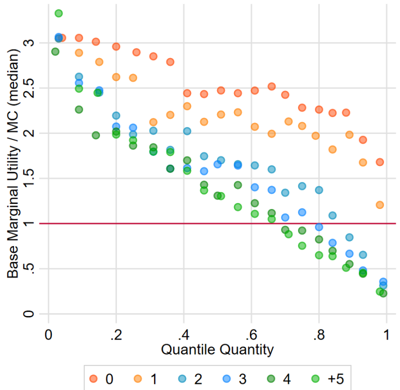
	mean	p10	p25	p50	p75	p90
β	0.56	0.30	0.48	0.61	0.76	0.82
k	171.23	9.00	17.24	39.64	86.61	282.40

Notes: This table reports distribution of estimated values for the ex-post parametrization of the return function.

OA-10.4 Economic Magnitudes: Base Marginal Return

Online Appendix Figure OA-20 presents a binscatter of the ratio of marginal revenue product (base marginal return) over marginal costs against the quantile of quantity, across sellers for tenure 0. It shows that the return of the input for the buyer is greater than the private marginal cost of providing it for the seller, for a majority of the buyers. For instance, the median buyer obtains 1.5 dollars of revenue for each dollar spent by the seller to produce the product.

Figure OA-20: **Base Marginal Return over Marginal Costs**



Notes: This figure plots the median of the ratio of base marginal return to marginal costs across sellers by quantile of quantity for each tenure.

OA-10.5 Model Fit

Online Appendix Figure OA-21 presents the statistical fit of the model across tenures. It plots a reordered equation I-EQ's left-hand side on the X-axis and the model's prediction using estimated coefficients of the right-hand side on the Y-axis.[69] Fit generally worsens for higher tenures; the results from Monte Carlo studies in Online Appendix OA-8 suggest that the decrease in statistical fit is driven by noise from using estimates for limited enforcement multipliers $\Gamma_s(\cdot)$ for earlier tenures s.

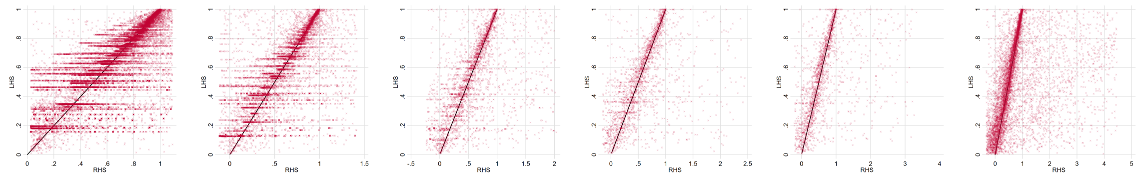
Online Appendix Figure OA-22 shows the fit in terms of quantities. To obtain quantities, I use the parametrization $v(q) = kq^\beta$, for $k > 0$ and $\beta \in (0, 1)$ and the closed-form formula in 8.

[69]Reorder equation I-EQ to obtain:

$$\alpha = \Gamma_\tau(\alpha) - \sum_{s=0}^{\tau-1}(1 - \Gamma_s(\alpha)) - \left[\frac{T'_\tau(q_\tau(\alpha)) - c_\tau}{T'_\tau(q_\tau(\alpha))} - \gamma_\tau(\alpha)\right]\frac{\theta_\tau(\alpha)}{\theta'_\tau(\alpha)},$$

and use the estimated analogues of the right-hand side to make the predictions.

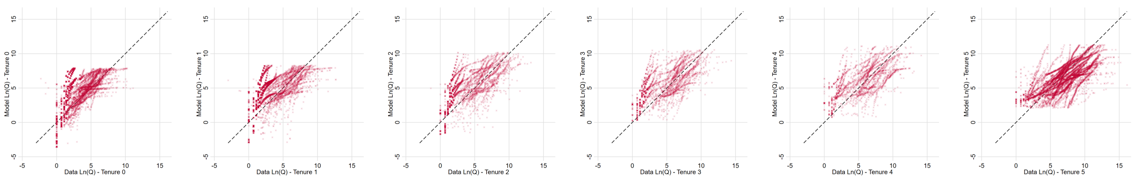
Figure OA-21: **Model Fit - Statistical**



(a) T0 (b) T1 (c) T2 (d) T3 (e) T4 (f) T5

Notes: These figures show binscatters of statistical fit of the model across tenures as implied by identification equation I-EQ. On the X-axis, it shows the predicted cumulative distribution function for the observation while on the Y-axis it plots the observed value.

