Q1

Given that,

$$y = \beta_1 x + \varepsilon \quad \varepsilon \sim N\left(0, \sigma^2\right)$$

σ^2 is unknown so it is estionated by σ^2

$$\sigma^2 = \frac{SSE}{(n-1)}, \quad SSE = (n-1)\sigma^2$$

where $SSE = \sum_{i=1}^{n} \left(y_i - \bar{y}\right)^2$ called

Sum of squares of y_i with $(n-1)$ in the denominator

σ^2 is the unbaised estimator of σ^2 if

$$E\left[\sigma^2\right] = \sigma^2$$

Since we have,

$$SSE = (n-1)\sigma^2$$

$$\Rightarrow E\left[\frac{SSE}{n-1}\right] =| \frac{(n-1)\sigma^2}{n-1} = \sigma^2$$

Therefore,

$$\sigma^2 \text{ is an unbiased estimator for } \sigma^2$$

Q5

Full model:
Use P value calculator, the critical value is 2.4753.
Here, the value of F test is 22.98, which is greater than the critical value. Therefore, the null hypothesis is rejected.
Reduced model:
Use P value calculator, the critical value is 3.9457.
Here, the value of F test is 18.6, which is greater than the critical value. Therefore, the null hypothesis is rejected.
Thus, it can be concluded that the null hypothesis is rejected at both full and reduced model.

Q3

1. The regression model 1

$$F = \beta_0 + \beta_1 * P1 + \text{ Error}$$

Regression Statistics	
Multiple R	0.895684
R Square	0.80225
Adjusted R Square	0.792363
Standard Error	5.081057
Observations	22

ANOVA					
	df	SS	MS	F	Significance F
Regression	1	2094.748	2094.748	81.13787	1.78E − 08
Residual	20	516.3428	25.81714		
Total	21	2611.091			

	Coefficients	Standard Error	t Stat	P-value
Intercept	−22.3424	11.56395	−1.93208	0.067641
P1	1.260516	0.139938	9.007656	1.78E − 08

Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
−46.4644	1.77955	−46.4644	1.77955
0.96861	1.552422	0.96861	1.552422

The Fitted model 1 is

$$\mathrm{F} = -22.34 + 1.26^{*}\mathrm{P1} + \mathrm{Error}$$

The regression model2 is

$$F = \beta_0 + \beta_1 * P2 + \text{ Error}$$

The fitted model 2 is

$$\mathrm{F} = -1.853 + 1.004^{*}\mathrm{P2} + \mathrm{Error}$$

The regression model3 is

$$F = \beta_0 + \beta_1 * P1 + \beta_2 * P2 + \text{ Error}$$

Therefore the fitted model 3 is

$$\mathrm{F} = -14.50 + 0.488^{*}\mathrm{P1} + 0.6720^{*}\mathrm{P2} + \text{ Error}$$

2. To test whether beta=0 or not we can use F-test of overall significance test which is specific form of F-test which compares the model fitted model with the regression model that contains no predictors or intercept only model.

   The hypothesis for F-test are Null Hypothesis:Ho: The fit of the intercept only model and the fitted model are equal or All regression coeficents are equal to zero Alternative Hyppthesis: H1:The fit of intercept only model is significanlty reduced compared to fitted model or At least one regression coficient of fitted model differes significantly from zero.
   Since level of significance is not given so i am choosing here standard level of significance= 5%
   For Model 1
   F-statstic = 81.13 (From regression ANOVA table of model 1) P-value for F-statistic at 5% level of significance for degrees of freedom(Regression DF=1,Residual DF=20)=1.78E-08 form F-distribution table. Here P-value is less than level of significance, hence we reject null hypothesis and conclude that at least one regression coeficent differes significantly from zero for model1.
   For Model 2
   F-statstic = 122.89 (From regression ANOVA table of model2) P-value for F-statistic at 5% level of significance for degrees of freedom(Regression DF=1,Residual DF=20)=5.44E-10 form F-distribution table. Here P-value is less than level of significance, hence we reject null hypothesis and conclude that at least one regression coeficent differes significantly from zero for model2.
   For Model 3
   F-statstic = 74.06 (From regression ANOVA table of model3) P-value for F-statistic at 5% level of significance for degrees of freedom(Regression DF=1,Residual DF=19)=1.06E-09 form F-distribution table. Here P-value is less than level of significance, hence we reject null hypothesis and conclude that at least one regression coeficent differes significantly from zero for model3.
3. Variable P2 is better than P1 as P2 has 0.9273 positve positve linear relationship with F as compared to P1 which has 0.8956. Also P2 has R_square=0.8600 i.e P2 explins 86% variation in F as compared to P1 which explins 80.02% of variation in F only.
4. Model 3 can be used for prediction final examination socres for a student as it is showing 91.14% positve linear realtionship between F and (P1,P2). Also from model 3 we are able to explin 88.63% of variation in F from (P1,P2) which higher as compared to other 2 model. The Predicted value of final scores of a studen who scores P1 = 78 and P2 = 85 is

