P3

September 11, 2021

$\mathrm{Cov}^2(X, Y) \leq \mathrm{Var}(X)\,\mathrm{Var}(Y)$

It follows from this Cauchy-Schwarz inequality that the correlation coefficient is between -1 and 1 .

$$-\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)} \leq \mathrm{Cov}(X, Y) \leq \sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}$$

Therefore,

$$-1 \leq \frac{\mathrm{Cov}(X, Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}} \leq 1$$

Actually the covariance is the inner product between two random variables and the standard deviation is the norm of a random variable. If we denote the inner product $\langle X, Y \rangle$ and the norm $|X|$, then the usual Cauchy-Schwarz inequality still holds: $< X, Y >^2 \leq |X|^2 |Y|^2$

The correlation coefficient is in fact the cosine of the angle between two variables:

$$\mathrm{Corr}(X, Y) = \frac{\langle X, Y \rangle}{|X||Y|} = \cos(\theta)$$

which is between -1 and 1.

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