Figure OA-22: **Model Fit - Quantities**

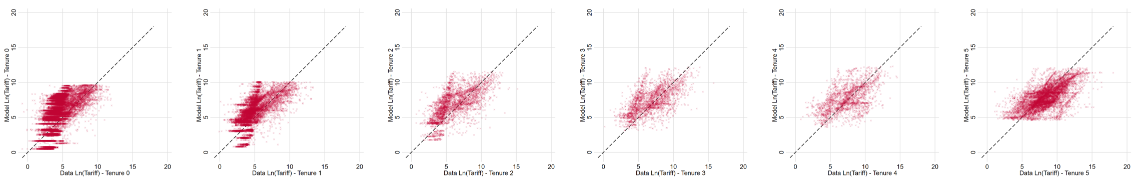


(a) T0 (b) T1 (c) T2 (d) T3 (e) T4 (f) T5

Notes: These figures display binscatters of model fit according to quantities. Estimated quantities use the close-form formula under the CES parametrization of the return function, as discussed in Appendix OA-8. The X-axis plots the observed (log) quantities and Y-axis model predicted (log) quantities.

Online Appendix Figure OA-23 shows the fit of tariffs. To generate tariffs in the model, I use the empirical equivalent of equation t-RULE.

Figure OA-23: **Model Fit - Tariffs**

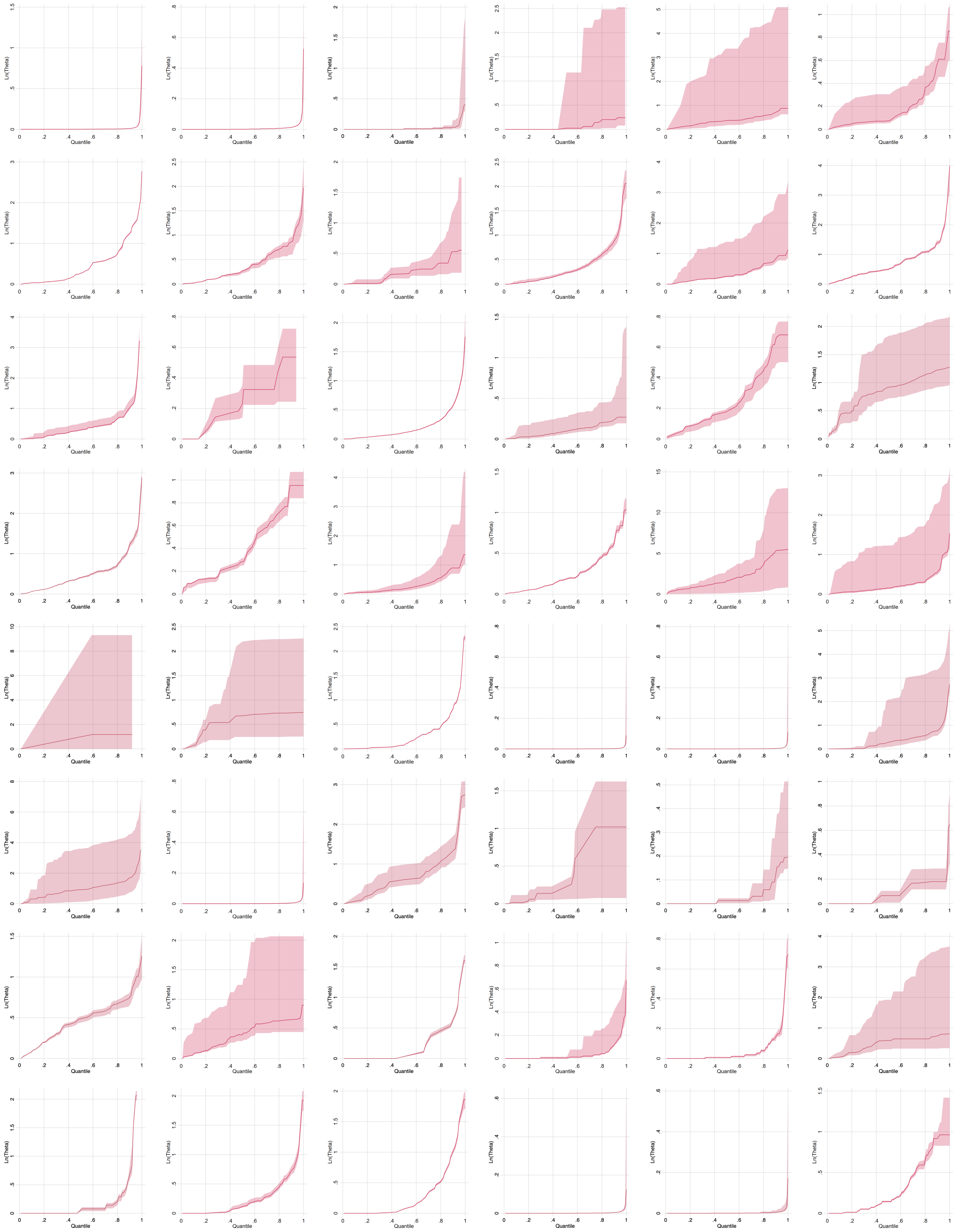


(a) T0 (b) T1 (c) T2 (d) T3 (e) T4 (f) T5

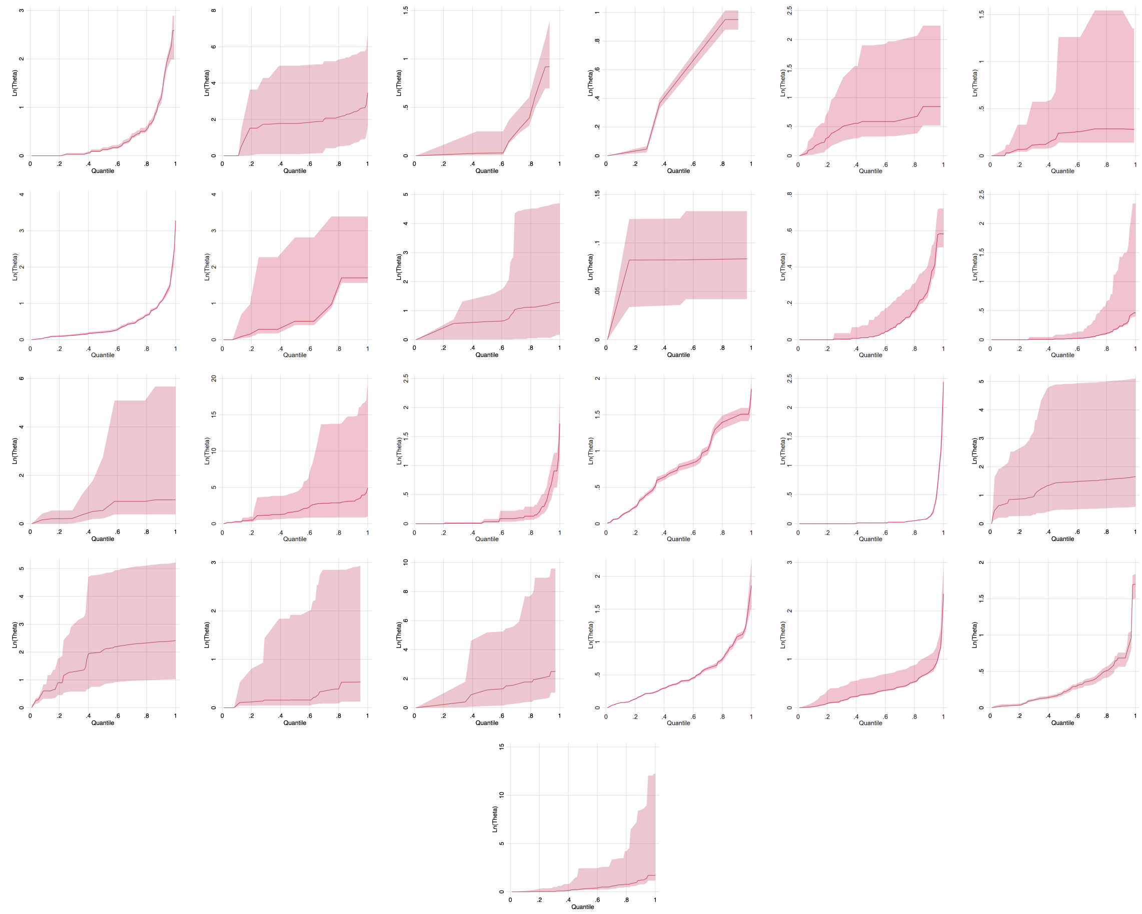
Notes: These figures display binscatters of model fit according to tariffs. Estimated tariffs are generated by using the empirical analogue of the transfer rule t-RULE, taking as inputs estimated parameters θ, the parametrized return function $v(\cdot)$, and model generated quantities. The X-axis plots the observed (log) tariffs and Y-axis model predicted (log) tariffs.

