Math454_A01_02

January 30, 2022

#1

The rightmost two graphs below are isomorphic. The third graph contains odd cycles, which is not isomorphic

#2

The rightmost two graphs below are isomorphic. The graph on the left is bipartite, cannot be isomorphic.

#3

Notice that the opposite corners are of the same color. So when remove those two squares, there are more squares left of one color (say red) than of the other (say white). A tiling of the board is a perfect matching between red squares and white squares.

Therefore, we can't have if the set of red squares is larger than the set of white squares.

Since this adjacency graph is two-colorable it is bipartite. A tiling is a choice of k disjoint edges which, as a set, cover all vertices. In general, this cannot be done if the two partites are of different sizes.

#4

Let G be k-regular of girth four, and chose $xy \in E(G)$. Girth 4 implies that G is simple and that x and y have no common neighbors. Thus the neighborhoods $N(x)$ and $N(y)$ are disjoint sets of size k, which forces at least $2k$ vertices into G. Possibly there are others.

Note also that $N(x)$ and $N(y)$ are independent sets, since G has no triangle. If G has no vertices other than these, then the vertices in $N(x)$ can have neighbors only in $N(y)$. Since G is k-regular, every vertex of $N(x)$ must be adjacent to every vertex of $N(y)$. Thus G is isomorphic to $K_{k,k}$, with partite sets $N(x)$ and $N(y)$. In other words, there is only one such isomorphism class for each value of k.

#5

Proof. Since A is symmetric, the i th diagonal entry of A is the dot product square of the vector

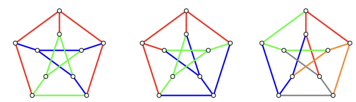
$$\mathrm{v}^{(i)} \quad \text{where } {}_j^{(i)} = \begin{cases} 1 & \text{if } v_i \text{ is adjacent to } v_j, \\ 0 & \text{otherwise.} \end{cases}$$

In general, the dot product square of a vector is the sum of the squares of the entries in the vector. Since all of the entries of v are 1 or $0, \mathrm{v} \cdot \mathrm{v}$ is just the sum of the entries of v, which is the degree of v_i. The i th diagonal entry of MM^T is the dot product square of

$$\mathrm{w}^{(i)} \qquad \text{where } \mathrm{w}_j^{(i)} = \begin{cases} 1 & \text{if } e_j \text{ is incident to } v_i \\ 0 & \text{otherwise.} \end{cases}$$

In A^2, the (i, j) entry is the dot-product $\mathrm{v}^{(i)} \cdot \mathrm{v}^{(j)}$, which will again be the sum of 1 's and 0 's: a 1 occurs in the k th summand if v_i and v_j share v_k as a neighbor. So (i, j) entry is the number of neighbors v_i and v_j have in common.

#1



Leftmost 2 fig: Decomposition into three copies of isomorphic graphs
Rightmost fig: Decomposition into five copies of P_4

#2

$K_{m,n}$ decomposes into two isomorphic subgraphs if and only if m and n are not both odd. The condition is necessary because the number of edges must be even. It is sufficient because $K_{m,n}$ decomposes into two copies of $K_{m,n/2}$ when n is even.

#3

Let P and Q be distinct u, v-paths. x is a vertex such that x, u-path P and x, v-path Q could get u, v-walks. Hense, graph is connected

Let $G = P_3$, with vertices u, x, v in order. The distinct u, v-walks with vertex lists u, x, u, x, v and u, x, v, x, v do not contain a cycle in their union.