P2

September 11, 2021

[1]:
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#2
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we have $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$ and $\hat{\beta}_1 = \frac{s_{xy}}{s_{xx}} = \sum yici \quad ci = \frac{x_i - \bar{x}}{s_{xx}}$

$s_{xy} = \sum y_i \left(x_i - \bar{x}\right)\ s_{xx} = \sum \left(x_i - \bar{x}\right)^2$

$$
\begin{aligned}
\operatorname{Cov}\left(\bar{y}_1 b_1\right) &= E[\bar{y} - (E(\bar{y}))]\left[\beta_1 - E\left(\hat{\beta}_1\right)\right] \\
&= E\left[\hat{E}\left(\Sigma c_i y_j - \beta_1\right)\right] \\
&= \frac{1}{n}\left[\left(\Sigma \epsilon_i\right)\left(\beta_0 \Sigma c_i + \beta_1 \Sigma c_i x_i + \varepsilon c_i \varepsilon_i\right)\right. \\
&= \frac{1}{n}[0 + 0 + 0 + 0] = 0
\end{aligned}
$$

$$
\begin{aligned}
\operatorname{Cov}\left(\bar{y}, \hat{\beta}_1\right) &= \operatorname{Cov}\left(\frac{1}{n}\sum_{i=1}^{n} y_i, \sum_{i=1}^{n} c_i y_i\right) \\
&= \frac{1}{n}\sum_{i=1}^{n} c_i \operatorname{Var}\left(y_i\right) \\
&= \frac{\sigma^2}{n}\sum_{i=1}^{n} c_i = 0
\end{aligned}
$$