OA-10.6 Bootstrapped Distribution of Types

Figure OA-24: Bootstrapped Distribution of Types



Bootstrapped Distribution of Types (Continued)



Notes: This figure plots distribution of types (log type $ln(\theta)$ by quantile of quantity) for each seller-year. The error bars show variation at the 90% confidence interval level, obtained from 30 bootstrapped simulations for each seller-year.

OA-11 Additional Counterfactual Results

This subsection presents comparisons of different counterfactual models relative to the baseline nonlinear pricing regime with limited enforcement. Online Appendix Table OA-9 shows all the results. The table present the *share of observations* in each percentile group for which each reported category (e.g., buyer's net return) is greater under the baseline than under the alternative. The main takeaways are the following.

Buyers. Small-quantity buyers tend to prefer limited enforcement of contracts over perfect enforcement. They can effectively use the threat of default to reap higher returns. In contrast, the median and top buyers prefer perfect enforcement in the short term but limited enforcement in the long term. Under weak enforcement of contracts, buyers prefer price discrimination over uniform pricing, as otherwise they would be excluded from trade (only median and top buyers prefer uniform pricing in the long term). However, if exclusion and default are restricted, most buyers prefer uniform pricing.

Sellers. Sellers prefer limited enforcement in the short term but perfect enforcement in the long term. Under weak enforcement of contracts, they enjoy the ability to price discriminate, as it allows them to sell to buyers that would otherwise be excluded from trade. In contrast, if enforcement is strong, sellers prefer uniform pricing in the short term but price discrimination in the long term. This preference is driven by the rapid increase in quantities, despite the decrease in unit prices offered to most buyers as an incentive not to default.