$$\mathrm{F} = -14.50 + 0.488{*}78 + 0.6720{*}85$$
$$\mathrm{F} = 80.71$$

Q4

In multiple linear regression model

$$y = x\beta + \varepsilon$$

with one predictor

$$y = x_1\beta + \varepsilon$$

Here,

$$s(\beta) = (y - x\beta)' (y - x_1\beta)$$

erdincay least square estimator of β,

$$\frac{\partial^2 S(\beta)}{\partial \beta} = 2x_1' x_i$$

The normal equation is, $2x_1' x\beta - 2x_1' y = 0$

$$\hat{\beta} = \left(x_1' x_1\right)^{-1} x_1' y$$

In simple linear regression normal equation are,

$$\Sigma y = n\beta_0 + \beta_1 \Sigma x_1$$
$$\Sigma x_1 = \beta_0 \Sigma x_1 + \beta_1 \Sigma x_1^2$$

In matrix form,

$$\begin{pmatrix} \sum y \\ \sum x_1 y \end{pmatrix} = \begin{pmatrix} n & \sum x_1 \\ \sum x_1 & \sum x_1^2 \end{pmatrix} \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{pmatrix}$$

Therefore,

$$x_1' y = (x_1' x)\, \hat{\beta}$$
$$\left(x_1' x_1\right)^{-1} x_1' y = \left(x_1' x_1\right)^{-1} \hat{\beta}$$
$$\hat{\beta} = \left(x_i x_i\right)^{-1} x_i y, \text{which is same as above}$$

In MLR variance covariance mattix of β,

$$\begin{aligned} v\left(\hat{\beta}_1\right) &= E(\hat{\beta} - \beta)\,(\beta - \beta')' \\ &= E\left[\left(x_1' x_1\right)^{-1} x_1' \varepsilon \varepsilon' x_1' \left(x : x_1\right)^{-1}\right] \\ &= \left(x_1' x\right)^{-1} x_1' E\left(\varepsilon \varepsilon'\right) x_1 \left(x_i x_1\right)^{-1} \\ &= \sigma^2 \left(x_i x_1\right)^{-1} x_i I x_1 \left(x_1' x_1\right)^{-1} \\ &= \sigma^2 \left(x_1' x_1\right)^{-1} \end{aligned}$$

Thus,

$$\operatorname{var}\left(\hat{\beta}_1\right) = \sigma^2 \left[\frac{1}{n} + \frac{x^{-2}}{sxx}\right]$$
$$\operatorname{var}\left(\hat{\beta}_1\right) = \frac{\sigma^2}{sxx}$$

which is same as their counterparts derived in simple linear regression.

Q4

In multiple linear regression model

$$y = x\beta + \varepsilon$$

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Here,

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The normal equation is, $2x_1'x\beta - 2x_1'y = 0$

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$$\Sigma x_1 = \beta_0\Sigma x_1 + \beta_1\Sigma x_1^2$$

In matrix form,

$$\begin{pmatrix} \sum y \\ \sum x_1 y \end{pmatrix} = \begin{pmatrix} n & \sum x_1 \\ \sum x_1 & \sum x_1^2 \end{pmatrix} \begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{pmatrix}$$

Therefore,

$$x_1'y = \left(x_1'x\right)\hat{\beta}$$
$$\left(x_1'x_1\right)^{-1} x_1'y = \left(x_1'x_1\right)^{-1}\hat{\beta}$$
$$\hat{\beta} = \left(x_ix_i\right)^{-1} x_iy, \text{which is same as above}$$

In MLR variance covariance mattix of β,

$$\begin{aligned} v\left(\hat{\beta}_1\right) &= E(\hat{\beta} - \beta)\left(\beta - \beta'\right)' \\ &= E\left[\left(x_1'x_1\right)^{-1} x_1'\varepsilon\varepsilon' x_1' \left(x : x_1\right)^{-1}\right] \\ &= \left(x_1'x\right)^{-1} x_1'E\left(\varepsilon\varepsilon'\right) x_1 \left(x_ix_1\right)^{-1} \\ &= \sigma^2 \left(x_ix_1\right)^{-1} x_iIx_1 \left(x_1'x_1\right)^{-1} \\ &= \sigma^2 \left(x_1'x_1\right)^{-1} \end{aligned}$$

Thus,

$$\operatorname{var}\left(\hat{\beta}_1\right) = \sigma^2\left[\frac{1}{n} + \frac{x^{-2}}{sxx}\right]$$
$$\operatorname{var}\left(\hat{\beta}_1\right) = \frac{\sigma^2}{sxx}$$

which is same as their counterparts derived in simple linear regression.