Untitled1

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1 2

1. Consider the two (minimal) cuts $C_1 = (U, U')$ and $C_2 = (W, W')$.
2. Then their symmetric difference contains all the edges in the cut $C_3 = (Z, Z')$ where $Z = (W \cap U) \cup (W' \cap U')$, and $Z' = V \backslash Z = (W \cap U') \cup (W' \cap U)$.
3. Assume it is not, the symmetric difference cut leaves an edge connecting $W \cap U$ to $W \cap U'$. This edge is in C_1 and not in C_2, so it must be included in the symmetric difference - which contradicts the initial assumption.
4. Hence the symmetric difference of two edge cuts is an edge cut.

2 3

1. An n-vertex cactus is a connected graph, so it has a spanning tree with $n-1$ edges.
2. Each additional edge completes a cycle using at least two edges in the tree.
3. Each edge of the tree is used in at most one such cycle. Hence there are at most $(n-1)/2$ additional edges, and the total number of edges is at most $n + \lfloor (n-1)/2 \rfloor$.
4. Hence the maximum number of edges in a simple n-vertex cactus is $\lfloor 3(n-1)/2 \rfloor$

3 4

1. A block with two vertices is an edge.
2. A block H with more than two vertices is 2 -connected and has an ear decomposition.
3. If H is not a single cycle, then the addition of the first ear to the first cycle creates a subgraph in which a pair of vertices is connected by three pairwise internally-disjoint paths.
4. By the pigeonhole principle, two of the paths have length of the same parity. Also, their union is an even cycle.
5. Hence H must be a single cycle.