Table OA-9: **Counterfactual Policies**

		Nonlinear + Perfect						**Uniform + Limited**						**Uniform + Perfect**					
		10%	25%	50%	75%	100%	Agg.	10%	25%	50%	75%	100%	Agg.	10%	25%	50%	75%	100%	Agg.
Buyer Return	*Tenure 0*	43.4	38.2	11.0	4.9	7.1	6.9	97.3	96.5	96.0	94.3	91.7	92.0	0.1	0.2	0.6	7.0	41.8	38.5
	Tenure 1	68.3	55.3	23.0	9.4	11.9	11.8	94.6	92.2	88.6	88.0	87.4	87.6	0.1	0.1	0.2	13.5	54.9	47.0
	Tenure 2	64.3	46.5	31.1	26.2	28.4	28.3	83.8	79.6	70.3	66.9	63.1	63.6	1.2	0.4	0.9	10.9	32.1	29.6
	Tenure 3	66.3	59.8	40.5	32.3	38.0	37.6	79.7	71.4	59.6	54.6	55.4	55.5	3.1	0.8	1.6	11.2	27.8	25.5
	Tenure 4	61.2	48.6	43.5	42.6	50.5	49.0	69.0	59.9	47.6	47.9	46.3	46.7	5.3	1.2	4.9	8.8	21.3	18.6
	Tenure 5	58.7	61.8	66.1	59.6	69.5	67.8	69.1	62.2	38.3	34.8	32.8	33.5	0.7	1.6	2.9	9.0	22.0	19.6
Seller Profit	*Tenure 0*	34.1	41.6	88.2	94.9	92.8	93.0	92.7	92.6	96.4	98.0	98.4	98.4	7.1	7.4	11.1	35.0	47.4	46.4
	Tenure 1	53.9	55.0	83.3	90.6	88.1	88.3	99.1	96.7	94.8	97.1	89.9	91.2	29.1	18.4	29.8	44.8	52.8	51.0
	Tenure 2	46.6	49.1	71.5	73.8	71.6	71.8	95.0	97.0	98.2	99.5	97.5	97.7	34.1	35.1	50.8	69.1	86.6	84.3
	Tenure 3	45.8	48.1	61.2	67.9	62.0	62.5	96.5	99.2	97.5	99.3	93.9	94.5	49.6	50.0	61.6	77.9	86.6	85.1
	Tenure 4	52.0	47.1	59.1	57.4	49.5	51.1	92.9	97.6	95.0	95.2	94.5	94.6	53.5	64.2	71.4	86.5	93.7	91.5
	Tenure 5	56.1	42.5	36.8	40.6	30.5	32.4	93.4	93.5	96.0	97.4	95.9	96.1	64.9	66.0	81.9	93.1	94.8	93.9
Surplus	*Tenure 0*	18.6	18.9	9.0	3.8	2.6	2.7	98.4	98.1	98.8	98.5	99.5	99.5	3.8	4.1	5.2	12.0	65.5	60.4
	Tenure 1	40.5	41.7	30.3	12.6	29.6	26.9	97.5	96.2	97.3	99.2	100.0	99.8	6.0	7.4	11.0	31.9	76.1	67.6
	Tenure 2	47.8	50.9	48.3	63.2	72.8	71.5	90.9	90.7	91.6	98.6	99.7	99.5	15.3	16.4	27.3	57.0	95.0	90.2
	Tenure 3	61.0	57.8	69.7	76.8	69.9	70.5	93.8	92.3	89.5	98.5	99.6	99.4	24.6	26.5	37.4	69.1	98.4	94.0
	Tenure 4	65.6	71.9	74.5	77.1	67.3	69.2	81.0	87.8	85.4	98.4	99.5	98.9	25.7	34.3	51.0	79.4	97.9	92.9
	Tenure 5	74.4	79.7	88.7	91.2	84.9	85.9	84.8	86.6	80.6	97.1	100.0	98.7	30.8	34.1	53.1	86.4	99.9	95.7
Unit Prices	*Tenure 0*	75.9	75.4	89.0	94.5	92.9	93.0	93.6	93.1	95.4	90.3	42.9	47.4	93.6	93.1	95.4	90.3	42.9	47.4
	Tenure 1	55.3	55.5	77.6	90.5	88.0	88.2	98.6	96.8	87.9	68.2	24.6	33.1	98.6	96.8	87.9	68.2	24.6	33.1
	Tenure 2	38.6	55.1	67.6	73.7	71.7	71.8	92.3	95.0	90.9	64.5	18.0	23.6	92.0	94.9	90.9	64.5	18.0	23.6
	Tenure 3	36.5	41.4	58.3	67.9	61.8	62.3	91.2	97.0	89.1	56.0	13.7	19.7	90.8	97.0	89.1	56.0	13.7	19.7
	Tenure 4	37.9	51.7	56.7	59.1	49.4	51.2	89.2	95.8	88.0	63.2	18.7	28.8	88.7	95.8	88.0	63.2	18.6	28.6
	Tenure 5	34.4	34.1	33.8	39.5	30.5	32.0	90.0	91.4	87.6	54.0	10.4	20.5	89.1	91.2	87.5	53.7	10.0	20.1
% Excluded	*Tenure 0*	-	-	-	-	-	-	97.3	96.4	95.8	94.1	90.5	90.9	-	-	-	-	-	-
	Tenure 1	-	-	-	-	-	-	93.4	91.9	88.6	87.3	85.8	86.1	-	-	-	-	-	-
	Tenure 2	-	-	-	-	-	-	81.5	77.8	70.1	65.7	61.3	61.9	-	-	-	-	-	-
	Tenure 3	-	-	-	-	-	-	76.9	69.0	59.5	51.5	50.0	50.4	-	-	-	-	-	-
	Tenure 4	-	-	-	-	-	-	66.8	58.1	47.5	44.7	43.5	50.0	-	-	-	-	-	-
	Tenure 5	-	-	-	-	-	-	65.3	58.8	37.5	29.8	25.4	26.7	-	-	-	-	-	-

Notes: This table reports the % share of observations for which the reported category (e.g., Buyer's Net Return) is greater under the observed nonlinear pricing regime than under the alternative policy. The values are reported across different tenures and percentiles in the distribution of types. The policies considered are 1) Nonlinear pricing with perfect enforcement, 2) Uniform monopolist pricing with limited enforcement, and 3) Uniform monopolist pricing with perfect enforcement. The reported categories are Buyer's Net Return, Seller's Profits, Total Surplus, Unit Prices, and percentage of Excluded Buyers.