FALL 2021 MATH 484/564 HOMEWORK #2

Due: September 25, 11:59PM, submit in blackboard

Homework solution is not required to be typed, but must be legible.

Problem 1 In regression through the origin (RTO) model $Y = \beta_1 X + \epsilon$, it is assumed $\epsilon \sim N(0, \sigma^2)$. If the common variance σ^2 is unknown, a possible estimator for σ^2 is $\hat{\sigma}^2 = SSE/(n-1)$. Prove that σ^2 is an unbiased estimator for σ^2.

Problem 2 Consider the *Supervisor Performance Data* in Table 3.3 on page 60 of the TEXT (Table 3.3 attached).

1) Estimate the regression coefficients vector $\hat{\boldsymbol{\beta}}$.
2) Verify that $\sum\limits_{i=1}^{n} \hat{y}_i = \sum\limits_{i=1}^{n} y_i$ for this dataset.
3) Does $\sum\limits_{i=1}^{n} \hat{y}_i = \sum\limits_{i=1}^{n} y_i$ hold true in general for multiple linear regression model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p + \epsilon$? Prove or disprove it.
4) Now consider $p = 2$, and only use X_3 and X_4 two predictors. The model becomes

$$Y = \beta_0 + \beta_3 X_3 + \beta_4 X_4 + \epsilon$$

Use the 3-step method described on page 63 to obtain the coefficient for X_3, and compare it with the coefficient of X_3 by regressing Y on X_3 and X_4 using the 2-predictor model above. Are they the same? Explain why or why not.

Problem 3 Exercise 3.3 from the TEXT (Table 3.10 attached).

Problem 4 Verify that for the multiple linear regression model $Y = X\boldsymbol{\beta} + \epsilon$ with one predictor variable, the least square estimate of $\boldsymbol{\beta}$ using the matrix form gives the same result as in SLR. Then use the matrix form to derive the variance-covariance matrix of $\hat{\boldsymbol{\beta}}$ and verify the variances $\text{Var}(\hat{\beta}_0)$ and $\text{Var}(\hat{\beta}_1)$ are the same as their counterparts derived in SLR.

Problem 5 Exercise 3.14 from the TEXT.