

Math574_A15

April 29, 2022

1 Q1

1. Let C be the set of vertices of color i , and consider $v \in C$.
2. If v has no neighbor of color j , then we can switch the color of v to j . Since we are changing colors only for vertices in C , moving several of them to color j in this way creates no conflicts, since C is an independent set.
3. After relabeling all vertices of C , we have obtained a proper coloring without using color i .
4. Hence, for each color i there is a vertex with color i that is adjacent to vertices of the other $k - 1$ colors.

2 Q3

1. If a connected graph is not a clique, then the shortest of all paths between nonadjacent pairs of vertices has length two, and the three vertices of this path induce a subgraph with exactly two edges.
2. Hence a simple graph G is a complete multipartite graph if and only if it has no 3-vertex induced subgraph with one edge.