Math454_A03

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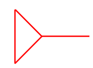
#1

Let v be a vertex of G. If the neighborhood of v is not a clique, then v has a pair x, y of nonadjacent neighbors; $\{x, v, y\}$ induces P_3. If the neighborhood of v is a clique, then since G is not complete there is some vertex y outside the set S consisting of v and its neighbors. Since G is connected, there is some edge between a neighbor w of v and a vertex x that is not a neighbor of v. Now the set $\{v, w, x\}$ induces P_3, since x is not a neighbor of v.

Counterexample: In the graph below, e lies in no such subgraph. e 

#2

If edge e in walk W does not lie on a cycle consisting of edges in W, then by our characterization of cut-edges, e is a cut-edge of the subgraph H consisting of the vertices and edges in W. This means that the walk can only return to e at the endpoint from which it most recently left e. This requires the traversals of e to alternate directions along e. Since a closed walk ends where it starts, the number of traversals of e by W must be even.

#3

First, we show that in the event that G is bipartite, at that point the condition on H holds. Leave A and B alone the arrangements of the bipartition of G. Each instigated subgraph H must contain k vertices from A and $|V(H)| - k$ vertices from B for some k. since $k + (V(H)\ | -k) = |V(H)|$, in any event, one of the two must be at any rate $|V(H)|/2$, so we just take all the vertices from the relating side since we realize A and B are every free set. Presently we show that if the condition on H holds, our graph must be bipartite. Assume the condition holds. We'll show by contrapositive that we can't have any odd cycles for this situation. Assume we have some odd cycle. At that point, we should likewise have some insignificant odd cycle C (one that doesn't contain any chords). Let H alone the actuated subgraph on the vertices of C. In any case, an autonomous set in an odd cycle can't be bigger than $(|V(C)| - 1)/2 = (|V(H)| - 1)/2 < |V(H)|/2$, so we've violated our condition. This reveals to us that if our condition holds we can't have any odd cycles, henceforth our graph must be bipartite.

#4

Let w be the first repetition of a vertex along W, arriving from v on edge e. From the first occurrence of w to the visit to v is a w, v walk, which is a cycle if $v = w$ or contains a nontrivial w, v-path P. This completes a cycle with e unless in fact P is the path of length 1 with edge e, in which case e repeats immediately in opposite directions.

#5

If G is a loopless graph and $\delta(G) \geq 3$, then G has a cycle of even length. An endpoint v of a maximal path P has at least three neighbors on P. Let x, y, z be three such neighbors of v in order on P. Consider three v, y-paths: the edge vy, the edge vx followed by the x, y-path in P, and the edge vz followed by the z, y-path in P.

These paths share only their endpoints, so the union of any two is a cycle. By the pigeonhole principle, two of these paths have lengths with the same parity. The union of these two paths is an even cycle.